Recent results on hadron structure from lattice QCD

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Extended Twisted Mass Collaboration

STIMULATE European Joint Doctorates

AQTIVATE European Joint Doctorate

HADRON 2023

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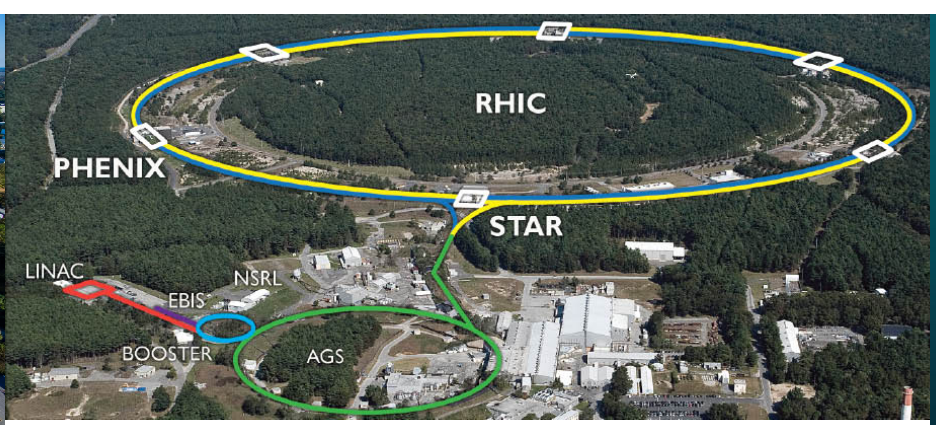
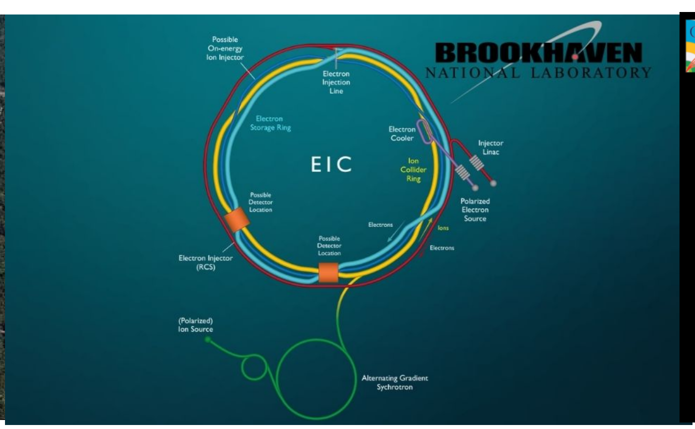
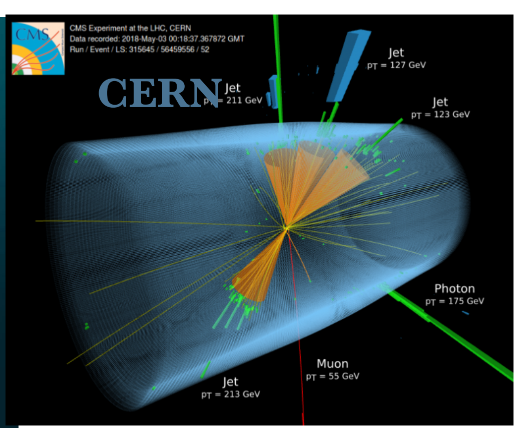
Outline

- **Introduction**
- **3D structure of the nucleon**
  - **Mellin moments —> charges, form factors and spin**
  - **Direct computation of GPDs**
- **Conclusions**

Relevant for interpreting and providing input for on-going and future experiments



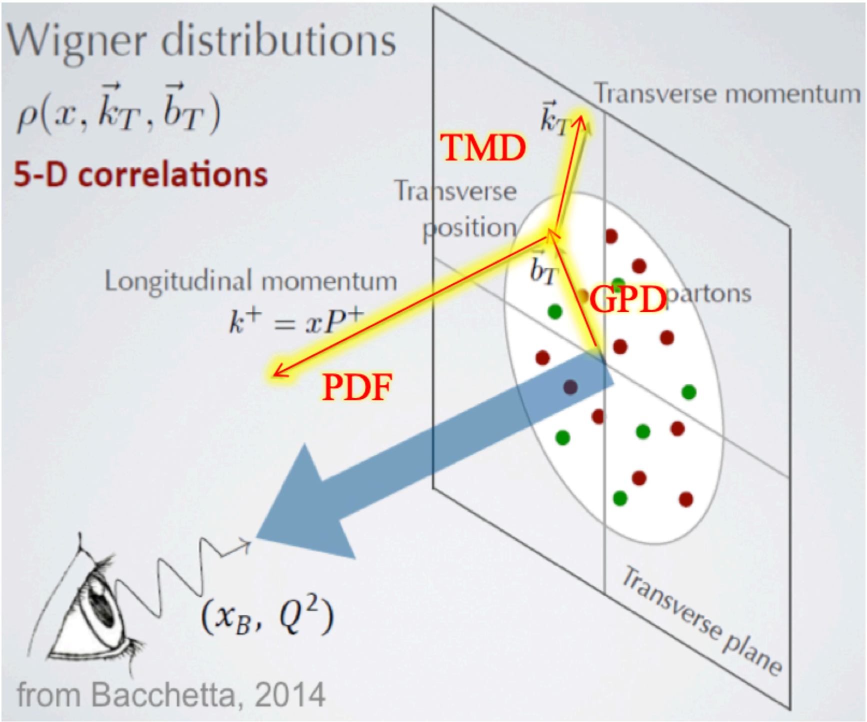




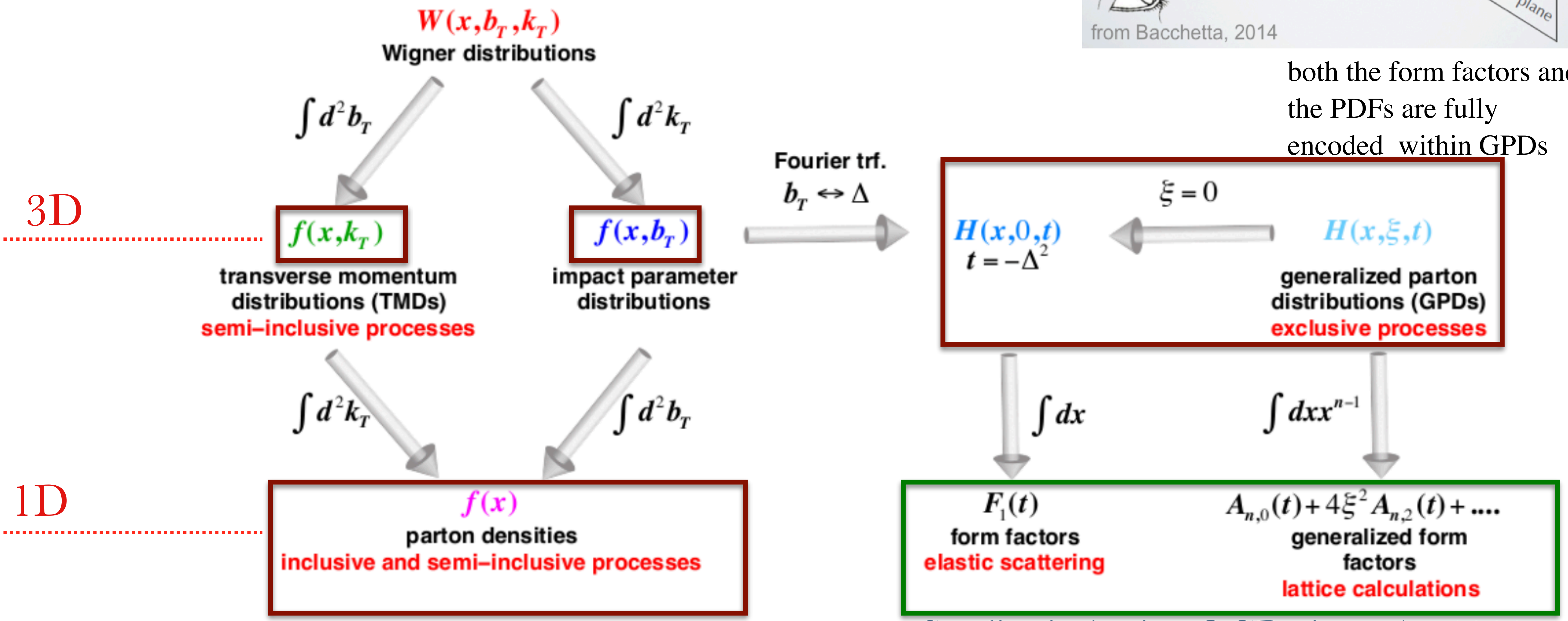
3D structure of the nucleon

- Understanding the 3D-structure of the nucleon from its fundamental constituents, the quarks and the gluons, is major goal of nuclear physics and a key aim of on-going experiments and the future EIC

- Lattice QCD can contribute towards this goal - many recent developments to compute Mellin moments but also directly parton distributions



both the form factors and the PDFs are fully encoded within GPDs



Studies in lattice QCD since the 1980s

EIC white paper, arXiv:1212.1701

Computation of Mellin moments of GPDs

- Light-cone matrix elements cannot be computed using a Euclidean lattice formulation of QCD
- Expansion of light-cone operator leads to a tower of local twist-2 operators —> connected to moments that can be computed in lattice QCD

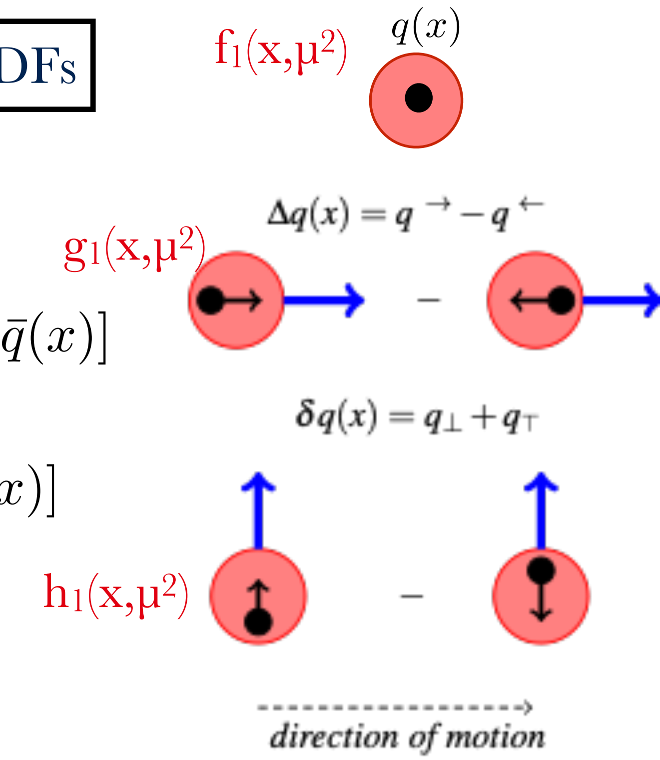
Forward matrix elements give moments of PDFs

$$\mathcal{O}^{\mu_1\dots\mu_n} = \bar{\psi}\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi \quad \overset{unpolarized}{\rightarrow} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n \left[q(x) - (-1)^n \bar{q}(x)\right]$$

$$\tilde{\mathcal{O}}^{\mu_1\dots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi \quad \overset{helicity}{\rightarrow} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n \left[\Delta q(x) + (-1)^n \Delta\bar{q}(x)\right]$$

$$\mathcal{O}_T^{\rho\mu_1\dots\mu_n} = \bar{\psi}\sigma^{\rho\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi \quad \overset{transversity}{\rightarrow} \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx\, x^n \left[\delta q(x) - (-1)^n \delta\bar{q}(x)\right]$$

$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$$



Twist-2 PDFs

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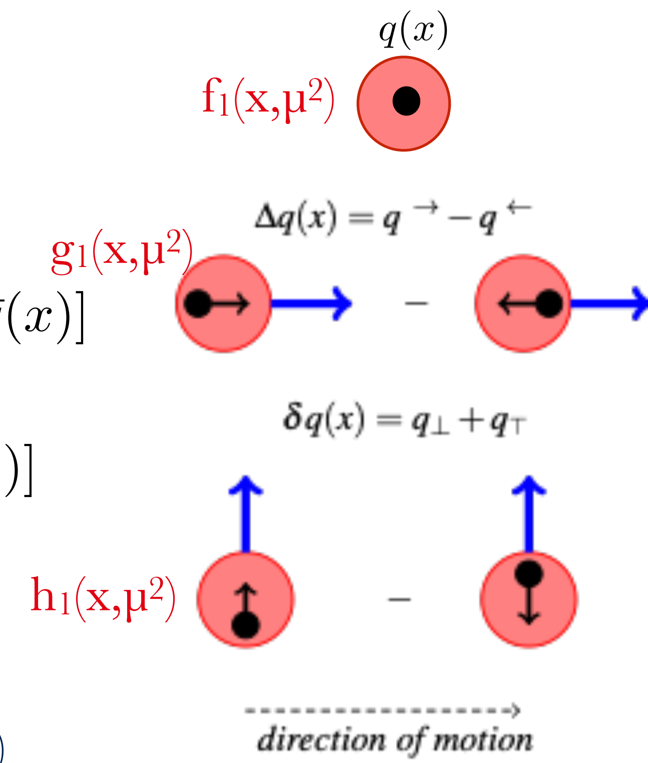

Twist-2 PDFs

- Off-diagonal matrix elements yield moments of GPDs or the generalised form factors (GFFs)

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1}\left[(2\xi)^i A_{ni}(\tau) + \text{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

$$\int_{-1}^{1} dx\, x^{n-1} E(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1}\left[(2\xi)^i B_{ni}(\tau) - \text{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

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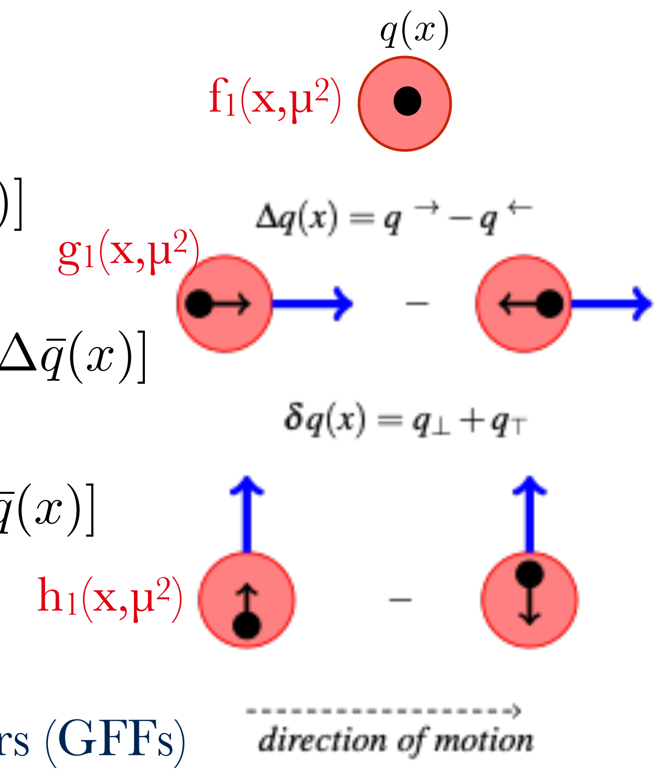

Twist-2 PDFs

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$$\int_{-1}^1 dx\, x^{n-1}E(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1}\left[(2\xi)^i B_{ni}(\tau) - \mathrm{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

Special cases: n=1,2 for the nucleon

- n=1: τ=0 —> charges g_V, g_A, g_T

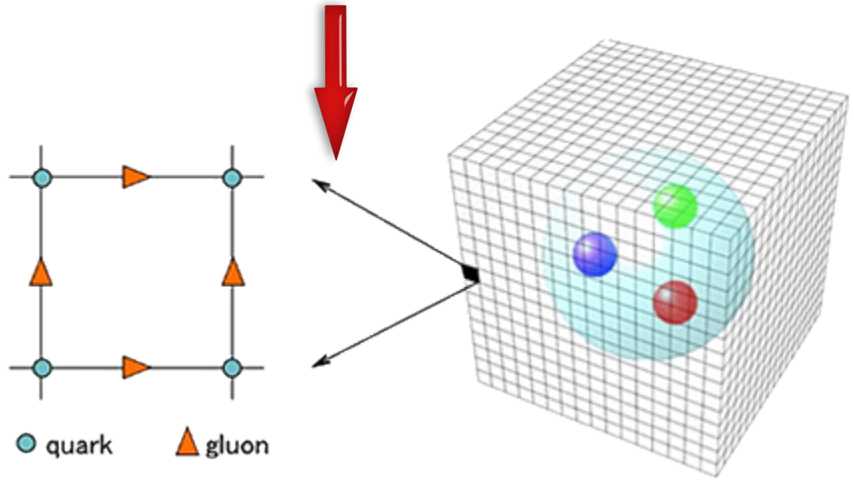
  $\tau\neq 0$ —> form factors: $A_{10}(\tau)=F_1(\tau)$, $B_{10}(\tau)=F_2(\tau)$, $\tilde A_{10}(\tau)=G_A(\tau)$, $\tilde B_{10}(\tau)=G_p(\tau)$
- n=2: generalised form factors: $A_{20}(\tau)$, $B_{20}(\tau)$, $C_{20}(\tau)$, $\tilde A_{20}(\tau)$, $\tilde B_{20}(\tau)$

  $\langle x\rangle_q = A_{20}(0)$, $\langle x\rangle_{\Delta q} = \tilde A_{20}(0)$, $\langle x\rangle_{\delta q} = A_{20}^T(0)$ and $J_q = \frac{1}{2}[A_{20}(0)+B_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$

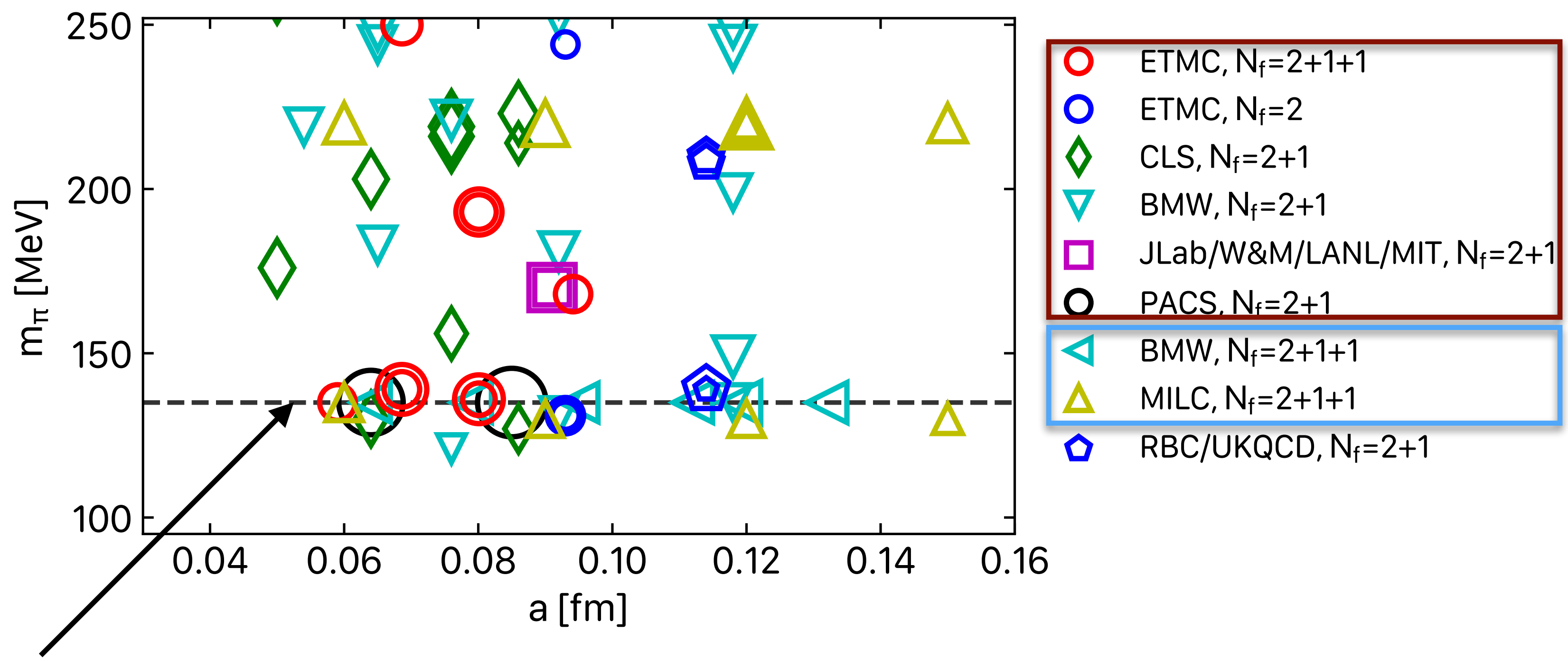
- Spin and momentum sums: $\sum_q\left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2}$, $\sum_q\langle x\rangle_q + \langle x\rangle_g = 1$

Simulations of lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



Simulation of gauge ensembles {U}: $P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$



Wilson-type

Staggered

Domain wall

Physical point ensembles

Simulations of lattice QCD

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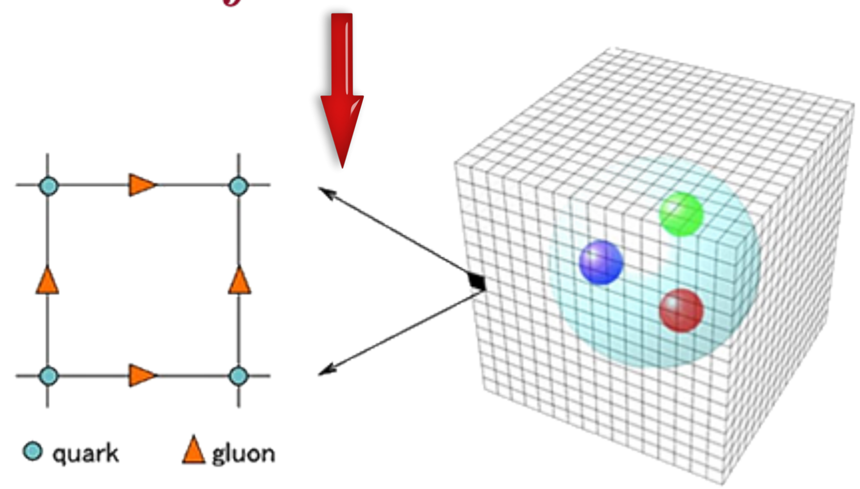

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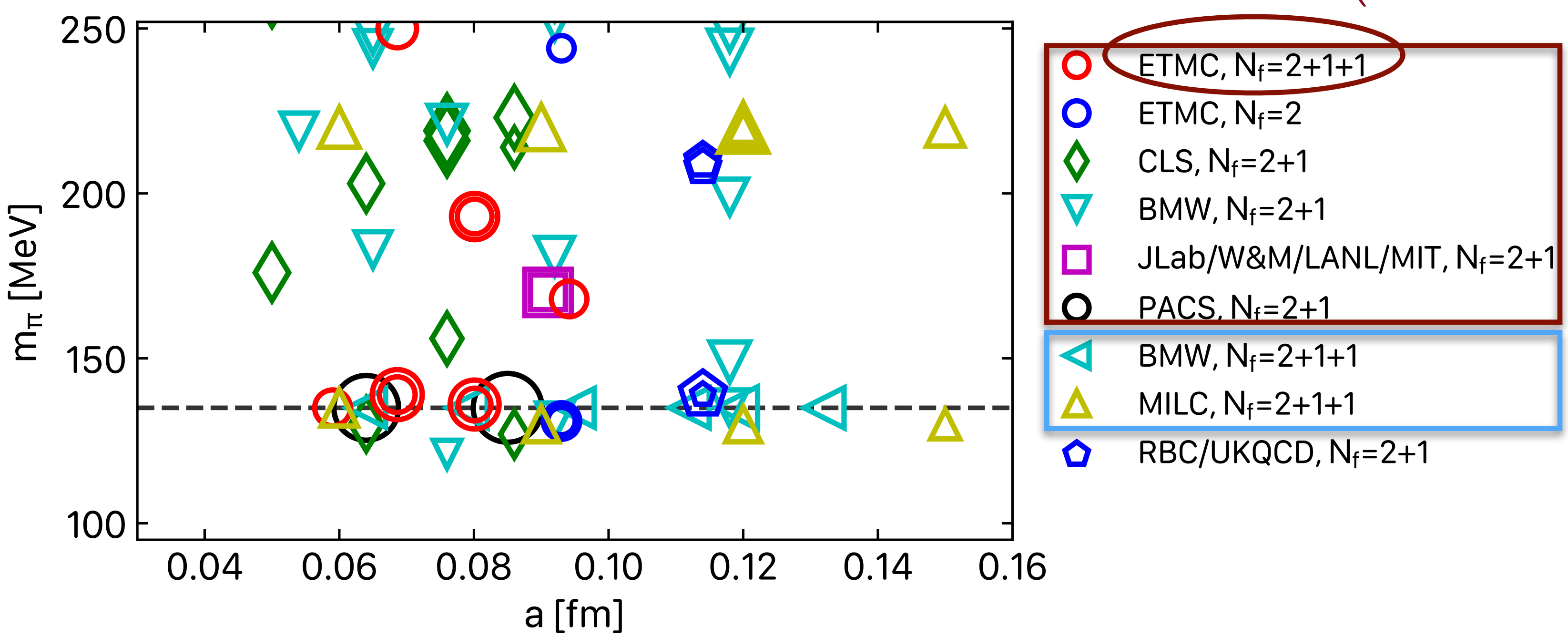

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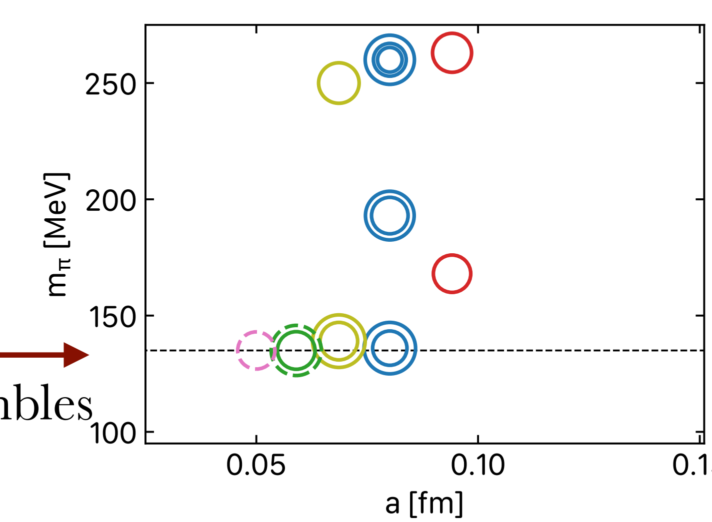
ETMC ensembles

- B-ensemble: 64^3 x 128, a~0.08 fm
- C-ensemble: 80^3x160, a~0.07 fm
- D-ensemble:96^3x192, a~0.06 fm

Physical point ensembles



Systematics & Challenges

- **Simulations directly at the physical point** ✓

  Systematic effects from chiral extrapolation are eliminated

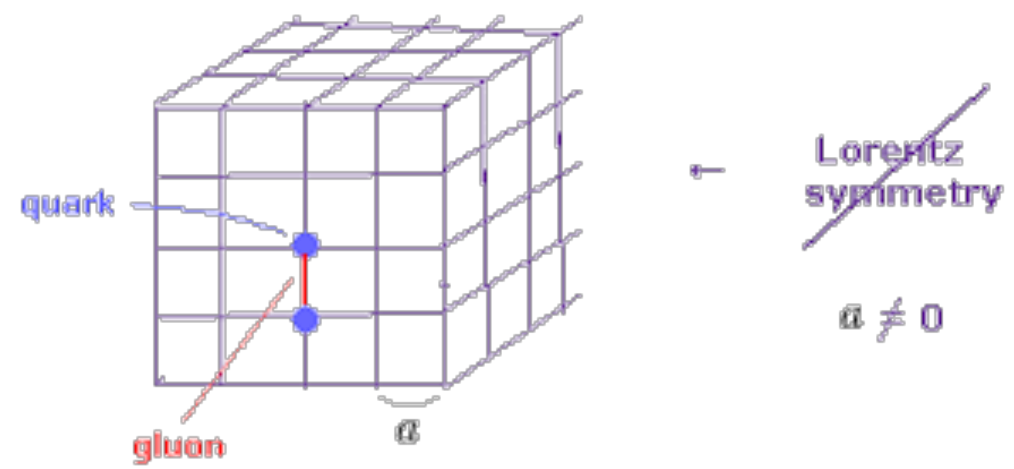
- **Discretisation effect:** Continuum limit

  —> need simulations for at least 3 lattice spacings

- **Finite volume effects:** Infinite volume limit

  —> need simulations for at least 3 volumes

Typically done using simulations for heavier than physical values of the pion mass



- In what follows we assume **isospin symmetry** i.e. up and down quarks have equal mass, and **neglect EM effects**

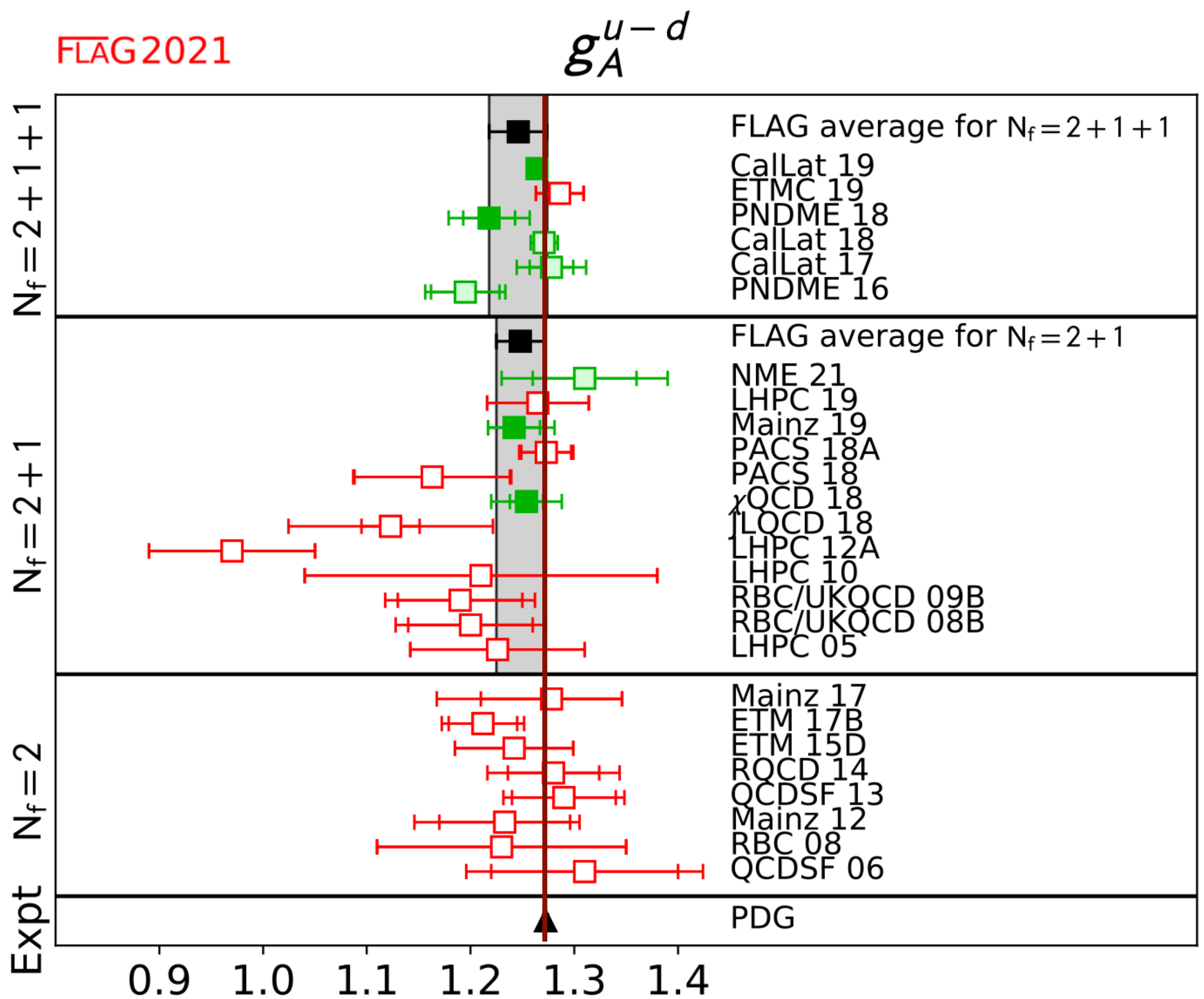
First Mellin moments

- Moments for n=1,2 are readily accessible in lattice QCD
- Computation of the low Mellin moments has a long history   G. Martinelli and Ch. Sachradja Phys. Lett. B217 (1989) 319
- Only recently we have results directly at the physical point (i.e. simulations with $m_\pi \sim 135$ +/-10 MeV)

Nucleon isovector charges

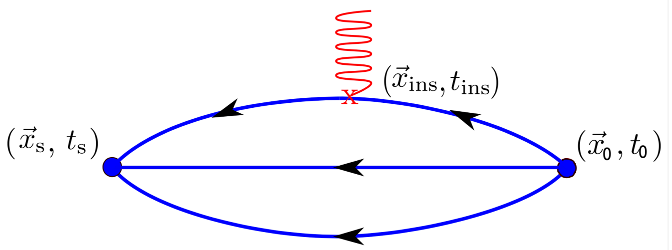
$$g_V = \langle 1 \rangle_{u-d}$$
$$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$$
$$g_T = \langle 1 \rangle_{\delta u - \delta d}$$

- g_V= 1
- g_A= 1.2723 ± 0.0023 reproduce
- g_T= 0.53 ± 0.25   M. Radici and A. Bacchetta. PRL 120 (2018) 192001

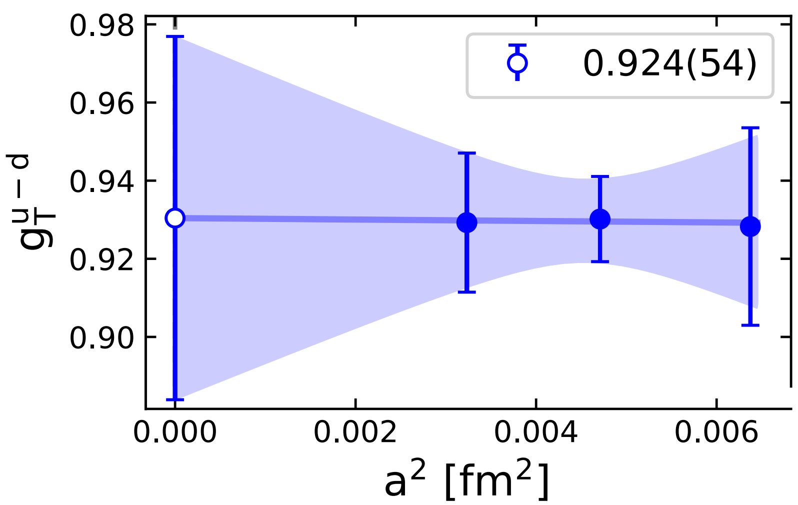


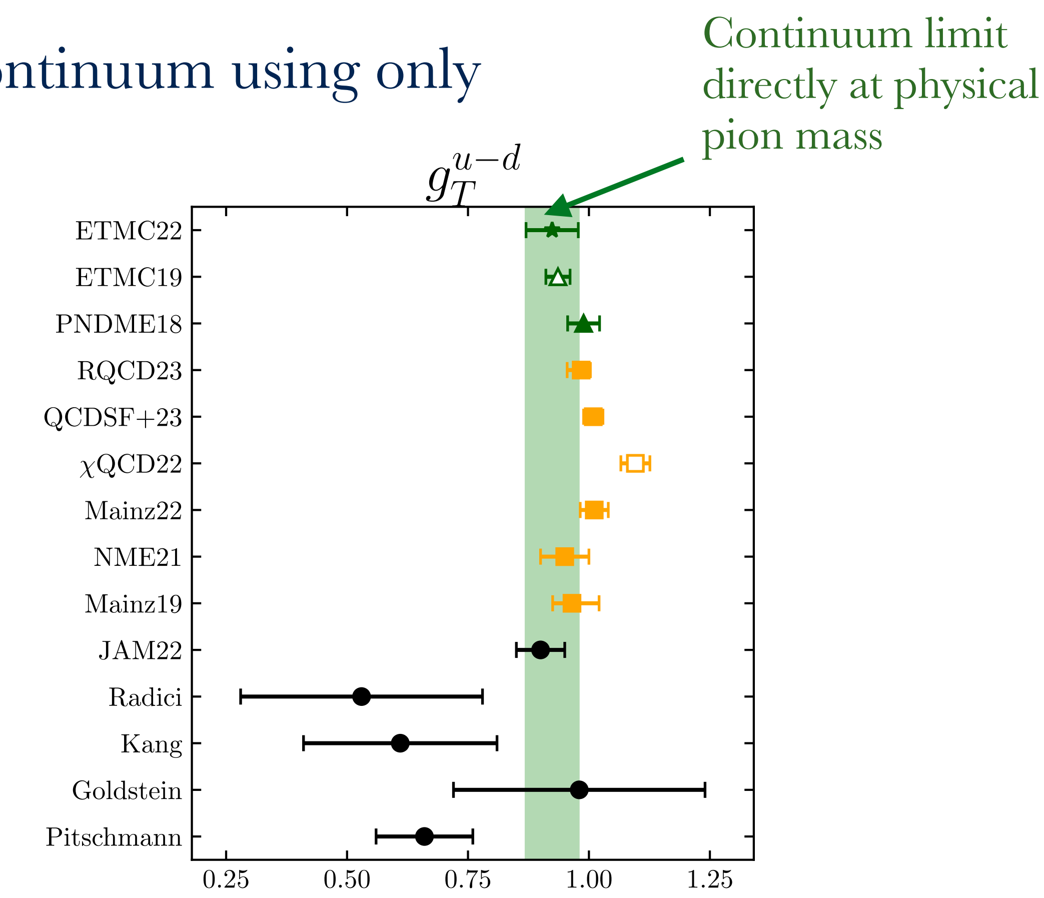
(1) Lattice QCD results on g_A consistent with experimental value

Nucleon isovector (u-d) tensor charge



- Only connected contributions
- Use three gauge ensembles generated using physical values of the light, strange and charm quarks:
  - B-ensemble: 64^3 x 128, a~0.08 fm
  - C-ensemble: 80^3x160, a~0.07 fm
  - D-ensemble:96^3x192, a~0.06 fm
- Obtain the tensor charge for the first time in the continuum using only physical point ensembles



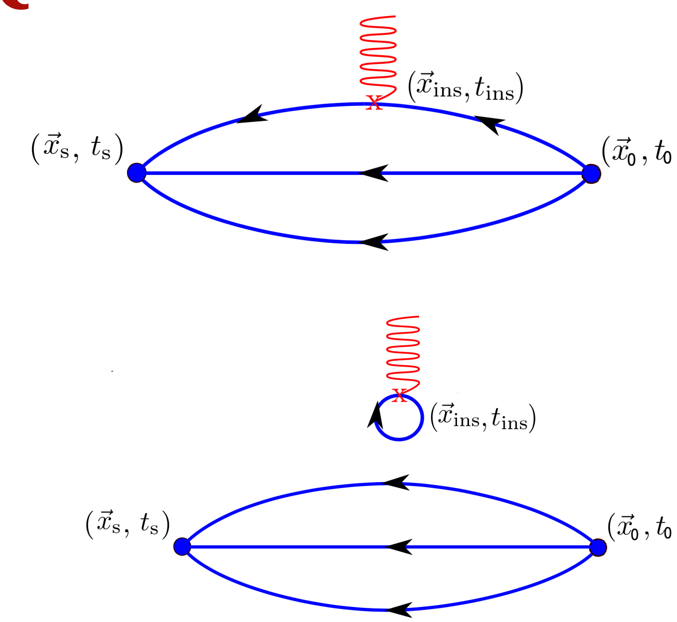


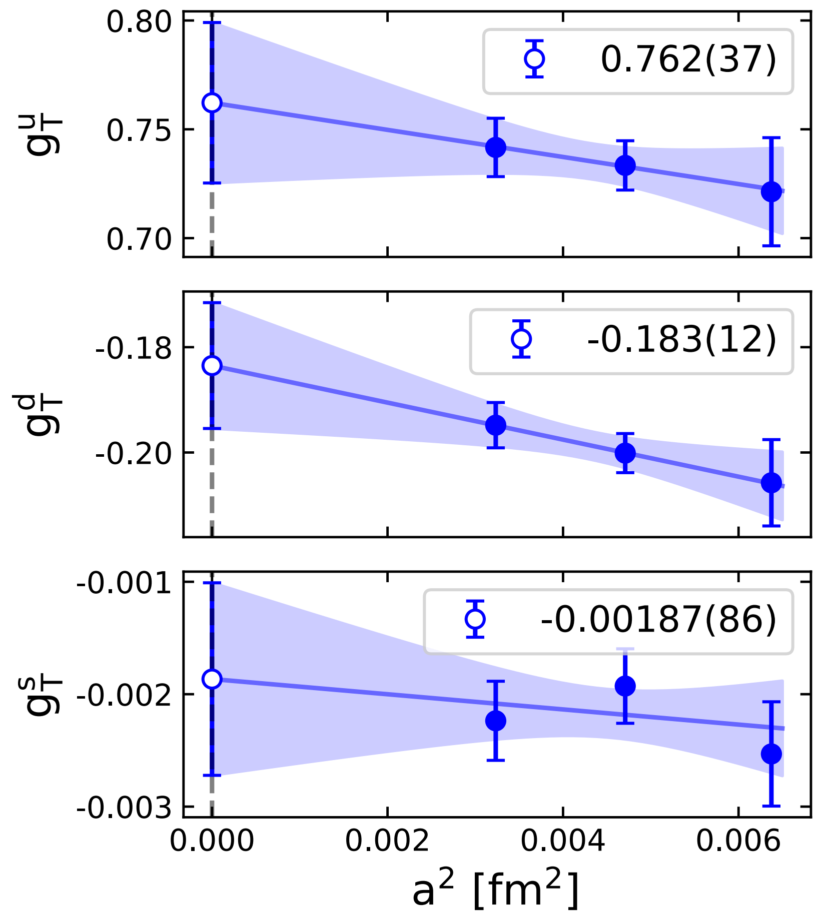
- Precision results on the isovector tensor charge - input for phenomenology e.g. JAM3D-22 analysis

Phys.Rev.D 106 (2022) 3, 034014, arXiv:2205.00999

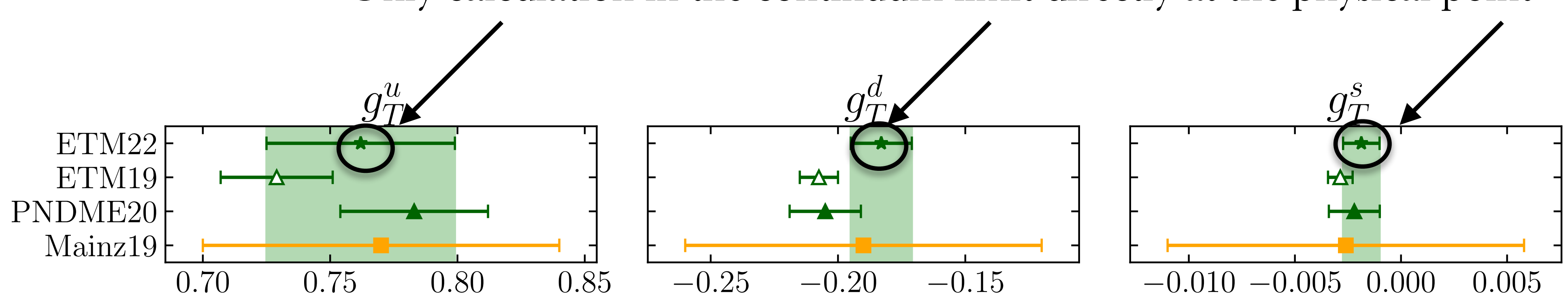
Flavor diagonal tensor charge

- Evaluate both connected and disconnected contributions
- Obtain flavor diagonal tensor charge for the first time in the continuum using only physical point ensembles - input for phenomenology





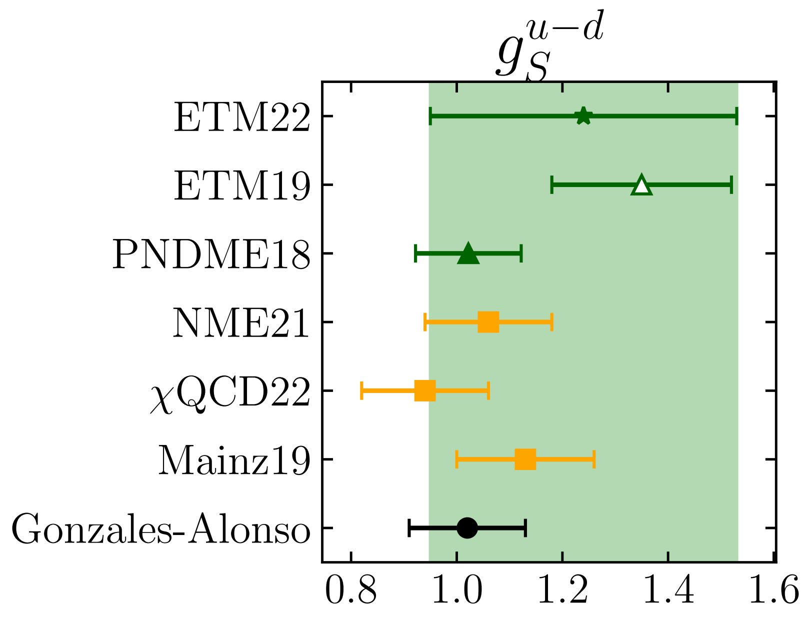
Only calculation in the continuum limit directly at the physical point

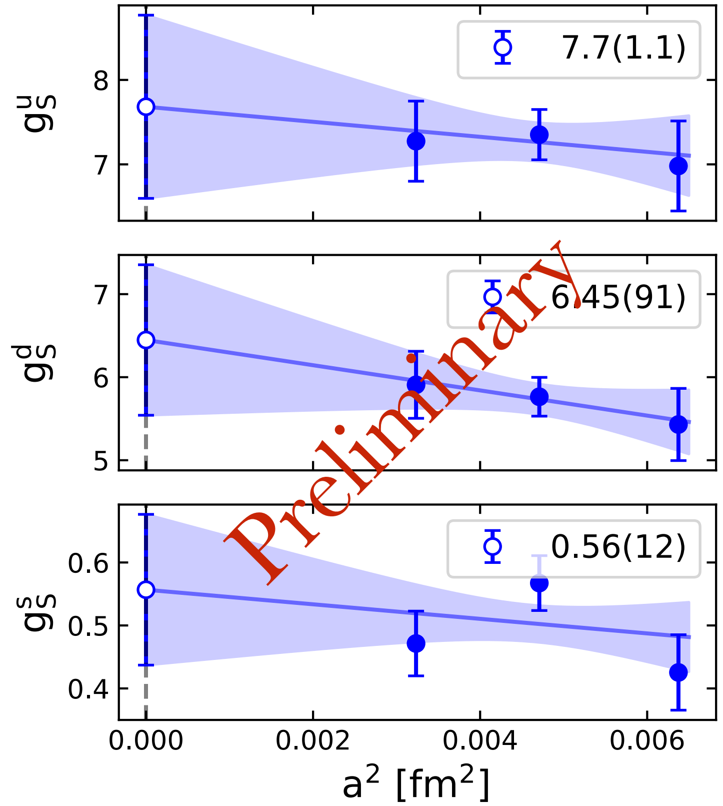


(2) Precision era of lattice QCD for first Mellin moments including flavor diagonal

Nucleon scalar charge and σ-terms (preliminary)

- Perform a similar analysis for the scalar charge - important input for direct dark matter searches





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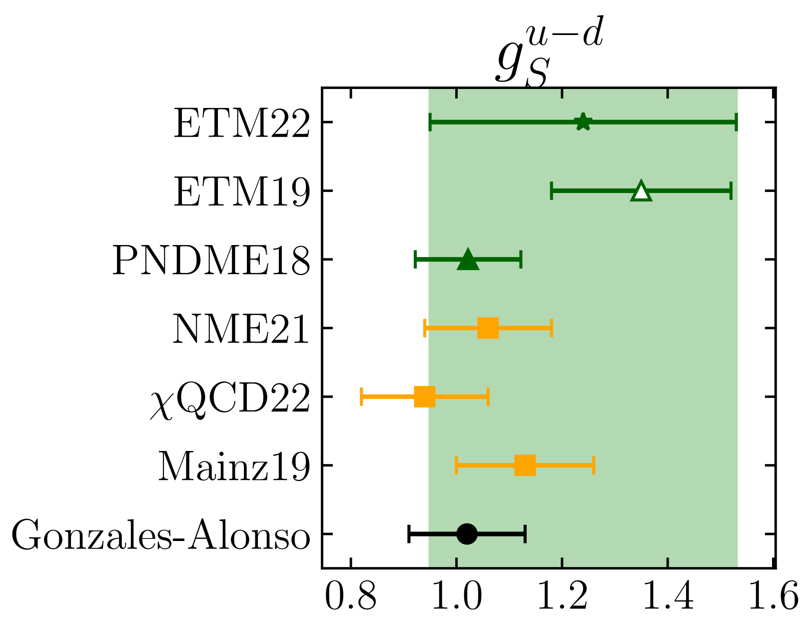
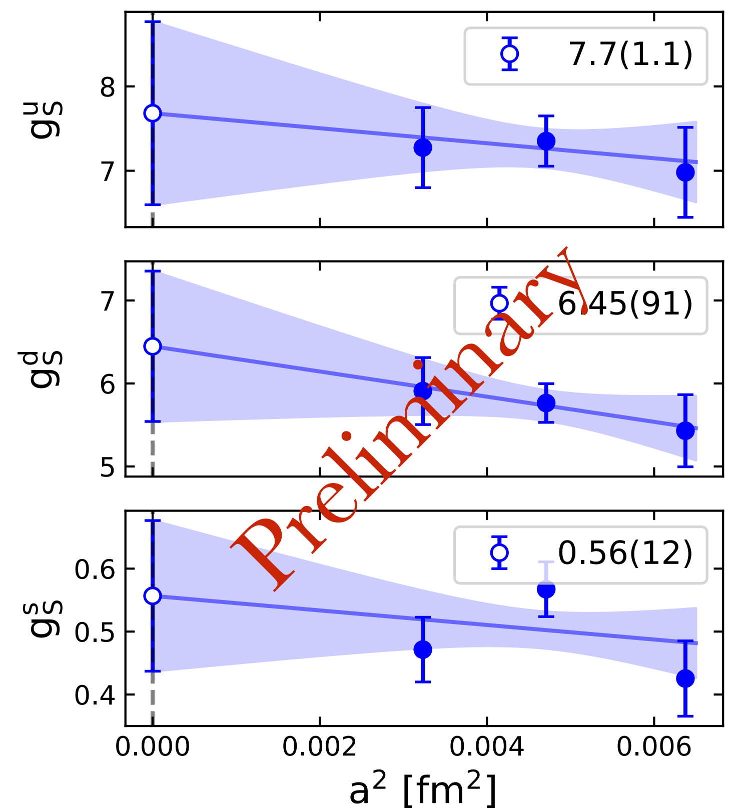
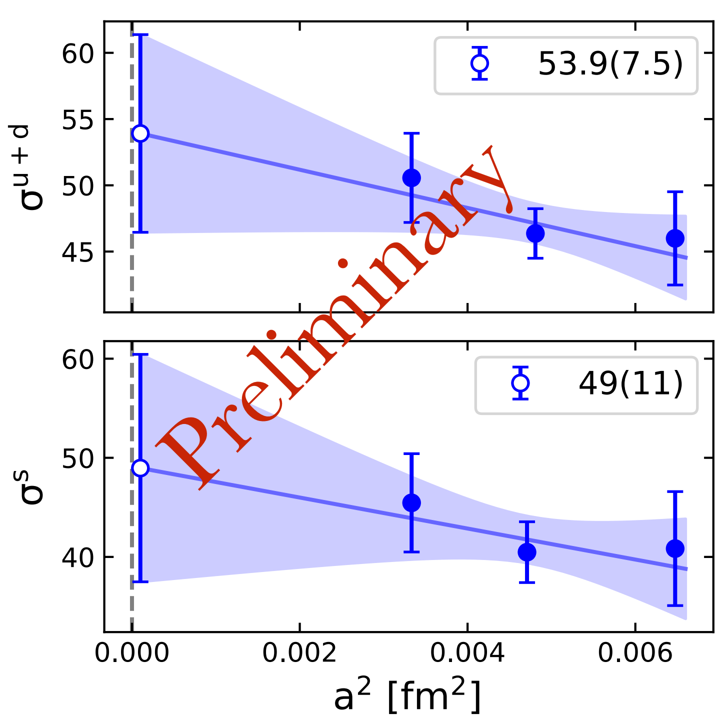
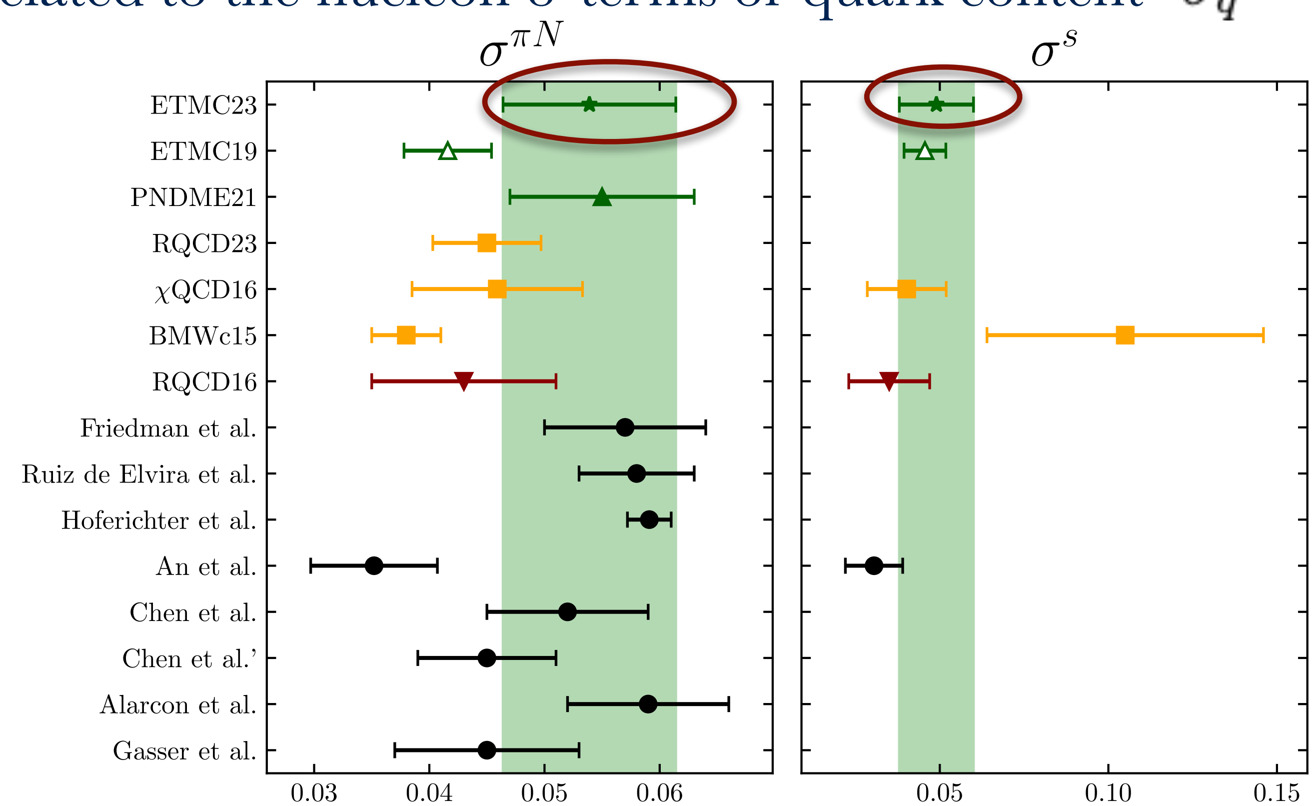




- Scalar charge is also directly related to the nucleon σ-terms or quark content $\sigma_q = m_q \langle N|\bar{q}q|N\rangle$





Only calculation in the continuum limit directly at the physical point

Second Mellin moments

- Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$
- Gluon unpolarised moment $\mathcal{O}^{\mu\nu,g} = F^{\{\mu\rho}F_{\rho}^{\nu\}}$ ← Field strength tensor

$$\langle N(p',s')|\mathcal{O}^{\mu\nu,q}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^q(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^q(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^q(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

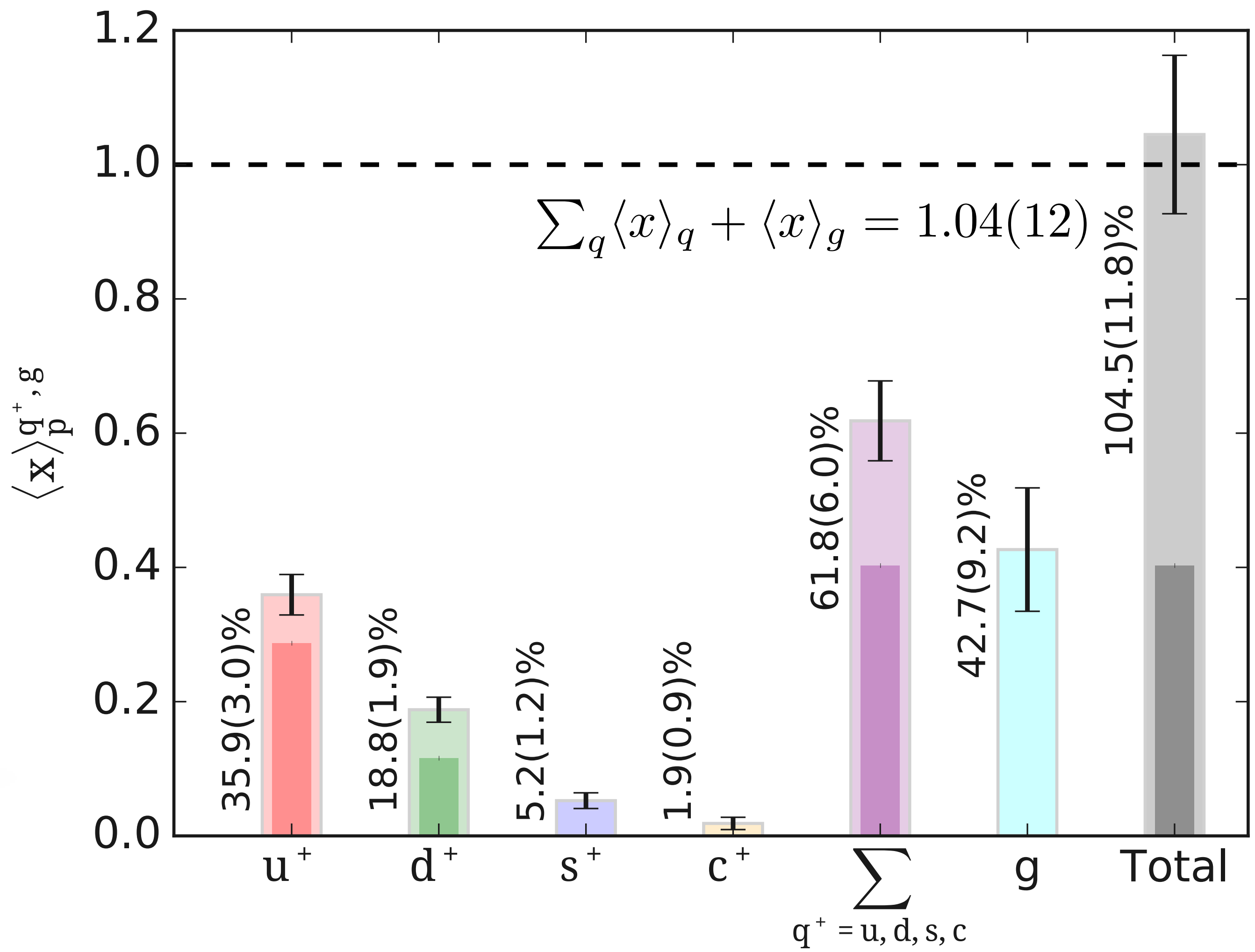
$$\langle x\rangle_q = A_{20}^q(0) \qquad J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

Momentum fraction carried by quark - best measured

- Equivalent expression for gluon

$$\langle x\rangle_g = A_{20}^g(0)$$

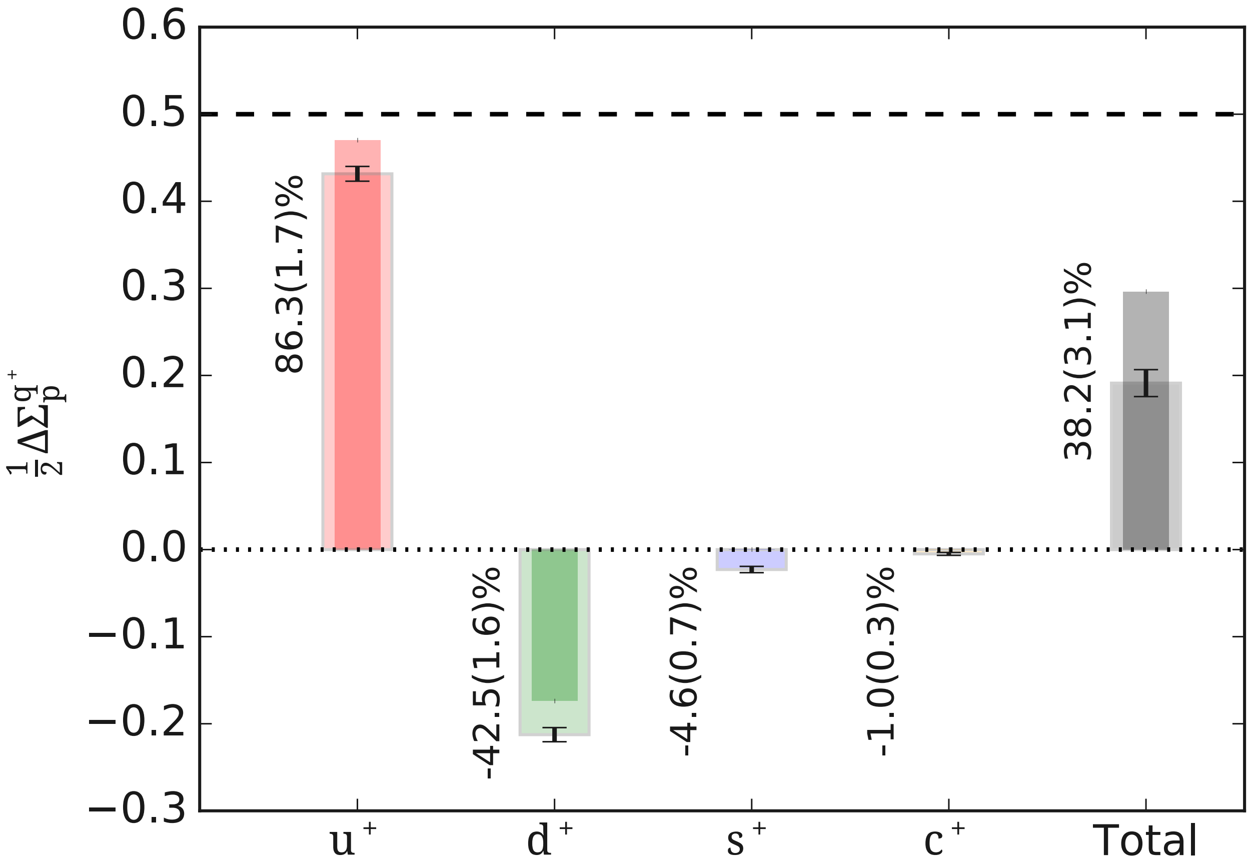
→ Nucleon momentum sum verified



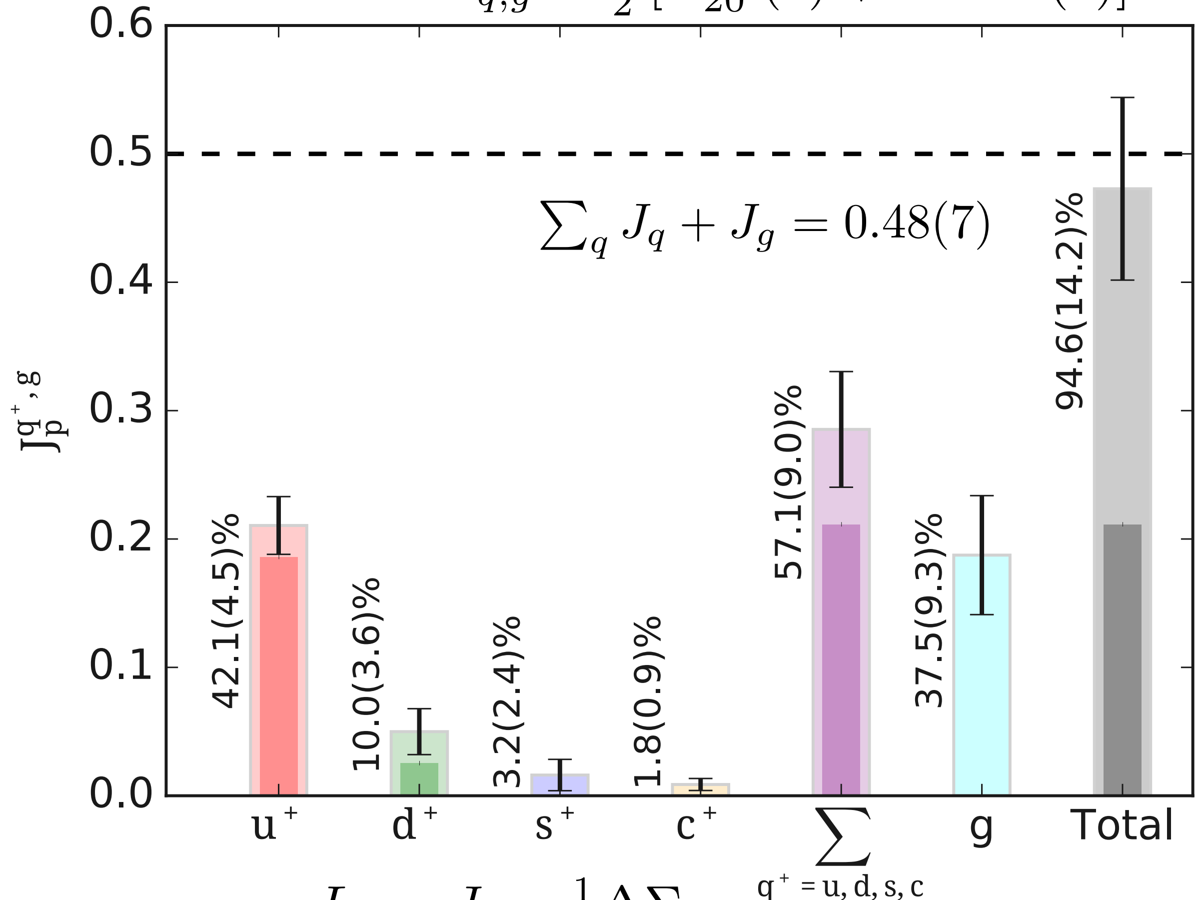
Momentum and spin sums

- Axial charge determines intrinsic spin carried by each quark

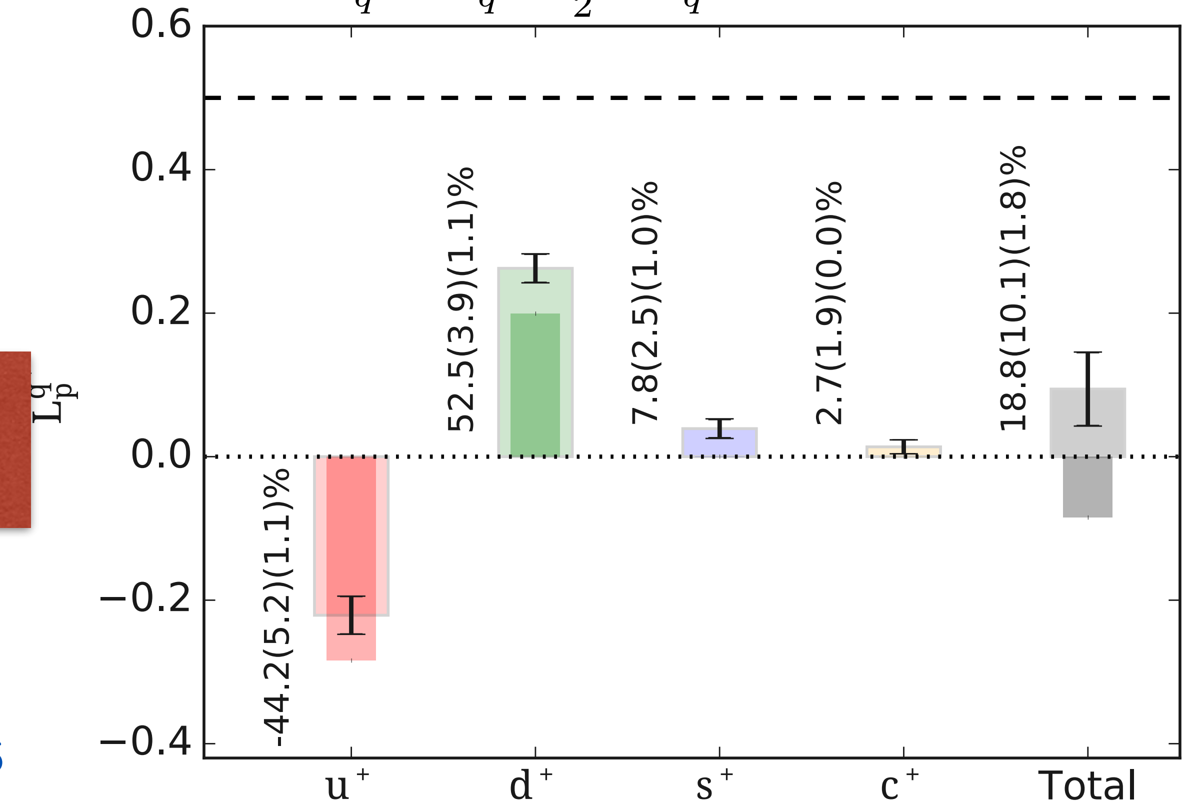
$$\Delta\Sigma_{q+}(\mu^2) = \int_0^1 dx\,\left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$



$$J_{q,g} = \frac{1}{2}\left[A_{20}^{q,g}(0) + B^{q,g}20(0)\right]$$



$$L_q = J_q - \frac{1}{2}\Delta\Sigma_q$$

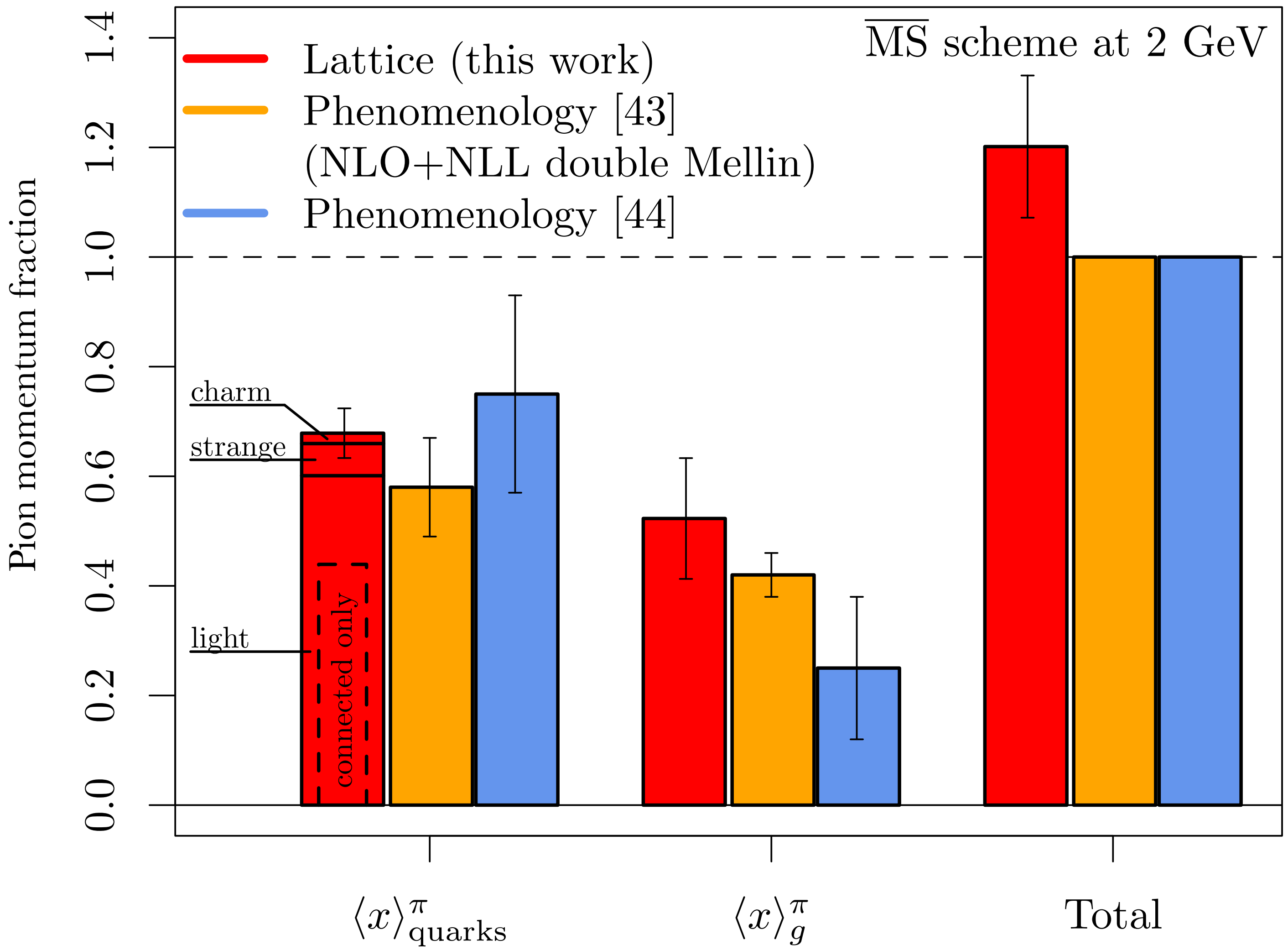


(3) Nucleon spin sum verified - lattice QCD solves a 30 year puzzle

C. A. *et al.* (ETMC) Phys. Rev. Lett. **119**, 142002, 1909.00485

C. A. *et al.* (ETMC) Phys.Rev.D **101** (2020) 9, 094513, 2003.08486

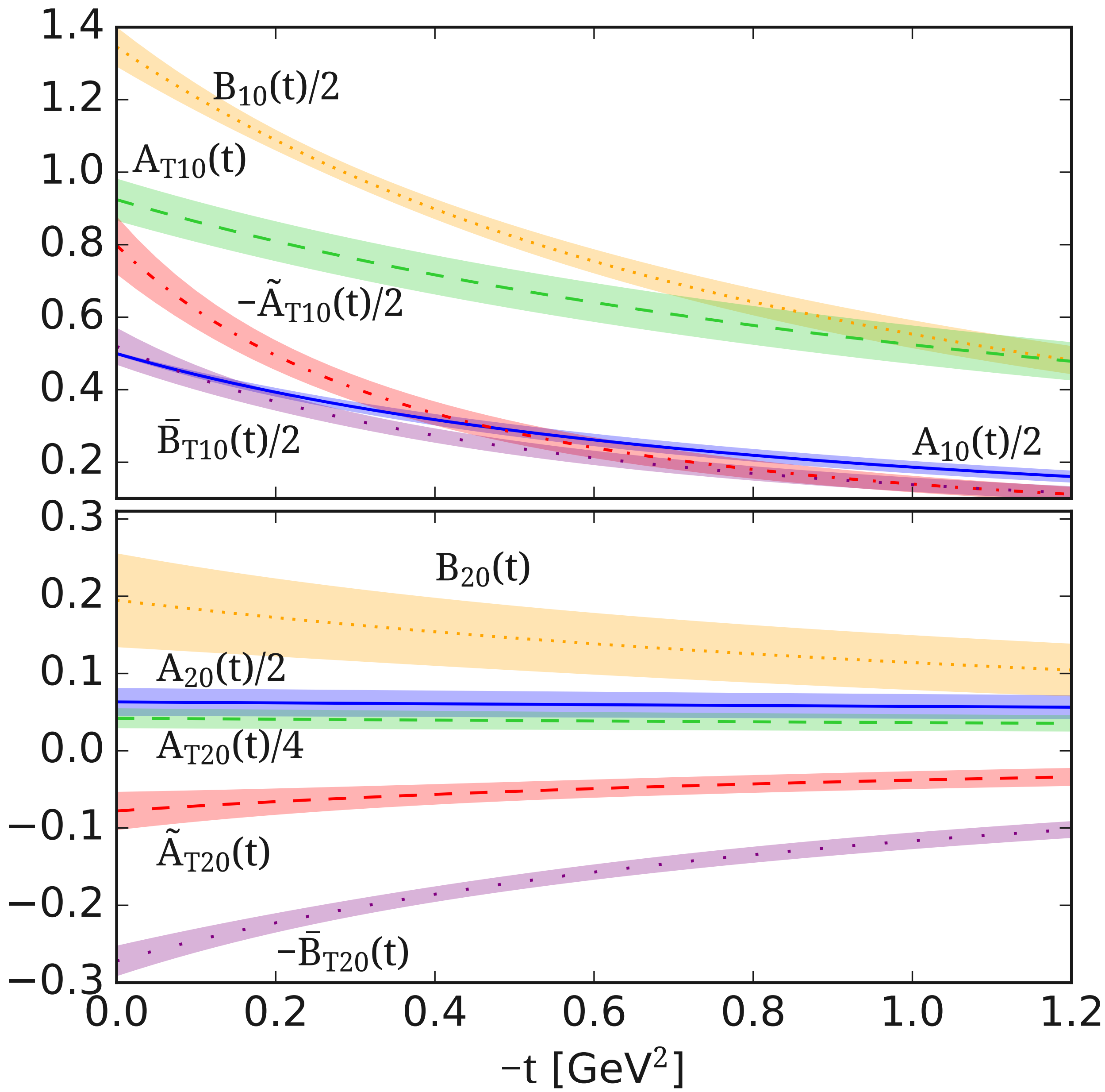
Pion momentum sum



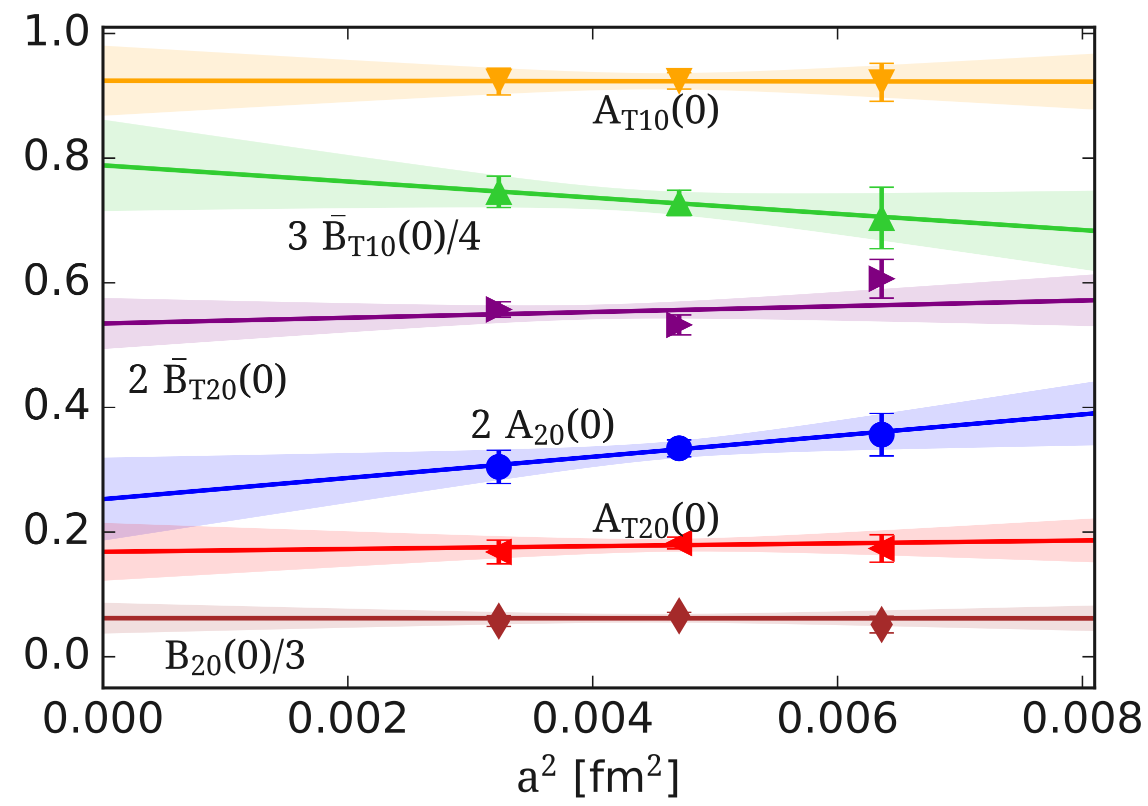
$x_{u+d} = 0.601(28)$, $x_s = 0.059(13)$, $x_c = 0.019(05)$, and $x_g = 0.52(11)$

C. A. *et al.* (ETMC), Phys.Rev.Lett. 127 (2021) 25, 252001, arXiv: 2109.10692

Nucleon transverse quark spin densities



- Compute twist-two matrix elements of the chiral-even unpolarized and chiral-odd transversity generalized form factors and Fourier transform to impact parameter space
- For the first we extrapolate to the continuum limit using 3 gauge ensembles directly at the physical pion mass

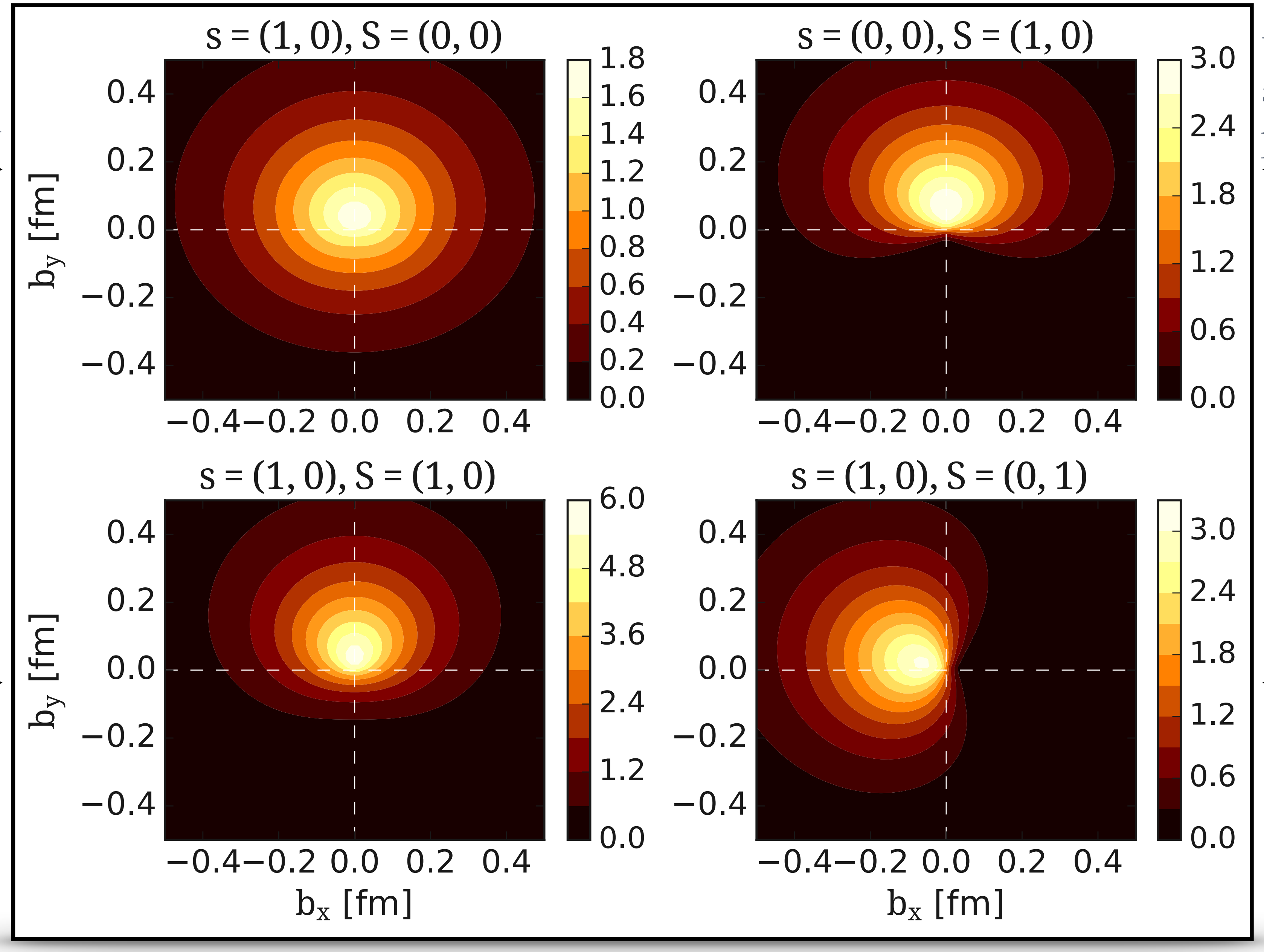


Moments of transverse density distributions (isovector)

$\langle x^{n-1}\rangle_\rho(\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) \equiv \int_{-1}^{1} dx\ x^{n-1}\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp),$

transversely polarized quarks in an unpolarized nucleon

transversely polarized quarks in a transversely polarized nucleon



unpolarized quarks in a transversely polarized nucleon

Large effect due to B_{10} (Sivers effect)

transversely polarized quarks in a perpendicularly polarized nucleon

Contours of the first moment (n=1) of the probability density, as a function of b_x and b_y

$$\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) = \frac{1}{2}\Big[H(x,b_\perp^2)+\frac{\mathbf{b}_\perp^j\epsilon^{ji}}{m_N}\left(\mathbf{S}_\perp^i E'(x,b_\perp^2)+\mathbf{s}_\perp^i \bar{E}'_T(x,b_\perp^2)\right)+\mathbf{s}_\perp^i\mathbf{S}_\perp^i\left(H_T(x,b_\perp^2)-\frac{\Delta_{b_\perp}\tilde{H}_T(x,b_\perp^2)}{4m_N^2}\right)+\mathbf{s}_\perp^i(2\mathbf{b}_\perp^i\mathbf{b}_\perp^j-\delta^{ij}b_\perp^2)\mathbf{S}_\perp^j\frac{\tilde{H}''_T(x,b_\perp^2)}{m_N^2}\Big]$$

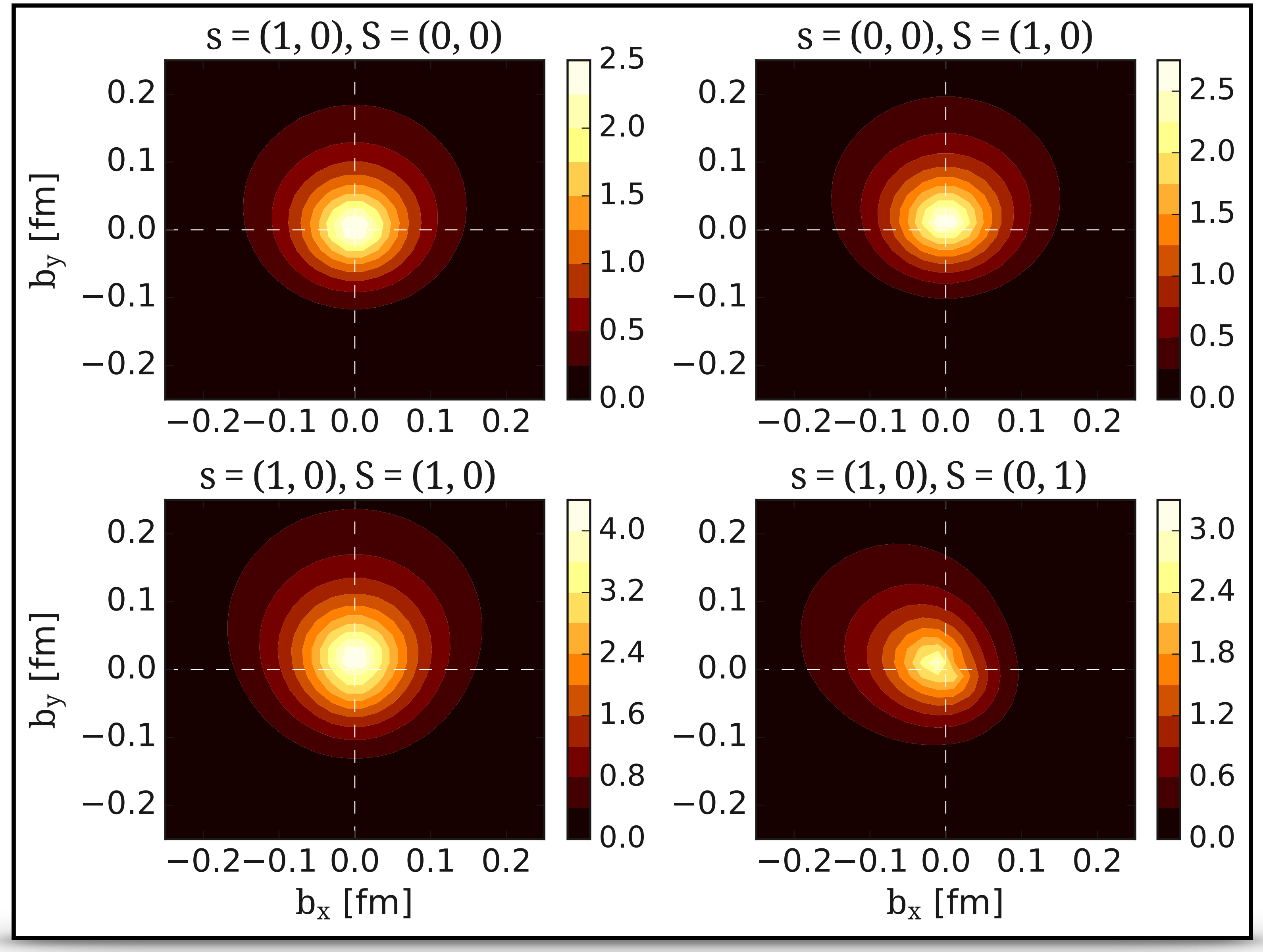
M. Diehl and Ph. Hägler, Eur. Phys. J. C44 (2005) 87, hep-ph/0504175

Transverse density distributions (isovector)

$\langle x^{n-1}\rangle_\rho(\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) \equiv \int_{-1}^{1} dx\; x^{n-1}\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp),$

transversely polarized quarks in an unpolarized nucleon →

unpolarized quarks in a transversely polarized nucleon ←



transversely polarized quarks in a transversely polarized nucleon →

transversely polarized quarks in a perpendicularly polarized nucleon ←

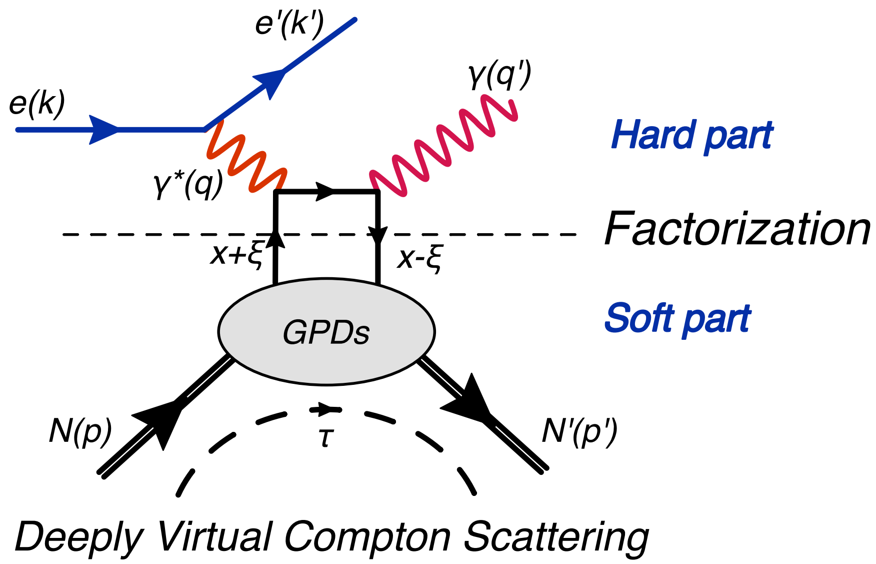
Contours of the second moment (n=2) of the probability density, as a function of b_x and b_y

Distortion is milder than for n=1 due to the milder dependence of $A_{20}(t)$ compared to $A_{10}(t)$

New era of direct computation of x-dependencncne of parton distributions

Generalised Parton Distributions (GPDs)

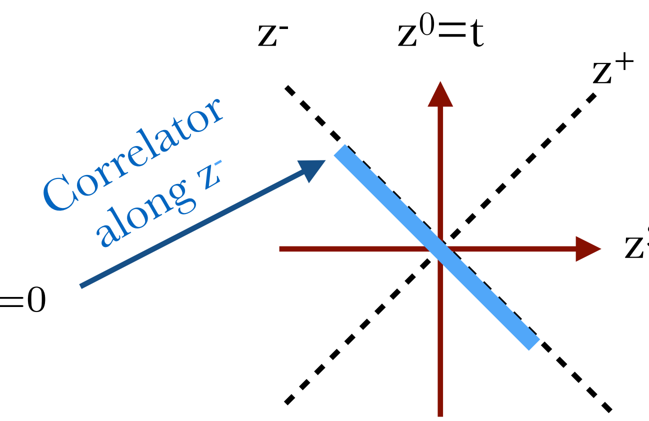
- High energy scattering processes: Factorization into a hard partonic subprocess, calculable in perturbation theory, and a universal non-perturbative parton distribution



- D. Mueller *et al.*, Fortschr. Phys. 42, 101 (1994)
- A. V. Radyushkin, Phys. Lett. B380, 417 (1996), hep-ph/9604317
- A. V. Radyushkin, Phys. Lett. B385, 333 (1996), hep-ph/9605431
- A. V. Radyushkin, Phys. Rev. D56, 5524 (1997), hep-ph/9704207
- X. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249.
- X. Ji, Phys. Rev. D55, 7114 (1997), hep-ph/9609381
- X. Ji, J. Phys. G24, 1181 (1998), hep-ph/9807358

- GPDs are light cone matrix elements

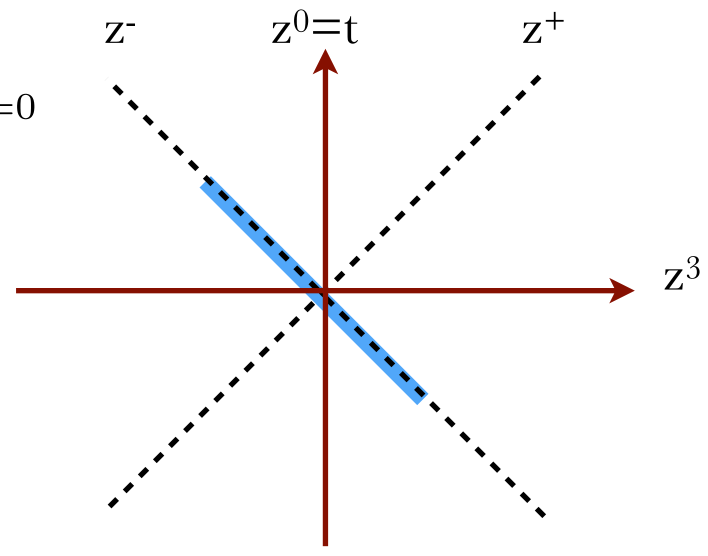
$$F_\Gamma(x,\xi,\tau) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle N(p')|\bar{\psi}(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$



- $P^+ = \frac{p'^+ + p}{2}$
- $\tau = -Q^2 = (p'-p)^2$
- $\xi = \frac{p^+ - p'^+}{2P^+}$: skewness

Direct computation of GPDs

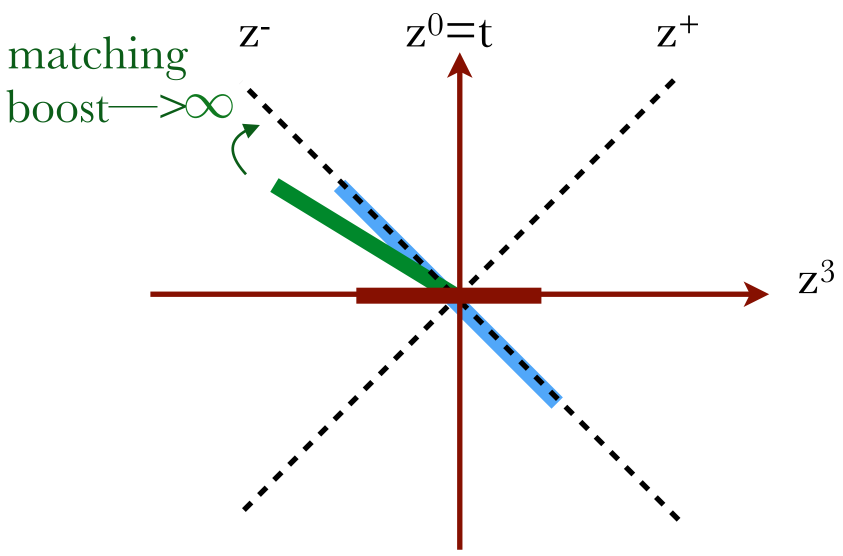
$$F_\Gamma(x,\xi,\tau) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle N(p')|\bar{\psi}(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$



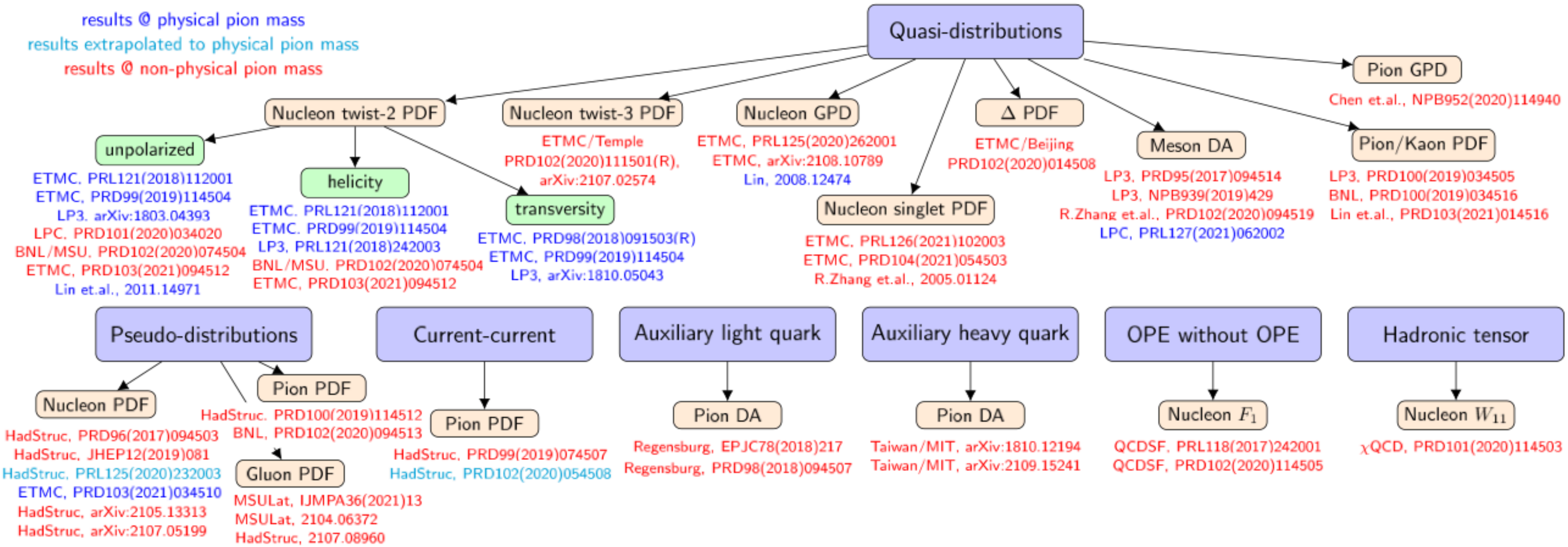
- Define spatial correlators e.g. along z^3 and boost nucleon state to large momentum

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

- Match to the infinite momentum frame using the matching kernel computed in perturbation theory (large momentum effective theory - LaMET)



Overview of results from different approaches



K. Cichy, arXiv:2111.04552

Reviews

- M. Constantinou *et al.*, "Snowmass", 2202.07193
- X. Ji, Y. Liu, J.-H. Zhang, Rev. Mod. Phys. **93**, 035005 (2021), 2004.03543
- M. Constantinou *et al.* (2020) 2006.08636
- K. Cichy and M. Constantinou, *Adv.High Energy Phys.* (2019) 3036904,1811.07248
- H.-W. Lin *et al.* Prog. Part. Nucl. Phys. (2018) 100, 107, 1711.07916

Computation of quasi-PDFs

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_{\Gamma}(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle|_{\mu}$$

Renormalise non-perturbatively, Z(z,μ)
Need to eliminate both UV and exponential divergences

- Match using LaMET

Perturbative kernel

$$\tilde{F}_{\Gamma}(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_{\Gamma}(y, \mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

Direct computation of PDFs (and GPDs)

See talk by M. Constantinou

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle |_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$
Need to eliminate both UV and exponential divergences

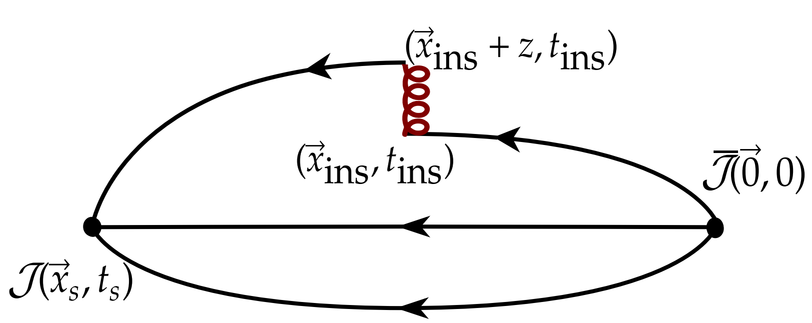
- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{QCD}^2}{P_3^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

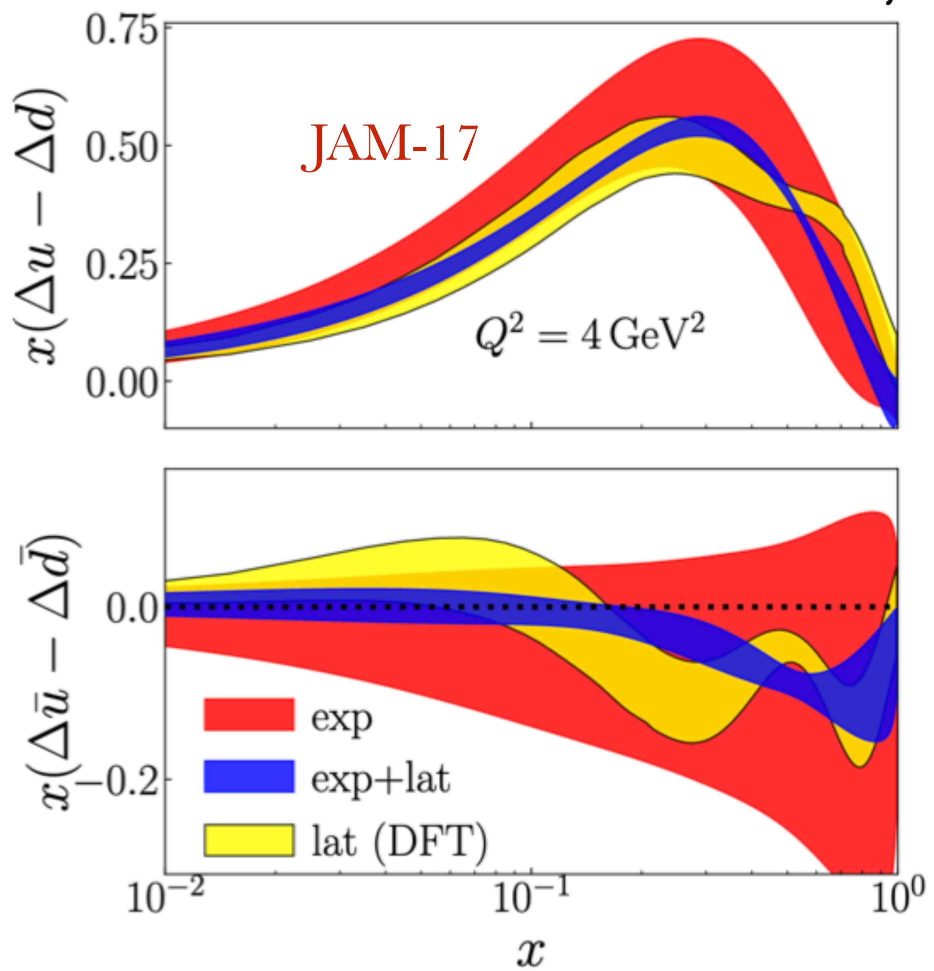
Isovector (u-d)



$\Gamma =$	γ_0	unpolarised
	$\gamma_5\gamma_3$	helicity
	$\sigma_{3i}, i = 1,2$	transversity

C.A. et al. (ETMC) Phys. Rev. Lett. **121**, 112001 (2018)

State-of-the-art results on helicity



(4) Parton distribution functions can be computed directly in lattice QCD

Computation of quasi-PDFs

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x,P_3,\mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle |_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$

Need to eliminate both UV and exponential divergences

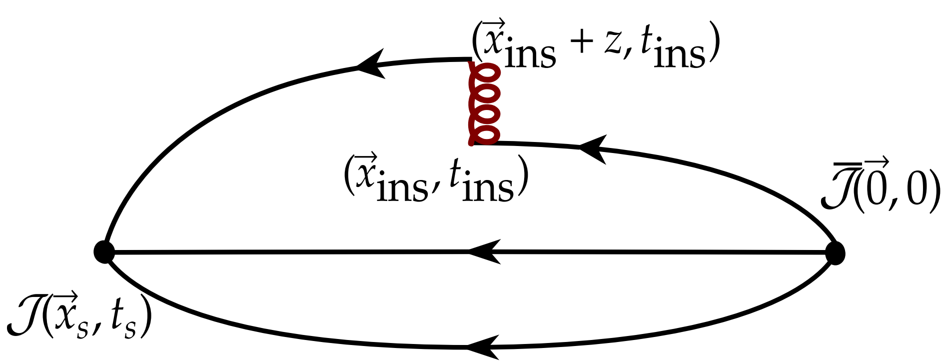
- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x,P_3,\mu) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y,\mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{QCD}^2}{P_3^2}\right)$$

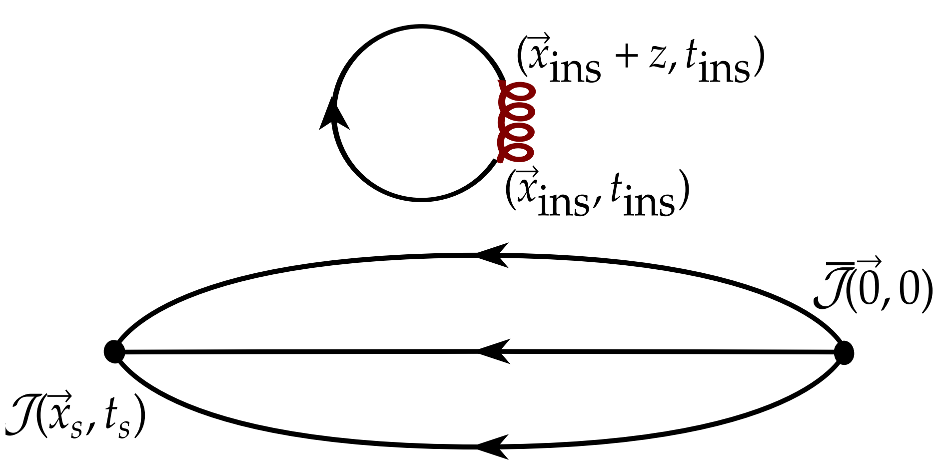
Isovector (u-d) and isoscalar (u+d) connected

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 [arXiv:1305.1539]

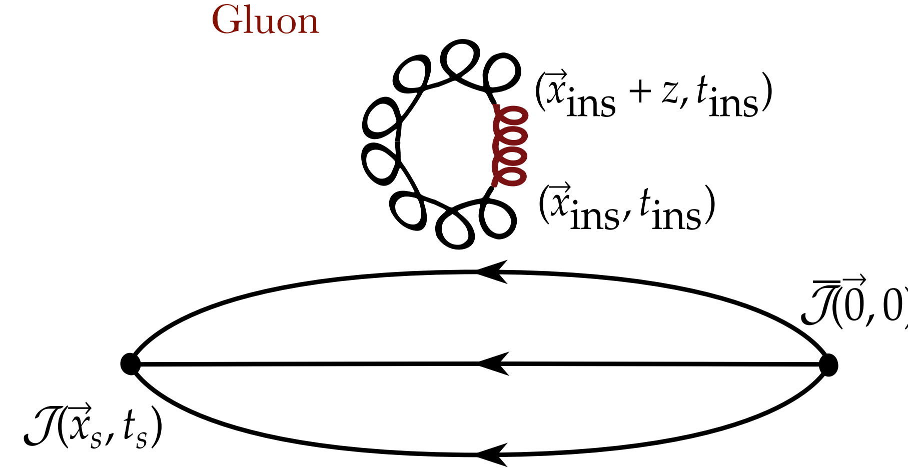


$$\Gamma = \begin{cases} \gamma_0 & \text{unpolarised} \\ \gamma_5\gamma_3 & \text{helicity} \\ \sigma_{3i}, i=1,2 & \text{transversity} \end{cases}$$

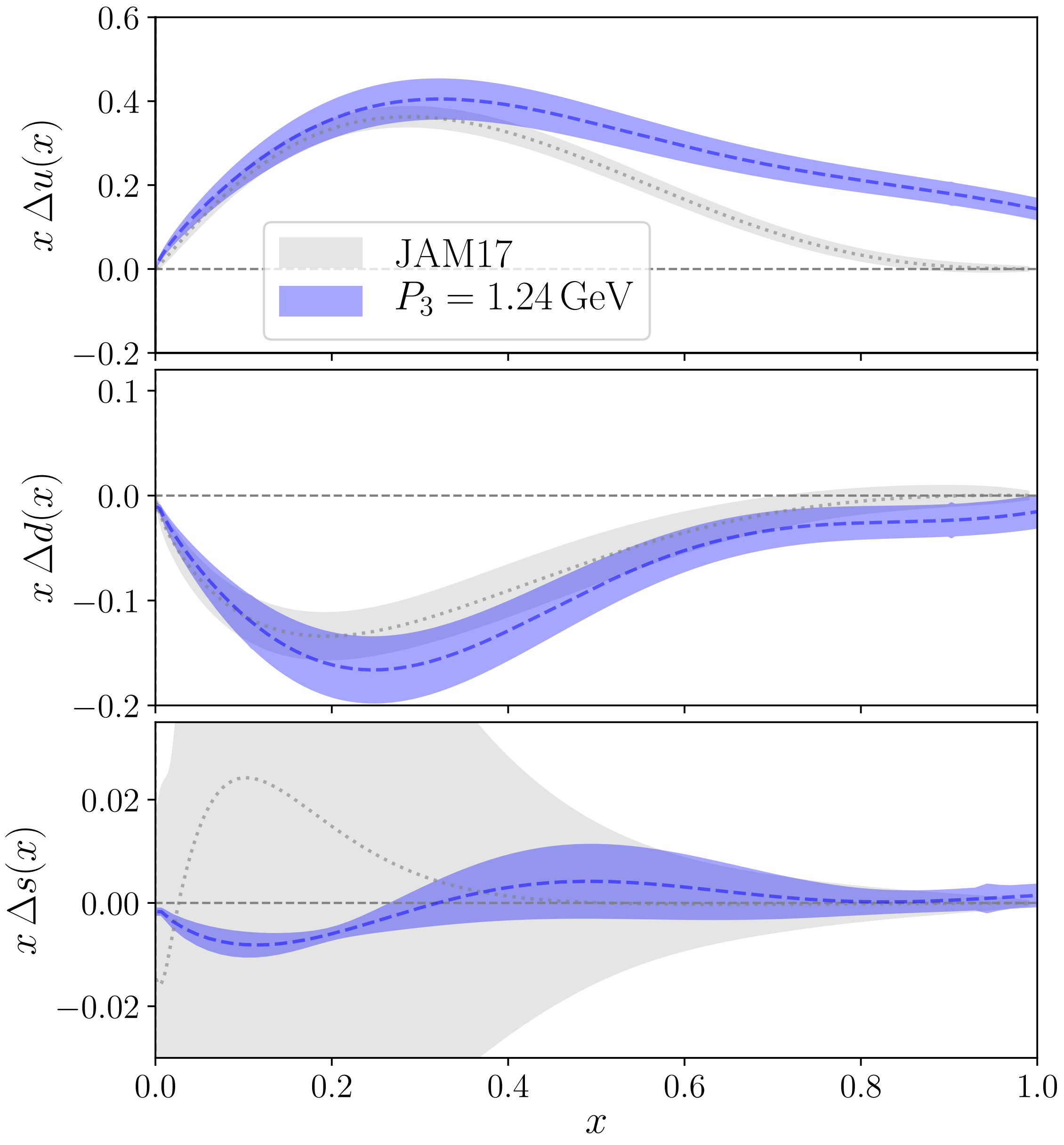
Isoscalar (u+d) disconnected, s and c



Gluon



Helicity distributions

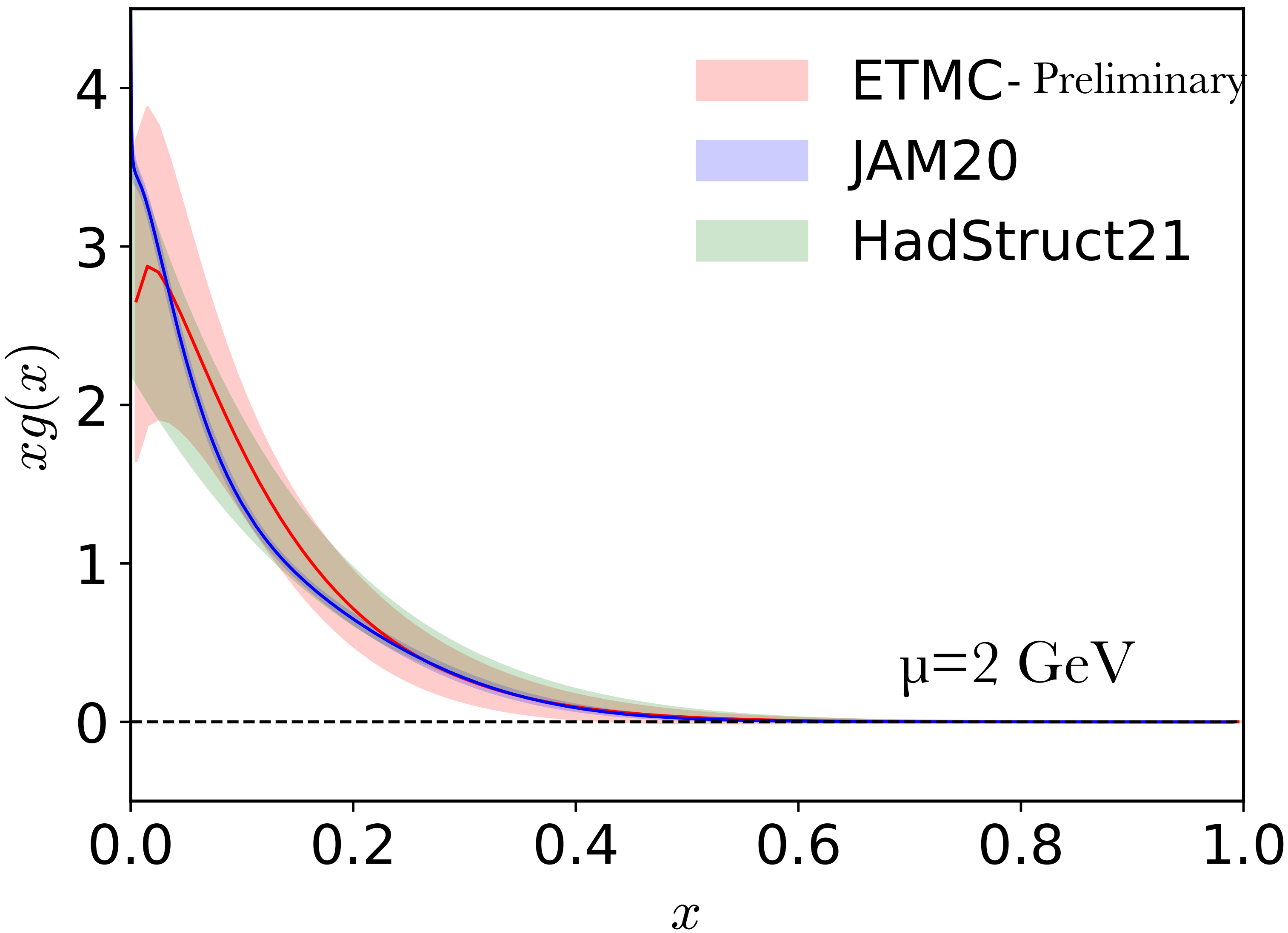


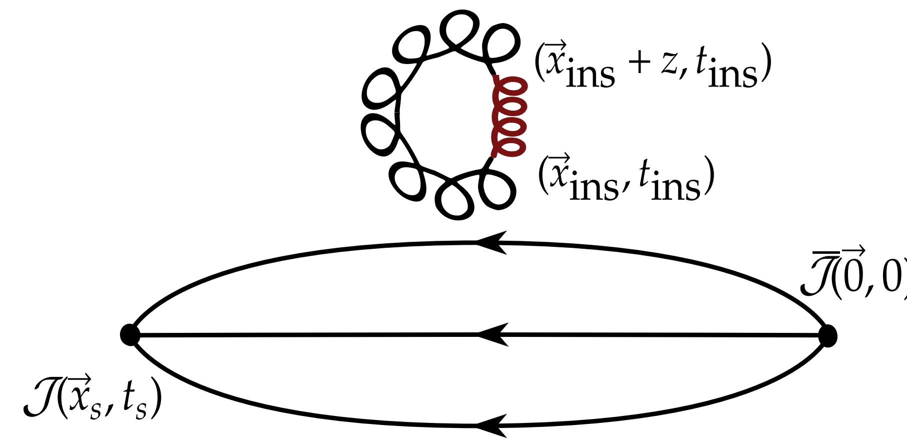
C. A., M. Constantinou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 10, 102003, arXiv:2009.13061
C.A., G. Iannelli, K. Jansen, F. Manigrasso, Phys. Rev. D 102 (2020) 9, 094508, arXiv:2007.13800

- Computation at the physical point is currently on-going

Unpolarized gluon PDF

- Calculate the matrix elements of a spin-averaged nucleon for two gluon fields connected by a Wilson line
- Use Wilson flow to reduce ultraviolet fluctuations
- Pseudo-PDF approach with pion mass 358 MeV





T. Khan, *et al.* (HadStruc Collaboration) Phys. Rev. D 101 (2021) 094516, 2107.08960

Generalised parton distributions

- Compute space-like matrix element with different initial and final nucleon boosts in the Breit frame

$$h_\Gamma(z,\tilde{\xi},Q^2,P_3) = \langle N(P_3\hat{e}_z+\vec{Q}/2)|\,\overline{\psi}(z)\,\Gamma W(z,0)\,\psi(0)|\,N(P_3\hat{e}_z-\vec{Q}/2)\rangle$$

$$\tilde{\xi} = -\frac{Q_3}{2P_3}\ :\ \mathrm{quasi-skewness} \quad \tilde{\xi} = \xi + \mathcal{O}(\frac{1}{P_3^2})$$

- Rest of the steps are the same as for quasi-PDFs: i.e. renormalise, take the Fourier transform and match

$$\tilde{F}_\Gamma(z,\tilde{\xi},Q^2,P_3,\mu^0,\mu_3^0) = \int_{-1}^{1}\frac{dy}{y}C_\Gamma\left(\frac{x}{y},\frac{\mu}{yP_3},\frac{\mu_3^0}{yP_3},\frac{(\mu^0)^2}{(\mu_3^0)^2}\right)F_\Gamma(y,Q^2,\xi,\mu)+\mathcal{O}\left(\frac{m^2}{P_3^2},\frac{Q^2}{P_3^2},\frac{\Lambda^2_{\rm QCD}}{x^2P_3^2}\right)$$

RI-scale

$\overline{\rm MS}$ – scale

Reduces to the matching kernel for ξ=0
Does not depend on Q^2

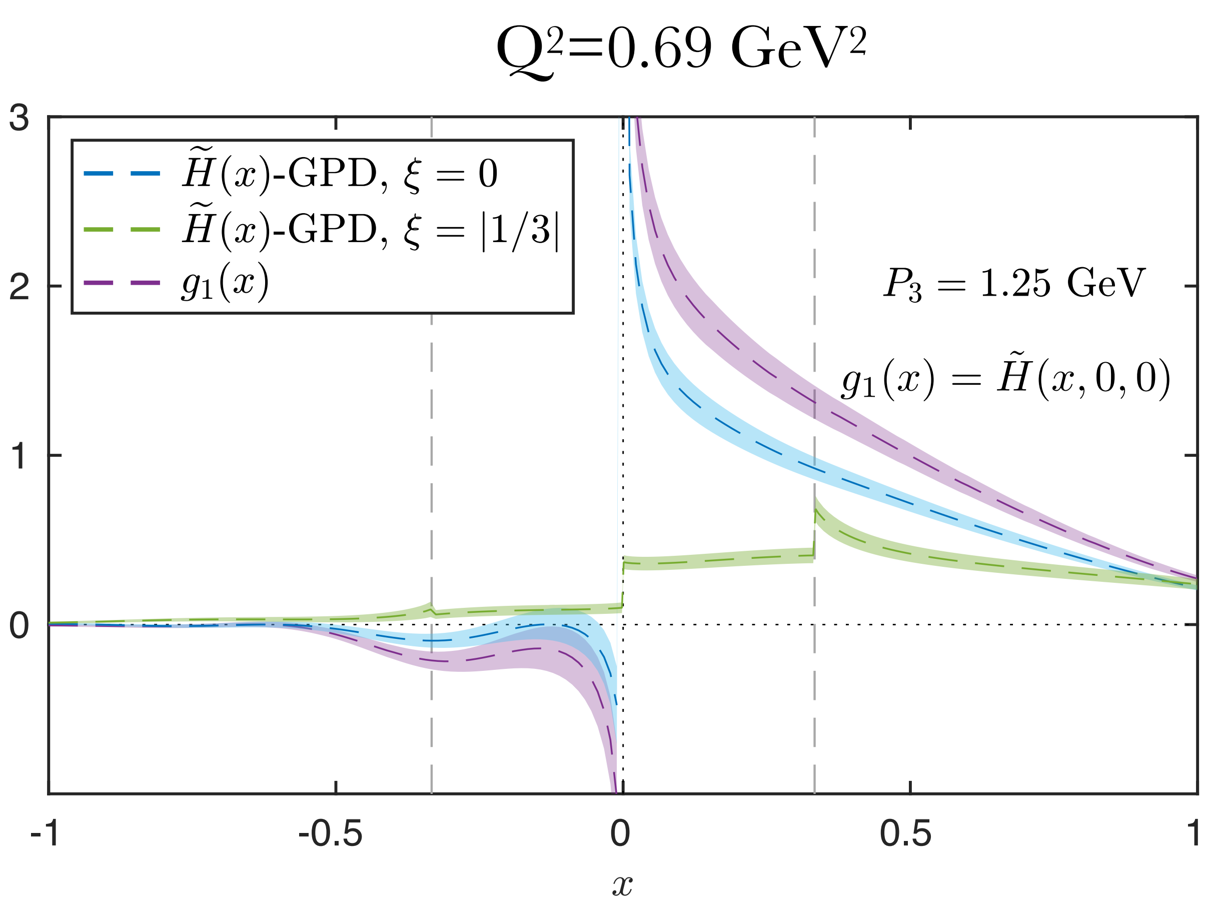
X.Ji *et al.*, Phys.Rev. D92 (2015) 014039
X.Xiong, J-H. Zhang, Phys.Rev. D92 (2015) 054037
Y-S. Liu *et al.*, Phys.Rev. D100 (2019), 034006

- First studies for pion and nucleon GPDs

J.W. Chen, H.W. Lin, J.H. Zhang, Nucl. Phys. B 952, 114940 (2020), 1904.12376
C. A. *et al.*, Phys.Rev.Lett. 125 (2020) 26, 262001, 2008.10573
H.-W. Lin, Phys. Rev. Lett. 127, 182001 (2021), 2008.12474
H.-W. Lin, Phys. Lett. B 824, 136821 (2022), 2112.07519

First results on helicity GPD

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$



C. A. *et al.*, Phys.Rev.Lett. 125 (2020) 26, 262001, 2008.10573

(5) GDPs can be computed directly in lattice QCD

Conclusions

(1) Lattice QCD yields precision results on e.g. nucleon axial charge, form factors, etc - reproduces benchmark quantities

—> Precision era of lattice QCD: A number of accurate results with controlled systematics on less known quantities provide valuable input for searches of new physics, e.g nucleon scalar and tensor charges including flavor diagonal, strangeness, …

(2) Lattice QCD provides insights on the distribution of spin among the quarks and the gluons in hadrons using Mellin moments, computes accurately strangeness etc

—> yields insights into the QCD dynamics

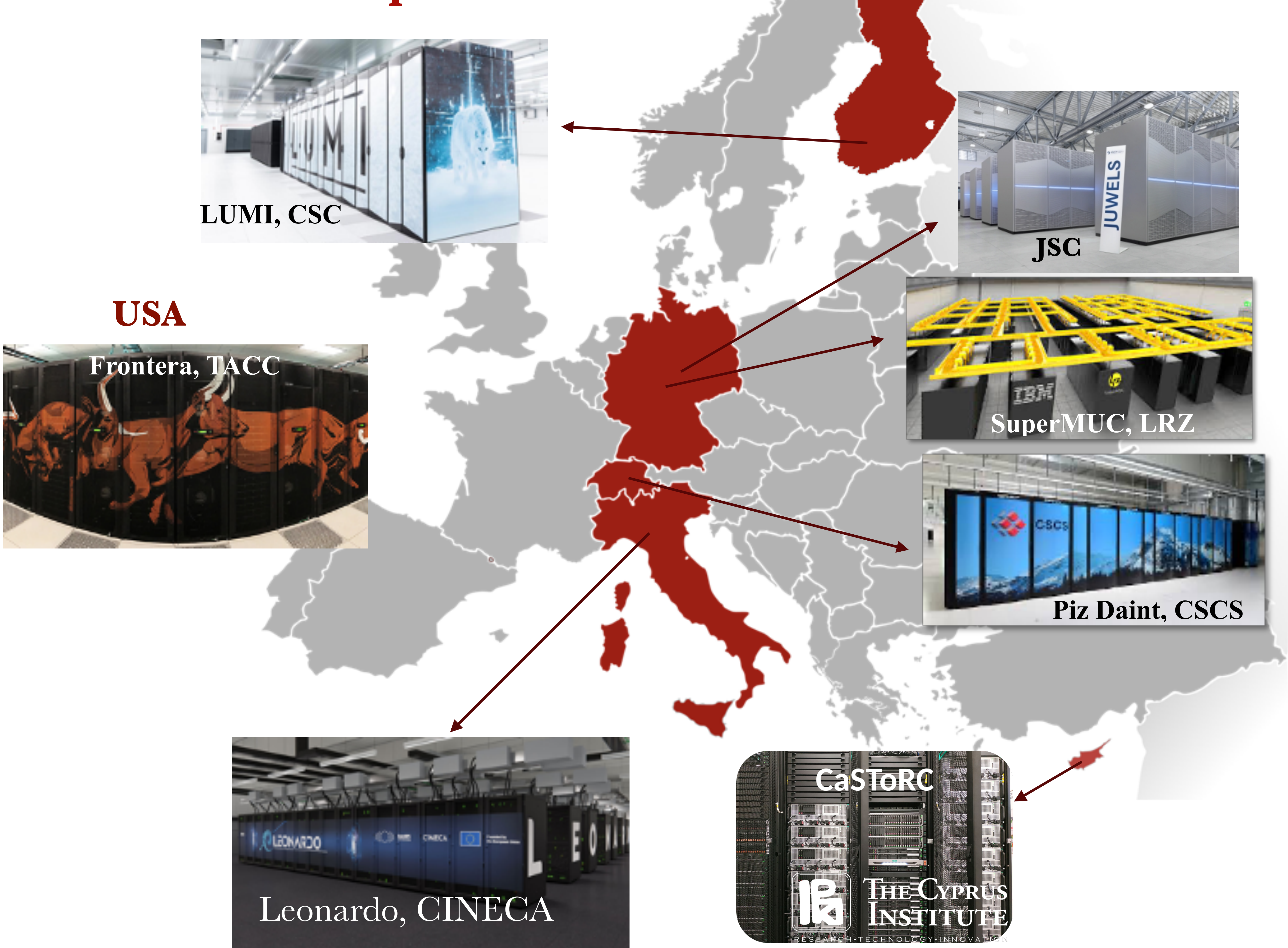
(3)Theoretical developments are enabling direct computation of GPDs and TMDs

—>Direct computation of PDFs, GPDs and TMDs probing the 3D structure of hadrons is a very active field Lattice QCD provides essential input to searchers beyond the standard model

(4) **Many other results are emerging, such as properties of resonances and exotics, phase diagram of QCD and nuclear equation of state, muon g-2, etc** (see a number of talks at this meeting)

Extended Twisted Mass Collaboration

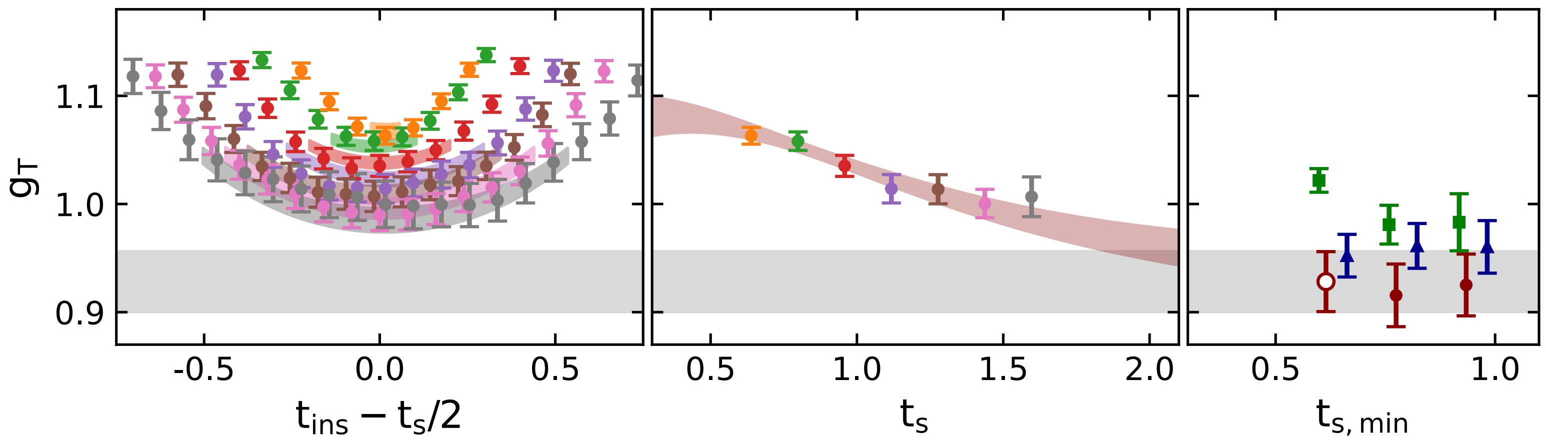
Computational resources



Backup slides

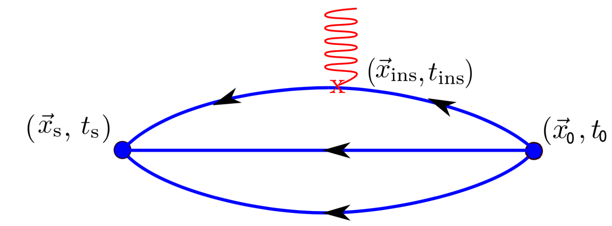
Extraction of isovector tensor charge

B-ensemble: 64^3 x 128, a~0.08 fm

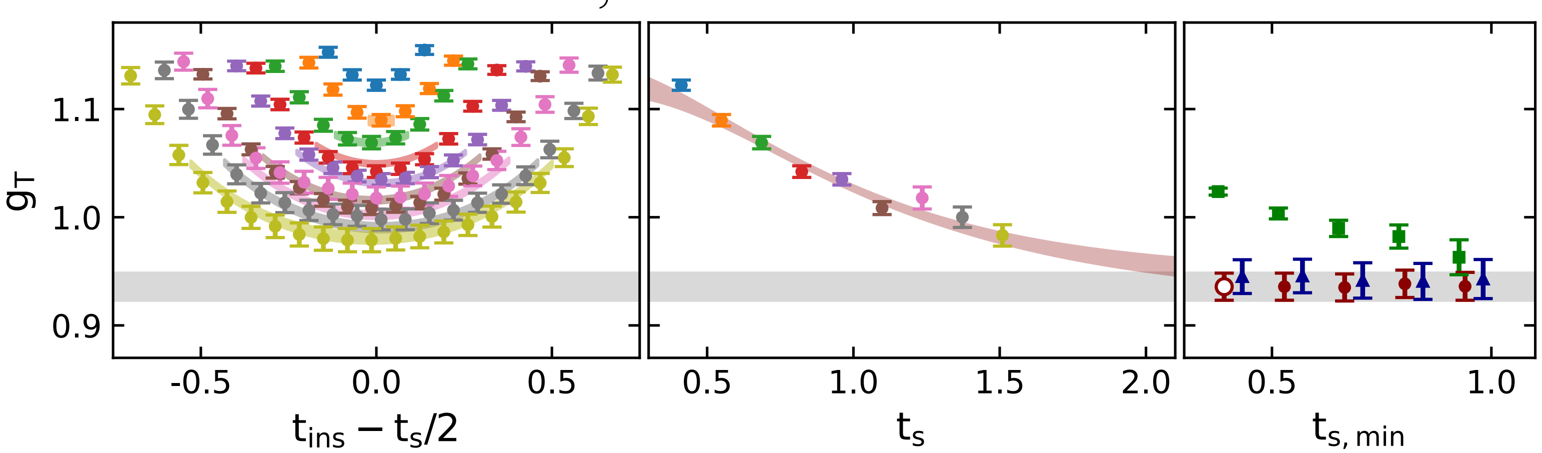


t_s/a	t_s [fm]	n_{srcs}
8	0.64	1
10	0.80	2
12	0.96	4
14	1.12	6
16	1.28	16
18	1.44	48
20	1.60	64
2-point		264

x 750 configurations = **15 M** inversions!



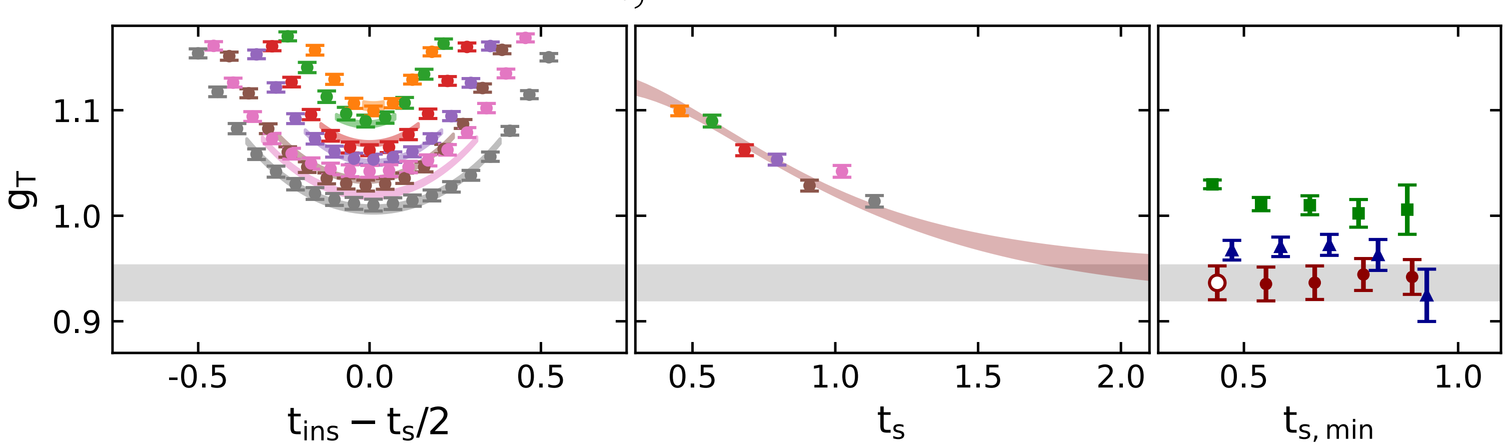
C-ensemble: 80^3 x 160, a~0.07 fm



t_s/a	t_s [fm]	n_{srcs}
8	0.549	1
10	0.686	2
12	0.823	5
14	0.961	11
16	1.098	24
18	1.235	45
20	1.372	116
22	1.509	246
2-point		650

x 401 configurations = **23.6 M** inversions!

D-ensemble: 96^3 x 192, a~0.06 fm



t_s/a	t_s [fm]	n_{srcs}
8	0.456	1
10	0.570	2
12	0.684	4
14	0.798	8
16	0.912	16
18	1.026	32
20	1.140	64
22	1.254	16
24	1.368	32
26	1.482	64
2-point		368

on-going

x 500 configurations = **15.9 M** inversions!

- Important to probe large t_s values keeping error approximately the same to reliably eliminate contributions from excited states