HADRON 2023

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Can Constituent Gluons Describe Glueballs and Hybrids?

Eric Swanson

ExoHad
Exotic Hadrons Topical Collaboration

University of Pittsburgh

References

Christian Farina

Light hybrid decays,
C. Farina & E.S. Swanson, in preparation.

Light hybrid mixing and phenomenology,
E.S. Swanson, arXiv:2302.01372.

Heavy hybrid decays in a constituent gluon model,
C. Farina, H.G. Tecocoatzi, A. Giachino, E. Santopinto, & E.S. Swanson,
***Phys.Rev.D* 102 (2020) 1, 014023.**

The low lying glueball spectrum,
A.P. Szczepaniak & E.S. Swanson, *Phys.Lett.B* 577 (2003) 61-66.

Coulomb gauge QCD, confinement, and the constituent representation,
A.P. Szczepaniak & E.S. Swanson, *Phys.Rev.D* 65 (2001) 025012.

Flavour Mixing in the Isoscalar Sector a la LGT

Toward the excited isoscalar meson spectrum from lattice QCD, [HadSpec] J.J. Dudek et al. Phys.Rev.D 88 (2013) 9, 094505.

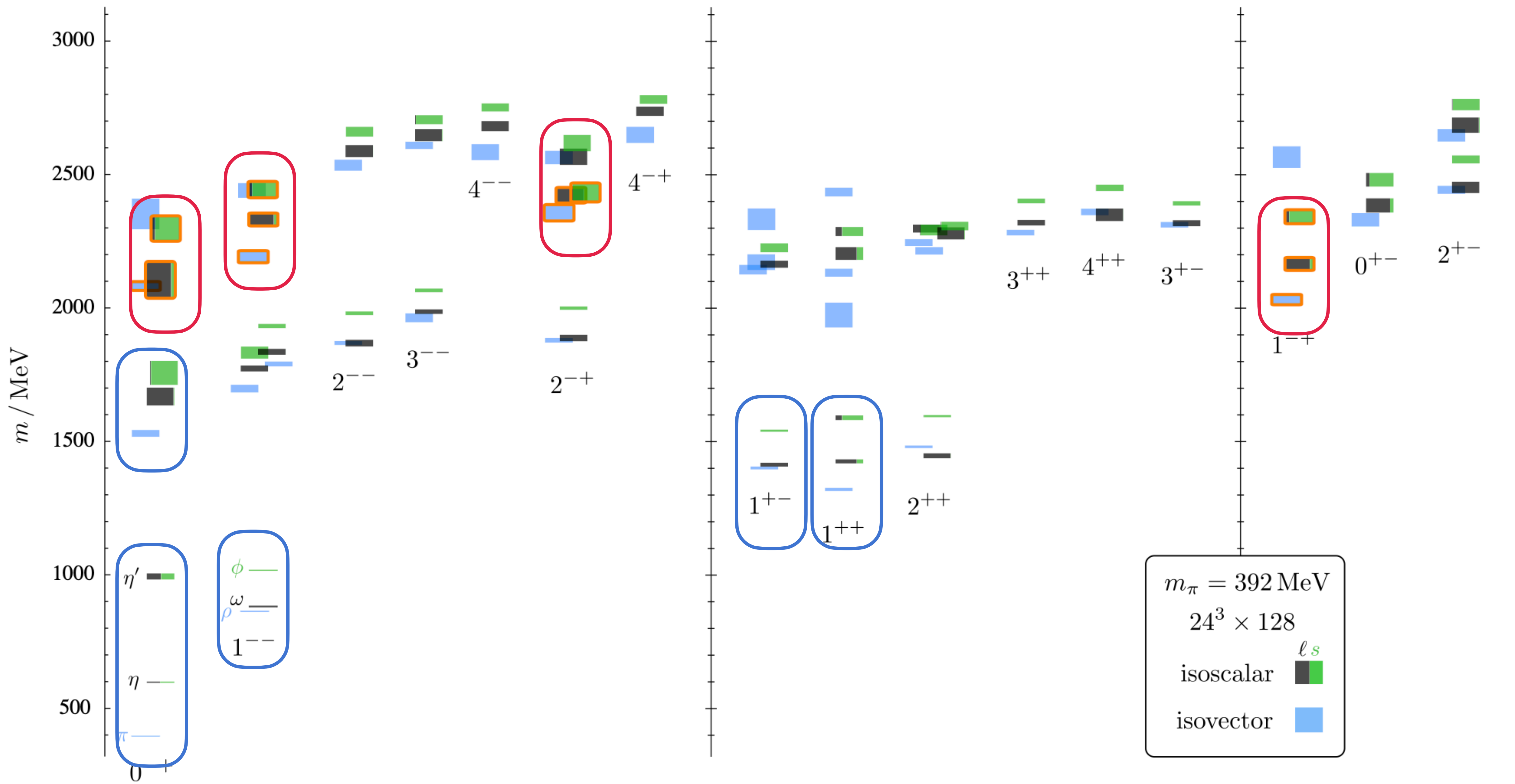
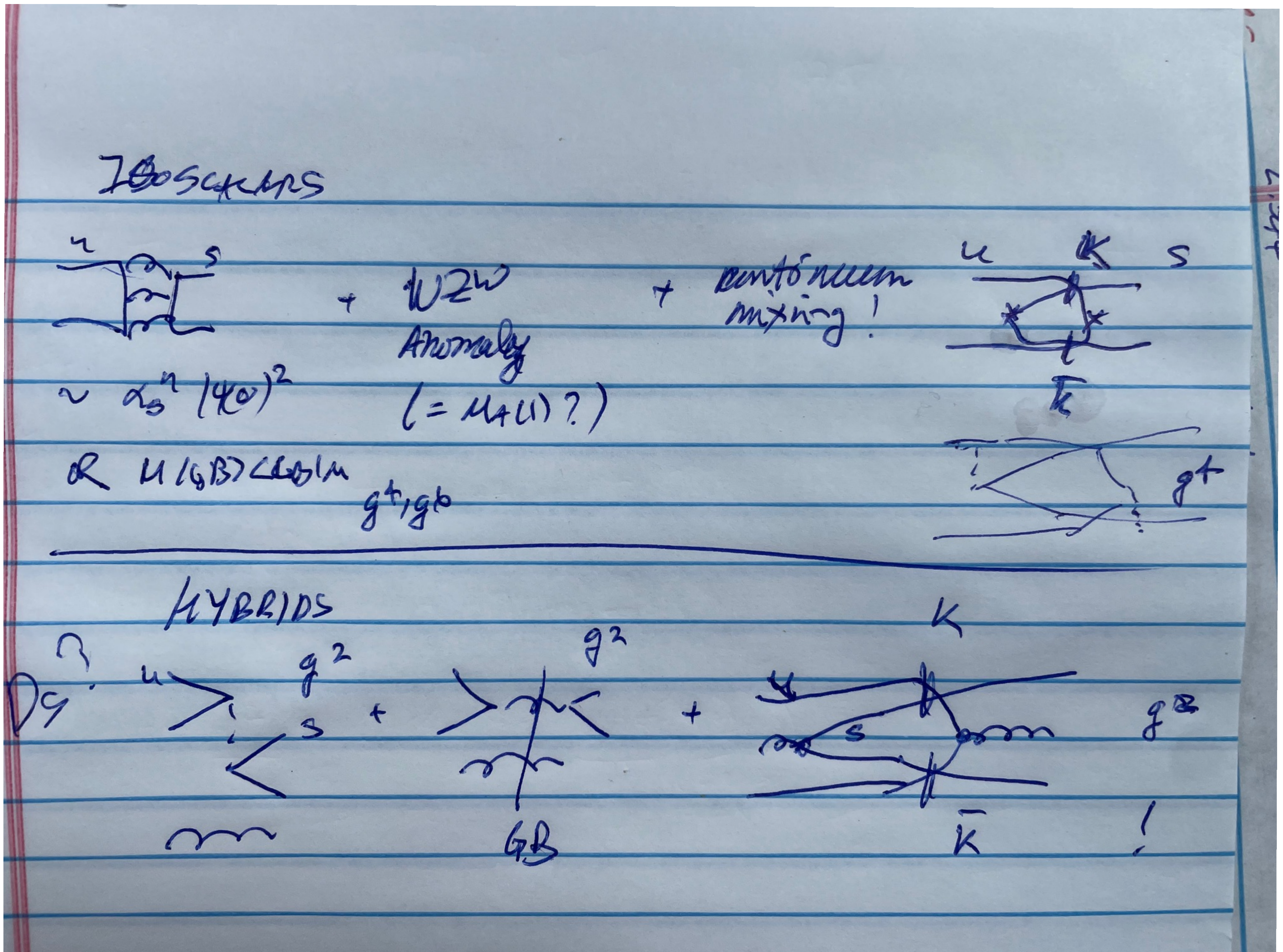


FIG. 11: Isoscalar (green/black) and isovector (blue) meson spectrum on the $m_\pi = 391$ MeV, $24^3 \times 128$ lattice. The vertical height of each box indicates the statistical uncertainty on the mass determination. States outlined in orange are the lowest-lying states having dominant overlap with operators featuring a chromomagnetic construction – their interpretation as the lightest hybrid meson supermultiplet will be discussed later.

Hybrid Flavour Mixing is Different!



We need a specific model for hybrids and glueballs, interactions, etc

The form of the gluonic structure present in the operators having good overlap with these states is chromomagnetic, having $J_g^{P_g C_g} = 1^{+-}$. With the $q\bar{q}$ pair in an internal S-wave this describes the observed J^{PC}. Heav-

The lightest hybrid meson supermultiplet in QCD
J.J. Dudek, Phys.Rev.D 84 (2011) 074023

Agrees with old bag models and other modelling

Heavy hybrids with constituent gluons,
E.S. Swanson & A.P. Szczepaniak, Phys.Rev.D 59 (1999) 014035

the Hamiltonian

$$H_{QCD} = \int d^3x \left[\psi^\dagger \left(-i\alpha \cdot \nabla + \beta m\right)\psi + \frac{1}{2}\left(\mathcal{J}^{-1/2}\Pi\mathcal{J} \cdot \Pi\mathcal{J}^{-1/2} + B \cdot B\right) - g\psi^\dagger\alpha \cdot A\psi\right] + H_C$$

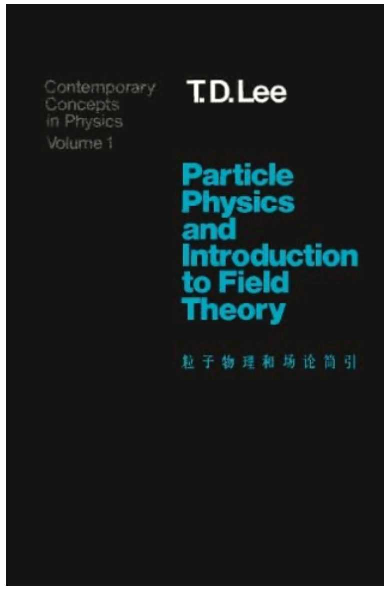
$$H_C = \frac{1}{2}\int d^3x\, d^3y\, \mathcal{J}^{-1/2}\rho^A(\mathbf{x}x)\mathcal{J}^{1/2}\hat{K}_{AB}(\mathbf{x},\mathbf{y};\mathbf{A})\mathcal{J}^{1/2}\rho^B(\mathbf{y})\mathcal{J}^{-1/2}$$

$$\mathcal{J} \equiv \det(\nabla \cdot D)$$

$$\rho^A(\mathbf{x}) = f^{ABC}\mathbf{A}^B(\mathbf{x}) \cdot \mathbf{\Pi}^C(\mathbf{x}) + \psi^\dagger(\mathbf{x})T^A\psi(\mathbf{x})$$

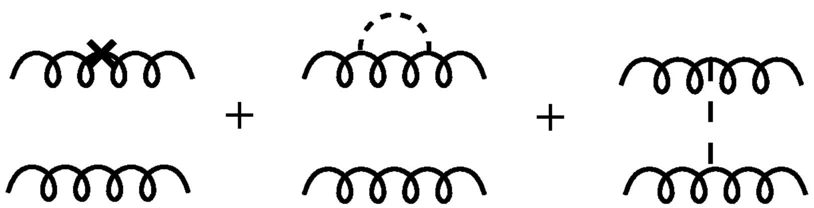
$$\hat{K}^{AB}(\mathbf{x},\mathbf{y};\mathbf{A}) \equiv \langle \mathbf{x}, A\,|\frac{g}{\nabla \cdot \mathbf{D}}(-\nabla^2)\frac{g}{\nabla \cdot \mathbf{D}}|\,\mathbf{y}, B\rangle\,.$$

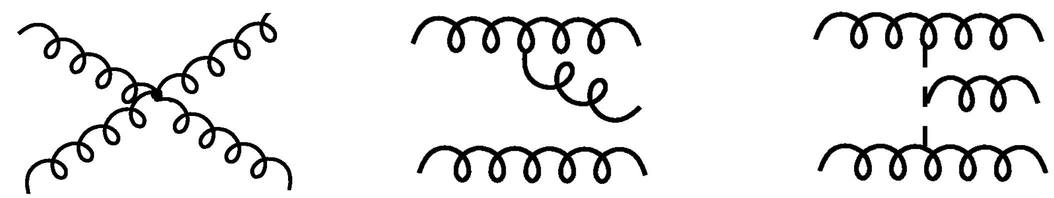
$$D^{AB} \equiv \delta^{AB}\nabla - gf^{ABC}A^C$$



Glueballs

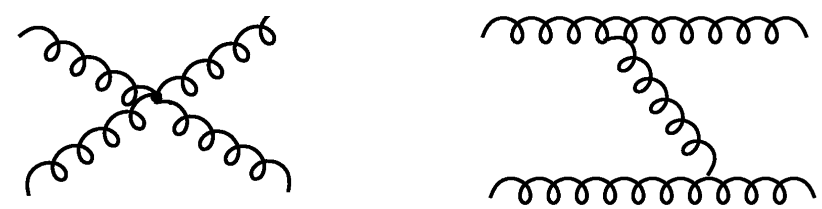
Glueballs





V_{4g} $\quad$ H_{ggg} $\quad$ $K^{(1)}$

$c(\Lambda)$



$\Delta E < \Lambda$ $\quad$ $\Delta E > \Lambda$

$$|JM;\lambda,\lambda'\rangle = \frac{1}{\sqrt{2(N_c^2-1)}}\sqrt{\frac{2J+1}{4\pi}}\int \frac{d^3k}{(2\pi)^3}\,\psi(k)\,D^{J*}_{M,\lambda-\lambda'}(\phi,\theta,-\phi)\,\Pi\, a^\dagger(k,\lambda,A)a^\dagger(-k,\lambda,A)\,|0\rangle$$

$$|JM;\eta\rangle = \frac{1}{\sqrt{2}}\left(|JM;\lambda,\lambda'\rangle + \eta\,|JM;-\lambda,-\lambda'\rangle\right)$$

Glueballs

$$E\int \frac{k^2dk}{(2\pi)^3}\,|\psi_i(k)|^2 = \int \frac{k^2dk}{(2\pi)^3} 2\omega(k)\,|\psi_i(k)|^2 + \frac{N_C}{2}\sum_i \int \frac{k^2dk}{(2\pi)^3}\frac{q^2dq}{(2\pi)^3}\frac{\omega(k)}{\omega(q)}\left[\frac{4}{3}V_0 + \frac{2}{3}V_2\right]|\psi_i(k)|^2$$

$$-\frac{N_C}{4}\int \frac{k^2dk}{(2\pi)^3}\frac{q^2dq}{(2\pi)^3}\frac{\left(\omega(k)+\omega(q)\right)^2}{\omega(k)\omega(q)}\psi_i^*(q)K_{ij}(q,k)\psi_j(k)$$

$J^P = (\text{odd} \geqslant 3)^+$ (there is no 1^+ gg glueball):

$$K = \frac{J+2}{2J+1}V_{J-1} + \frac{J-1}{2J+1}V_{J+1};$$

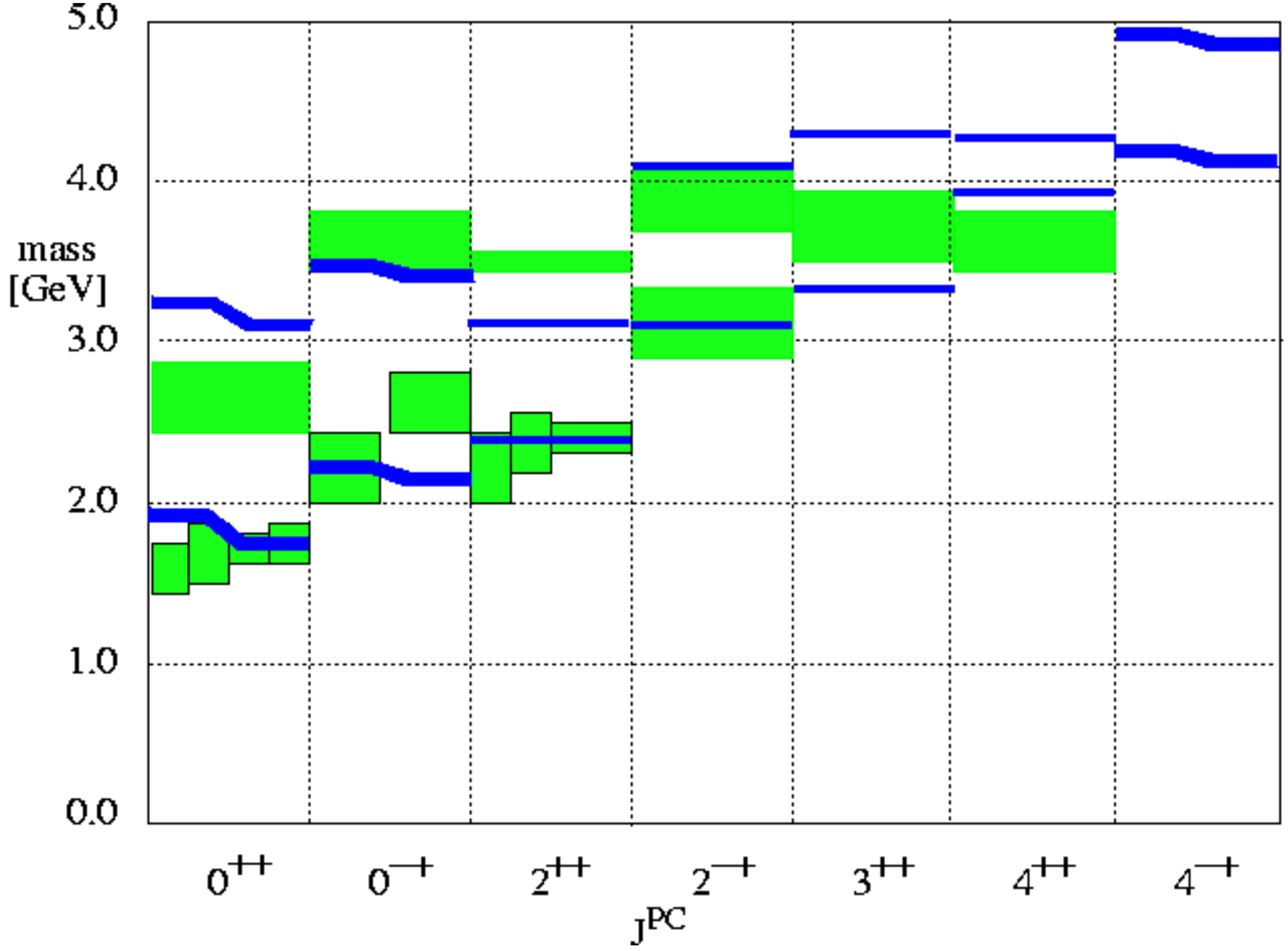
$J^P = (\text{even} \geqslant 0)^-$:

$$K = \frac{J}{2J+1}V_{J-1} + \frac{J+1}{2J+1}V_{J+1};$$

$J^P = 0^+$:

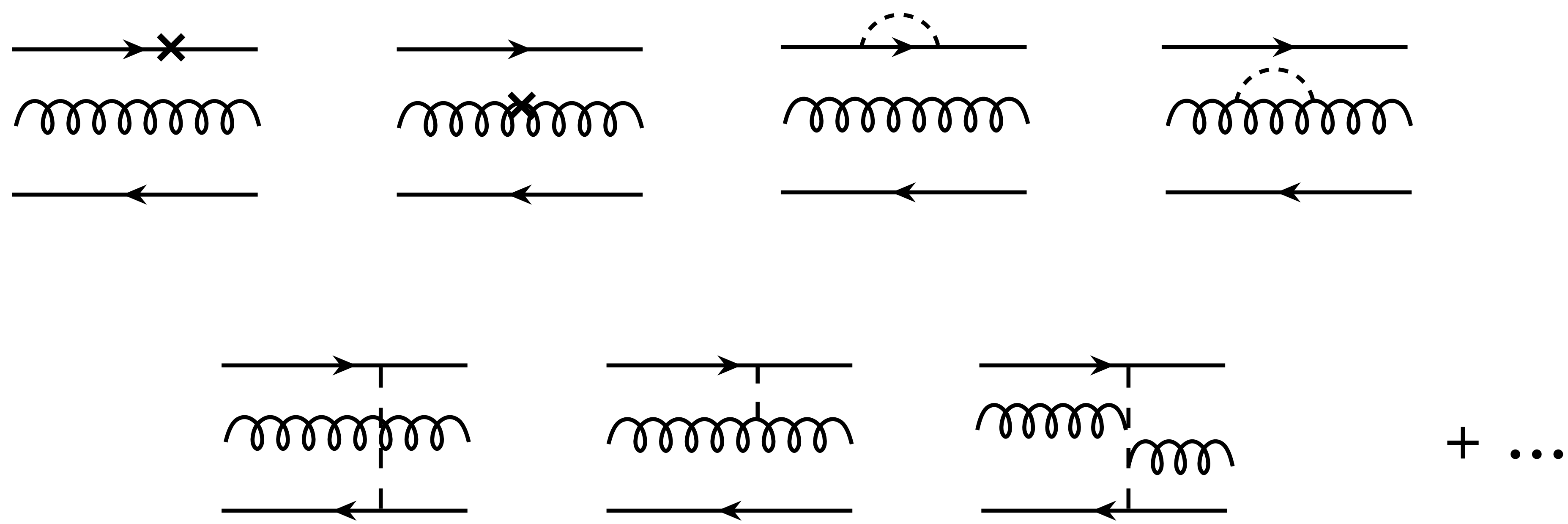
$$K = \frac{2}{3}\left(V_0 + \frac{V_2}{2}\right).$$

Glueballs



Hybrids

Hybrids



$$|JM[LS\ell j_g\xi]\rangle = \frac{1}{2}T^A_{ij}\int \frac{d^3q}{(2\pi)^3}\,\frac{d^3k}{(2\pi)^3}\,\Psi_{j_g;\ell m_\ell}(\mathbf{k},\mathbf{q})\,\sqrt{\frac{2j_g+1}{4\pi}}\,D^{j_g\,*}_{m_g\mu}(\hat{k})\,\chi^{(\xi)}_{\mu,\lambda}$$

$$\times\,\langle\frac{1}{2}m\frac{1}{2}\bar{m}|SM_S\rangle\,\langle\ell m_\ell, j_g m_g|LM_L\rangle\,\langle SM_S, LM_L|JM\rangle\,b^\dagger_{\mathbf{q}-\frac{\mathbf{k}}{2},i,m}\,d^\dagger_{-\mathbf{q}-\frac{\mathbf{k}}{2},j,\bar{m}}\,a^\dagger_{\mathbf{k},A,\lambda}|0\rangle.$$

Hybrids

TABLE II: J^{PC} Hybrid Multiplets.

multiplet	operator	ξ	j_g	ℓ	L	J^{PC} $S=0$ $(S=1)$
H_1	$\psi^\dagger \mathbf{B}\chi$	-1	1	0	1	1^{--}, $(0,1,2)^{-+}$
H_2	$\psi^\dagger \boldsymbol{\nabla}\times\mathbf{B}\chi$	-1	1	1	1	1^{++}, $(0,1,2)^{+-}$
H_3	$\psi^\dagger \boldsymbol{\nabla}\cdot\mathbf{B}\chi$	-1	1	1	0	0^{++}, (1^{+-})
H_4	$\psi^\dagger [\boldsymbol{\nabla}\mathbf{B}]_2\chi$	-1	1	1	2	2^{++}, $(1,2,3)^{+-}$

Hybrids

Hartree-Fock-type method

$$\Psi_{j_g;\ell m_\ell}(\mathbf{k},\mathbf{q}) = \chi_{j_g}(k)\,\varphi_\ell(q) Y_{\ell,m_\ell}(\hat{q})\,.$$

$$K_q\varphi + \int \chi^* K_g \chi \cdot \varphi + \int \chi^* V \chi \cdot \varphi = E\varphi$$

$$K_g\chi + \int \varphi^* K_q \varphi \cdot \chi + \int \varphi^* V \varphi \cdot \chi = E\chi$$

Hybrids

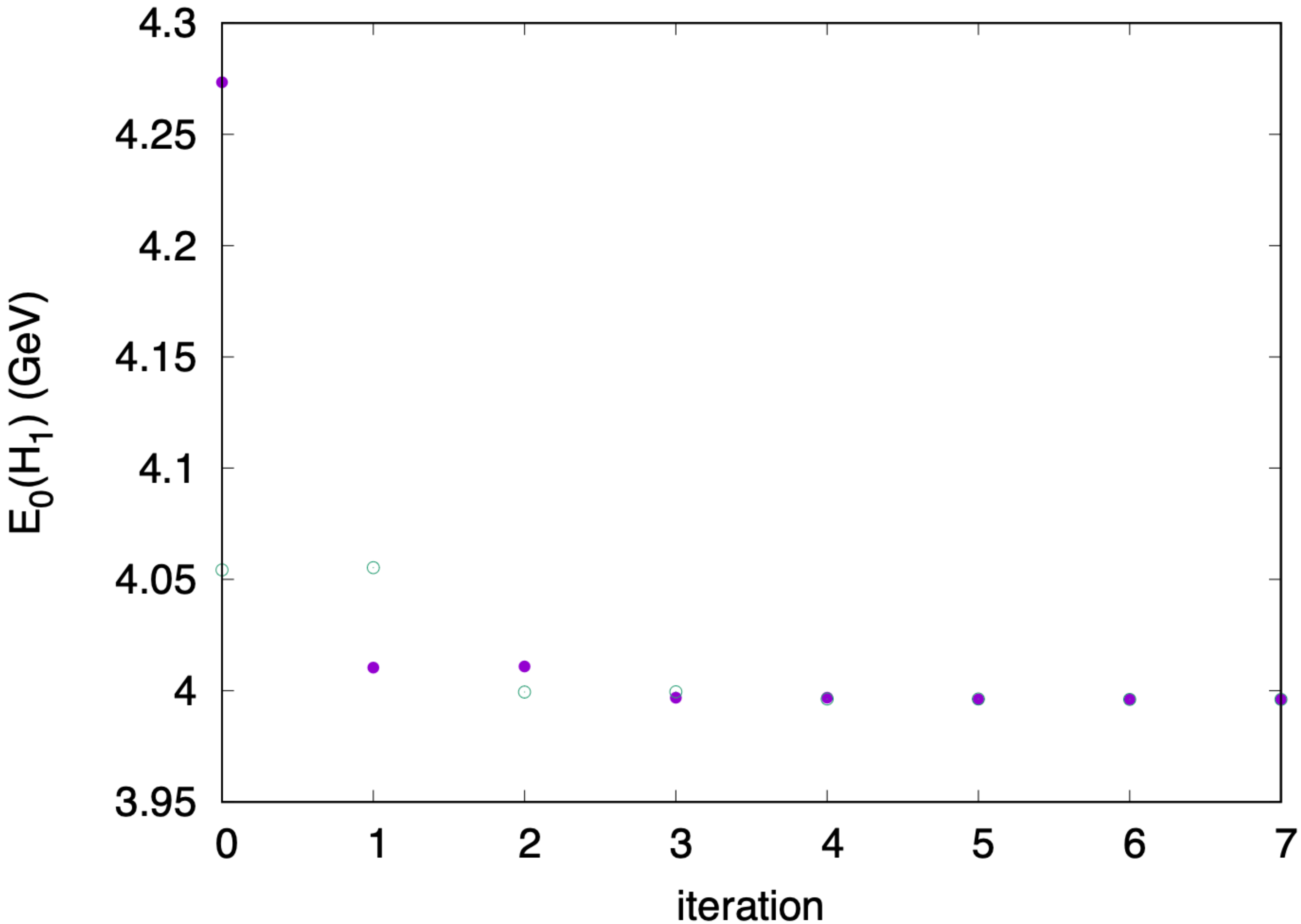
Hartree-Fock-type method

$$\left[2M + \frac{q^2}{M}\right] \phi_\ell(q) + \int \left(\frac{k^2}{4M} + \Omega(k)\right) |\psi_j(k)|^2 \cdot \phi_\ell(x) + \frac{1}{6} V(x)\phi_\ell(x)$$
$$-3 \int \left[V_0(y, x/2) + dV_2(y, x/2)\right] |\psi_j(y)|^2 \cdot \phi_\ell(x) = E\phi_\ell(x) \tag{63}$$

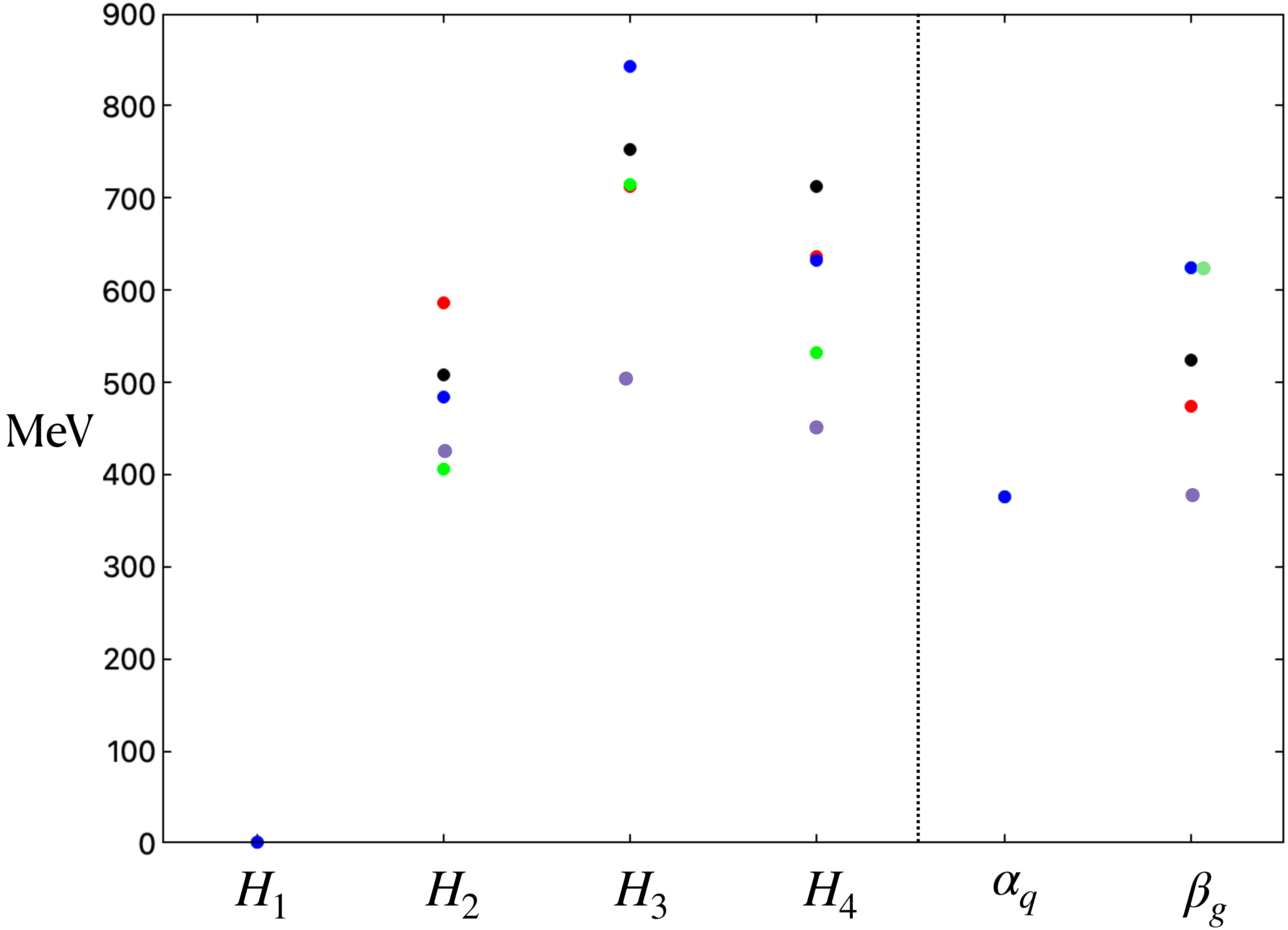
$$\left[\frac{k^2}{4M} + \Omega(k)\right] \psi_j(k) + \int \frac{q^2}{M} |\phi_\ell(q)|^2 \cdot \psi_j(x) + \frac{1}{6} \int V(y)|\phi_\ell(y)|^2 \cdot \psi_j(x)$$
$$-3 \int \left[V_0(x, y/2) + dV_2(x, y/2)\right] |\phi_\ell(y)|^2 \cdot \psi_j(x) = E\psi_j(x) \tag{64}$$

Hybrids

Hartree-Fock-type method

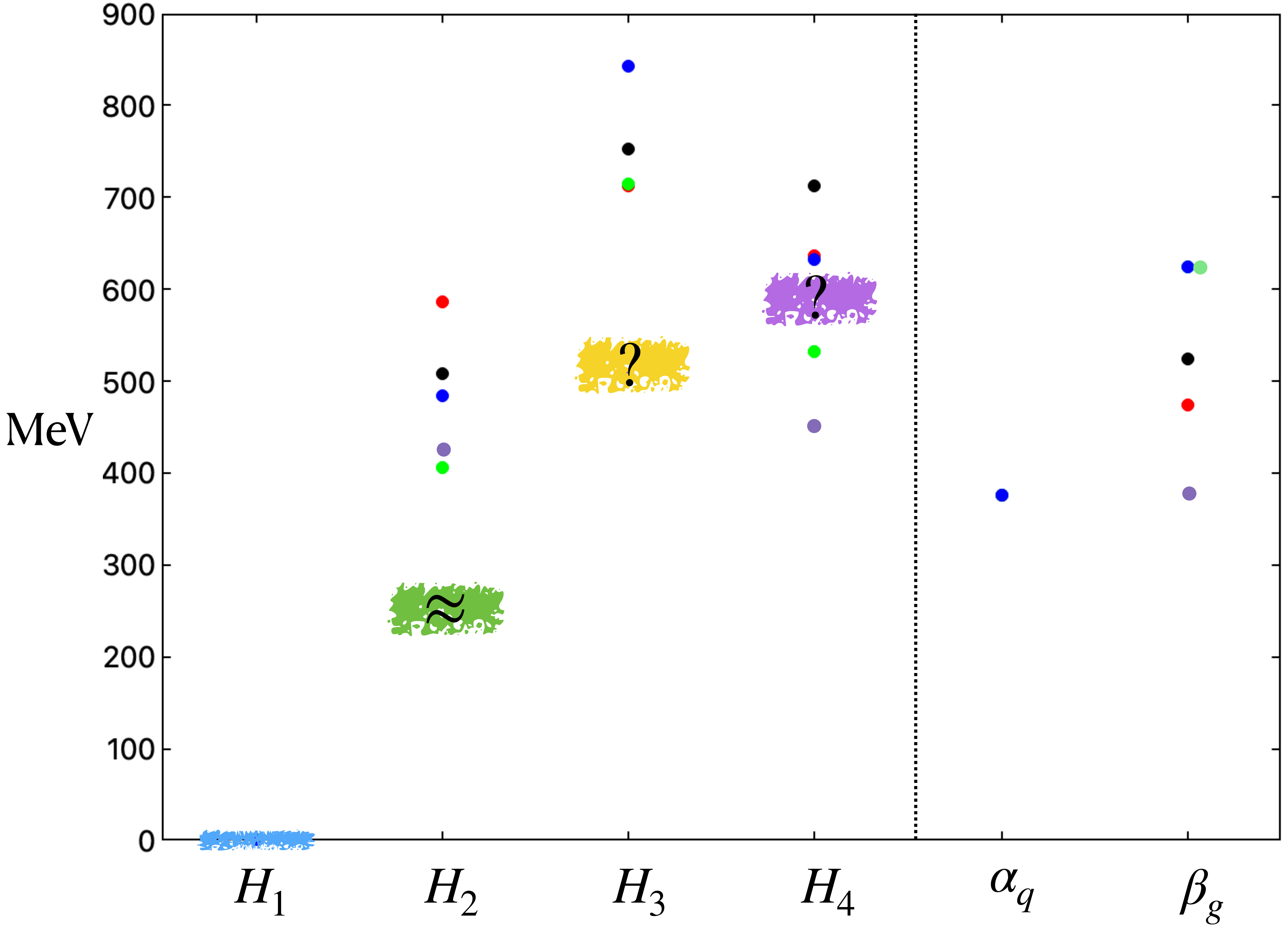


Hybrids



900
800
700
600
500
400
300
200
100
0
MeV
H_1
H_2
H_3
H_4
α_q
β_g

Hybrids



MeV
900
800
700
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500
400
300
200
100
0
≈
?
?
H_1
H_2
H_3
H_4
α_q
β_g

Hybrid Flavour Mixing

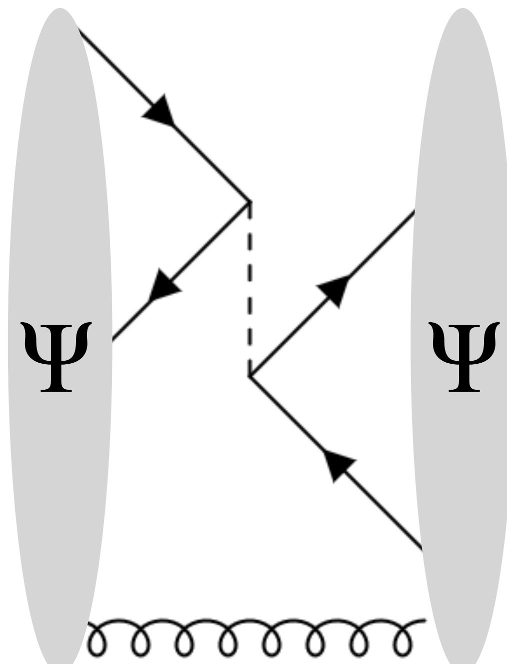
Mixing

$$\begin{array}{cc} & \begin{array}{cccc} |\,u\bar{u}\rangle & |\,d\bar{d}\rangle & |\,s\bar{s}\rangle & |\,gg\rangle \end{array} \\ H_{uds} = & \begin{pmatrix} m + A_{nn} & A_{nn} & A_{ns} & \mathscr{A}_n^{(0)} \\ A_{nn} & m + A_{nn} & A_{ns} & \mathscr{A}_n^{(0)} \\ A_{ns} & A_{ss} & m + \Delta m + A_{ss} & \mathscr{A}_s^{(0)} \\ \mathscr{A}_n^{(0)} & \mathscr{A}_n^{(0)} & \mathscr{A}_s^{(0)} & M_{gb} \end{pmatrix}. \end{array}$$

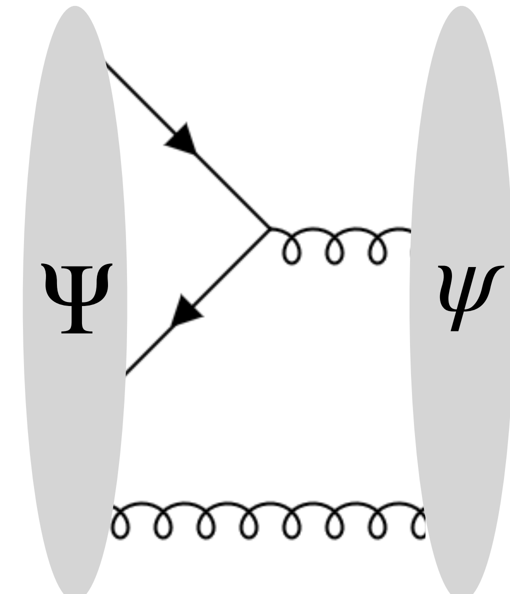
$$H_{iso} = \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & m + 2A_{nn} & \sqrt{2}A_{ns} & \sqrt{2}\mathscr{A}_n^{(0)} \\ 0 & \sqrt{2}A_{ns} & m + \Delta m + A_{ss} & \mathscr{A}_s^{(0)} \\ 0 & \sqrt{2}\mathscr{A}_n^{(0)} & \mathscr{A}_s^{(0)} & M_{gb} \end{pmatrix}.$$

truncate sums over glueballs

Mixing



$\ell = 0;\ S = 1;\ H_1$ only



$$A_{ff'} = \frac{1}{m_f m_{f'}} \int \frac{k^2 dk}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{d^3q'}{(2\pi)^3} \Psi_f(k, \mathbf{q}) \Psi^*_{f'}(k, \mathbf{q}') k^2 V(k)\, B_J$$

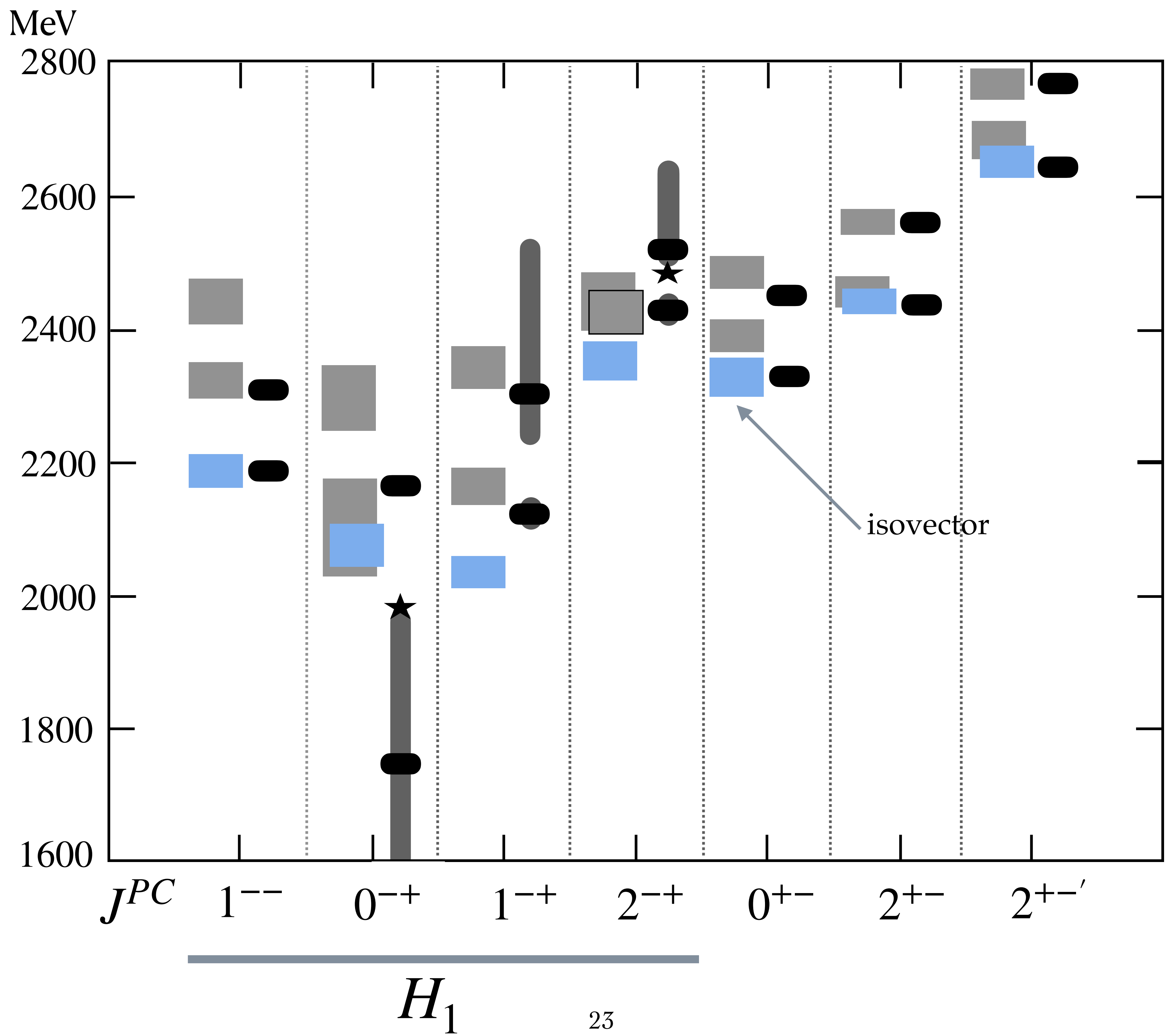
$$= \frac{F_f F_{f'}}{8 m_f m_{f'}} \int \frac{k^2 dk}{(2\pi)^3} \, |\chi_1(k)|^2 k^2 V(k)\, B_J\,.$$

$$\mathcal{A}_f^{(n)} = -\frac{i[gF_f]}{4} \int \frac{k^2 dk}{(2\pi)^3} \frac{\psi_n^*(k)\chi(k)}{\sqrt{\omega(k)}} C_J,$$

$$[gF_f] = \int \frac{d^3q}{(2\pi)^3} \sqrt{4\pi\alpha_V(q)}\, \phi_{\ell=0}(q)$$

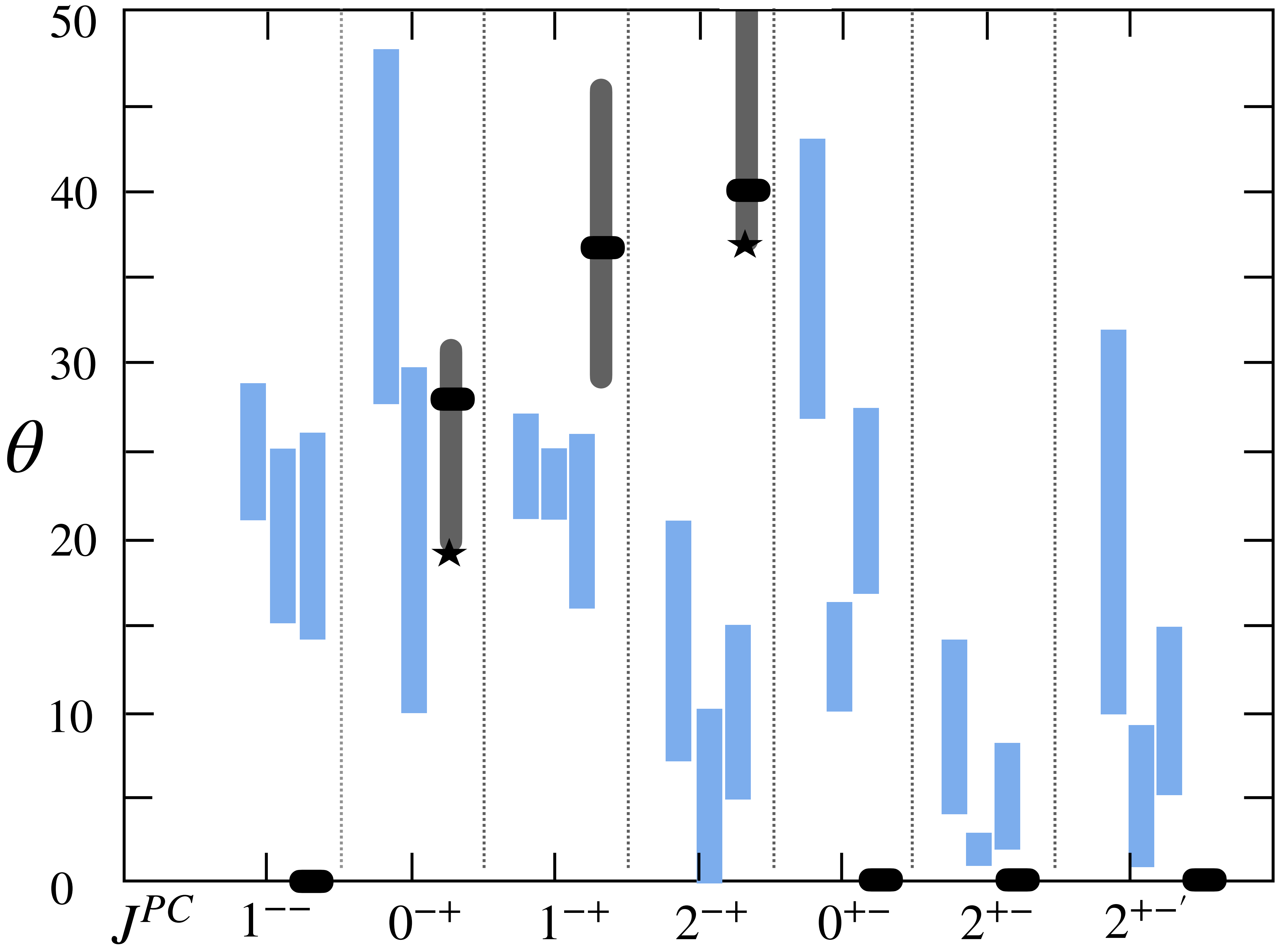
"octet decay constant, F"

Mixing



MeV
2800
2600
2400
2200
2000
1800
1600
isovector
J^{PC} 1^{--} 0^{-+} 1^{-+} 2^{-+} 0^{+-} 2^{+-} $2^{+-\prime}$
H_1

Mixing



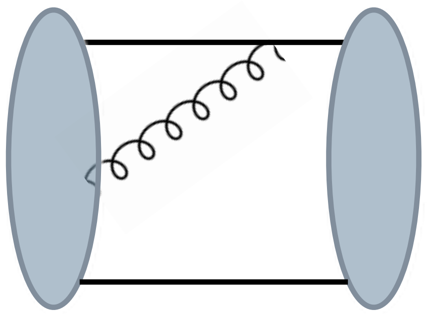
Mixing

J^{PC}	nominal state	$u\bar{u}g$	$s\bar{s}g$	gg
1^{--}	light	≈ 100	≈ 0	≈ 0
	heavy	≈ 0	≈ 100	≈ 0
	glueball	≈ 0	≈ 0	≈ 100
0^{-+}	light	62 [87]	17 [6]	21 [7]
	heavy	24 [8]	75 [91]	0.5 [1]
	glueball	13 [5]	8 [3]	78 [92]
1^{-+}	light	37	63	0
	heavy	63	37	0
	glueball	0	0	100
2^{-+}	light	54 [59]	46 [41]	0.1 [0]
	heavy	32 [38]	67 [61]	1 [1]
	glueball	2 [0.4]	1 [0.6]	97 [99]

LGT assumptions are validated

Vector-Hybrid Vector Mixing

Vector-Hybrid Vector Mixing



$$\mathscr{H} \equiv \langle q\bar{q}g \,|\, ig \int \psi^\dagger \alpha \cdot A \psi \,|\, q\bar{q} \rangle$$

$$\mathscr{H}_n = -i\frac{g}{m}\frac{2\sqrt{4\pi}}{3}\int \frac{d^3q}{(2\pi)^3}\frac{k^2dk}{(2\pi)^3}\frac{k}{\sqrt{\omega(k)}}\Psi^*(k,q)\psi_n(q+k/2)\,.$$

$$\mathscr{H}_{1S} = -ig\begin{cases} 84\ \mathrm{MeV}^2/m_q, & \rho \\ 190\ \mathrm{MeV}^2/m_c, & J/\psi \\ 225\ \mathrm{MeV}^2/m_b, & \Upsilon \end{cases} \approx -i\begin{cases} 210\ \mathrm{MeV}, & \rho \\ 60\ \ \mathrm{MeV}, & J/\psi\,. \\ 20\ \ \mathrm{MeV}, & \Upsilon \end{cases}$$

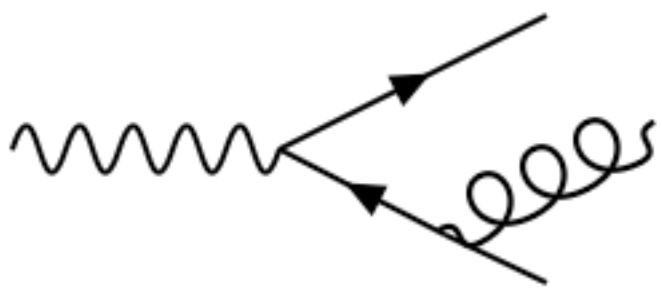
Hybrid configuration content of heavy S-wave mesons,
[MILC] T. Burch & D. Toussaint, Phys. Rev. 68, 094504 (2003).

$$\mathscr{H}_{NRQCD} \approx 170\ \mathrm{MeV}\ (J/\psi)$$

$$\mathscr{H}_{NRQCD} \approx 70\ \mathrm{MeV}\ \ (\Upsilon)\,.$$

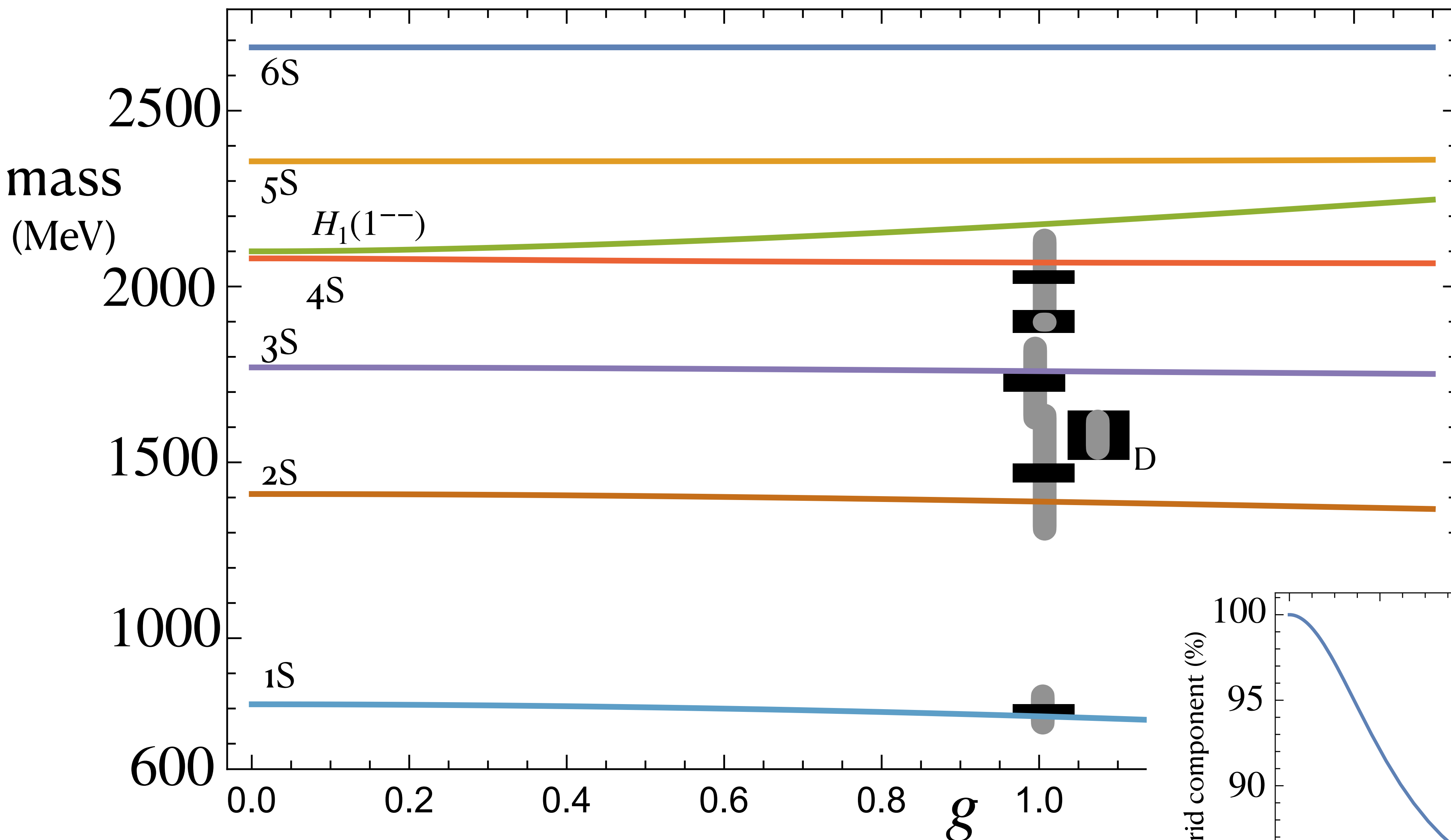
Vector Hybrid Production/Decay Constant

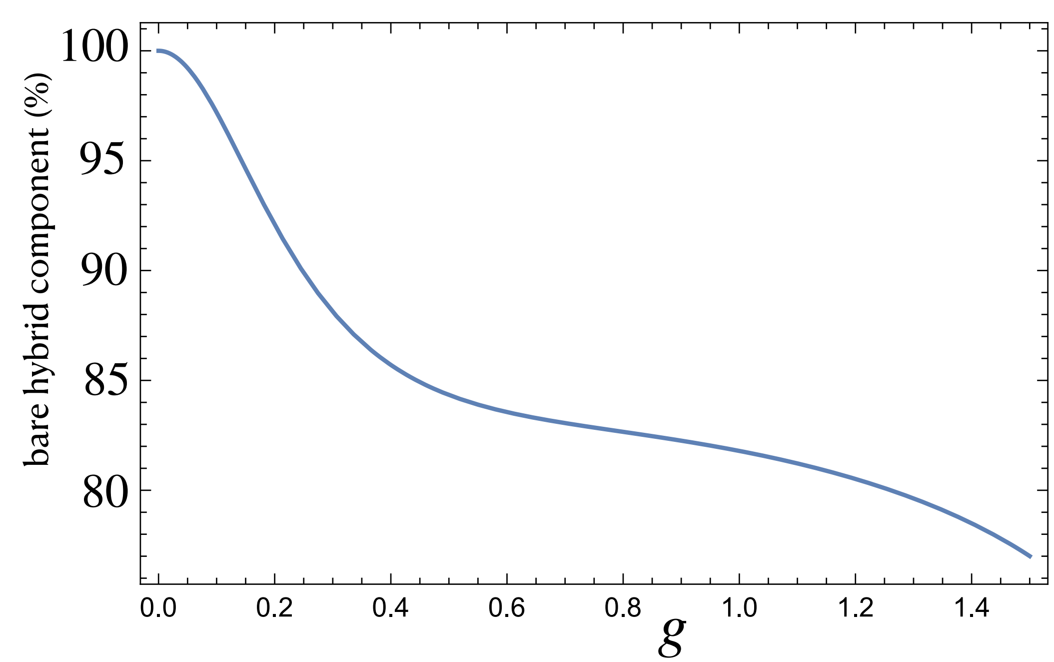
Vector Hybrid Production/Decay Constant



$$\delta H = \begin{pmatrix} m_1 & & & & & & \mathscr{H}_{1S} \\ & m_2 & & & & & \mathscr{H}_{2S} \\ & & m_3 & & & & \mathscr{H}_{3S} \\ & & & m_4 & & & \mathscr{H}_{4S} \\ & & & & m_5 & & \mathscr{H}_{5S} \\ & & & & & m_6 & \mathscr{H}_{6S} \\ \mathscr{H}_{1S} & \mathscr{H}_{2S} & \mathscr{H}_{3S} & \mathscr{H}_{4S} & \mathscr{H}_{5S} & \mathscr{H}_{6S} & M_H \end{pmatrix}.$$

Vector Hybrid Production/Decay Constant





Vector Hybrid Production/Decay Constant

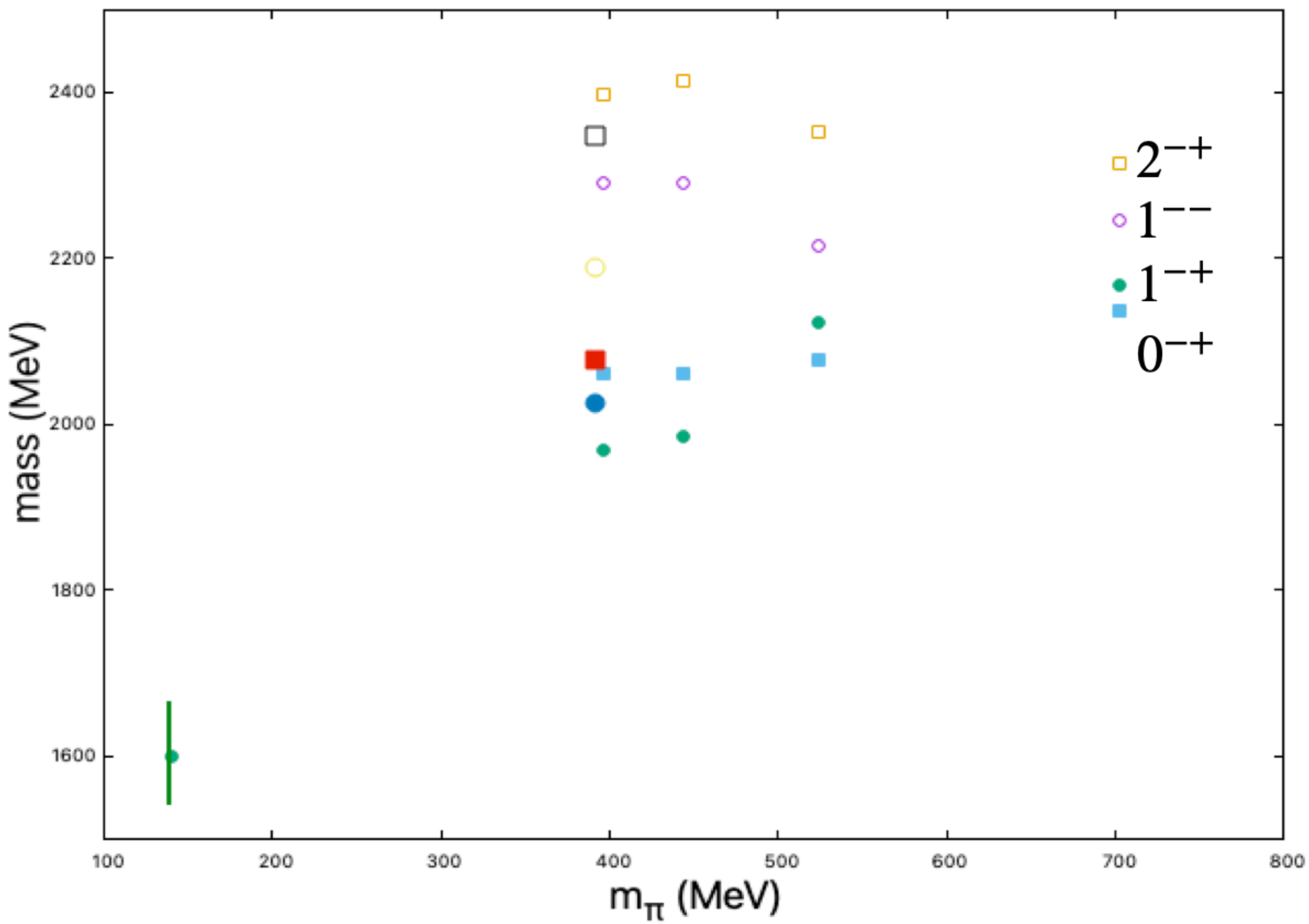
$$f_H = \frac{1}{\sqrt{M_H}} \sum_{n \neq H} \sqrt{M_n} f_V^{(n)} C_n \qquad\qquad C_n = \langle nS | H_1(1^{--}) \rangle$$

$$f_V^{(n)} = \sqrt{\frac{3}{M_n}} \int \frac{d^3k}{(2\pi)^3} \psi^{(n)}(\vec{k}) \sqrt{1 + \frac{m_q}{E_k}} \sqrt{1 + \frac{m_{\bar{q}}}{E_{\bar{k}}}} \left(1 + \frac{k^2}{3(E_k + m_q)(E_{\bar{k}} + m_{\bar{q}})} \right).$$

$$f_{H_1(1^{--})} \approx 20 \text{ MeV}.$$

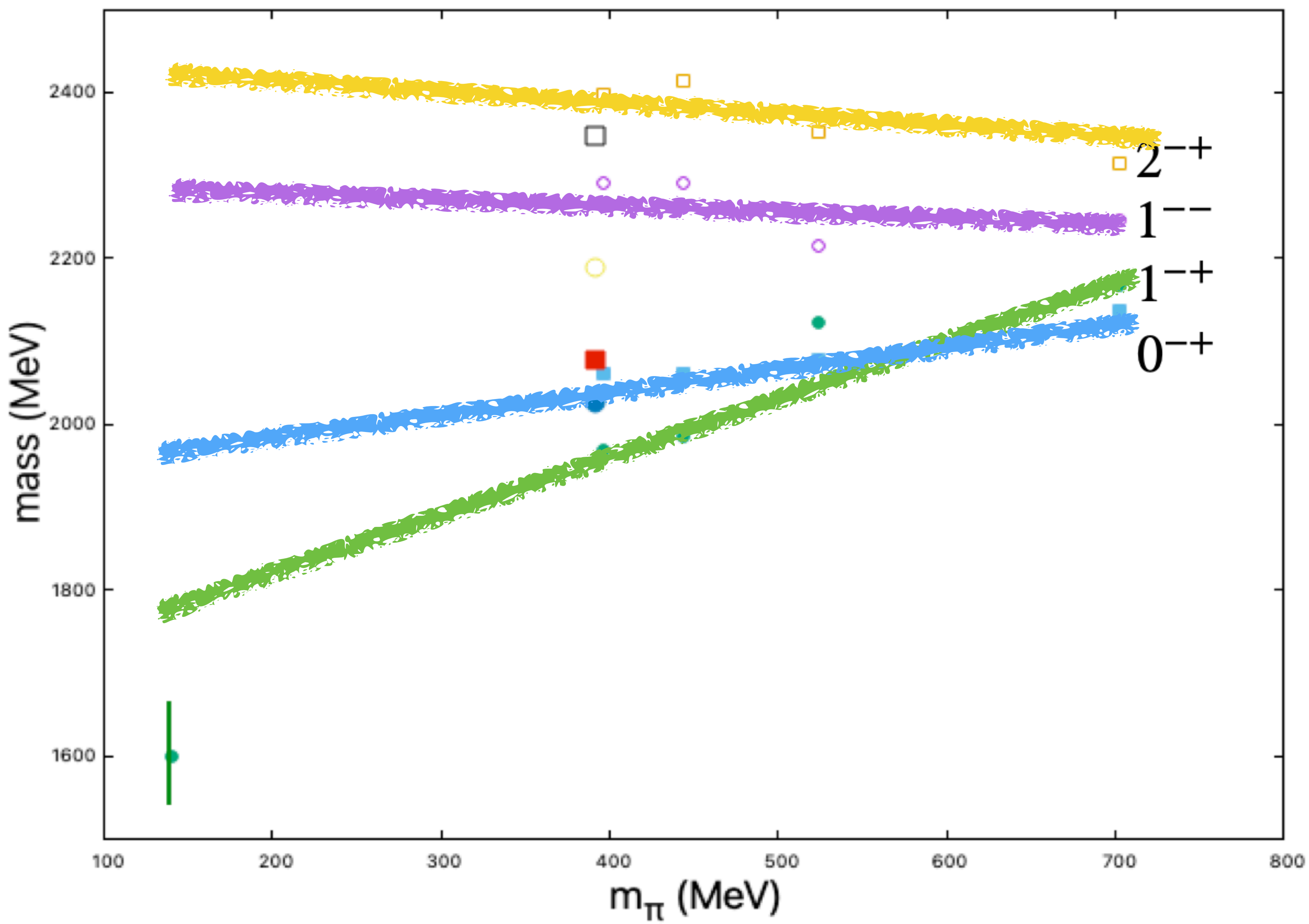
Phenomenology

lattice hybrid masses vs quark mass



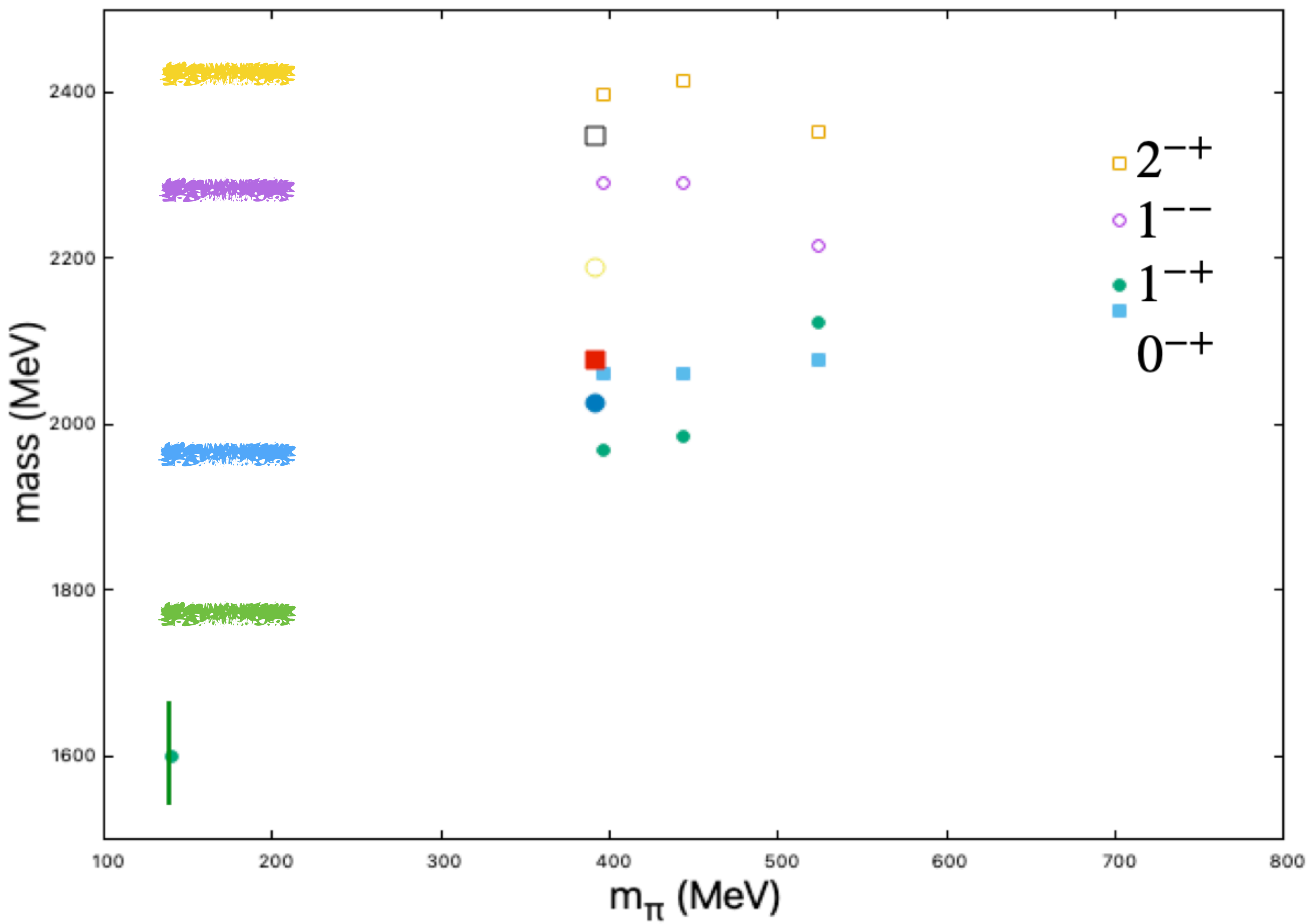
Phenomenology

lattice hybrid masses vs quark mass



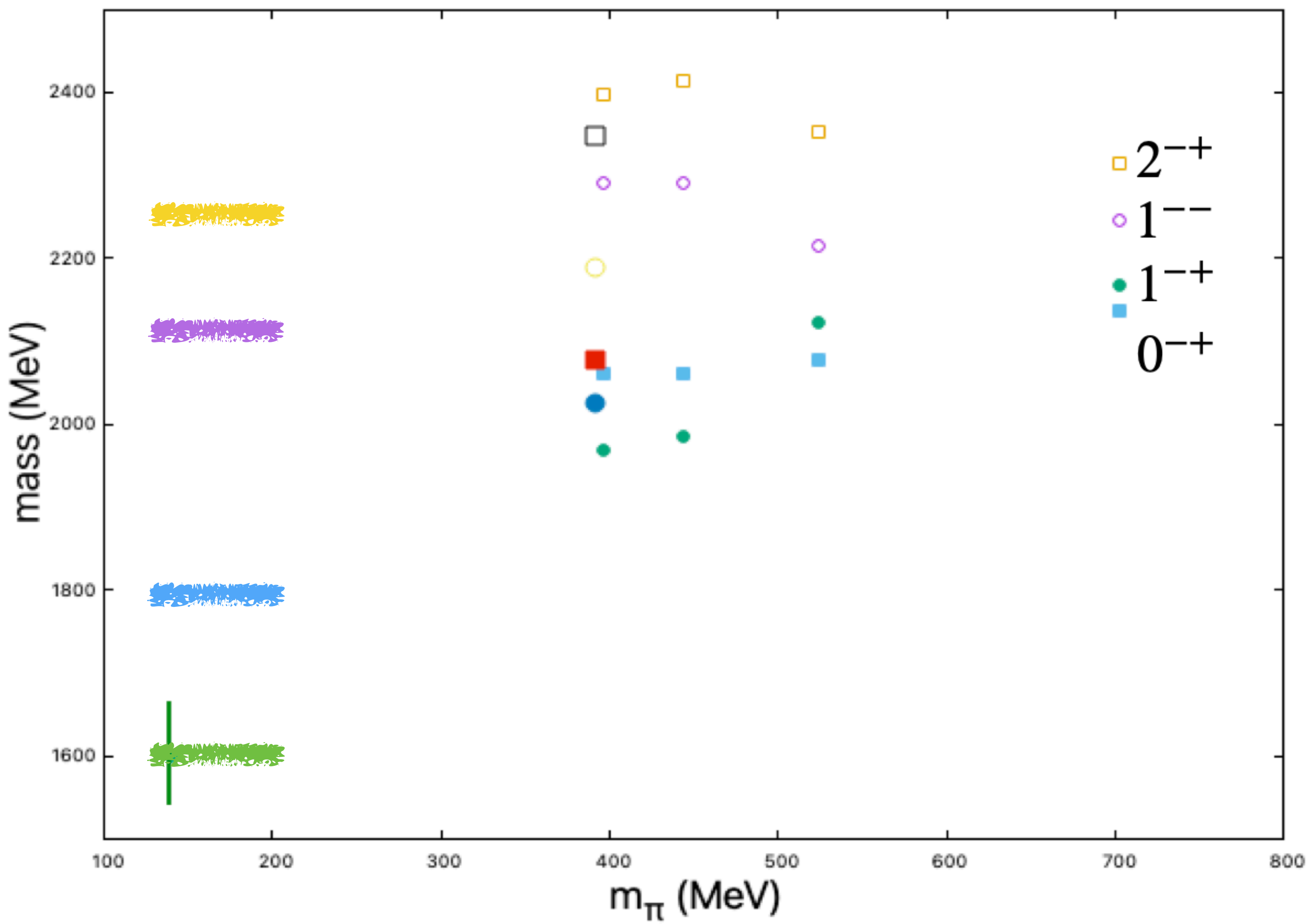
Phenomenology

lattice hybrid masses vs quark mass



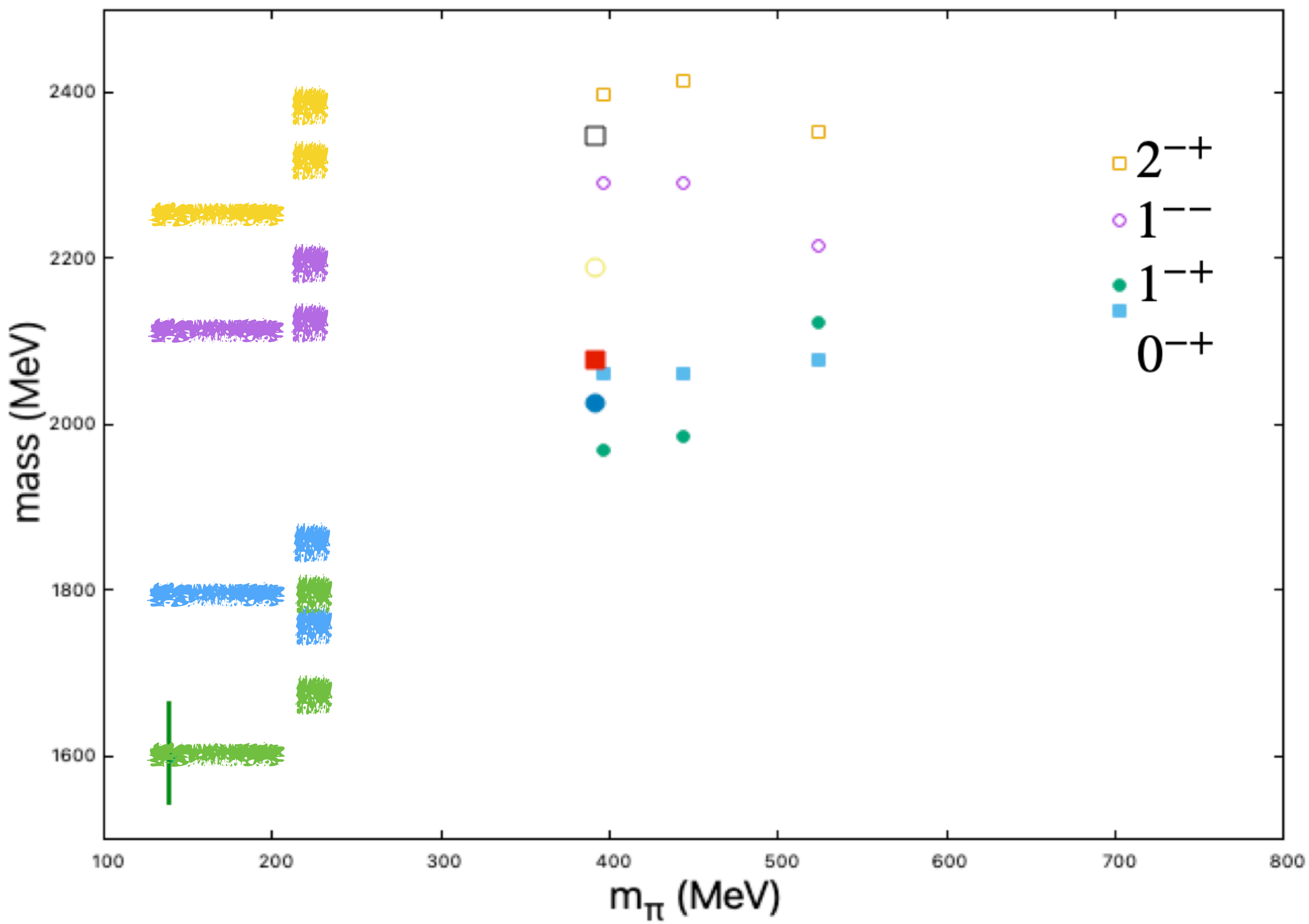
Phenomenology

lattice hybrid masses vs quark mass



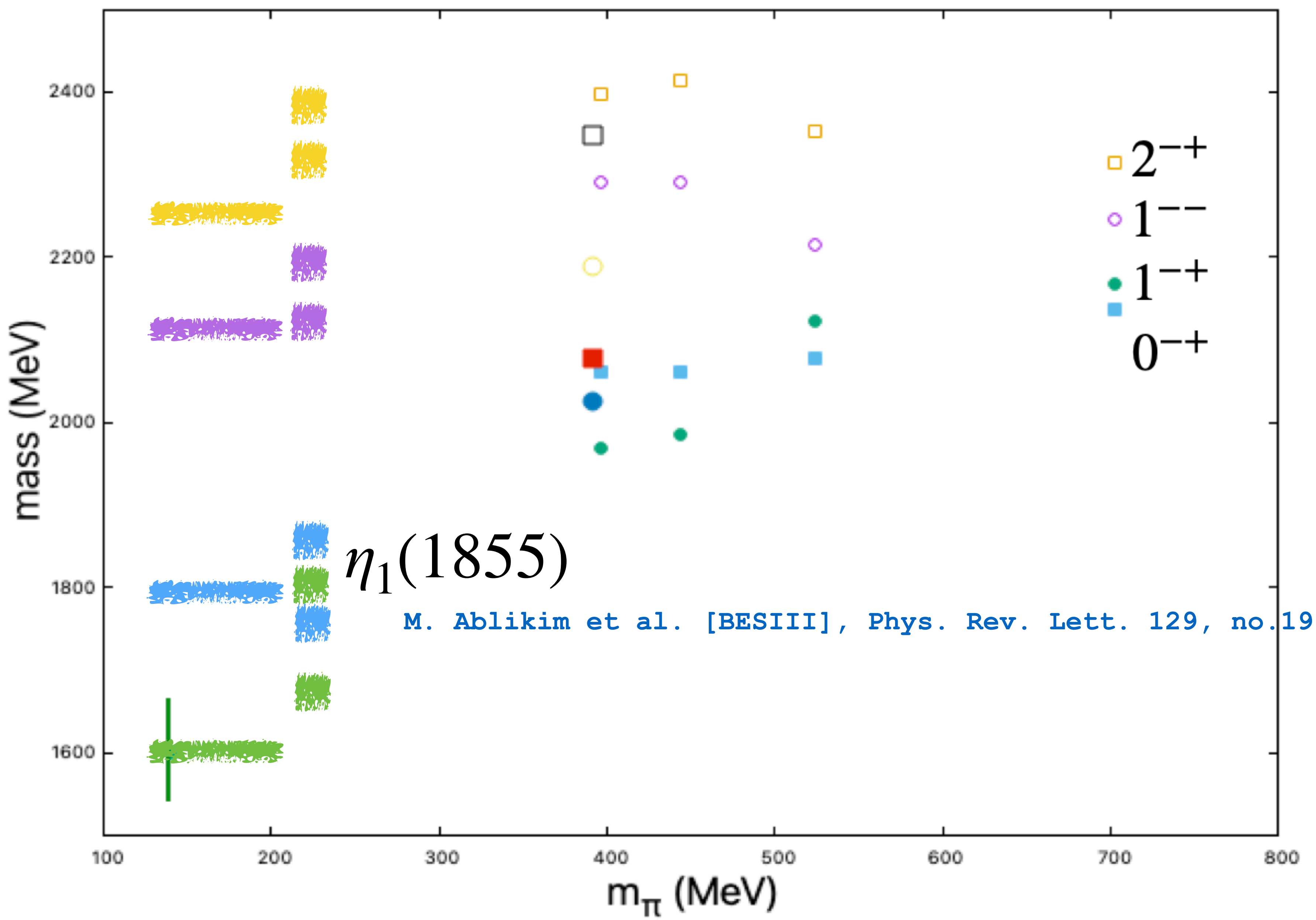
Phenomenology

lattice hybrid masses vs quark mass



Phenomenology

lattice hybrid masses vs quark mass



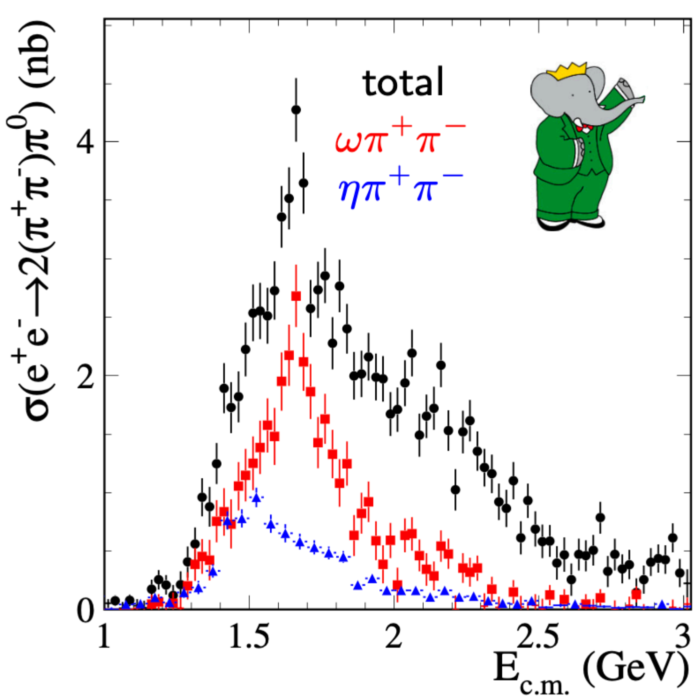
M. Ablikim et al. [BESIII], Phys. Rev. Lett. 129, no.19, 192002 (2022)

Phenomenology

TABLE IV: Model Assignments and Experimental Isovector Vector States (MeV).

state	Ref.	mass	width	model (iv)	mass	model (v)	mass
$\rho(770)$	PDG	775.2 ± 0.2	147.4 ± 0.8	1^3S_1	720	1^3S_1	810
$\rho(1450)$	PDG	1465 ± 25	400 ± 60	2^3S_1	1440	2^3S_1	1405
$\rho(1570)$	PDG	1570 ± 70	144 ± 90	1^3D_1	1510	1^3D_1	1497
$\rho(1700)$	PDG	1720 ± 20	250 ± 100	$H_1(1^{--})$	1760	3^3S_1	1770
						2^3D_1	1835
				3^3S_1	1850		
$\rho(1900)$	[32]	1900 ± 30	50 ± 30	2^3D_1	1910	4^3S_1	2080
$\rho(2150)$	[33]	2034 ± 16	234 ± 39	4^3S_1	2170	$H_1(1^{--})$	2100
				3^3D_1	2220	3^3D_1	2130

$e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^0$



Phys.Rev. D76 (2007) 092005 [3]

Conclusions

Conclusions

-a reasonable approximation to lattice glueball and hybrid spectra is obtained (clearly room for improvement)

-hybrid flavour mixing is roughly reproduced (except for 1--)

-hybrid-vector mixing is roughly reproduced

-the constituent picture looks to be a good starting point for detailed modelling; it has the benefit of being a well-constrained, unified description of hadrons and their interactions

Conclusions

- vector hybrid near 2100 w/ decay constant ~ 20 MeV

- partner states at 2100-2250 and 2220-2350.

- with a $\pi_1(1600)$ expect partner states at 1750-1780 and ~1900 (near the $\eta_1(1855)$).

~thank you~