The Sill distribution
and
its application to exotic hadrons

Francesco Giacosa
in collaboration with V. C. Shastry and A. Okopinska

UJK Kielce (Poland) **& Goethe U Frankfurt** (Germany)

Hadron 2023

5-9/6/2023 Genoa italy

Outline

- Breit-Wigner distribution: brief recall
- How to introduce an energy threshold? Naive and proper treatment
- Relativistic Breit-Wigner
- Sill, deifnition and examples
- Sill: novel applications
- Sill and hybrids

Beyond Breit-Wigner

Eur. Phys. J. A (2021) 57:336
https://doi.org/10.1140/epja/s10050-021-00641-2

THE EUROPEAN PHYSICAL JOURNAL A

Check for updates

Regular Article - Theoretical Physics

A simple alternative to the relativistic Breit–Wigner distribution

Francesco Giacosa[1,2], Anna Okopińska[1], Vanamali Shastry[1,a]

ArXiv: 2106.03749

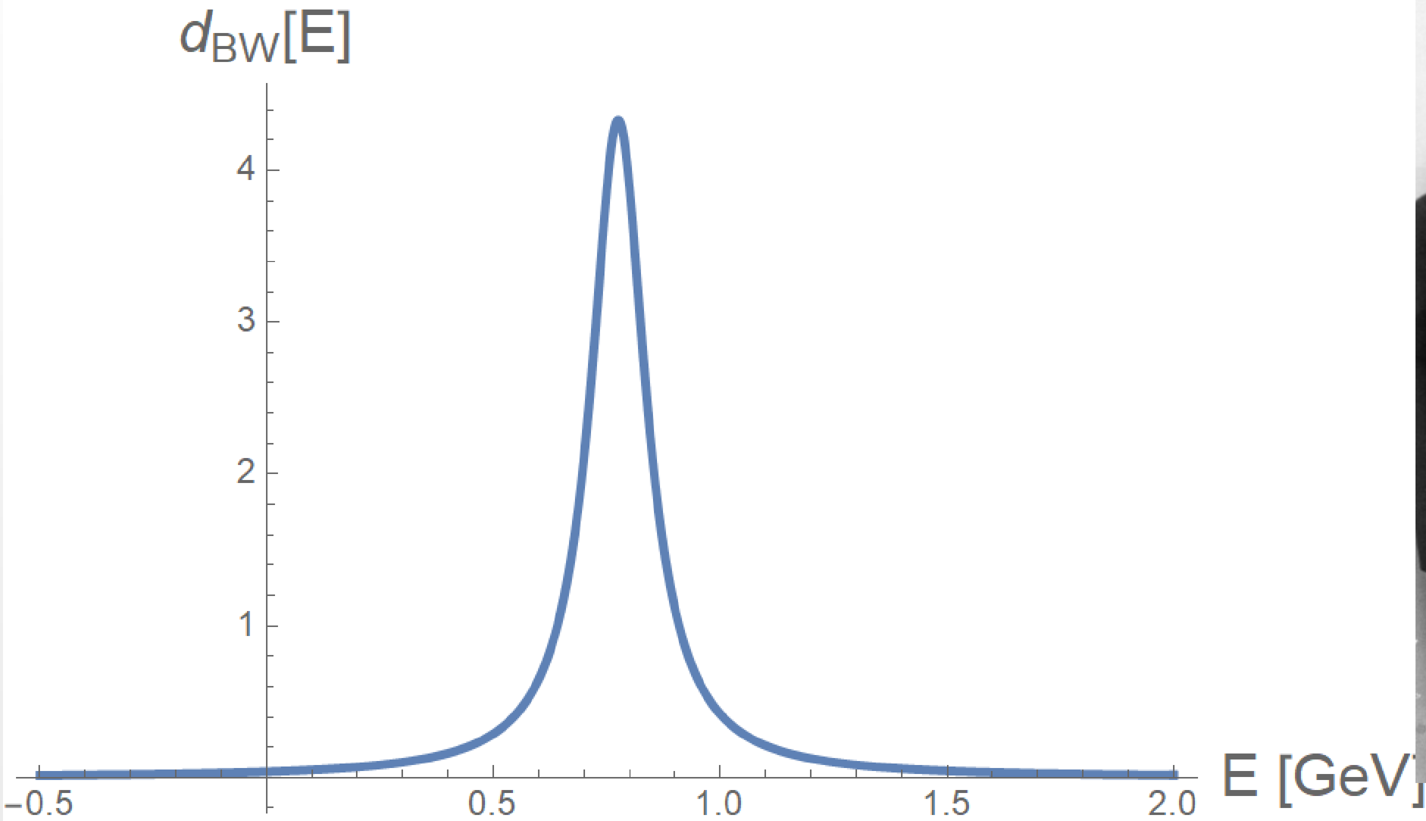
$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2 - M^2)^2 + (M\Gamma)^2} \theta(E)$$

$$d_S^{\mathrm{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2 - M^2)^2 + (\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma})^2} \theta(E - E_{th})$$

Breit-Wigner distribution

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$



Rho-meson as example.

BW extends from –inf to +inf. There is no left threshold.

BW: properties

BW-distribution: $$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

Pole:

$$z_{pole}^{\mathrm{BW}} = M - i\Gamma/2$$

BW-propagator: $$G_S^{\mathrm{BW}}(E) = \frac{1}{E - M + i\Gamma/2 + i\varepsilon}$$

Link prop-dist: $$d_S^{\mathrm{BW}}(E) = -\frac{1}{\pi} \mathrm{Im}[G_S^{\mathrm{BW}}(E)] = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

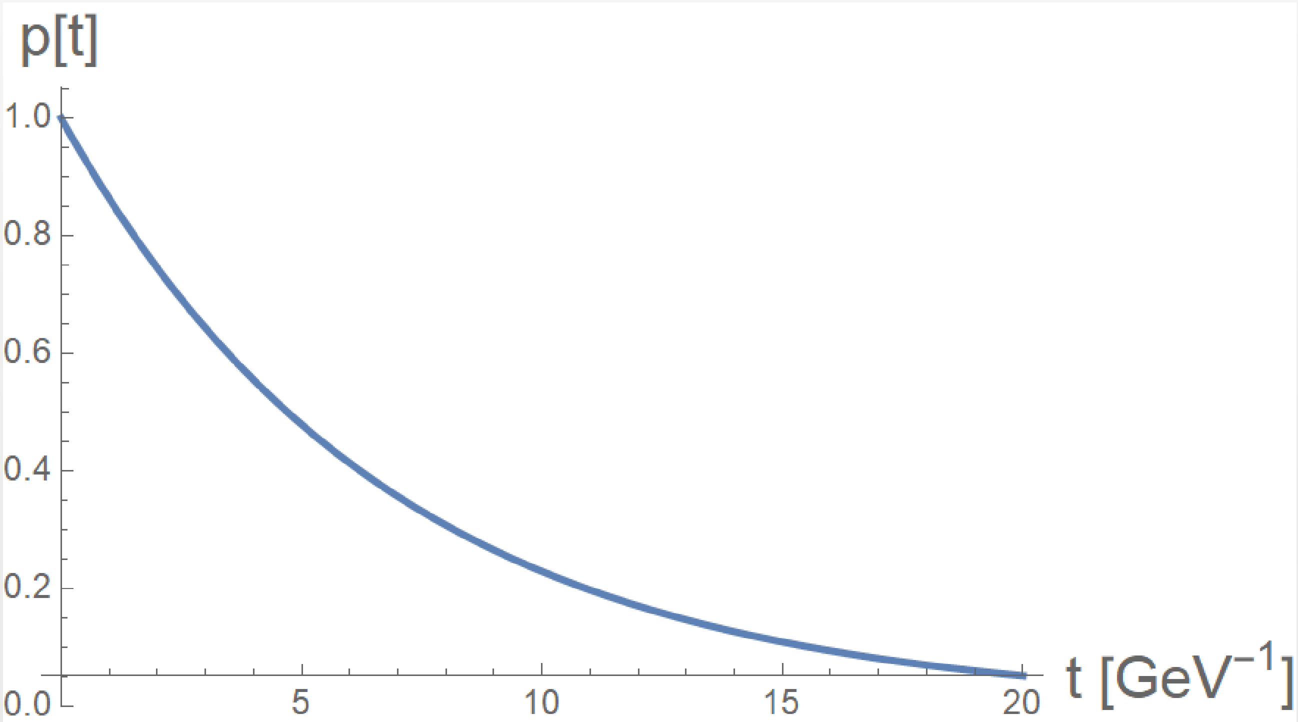
Normalization:
(important for prob. interpretation)

$$\int_{-\infty}^{+\infty} d_S^{\mathrm{BW}}(E) dE = 1$$

BW corresponds to exp. decay

$$a_S^{\mathrm{BW}}(t) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} \mathrm{dE} G_S^{\mathrm{BW}}(E) e^{-iEt} = \int_{E_{th}}^{+\infty} \mathrm{dE} d_S^{\mathrm{BW}}(E) = e^{-iMt-\Gamma t/2}$$

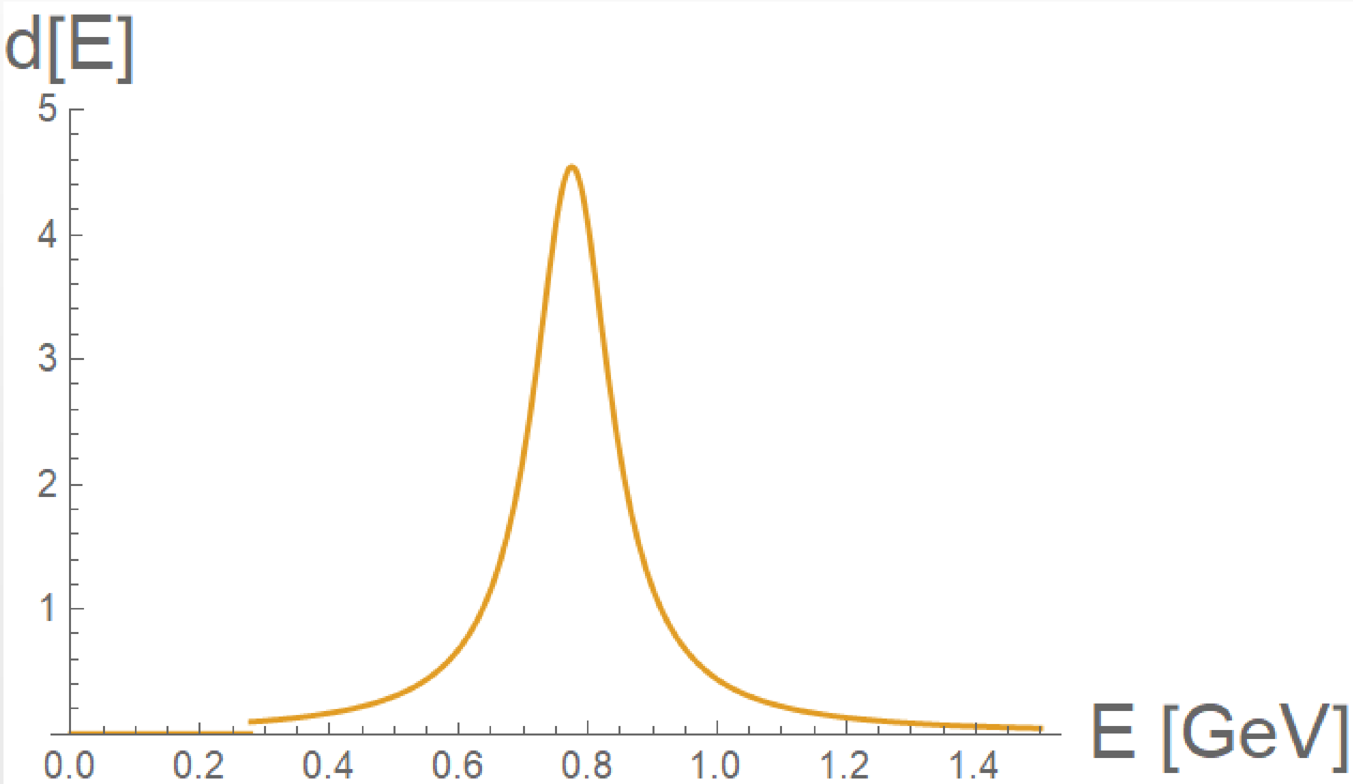
$$p^{\mathrm{BW}}(t) = \left|a_S^{\mathrm{BW}}(t)\right|^2 = e^{-\Gamma t}$$



How to introudce a threshold?
BW with threshold (naive threatment)

$$d_S(E) = N\frac{\Gamma}{2\pi}\frac{1}{(E-M)^2+\frac{\Gamma^2}{4}}\theta(E-E_{th})$$

N is needed because the normalization is lost!

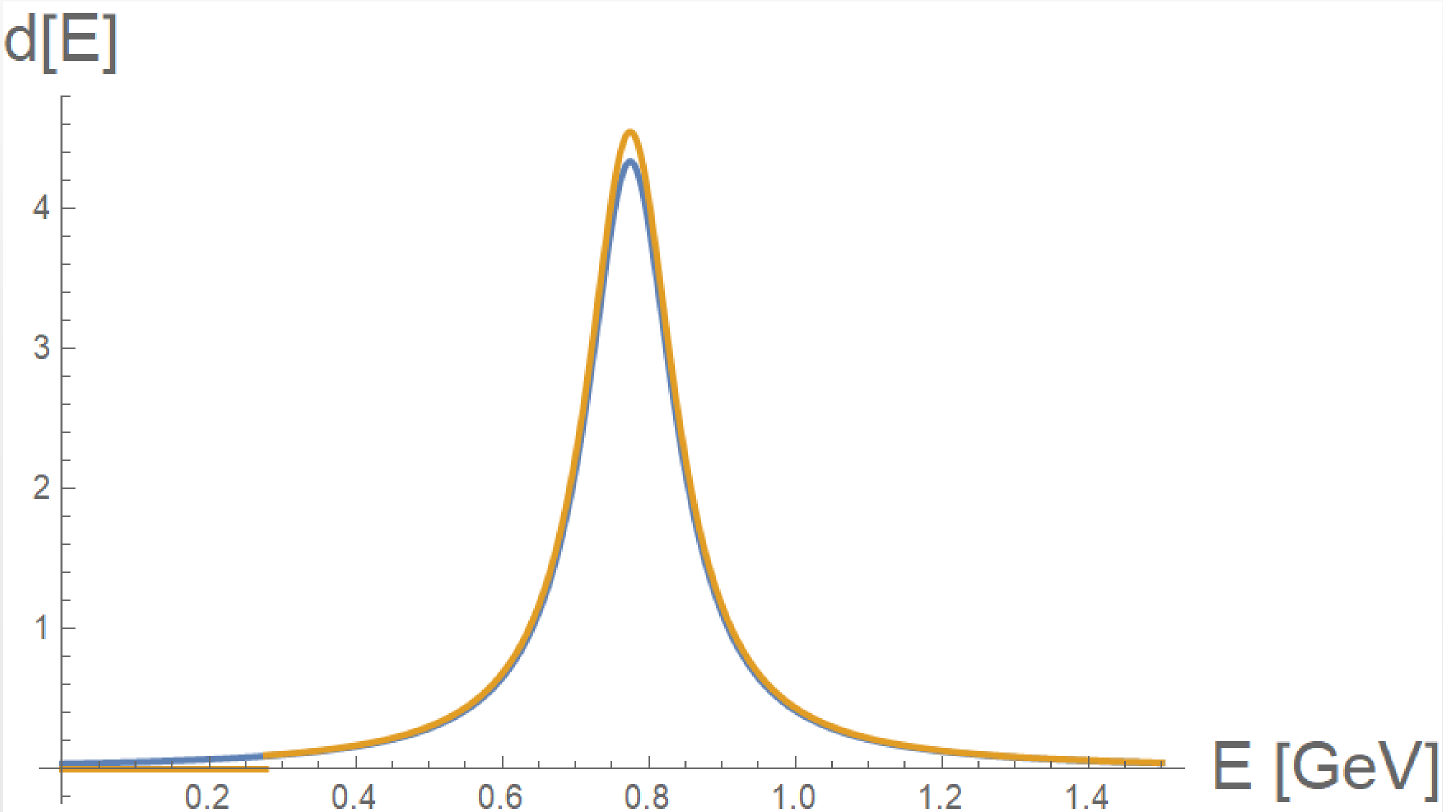


BW with threshold (naive treatment)

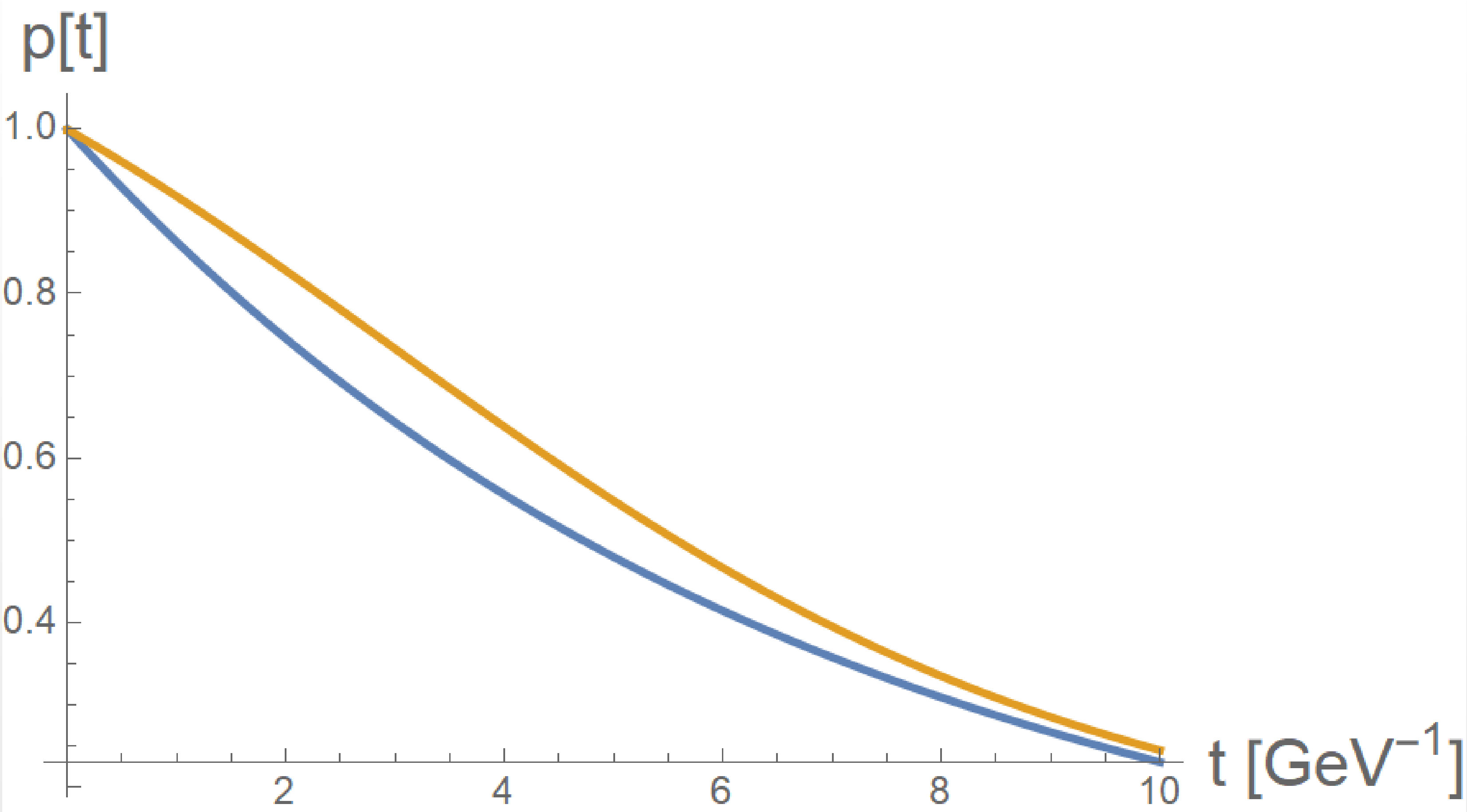
$$d_S(E) = N \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}} \theta(E - E_{th})$$

N is needed because the normalization is lost!

For the ρ meson, we get $N = 1.05$



Time evolution



Blue: plain BW, yellow: BW with threshold (naive)

Comments

- The ‘brute force’ threshold can be good as a first approximation, but it is just an ‘ad hoc’ modification of Breit-Wigner
- Which is the correct propagator that contains a threshold?
- Which is the correct energy distribution?

General non-relativistic approach

Propagator

$$G_S(E) = \frac{1}{E - M + \Pi(E) + i\varepsilon}$$

Self-energy (or loop)
E_{th} is the threshold energy

$$\Pi(E) = -\int_{E_{th}}^{\infty} \frac{1}{\pi} \frac{\mathrm{Im}\,\Pi(E')}{E - E' + i\varepsilon} dE'$$

Energy dependent ‘decay width’

$$\Gamma(E) = 2\,\mathrm{Im}\,\Pi(E)$$

Energy distribution (or spectral function)

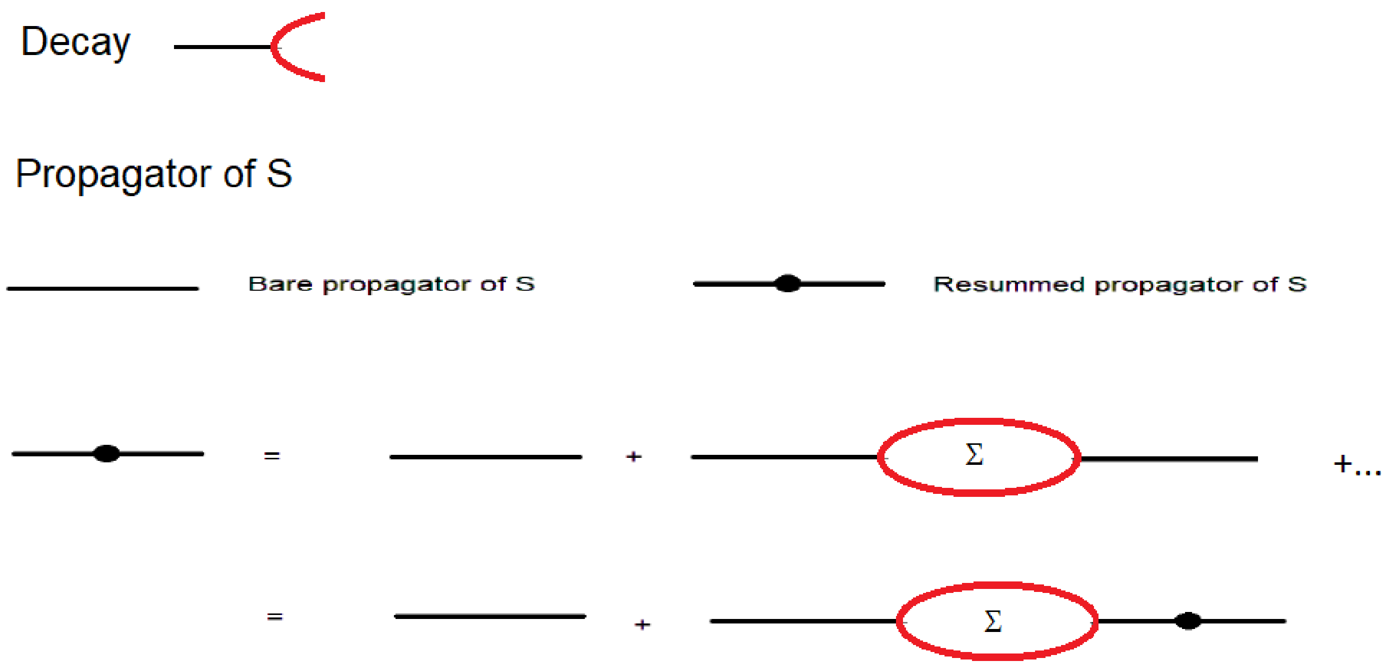
$$d_S(E) = -\frac{1}{\pi}\mathrm{Im}[G_S(E)] = \frac{1}{\pi} \frac{\mathrm{Im}\,\Pi(E)}{(E - M + \mathrm{Re}\,\Pi(E))^2 + (\mathrm{Im}\,\Pi(E))^2}$$

Link between propagator and distribution

The propagator can be expressed as (H being the full Hamiltonian)

$$G_S(E) = \langle S| \frac{1}{E - H + i\varepsilon} |S\rangle = \frac{1}{E - M + \Pi(E) + i\varepsilon} = \int_{E_{th}}^{+\infty} \mathrm{dE}' \frac{d_S(E')}{E - E' + i\varepsilon}$$

out of which $d_S(E) = -\frac{1}{\pi} \mathrm{Im}[G_S(E)]$



Normalization and its heuristic justification

One can show that under quite general conditions

$$\int_{E_{th}}^{\infty} \mathrm{dE}\, d_S(E) = 1$$

Brief QM recall

$$|S\rangle = \int_{E_{th}}^{\infty} \mathrm{dE}\, a_S(E)\, |E\rangle$$

Eigenstates of Hamilton H

$$H\,|E\rangle = E\,|E\rangle$$

The quantity $d_S(E) = |a_S(E)|^2$ is the 'spectral function'

$$1 = \langle S|S\rangle = \int_{E_{th}}^{\infty} \mathrm{dE}\, d_S(E)$$

$$a_S(t) = \langle S\,|e^{-iHt}|\,S\rangle = \int_{E_{th}}^{+\infty} \mathrm{dE}\, d_S(E) e^{-iEt}$$

Time-evolution (general)

$$a_S(t) = Ze^{-iz_{pole}t} + \ldots,$$

The dots describe short- and long-time deviations from the exponential decay

The pole:

$$z_{pole} - M + \Pi_{II}(z_{pole}) = 0\ ,$$

where II refers to the second Riemann sheet. Then:

$$z_{pole} = M_{pole} - i\frac{\Gamma_{pole}}{2}\ .$$

BW with threshold **properly done**

We assume that:

$$\mathrm{Im}\,\Pi(E) = \begin{cases} \frac{\Gamma}{2} \text{ for } E \in (E_{th}, \Lambda) \\ 0 \text{ otherwise} \end{cases}$$

In the limit $\Lambda \to \infty$ and by using one subtraction we get:

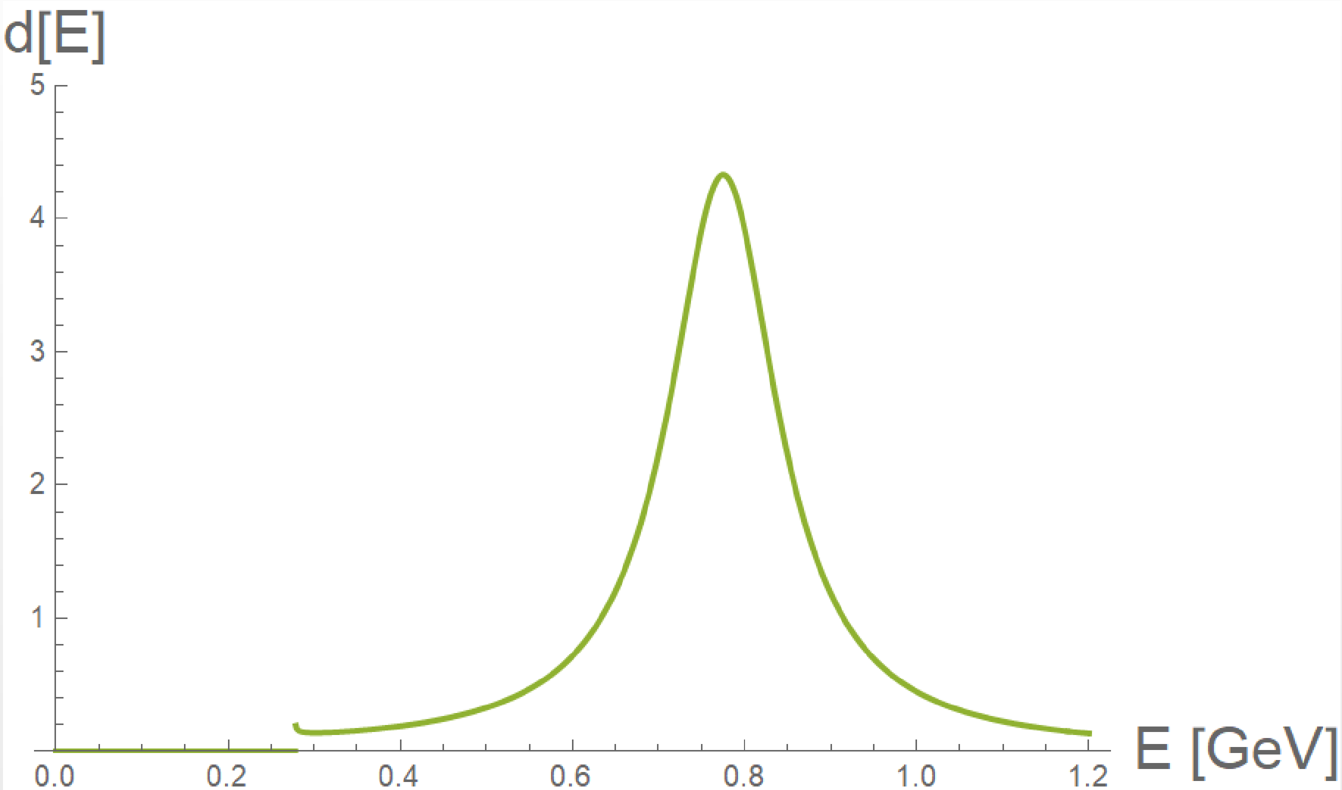
$$\Pi(E) = \frac{\Gamma}{2\pi} \ln\left(\frac{-E_{th} + M}{E_{th} - E}\right)$$

Then:

$$d_S(E) = \frac{\Gamma}{2\pi} \frac{1}{\left[E - M + \frac{\Gamma}{2\pi}\ln\left(\frac{M - E_{th}}{E_{th} - E}\right)\right]^2 + \frac{\Gamma^2}{4}} \theta(E - E_{th})$$

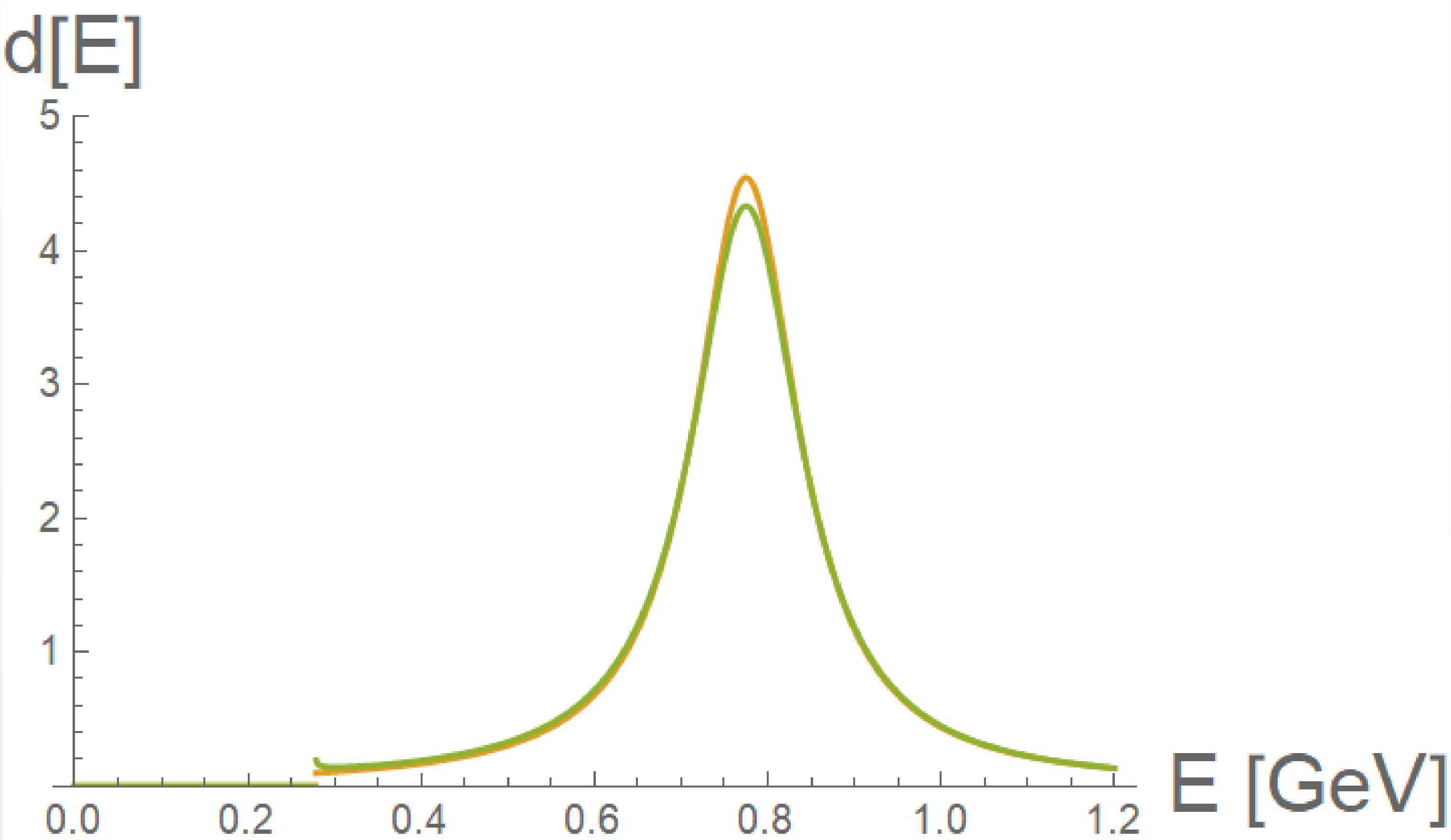
This is actually the correct Breit-Wigner with threshold!
Correctly normalized to unity, no need of an extra N…but somewhat not handy

BW with threshold **properly done**



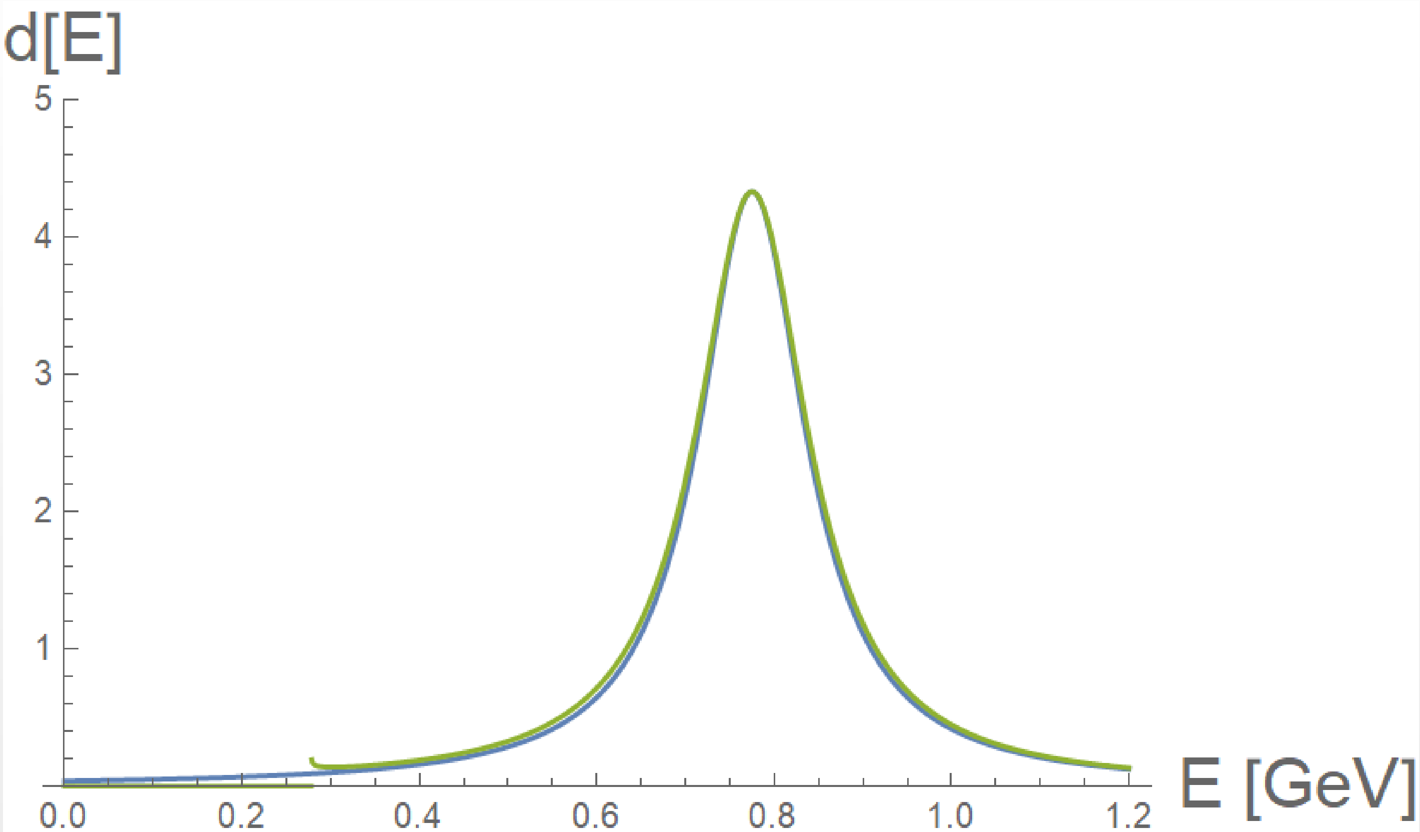
Correct BW with threshold

BW with threshold **properly done**



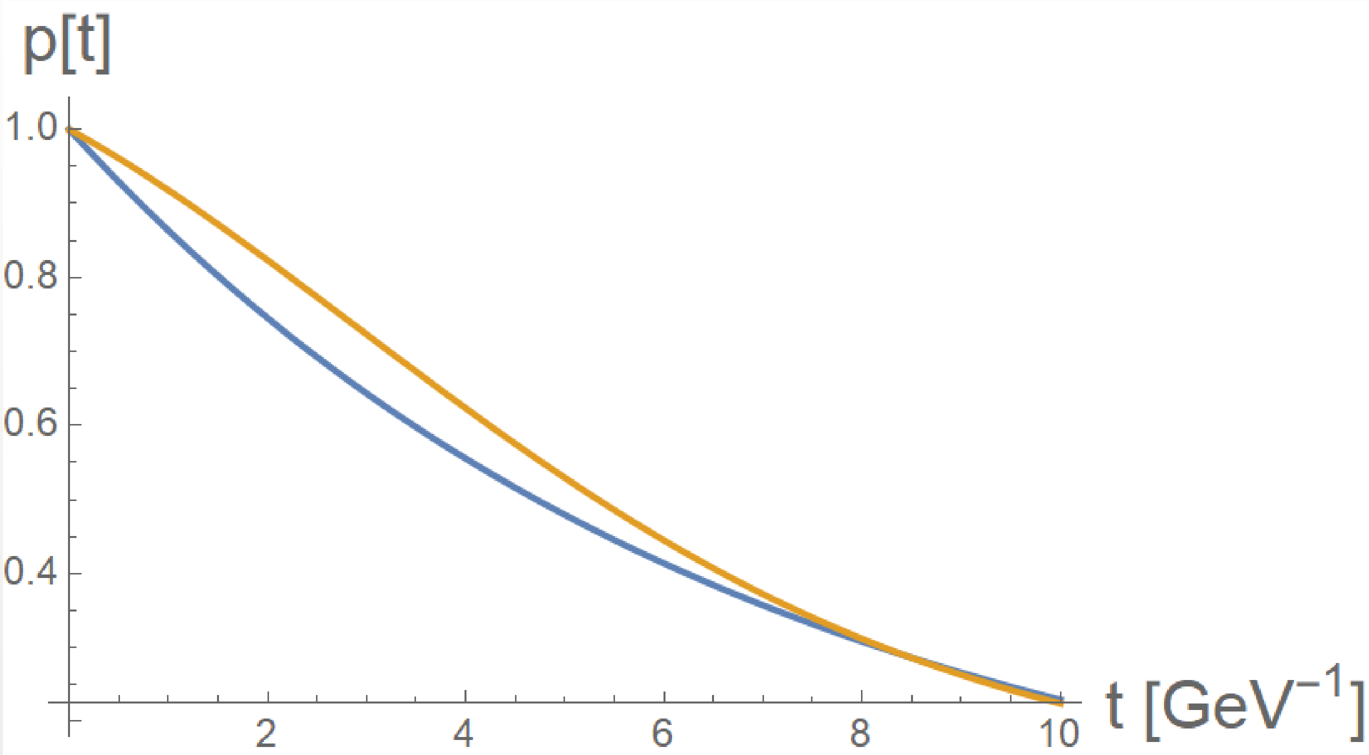
Comparision with ‘naive’ BW with threshold

BW with threshold **properly done**



Comparision with plain BW: indeed very similar!

BW with threshold **(properly done)** and time-evolution



Blue: BW, yellow: BW with threshold (properly done)

Relativistic Breit-Wigner (rBW)

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2 - M^2)^2 + (M\Gamma)^2} \theta(E)$$

In a relativistic framework there is always a threshold! (eventually zero).

Function above not normalized as it stands.

From above often used in various applications.

Relativistic propagator

Uniwersytet
Jana Kochanowskiego w Kielcach

The relativistic case can be easily obtained by replacing the variable E with the variable $s = E^2$, hence the propagator (as function of s) takes the form

$$G_S(s) = \frac{1}{s - M^2 + \Pi(s) + i\varepsilon} \, .$$

$$\Pi(s) = -\int_{s_{th}}^{\infty} \frac{1}{\pi} \frac{\operatorname{Im}\Pi(s')}{s - s' + i\varepsilon} ds'.$$

$$\operatorname{Im}\Pi(s) = \sqrt{s}\,\Gamma(s)$$

the pole position s_{pole} is such that

$$s_{pole} - M^2 + \Pi_{II}(s_{pole}) = 0$$

and leads to the pole mass and decay width defined as:

$$\sqrt{s_{pole}} = M_{pole} - i\frac{\Gamma_{pole}}{2} \, .$$

rBW: derivation

In order to obtain possible expressions for the relativistic BW distribution, let us consider

$$\mathrm{Im}\,\Pi(s) = M\Gamma\theta(s)$$

With a proper subtraction

$$\Pi(s) = \frac{M\Gamma}{\pi}\ln\frac{-M^2}{s}$$

which assures that $\mathrm{Re}\,\Pi(s = M^2) = 0$

The propagator reads

$$G_S(s) = \frac{1}{s - M^2 + \frac{M\Gamma}{\pi}\ln\frac{M^2}{s} + iM\Gamma\theta(s) + i\varepsilon}$$

rBW: derivation

$$G_S(s) = \frac{1}{s - M^2 + \frac{M\Gamma}{\pi}\ln\frac{M^2}{s} + iM\Gamma\theta(s) + i\varepsilon}$$

$$d_S(s) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(s)] = \frac{1}{\pi}\frac{M\Gamma}{(s - M^2 + \frac{M\Gamma}{\pi}\ln\frac{M^2}{s})^2 + (M\Gamma)^2}\theta(s)$$

The propagator for the relativistic BW approximation is obtained by approximating the previous propagator upon setting the real part of the loop artificially to zero, thus finding:

$$G_S(s) = G_S^{BW}(s) = \frac{1}{s - M^2 + iM\Gamma + i\varepsilon}$$

$$d_S^{\mathrm{rBW}}(s) = \frac{1}{\pi}\frac{M\Gamma}{(s - M^2)^2 + (M\Gamma)^2}\theta(s)$$

$$d_S(E) = d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi}\frac{M\Gamma}{(E^2 - M^2)^2 + (M\Gamma)^2}\theta(E)$$

Sill

Let us consider a resonance with mass M decaying into twoparticles:

$$E_{th} = m_1 + m_2 = \sqrt{s_{th}}$$

$$s_{th} = E_{th}^2$$

We **assume** that:

$$\mathrm{Im}\,\Pi(s) = \sqrt{s - s_{th}}\,\tilde{\Gamma}\,\theta(s - s_{th})$$

$$\Gamma M = \tilde{\Gamma}\sqrt{M^2 - E_{th}^2}$$

Decay width as function of the energy:

$$\Gamma(s) = \frac{\sqrt{s - s_{th}}}{\sqrt{s}}\tilde{\Gamma}$$

Note, it saturates for large s

Sill

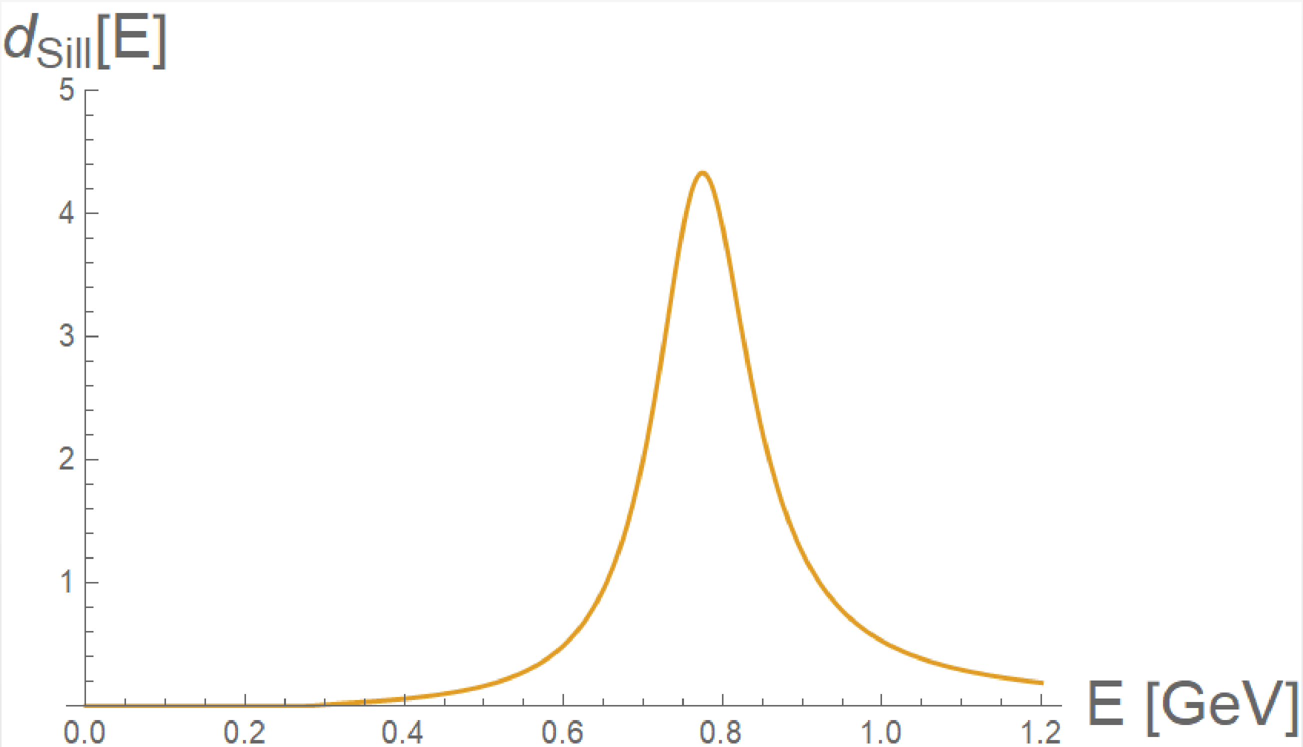
$$\Pi(s) = i\tilde{\Gamma}\sqrt{s - s_{th}}$$

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}\sqrt{s - s_{th}} + i\varepsilon}$$

$$\begin{aligned} d_S(s) &= -\frac{1}{\pi}\,\mathrm{Im}\,\frac{1}{s - M^2 + i\tilde{\Gamma}\sqrt{s - s_{th}} + i\varepsilon} \\ &= \frac{1}{\pi}\frac{\sqrt{s - s_{th}}\,\tilde{\Gamma}}{(s - M^2)^2 + (\sqrt{s - s_{th}}\,\tilde{\Gamma})^2}\theta(s - s_{th}) \end{aligned}$$

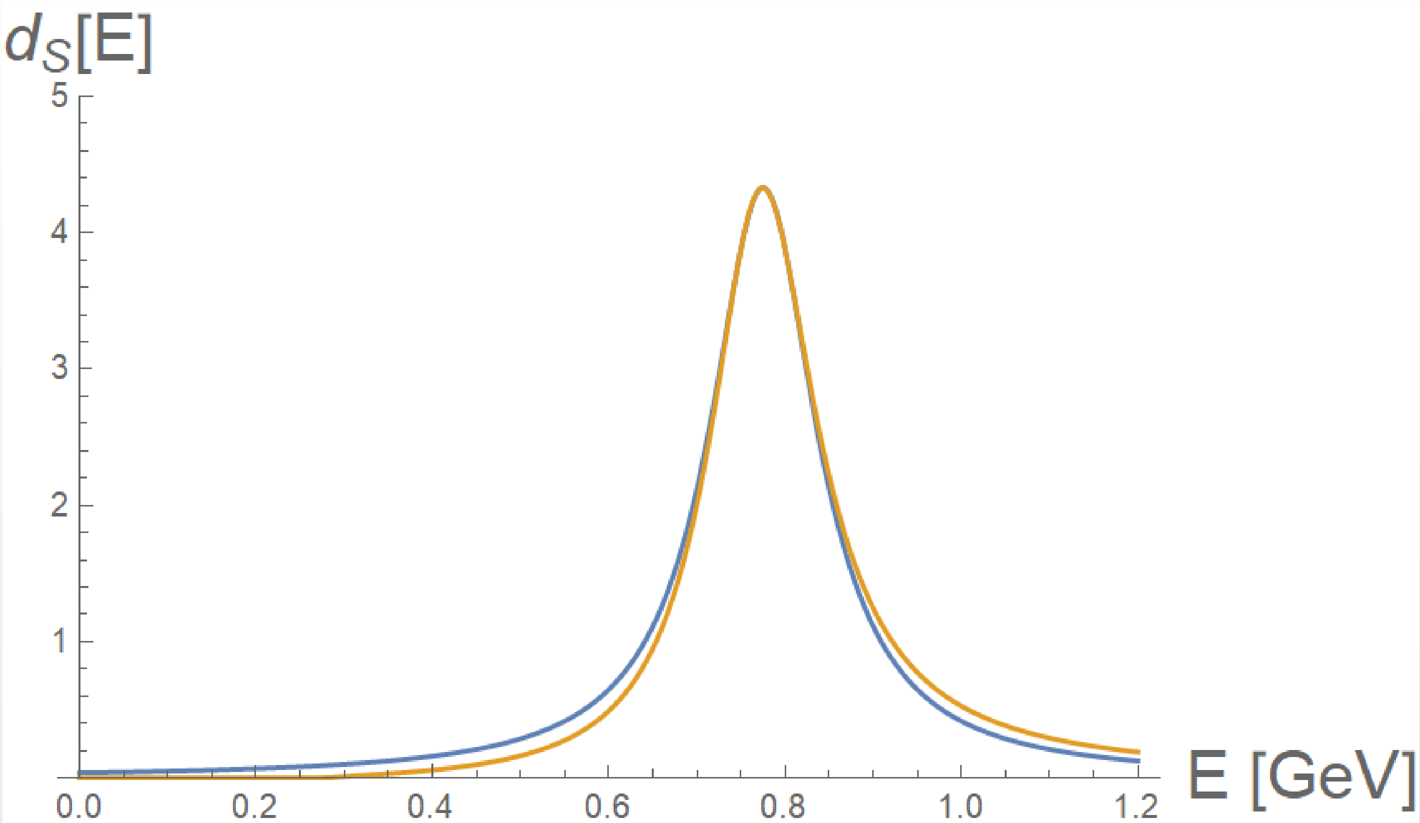
Sill

$$d_S(E) = d_S^{\text{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2 - M^2)^2 + \left(\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}\right)^2} \theta(E - E_{th})$$



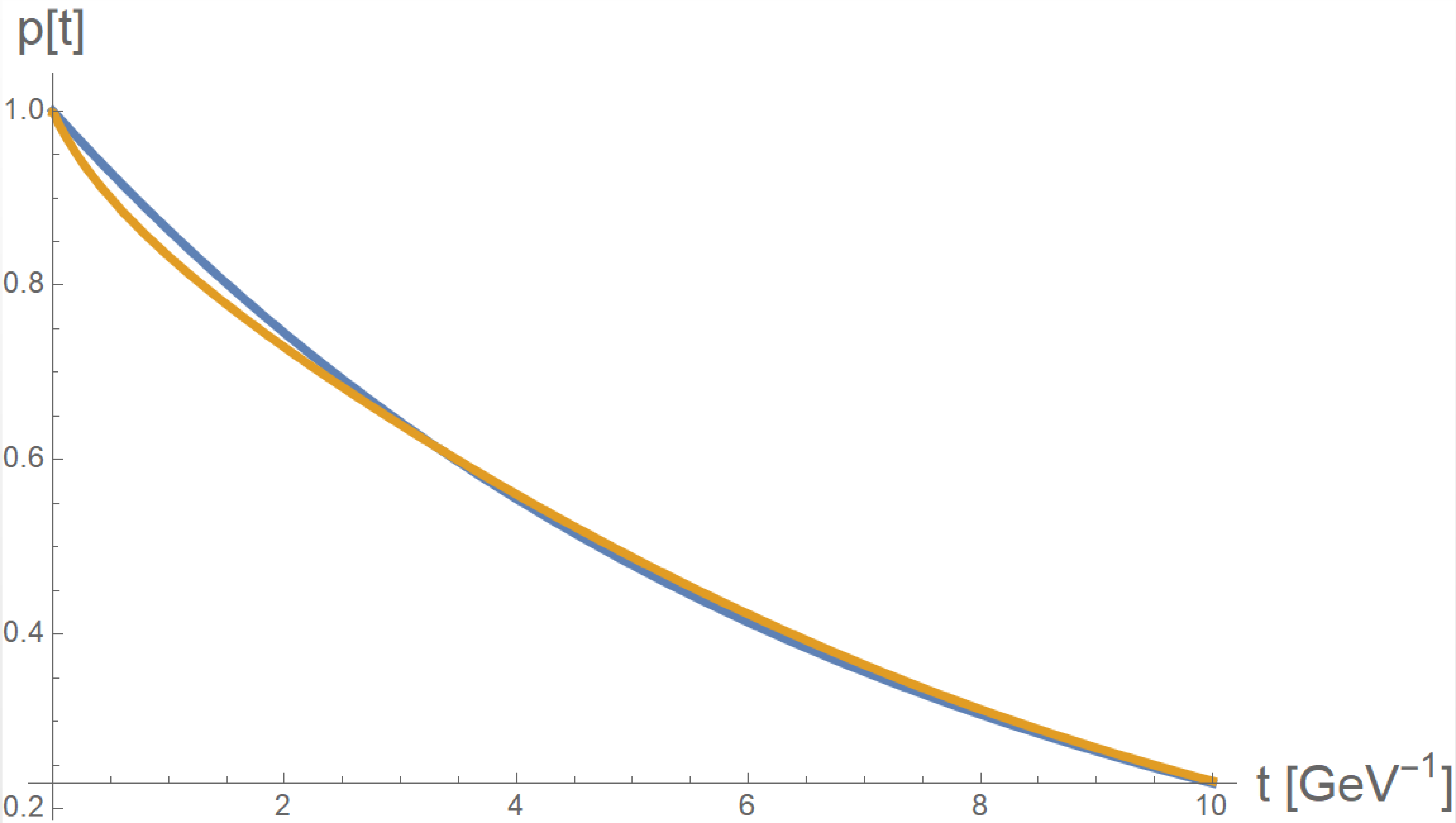
Sill for the rho-meson

Sill vs BW



Sill vs BW, distributions (rho-meson example)

Sill vs BW: time evolution



Time evolution

Comments

$$s_{pole} = M^2 - \frac{\tilde{\Gamma}^2}{2} - i\sqrt{(M^2 - s_{th})\tilde{\Gamma}^2 + \frac{\tilde{\Gamma}^4}{4}} \, .$$

Note, for $\tilde{\Gamma}^2$ sufficiently smaller than $M^2 - s_{th}$, the pole of s can be approximated as

$$s_{pole} \simeq M^2 - i\sqrt{(M^2 - s_{th})}\tilde{\Gamma} = M^2 - iM\Gamma \, ,$$

The normalization

$$\int_{E_{th}}^{+\infty} \mathrm{dE} \, d_S^{\mathrm{Sill}}(E) = 1$$

for any E_{th}, M, and $\tilde{\Gamma}$ is a consequence of the proper treatment of the real part of the loop

Comment

The Sill is Flatte-like, but not equal.

Flatté-like distributions and the $a_0(980)/f_0(980)$ mesons

V. Baru[1], J. Haidenbauer[2], C. Hanhart[2], A. Kudryavtsev[1], Ulf-G. Meißner[2,3]

Eur.Phys.J.A 23 (2005) 523-533e-Print: nuclth/0410099 [nucl-th]

$$\frac{d\sigma_i}{dm} \propto \left| \frac{m_R \sqrt{\Gamma_{\pi\eta} \Gamma_i}}{m_R^2 - m^2 - i m_R (\Gamma_{\pi\eta} + \Gamma_{K\bar{K}})} \right|^2 ,$$

with the partial widths $\Gamma_{\pi\eta} = \bar{g}_\eta q_\eta$ and

$$\Gamma_{K\bar{K}} = \bar{g}_K \sqrt{m^2/4 - m_K^2}$$

above threshold and

$$\Gamma_{K\bar{K}} = i \bar{g}_K \sqrt{m_K^2 - m^2/4}$$

The Sill is as Flatte along KK
(but not along pion-eta)

Comments

Uniwersytet Jana Kochanowskiego w Kielcach

The Sill is Flatte-like, but not equal.

PHYSICAL REVIEW D **99**, 093007 (2019)

Isovector scalar $a_0(980)$ and $a_0(1450)$ resonances in the $B \to \psi(K\bar{K}, \pi\eta)$ decays

Zhou Rui,* Ya-Qian Li, and Jie Zhang

$$M_{a_0(980)}(\omega^2) = \frac{m_0^2}{m_0^2 - \omega^2 - i(g_{\pi\eta}^2 \rho_{\pi\eta} + g_{KK}^2 \rho_{KK})}$$

$$\rho_{\pi\eta} = \sqrt{\left[1 - \left(\frac{m_\eta - m_\pi}{\omega}\right)^2\right]\left[1 - \left(\frac{m_\eta + m_\pi}{\omega}\right)^2\right]},$$

$$\rho_{K\bar{K}} = \frac{1}{2}\sqrt{1 - \frac{4m_{K^\pm}^2}{\omega^2}} + \frac{1}{2}\sqrt{1 - \frac{4m_{K^0}^2}{\omega^2}}.$$

It does not reduce to Flatte (even not in the KK channel)

BW, rBW, Sill in comparison: rho and a1 case

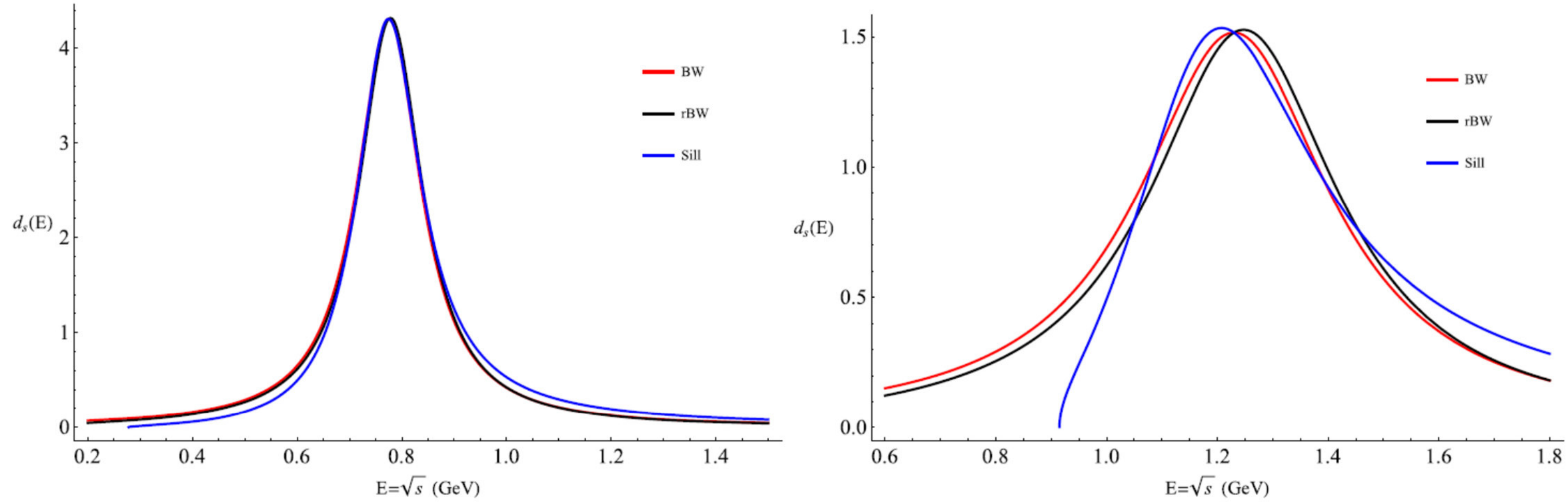
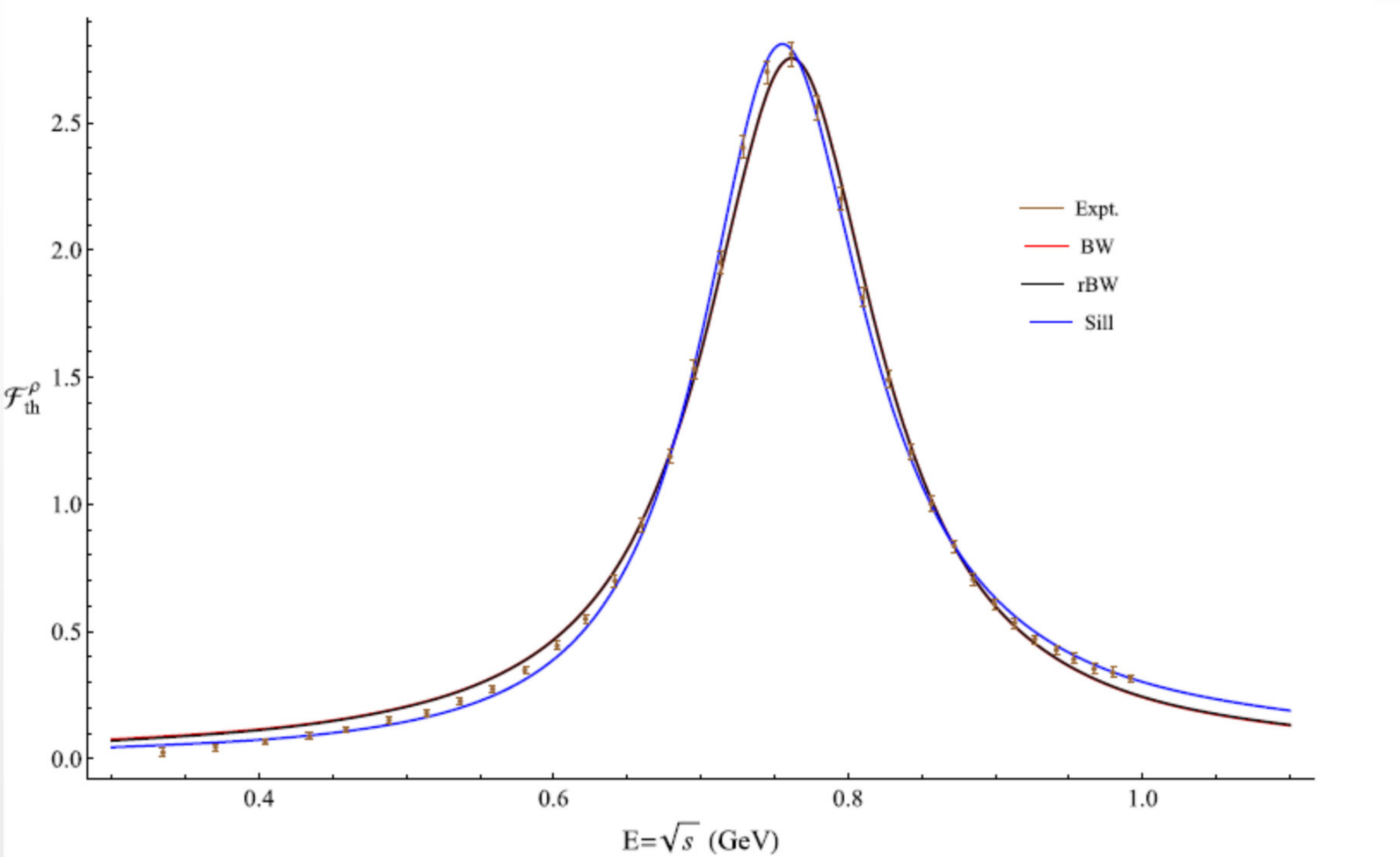


Fig. 1 Illustrative comparison of the four distributions discussed in the paper. Left panel, peak far away from the threshold ($\rho(770)$, $M = 0.775$ GeV, $\Gamma = 0.1478$ GeV, and $E_{th} = 2m_\pi$); right panel, peak near the threshold ($a_1(1260)$, $M = 1.230$ GeV, $\Gamma = 0.5$ GeV, and $E_{th} = m_\rho + m_\pi$)

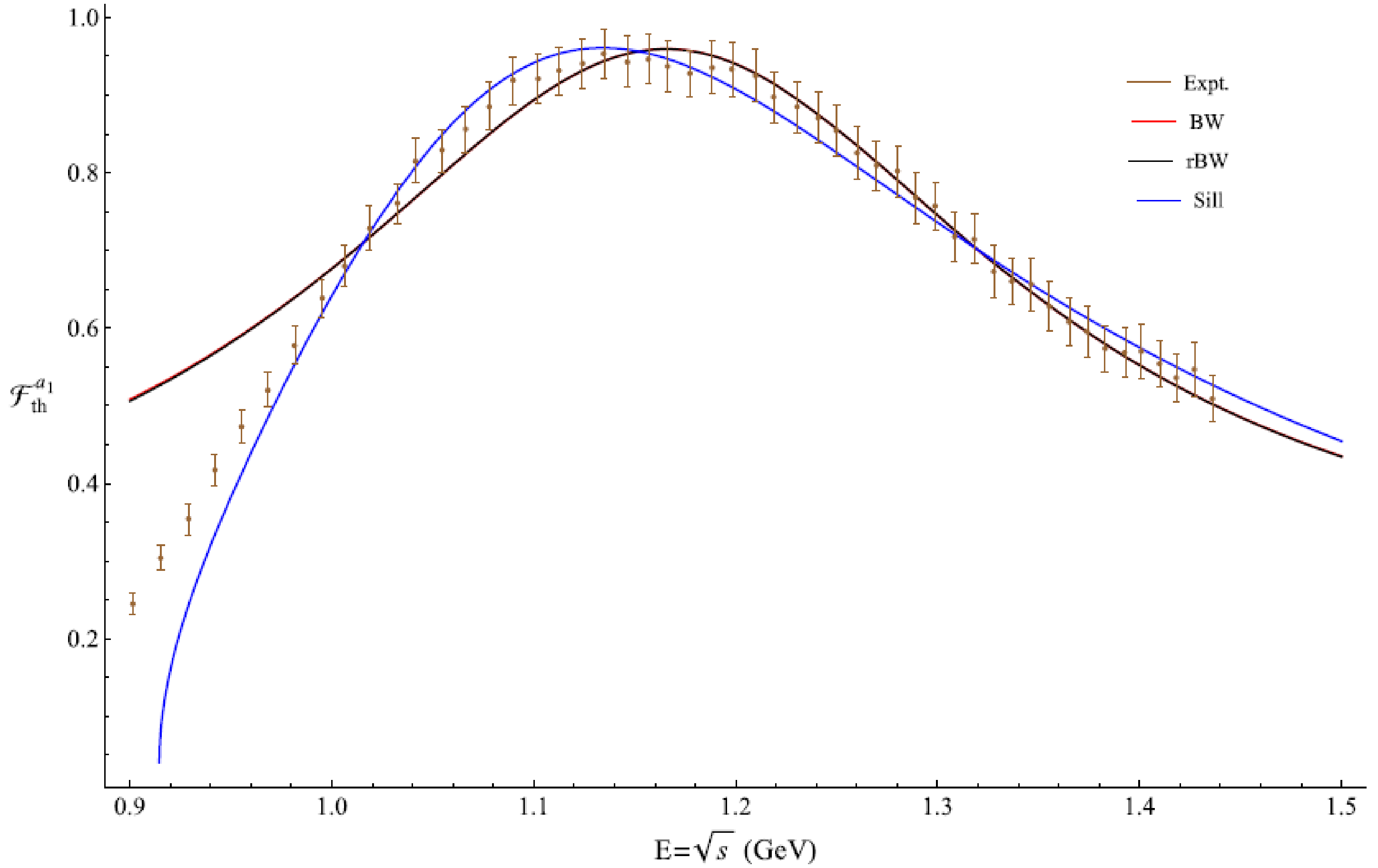
Rho meson



Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	761.64 ± 0.32	144.6 ± 1.3	10.16	$761.6 - i\,72.3$
Relativistic BW	758.1 ± 0.33	145.2 ± 1.3	9.42	$761.5 - i\,72.3$
Sill	755.82 ± 0.33	137.3 ± 1.1	3.52	$751.7 - i\,68.6$

a1 meson

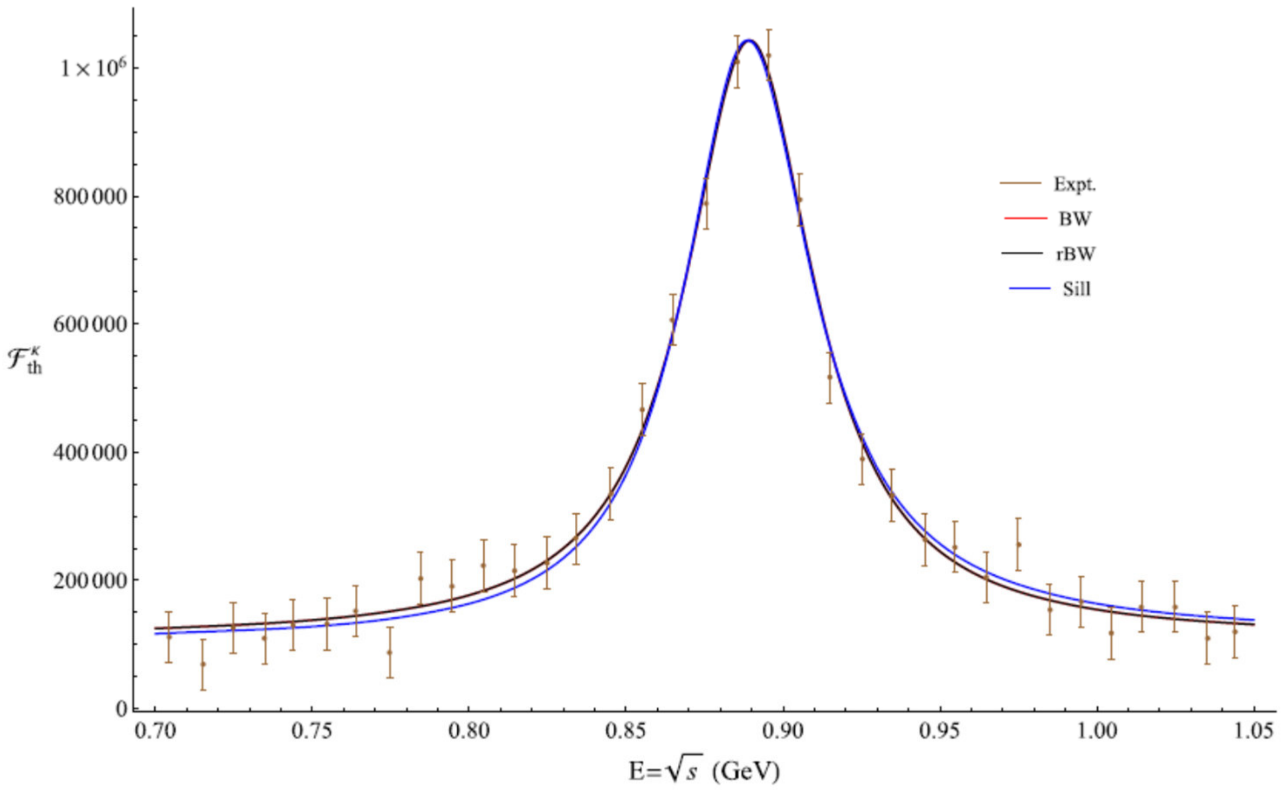


Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1165.6 ± 1.2	415 ± 15	4.31	$1166 - i\,208$
Relativistic BW	1146.5 ± 1.6	424 ± 16	4.25	$1165 - i\,209$
Sill	1181.3 ± 3.4	539 ± 27	3.52	$1046 - i\,250$

The K*(892) meson: basically no difference

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	889.37 ± 0.43	50.1 ± 1.6	1.78	$889.4 - i\,25.0$
Relativistic BW	889.01 ± 0.43	50.1 ± 1.6	1.78	$890.1 - i\,24.9$
Sill	889.06 ± 0.43	49.9 ± 1.6	2.08	$888.0 - i\,25.0$



J. Adam *et al.* [ALICE], arXiv:1601.07868

The Delta baryon

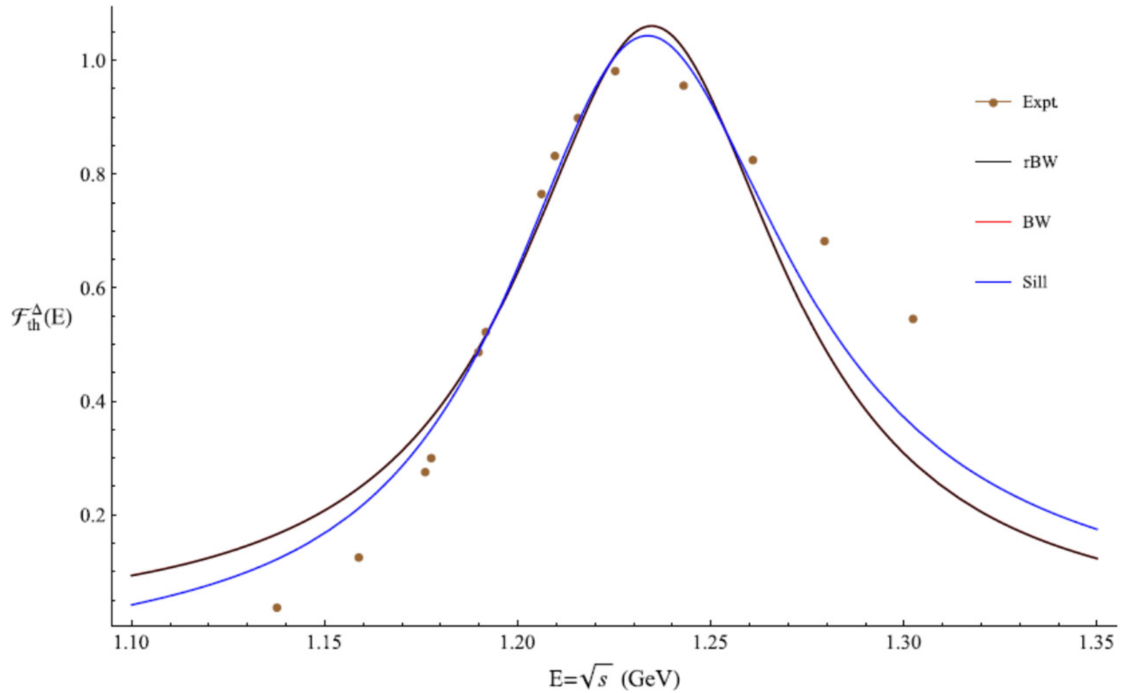


Fig. 6 The spectral function for the $\Delta(1232)$. Experimental data from [82]. It is visible that the Sill fairs marginally better than the (r)BW distributions

Table 5 Mass and width of $\Delta(1232)$ fitted using the three distributions discussed in the text, their error estimates, and the poles (as described in the text)

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1234.6 ± 1.3	83.6 ± 4.1	2.92	$1234.7 - i\,41.8$
Relativistic BW	1233.9 ± 1.2	83.7 ± 4.2	2.92	$1234.7 - i\,41.8$
Sill	1236.2 ± 1.5	90.4 ± 4.9	1.53	$1235.4 - i\,45.2$

Data from: J.R. Haskins, Am. J. Phys. **53**, 988–991 (1985)

More than a single channel

The extension to the N channels is straightforward::

$$G_S(s) = \frac{1}{s - M^2 + i\sum_{k=1}^{N} \tilde{\Gamma}_k \sqrt{s - s_{k,th}} + i\varepsilon}$$

with

$$\tilde{\Gamma}_k = \Gamma_k \frac{M}{\sqrt{M^2 - E_{k,th}^2}} \text{ and}$$

$$s_{1,th} = E_{1,th}^2 \leq s_{2,th} \leq \ldots \leq_{N,th} = E_{N,th}^2 \, .$$

$$d_s^k(s) = \frac{1}{\pi} \frac{\sqrt{s - s_{\rm th,k}}\; \tilde{\Gamma}_k}{(s - M^2 - \sum_{i=1}^{Q} \sqrt{s_{\rm th,i} - s}\; \tilde{\Gamma}_i)^2 + \sum_{i=Q+1}^{N} (\sqrt{s - s_{\rm th,i}}\; \tilde{\Gamma}_i)^2} \theta(s - s_{\rm th,k})$$

where, $s_{\rm th,k}$ is the $k^{\rm th}$ threshold, and the integer Q is such that, for all $i < Q$, $s_{th,i} < s_{th,k}$

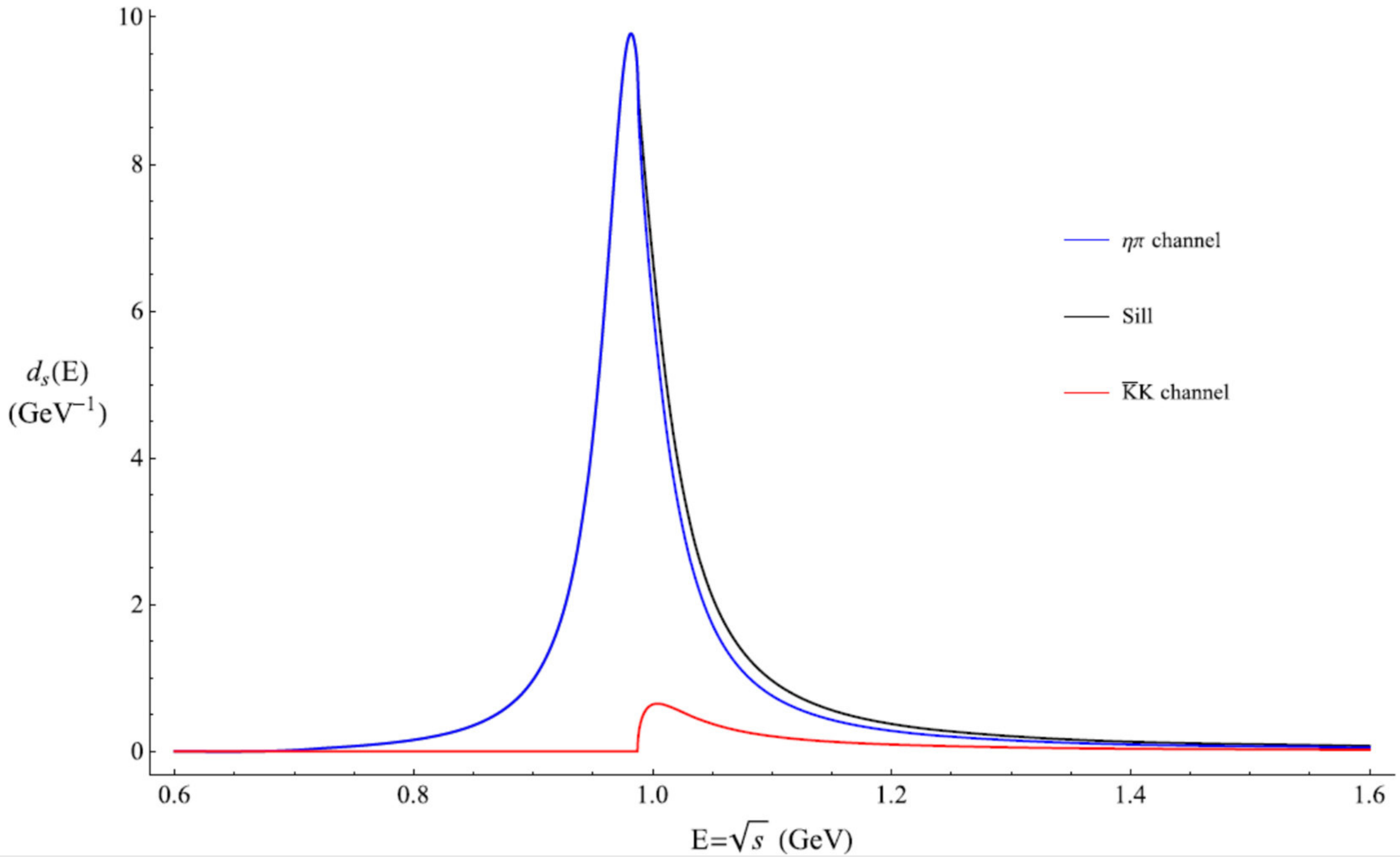
Two-channel case

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}_1\sqrt{s - s_{1,th}} + i\tilde{\Gamma}_2\sqrt{s - s_{2,th}} + i\varepsilon}.$$

$$d_S(s) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(s)] = \begin{cases} \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}}}{(s-M^2)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}})^2} & \text{for } s > s_{2,th} \\ \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}}{\left(s-M^2-\tilde{\Gamma}_2\sqrt{s_{2,th}-s}\right)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}})^2} & \text{for } s_{1,th} \le s \le s_{2,th} \\ 0 & \text{for } s < s_{1,th} \end{cases}$$

a0(980) example

Fig. 8 The Sill distribution of the $a_0(980)$ and the $\eta\pi$ and $\bar{K}K$ channels. The non-BW form due to the KK threshold is evident



Multichannel decay law

Physics Letters B 831 (2022) 137200

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

ELSEVIER

Multichannel decay law

Francesco Giacosa[a,b,*]

w1(t) is the probability that the decay has occurred in the first channel between (0,t)

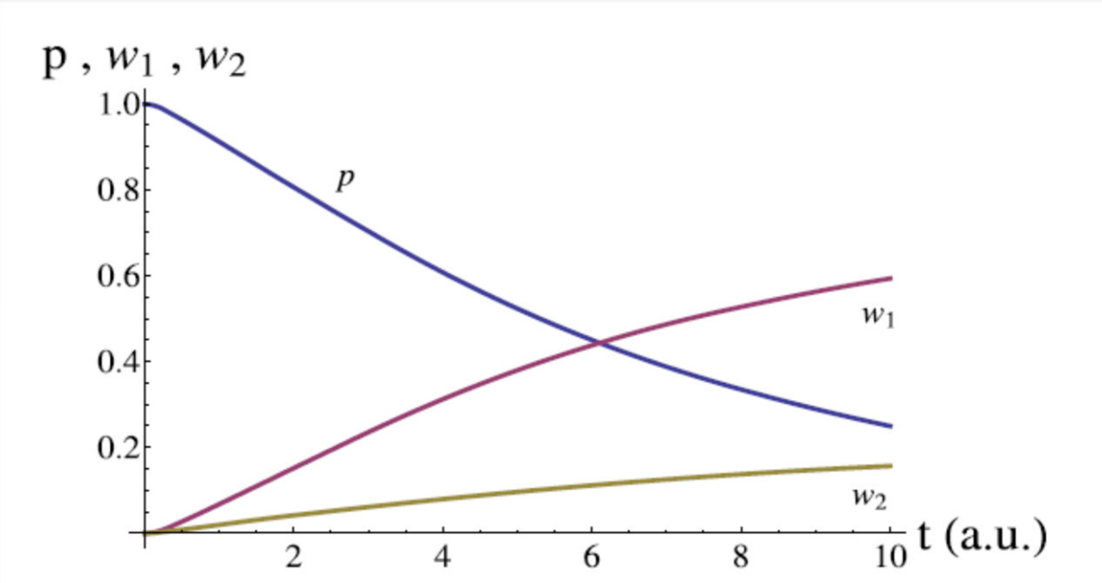
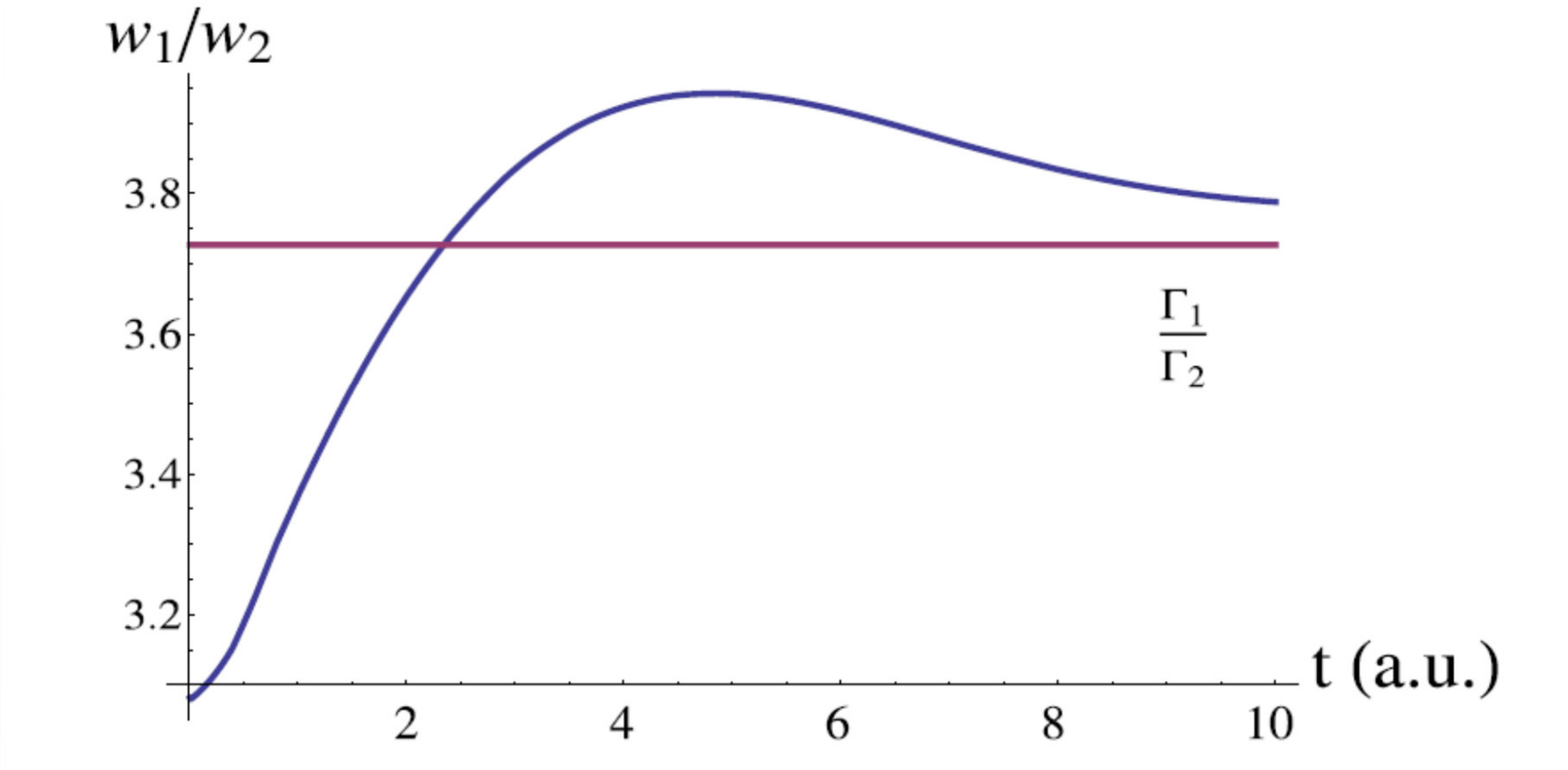


Fig. 1. The survival probability $p(t)$ of Eq. (1) and the decay probabilities $w_1(t)$ and $w_2(t)$ of Eq. (14) are plotted as function of t. The constraint $p + w_1 + w_2 = 1$ holds. Note, t is expressed in a.u. of $[M^{-1}]$.

$$\sum_{i=1}^{N} w_i = 1 - p(t)$$

$$w_i(t) = \int_{E_{th,i}}^{\infty} \mathrm{dE} \frac{2E^2 \Gamma_i(E)}{\pi} \left| \int_{E_{th,1}}^{\infty} \mathrm{dE}' d_S(E') \frac{e^{-iE't} - e^{-iEt}}{E'^2 - E^2} \right|^2$$

w1/w2 is not a constant



Recent Sill application/1

PHYSICAL REVIEW D **106**, 094009 (2022)

XYZ spectroscopy at electron-hadron facilities. II. Semi-inclusive processes with pion exchange

D. Winney,[1,2,*] A. Pilloni,[3,4,†] V. Mathieu,[5,‡] A. N. Hiller Blin,[6,7] M. Albaladejo,[8] W. A. Smith,[9,10] and A. Szczepaniak[9,10,11]

(Joint Physics Analysis Center)

description of the πp mass distribution in the Δ mass region:

$$d_{\Delta\to\pi p}(M^2) = \frac{1}{\pi}\frac{\rho(M^2)\tilde{\Gamma}_\Delta}{[M^2 - m_\Delta^2]^2 + [\rho(M^2)\tilde{\Gamma}_\Delta]^2}, \quad (39)$$

with $\rho(M^2) = \sqrt{M^2 - M_{\rm min}^2}$ and $\tilde{\Gamma}_\Delta = \Gamma_\Delta m_\Delta / \rho(m_\Delta^2)$. Interestingly, this function is normalized across the mass

Recent Sill application/2

First measurement of hard exclusive $\pi^-\Delta^{++}$ electroproduction beam-spin asymmetries off the proton

(The CLAS Collaboration)

ArXiv: 2303.11762

As a second completely independent method, a bin-by-bin background subtraction was performed based on a fit of the complete distribution (signal + background) with a so-called "Sill" function, which is a Breit-Wigner distribution including threshold effects [28] plus a fifth-order polynomial background in each Q^2, x_B, $-t$ and ϕ bin and for each helicity state. After the combined fit, the signal and background contributions were separated and the asymmetry was calculated based on the pure signal events. It was found that both methods provided consistent results for the signal asymmetry within the statistical uncertainty.

Recent Sill application/3: Xi(16260)

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ALICE

CERN

CERN-EP-2023-106
30 May 2023

Accessing the strong interaction between Λ baryons and charged kaons with the femtoscopy technique at the LHC

ALICE Collaboration*

ArXiv: 2305.19093

$\Xi(1620)$

$I(J^P) = \frac{1}{2}(?^?)$ Status: $*$
J, P need confirmation.

OMITTED FROM SUMMARY TABLE

What little evidence there is consists of weak signals in the $\Xi\pi$ channel. A number of other experiments (e.g., BORENSTEIN 72 and HASSALL 81) have looked for but not seen any effect.

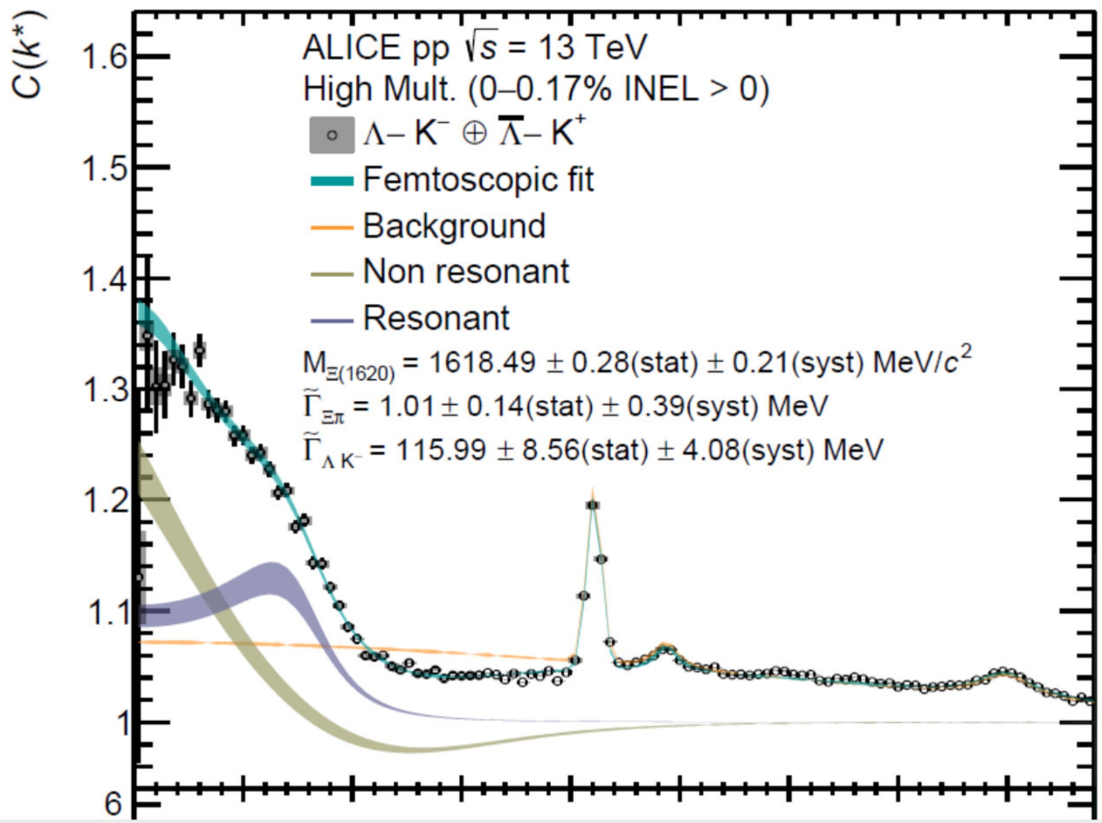
$\Xi(1620)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
≈ 1620 OUR ESTIMATE				

$\Xi(1620)$ DECAY MODES

	Mode
Γ_1	$\Xi\pi$

$$f(k^*) = \frac{-2\tilde{\Gamma}_{\Lambda K^-}}{E^2 - M^2 + i\tilde{\Gamma}_{\Xi\pi}\sqrt{E^2 - E^2_{\text{thr.}\Xi\pi}} + i\tilde{\Gamma}_{\Lambda K^-}\sqrt{E^2 - E^2_{\text{thr.}\Lambda K^-}}}$$



Hybrid meson and the Sill

Toward a nonet of hybrid state/PDG

Uniwersytet Jana Kochanowskiego w Kielcach

$\pi_1(1600)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Spectroscopy of Light Meson Resonances" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

$\pi_1(1600)$ T-Matrix Pole $\sqrt{s}$

$\pi_1(1600)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1661^{+15}_{-11} OUR AVERAGE	Error includes scale factor of 1.2.			

$\pi_1(1600)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
240± 50 OUR AVERAGE	Error includes scale factor of 1.7. See the ideogram below.			

$\pi_1(1600)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi\pi$	seen
Γ_2	$\rho^0\pi^-$	seen
Γ_3	$f_2(1270)\pi^-$	not seen
Γ_4	$b_1(1235)\pi$	seen
Γ_5	$\eta'(958)\pi^-$	seen
Γ_6	$\eta\pi$	
Γ_7	$f_1(1285)\pi$	seen

$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1400)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1354 ±25 OUR AVERAGE	Error includes scale factor of 1.8. See the ideogram below.				

$\pi_1(1400)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
330 ±35 OUR AVERAGE					

$\pi_1(1400)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi^0$	seen
Γ_2	$\eta\pi^-$	seen

A unique I=1 hybrid state

PHYSICAL REVIEW LETTERS **122**, 042002 (2019)

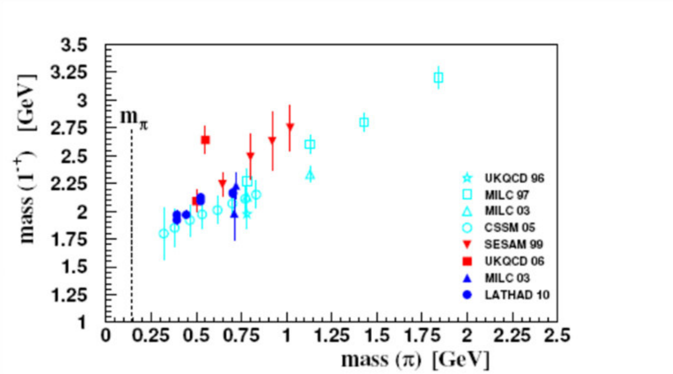
Determination of the Pole Position of the Lightest Hybrid Meson Candidate

A. Rodas,[1,*] A. Pilloni,[2,3,†] M. Albaladejo,[2,4] C. Fernández-Ramírez,[5] A. Jackura,[6,7] V. Mathieu,[2] M. Mikhasenko,[8] J. Nys,[9] V. Pauk,[10] B. Ketzer,[8] and A. P. Szczepaniak[2,6,7]

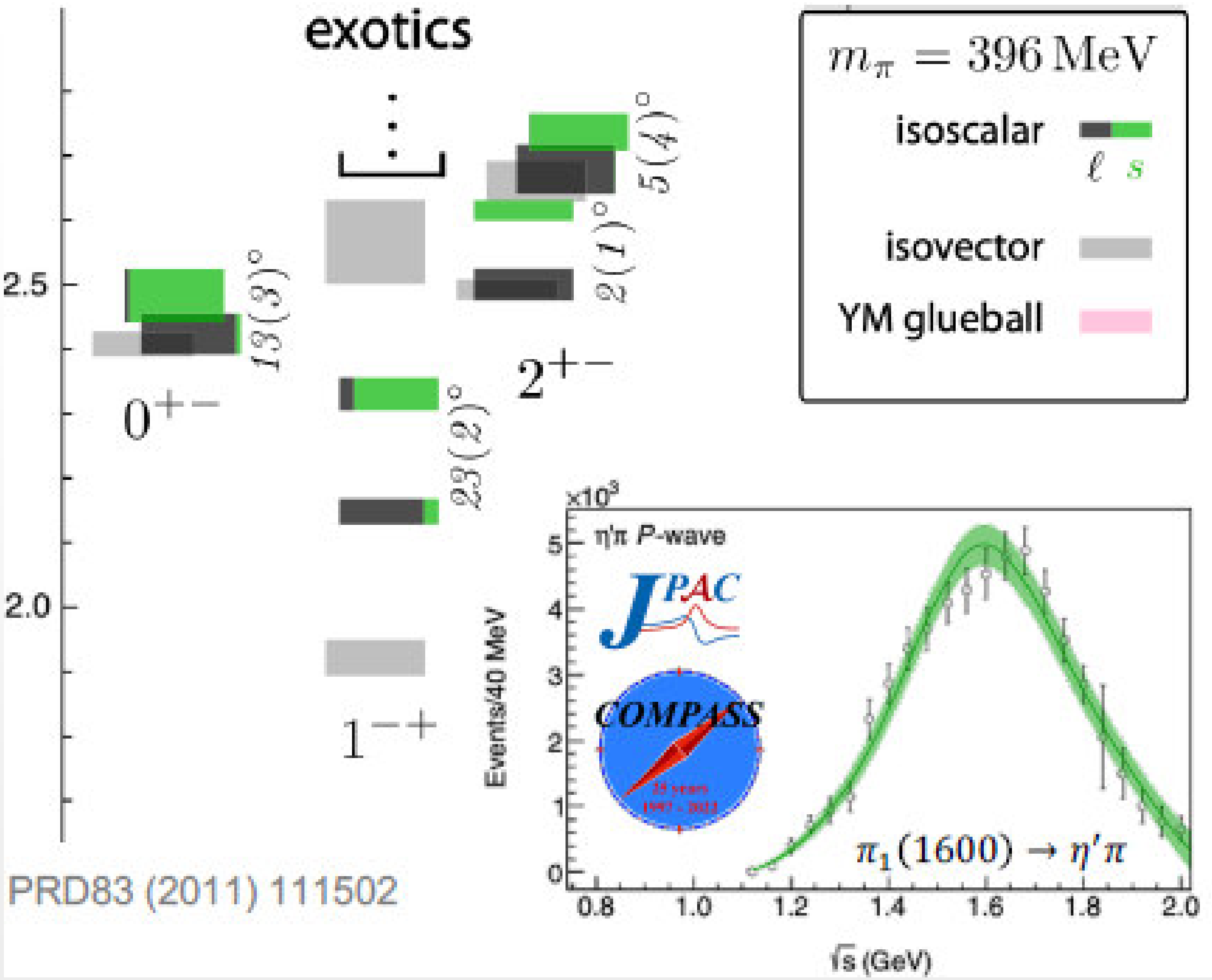
Mapping states with explicit gluonic degrees of freedom in the light sector is a challenge, and has led to controversies in the past. In particular, the experiments have reported two different hybrid candidates with spin-exotic signature, $\pi_1(1400)$ and $\pi_1(1600)$, which couple separately to $\eta\pi$ and $\eta'\pi$. This picture is not compatible with recent Lattice QCD estimates for hybrid states, nor with most phenomenological models. We consider the recent partial wave analysis of the $\eta^{(\prime)}\pi$ system by the COMPASS Collaboration. We fit the extracted intensities and phases with a coupled-channel amplitude that enforces the unitarity and analyticity of the S matrix. We provide a robust extraction of a single exotic π_1 resonant pole, with mass and width $1564 \pm 24 \pm 86$ and $492 \pm 54 \pm 102$ MeV, which couples to both $\eta^{(\prime)}\pi$ channels. We find no evidence for a second exotic state. We also provide the resonance parameters of the $a_2(1320)$ and $a_2'(1700)$.

π1(1600) and π1(1400) are the same state
(in agreement with various models and lattice QCD)

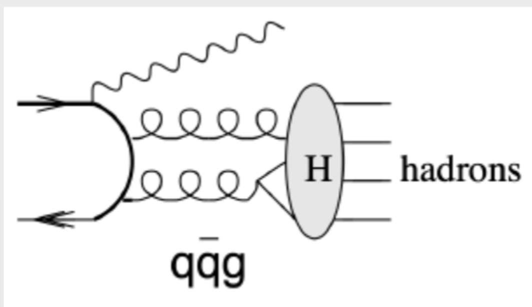
C. Meyer and E. Swanson,
Hybrid Mesons,
Prog. Part. Nucl. Phys. 82 (2015) 21
[arXiv:1502.07276 [hep-ph]].



Uniwersytet Jana Kochanowskiego w Kielcach



PRD83 (2011) 111502



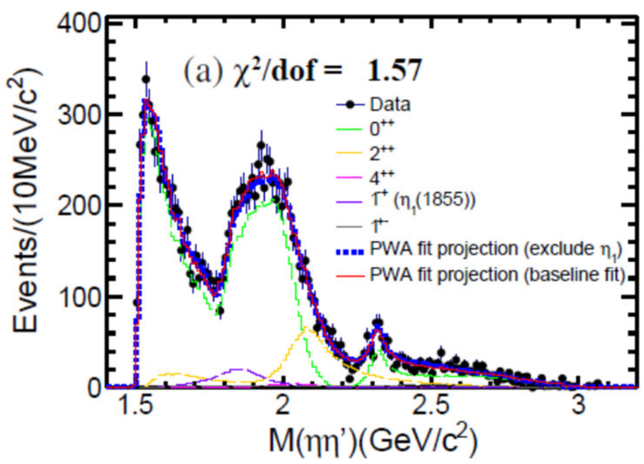
New experimental finding: η_1(1855)

Observation of an isoscalar resonance with exotic $J^{PC} = 1^{-+}$ quantum numbers in $J/\psi \to \gamma\eta\eta'$

M. Ablikim[1], M. N. Achasov[10,b], P. Adlarson[68], S. Ahmed[14], M. Albrecht[4], R. Aliberti[28], A. Amoroso[67A,67C], M. R. An[32],

Using a sample of $(10.09\pm0.04)\times10^9$ J/ψ events collected with the BESIII detector operating at the BEPCII storage ring, a partial wave analysis of the decay $J/\psi \to \gamma\eta\eta'$ is performed. The first observation of an isoscalar state with exotic quantum numbers $J^{PC} = 1^{-+}$, denoted as $\eta_1(1855)$, is reported in the process $J/\psi \to \gamma\eta_1(1855)$ with $\eta_1(1855) \to \eta\eta'$. Its mass and width are measured to be $(1855\pm9^{+6}_{-1})$ MeV/c^2 and $(188\pm18^{+3}_{-8})$ MeV, respectively, where the first uncertainties are statistical and the second are systematic, and its statistical significance is estimated to be larger than 19σ.

Phys.Rev.Lett. 129 (2022) 19, 192002 2202.00621 [hep-ex]



A nonet of hybrid states?

Physics Letters B 834 (2022) 137478

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry [a,*], Christian S. Fischer [b,c], Francesco Giacosa [a,d]

arXiv:2203.04327

Beides π1(1600) and η1(1855), we expect also:
K1(1750) and η1(1660). The last two not yet seen.

	M (MeV)
K_1^{hyb}	1761
η_1^L	1661
η_1^H	1855

$\pi_1(1600)$

Channel	Width (MeV)	Channel	Width (MeV)
$\Gamma_{b_1\pi}$	220 ± 34	$\Gamma_{f_1\pi}$	16.2 ± 3.1
$\Gamma_{\rho\pi}$	7.1 ± 1.8	$\Gamma_{f_1'\pi}$	0.83 ± 0.16
Γ_{K^*K}	1.2 ± 0.3	$\Gamma_{\eta\pi}$	0.37 ± 0.08
$\Gamma_{\rho\omega}$	0.08 ± 0.03	$\Gamma_{\eta'\pi}$	4.6 ± 1.0
		Γ_{tot}	250 ± 34

Predictions for hybrids

Uniwersytet Jana Kochanowskiego w Kielcach

$\eta_1^{hyb}(1660)$

Channel	Width (MeV)
	Set-1
$\Gamma_{a_1\pi}$	80 ± 15
Γ_{K^*K}	0.29 ± 0.075
$\Gamma_{\eta'\eta}$	0.41 ± 0.09
$\Gamma_{K_1(1270)K}$	0
$\Gamma_{\rho\rho}$	0.081 ± 0.028
$\Gamma_{K^*K^*}$	0
$\Gamma_{\omega\phi}$	0
$\Gamma_{f_1\eta}$	0
Γ_{tot}	81 ± 15

$\eta_1(1855)$

Channel	Width (MeV)
	Set-1
$\Gamma_{K_1(1270)K}$	253 ± 92
Γ_{K^*K}	1.45 ± 0.37
$\Gamma_{\eta'\eta}$	2.28 ± 0.51
$\Gamma_{a_1\pi}$	0
$\Gamma_{\rho\rho}$	0
$\Gamma_{K^*K^*}$	0.075 ± 0.027
$\Gamma_{\omega\phi}$	$\sim 10^{-4}$
$\Gamma_{f_1\eta}$	2.15 ± 0.56
Γ_{tot}	259 ± 92

$K_1^{hyb}(1750)$

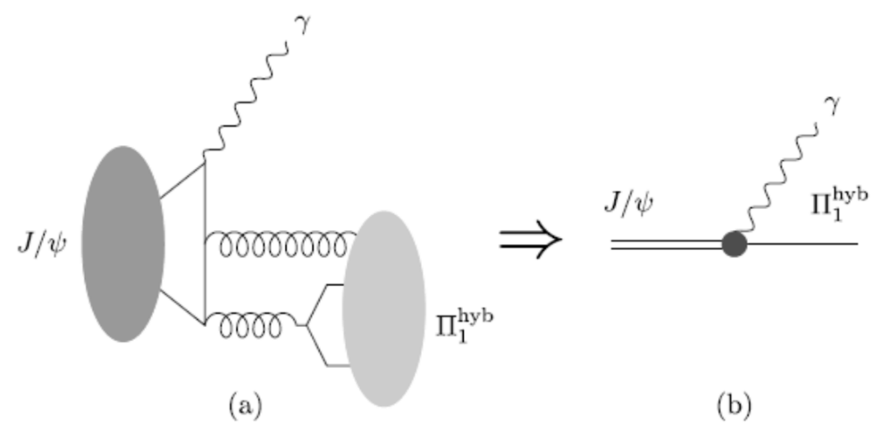
Channel	Width (MeV)
	Set-1
$\Gamma_{K_1(1270)\pi}$	125 ± 42
$\Gamma_{K_1(1400)\pi}$	103 ± 45
$\Gamma_{h_1(1170)K}$	1.53 ± 0.28
$\Gamma_{\eta K}$	0.29 ± 0.07
$\Gamma_{\eta' K}$	2.77 ± 0.62
$\Gamma_{\rho K^*}$	0.045 ± 0.016
$\Gamma_{a_1 K}$	11.0 ± 2.32
$\Gamma_{\rho K}$	2.18 ± 0.56
$\Gamma_{\omega K}$	0.82 ± 0.21
$\Gamma_{\phi K}$	0.49 ± 0.12
$\Gamma_{K^*\pi}$	0.67 ± 0.17
$\Gamma_{K^*\eta}$	0.30 ± 0.08
$\Gamma_{\omega K^*}$	0.011 ± 0.004
$\Gamma_{b_1 K}$	64 ± 14
Γ_{tot}	312 ± 97

Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics A 1037 (2023) 122683
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

Uniwersytet Jana Kochanowskiego w Kielcach

Radiative production and decays of the exotic $\eta_1'(1855)$ and its siblings

Vanamali Shastry [a,*], Francesco Giacosa [a,b]



J/Psi decay and the Sill

Production Channel $(\phi_1\phi_2)$	Branching ratio (10^{-4}) Set-1 $\theta_h = 0^\circ$
$a_1\pi$	4.8 ± 1.4
K^*K	$(1.73 \pm 0.49) \times 10^{-2}$
$\eta'\eta$	$(2.28 \pm 0.65) \times 10^{-2}$
$\rho\rho$	$(4.4 \pm 1.3) \times 10^{-3}$
$K_1(1270)K$	2.45 ± 0.70
K^*K	$(1.86 \pm 0.53) \times 10^{-2}$
K^*K^*	$(7.2 \pm 2.1) \times 10^{-4}$
$f_1(1285)\eta$	$(27.6 \pm 7.9) \times 10^{-3}$
$\eta\eta'$	$(2.70 \pm 0.76) \times 10^{-2}$ [10]

The branching ratios of the $J/\psi \to \gamma\eta_1^{hyb}(1660) \to \gamma\phi_1\phi_2$ and $J/\psi \to \gamma\eta_1'(1855) \to \gamma\phi_1\phi_2$

$$\Gamma_{A\to BC_1C_2} = \int_{s_{th}}^{(\Delta M_{AC_2})^2} ds\, \Gamma_{A\to\mathcal{R}^*C_2}(s) d_s^i(s)$$

Sill implemented in all decays above

Conclusions and outlook

Features of the Sill

- simple Flatte-like relativistic implementation of threshold(s);
- generalization of BW
- Normalization, simple propagator

Applications to (un)conventional states

- Rho, a1, K*(892), and a0 mesons as examples
- Delta baryon
- Xi(1620)baryon
- J/Psi decay into hybrids

Thanks!

Symmetries of QCD

Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

Trace anomaly: the emergence of a dimension

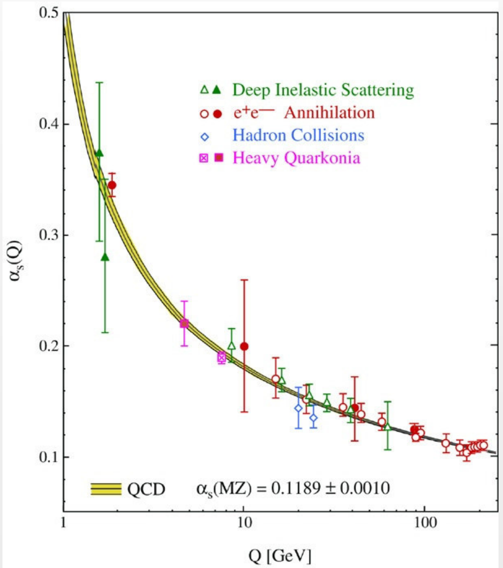
Chiral limit: $m_q = 0$

$$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu$$

is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250 \text{ MeV}$

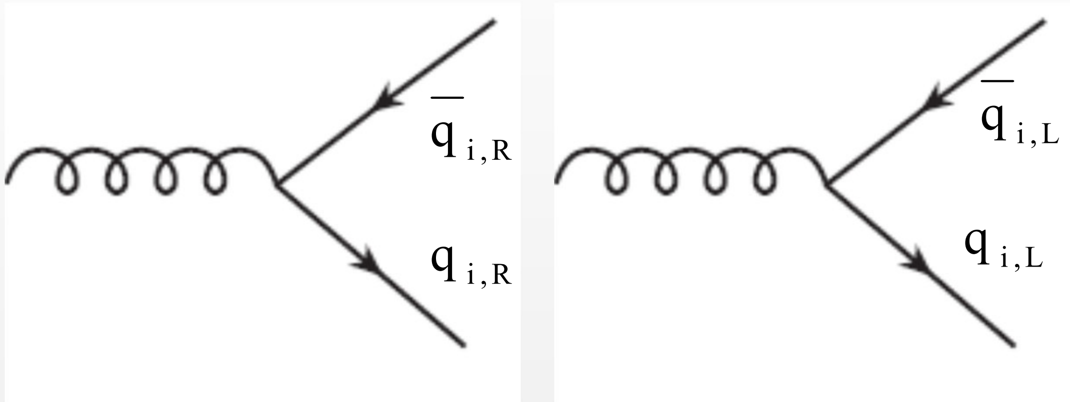
$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \rightarrow m^*_{gluon} \approx 500 - 800 \text{ MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Chiral symmetry



$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \to U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\partial^\mu (\bar{q}^i \gamma_\mu \gamma_5 q^i) = \frac{3g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}(G_{\mu\nu} G_{\rho\sigma})$$

Conventional mesons: quark-antiquark states

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

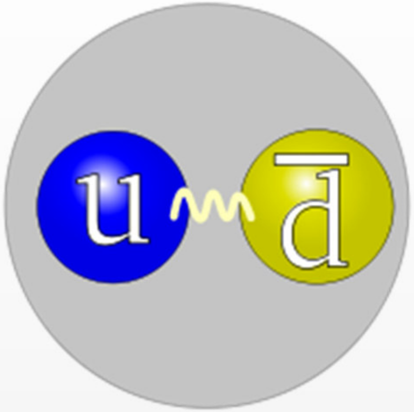
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + ...\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

Rho-meson $m_{\rho^+} = 775 \text{ MeV}$

Pion $m_{\pi^+} = 139 \text{ MeV}$

$$m_u + m_d \approx 7 \text{ MeV}$$

Mass generation in QCD is a nonpert. penomenon based on SSB
(mentioned previusly).

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'(958)}$	−11.3	−24.5
$1\,^3S_1$	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$	39.2	36.5
$1\,^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^\dagger$	$h_1(1380)$	$\boldsymbol{h_1(1170)}$		
$1\,^3P_0$	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$		
$1\,^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^\dagger$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$		
$1\,^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^\dagger$	$\eta_2(1870)$	$\boldsymbol{\eta_2(1645)}$		
$1\,^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}$		$\boldsymbol{\omega(1650)}$		
$1\,^3D_2$	2^{--}		$\boldsymbol{K_2(1820)}$				
$1\,^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$	31.8	30.8
$1\,^3F_4$	4^{++}	$\boldsymbol{a_4(2040)}$	$\boldsymbol{K_4^*(2045)}$		$\boldsymbol{f_4(2050)}$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$		
$2\,^3S_1$	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}$	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$		
$3\,^1S_0$	0^{-+}	$\boldsymbol{\pi(1800)}$			$\eta(1760)$		

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	θ_{quad} [°]	θ_{lin} [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	-11.3	-24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

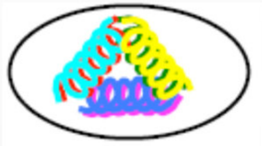
J^{PC}, ${}^{2S+1}L_J$	$I = 1(\bar{u}d, \bar{d}u, \frac{\bar{d}d - \bar{u}u}{\sqrt{2}})$ $I = 1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$ $I = 0(\frac{\bar{u}u + \bar{d}d}{\sqrt{2}}, \bar{s}s)^{**}$	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_\mathrm{L} \times SU(3)_\mathrm{R} \times \times U(1)_\mathrm{A}$
0^{-+}, ${}^1S_0$	π K $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\gamma^5 q^i$	$\Phi = S + \mathrm{i}P$ $(\Phi^{ij} = \bar{q}^j_\mathrm{R} q^i_\mathrm{L})$	$\Phi \to \mathrm{e}^{-2\mathrm{i}\alpha} U_\mathrm{L} \Phi U_\mathrm{R}^\dagger$
0^{++}, ${}^3P_0$	$a_0(1450)$ $K_0^*(1430)$ $f_0(1370)$, $f_0(1710)^*$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
1^{--}, ${}^1S_1$	$\rho(770)$ $K^*(892)$ $\omega(782)$, $\phi(1020)$	$V^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L^{ij}_\mu = \bar{q}^j_\mathrm{L} \gamma_\mu q^i_\mathrm{L})$	$L_\mu \to U_\mathrm{L} L_\mu U_\mathrm{L}^\dagger$
1^{++}, ${}^3P_1$	$a_1(1260)$ $K_{1,A}$ $f_1(1285)$, $f_1(1420)$	$A^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R^{ij}_\mu = \bar{q}^j_\mathrm{R} \gamma_\mu q^i_\mathrm{R})$	$R_\mu \to U_\mathrm{R} R_\mu U_\mathrm{R}^\dagger$
1^{+-}, ${}^1P_1$	$b_1(1235)$ $K_{1,B}$ $h_1(1170)$, $h_1(1380)$	$P^{ij}_\mu = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi^{ij}_\mu = \bar{q}^j_\mathrm{R} \mathrm{i}\overleftrightarrow{D}_\mu q^i_\mathrm{L})$	$\Phi_\mu \to \mathrm{e}^{-2\mathrm{i}\alpha} U_\mathrm{L} \Phi_\mu U_\mathrm{R}^\dagger$
1^{--}, ${}^3D_1$	$\rho(1700)$ $K^*(1680)$ $\omega(1650)$, $\phi(?)$	$S^{ij}_\mu = \frac{1}{2}\bar{q}^j \mathrm{i}\overleftrightarrow{D}_\mu q^i$		
2^{++}, ${}^3P_2$	$a_2(1320)$ $K_2^*(1430)$ $f_2(1270)$, $f_2'(1525)$	$V^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\mu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{L}(\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots)q^i_\mathrm{L})$	$L_{\mu\nu} \to U_\mathrm{L} L_{\mu\nu} U_\mathrm{L}^\dagger$
2^{--}, ${}^3D_2$	$\rho_2(?)$ $K_2(1820)$ $\omega_2(?)$, $\phi_2(?)$	$A^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma^5 \gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{R}(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_\mathrm{R})$	$R_{\mu\nu} \to U_\mathrm{R} R_{\mu\nu} U_\mathrm{R}^\dagger$
2^{-+}, ${}^1D_2$	$\pi_2(1670)$ $K_2(1770)$ $\eta_2(1645)$, $\eta_2(1870)$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\mathrm{i}\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{R}(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_\mathrm{L})$	$\Phi_{\mu\nu} \to \mathrm{e}^{-2\mathrm{i}\alpha} U_\mathrm{L} \Phi_{\mu\nu} U_\mathrm{R}^\dagger$
2^{++}, ${}^3F_2$	$a_2(?)$ $K_2^*(?)$ $f_2(?)$, $f_2'(?)$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$		
3^{--}, ${}^3D_3$	$\rho_3(1690)$ $K_3^*(1780)$ $\omega_3(1670)$, $\phi_3(1850)$	⋮	⋮	⋮

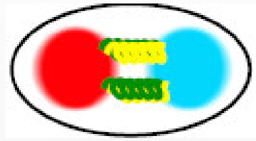
Uniwersytet Jana Kochanowskiego w Kielcach

Table from:

F.G., R. Pisarski,
A. Koenigstein
Phys.Rev.D 97 (2018) 9, 091901
e-Print: 1709.07454

Non-conventional mesons: beyond qq

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

(Some) novel results for conventional mesons

Strategy

- For a given nonet , write down the corresponding model-Lagrangian respecting flavor (or if possible chiral) symmetry.
- Consider only C, P, invariant terms
- Calculate decays in all possible channels (first at tree-level, in some selected case including finite width or loop effects;
- Fit free parameters to known experimental value;
- Make postdictions and predictions.

Tensor and (axial-)tensors

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

PHYSICAL REVIEW D **106,** 036008 (2022)

From well-known tensor mesons to yet unknown axial-tensor mesons

Shahriyar Jafarzade,[1] Arthur Vereijken,[1] Milena Piotrowska,[1] and Francesco Giacosa[1,2]
[1]*Institute of Physics, Jan Kochanowski University, ulica Uniwersytecka 7, 25-406 Kielce, Poland*
[2]*Institute for Theoretical Physics, J. W. Goethe University,*
Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany

(Received 4 April 2022; accepted 25 July 2022; published 10 August 2022)

While the ground-state tensor ($J^{PC} = 2^{++}$) mesons $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, and $f_2'(1525)$ are well known experimentally and form an almost ideal nonet of quark-antiquark states, their chiral partners, the ground-states axial-tensor ($J^{PC} = 2^{--}$) mesons are poorly settled: only the kaonic member $K_2(1820)$ of the nonet has been experimentally found, whereas the isovector state ρ_2 and two isoscalar states ω_2 and ϕ_2 are still missing. Here, we study masses, strong, and radiative decays of tensor and axial-tensor mesons within a chiral model that links them: the established tensor mesons are used to test the model and to determine its parameters, and subsequently various predictions for their chiral partners, the axial-tensor mesons, are obtained. The results are compared to current lattice QCD outcomes as well as to other theoretical approaches and show that the ground-state axial-tensor mesons are expected to be quite broad, the vector-pseudoscalar mode being the most prominent decay mode followed by the tensor-pseudoscalar one. Nonetheless, their experimental finding seems to be possible in ongoing and/or future experiments.

DOI: 10.1103/PhysRevD.106.036008