Spin-1 quarkonia in a rotating frame and their spin contents

ref) PLB 843 (2023) 137986

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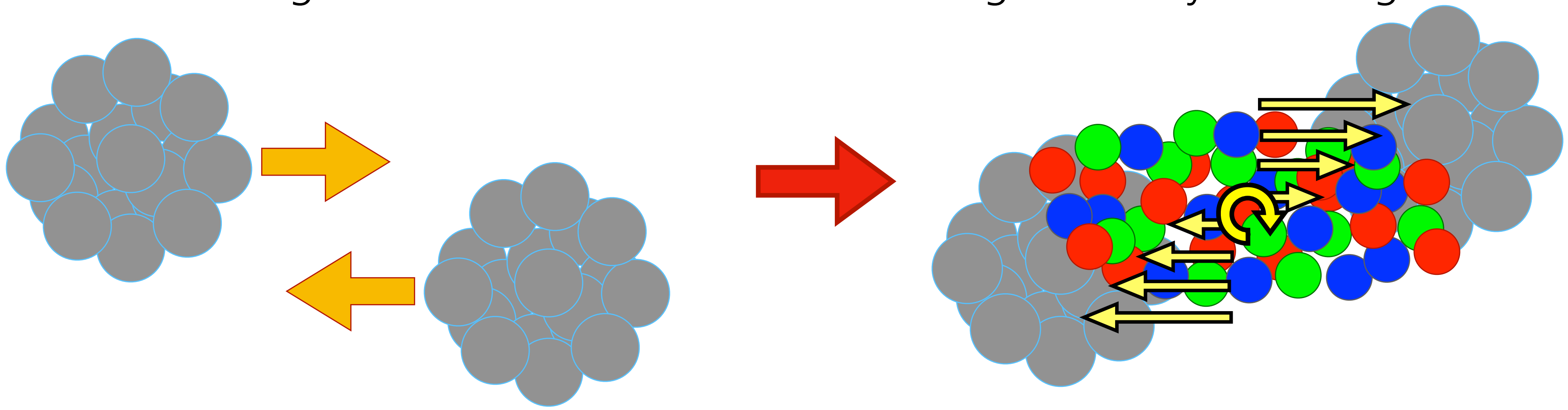
@HADRON2023, Genova, Italy, 2023.06.08

Introduction - Experimental Motivation

- **Vector mesons' spin alignment in non-central HICs**

initial angular momentum

Large vorticity & Strong B field



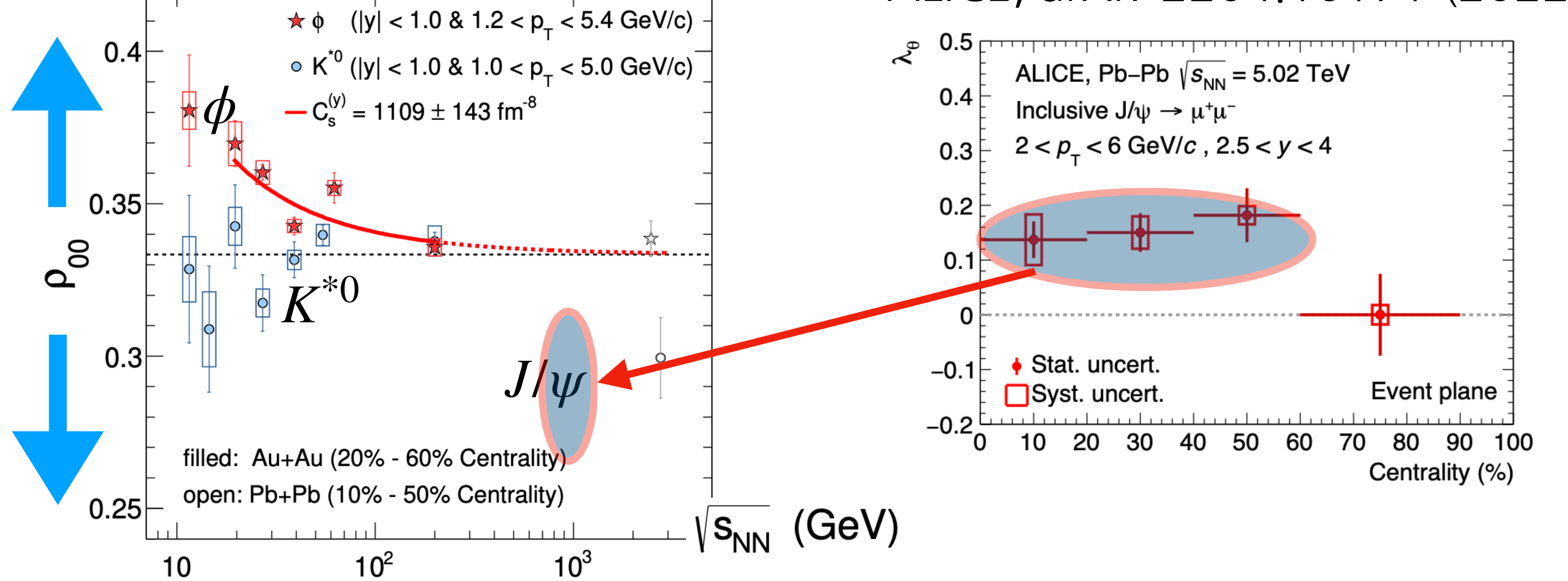
STAR, arXiv:2204.02302 (2022)

ALICE, arXiv:2204.10171 (2022)

Longitudinal-like

Unpolarized

Transverse-like



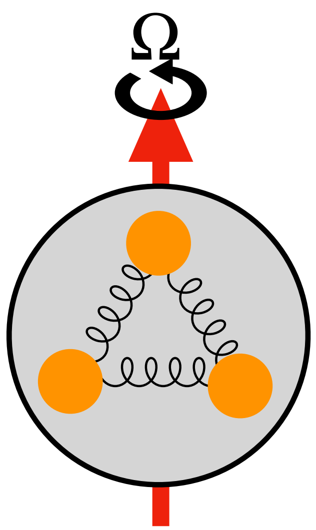
not yet clearly understood.

Overall polarization depends on both total spin and spin content

Deeper understanding of the spin content of hadrons would be required.

Introduction - Theoretical Motivation

- **Spin-Rotation Coupling(SRC)**
  the most responsible theory for spin alignment in a rotating medium



$$H_{\text{SRC}} = H_{\text{r}} - H_{\text{i}} = -S \cdot \Omega$$

H_i : Energy in an inertial frame
H_r : Energy in a rotating frame
S : total spin of the system
Ω : angular velocity

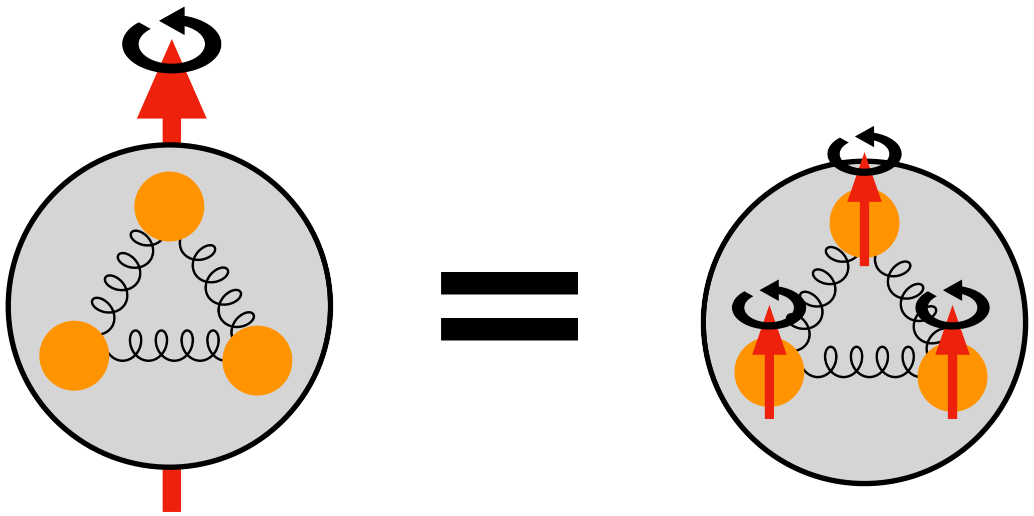
- Spin1/2 : Derived from Dirac eq. in a rotating frame using G.R.
- Spin1 : No strict derivation using G.R. until recently!
- Phys.Rev.D 102 (2020) 12, 125028 - J.Kapusta, E.Rrapaj, S.Rudaz
  - Proca eq. for massive spin-1 particle using G.R.
  - $H_{\text{SRC}}^{\text{spin1}} = -\frac{1}{2} S \cdot \Omega$!
  - contradictory to naive expectation and quark model
- **We need to clarify the strength of SRC of spin-1 particle!**

Outline

- As a first step, we study **spin-1 heavy $Q\bar{Q}$ system**
- Introduce a **free parameter** "g_Ω" which indicates the strength of SRC,

$$H_{\mathrm{SRC}} = -\, g_\Omega \, S \cdot \Omega$$

- Since hadron is a composite particle, we describe **total SRC of a hadron by collecting all responses of quarks and gluons in a rotating frame**



- We prove that $g_\Omega = g_\Omega^{quark}(Q^2) + g_\Omega^{gluon}(Q^2) = 1$
- We recognize that
  the fraction of g_Ω carried by each quarks and gluons = spin content
- We study the spin content of $J/\psi, \Upsilon(1S)$(vector) and χ_{c1}, χ_{b1}(axialvector)

How to extract g_Ω?

1. Consider the correlation function in a rotating frame

$$\Pi^{\mu\nu}(q) = i\int d^4x e^{iqx}\langle 0|\mathrm{T}\{\mathrm{j}^\mu(\mathrm{x})\mathrm{j}^\nu(0)\}|0\rangle$$

2. Put the system at the center of the rotation -> no external OAM
   Pick out a right circularly polarized state

$$\Pi^+(\omega) = \epsilon^+_\mu \epsilon^{+*}_\nu \Pi^{\mu\nu}(\omega,0) \qquad q_\mu = (\omega,\vec{0}), \quad \epsilon^+_\mu = (0,1,i,0)/\sqrt{2}$$

3. Keep Ω linear terms

$$\Pi^+(\omega) = \omega^2\,\Pi^{vac}(\omega^2) + \omega\,\Pi^{rot}(\omega^2)\,\Omega + \mathcal{O}(\Omega^2)$$

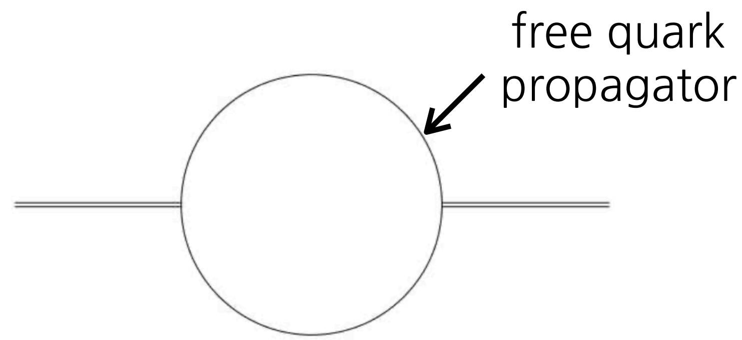
   Π^{vac} : ordinary vacuum invariant ftn. vacuum properties ex) mass
   Π^{rot} : new function appearing in a rotating frame. spin information

4. Extract g_Ω by comparing two different descriptions of Π^{rot}
   (a) Direct OPE computation in a rotating frame
   (b) Phenomenological derivation from Π^{vac}

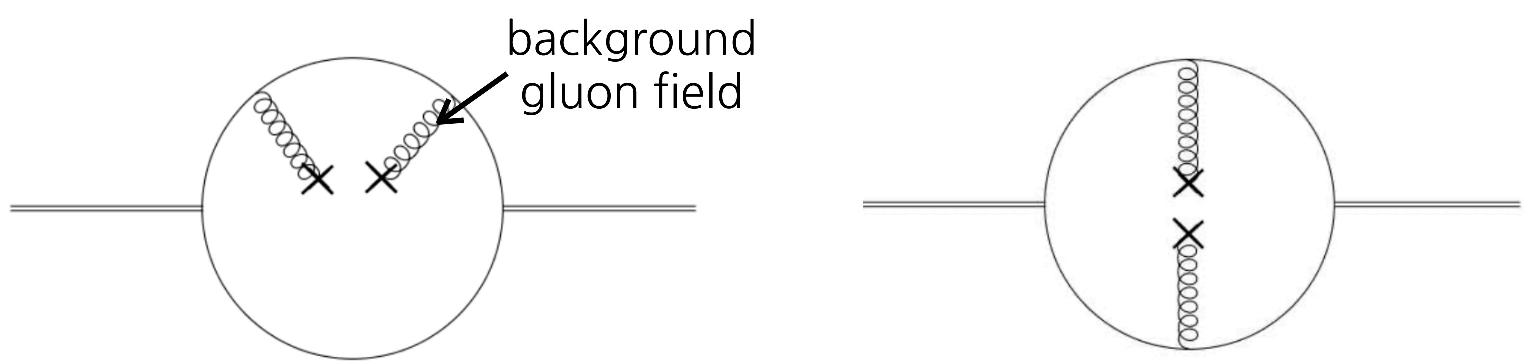
(a) Direct computation in a rotating frame

- Feynman diagrams in Operator Product Expansion(OPE)

Leading perturbative diagram



Leading non-perturbative diagrams : Gluon condensates



- Compute in an inertial frame → Ω independent terms → Π^{vac}
- Compute in a rotating frame → collect Ω linear terms → Π^{rot}

Quarks in a rotating frame

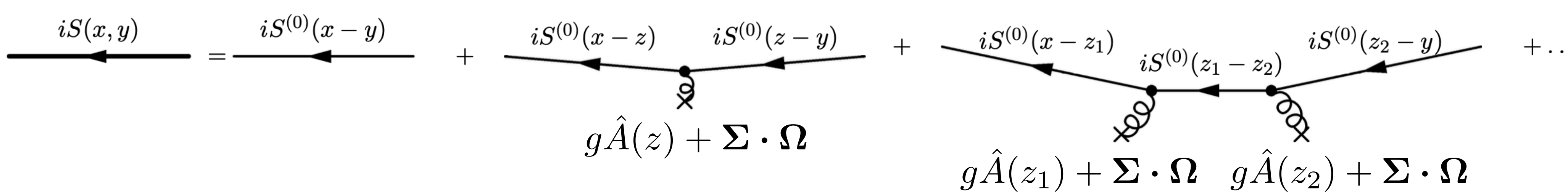
- Recall Dirac eq. in a rotating frame

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma}\cdot\mathbf{\Omega} - m\right]\Psi(x) = 0 \text{ where } \mathbf{\Sigma} = \gamma^0(\mathbf{S}+\mathbf{L})$$

- Quark propagator in a rotating frame

$$\left[i\partial\!\!\!/_x + gA\!\!\!/(x) + \mathbf{\Sigma}\cdot\mathbf{\Omega} - m\right]S(x) = \delta(x)$$

- difficult to find full propagator
- expansion in terms of 'g' and '$\mathbf{\Omega}$'



$$S^{\text{full}} \approx S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)}\left[gA\!\!\!/ + \Sigma\cdot\Omega\right]S^{(0)} + \cdots$$

- use $\mathbf{\Omega}$ linear terms to compute Π^{rot}

Gluons in a rotating frame

- Covariant derivatives in curved space-time(Γ^a_{bc} : Christoffel symbols)
  $D_c G_{ab} = \partial_c G_{ab} - \Gamma^d_{ca} G_{db} - \Gamma^d_{cb} G_{ad}$

- In Fock-Schwinger gauge,

$$A_\mu(x) = -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha \underline{\partial_\alpha} G_{\mu\nu} + \cdots$$

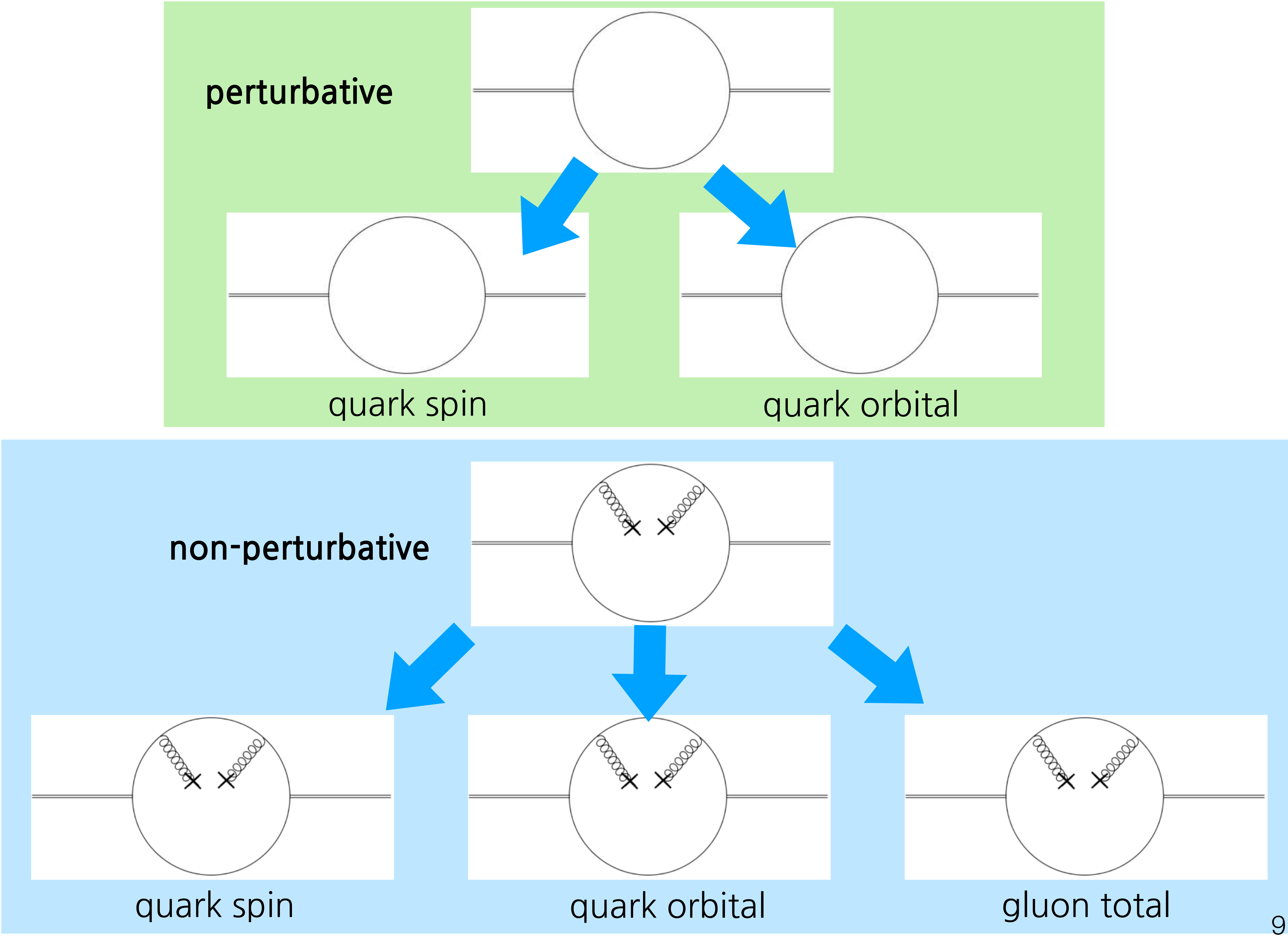
$$= -\frac{1}{2} x^\nu G_{\mu\nu} - \frac{1}{3} x^\nu x^\alpha \underline{D_\alpha} G_{\mu\nu} \underline{- \frac{1}{3} x^\nu x^\alpha (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d})} + \cdots$$

additional contribution in curved space-time

- $\Gamma^2_{01} = \Omega, \Gamma^1_{02} = -\Omega$ in a rotating frame.
  $\not{A}^{\text{new}}(x) = -\frac{1}{3} x^\nu x^\alpha \gamma^\mu (\Gamma^d_{\alpha\mu} G_{d\nu} + \Gamma^d_{\alpha\nu} G_{\mu d}) \propto \vec{x} \times (\vec{E} \times \vec{B}) \cdot \Omega = J_g \cdot \Omega$

- Kapusta et al. thought that $D_c G_{ab} = \partial_c G_{ab}$ in a rotating frame.
  Their result might be wrong!

Feynman diagrams in a rotating frame



(b) Phenomenological derivation of Π^{rot}

- In an inertial frame

$$\Pi^{+}(\omega) = \epsilon_{+}^{\mu}\epsilon_{+}^{\nu *}\Pi_{\mu\nu}(\omega,0) = \omega^2\,\Pi^{vac}(\omega^2)$$

- Energy shift of all right circularly polarized state in a rotating frame

$$=\!> \omega \to \omega - g_{\Omega}\,\Omega \qquad\qquad H_{\mathrm{SRC}} = -\,g_{\Omega}\,S\cdot\Omega$$

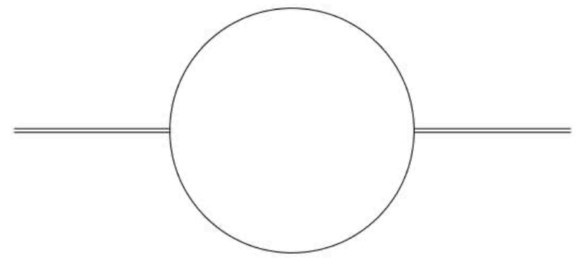
$$\begin{aligned}\Pi^{+}(\omega + g_{\Omega}\Omega) &= (\omega + g_{\Omega}\Omega)^2\,\Pi^{vac}((\omega + g_{\Omega}\Omega)^2) \\ &= \omega^2\,\Pi^{vac}(\omega^2) + \omega\,\Pi^{rot}(\omega^2)\,\Omega\end{aligned}$$

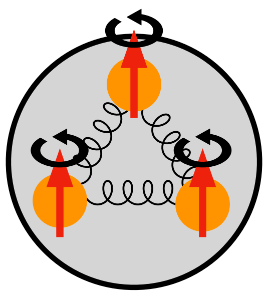
- Simple expression of rotating part in terms of vacuum invariant ftn.

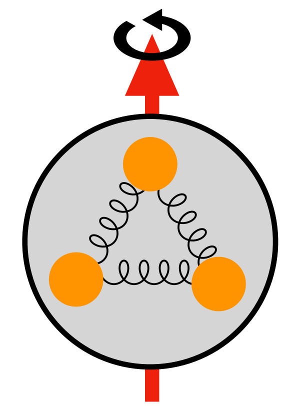
$$\Pi^{rot}_{(b)}(\omega^2) = 2g_{\Omega}\left\{\Pi^{vac}(\omega^2) + \omega^2\frac{\partial\Pi^{vac}(\omega^2)}{\partial\omega^2}\right\}$$

=> We can directly derive Π^{rot} from Π^{vac} but it has unknown g_{Ω}

g_Ω in perturbative region

$\Pi^{vac}_{pert} =$ 





(a) **Direct OPE computation**
collect all responses of quarks
in a rotating frame
quark's spin + orbital AM

(b) **Phen. derivation**
energy shift of total system by "-$g_\Omega \Omega$".
total spin of the system

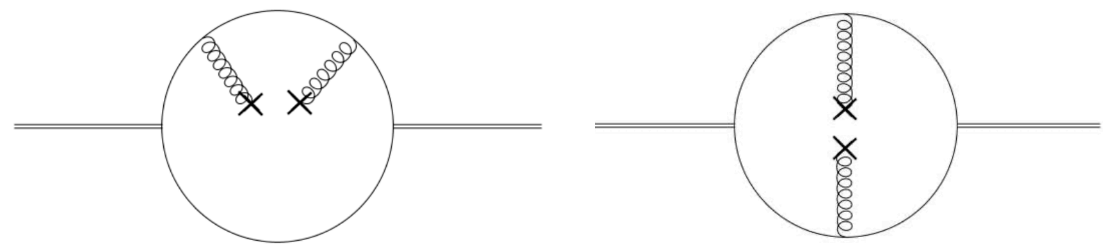
$$\Pi^{rot}_{(a)}(q_0^2) = \frac{2}{\pi}\int_{4m^2}^{\infty} ds \frac{s\,\mathrm{Im}\Pi^{vac}_{pert}(s)}{(s-q_0^2)^2} \quad = \quad \Pi^{rot}_{(b)}(q_0^2) = \frac{2g_\Omega}{\pi}\int_{4m^2}^{\infty} ds \frac{s\,\mathrm{Im}\Pi^{vac}_{pert}(s)}{(s-q_0^2)^2}$$

In the perturbative region,

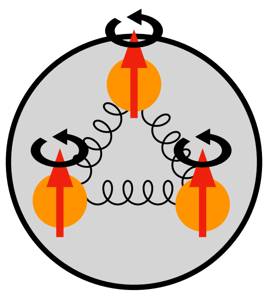
$$g_\Omega = g_\Omega^{\mathrm{quarkspin}}(Q^2) + g_\Omega^{\mathrm{quarkorbit}}(Q^2) = 1$$

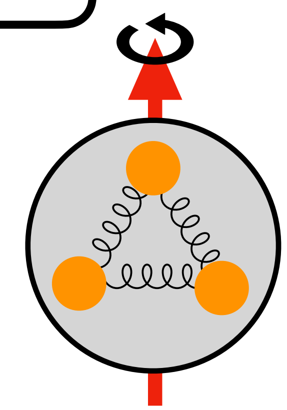
g_Ω in non-perturbative region

$\Pi^{vac}_{G0} =$



G0 : gluon condensate





(a) **Direct OPE computation**
all responses of quarks and gluons in a rotating frame

(b) **Phen. derivation**
energy shift of total system by "-$g_\Omega\Omega$".
total spin of the system

$$\Pi^{rot}_{(a)}(\omega^2) = 2\left\{\Pi^{vac}_{G_0}(\omega^2) + \omega^2\frac{\partial\Pi^{vac}_{G_0}(\omega^2)}{\partial\omega^2}\right\}$$

=

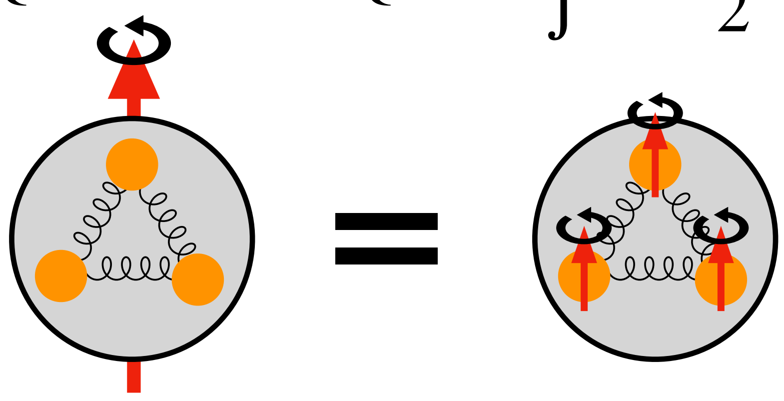
$$\Pi^{rot}_{(b)}(\omega^2) = 2g_\Omega\left\{\Pi^{vac}_{G_0}(\omega^2) + \omega^2\frac{\partial\Pi^{vac}_{G_0}(\omega^2)}{\partial\omega^2}\right\}$$

Even in the non-perturbative region,

$$g_\Omega = g_\Omega^{\text{quarkspin}}(Q^2) + g_\Omega^{\text{quarkorbit}}(Q^2) + g_\Omega^{\text{gluon}}(Q^2) = 1$$

Physical meaning of $g_\Omega = 1$?

- **What we computed in method (b)?**
  = SRC of the total system
  = $g_\Omega \langle \vec{S} \rangle$ where $\vec{S}$ is spin-1 operator where $\langle \cdots \rangle = \int d^4x e^{iq\cdot x} \langle 0 | T[j(x) \cdots j(0)] | 0 \rangle$
- **What we computed in method (a)?**
  = Ω linear terms in all responses of quarks and gluon in a rotating frame
  = $\langle \vec{J}_{\rm QCD} \rangle$ where $\vec{J}_{\rm QCD} = \int d^3x (\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \psi^\dagger(\vec{x} \times (-i\vec{D}))\psi + \vec{x} \times (\vec{E} \times \vec{B}))$



$$g_\Omega \langle \vec{S} \rangle = \langle \vec{J}_{\rm QCD} \rangle$$

- Therefore, we can conclude that $g_\Omega = \langle \vec{J}_{\rm QCD} \rangle / \langle \vec{S} \rangle$
  - $g_\Omega = 1$ means $\langle \vec{S} \rangle = \langle \vec{J}_{\rm QCD} \rangle$
  - This should be valid for any Feynman diagram ($\because$ AM conservation)
  - From Kallen-Lehmann rep, $g_\Omega = 1$ for all physical states that can couple to $j(x)$
  - By studying g_Ω of bound state, we can investigate the spin content of hadron

g_Ω of ground states

With the help of 'QCD sum rule' + simple 'pole+continuum' ansatz.

sum rule1 : $\mathscr{M}^{vac}(M^2) \equiv \mathscr{B}\Pi^{vac}_{OPE}(M^2) - \int_{s_0}^{\infty} ds e^{-s/M^2}\,\mathrm{Im}\Pi^{vac}_{pert}(s) = -\pi |\langle J/\psi | j^+ \rangle|^2 \frac{e^{-m^2_{J/\psi}/M^2}}{m^2_{J/\psi}}$

sum rule2 : $\mathscr{M}^{rot}(M^2) \equiv \mathscr{B}\Pi^{rot}_{OPE}(M^2) - \frac{2}{M^2}\int_{s_0}^{\infty} ds e^{-s/M^2} s\,\mathrm{Im}\Pi^{vac}_{pert}(s) = -2\pi g_\Omega |\langle J/\psi | j^+ \rangle|^2 \frac{e^{-m^2_{J/\psi}/M^2}}{M^2}$

For ground state, $g_\Omega^{\mathrm{ground}} = -\frac{M^2}{2}\sum_{i=S_q,L_k,L_p,J_g} \frac{\mathscr{M}_i^{rot}}{\mathscr{M}^{vac\prime}} = 1$

We are interested in the fraction of g_Ω carried by each angular momentum inside the ground state

$$g_\Omega^{\mathrm{ground}} = \frac{\langle S_q \rangle + \langle L_k \rangle + \langle L_p \rangle + \langle J_g \rangle}{\langle S_{\mathrm{tot}} \rangle}$$

$S_q = \frac{1}{2}\gamma^1\gamma^2$: quark spin,

$L_k = r \times p$: kinetic part of quark orbital angular momentum,

$L_p = r \times gA$: potential part of quark orbital angular momentum,

$J_g = r \times (E \times B)$: gluon total angular momentum

Result - spin contents of spin-1 quarkonia

This work ($\%$)

			Vector		Axialvector	
			J/ψ	$\Upsilon(1S)$	χ_{c1}	χ_{b1}
Quark	spin	S_q	88	92	40	43
Quark	$r \times p$	L_k	11	7.6	61	57
Quark	$r \times gA$	L_p	0.2	0.003	0.08	-0.001
Gluon	$r \times (E \times B)$	J_g	0.8	0.015	-1.5	-0.005

cf) Classical picture ($\%$)

	$^3S_1$	$^3P_1$
S_q	100	50
L_k	0	50
L_p	0	0
J_g	0	0

- Total sum of 4 pieces becomes 1 $\langle S_q \rangle + \langle L_k \rangle + \langle L_p \rangle + \langle J_g \rangle = 1$
- Classical picture from the naive Q.M.
  S-wave: quark spin(100%) , P-wave: quark spin(50%) quark oam(50%)
- Spin contents are slightly different from the classical picture.
  As the quark mass becomes lighter, spin contents deviate more from the classical picture
  ex) J/ψ is considered as S-wave but quarks do not carry all of the total spin
  $\Upsilon(1S)$ is still comparable with the classical picture

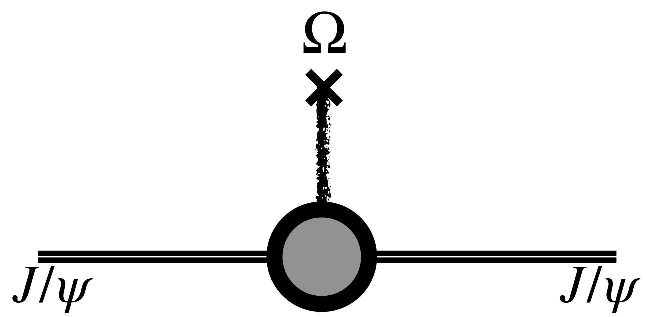
Summary

- We describe SRC of spin-1 system in terms of quarks and gluons in a rotating frame
- We show that $g_\Omega = 1$ through leading pert. and non-pert. diagrams.
- We recognize total spin of system in terms of total angular momenta of quarks and gluons
- We study spin contents of J/ψ, $\Upsilon(1S)$, χ_{c1}, χ_{b1} using QCDSR
- More accurate analysis requires α_s-correction of Π^{rot}
- The same strategy can be applied to study spin content of other hadron

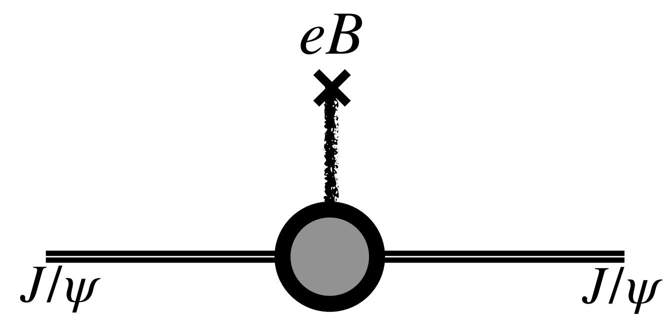
Back up

Ω v.s. B

- Linear term



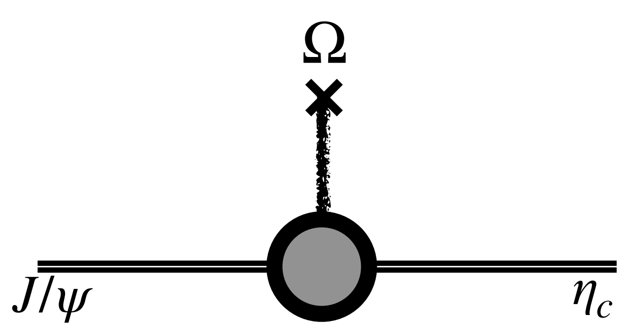
$\neq 0$



$= 0$

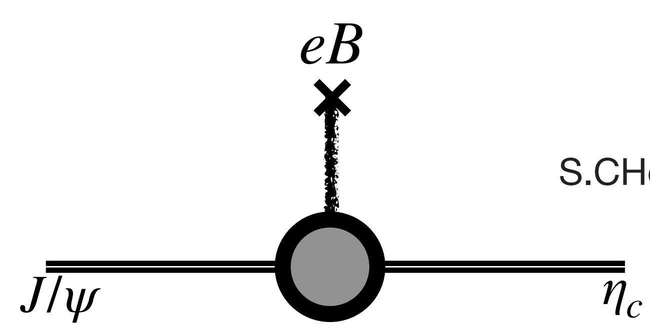
(at least LO)

- Mixing between η_c and J/ψ by Ω?



$= 0$

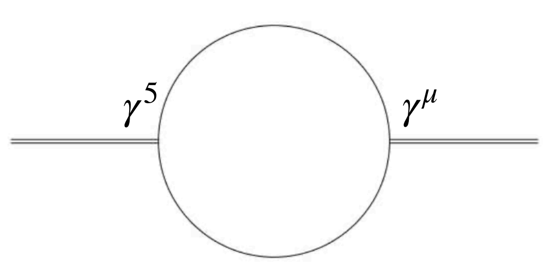
S.CHo,K.Hattori,S.H.Lee et al. PRL113.172301



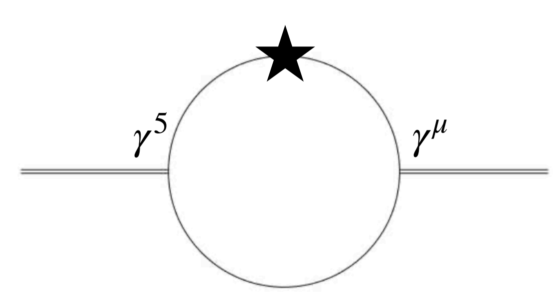
$\neq 0$

This can be simply understood by $\Pi^{rot}(\omega^2) = 2\left\{ \Pi^{vac}(\omega^2) + \omega^2 \frac{\partial \Pi^{vac}(\omega^2)}{\partial \omega^2} \right\}$

Since



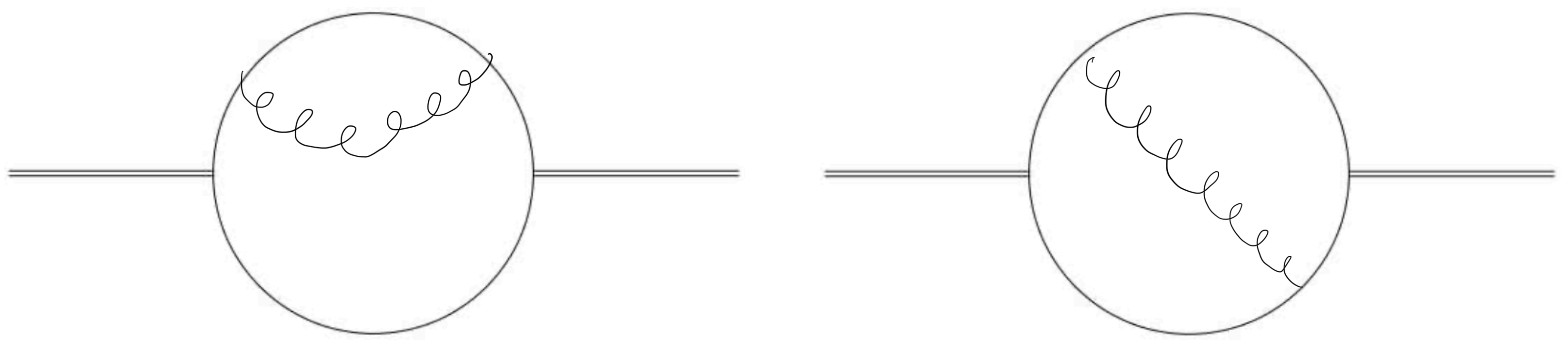
$= 0$,



$= 0$

★ indicates insertion of Ω linear terms

Q. α_s-correction?

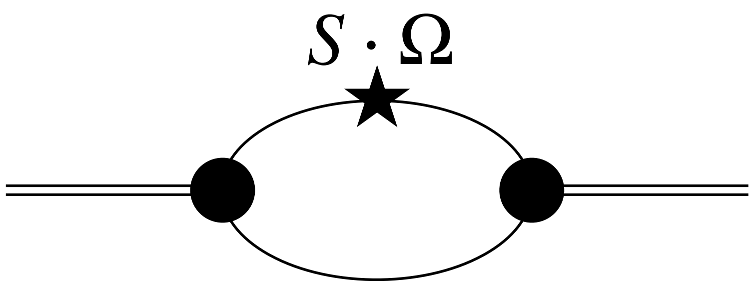


- In perturbative region, $L_p = r \times gA$ and $J_g = r \times (E \times B)$ start to contribute from the α_s corrections. Therefore, more accurate analysis requires α_s correction of Π^{rot}

- The signs of L_p and J_g will also be clear

Bethe-Salpeter approach: infinite heavy quark limit

self energy caused by quark's spin-rotation coupling



$$\Gamma^{\mu} = \left(\epsilon_0 + \frac{\vec{p}^2}{m}\right)\sqrt{\frac{m_V}{N_c}}\psi(\vec{p})P_+\gamma^{\mu}P_-, \quad \Gamma^{\mu\dagger} = -\left(\epsilon_0 + \frac{\vec{p}^2}{m}\right)\sqrt{\frac{m_V}{N_c}}\psi(\vec{p})^*P_-\gamma^{\mu}P_+$$

$$S^{(1)}_{\Omega} = -S^{(0)}(p)\Sigma\cdot\Omega S^{(0)}(p) = \frac{1}{2}S^{(0)}(p)\gamma^{\sigma}\gamma^{5}S^{(0)}(p)\Omega_{\sigma}$$

$$i\mathcal{M}^{\mu\nu}_a = -\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[1_c]\mathrm{Tr}[\Gamma^{\mu\dagger}(p-q,p)S^{(1)}_{\Omega}(p)\Gamma^{\nu}(p,p-q)S^{(0)}(p-q)]$$

$$= -m_V\epsilon^{0\mu\nu3}\Omega$$

$$i\mathcal{M}^{\mu\nu} = 2\times i\mathcal{M}^{\mu\nu}_a = -2m_V\epsilon^{0\mu\nu3}\Omega$$

$$D^{+} = D^{\mu\nu}\epsilon^{+}_{\mu}\epsilon^{-}_{\nu} = \frac{i}{q^2-m_V^2} - \frac{2im_V\Omega}{(q^2-m_V^2)^2} + \cdots \approx \frac{i}{(q+\Omega)^2-m_V^2} + \cdots$$

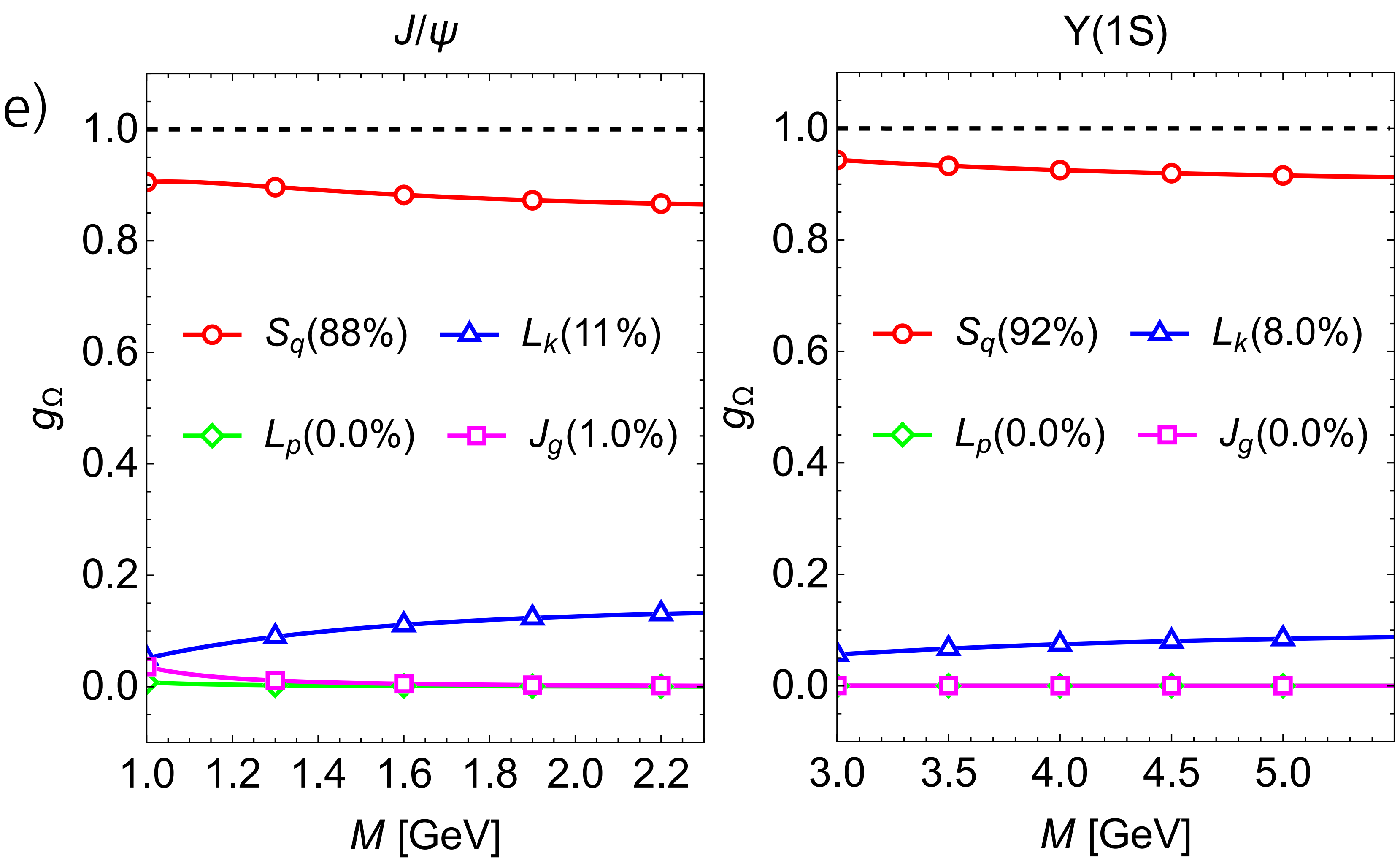
=> $m_V \to m_V - \Omega$: mass of vector meson in a rotating frame

=> quark's spin is dominant in the infinite heavy-quark limit.

=> Comparable with Quark model, $\Upsilon(1S)$ result

Typical QCDSR analysis

Example)



Even for bound states, total $g_\Omega = 1$
Average over a reliable window of Borel Mass 'M'

	Vector		Axial vector	
	J/ψ	$\Upsilon(1\mathrm{S})$	χ_{c1}	χ_{b1}
S_q	$0.88_{1.8\text{e-}4}$	$0.92_{7.6\text{e-}5}$	$0.40_{8.2\text{e-}5}$	$0.43_{1.1\text{e-}5}$
L_k	$0.11_{4.9\text{e-}4}$	$0.076_{7.8\text{e-}5}$	$0.61_{5.8\text{e-}6}$	$0.57_{1.0\text{e-}5}$
L_p	$2.0\text{e-}3_{2.9\text{e-}6}$	$3.5\text{e-}5_{3.0\text{e-}10}$	$8.2\text{e-}4_{2.3\text{e-}8}$	$-1.0\text{e-}5_{3.4\text{e-}10}$
J_g	$8.0\text{e-}3_{5.9\text{e-}5}$	$1.5\text{e-}4_{7.3\text{e-}9}$	$-0.015_{5.2\text{e-}5}$	$-5.2\text{e-}5_{2.3\text{e-}8}$

Dirac eq in a rotating frame

- By EEP, non-inertial frame ~ curved space-time
- Dirac eq in curved space-time

$$\left[i\gamma^a e_a^\mu \left(\partial_\mu - iqA_\mu - \frac{i}{4}\omega_\mu^{bc}\sigma^{bc}\right) - m\right]\Psi = 0$$

$\mu, \nu, \ldots$: curved space-time,

$a, b, \ldots$: flat space-time

$e_a^\mu(x)$: vierbein s.t. $g^{\mu\nu}(x) = e_a^\mu(x) e_b^\nu(x) \eta^{ab}$

$\omega_\mu^{ab} = e_\nu^a(\partial_\mu + \Gamma_{\mu\sigma}^\nu e^{\sigma b})$: spin connection,

- In a rotating frame,

$$ds^2 = (-1 + (\mathbf{\Omega} \times \mathbf{r})^2)dt^2 + 2(\mathbf{\Omega} \times \mathbf{r})d\mathbf{r}dt + d\mathbf{r}^2$$

$$\Rightarrow \left[i\partial\!\!\!/_x + gA\!\!\!/(x) - m - \gamma^0\left\{\frac{1}{2}\begin{pmatrix}\boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma}\end{pmatrix} + \vec{x} \times (i\vec{\partial}_x + g\vec{A}(x))\right\} \cdot \vec{\Omega}\right]\Psi(x) = 0$$

$$\Rightarrow \left[i\partial\!\!\!/_x + gA\!\!\!/(x) - m + \gamma^0(L_q + S_q) \cdot \Omega\right]\Psi(x) = 0$$