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On the molecular $\eta_1(1855)$ and its SU(3) partners

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Motivation

Observation and explanations on $\eta_1(1855)$

- $\eta_1(1855)$ reported with mass and width are $1855\,\mathrm{MeV}$ and $188\,\mathrm{MeV}$ and decays into $\eta\eta'$. BESIII, PRL.129 (2022)
- Hybrid$_{\mathrm{latt.}}$: $m \sim 2.00/2.24\,\mathrm{GeV}$, Had. Spec. PRD88(2013), F. Chen et al,PRD 107 (2023)
- Hybrid$_{\mathrm{theo.}}$: L. Qiu et al, CPC46 (2022); H.X. Chen et al, CPL39 (2022); V. Shastry et al, PLB834(2022); E.Swanson,PRD107(2023); B. Chen et al, 2302.06785
- Tetraquark: B. D. Wan et al,PRD106 (2022)
- $K_1(1400)\bar{K}$ form $\eta_1(1855)$ in OBE, X.K. Dong et al, Sci.China Phys.Mech 65 (2022)

Observation and explanations on $\pi_1(1600)$

- $\pi_1(1600)$ is firstly reported in $\pi^- p \to 3\pi p$, BNL-E0852,PRL81(1998)
- $\pi_1(1600)$ decays into $\rho\pi$ in P-wave, COMPASS,PRD105(2022) .
- $\left(K^*\bar{K}\right)_{f_1(1285)}\pi$ in Fixed Center Approach, X. Zhang et al, PRD95(2017)

Molecular candidate $\eta_1(1855)$: LO χPT

Weinberg-Tomozawa term

$$\mathcal{L}_I = -\frac{1}{4f_\pi^2}\langle[\Phi^\mu, \partial^\nu\Phi_\mu]\,[\phi, \partial_\nu\phi]\rangle, \;\; \Phi^8 = \{A_1, B_1\}\,.$$

$$A_1\,(1^{++}) = \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_1^8}{\sqrt{6}} & a_1^+ & K_{1A}^+ \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_1^8}{\sqrt{6}} & K_{1A}^0 \\ K_{1A}^- & \bar{K}_{1A}^0 & -\frac{2f_1^8}{\sqrt{6}} \end{pmatrix},$$

$$B_1\,(1^{+-}) = \begin{pmatrix} \frac{b_1^0}{\sqrt{2}} + \frac{h_1}{\sqrt{6}} & b_1^+ & K_{1B}^+ \\ b_1^- & -\frac{b_1^0}{\sqrt{2}} + \frac{h_1}{\sqrt{6}} & K_{1B}^0 \\ K_{1B}^- & \bar{K}_{1A}^0 & -\frac{2}{\sqrt{6}}h_1 \end{pmatrix}.$$

There are mixtures in physical mesons,

$$
\begin{aligned}
\eta &= \cos\theta_{\rm P}\eta^8 - \sin\theta_{\rm P}\eta^1, \\
\eta' &= \sin\theta_{\rm P}\eta^8 + \cos\theta_{\rm P}\eta^1, \\
f_1(1285) &= \cos\theta_{^3{\rm P}_1} f_1^1 + \sin\theta_{^3{\rm P}_1} f_1^8, \\
f_1(1420) &= -\sin\theta_{^3{\rm P}_1} f_1^1 + \cos\theta_{^3{\rm P}_1} f_1^8, \\
h_1(1170) &= \cos\theta_{^1{\rm P}_1} h_1^1 + \sin\theta_{^1{\rm P}_1} h_1^8, \\
h_1(1415) &= -\sin\theta_{^1{\rm P}_1} h_1^1 + \cos\theta_{^1{\rm P}_1} h_1^8, \\
K_1(1270) &= K_{1A}\sin\theta_{K_1} + K_{1B}\cos\theta_{K_1}, \\
K_1(1400) &= K_{1A}\cos\theta_{K_1} - K_{1B}\sin\theta_{K_1}.
\end{aligned}
$$

	θ_{K_1}	$\theta_{^3P_1}$	$\theta_{^1P_1}$	θ_P
$Set-A$	57°	52.0°	-17.5°	-17°
$Set-B$	34°	23.1°	28.0°	-17°

The mixing angles $\theta_{K1,^1P_1,^3P_1}$ are correlated in Ref. H.Y. Cheng,PLB707(2012)

Molecular candidate $\eta_1(1855)$: ChUA

Bethe-Salpeter equation

$$T = \left[1 + V\hat{G}\right]^{-1} (-V)\,\vec{\epsilon}\cdot\vec{\epsilon}\,',$$

with

$$\begin{aligned} V_{ij}(s) &= -\frac{\epsilon\cdot\epsilon'}{8f_\pi^2} C_{ij} \left[3s - \left(M^2 + m^2 + M'^2 + m'^2\right)\right. \\ &\quad \left. -\frac{1}{s}\left(M^2 - m^2\right)\left(M'^2 - m'^2\right)\right], \\ \hat{G} &= G\left(1 + \frac{1}{3}\frac{q_j^2}{M_j^2}\right) \end{aligned}$$

where G is dimensional regularized without finite widths of propagated Φ. M and m indicate Φ and ϕ masses, respectively. C_{ij} are the coefficients derived from the isospin basis.

Isoscalar scattering: $q_{max} = 700\,\mathrm{MeV}$

C_{ij}	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$a_1\pi$	-4	$\sqrt{\frac{3}{2}}\sin\theta_{K_1}$	0	$\sqrt{\frac{3}{2}}\cos\theta_{K_1}$	0
$K_1(1270)\bar{K}$		-3	$-\frac{3}{\sqrt{2}}\sin\theta_{^3P_1}\sin\theta_{K_1}$	0	$-\frac{3}{\sqrt{2}}\cos\theta_{^3P_1}\sin\theta_{K_1}$
$f_1(1285)\eta$			0	$-\frac{3}{\sqrt{2}}\cos\theta_{K_1}\sin\theta_{^3P_1}$	0
$K_1(1400)\bar{K}$				-3	$-\frac{3}{\sqrt{2}}\cos\theta_{^3P_1}\cos\theta_{K_1}$
$f_1(1420)\eta$					0

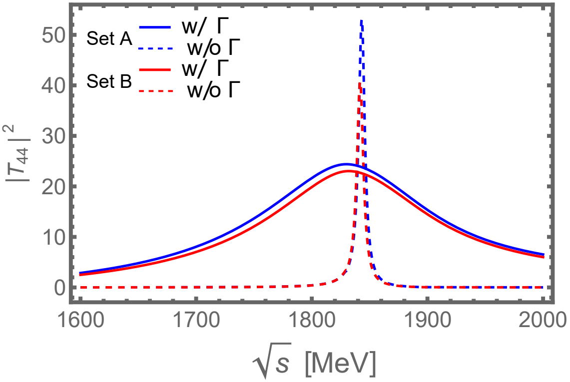
Isoscalar scattering: $q_{max} = 700\,\mathrm{MeV}$

C_{ij}	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$a_1\pi$	-4	$\sqrt{\frac{3}{2}}\sin\theta_{K_1}$	0	$\sqrt{\frac{3}{2}}\cos\theta_{K_1}$	0
$K_1(1270)\bar{K}$		-3	$-\frac{3}{\sqrt{2}}\sin\theta_{^3P_1}\sin\theta_{K_1}$	0	$-\frac{3}{\sqrt{2}}\cos\theta_{^3P_1}\sin\theta_{K_1}$
$f_1(1285)\eta$			0	$-\frac{3}{\sqrt{2}}\cos\theta_{K_1}\sin\theta_{^3P_1}$	0
$K_1(1400)\bar{K}$				-3	$-\frac{3}{\sqrt{2}}\cos\theta_{^3P_1}\cos\theta_{K_1}$
$f_1(1420)\eta$					0

Poles (Set A)			Channels		
1.84 $-i$0.03 $(---++)$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
g_l	$0.07+i0.28$	$0.69+i0.55$	$1.68+i0.08$	$\mathbf{9.33+i0.15}$	$1.16-i0.06$

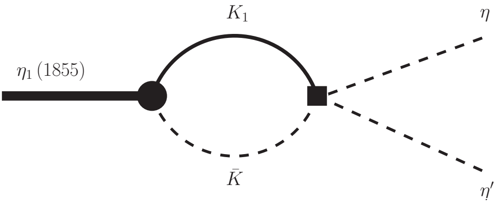
Poles (Set B)			Channels		
1.84 $-i$0.03 $(---++)$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
g_l	$0.15+i0.62$	$0.33-i0.27$	$1.83+i0.09$	$\mathbf{9.05+i0.17}$	$3.81-i0.20$

Including meson width in the scattering



- $M_i \to M_i - i\Gamma_i/2$ in G-loop.
- Peaks and FWHM: $\mathbf{(1.84, 0.16)}^A$ and $\mathbf{(1.85, 0.18)}^B\,\mathrm{GeV}$.

Two-body decay in RχT



$$\mathcal{L} = g\left[\langle A_{\mu\nu}\left(u^\mu u_\alpha h^{\nu\alpha} + h^{\nu\alpha} u_\alpha u^\mu\right)\rangle + \langle A_{\mu\nu}\left(u_\alpha u^\mu h^{\nu\alpha} + h^{\nu\alpha} u^\mu u_\alpha\right)\rangle + \langle A_{\mu\nu}\left(u^\mu h^{\nu\alpha} u_\alpha + u_\alpha h^{\nu\alpha} u^\mu\right)\rangle\right],$$

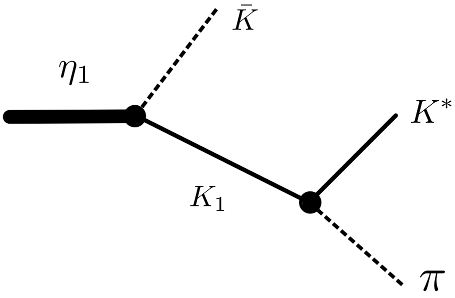
$$\mathcal{M}_{\eta_1\to\eta\eta'} = -\frac{4m_{\eta_1}^2}{3F_\pi^3 m_{K_1}} g g_{K_1\bar{K}} G\left[\left(\alpha p_{\eta'}^2 + \frac{1}{\sqrt{2}}\beta p_\eta^2\right)\varepsilon_{\eta_1}\cdot p_\eta + \left(p_\eta \leftrightarrow p_{\eta'}\right)\right],$$

$$\Gamma_{\eta\eta'} = (19 \pm 4\,\mathrm{MeV})^A \ or \ (7 \pm 2\,\mathrm{MeV})^B,$$

$$\alpha = \cos 2\theta_P + 2\sqrt{2}\sin 2\theta_P,\ \beta = 2\sqrt{2}\cos 2\theta_P - \sin 2\theta_P,\ g = 0.025\,\mathrm{GeV}^{-1}.$$

J.A. Miranada et al, PRD 102(2020), 2007.11019.

Three-body decay



- $\mathcal{M}_{3B} = g_{K_1(1400)\bar{K}} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{M_{K_1}^2} \right) \frac{1}{p^2 - M_{K_1}^2 + i\, M_{K_1} \Gamma_{K_1}}\, g_{K^*\pi} \varepsilon_{\eta_1}^{\mu} \varepsilon_{K^*}^{\nu}$
- $\frac{d\Gamma}{dM_{K_1\bar{K}}} = \frac{1}{(2\pi)^3} \frac{p_K \tilde{p_\pi}}{4M_{\eta_1}^2} |\mathcal{M}_{3B}|^2 \frac{1}{2J+1}$
- $\Gamma_{3B} = \left(81^{+11}_{-24}\mathrm{MeV}\right)^A$, $\Gamma_{3B} = \left(74^{+12}_{-24}\mathrm{MeV}\right)^B$.
- $\frac{\Gamma_{2B}}{\Gamma_{3B}} = \left(0.23^{-0.08}_{+0.16}\right)^A$ or $\left(0.10^{-0.03}_{+0.08}\right)^B$.

Isoscalar partners of $\eta_1(1855)$

Poles (Set A)	Channels				
$\mathbf{1.39 \pm 0.01 - i(0.04 \pm 0.01)}$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$(-++++)$					
g_I	$\mathbf{5.21 + i3.01}$	$1.22 + i0.78$	$0.01 + i0.02$	$0.36 + i0.35$	0.00
$\mathbf{(1.39, 0.24)}$					
$\mathbf{1.69 \pm 0.03}$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$(-++++)$					
g_I	$0.36 + i0.98$	$\mathbf{8.16 - i0.17}$	$3.64 + i0.01$	$0.09 - i0.15$	$2.46 + i0.01$
$\mathbf{(1.69, 0.08)}$					
$\mathbf{1.84 \pm 0.03}$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$(---++)$					
g_I	$0.07 + i0.28$	$0.69 + i0.55$	$1.68 + i0.08$	$\mathbf{9.33 + i0.15}$	$1.16 + i0.06$
$\mathbf{(1.84, 0.16)}$					
Poles (Set B)	Channels				
$\mathbf{1.39 \pm 0.01 - i(0.04 \pm 0.01)}$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$(-++++)$					
g_I	$\mathbf{5.21 + i3.03}$	$0.81 + i0.53$	0.00	$0.55 + i0.54$	0.00
$\mathbf{(1.42, 0.34)}$					
$\mathbf{1.70 \pm 0.02}$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$(-++++)$					
g_I	$0.25 + i0.67$	$\mathbf{8.34 - i0.08}$	$1.27 - i0.01$	$0.37 + i0.17$	$2.58 - i0.01$
$\mathbf{(1.70, 0.10)}$					
$\mathbf{1.84 \pm 0.03}$	$a_1\pi$	$K_1(1270)\bar{K}$	$f_1(1285)\eta$	$K_1(1400)\bar{K}$	$f_1(1420)\eta$
$(---++)$					
g_I	$0.15 + i0.62$	$0.33 - i0.27$	$1.83 + i0.09$	$\mathbf{9.05 + i0.17}$	$3.81 - i0.20$
$\mathbf{(1.85, 0.18)}$					

$SU(3)$ partners of $\eta_1(1855)$: $I=1$ sector

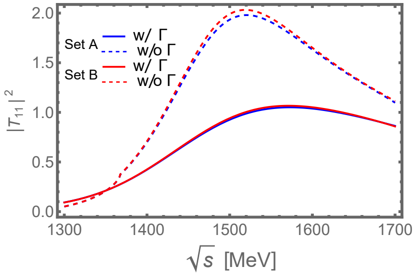
C_{ij}	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K1(1400)\bar{K}$
$b_1\pi$	-2	0	0	$\cos\theta_{K_1}$	0	$-\sin\theta_{K_1}$
$f_1(1285)\pi$		0	0	$\sqrt{\frac{3}{2}}\sin\theta_{K_1}\sin\theta_{^3P_1}$	0	$\sqrt{\frac{3}{2}}\cos\theta_{K_1}\sin\theta_{^3P_1}$
$f_1(1420)\pi$			0	$\sqrt{\frac{3}{2}}\cos\theta_{^3P_1}\sin\theta_{K_1}$	0	$\sqrt{\frac{3}{2}}\cos\theta_{K_1}\cos\theta_{^3P_1}$
$K_1(1270)\bar{K}$				-1	$-\sqrt{\frac{3}{2}}\sin\theta_{K_1}$	0
$a_1\eta$					0	$-\sqrt{\frac{3}{2}}\cos\theta_{K_1}$
$K_1(1400)\bar{K}$						-1

Channel	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
Threshold	1367	1419	1564	1748	1777	1895
	b_1	$f_1(1285)$	$f_1(1420)$	$K_1(1270)$	a_1	$K_1(1400)$
Γ	142	22.7	54.5	90	300	174

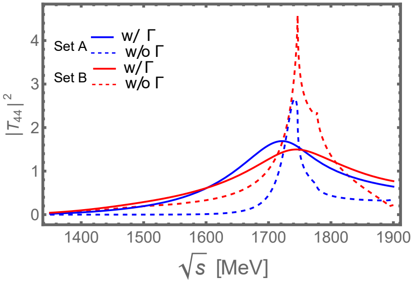
$SU(3)$ partners of $\eta_1(1855)$: $I = 1$ sector

Poles (Set A)	Channels					
$\mathbf{1.47 \pm 0.01 - i(0.12 \pm 0.02)}$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
$(--++++)$						
g_I	$\mathbf{5.22 + i4.40}$	$0.02 - i0.09$	$0.03 - i0.05$	$1.25 + i1.27$	$0.02 - i0.12$	$1.33 + i1.63$
$\mathbf{(1.56, 0.46)}$						
$\mathbf{1.75 \pm 0.02 - i(0.02 \pm 0.01)}$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
$(---+++)$						
g_I	$0.10 + i0.95$	$2.73 - i0.02$	1.89	$\mathbf{5.84 - i1.85}$	$3.49 - i0.03$	$2.65 - i0.53$
$\mathbf{(1.74, 0.30)}$						
Poles (Set B)	Channels					
$\mathbf{1.47 \pm 0.01 - i(0.12 \pm 0.02)}$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
$(--++++)$						
g_I	$\mathbf{5.27 + i4.31}$	$0.01 - i0.03$	$0.03 - i0.06$	$1.97 - i1.81$	$0.02 - i0.08$	$0.91 + i1.07$
$\mathbf{(1.57, 0.50)}$						
$\mathbf{1.77 \pm 0.01 - i(0.01 \pm 0.01)}$	$b_1\pi$	$f_1(1285)\pi$	$f_1(1420)\pi$	$K_1(1270)\bar{K}$	$a_1\eta$	$K_1(1400)\bar{K}$
$(---+++)$						
g_I	$0.13 + i1.44$	$1.37 - i0.25$	$2.86 - i0.50$	$\mathbf{4.80 - i2.29}$	$3.53 - i0.64$	$4.54 - i1.77$
$\mathbf{(1.72, 0.20)}$						

$SU(3)$ partners of $\eta_1(1855)$: $I = 1$ sector



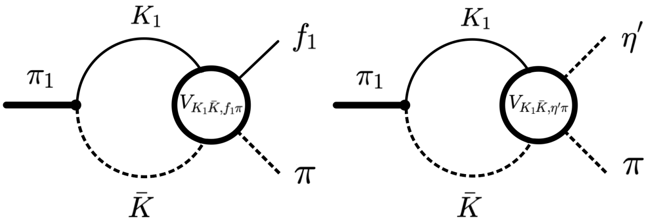
(a) Modulus square of $b_1\pi$ scattering.



(b) Modulus square of $K_1\bar{K}$ scattering.

The lower pole relating to $\pi_1(1400)$ is very broad w/o axial-vector width.

$SU(3)$ partners of $\eta_1(1855)$: $I = 1$ sector



In order to compare our findings with the ones listed in PDG, we evaluate the ratio,

$$\mathcal{R}_1 = \frac{|\mathcal{M}_{f_1(1285)\pi}|^2\, q}{|\mathcal{M}_{\eta'\pi}|^2\, \tilde{q}},$$

$$\mathcal{R}_1 = \begin{cases} \left(2.4^{+0.8}_{-0.6}\right)^A \\ \left(2.1^{+0.4}_{-0.3}\right)^B \end{cases},$$

where the one listed in PDG is 3.80 ± 0.78 for the $\pi_1(1600)$ state.

$SU(3)$ partners of $\eta_1(1855)$: $I = 1/2$ sector

C_{ij}	$a_1 K$	$f_1(1285)K$	$K_1(1270)\eta$	$f_1(1420)K$	$K_1(1400)\eta$
$a_1 K$	-2	0	$-\frac{3}{2}\sin\theta_{K_1}$	0	$-\frac{3}{2}\cos\theta_{K_1}$
$f_1(1285)K$		0	$\frac{3}{2}\sin\theta_{K_1}\sin\theta_{^3P_1}$	0	$\frac{3}{2}\sin\theta_{K_1}\cos\theta_{K_1}$
$K_1(1270)\eta$			0	$\frac{3}{2}\cos\theta_{^3P_1}\sin\theta_{K_1}$	0
$f_1(1420)K$				0	$\frac{3}{2}\cos\theta_{^3P_1}\cos\theta_{K_1}$
$K_1(1400)\eta$					0

C_{ij}	$h_1(1170)K$	$b_1 K$	$K_1(1270)\eta$	$h_1(1415)K$	$K_1(1400)\eta$
$h_1(1170)K$	0	0	$\frac{3}{2}\cos\theta_{K_1}\sin\theta_{^1P_1}$	0	$\frac{3}{2}\sin\theta_{K_1}\sin\theta_{^1P_1}$
$b_1 K$		-2	$-\frac{3}{2}\cos\theta_{K_1}$	0	$-\frac{3}{2}\sin\theta_{K_1}$
$K_1(1270)\eta$			0	$\frac{3}{2}\cos\theta_{K_1}\cos\theta_{^1P_1}$	0
$h_1(1415)K$				0	$\frac{3}{2}\sin\theta_{K_1}\cos\theta_{^1P_1}$
$K_1(1400)\eta$					0

$SU(3)$ partners of $\eta_1(1855)$: $I = 1/2$ sector

C_{ij}	$a_1 K$	$f_1(1285)K$	$K_1(1270)\eta$	$f_1(1420)K$	$K_1(1400)\eta$
$a_1 K$	-2	0	$-\frac{3}{2}\sin\theta_{K_1}$	0	$-\frac{3}{2}\cos\theta_{K_1}$
$f_1(1285)K$		0	$\frac{3}{2}\sin\theta_{K_1}\sin\theta_{^3P_1}$	0	$\frac{3}{2}\sin\theta_{K_1}\cos\theta_{K_1}$
$K_1(1270)\eta$			0	$\frac{3}{2}\cos\theta_{^3P_1}\sin\theta_{K_1}$	0
$f_1(1420)K$				0	$\frac{3}{2}\cos\theta_{^3P_1}\cos\theta_{K_1}$
$K_1(1400)\eta$					0

C_{ij}	$h_1(1170)K$	$b_1 K$	$K_1(1270)\eta$	$h_1(1415)K$	$K_1(1400)\eta$
$h_1(1170)K$	0	0	$\frac{3}{2}\cos\theta_{K_1}\sin\theta_{^1P_1}$	0	$\frac{3}{2}\sin\theta_{K_1}\sin\theta_{^1P_1}$
$b_1 K$		-2	$-\frac{3}{2}\cos\theta_{K_1}$	0	$-\frac{3}{2}\sin\theta_{K_1}$
$K_1(1270)\eta$			0	$\frac{3}{2}\cos\theta_{K_1}\cos\theta_{^1P_1}$	0
$h_1(1415)K$				0	$\frac{3}{2}\sin\theta_{K_1}\cos\theta_{^1P_1}$
$K_1(1400)\eta$					0

The transition in $\mathcal{O}\left(p^2\right)$ order reads

$$\mathcal{L}_{mix} \quad \propto \quad \langle A_{1\mu}\left[B_{1\nu}, \left[u^\mu, u^\nu\right]\right]\rangle$$

with $u_\mu = i\left(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger\right)$.

$SU(3)$ partners of $\eta_1(1855)$: $I = 1/2$ sector

Poles (Set A)			Channels		
$\mathbf{1.69 \pm 0.02}$	$a_1 K$	$f_1(1285)K$	$K_1(1270)\eta$	$f_1(1420)K$	$K_1(1400)\eta$
$(+++++)$					
g_I	**6.89**	0.89	3.75	0.54	2.10
$\mathbf{(1.70, 0.28)}$					
Poles (Set B)			Channels		
$\mathbf{1.70 \pm 0.02}$	$a_1 K$	$f_1(1285)K$	$K_1(1270)\eta$	$f_1(1420)K$	$K_1(1400)\eta$
$(+++++)$					
g_I	**6.58**	0.25	2.45	0.27	3.15
$\mathbf{(1.70, 0.30)}$					

Poles (Set A)			Channels		
$\mathbf{1.70 \pm 0.02}$	$h_1(1170)K$	$b_1 K$	$K_1(1270)\eta$	$h_1(1415)K$	$K_1(1400)\eta$
$(-++++)$					
g_I	0.20	**6.46**	$2.38 - i0.01$	0.50	$3.21 - i0.02$
$\mathbf{(1.70, 0.14)}$					
Poles (Set B)			Channels		
$\mathbf{1.69 \pm 0.02}$	$h_1(1170)K$	$b_1 K$	$K_1(1270)\eta$	$h_1(1415)K$	$K_1(1400)\eta$
$(-++++)$					
g_I	$0.55 - i0.01$	$\mathbf{6.78 + i0.02}$	$3.69 - i0.06$	$0.83 - i0.01$	$2.17 - i0.04$
$\mathbf{(1.70, 0.14)}$					

Summary

- Weinberg-Tomzawa term is applied to study GB scattering off 1^+ axial-vector mesons and several poles are found. One of the isoscalar poles may couple to $\eta_1(1855)$ and the isovector ones may relate to $\pi_1(1400/1600)$. In addition, two poles in the $I = 1/2$ sector may contribute to $K(1680)$.
- When the finite widths of axial-vector mesons are included, the nontrivial peaks in $|T_{ii}|^2$ are kept.
- In RχT, width of $\eta_1(1855) \to \eta\eta'$ is predicted and the ratio between $\pi_1\,(1600)$ decaying into $f_1(1285)\pi$ and $\eta'\pi$ matches the one in PDG.

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Thanks !

Backup: Dimensional regularization

$$G_j^{\text{Dim.}}(s) = \frac{1}{16\pi^2}\Bigg\{a(\mu) + \ln\frac{M_j^2}{\mu^2} + \frac{s - M_j^2 + m_j^2}{2s}\ln\frac{m_j^2}{M_j^2}$$
$$+\frac{\kappa_j}{2s}[\ln(s - m_j^2 + M_j^2 + \kappa_j) - \ln(-s + m_j^2 - M_j^2 + \kappa_j) +$$
$$\ln(s + m_j^2 - M_j^2 + \kappa_j) - \ln(-s - m_j^2 + M_j^2 + \kappa_j)]\Bigg\}.$$

The loop function G_j can also be regularized by a three-momentum cutoff q_{max}, and the corresponding regularized function $G_j^{\text{Cut}}(s, q_{max})$ is given by

$$G_j^{\text{Cut}}(s) = \frac{1}{4\pi^2}\int_0^{q_{max}} \mathrm{dq}\frac{\mathrm{q}^2}{\omega_1\omega_2} \times \frac{\omega_1+\omega_2}{(\sqrt{s}-(\omega_1+\omega_2))(\sqrt{s}+\omega_1+\omega_2)},$$

with $\omega_1 = \sqrt{m_j^2 + q^2}$ and $\omega_2 = \sqrt{M_j^2 + q^2}$.