HADRON 2023

5-9 June 2023
Genova, Italy

Spatial densities, form factors and internal properties of Hadrons

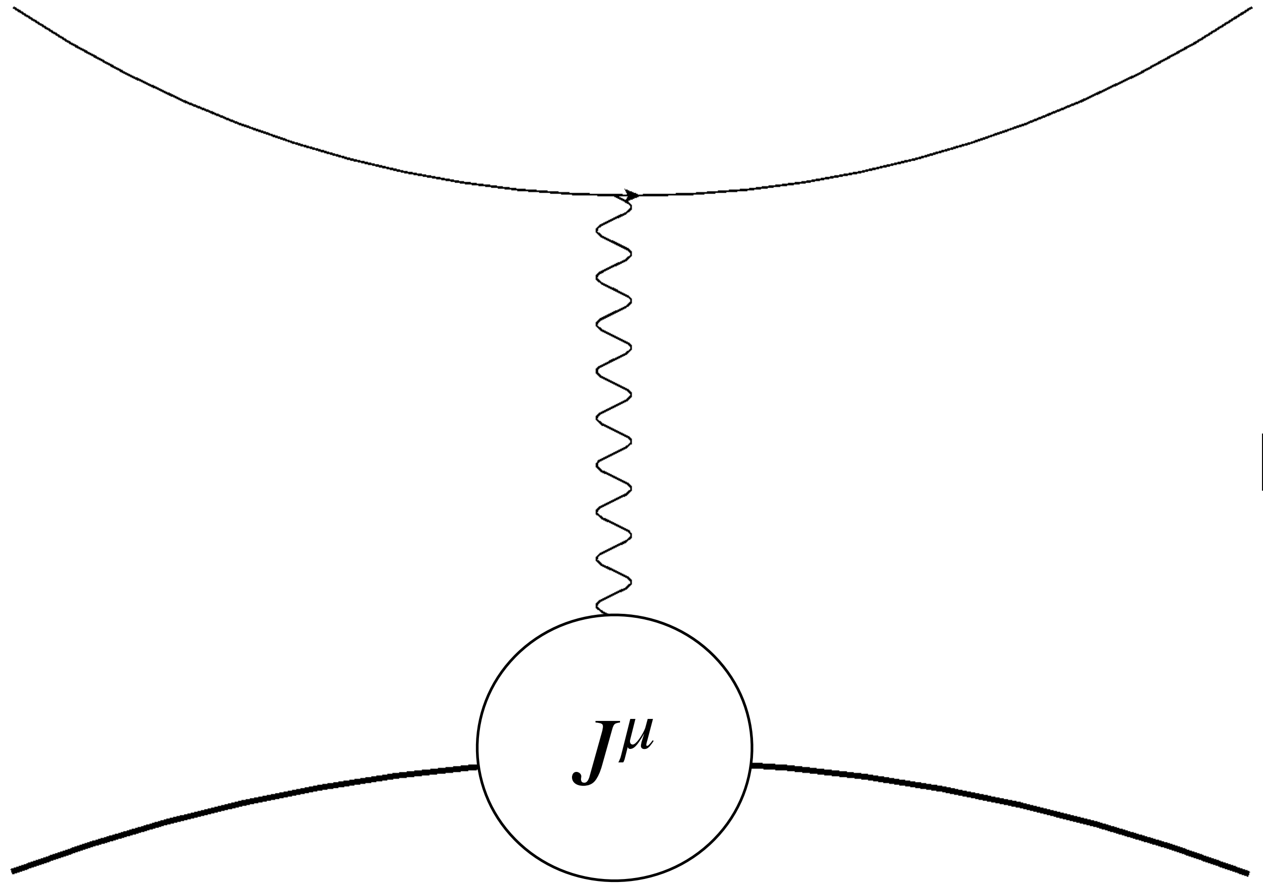
Julia Panteleeva
Ruhr-University Bochum
Germany

Based on:
M.V. Polyakov, P. Schweizer [Int. J. Mod. Phys. A 33] (2018)
V.Burkert, L. Elouadrhiri, F.X.Girod, C. Lorce, P. Schweitzer, P.E.Shanahan [e-Print: 2303.08347] (2023)

Epelbaum, Gegelia, Lange, Meißner, Polyakov [Phys.Rev.Lett.129, 012001] (2022)
Panteleeva, Epelbaum, Gegelia, Meißner [PhysRevD.106, 056019] (2022)
Panteleeva, Epelbaum, Gegelia, Meißner [e-Print: 2211.09596] (2022)

Juliia.Panteleeva@rub.de

EM structure of a particle



virtual
photon

$$d\sigma/d\Omega = \left(d\sigma/d\Omega\right)_{pointlike} \times \left(F_1^2(q^2) + \frac{q^2}{4m^2}(F_2^2(q^2) + \ldots) \right)$$

For spin-1/2

$$\langle p', s' \,|\, \hat{j}^\mu(0) \,|\, p, s\rangle = \bar{u}(p', s')\left[\gamma^\mu F_1(q^2) + \frac{1}{2} i\sigma^{\mu\nu} q_\nu\, F_2(q^2)\right] u(p, s)$$

[Rosenbluth, 1950
Hofstadter et al. 1953]

- electric charge

$$F_1(0) = e$$

- anomalous magnetic moment

$$1/2(F_1(0) + F_2(0)) = \mu$$

- conserved

$$\partial^\mu j_\mu = 0$$

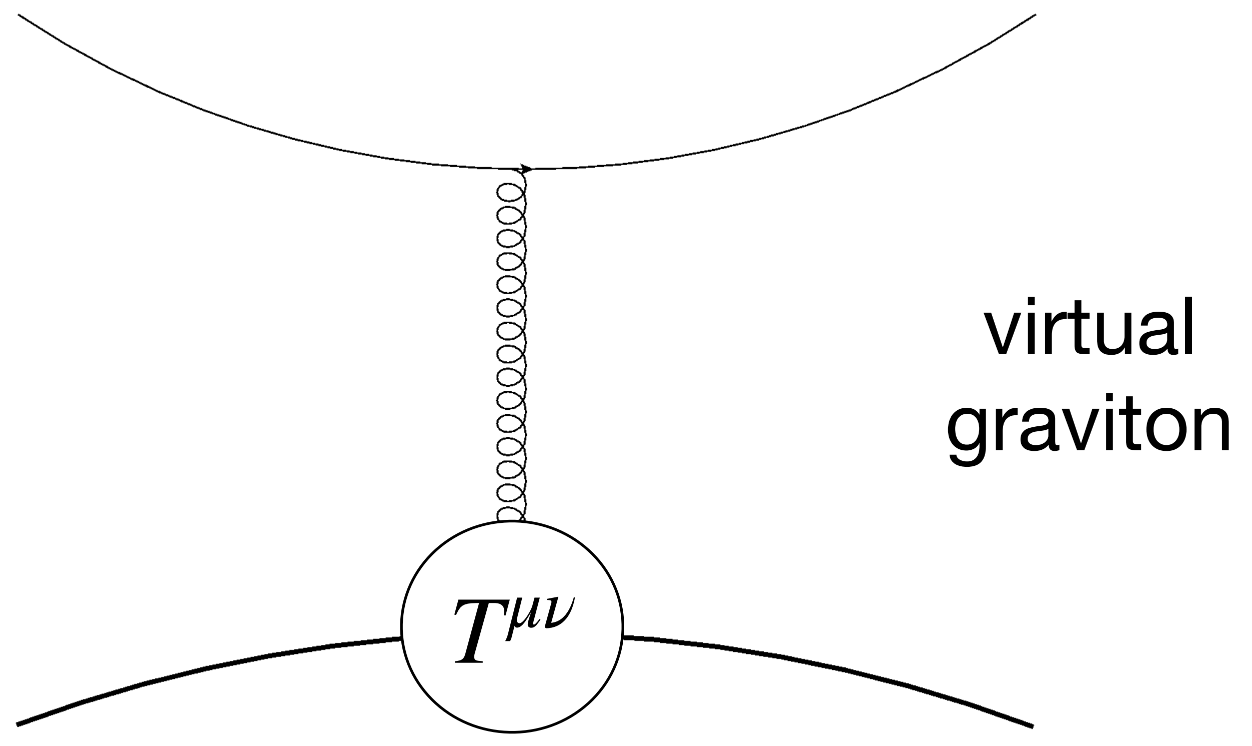
- gauge invariant

Gravitational structure of hadrons

[Kobzarev, Okun (1962)
Pagels (1966)]

For spin-1/2

$$\langle p'|\hat{T}^{\mu\nu}(0)|p\rangle = \bar{u}\left[A(q^2)\frac{P_\mu P_\nu}{m} + iJ(q^2)\frac{(P_\mu\sigma_{\nu\alpha}+P_\nu\sigma_{\mu\alpha})q^\alpha}{4m} + D(q^2)\frac{q_\mu q_\nu - \eta_{\mu\nu}q^2}{4m}\right]u$$



No direct experiment for detection of the matter-graviton interaction

$$T_{\mu\nu}(x) \sim \frac{\delta S_M}{\delta g^{\mu\nu}(x)}$$

Gravity couples to matter due to EMT

- **mass** $m = \int d^3r T_{00}(r)$

$$A(0) = 1$$

- **spin** $J^i = \epsilon^{ijk}\int d^3r r^j T_{0k}(r)$

$$J(0) = 1/2$$

- **anomalous magnetic moment**

$$2J(t) = A(t) + B(t) \quad B(0) = 0$$

- symmetric
- conserved

$$\partial^\mu T_{\mu\nu} = 0$$

- gauge invariant

[Xiang-Dong Ji, Phys.Rev.D 58 (1998)
Xiang-Dong Ji, Phys.Rev.Lett. 78 (1997)]

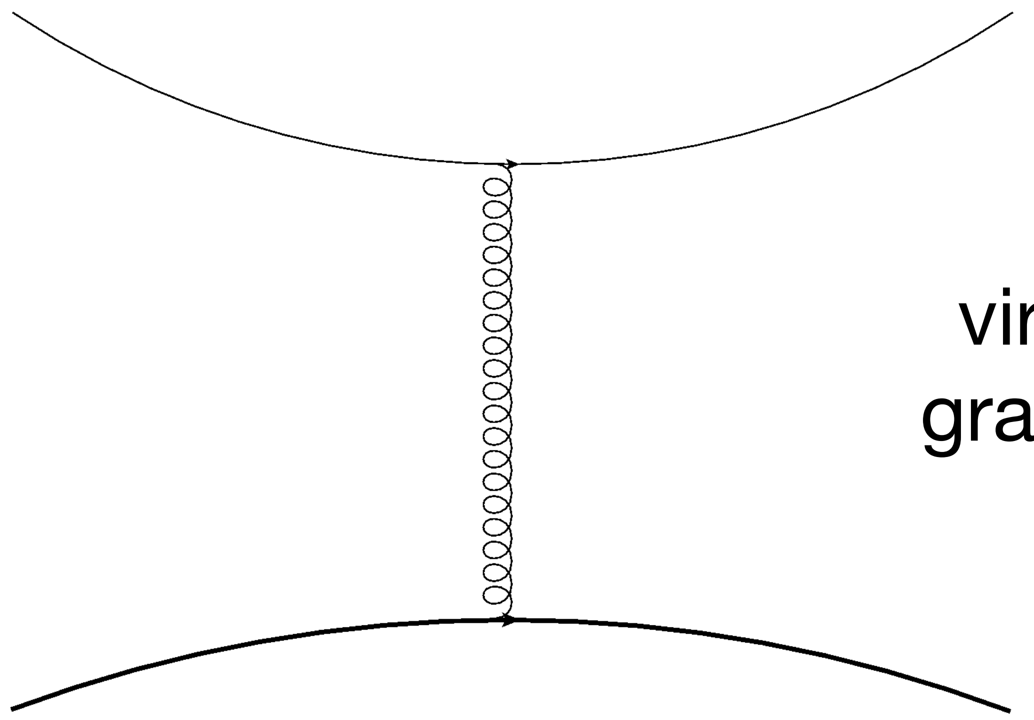
- **D-term:**

$$D = D(0) = -\frac{m}{2}\int d^3r\left(r^i r^j - \frac{1}{3}\delta_{ij}\right)T_{ij}(r)$$

[M.V.Polyakov,
Phys. Lett.B 555, 57 (2003)]

*…D-term as fundamental as mass and spin!
It is necessary connected with the true gravity.*

How to measure GFFs?



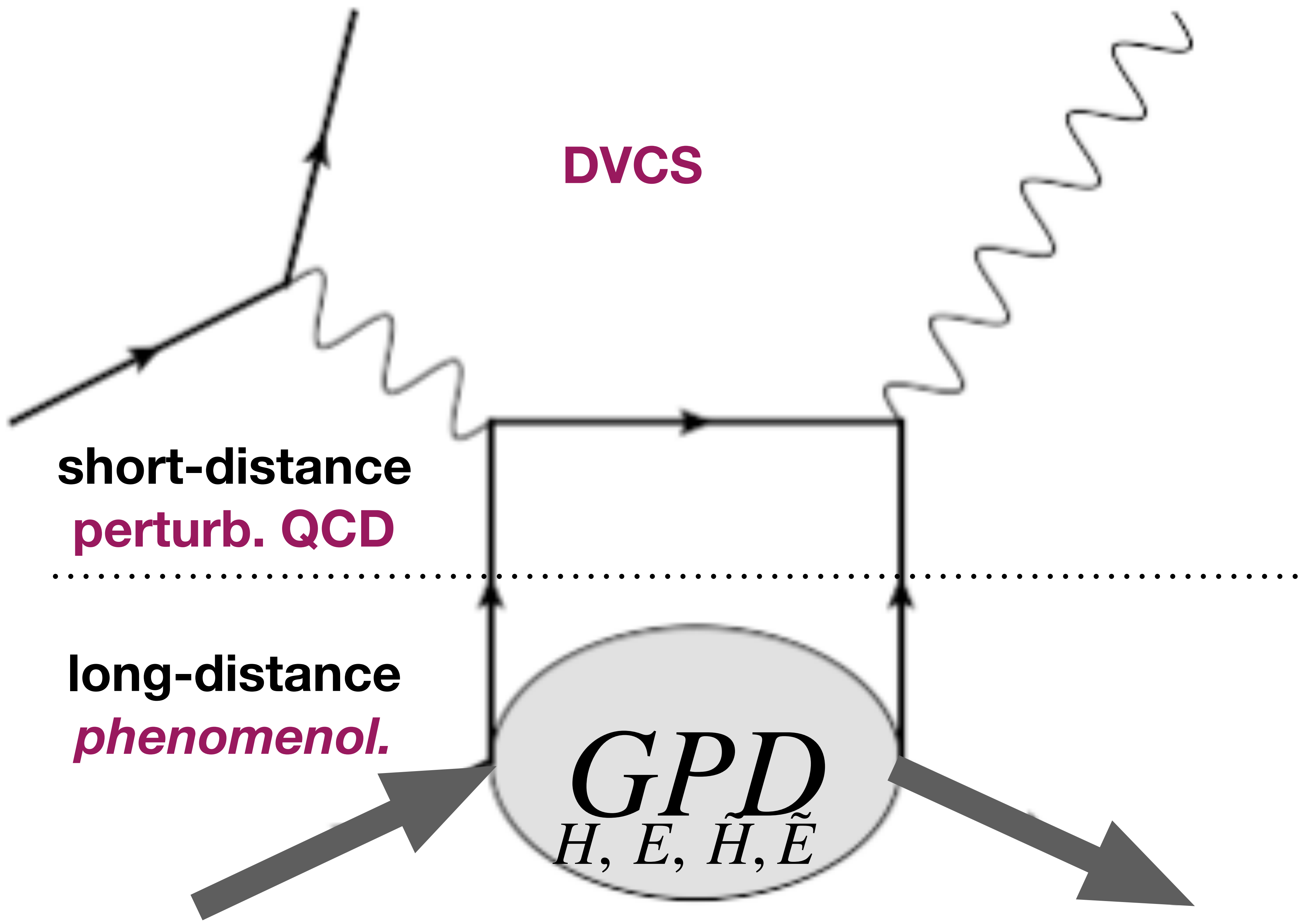
No direct experiment for detection of the matter-graviton interaction

However, it is possible with 2 photons

$$d\sigma/d\Omega = L_{\mu\nu}W^{\mu\nu}$$

GPDs

$$W^{\mu\nu} \sim H, E, \tilde{H}, \tilde{E}$$



$$\int_{-1}^{1} dx\ xH(x,\xi,t) = A(t) + \xi^2 D(t)$$

$$\int_{-1}^{1} dx\ xE(x,\xi,t) = B(t) - \xi^2\ D(t)$$

Details in
[D. Müller et al., F.Phys. 42,1994,
X. Ji, PRL 78, 610, 1997
A. Radyushkin, PLB 380, 1996]

In DVCS $d\sigma/d\Omega \sim \mathscr{H}$ **Compton form factor**

$$\mathscr{H}(\xi, t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q(x, \xi, t)$$

A(t) and J(t) can be extracted

$$Re\mathscr{H}(\xi, t) = \frac{1}{\pi}\mathbf{P.V.}\int_{-1}^{1} dx \left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right] ImH_q(x, \xi, t) + \Delta(t, \mu^2)$$

D(t) can be extracted

$$\Delta(t, \mu^2 \to \infty) \sim D^q(t)$$

Details in

M.V.Polyakov, PLB 555 (2003)

Anikin, Teryaev, PRD76 (2007)

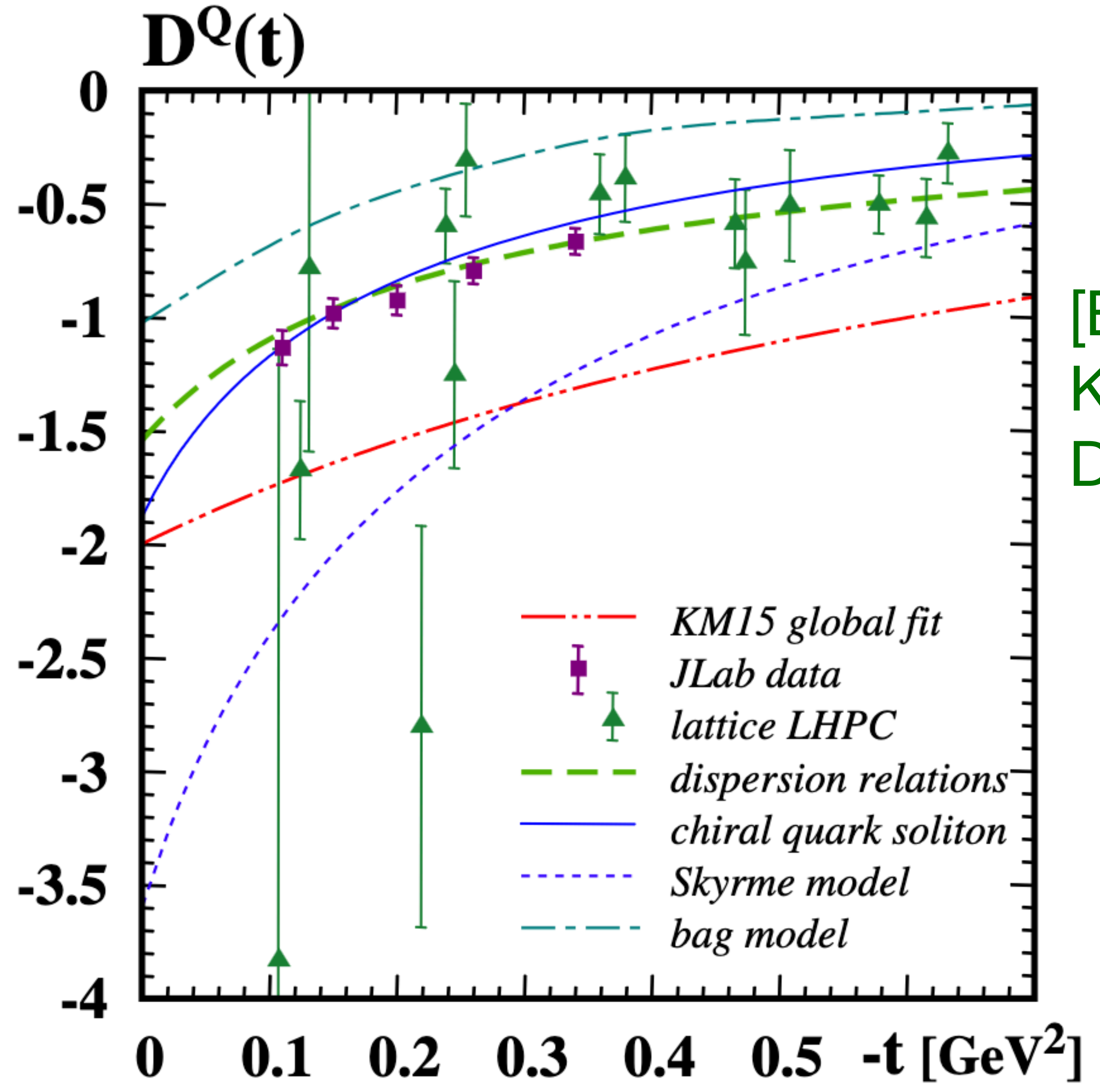
Diehl and Ivanov, EPJC52 (2007)

Radyushkin, PRD83, 076006 (2011)

Bertone et al., PRD 103 (2021)

Results for GFFs

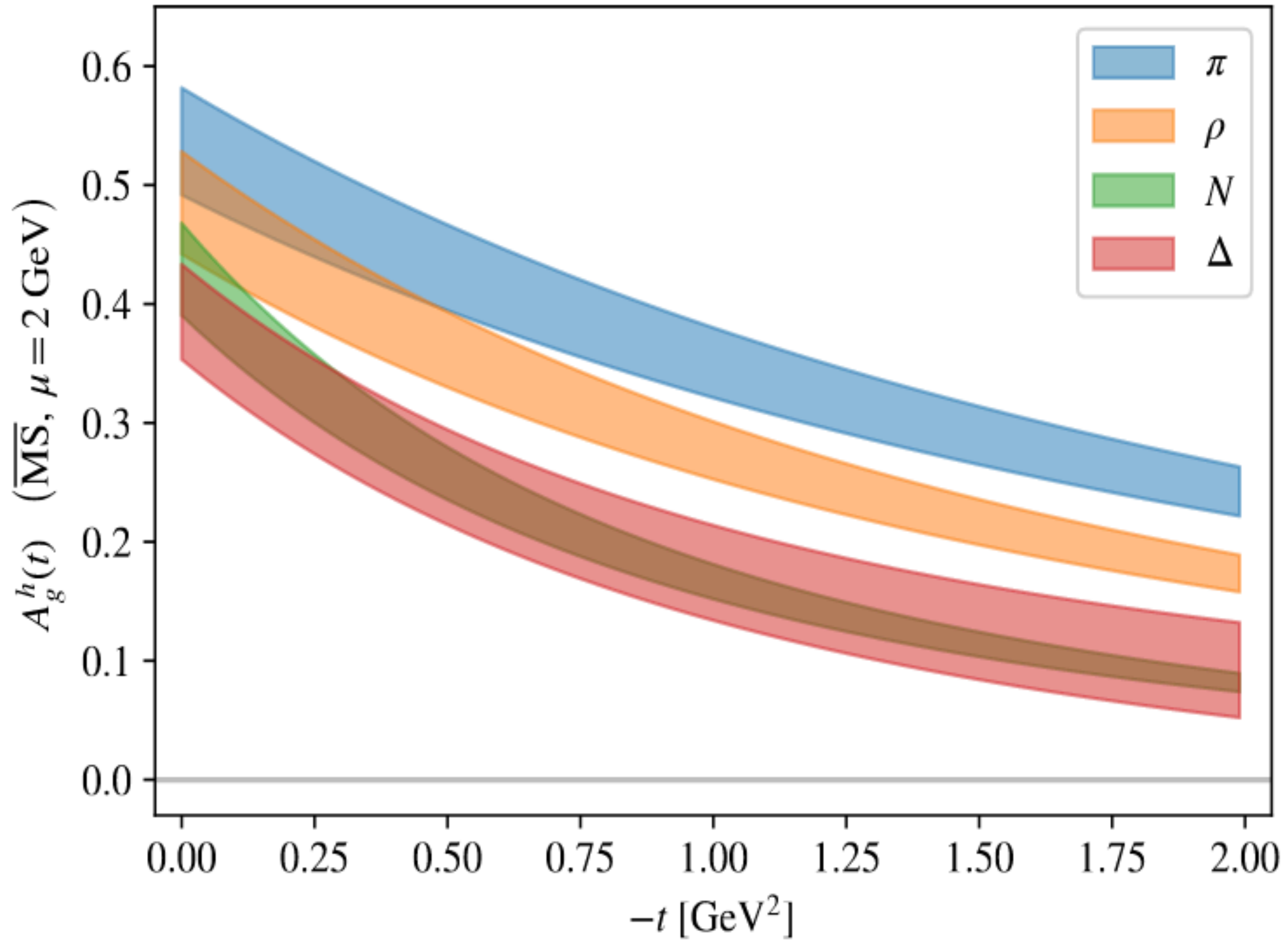
From Experiment



Comparison of experimental data with lattice data and model calculations
[M.V. Polyakov, P. Schweitzer, I nt.J.Mod.Phys.A 33 (2018)]

Details in
[Burkert et al., Nature 557 (2018)
Kumeticki, Nature 570 (2019)
Dutrieux et al., Eur.Phys.J C 81 (2021)]

From QCD



Gluon contribution to GFF A(t) for various hadrons from lattice QCD study
with pion mass mπ = 450(5) MeV
[Pefkou et all. *Phys.Rev.D* 105 (2022)]

Details in
[Detmold et al.
Phys.Rev.Lett. 126 (2021)
Alexandrou et al.
Phys.Rev.D 105 (2022)]

From ChPT

Details in
Alharazin et al., *Phys.Rev.D* 102 (2020)
Epelbaum et al. Phys.Rev.D 105 (2022)
Alharazin et al *Eur.Phys.J.C* 82 (2022)

How to use FFs?

for non-relativistic (heavy) systems

[Hofstadter et. all,
Rev. Mod. Phys. 30, 482 (1958)]

$$F(Q^2) = \int d^3 r \rho(\mathbf{r}) e^{i\vec{Q}\cdot\vec{r}}$$

charge density
of proton

[Sachs,
Phys. Rev.126, 2256-2260 (1962)]

Breit frame
$Q^2 = -\vec{q}^{\,2}$

$$\rho(r) \equiv \int \frac{d^3 Q}{(2\pi)^3} G_E(Q^2) e^{-i\vec{Q}\cdot\vec{r}}$$

[M.V.Polyakov,
Phys. Lett.B 555, 57 (2003)]

$$T_{\mu\nu}(\mathbf{r}, s) = \frac{1}{2E} \int \frac{d^3 Q}{(2\pi)^3} e^{i\vec{Q}\cdot\vec{r}} \langle p', s' \,|\, \hat{T}_{\mu\nu}(0) \,|\, p, s \rangle$$

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \vert J^\mu_{\mathbf{em}} \vert N \rangle$	$\longrightarrow$	Q	$= 1.602176487(40) \times 10^{-19}\text{C}$
				μ	$= 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\mathbf{weak}} \vert N \rangle$	$\longrightarrow$	g_A	$= 1.2694(28)$
				g_p	$= 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\mathbf{grav}} \vert N \rangle$	$\longrightarrow$	m	$= 938.272013(23)\,\text{MeV}/c^2$
				J	$= \frac{1}{2}$
				$\boldsymbol{D}$	$= \mathbf{?}$

Last global unknown property

…Sachs's derivation assumes delocalised wave packet, resulting in moments of the charge density governed by the size of the wave packet

[M. Burkardt
Phys. Rev. D 66 (2002), 119903(E),
G. Miller
Phys. Rev. Lett. 99, 112001 (2007)
Phys. Rev. C 79, 055204 (2009)
Ann. Rev. Nucl. Part. Sci. 60 (2010), 1-25
Phys. Rev. C99, no.3, 035202 (2019),
A. Freese and G. Miller
Phys. Rev. D103, 094023 (2021)],
arxiv.org/pdf/2210.03807(2022)]
[R.L.Jaffe, Phys. Rev. D103 no.1, 016017 (2021)]

…the meaningful way to obtain the fully relativistic spatial densities is through 2D Fourier transform at fixed light-front time

…this interpretation is not valid for system $\Delta \sim 1/m$

How to define spatial densities?

- **3D Breit frame approach is not exact, valid only for heavy system with $\Delta > 1/m$**
- **2D light-front approach is exact, for all systems**
- **the 3D phase-space approach is exact, for all systems, but has no probabilistic interpretation**
- **3D novel approach of sharp localisation**

C. Lorce,
Phys. Rev. Lett. 125, no.23, 232002 (2020),
C. Lorce, P. Schweitzer and K. Tezgin,
Phys.Rev. D 106, 014012 (2022)
Y. Guo, X. Ji and K. Shiells,
Nucl. Phys. B 969, 115440 (2021),
C. Lorce, H. Moutarde and A. P. Trawinski,
Eur. Phys. J. C 79, no.1, 89 (2019).1, 016017 (2021)

Construction of electromagnetic densities for a spin-1/2 particle

Matrix element of electromagnetic current operator at t=0:

$$\langle p', s' | \hat{j}^\mu(\mathbf{x},0) | p, s\rangle = e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}}\,\bar{u}(p',s')\left[\gamma^\mu F_1(q^2) + \frac{1}{2} i\sigma^{\mu\nu} q_\nu F_2(q^2)\right] u(p,s)$$

$F_1(0) = 1,\ F_2(0) = \kappa/m$

$q = p' - p$

Normalised Heisenberg-picture state: $$|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\,\phi(s,\mathbf{p})\,e^{-i\mathbf{p}\cdot\mathbf{X}}\,|p,s\rangle$$

Profile function: **spherically symmetric**

$\phi(s,\mathbf{p}) = \phi(\mathbf{p}) = R^{3/2}\tilde{\phi}(R|\mathbf{p}|)$

sharp localization: $R \to 0$

$\int d^3p\,|\phi(s,\mathbf{p})|^2 = 1$

$$\boxed{j^\mu_\phi(\mathbf{r}) = \langle \Phi, \mathbf{X}, s' | \hat{j}^\mu(\mathbf{x},0) | \Phi, \mathbf{X}, s\rangle}$$

$\mathbf{X}$ — position of the charge and magnetisation centers

ZAMF - zero average momentum frame, where $\langle \Phi, \mathbf{X}, s | \mathbf{p} | \Phi, \mathbf{X}, s\rangle = 0$

$\mathbf{P} = (\mathbf{p}' + \mathbf{p})/2,\ \mathbf{q} = \mathbf{p}' - \mathbf{p}$

$$j^\mu_\phi(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}}\bar{u}(p',s')\left[\gamma^\mu F_1((E-E')^2 - \mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2} F_2((E-E')^2 - \mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$E = \sqrt{m^2 + \mathbf{P}^2 - \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4}$ $\qquad E' = \sqrt{m^2 + \mathbf{P}^2 + \mathbf{P}\cdot\mathbf{q} + \mathbf{q}^2/4}$

Current densities in static approximation

$$j_\phi^\mu(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}} \bar{u}(p',s')\left[\gamma^\mu F_1((E-E')^2-\mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2} F_2((E-E')^2-\mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

[R.L. Jaffe, 2021]

taking $m \to \infty$ and after that $R \to 0$ using method of dimensional counting (= strategy of regions):

[J. Gegelia, G.Sh. Dzaparidze and K.Sh. Turashvili, Theor. Math. Phys.101, 1313-1319 (1994)]

$$J^0_{\text{static}}(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_E(-\mathbf{q}^2) \equiv \rho^{\text{ch}}_{\text{static}}(r)$$

$$\mathbf{J}_{\text{static}}(\mathbf{r}) = \frac{\nabla_\mathbf{r}\times\boldsymbol{\sigma}}{2m}\int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} G_M(-\mathbf{q}^2) \equiv \frac{\nabla_\mathbf{r}\times\boldsymbol{\sigma}}{2m}\rho^{mag}_{static}(r)$$

- coincide with Breit Frame expressions
- no dependence on wave packet
- valid for heavy systems with $\Delta \gg R \gg 1/m$
- this approximation is doubtful for light hadrons, $\Delta \lesssim 1/m$

[R.L. Jaffe, Phys. Rev. D103 no.1, 016017, (2021)]

[Sachs,
Phys. Rev.126, 2256-2260 (1962)]

Novel definition of the current densities

$$j^{\mu}_{\phi}(\mathbf{r}) = \int \frac{d^3P d^3q}{(2\pi)^3\sqrt{4EE'}} \bar{u}(p', s')\left[\gamma^{\mu} F_1((E-E')^2 - \mathbf{q}^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2} F_2((E-E')^2 - \mathbf{q}^2)\right] u(p,s) \times \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^{\star}\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

taking $R \to 0$ for arbitrary m, using method of dimensional counting:

[Epelbaum et al. [Phys.Rev.Lett.129, 012001](2022)]

$$\mathbf{J}(\mathbf{r}) = \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m} \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{+1} d\alpha \frac{1}{4}\left(1+\alpha^2\right) m F_2\left[(\alpha^2 - 1)\mathbf{q}^2\right] \equiv \frac{\nabla_{\mathbf{r}} \times \boldsymbol{\sigma}}{2m} \rho_2(r)$$

★

$$J^0(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \int_{-1}^{+1} d\alpha \frac{1}{2} F_1\left[(\alpha^2 - 1)\mathbf{q}^2\right] \equiv \rho_1(r)$$

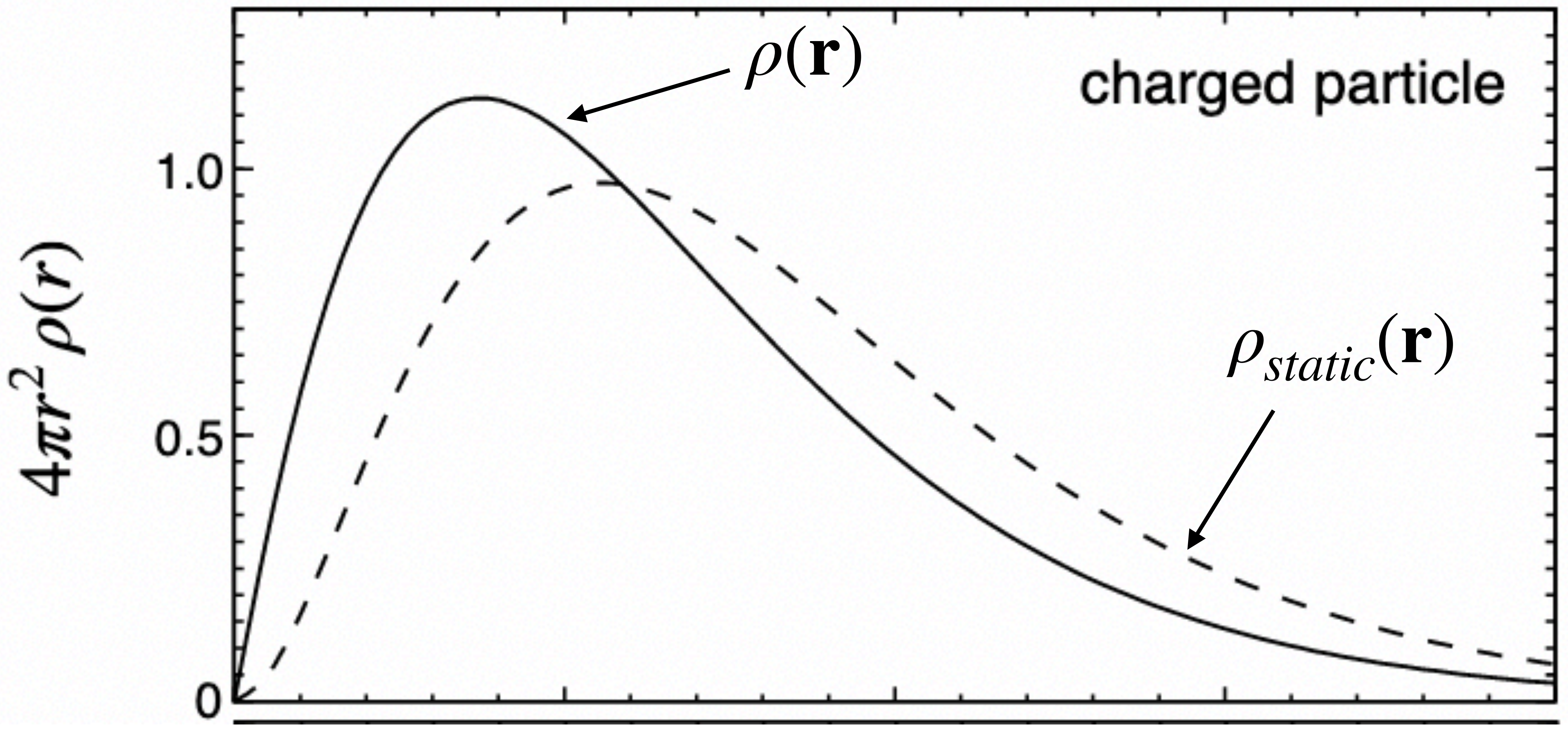
[G.N.Fleming, Physical Reality Math. Descrip., 357 (1974)]

$$\sqrt{\langle r^2\rangle_{\text{static}}} = \sqrt{6\left(F_1'(0)\right)} \simeq 0.8409(4)\ , \ \sqrt{\langle r^2\rangle} = \sqrt{4F_1'(0)} \simeq 0.62649,$$

$\Delta \gg R \gg 1/m$

$R \to 0$

[G. A. Miller, Phys. Rev. C 99, no.3, 035202 (2019).]

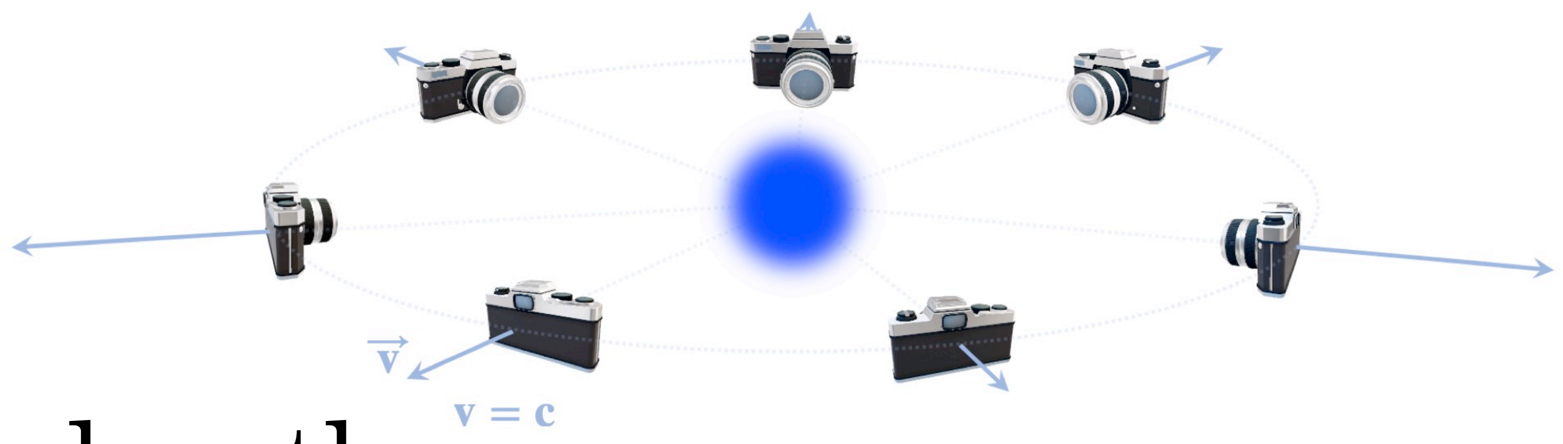


Connection with IMF densities

In moving frame:

$$\boxed{j^{\mu}_{\phi,v}(\mathbf{r}) = {}_{\mathbf{v}}\langle \Phi, \mathbf{X}, s' \,|\, \hat{j}^{\mu}(\mathbf{r},0) \,|\, \Phi, \mathbf{X}, s \rangle_{\mathbf{v}}}$$

$$J^0_{ZAMF}(\mathbf{r}) = \frac{1}{4\pi}\int d\hat{\mathbf{v}}\, J^0_{IMF}(\mathbf{r})\,, \qquad \mathbf{J}_{ZAMF}(\mathbf{r}) = 2\times\frac{1}{4\pi}\int d\hat{\mathbf{v}}\, \mathbf{J}_{IMF}(\mathbf{r})\,.$$



- described only by intrinsic properties of system
- valid for any systems independently on the Compton wavelength
- holographic relation between ZAMF and IMF

[Epelbaum et al. [Phys.Rev.Lett.129, 012001](2022)]

Gravitational spatial densities for spin-1/2

$$\langle p', s' | \hat{T}_{\mu\nu}(\mathbf{x},0) | p, s\rangle = e^{-i\mathbf{q}\cdot\mathbf{x}}\, \bar{u}(p', s') \left[A(q^2)\frac{P_\mu P_\nu}{m} + iJ(q^2)\frac{P_\mu \sigma_{\nu\alpha}q^\alpha + P_\nu \sigma_{\mu\alpha}q^\alpha}{2m} + D(q^2)\frac{q_\mu q_\nu - \eta_{\mu\nu}q^2}{4m}\right] u(p, s)\,,$$

$$\boxed{t^{\mu\nu}_\phi(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' | \hat{T}^{\mu\nu}(\mathbf{x},0) | \Phi, \mathbf{X}, s\rangle}$$

[Panteleeva, Epelbaum, Gegelia, Meißner, arXiv: 2211.09596]

$$t^{00}_\phi(\mathbf{r}) = \frac{N_{\phi,\infty}}{4\pi}\int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} A\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$\boxed{\begin{aligned} N_{\phi,\infty} &= \frac{1}{R}\int d^3\tilde{P}\tilde{P}\, |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2 \\ N_{\phi,0} &= R\int d^3\tilde{P}\frac{1}{\tilde{P}}|\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2 \end{aligned}}$$

$$t^{0i}_\phi(\mathbf{r}) = \frac{N_{\phi,\infty}}{4\pi}\int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3}\left[\frac{iJ\left(-\mathbf{q}_\perp^2\right)}{2m}\left((\boldsymbol{\sigma}_\perp \times \mathbf{q})^i + \hat{\mathbf{n}}\cdot(\boldsymbol{\sigma}_\perp\times\mathbf{q})\hat{n}^i\right)\right] e^{-i\mathbf{q}\cdot\mathbf{r}}$$

$$t^{ij}_\phi(\mathbf{r}) = \frac{N_{\phi,\infty}}{4\pi}\int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3}\hat{n}^i\hat{n}^j A\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}} + \frac{1}{4}\,\frac{N_{\phi,0}}{4\pi}\int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3}\left(q^i q^i - \delta^{ij}\mathbf{q}_\perp^2\right) D\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

flow tensor ***stress tensor***

★

Mass and energy distribution

Interpretation

$$\boxed{t_\phi^{00}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' \,|\, \hat{T}^{00}(\mathbf{r},0) \,|\, \Phi, \mathbf{X}, s \rangle}$$

For sharply localised packet $R \to 0$ and arbitrary m

$$t_\phi^{00}(\mathbf{r}) = N_{\phi,\infty} \int \frac{d^2\hat{n}\, d^2 q_\perp}{(2\pi)^3(4\pi)} A\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}_\perp\cdot\mathbf{r}_\perp} \delta(r_\parallel)$$

Energy distribution

$$N_{\phi,\infty} = \frac{1}{R} \int d^3\tilde{P}\tilde{P}\,|\,\tilde{\phi}(\,|\,\tilde{\mathbf{P}}\,|\,)\,|^2$$

for $R \to 0$ and $\mathbf{P} \sim 1/R$

the energy $E = \sqrt{m^2 + \mathbf{P}^2} \sim \dfrac{1}{R}$

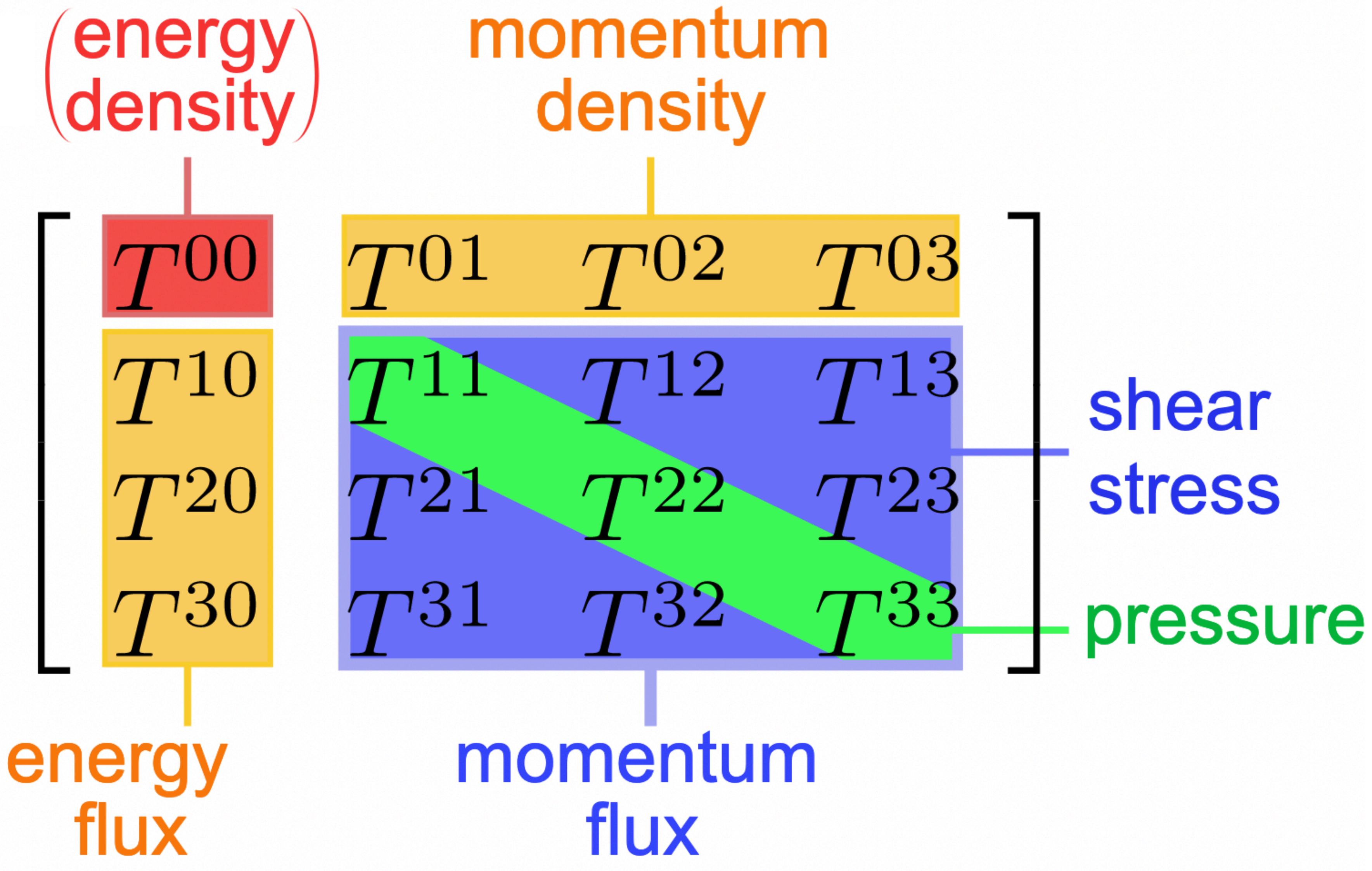
Static approximation $(m \to \infty, R \to 0)$: $R \gg 1/m$

$$t_{static}^{00}(\mathbf{r}) = m \int \frac{d^3q}{(2\pi)^3} A(-\mathbf{q}^2) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

Mass distribution

for $m \to \infty$ $R \gg 1/m$,
$\mathbf{P} \sim 1/R \ll m$

$$E = \sqrt{m^2 + \mathbf{P}^2} \simeq m + O(\mathbf{P}^2/(2m))$$



Pressure and shear force distributions

$$\boxed{t^{ij}_{\phi}(\mathbf{r}) \equiv \langle \Phi, \mathbf{X}, s' \,|\, \hat{T}^{ij}(\mathbf{x},0) \,|\, \Phi, \mathbf{X}, s \rangle = t^{ij}_{\phi,0}(\mathbf{r}) + t^{ij}_{\phi,2}(\mathbf{r})}$$

Interpretation

For sharply localised packet $(R \to 0$ and arbitrary $m)$

$$t^{ij}_{\phi,2}(\mathbf{r}) = \frac{1}{4} N_{\phi,0} \int \frac{d^2\hat{n}}{4\pi} \frac{d^3q}{(2\pi)^3} \left(q^i q^j - \delta^{ij} \mathbf{q}_\perp^2\right) D\left(-\mathbf{q}_\perp^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

Static approximation $(m \to \infty, R \to 0)$: $R \gg 1/m$

$$t^{ij}_{static,2}(\mathbf{r}) = \frac{1}{4m} \int \frac{d^3q}{(2\pi)^3} \left(-\mathbf{q}^2 \delta^{ij} + q^i q^j\right) D\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}$$

pressure **shear force**

$$t^{ij}_2(r) = \delta^{ij} p(r) + \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r)$$

$$p(\mathbf{r}) = \frac{N_{\phi,0}}{4} \int \frac{d^2\hat{n}}{4\pi} \left(\frac{1}{r_\perp^2} \frac{d}{dr_\perp} r_\perp^2 \frac{d}{dr_\perp} - \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr}\right) \left(\delta(r_\parallel) \tilde{D}\left[\mathbf{r}_\perp\right]\right)$$

$$p_{static}(\mathbf{r}) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}\left[\mathbf{r}\right]$$

$$s(\mathbf{r}) = -\frac{N_{\phi,0}}{4} \int \frac{d^2\hat{n}}{4\pi} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \left(\delta(r_\parallel) \tilde{D}\left[\mathbf{r}_\perp\right]\right)$$

$$s_{static}(\mathbf{r}) = -\frac{1}{4m} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}\left[\mathbf{r}\right]$$

2D Fourier transformation

3D Fourier transformation

Mechanical properties of hadrons

D-term via the static approximation

$$D = -\frac{m}{2}\int d^3r \left(r^i r^j - \frac{1}{3}\delta_{ij}\right) T_{ij}^{static}(r) = m\int d^3r r^2 p_{static}(r) = -\frac{4}{15}m\int d^3r r^2 s_{static}(r)$$

[M.V.Polyakov, Phys. Lett.B 555, 57 (2003)]

D-term via the sharp localisation

$$D = -\frac{4}{15\ N_{\phi,0}}\int d^3r r^2 s(r)$$

$$\partial_i T_{ij}(r) = 0$$

$$\int d^3r\ p(r) = 0$$

the von Laue stability condition

[Laue, Ann. Phys. 340, 524 (1911)]

$$\frac{2}{3}s'(r) + p'(r) + \frac{2}{r}s(r) = 0$$

equilibrium equation

$$F^i(r) = T^{ij}(r)dS\ n^j = \left(\frac{2}{3}s(r) + p(r)\right) dS^i = F_n(r)dS^i$$

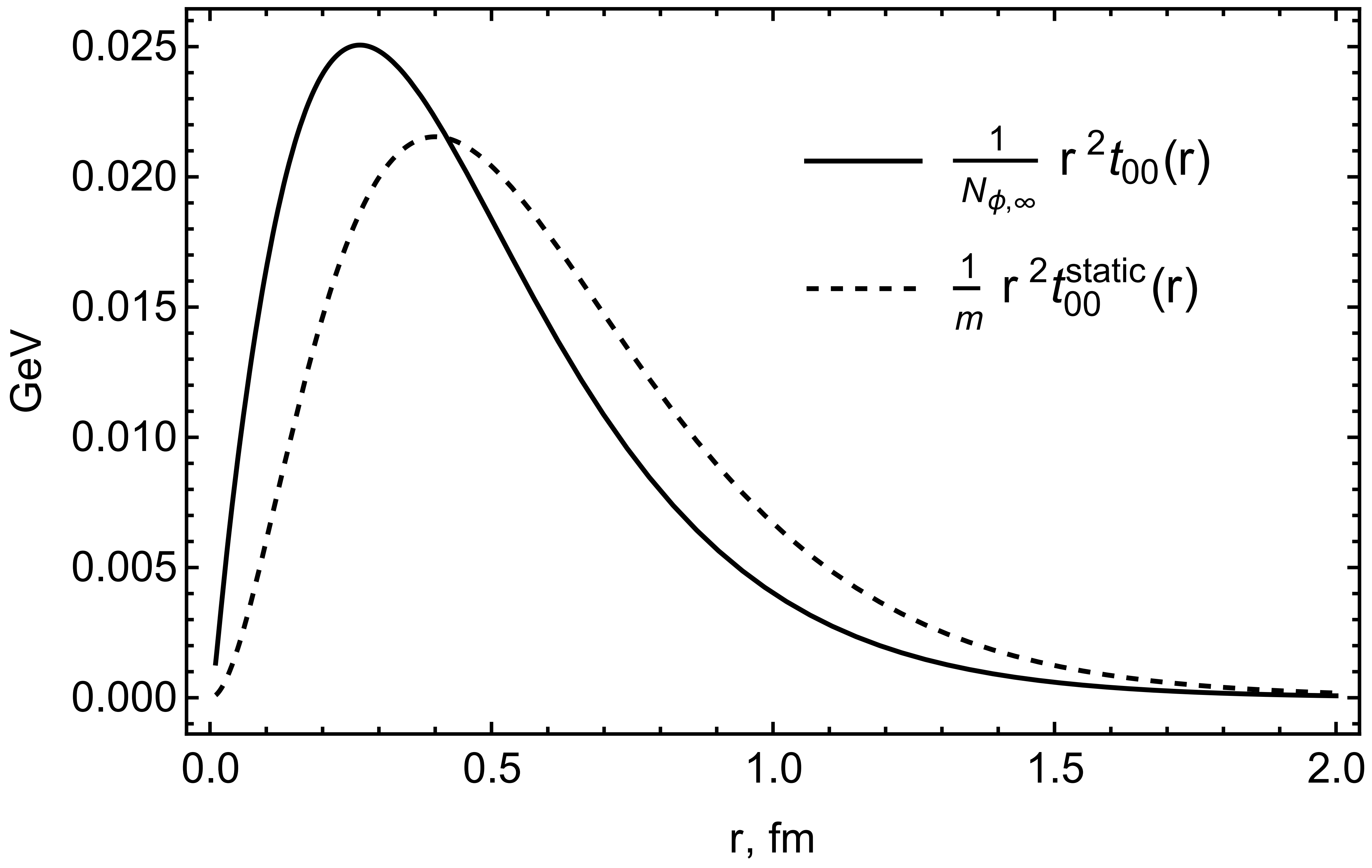
the normal forces

$$\frac{2}{3}s(r) + p(r) > 0$$

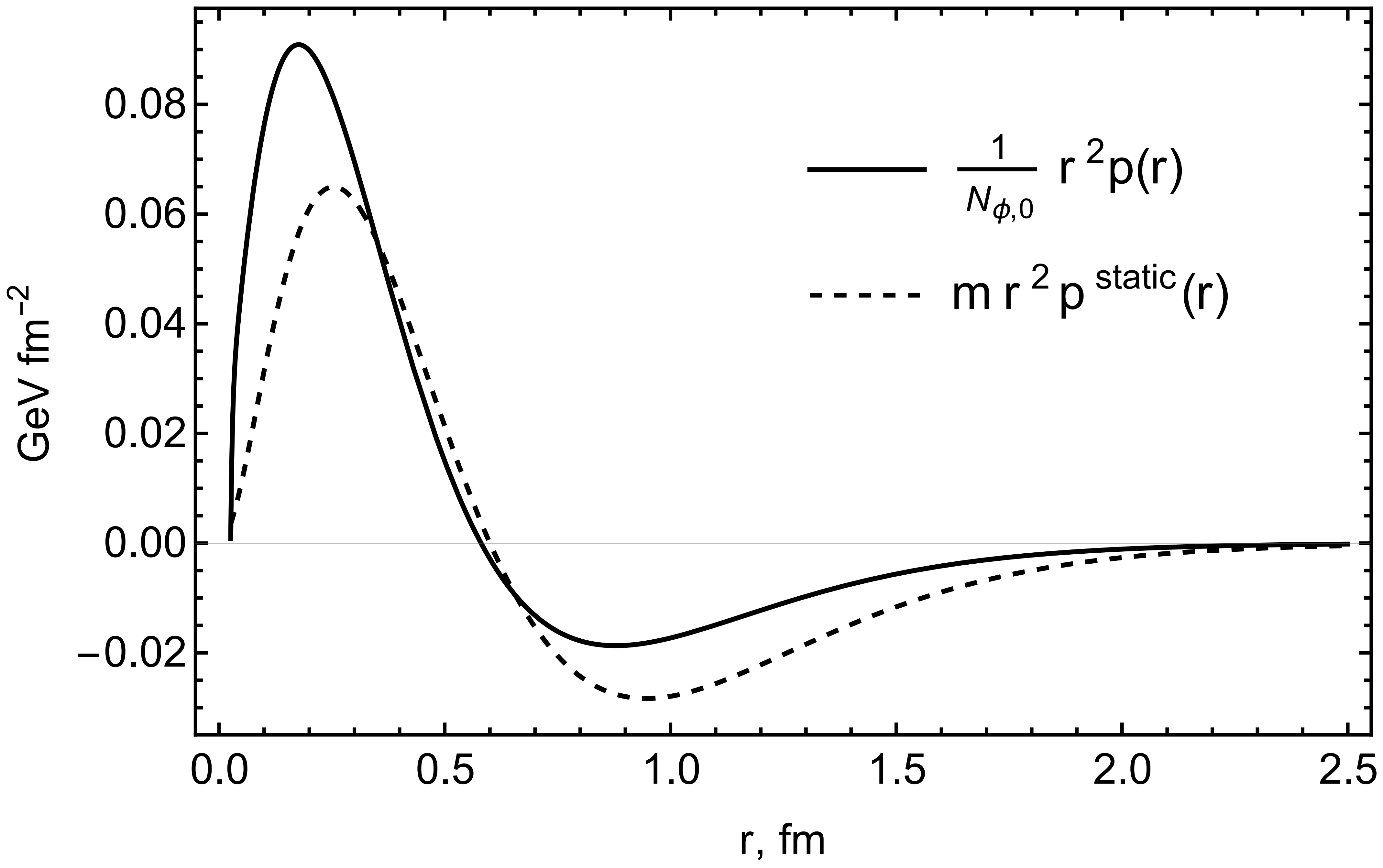
local stability condition

$$D < 0$$

[Perevalova, Polyakov, Schweitzer, Phys. Rev. D 94, 054024 (2016)]



Comparison of energy densities



Comparison of pressure distributions

$$\langle r_E^2 \rangle = \frac{\int d^3 r r^2 \rho_E^{static}(r)}{\int d^3 r \rho_E^{static}(r)} = 6A'(0)$$

$$\langle r_E^2 \rangle = \frac{\int d^3 r r^2 \rho_E(r)}{\int d^3 r \rho_E(r)} = 4A'(0)$$

[G. A. Miller, Phys. Rev. C 99, no.3, 035202 (2019).]

$$\langle r_{mech}^2 \rangle = \frac{\int d^3 r r^2 F_n^{static}(r)}{\int d^3 r F_n^{static}(r)} = \frac{6D}{\int_0^\infty dt D(t)}$$

[M.V. Polyakov, P. Schweitzer, Int.J.Mod.Phys.A 33 (2018)]

$$\langle r_{mech}^2 \rangle = \frac{\int d^3 r r^2 F_n(r)}{\int d^3 r F_n(r)} = \frac{16D}{\int_0^\infty dt \int_{-1}^{1} d\alpha D\left(t[1-\alpha^2]\right)}$$

- Form factors help to understand the structure of particles

- The definition of spatial densities is important for studying the structure of particles

- The sharp localisation approach suggests 3D definition of local densities and is valid for any system

Thank you for your attention!