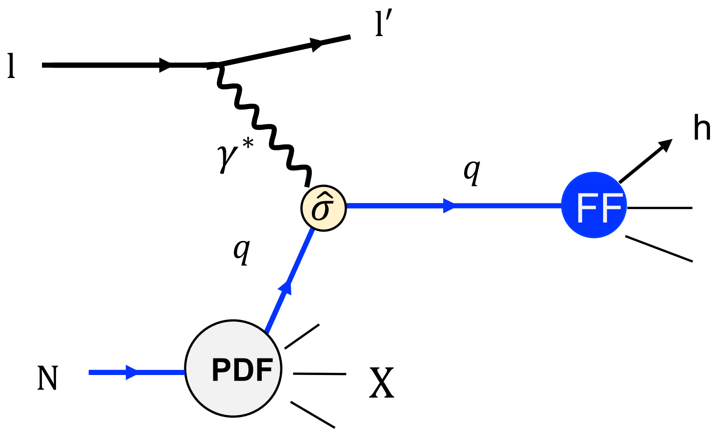
Modeling spin effects in electron-positron annihilation to hadrons

Albi Kerbizi
University of Trieste and INFN Trieste

work done in the context of the POLFRAG project

Nucleon structure and hadronization



Semi-inclusive deep inelastic scattering (SIDIS)
powerful tool used to study the partonic structure of nucleons

Couples PDFs and fragmentation functions (FFs)

transverse spin structure → involves the fragmentation of transversely polarized quarks
described by the **Collins FF** $\mathbf{H_{1q}^{\perp h}}$

Collins, NPB 396, 161 (1993).

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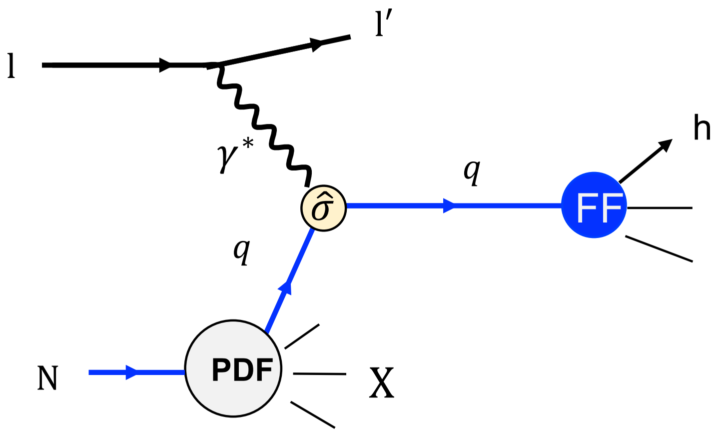

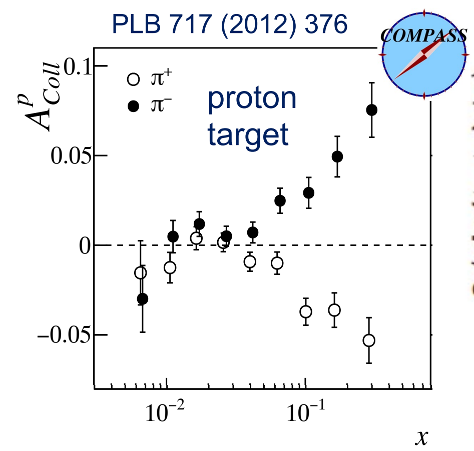

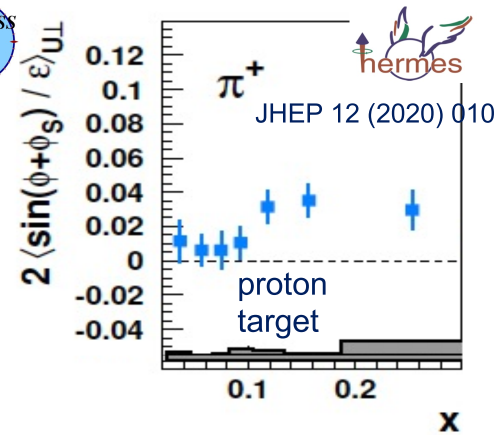

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Example: Collins asymmetry

$$A_{UT}^{\sin\phi_h+\phi_S-\pi} = \frac{\sum_q e_q^2 h_1^q \otimes H_{1q}^{\perp h}}{\sum_q e_q^2 f_1^q \otimes D_{1q}^h}$$

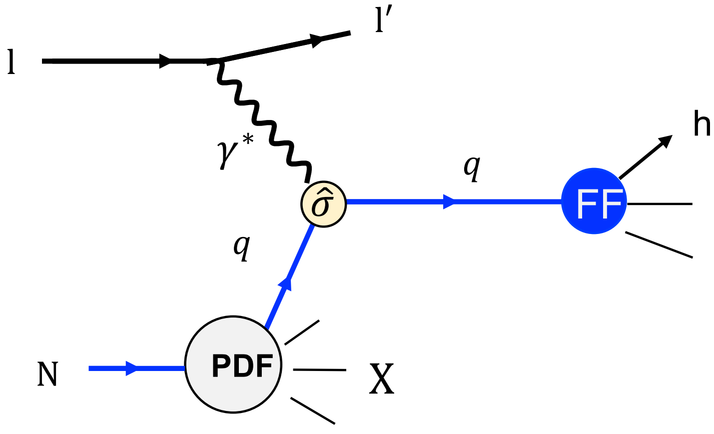
h_1^q → transversity PDF

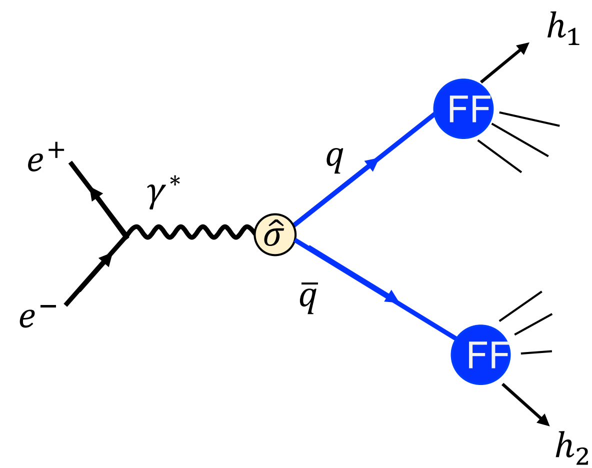
transverse polarization of quarks in a transversely polarized nucleon

Measured by HERMES (p), COMPASS (p,d), Jlab (n)

To extract transversity the Collins FF is needed

Nucleon structure and hadronization





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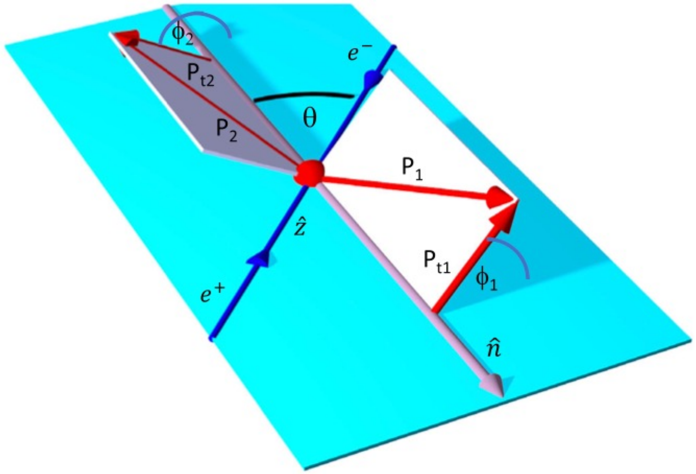
e^+e^- annihilation to hadrons
an important tool to study hadronization

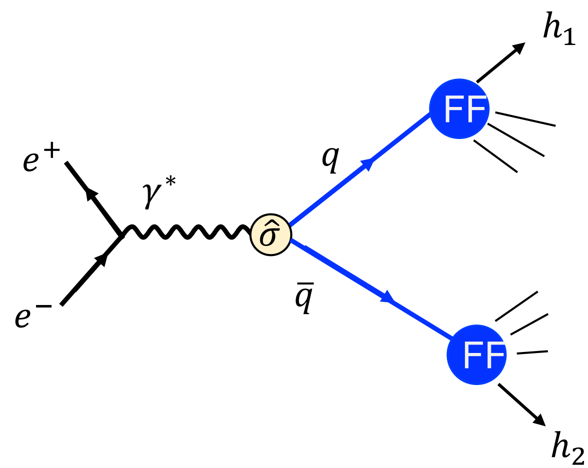
q and $\bar{q}$ have correlated transverse polarizations

→ access to Collins FF $\mathbf{H_{1q}^{\perp h}}$ via the Collins asymmetry in e^+e^-

other FFs as well: unpolarized, interference..

Nucleon structure and hadronization





Cross section for the production of two back-to-back hadrons h_1 and h_2

Boer, NPB, 806:23–67, 2009
D'Alesio et al., JHEP 10 (2021) 078

$$d\sigma^{e^+e^-\to h_1h_2X} \propto 1 + \frac{\sin^2\theta}{1+\cos^2\theta} A_{12}\cos(\phi_1+\phi_2)$$

Collins asymmetry

$$A_{12}(z_1, z_2, P_{T_1}, P_{T2}) = \frac{\sum_q e_q^2 \; H_{1q}^{\perp h_1}(z_1, P_{T1}) \times H_{1\bar{q}}^{\perp h_2}(z_2, P_{T2})}{\sum_q e_q^2 \; D_{1q}^{h_1}(z_1, P_{T1}) \times \; D_{1\bar{q}}^{h_2}(z_2, P_{T2})}$$

$z_i = 2E_{h_i}/\sqrt{s}$

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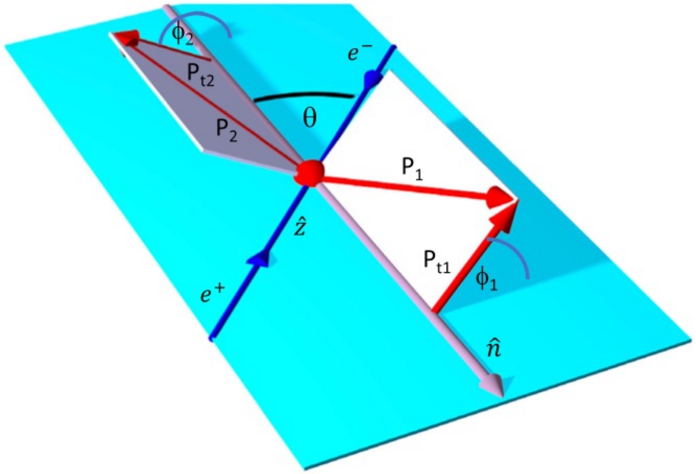

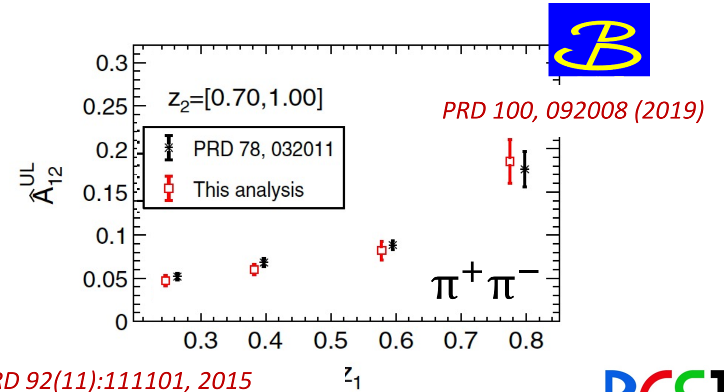

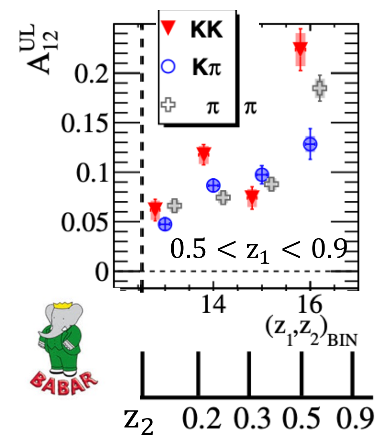

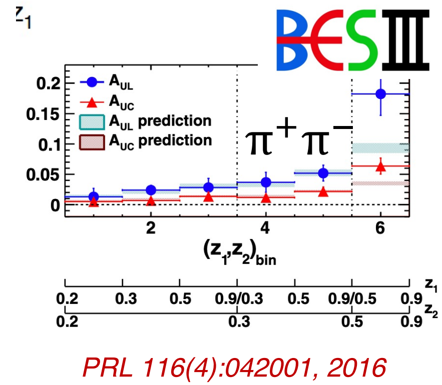

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$$z_i = 2E_{h_i}/\sqrt{s}$$

many measurements by *BELLE, BABAR, BESIII*

Used for the extractions of transversity PDFs

Anselmino et al, PRD 92 (11) (2015) 114023
Martin et al., PRD 91(1):014034, 2015
Kang et al., PRD 93 (1) (2016) 014009
…

Benchmark for hadronization models!

Modeling hadronization

We have developed a model for the simulation of the fragmentation of a quark with a given polarization → string+$^3P_0$ model

AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018)	PS mesons
AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019)	PS mesons
AK, Artru, Martin, PRD 104, 114038 (2021)	PS + VM

Implemented in Pythia for SIDIS → StringSpinner

AK, L. Lönnblad, CPC **272** (2022) 108234	PS, Pythia 8.2
AK, L. Lönnblad, arXiv: 2305.05058	PS + VM, Pythia 8.3

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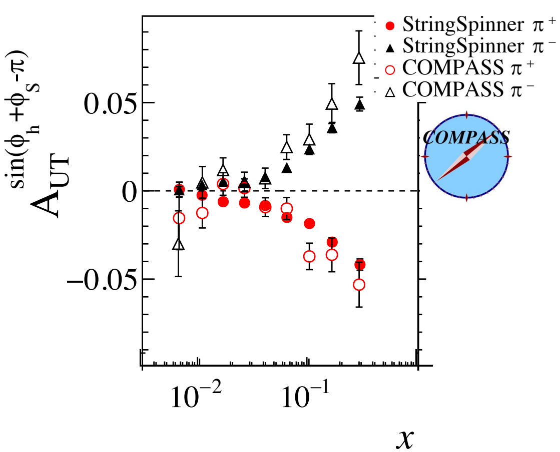
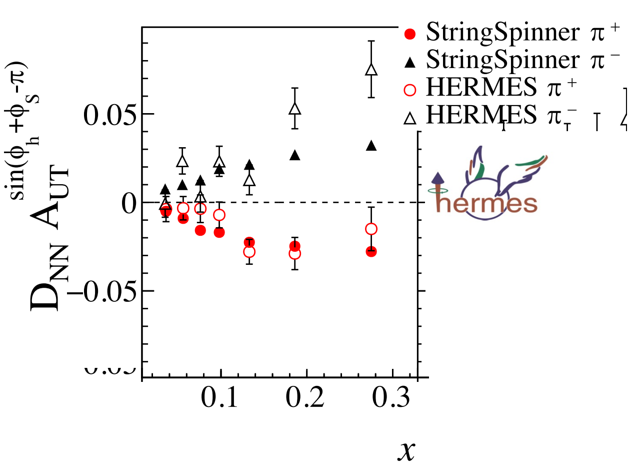
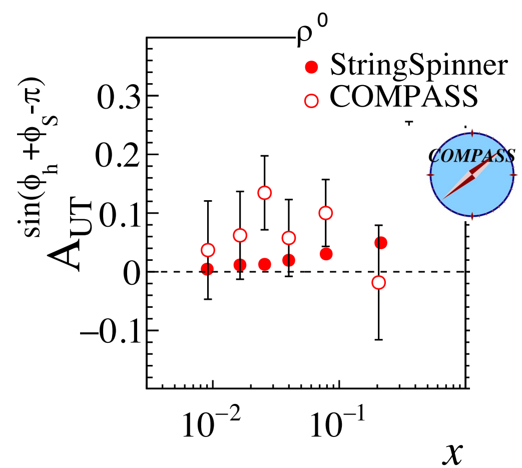






Promising results for SIDIS! (more results in arXiv: 2305.05058)

A similar work for e^+e^- annihilation does not exist

requires extension of string+$^3P_0$ model → this talk

In the following slides

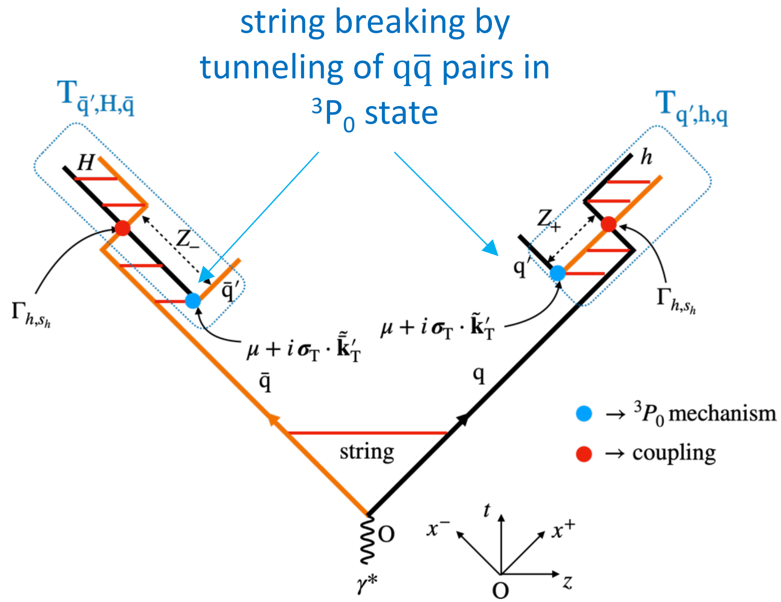
i) recall of the string+$^3P_0$ model

ii) recipe for the simulation of $e^+e^- \rightarrow q\bar{q} \rightarrow$ hadrons

application of the model to the hadronization of a quark-antiquark pair with correlated spin states

in collaboration with X. Artru

The hadronization model: **string+$^3P_0$**



the hadronization of the $q\bar{q}$ pair is described in the string fragmentation framework supplemented with the $^3P_0$ model of quark tunneling

extension of the Lund string Model (Pythia)

AK, Artru, Martin, PRD 104, 114038 (2021)

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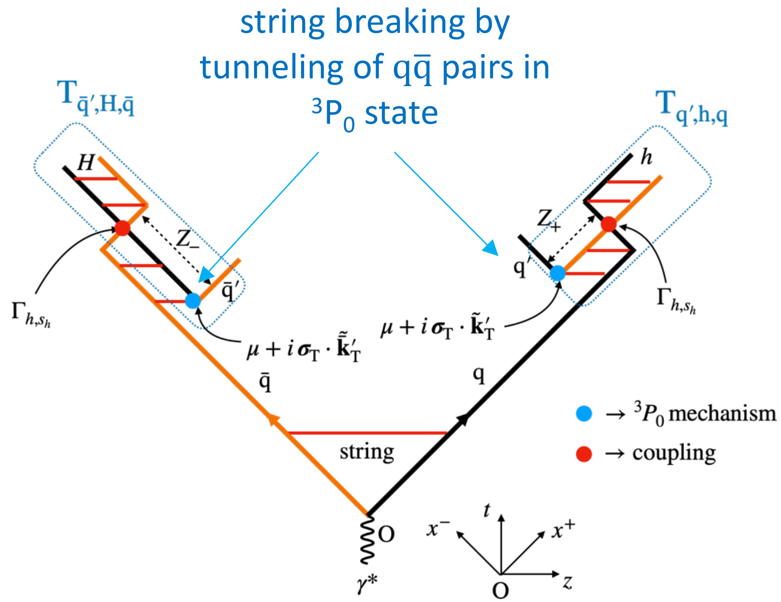
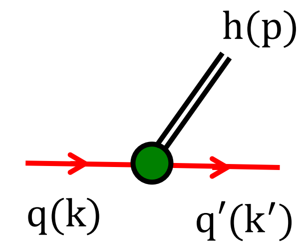




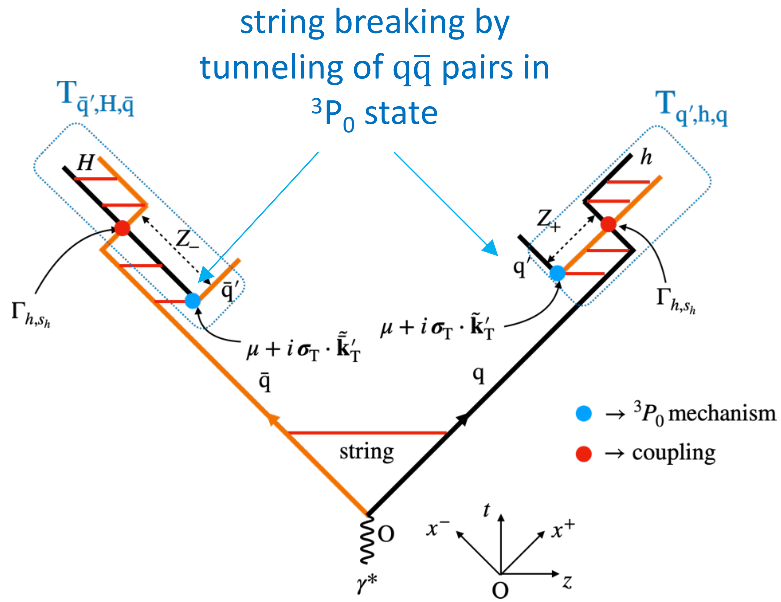
quark splitting $q \to h + q'$

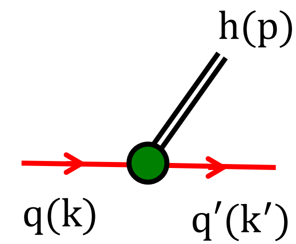
Relevant variables:
$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$
$Z_+ = p^+/k^+$
$\varepsilon_h^2 = M^2 + p_T^2$

Transverse vectors defined w.r.t. string axis

AK, Artru, Martin, PRD 104, 114038 (2021)

The hadronization model: string+$^3P_0$





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Quark splitting amplitude in the string+$^3P_0$ model

$$T_{q',h,q} \propto C_{q',h,q}\, D_h(M^2) \left(\frac{1-Z_+}{\varepsilon_h^2}\right)^{\frac{a}{2}} \exp\left[-\frac{b_L \varepsilon_h^2}{2Z_+}\right] N_a^{-\frac{1}{2}}(\varepsilon_h^2)\, e^{-\frac{b_T k'^2_T}{2}} \quad [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \quad \Gamma_{h,s_h}$$

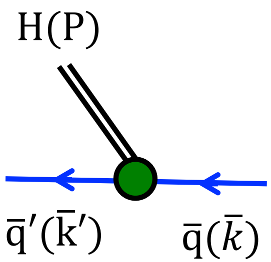
flavor — mass — longitudinal momentum — transverse momentum (w.r.t string axis) — $^3P_0$ mechanism [μ complex mass paramter] — Coupling e.g. $\Gamma_{h=PS} = \sigma_z$

Free param. Lund

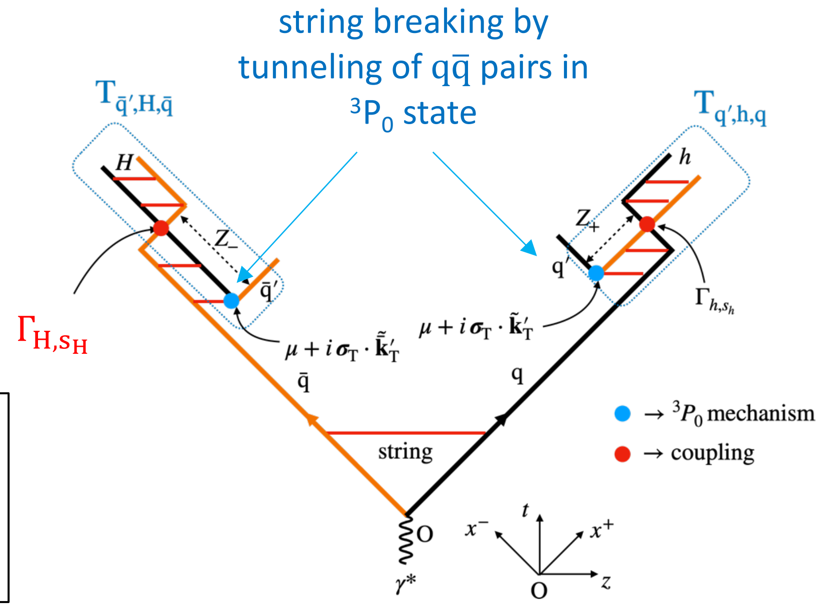
Free param. string+$^3P_0$

AK, Artru, Martin, PRD 104, 114038 (2021)

The hadronization model: **string+$^3P_0$**



antiquark splitting
$\bar{q} \rightarrow H + \bar{q}'$



Relevant variables:

$\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$

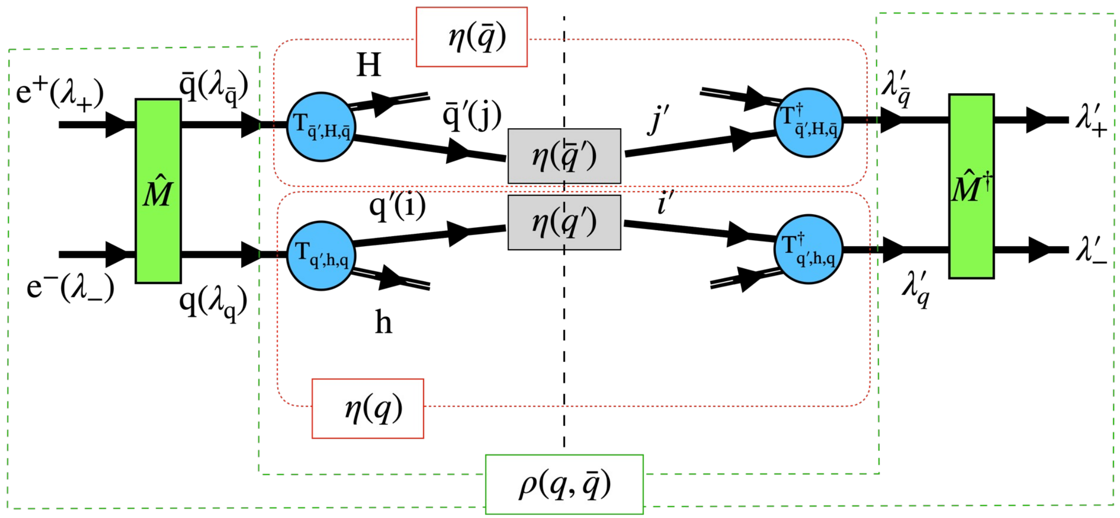
$Z_- = P^-/\bar{k}^-$

$\varepsilon_H^2 = M^2 + P_T^2$

Antiquark splitting amplitude in the string+$^3P_0$ model obtained by the quark one by

$$\{q, h, q'\} \rightarrow \{\bar{q}, H, \bar{q}'\}, \qquad Z_+ \rightarrow Z_-, \qquad \{\mathbf{k}_T, \mathbf{p}_T \mathbf{k}'_T\} \rightarrow \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$$

The ingredients needed for e^+e^- annihilation



Squared amplitude associated to the unitarity diagram (factorized approach)

$$|A(e^+e^- \to h\,H\,X)|^2 = \langle |M|^2 \rangle \times \mathrm{Tr}_{q\bar{q}}\, \rho(q,\bar{q})\, \eta(q) \otimes \eta(\bar{q})$$

Artru et al., Phys. Rept., 470:1–92, 2009

Spin-averaged squared matrix element for the hard scattering $e^+e^- \to q\bar{q}$

Joint spin density matrix of $q\bar{q}$

$$\rho(q,\bar{q}) = C^{q\bar{q}}_{\alpha\beta}\, \sigma^\alpha_q \otimes \sigma^\beta_{\bar{q}}$$

implements spin correlations

$C^{q\bar{q}}_{\alpha\beta} \to$ correlation coefficients

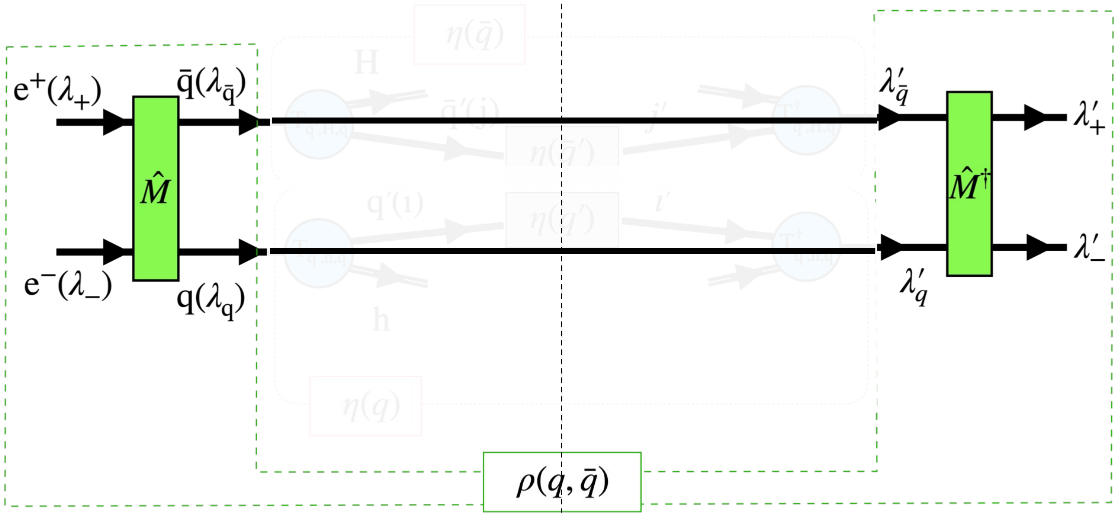
Acceptance matrix of q

information coming to q from "future" emissions

$$\eta(q) = \begin{cases} 1_q & \text{(no info)} \\ T^\dagger_{q',h,q}\, \eta(q')\, T_{q',h,q} & q \to h + q' \end{cases}$$

similarly for $\eta(\bar{q})$

The recursive recipe for simulating e^+e^- annihilation



Integrate over the emissions of q and $\bar{q}$ and set up the pair

no info to q or $\bar{q} \rightarrow \eta(q) = 1_q, \eta(\bar{q}) = 1_{\bar{q}}$

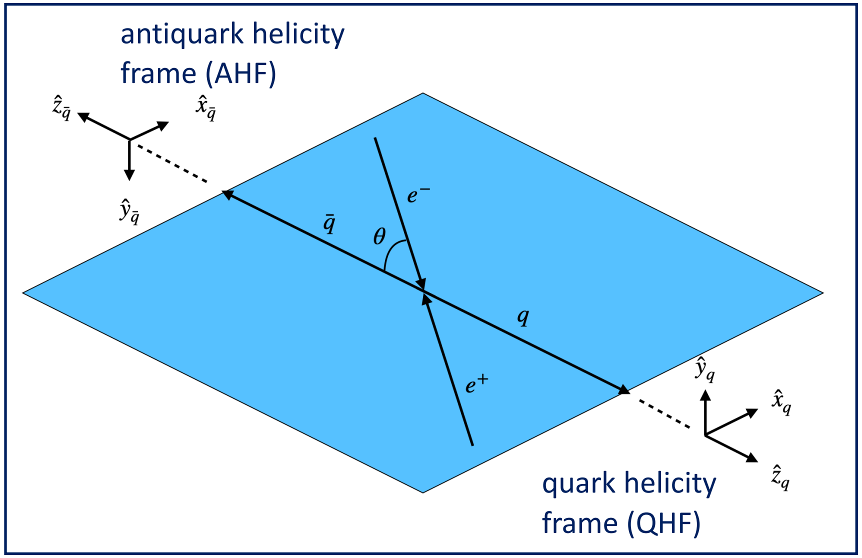
i. generate the quark flavors and kinematics using

$$d\hat{\sigma}(q\bar{q})/d\cos\theta \propto \left\langle |\hat{M}|^2 \right\rangle$$

ii. set up the joint spin density matrix

$$\rho(q,\bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma_q^z \otimes \sigma_{\bar{q}}^z + \sin^2\theta \, [\sigma_q^x \otimes \sigma_{\bar{q}}^x + \sigma_q^y \otimes \sigma_{\bar{q}}^y]/(1+\cos^2\theta)$$

$\sigma_a^\alpha \rightarrow$ Pauli matrices along $\alpha = 0, x, y, z$ in the helicity frame of $a = q, \bar{q}$



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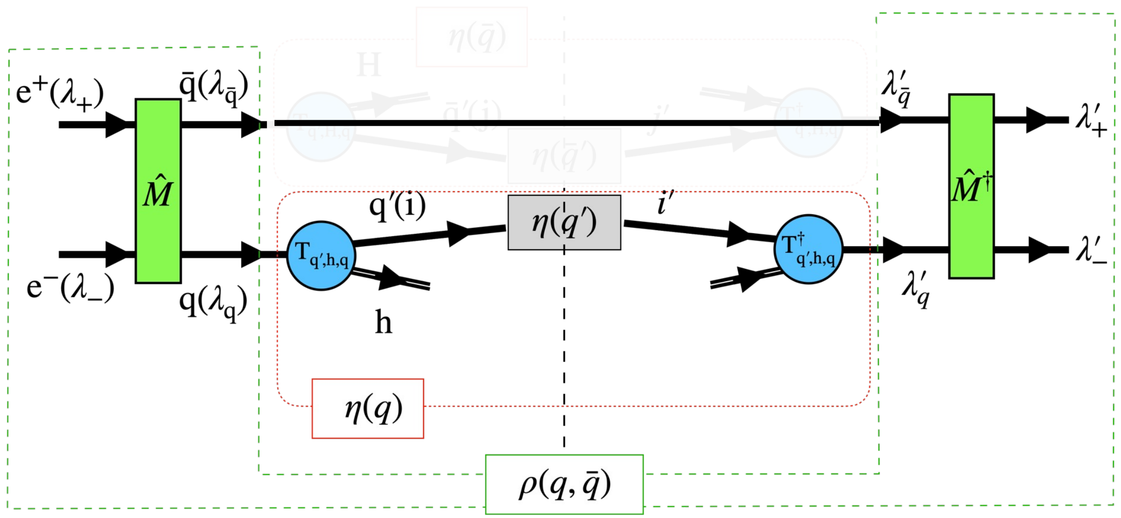


Emit the first hadron the q side

assuming h=PS from

$\rightarrow \eta(q) = T^\dagger_{q',h,q}\, \eta(q')\, T_{q',h,q}$ (no info to q' $\rightarrow$ $\eta(q') = 1_{q'}$)

Emission probability density (splitting function)

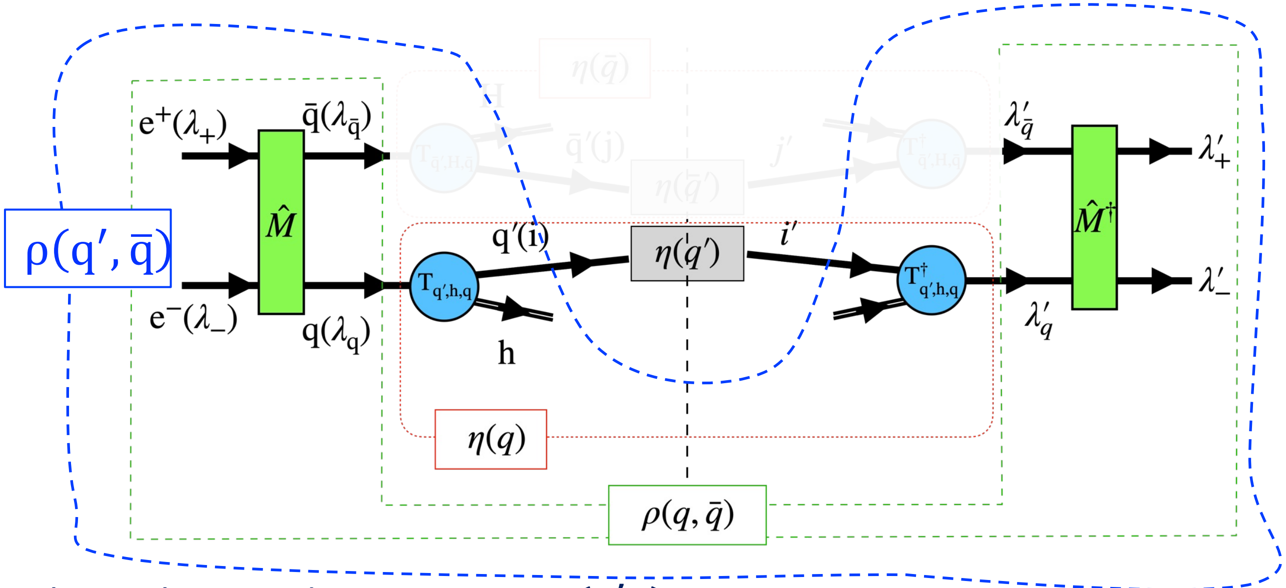
$$\frac{dP(q \rightarrow h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2 p_T} = \mathrm{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^\dagger_{q',h,q} = F_{q',h,q}(Z_+, p_T; k_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$

emission of a vector meson h=VM more involved (but similar steps)

→ see backup

The recursive recipe for simulating e^+e^- annihilation

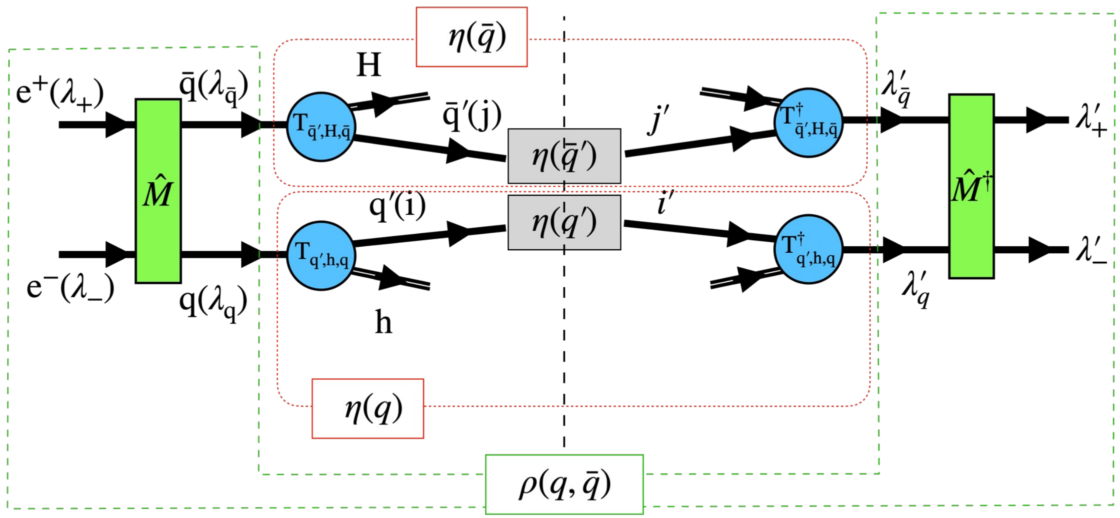


Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q',\bar{q}) = \mathbf{T}_{q',h,q}\,\rho(q,\bar{q})\mathbf{T}^\dagger_{q',h,q}$$

includes the information on the emission of h

The recursive recipe for simulating e^+e^- annihilation



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includes the information on the emission of h

Emit a hadron H, e.g. H = PS, from the antiquark

$$\rightarrow \eta(\bar{q}) = T^\dagger_{\bar{q}',H,\bar{q}}\, \eta(\bar{q}')\, T_{\bar{q}',H,\bar{q}} \qquad \text{(no info to } \bar{q}' \rightarrow \eta(\bar{q}') = 1_{\bar{q}'}\text{)}$$

which gives the emission probability density

$$\frac{dP(\bar{q} \rightarrow H + \bar{q}'; q'\bar{q})}{dZ_- Z_-^{-1} d^2P_T} = \mathrm{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}',H,\bar{q}}\, \rho(q',\bar{q})\, \mathbf{T}^\dagger_{\bar{q}',H,\bar{q}} = F_{\bar{q}',H,\bar{q}}(Z_-, P_T; \bar{k}_T, C^{q'\bar{q}})$$

Depend on the azimuthal angle h

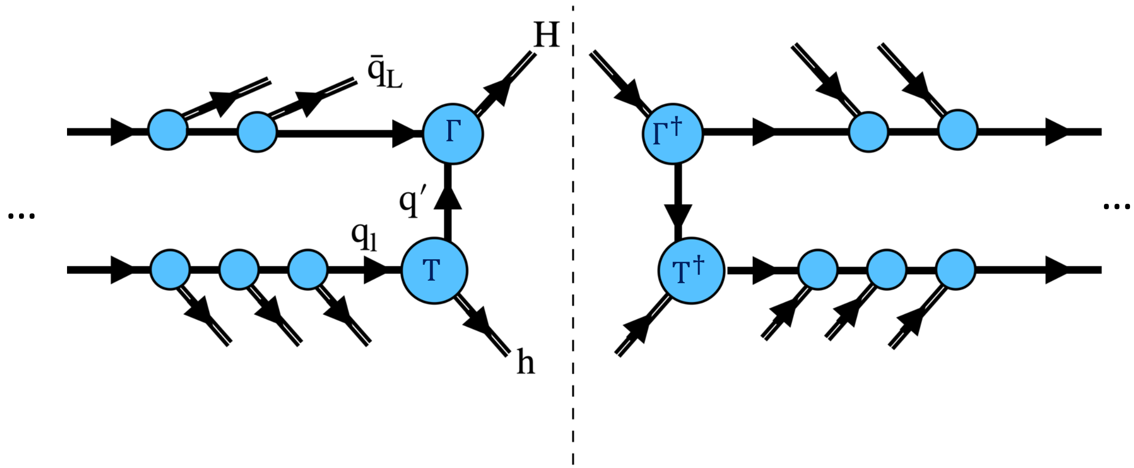
Expressed in the AHF

conditional probability of emitting H, having emitted h

→ correlations between their transverse momenta

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

The recursive recipe for simulating e^+e^- annihilation: exit condition



After several emissions hadronize the last pair $q_l\bar{q}_L$

joint spin-density matrix $\rho(q_l,\bar{q}_L)$

Emit the hadron $h = q_l\bar{q}'$ from q_l and project $\bar{q}_L q'$ to the state H

$$dP(q_l \to h + q'; q_l\bar{q}_L) = \mathrm{Tr}_{q'\bar{q}_L}\left[T_{q',h,q_l} \otimes \Gamma_{H,s_H}\right] \quad \rho(q_l,\bar{q}_L) \quad \left[T^\dagger_{q',h,q_l} \otimes \Gamma^\dagger_{H,s_H}\right]$$

or emit the hadron $H = q'\bar{q}_L$ from $\bar{q}_L$ and project $q_l\bar{q}'$ to the state h

Application of the recipe to the first two hadrons produced

Application of the recipe to $e^+e^- \to h\,H\,X$

h = PS and H = PS being the first two hadrons produced

$$dP(e^+e^- \to h\,H\,X) = \hat{\sigma}^{-1}\frac{d\hat{\sigma}}{d\cos\theta} \times F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}}) \times F_{\bar{q}',H,\bar{q}}(Z_-, \bar{\mathbf{p}}_T; \bar{\mathbf{k}}_T, C^{q'\bar{q}})$$

$\text{Prob}(e^+e^- \to q\bar{q})$ $\quad$ $\text{Prob}(q \to h + q')$ $\quad$ $\text{Prob}(\bar{q} \to H + \bar{q}'; q \to h + q')$

$$\propto (1+\cos^2\theta) \times (\dots) \times \left[1 + \frac{\sin^2\theta}{1+\cos^2\theta}\,\frac{2\mathrm{Im}(\mu)p_T}{|\mu|^2+p_T^2}\,\frac{2\mathrm{Im}(\mu)P_T}{|\mu|^2+P_T^2}\cos(\phi_h+\phi_H)\right]$$

expected form for the azimuthal distribution of back-to-back hadrons!

For quantitative results and phenomenology

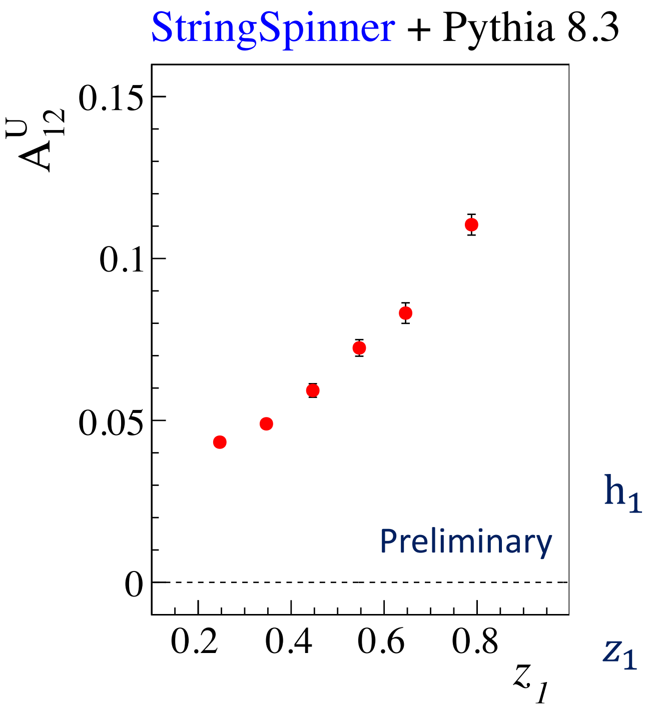
→ implementation of the model in Pythia 8.3 for of $e^+e^- \to$ hadrons

we are extending the StringSpinner package (currently for SIDIS) to simulate also e^+e^-

ongoing work in collaboration with L. Lönnblad and A. Martin

Example of simulation results with Pythia 8.3

Collins asymmetry for back-to-back $\pi^+$and π^- in $e^+e^- \rightarrow u\bar{u} \rightarrow \pi^+\pi^- X$ at $\sqrt{s} = 10$ GeV
only PS mesons in simulations



$h_1 = \pi^{\pm}, \quad h_2 = \pi^{\pm}$

$$z_i = \frac{2E_i}{\sqrt{s}}$$

$z_1 > 0.2, \quad z_2 > 0.2$

The asymmetry reproduces the main features of the data
- positive sign
- rising trend with fractional energy

more quantitative analysis will be performed

Conclusions

We generalized the string+$^3P_0$ model of hadronization to $e^+e^- \rightarrow q\bar{q} \rightarrow$ hadrons

recursive quantum mechanical recipe

The recipe is general, independent on the production mechanism of the $q\bar{q}$ pair

The implementation in Pythia 8.3 is ongoing

preliminary Collins asymmetry for back-to-back pions is as expected

(More) phenomenological studies ongoing

the goal is to publish the results in few months..

Backup

Relevant free parameters for string fragmentation used in simulations

(see AK, L. Lönnblad, arXiv: 2305.05058)

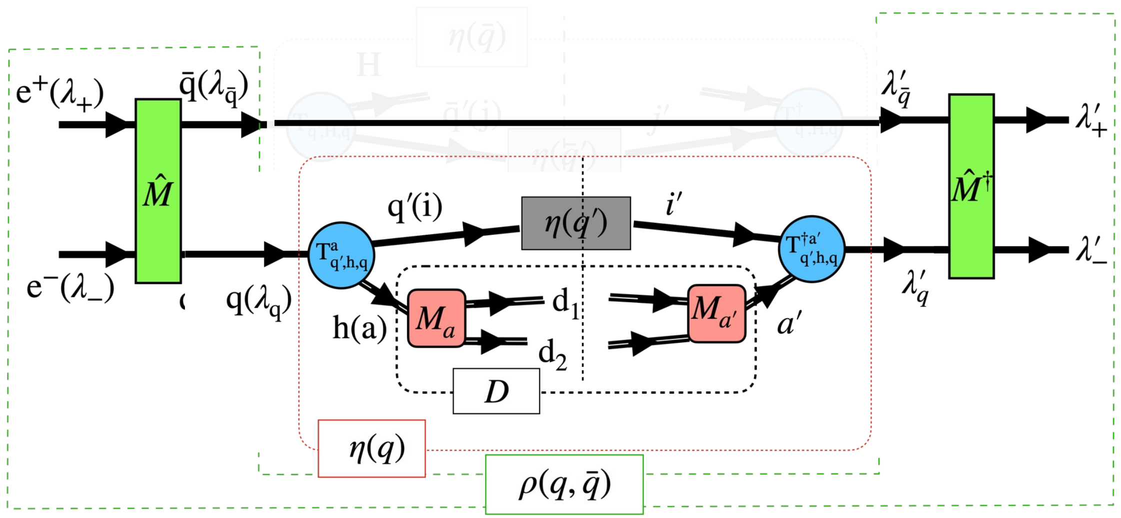
Pythia parameters

StringZ:aLund	default
StringZ:bLund	default
StringPT:sigma	default
StringPT:enhancedFraction	0.0
StringPT:enhancedWidth	0.0 $\mathrm{GeV/c}$

String+$^3P_0$ parameters

$\mathrm{Re}(\mu)$	0.42 $\mathrm{GeV/c^2}$
$\mathrm{Im}(\mu)$	0.76 $\mathrm{GeV/c^2}$
f_L	0.93
θ_{LT}	0

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson h=VM

$$\rightarrow \eta(q) = \mathbf{T}^{a'\dagger}_{q',h=VM,q}\, \eta(q')\, \mathbf{T}^{a}_{q',h=VM,q} D_{a'a},\ \eta(q') = 1_{q'} \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \rightarrow h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2p_T} = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^{a}_{q',h,q}\ \rho(q,\bar{q})\ \mathbf{T}^{a\dagger}_{q',h,q} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of h=VM, and decay the meson

$$\rho_{aa'}(h) = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^{a}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{a'\dagger}_{q',h,q}$$

iii) Decay the meson $p \rightarrow p_1\, p_2 ..$

$$dN(p_1, p_2 ..)/d\Omega \propto M^{a}_{dec.}(p \rightarrow p_1 p_2 ..)\ \rho_{aa'}(h) M^{\dagger a'}_{dec.}(p \rightarrow p_1 p_2 ..)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, ..) = M^{\dagger a'}_{dec.}(p \rightarrow p_1 p_2 ..)\ M^{a}_{dec.}(p \rightarrow p_1 p_2 ..)$