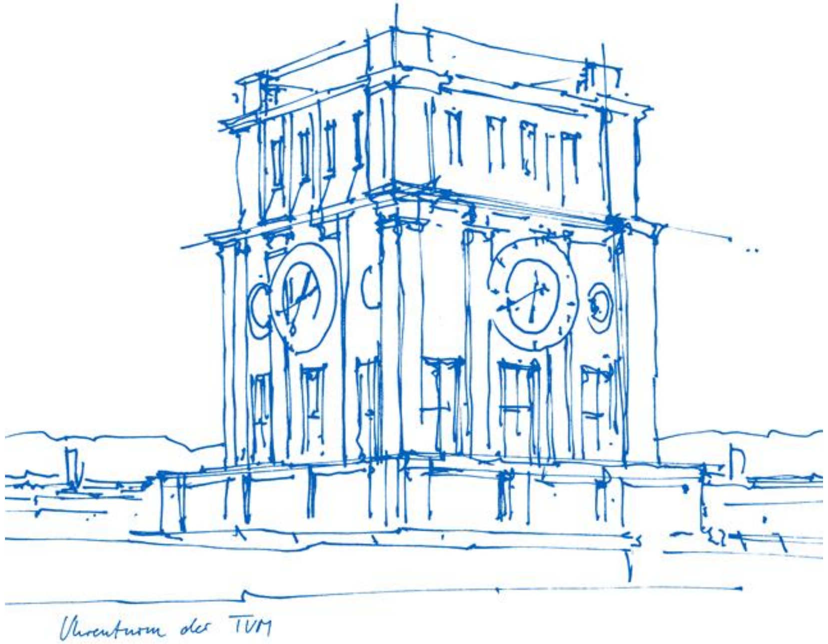
Testing Predictions of the Chiral Anomaly in Primakoff Reactions at COMPASS

Dominik Ecker, Andrii Maltsev **on behalf of the COMPASS collaboration**

HADRON 08/06/2023



The chiral anomaly

- Lagrange density of QCD:

$$\mathcal{L}_{QCD} = \sum_{f=\substack{u,d,s,\\c,b,t}} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

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Symmetry breaking term: $m_f = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$

Chiral limit: $m_u, m_d, m_s = 0$

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$$q(x) \rightarrow e^{i\theta\gamma_5}q(x)$$

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- Features *axial* $U(1)$-symmetry in chiral limit:

$$q(x) \rightarrow e^{i\theta\gamma_5} q(x) \xrightarrow{\text{Noether}} \partial_\mu A_0^\mu = \sum_{f=u,d,s} i2m_f \bar{q}_f \gamma_5 q^f$$

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Noether

violates color symmetry

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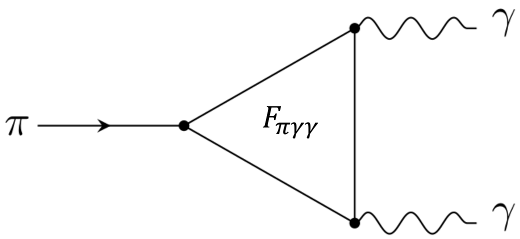

- **Anomaly:** Symmetry of classical Lagrangian violated at quantum level (by renormalization choice)

- Adler, Bell, Jackiw 1969: $\tau_{\mathrm{anom}}(\pi^0) = (9.5 \pm 1.5) \cdot 10^{-17}\mathrm{s} \neq \tau_{\mathrm{theory}}(\pi^0) \approx 10^{-13}\mathrm{s}$

The chiral anomaly

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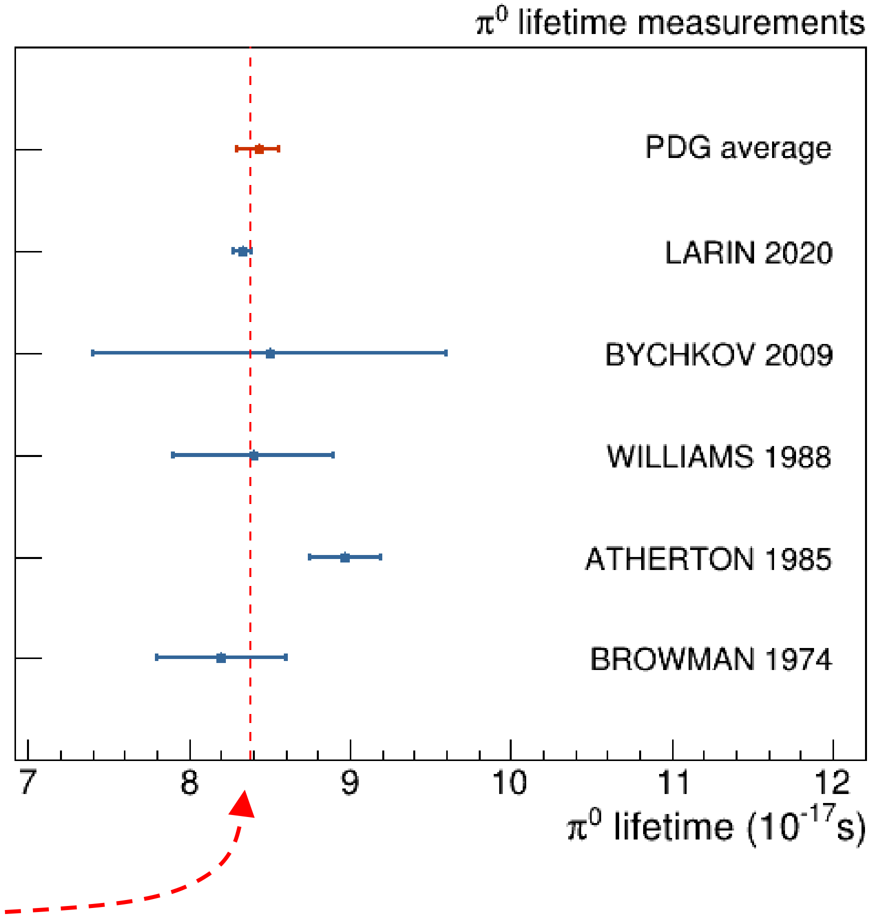
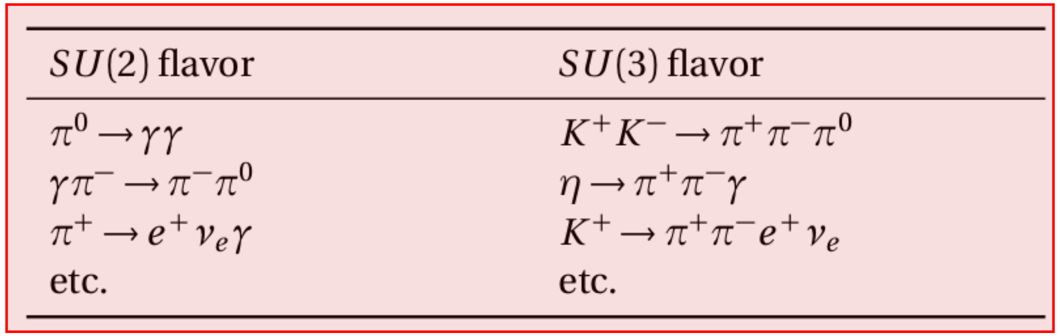


well tested in π^0 decay

Anomalous processes

- Chiral anomaly governs couplings of odd number of Goldstone bosons:

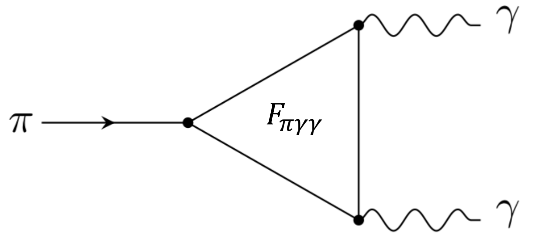


$SU(2)$ flavor	$SU(3)$ flavor
$\pi^0 \to \gamma\gamma$	$K^+K^- \to \pi^+\pi^-\pi^0$
$\gamma\pi^- \to \pi^-\pi^0$	$\eta \to \pi^+\pi^-\gamma$
$\pi^+ \to e^+\nu_e\gamma$	$K^+ \to \pi^+\pi^- e^+\nu_e$
etc.	etc.

- On tree-level: low-energy theorems with few parameters, e.g. pion decay constant F_π measured from leptonic decays of the charged pion $(\pi^\pm \to \mu^\pm + \nu)$

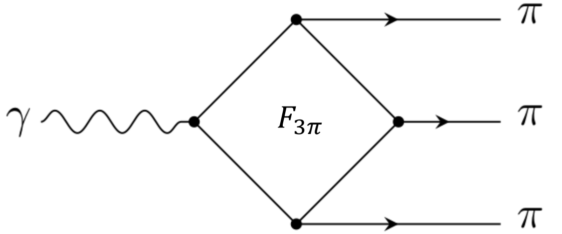
- Higher order corrections via Chiral Perturbation Theory (ChPT)

$F_{\pi\gamma\gamma}$



- $F_{\pi\gamma\gamma} = \frac{e^2 N_C}{12\pi^2 F_\pi} = 2.52 \cdot 10^{-2}\mathrm{GeV}^{-1}$

$F_{3\pi}$

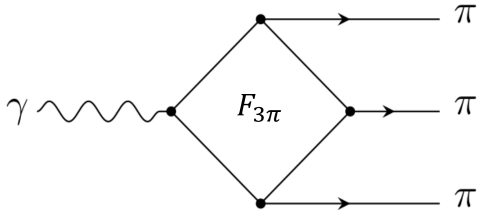


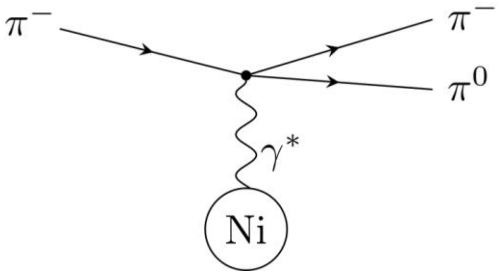
- $F_{3\pi} = \frac{e N_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05)\mathrm{GeV}^{-3}$

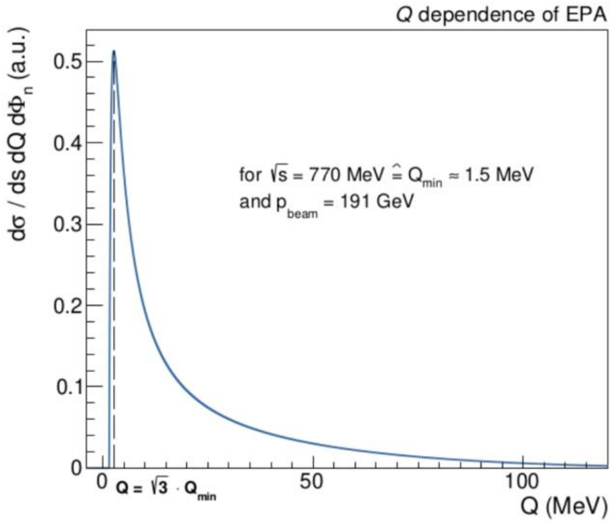
Testing the chiral anomaly - $F_{3\pi}$

- $F_{3\pi}$: Direct coupling of γ to 3π - process proceeds primarily via the chiral anomaly

- Accessible in Primakoff reactions via: $\pi^-\gamma^* \rightarrow \pi^-\pi^0$ ultra-relativistic pion scatters in e.m. field of nucleus (characterized by very low momentum transfer)







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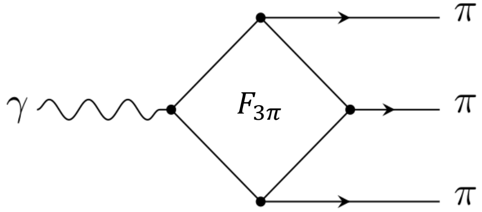

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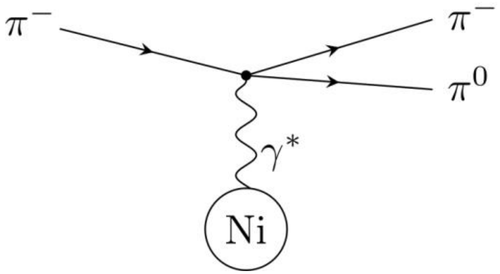
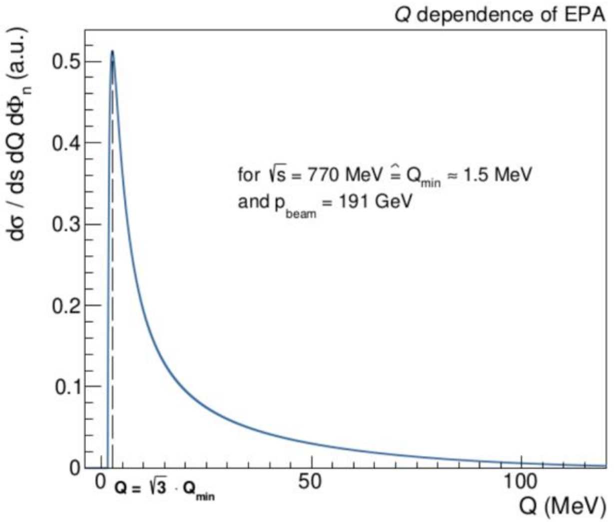


- Problem of explicit chiral symmetry breaking:

$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05)\mathrm{GeV}^{-3} = F(s=t=u=0)$$

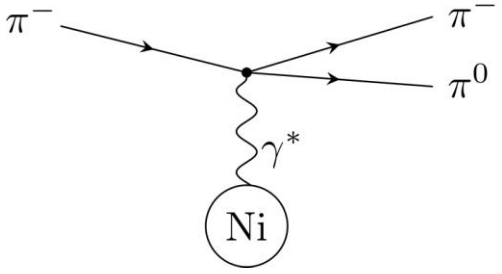
We measure at $s > (2m_\pi)^2$: use ChPT to bridge „gap“

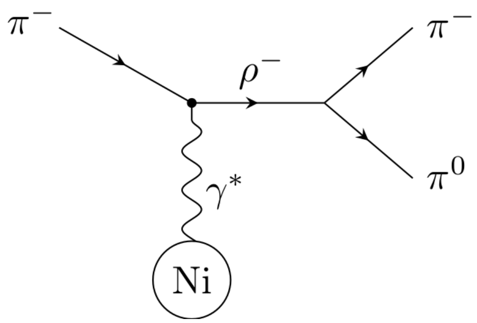
$$F_{3\pi}(s,t,u) = F_{3\pi}\left(f^{(0)}(s,t,u) + f^{(1)}(s,t,u) + f^{(2)}(s,t,u) + \ldots\right)$$



Radiative width of ρ-meson

- Cross section of $\pi^-\pi^0$ final state result of two coherent processes:

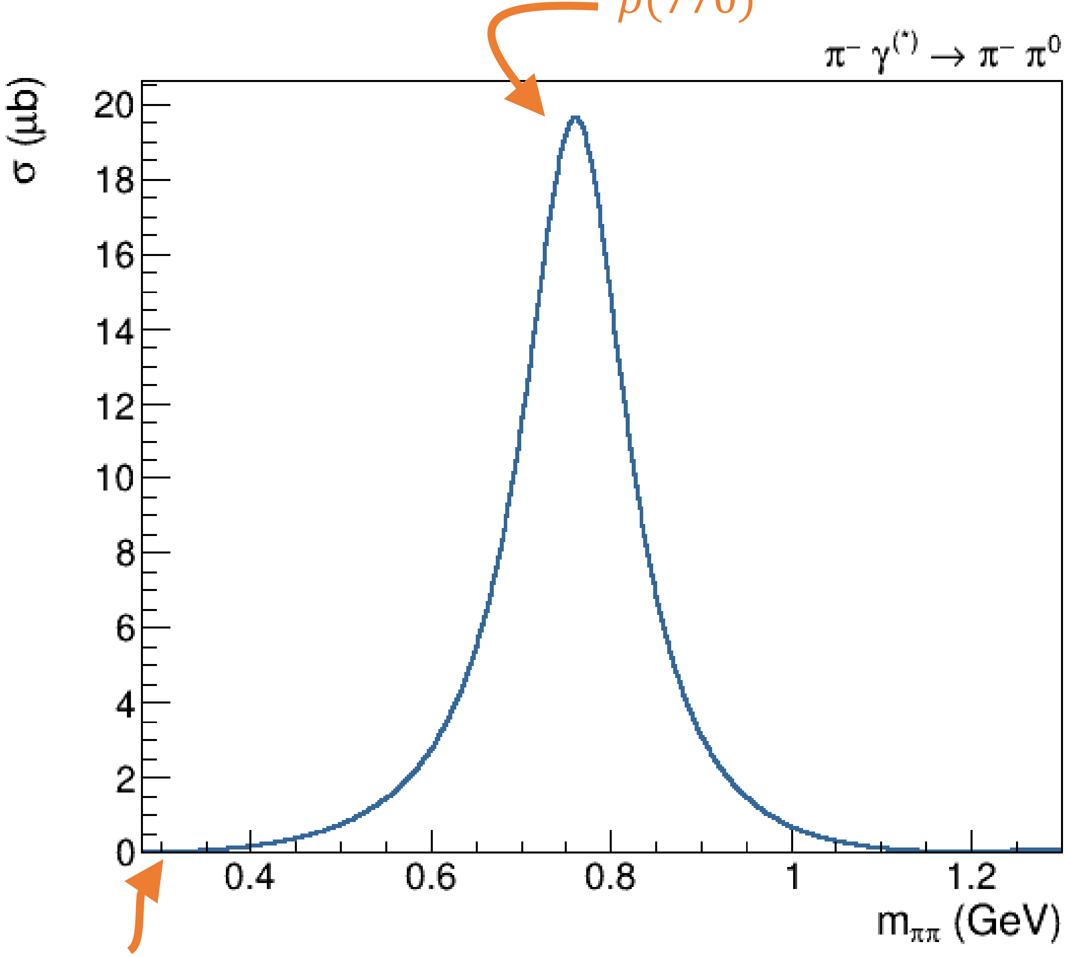




- At kinematic threshold: dominated by chiral anomaly
- Interference between Chiral Anomaly and ρ gives additional information

$\Rightarrow$ possibility of extraction of radiative width of ρ-meson: $\Gamma_{(\rho\to\pi\gamma)}/\Gamma_{\text{tot}} \approx 4.5 \cdot 10^{-4}$

$\rho(770)$



Low-mass tail:
mainly driven by $F_{3\pi}$

Radiative width of $\boldsymbol{\rho}$-meson

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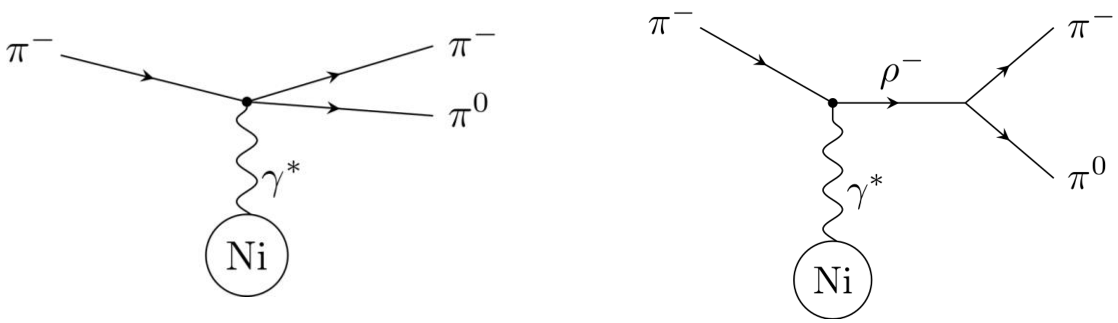

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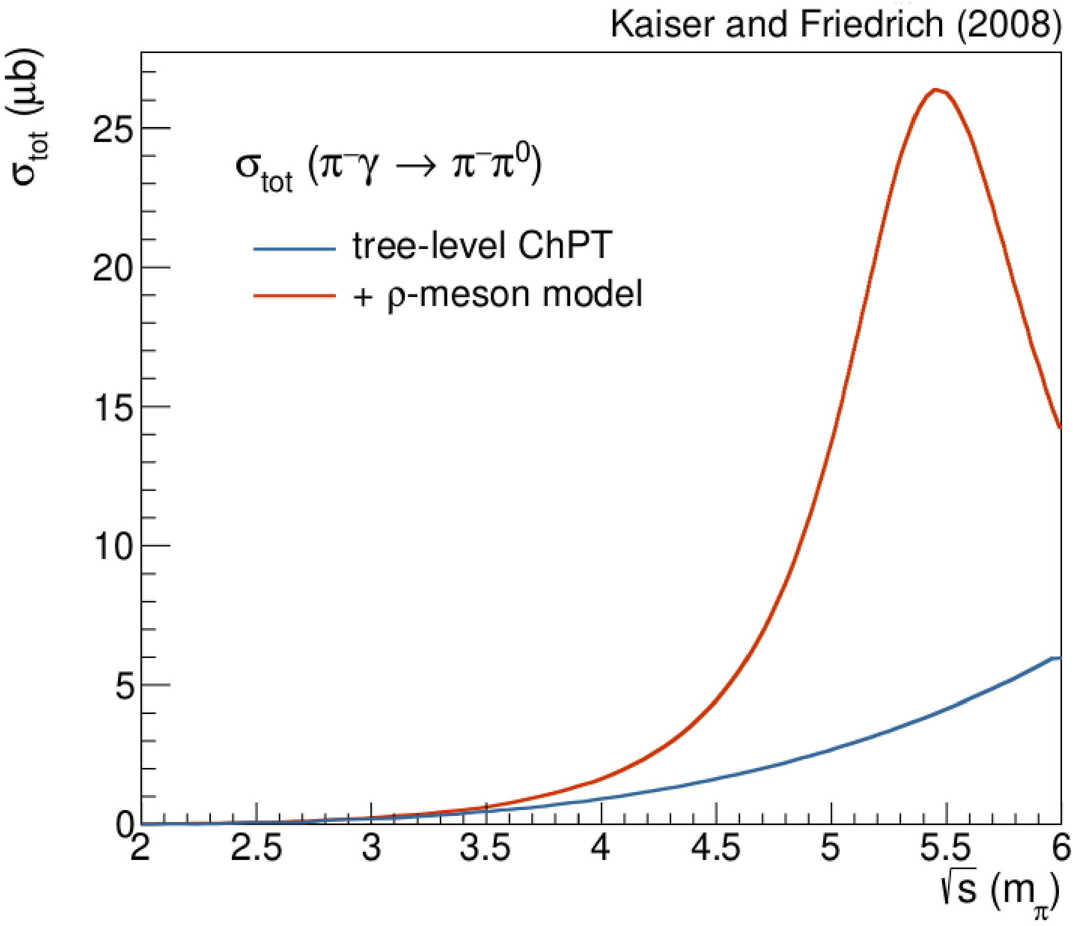

Kaiser, N. and Friedrich, J. M., EPJA 36 no. 2, (2008) 181–188

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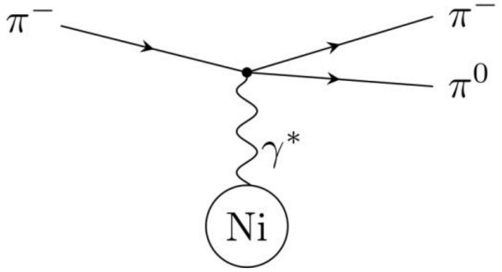

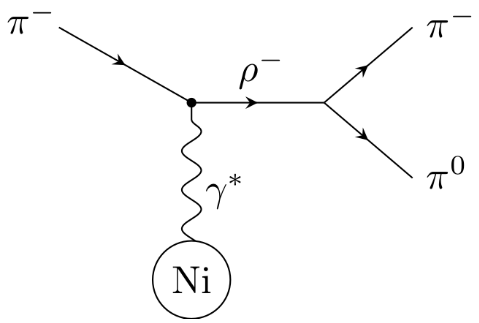

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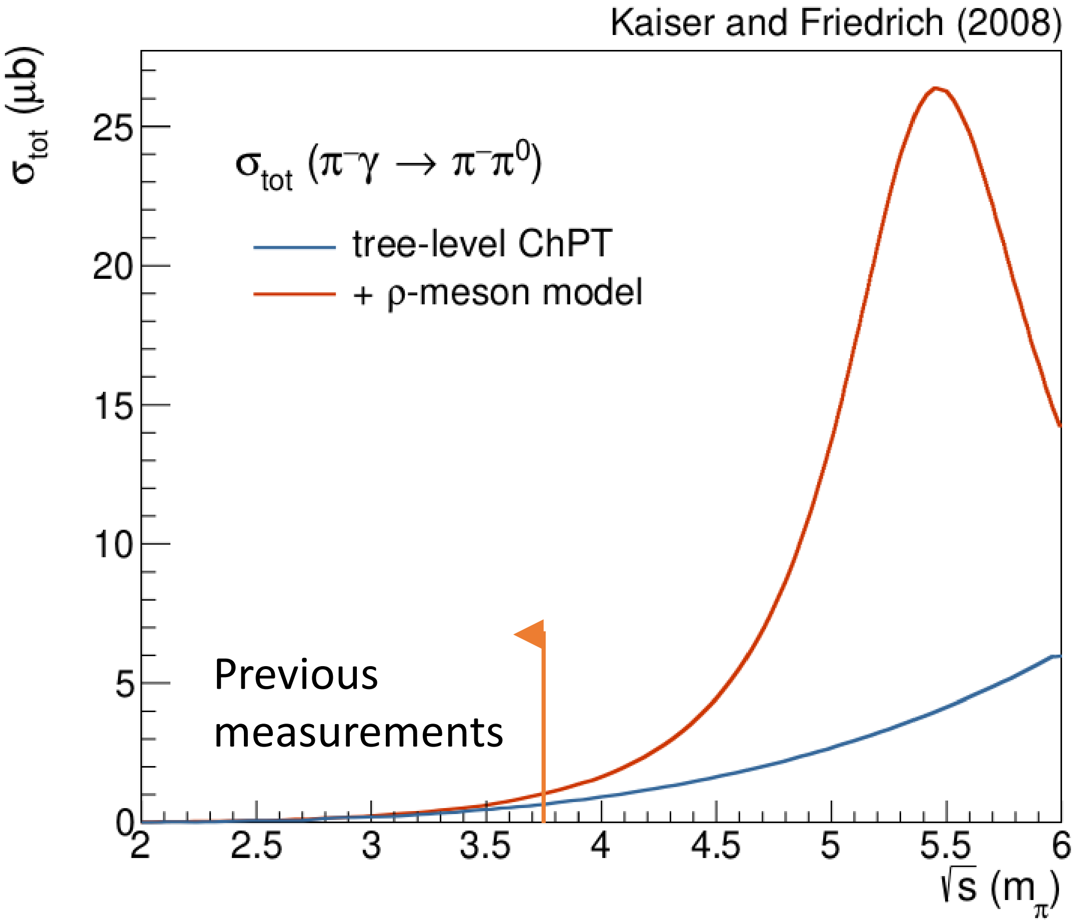
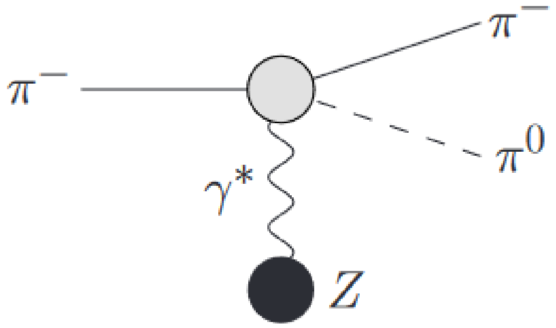
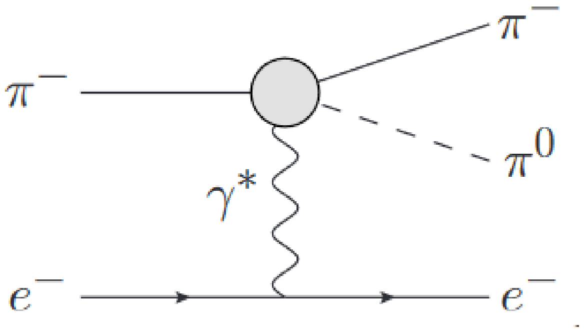


Kaiser, N. and Friedrich, J. M., EPJA 36 no. 2, (2008) 181–188

Previous measurements – $F_{3\pi}$





Antipov, Y. *et al.* PRD 36 (1987) 101103
and reanalyzed by
Ametller, L. *et al.* PRD 64 (2001) 094009

$$F_{3\pi} = (10.7 \pm 1.2)\ \mathrm{GeV}^{-3}$$

- Neglecting s-channel production of ρ meson
- No proper consideration of systematics

Giller, I. *et al.* EPJ. A25 (2005) 229-240
from cross-section data of
Amendolia, S.R. *et al.*, PLB 155, 457 (1985)

$$F_{3\pi} = (9.6 \pm 1.1)\ \mathrm{GeV}^{-3}$$

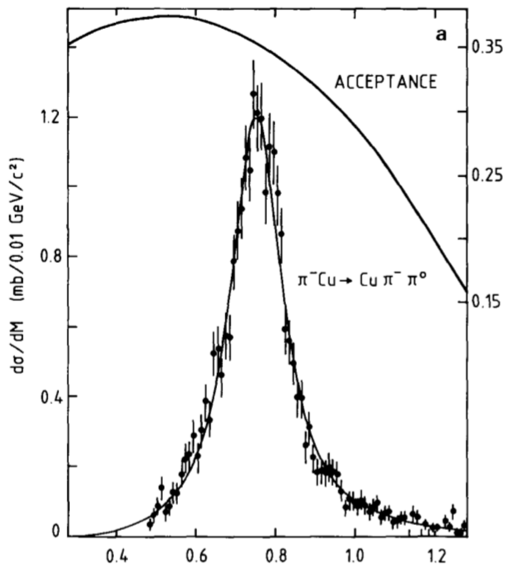
- Neglecting s-channel production of ρ meson
- No proper consideration of systematics
- Dominant background of elastically scattered pions

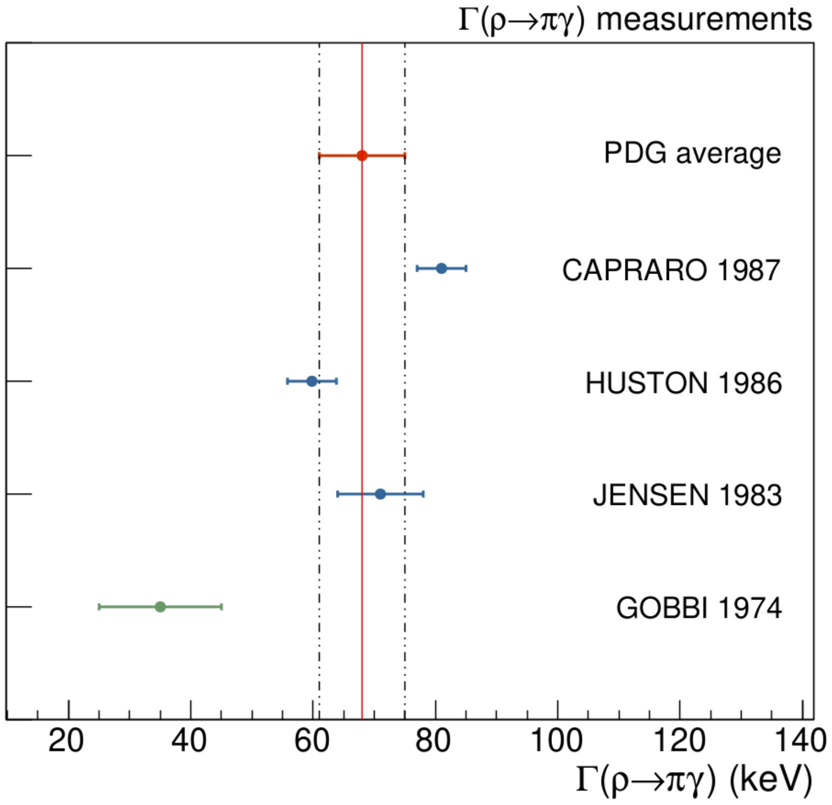
Previous measurements – $\Gamma_{\rho\to\pi\gamma}$

Radiative width of ρ-meson:

Capraro, L. *et al.* Nucl.Phys. B288 (1987) 659-680
at CERN (SPS):

- From fit to cross section (BW shape):
  $\Gamma(\rho\to\pi\gamma) = (81 \pm 4 \pm 4)\ \mathrm{keV}$





$\pi\gamma \rightarrow \pi\pi$ from dispersion relations

- Cross section:

$$\sigma(s) = \frac{(s-4m_\pi^2)^{\frac{3}{2}}(s-m_\pi^2)}{1024\pi\sqrt{s}} \int_{-1}^{+1} \mathrm{dz}\,(1-z^2)|F_{3\pi}(s,t,u)|^2$$

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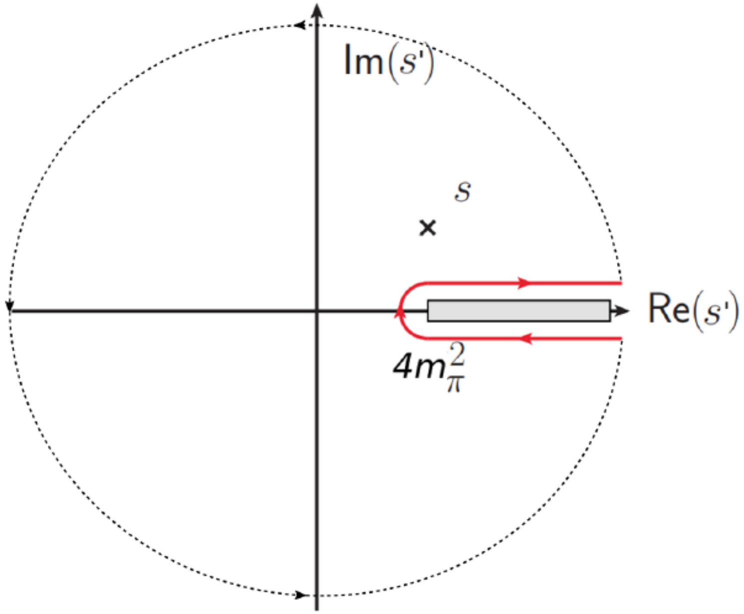
- Dispersive framework to deduce $F_{3\pi}$ from a fit to the full data set up to 1.0 GeV including the $\rho(770)$-resonance:

$$F_{3\pi}^{\mathrm{DR}}(s) = \frac{1}{3}\left(C_2^{(1)} + C_2^{(2)} s\right) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{s'^2}\frac{s^2}{s'-s} \times \left(C_2^{(1)}\mathrm{Im}\mathcal{F}_2^{(1)}(s') + C_2^{(2)}\mathrm{Im}\mathcal{F}_2^{(2)}(s')\right)$$



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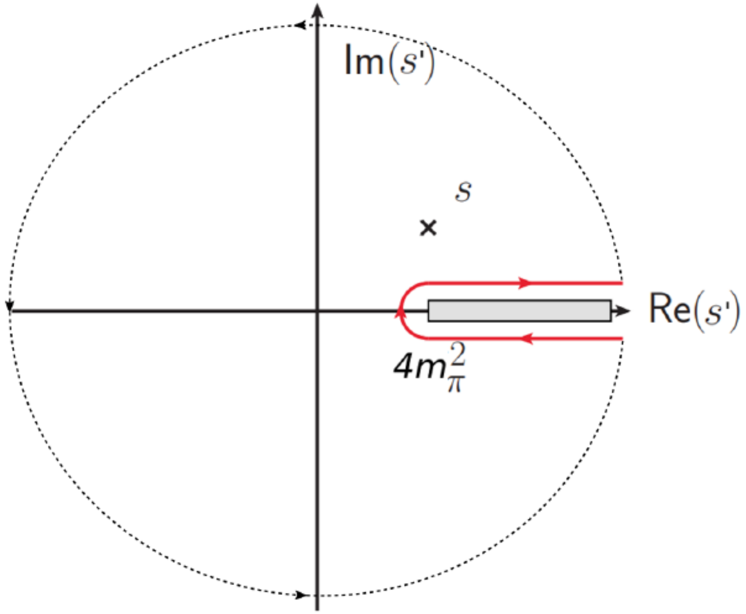
Fit parameters

Basis functions provided in:



M. Hoferichter, B. Kubis, and D. Sakkas, *PRD* **86** (2012) 116009

M. Hoferichter, B. Kubis, and M. Zanke, *PRD* **96** (2017) 114016

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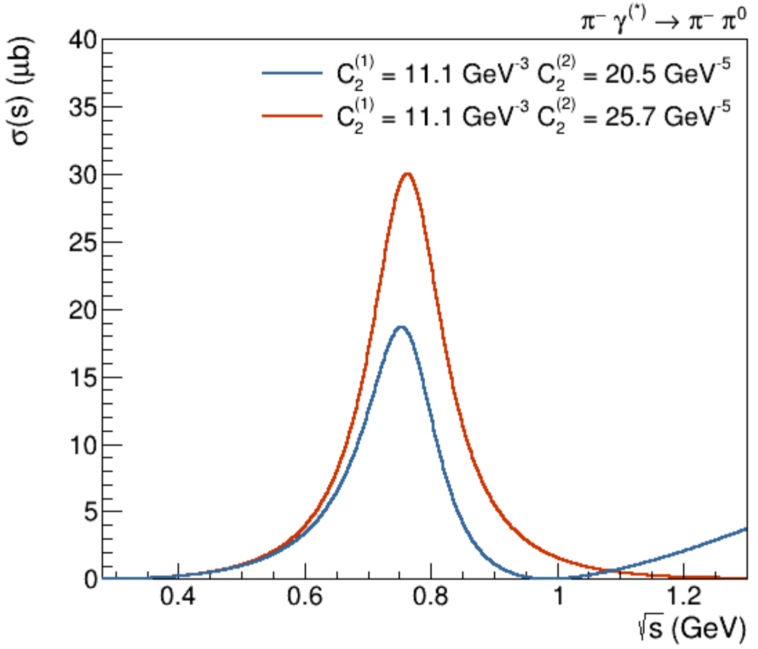

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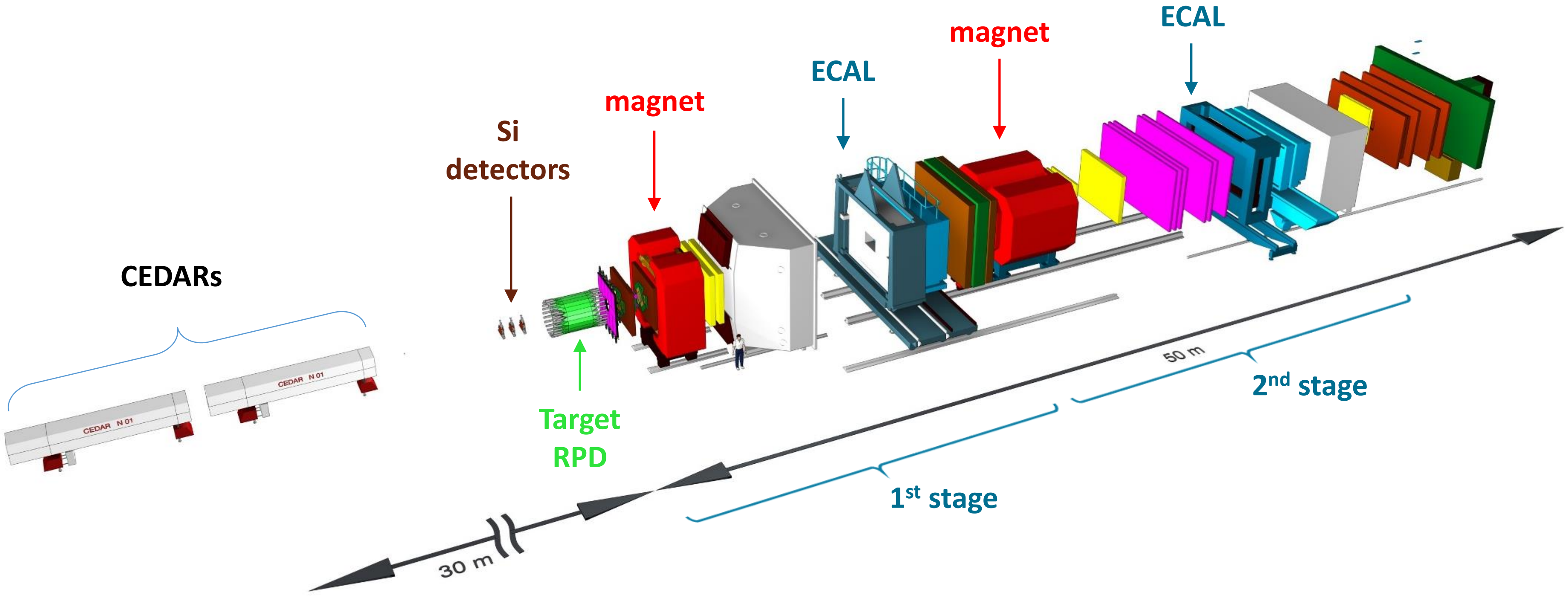
COmmon Muon and Proton Apparatus for Structure and Spectroscopy

COMPASS
25 years
1997 - 2022

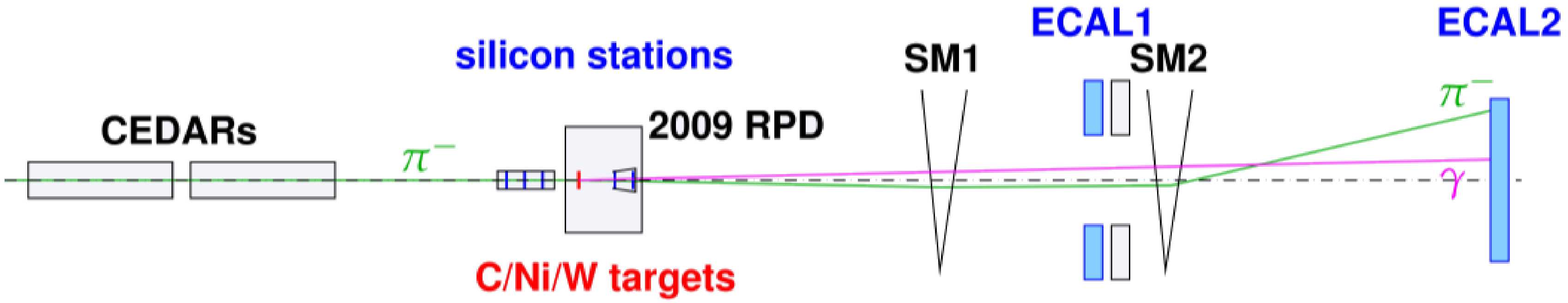
COMPASS spectrometer



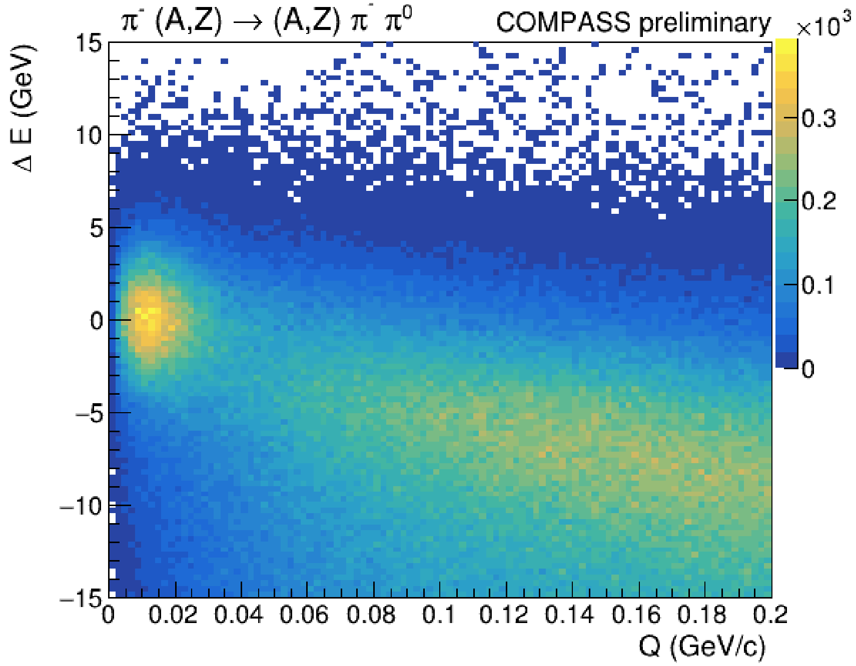
Abbon, P. *et al.* NIM A 779 (2014) 69–115

- 190 GeV negative hadron beam: $96.8\%\ \pi^-, 2.4\%\ K^-, 0.8\%\ \bar{p}$
- Beam PID by Cherenkov detectors
- Two stage magnetic spectrometer
- 4mm Ni target disk ($\approx 25\%\ \ X/X_0$)
- Calorimetric trigger on photons

Principle of measurement



- Measure scattered π^- and photons of π^0 decay
- Select exclusive events at very low Q^2



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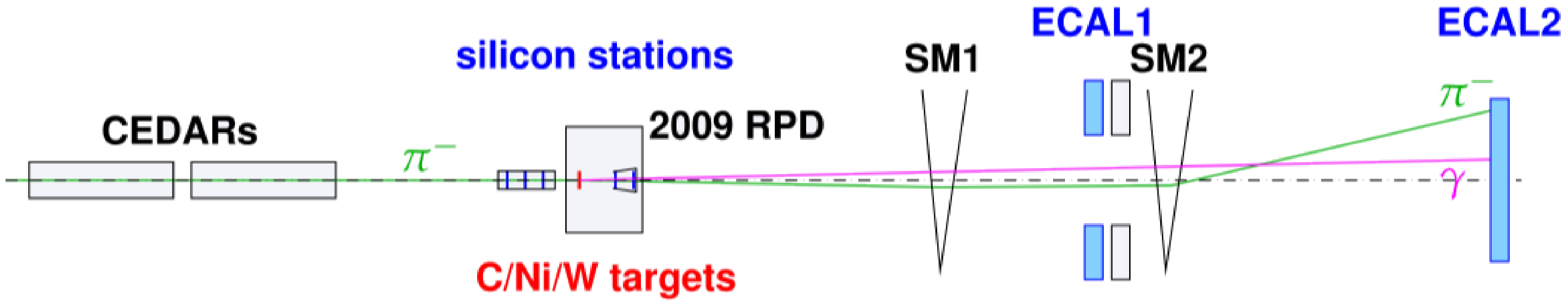

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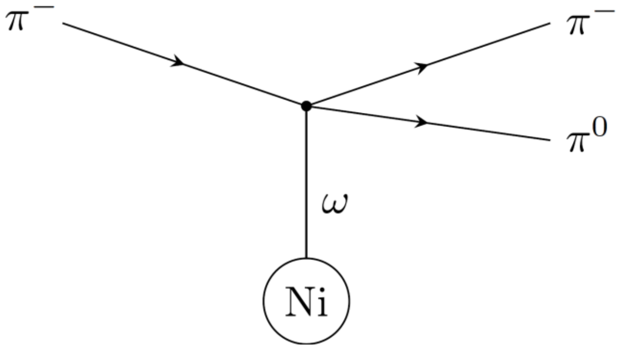
- Select exclusive events at very low Q^2
- For absolute cross-section measurements: Luminosity

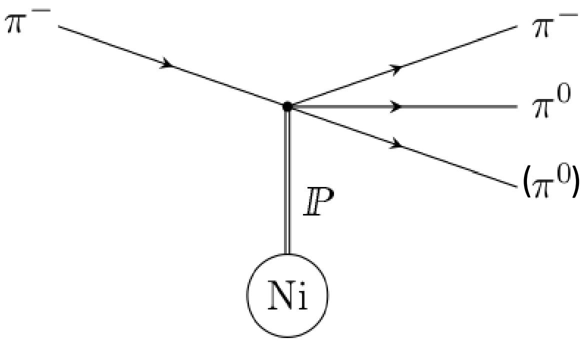
  Indirect determination of luminosity via free Kaon decays

$$(K^- \longrightarrow \pi^-\pi^0 \text{ or } K^- \longrightarrow \pi^-\pi^0\pi^0)$$

$$\int L\,\mathrm{dt} = \left(5.21 \pm 0.04_{\mathrm{stat}} \pm 0.48_{\mathrm{syst}}\right)\,\mathrm{nb}^{-1}$$

Potential background processes to $\pi\gamma \rightarrow \pi\pi$





$\pi^-\pi^0$ via strong interaction

- Pomeron exchange: forbidden by G-parity conservation
- π and ω exchange:
  low cross section at COMPASS beam energies

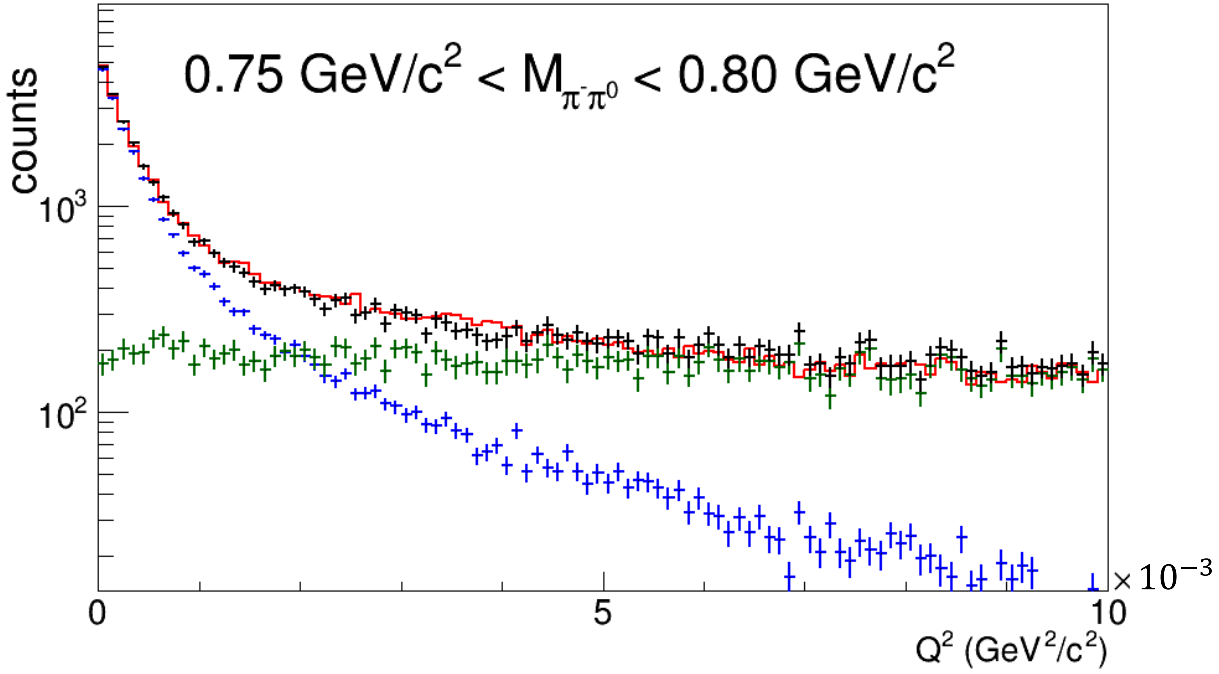
$\pi^-\pi^0\pi^0$ via Pomeron exchange

- Large cross section
- Main background: loss of one (soft) π^0
- Approach:
  - Using the model from COMPASS $\pi^-\pi^0\pi^0$ data
  - Apply $\pi^-\pi^0$ event selection -> realistic distributions of leakage in $\pi^-\pi^0$

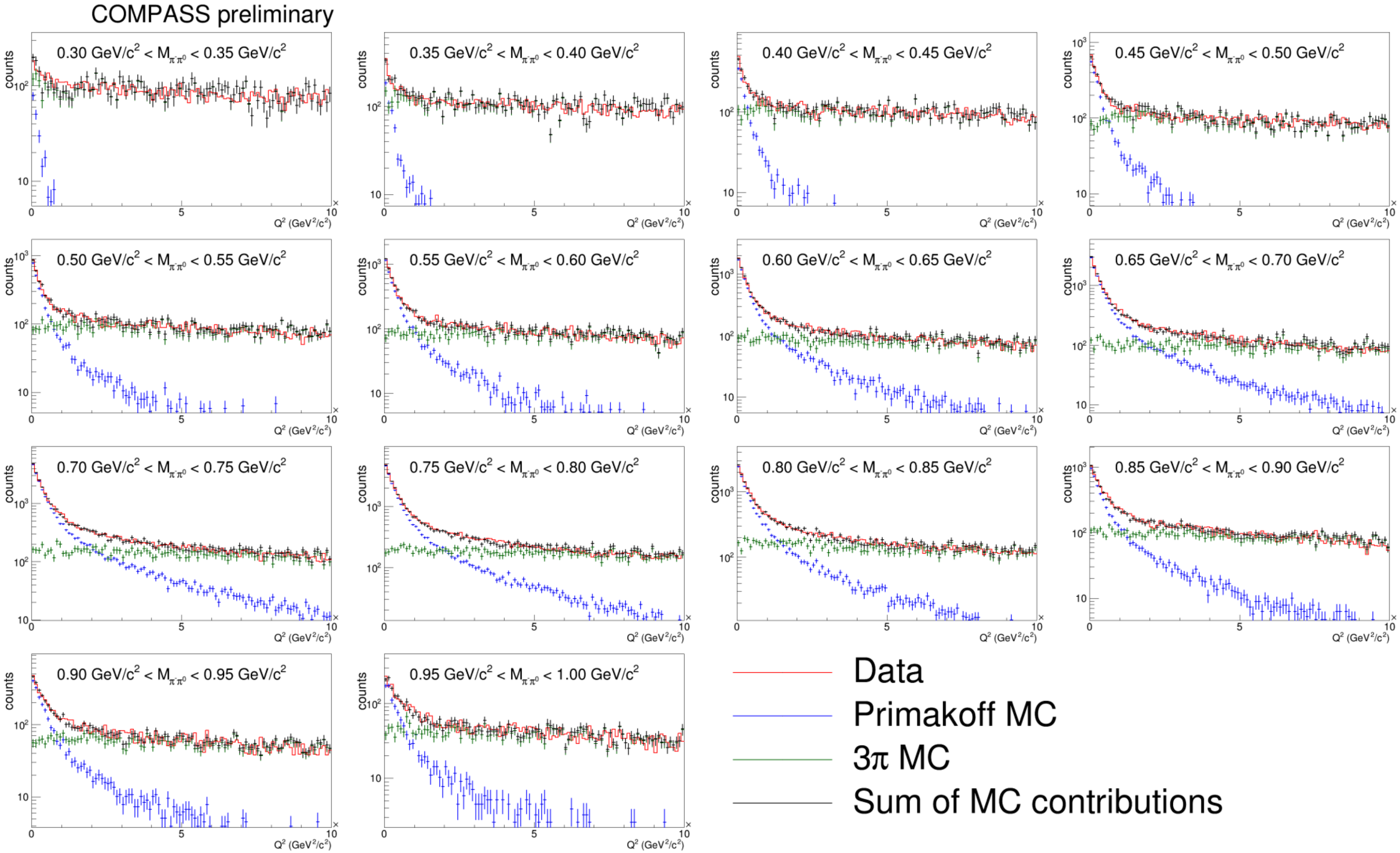
Subtraction of 3π background

Model from COMPASS $\pi^-\pi^0\pi^0$ data:

- Realistic shapes for signal and background contributions
- Fit yields (signal vs background) to match observed momentum transfer distribution



Subtraction of 3π background



Results of dispersive fits

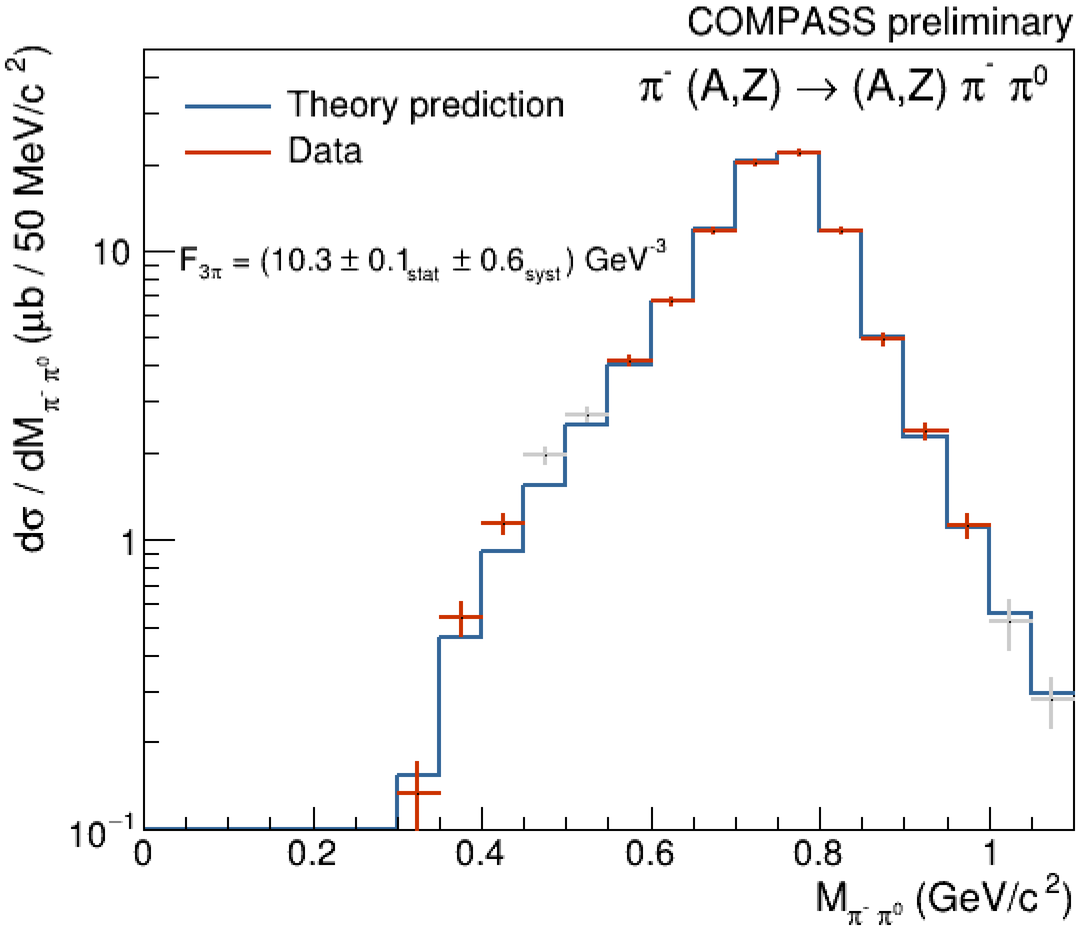
- Determine subtraction constants from fit
    - Use data up to 1 GeV/c^2
    - Exclude data around 500 MeV/c^2 due to background of free kaon decay

$$C_2^{(1)} = (10.5 \pm 0.1_{stat} \pm 0.6_{syst})\mathrm{GeV}^{-3}$$
$$C_2^{(2)} = \left(24.5 \pm 0.1_{stat}{}^{+1.6}_{-1.4}{}_{syst}\right)\mathrm{GeV}^{-5}$$

- Use ChPT expansion (NLO) to determine $F_{3\pi}(0,0,0)$:

$$F_{3\pi} = (10.3 \pm 0.1_{stat} \pm 0.6_{syst})\mathrm{GeV}^{-3}$$

$$\Gamma_{\rho\to\pi\gamma} = \left(76 \pm 1_{stat}{}^{+10}_{-8}{}_{syst}\right)\mathrm{keV}$$



Comparison to previous measurements

- COMPASS: **<u>First combined</u>** measurement of $F_{3\pi}$ and $\Gamma_{\rho\to\pi\gamma}$

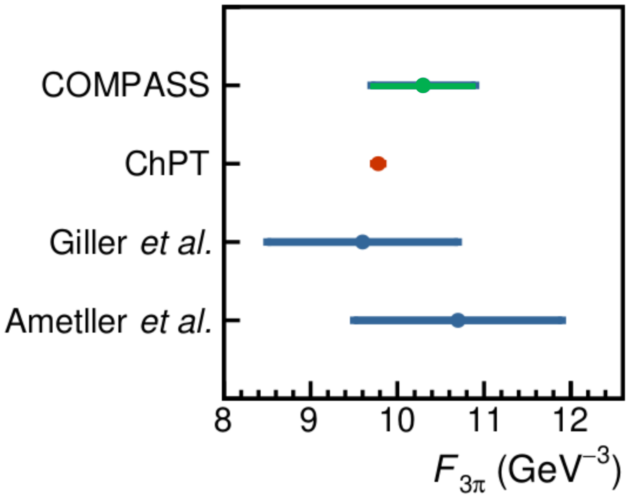
$$F_{3\pi} = (10.3 \pm 0.1_{stat} \pm 0.6_{syst})\mathrm{GeV}^{-3}$$

$$\Gamma_{\rho\to\pi\gamma} = \left(76 \pm 1_{stat}{}^{+10}_{-8}{}_{syst}\right)\mathrm{keV}$$

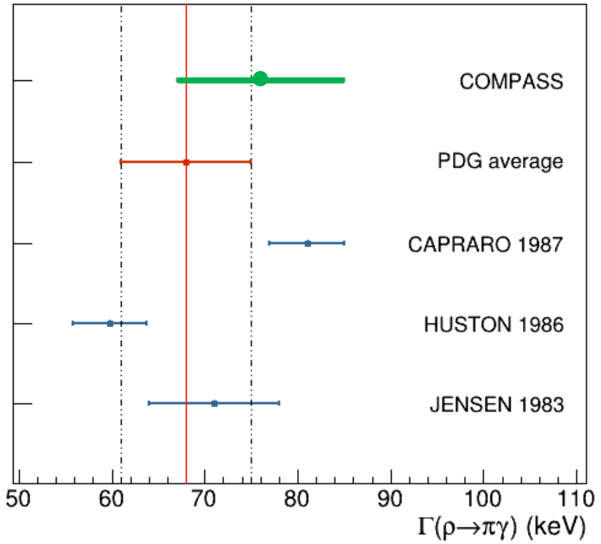
- Intensive test of systematics (dominant contributions):
  - Luminosity
  - Radiative corrections
  - Background of ω, π exchange
  - Background from $\pi\gamma$ final state

- Accompanied with intensive analysis of $\pi^-\mathrm{Ni} \to \pi^-\pi^0\pi^0\mathrm{Ni}$ for background estimation

chiral anomaly



ρ radiative width



Conclusion and outlook

- Measurement of $F_{3\pi}$ - fundamental test of low-energy QCD

- COMPASS did first combined measurement of $F_{3\pi}$ and $\Gamma_{\rho \to \pi\gamma}$

- Result for $F_{3\pi}$ is in agreement with prediction from ChPT

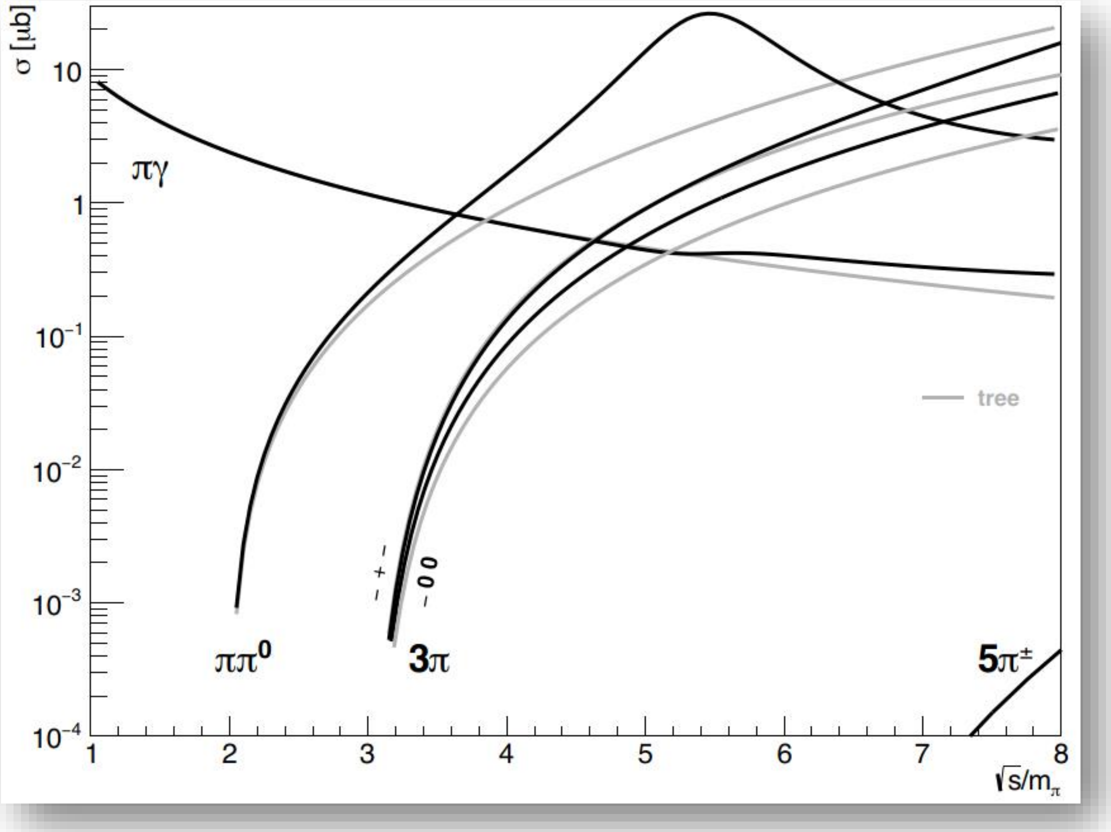
- Results dominated by systematic uncertainties -> improvement expected
  - Background prediction
  - Luminosity determination

- On the future program of successor experiment AMBER: similar program on kaon sector (see talk by Oleg Denisov, “From COMPASS to AMBER”, Fri 14:00)

Thank you for your attention

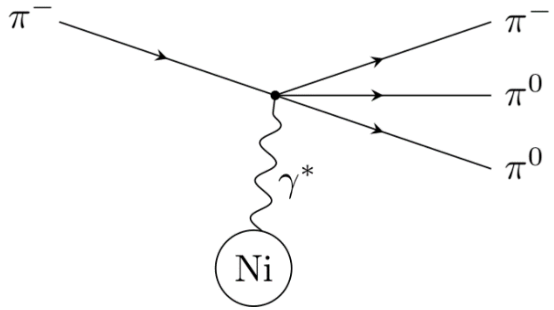
Backup

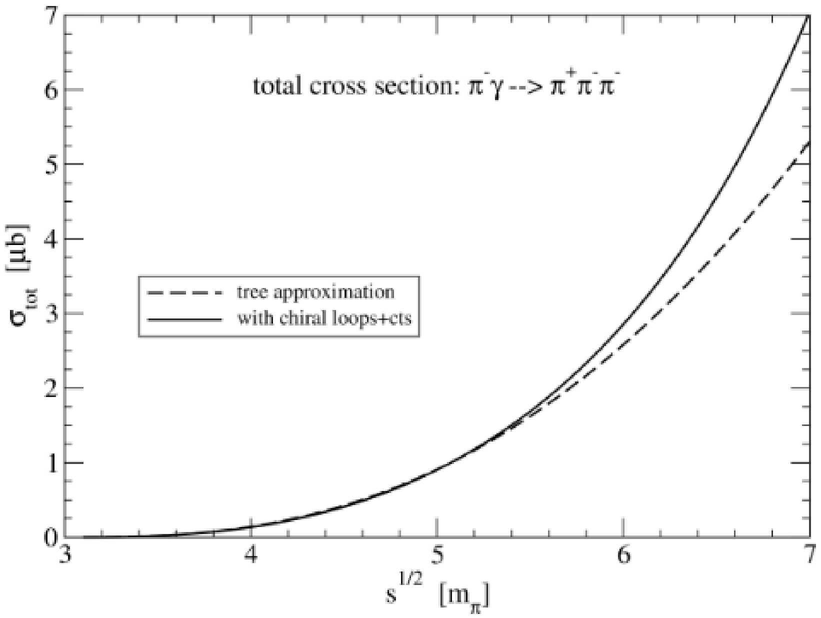
Cross sections for Primakoff effect



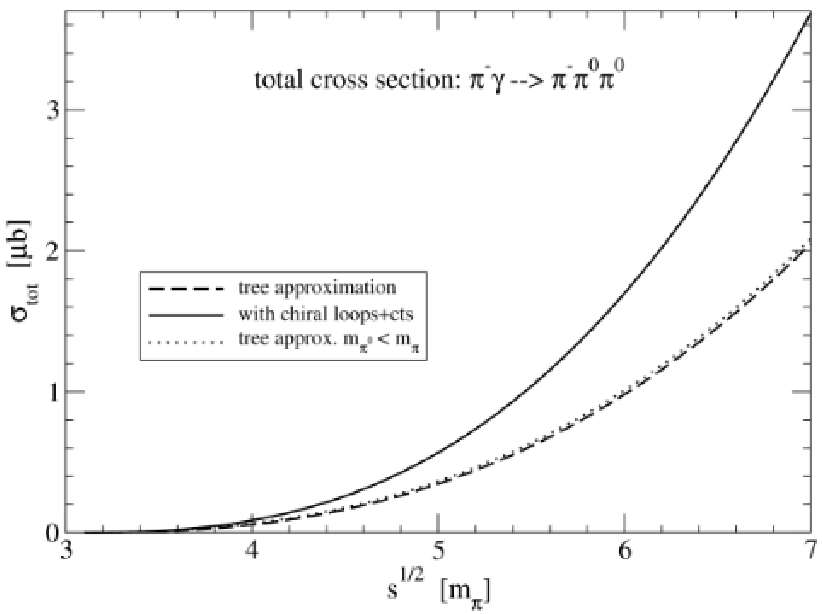
Chiral tree, chiral loop

- Direct (point-like) coupling of photon to 4 pions
- Prediction from ChPT at tree- and loop-level available





[Grabmüller S. (2012). Cryogenic Silicon Detectors and Analysis of Primakoff Contributions to the Reaction $\pi^- Pb \longrightarrow$]



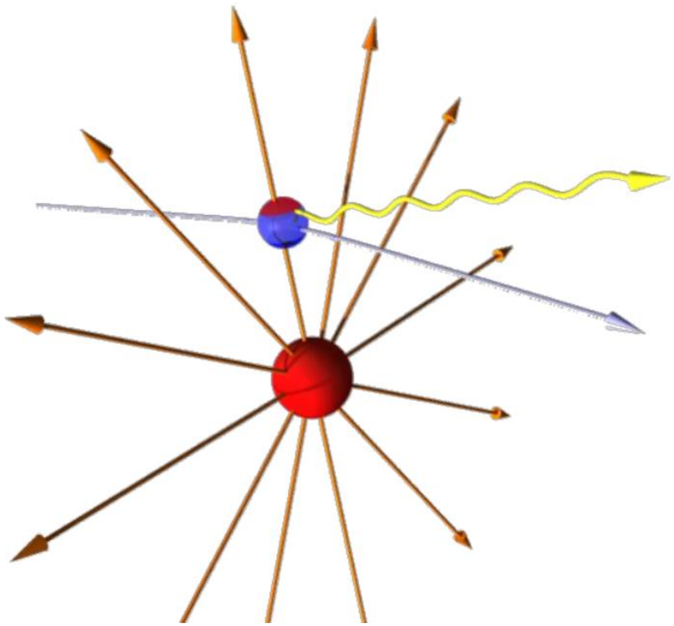
[Krämer M. (2016) Evaluation and Optimization of a digital calorimetric trigger and analysis of $\pi^- Ni \longrightarrow$]

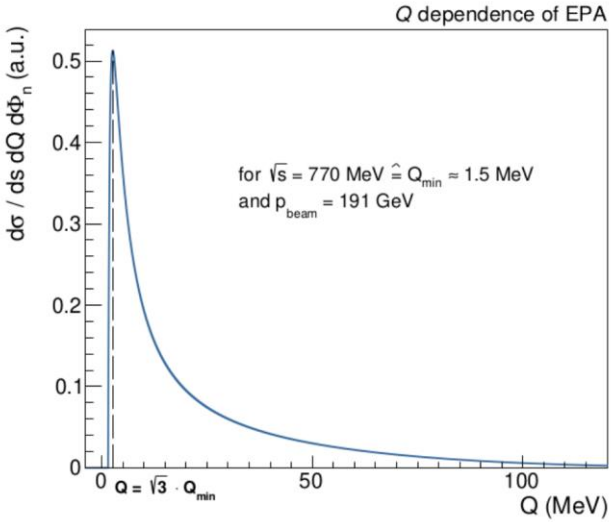
Pion-photon reactions through the Primakoff technique

- Photon is provided by the strong Coulomb field of a nucleus (typical field strength at $d = 5R_{Ni}$: $E \approx 300\ \mathrm{kV/fm}$)

- Coulomb field of nucleus is a source of quasi-real ($P_\gamma^2 \ll m_\pi^2$) photons

- Large impact parameters (ultra-peripheral scattering)





$$\frac{\mathrm{d}\sigma}{\mathrm{d}s\,\mathrm{d}Q^2\,\mathrm{d}\Phi_n} = \underbrace{\frac{Z^2\alpha}{\pi(s-m_\pi^2)} F^2(Q^2) \frac{Q^2 - Q^2_{\mathrm{min}}}{Q^4}}_{\text{Flux of quasi-real photons}} \cdot \underbrace{\frac{\mathrm{d}\sigma_{\pi\gamma\to X}}{\mathrm{d}\Phi_n}}_{\pi\gamma \text{ scattering cross section}}$$

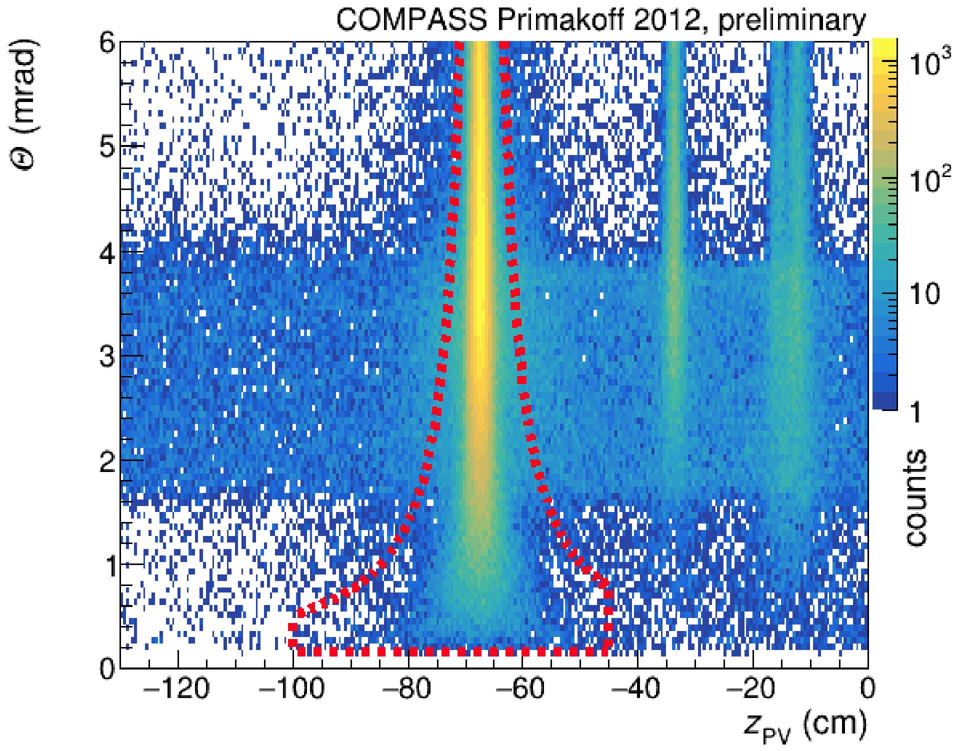
Integrated luminosity

- Needed for absolute cross section measurement: effective integrated luminosity

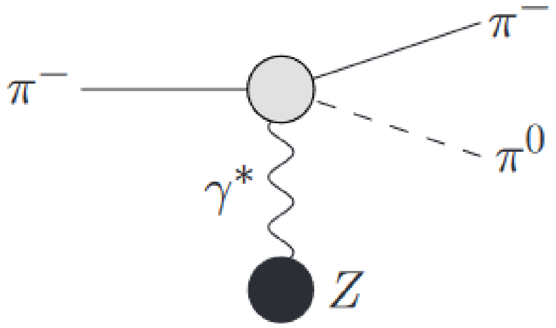
$$L_{\mathrm{eff}} = L \cdot \left(1 - \epsilon_{\mathrm{DAQ}}\right)$$

- Can be determined via free kaon decays:
  - Use CEDAR detectors for beam PID
  - Free decays where no material
  - Exclusive events with no momentum transfer

$$\overset{\text{Number of detected kaon decays}}{N_K} = \underset{\text{Pion flux}}{\Phi_\pi} \overset{\text{Kaon/pion flux ratio}}{\frac{n_K}{n_\pi}} \underbrace{BR(K \to X)}_{\text{Branching ratio}} \overbrace{\left(1 - e^{-\frac{L}{\gamma c \tau}}\right)}^{\text{Decay probability of kaon to decay in } L} \underset{\text{Acceptance for kaon decays for final state X}}{\epsilon_{K \to X}}$$



Previous measurements – $F_{3\pi}$



Antipov, Y. *et al.* PRD 36 (1987) 101103

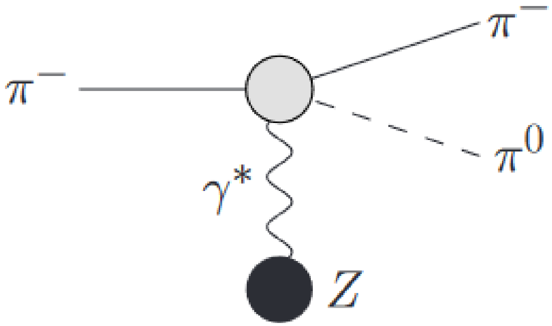
$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05)\mathrm{GeV}^{-3}$$

tension

$$F_{3\pi} = (12.9 \pm 0.9)\ \mathrm{GeV}^{-3}$$

- Assuming $F_{3\pi} = \bar{F}_{3\pi}(s, t, u)$
- Neglecting s-channel production of ρ meson
- No proper consideration of systematics

Previous measurements – $F_{3\pi}$



Antipov, Y. *et al.* PRD 36 (1987) 101103
and reanalyzed by
Ametller, L. *et al.* PRD 64 (2001) 094009

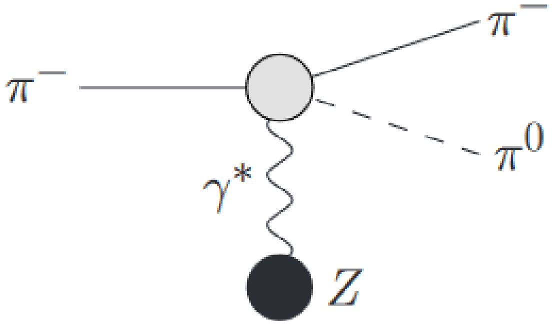
$$F_{3\pi} = (11.4 \pm 1.3)\ \mathrm{GeV}^{-3}$$

tension

$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05)\mathrm{GeV}^{-3}$$

- Neglecting s-channel production of ρ meson
- No proper consideration of systematics
- Using ChPT to extrapolate to chiral limit (NNLO)

Previous measurements – $F_{3\pi}$



Antipov, Y. *et al.* PRD 36 (1987) 101103
and reanalyzed by
Ametller, L. *et al.* PRD 64 (2001) 094009

$$F_{3\pi} = (10.7 \pm 1.2)\ \mathrm{GeV}^{-3}$$

~~tension~~

$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_\pi^3} = (9.78 \pm 0.05)\mathrm{GeV}^{-3}$$

- Neglecting s-channel production of ρ meson
- No proper consideration of systematics
- Using ChPT to extrapolate to chiral limit (NNLO)
- Considering dominant correction