ELECTROWEAK STRUCTURE OF THE NUCLEON

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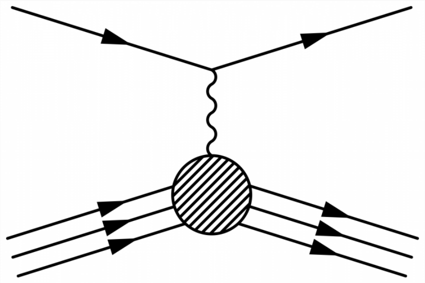
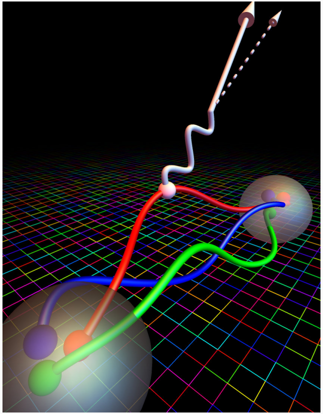
Electroweak structure of the nucleon

- $F_{V(\mathrm{EM})} \to$ charge distribution and magnetism
  $F_A \to$ spin distribution
  Weak interaction is $V - A$
  - $V^\mu = \bar{q}Q\gamma^\mu q$, $\langle N(p')|V^\mu(0)|N(p)\rangle = \bar{u}'\left[\gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F_2^N(q^2)\right]u,$
  - $A^\mu = \bar{q}\gamma^\mu\gamma_5 q$, $\langle N(p')|A^\mu(0)|N(p)\rangle = \bar{u}'\left\{\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N}G_P(q^2)\right\}\gamma_5 u$
    $q = p' - p \quad N = p, n$

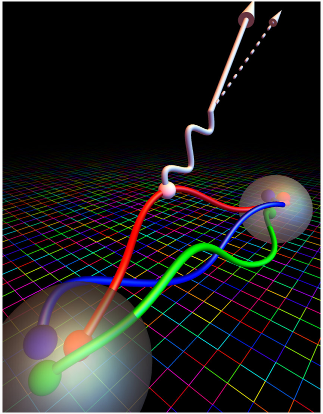




- Experimental determinations
  - Proton radius puzzle [Pohl et al., Nature 466, 213 (2010)]
- Lattice QCD
  - Nucleon f.f. is a benchmark for LQCD
  - Uncertainties reduced for unphysical large M_π
  - Technical difficulties → recent progress (cite)
  - Experimental and lattice q^2 parametrisation:

- dipole ansatz
- z-expansion
- ...

$\Longrightarrow$ different $\langle r_1^2 \rangle$, and F_{EM} in general



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$$\left.\begin{array}{l}\text{- dipole ansatz}\\ \text{- z-expansion}\\ \text{- ...}\end{array}\right\} \Longrightarrow \text{different } \langle r_1^2\rangle \text{, and } F_{\mathrm{EM}} \text{ in general}$$

- Chiral Perturbation Theory (χPT)
  - EFT for QCD at low energy
  - QCD based parametrization of q^2 and M_π dependencies $\Longrightarrow$ extrapolate lattice results to the phys. point and extract $\langle r_i^2\rangle$ and κ from the lattice simulations
  - Account for finite volume, lattice spacing and excited states
  - Determining χPT LECs from the lattice $\Longrightarrow$ predicting other observables
- Dispersion theory
  - Enlarge the q^2 range of plain χPT
- **Goal**: Disp+χPT = good q^2 and M_π description

- Enlarge the q^2 range of χPT (ρ dynamics)

1. Disp. rel. (Cauchy)

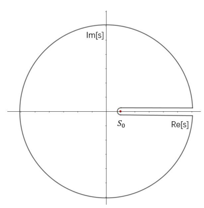
$$F(q^2) = \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{\mathrm{Im}\, F(s)}{s - q^2 - i\varepsilon}$$

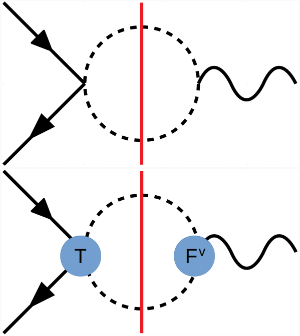
2. Unitarity$\Rightarrow$ $\mathrm{Im}\, F = \frac{1}{2} \sum_n T_{\gamma^* n} T^{\dagger}_{n \bar{N} N}$, $n = \pi^+ \pi^-, \ldots$
   - $\ell = 1$, $\pi\pi$ must be iso-vector
     $\Rightarrow F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\sigma^3}{2}$

3. Using full $N\bar{N}\pi\pi$ and $\gamma^* \pi\pi$ vertices with M_π dep.

$$F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T\, p_{\mathrm{cm}}^3\, F_\pi^{V*}}{s^{1/2}(s - q^2 - i\varepsilon)},$$

[Granados et al, EPJ A 53 (2017)]

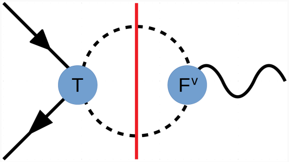


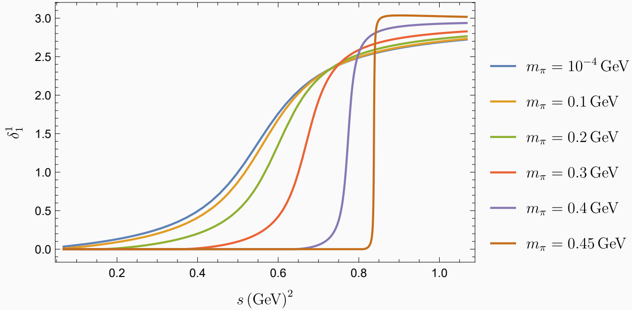


Dispersion theory

- Our two vertices, T and F_π^V, include nonperturbatively the $\pi\pi$ scattering amplitude, t
  - $\mathcal{A}(s) = \frac{8}{\pi}\sum_\ell (2\ell+1) P_\ell(\cos\theta) t_\ell(s)$
  - $\ell = 1$, $t = \frac{\sqrt{s}}{2p_{\rm cm}} \sin\delta e^{i\delta}$
  - $\Omega(s) = \exp\left\{ s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'(s'-s-i\varepsilon)} \right\}$
  - We fit $t_{\rm IAM}$ to δ from [Garcia-Martin PRD 83(2011)]
  - We check that the M_π dependence is realistic
  - $\boxed{F_\pi^V(s) = [1+\alpha_V s]\,\Omega(s)}$
  - $\boxed{T_i(s) = K_i(s) + \Omega(s) P_i + I_i(s)}$,
    - $I_i(s) = \Omega(s) s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{K_i(s')\sin\delta(s')}{|\Omega(s')|(s'-s-i\varepsilon)s'}$
    - K and P from $N\bar{N} \to \pi\pi$ in χPT
      $\mathcal{M} = A\bar{v}u - \frac{1}{2} B\bar{v}\not{k}u \longleftrightarrow K, P$
  -

$$\Rightarrow \boxed{F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T\, p_{\rm cm}^3\, F_\pi^{V*}}{s^{1/2}(s-q^2-i\varepsilon)}},$$





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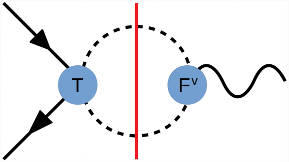
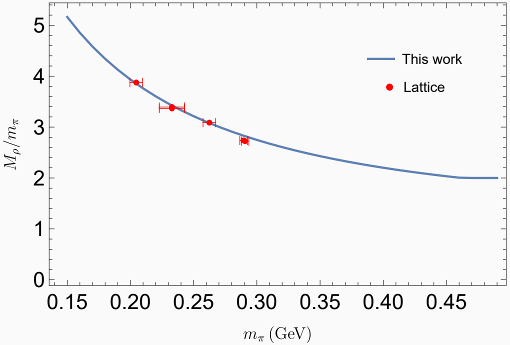
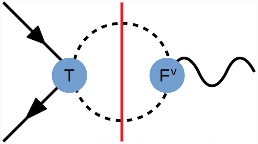
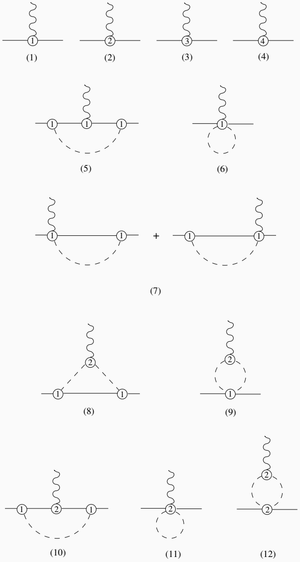




- Disp+χPT= $F_{\mathrm{EM}}^{\mathrm{disp}}$+ diagrams without 2π cut from χPT
  - $F_{\mathrm{EM}}^{\mathrm{disp}}$



  - $+\chi$PT diagrams



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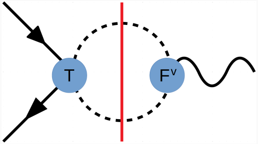
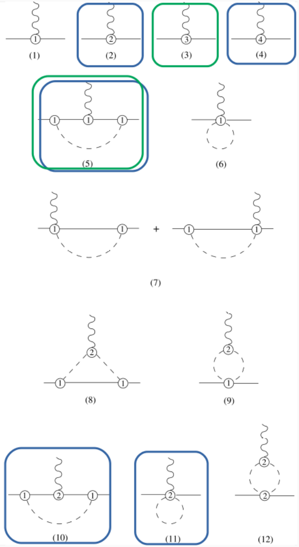
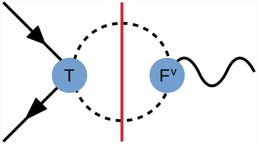


  - $+\chi$PT diagrams
    - Relativistic and with explicit $\Delta(1232)$ [Bauer et al., PRC 86 (2012)]
    - green: $F_1 - F_1(0)$ (the charge is trivial)
    - blue: $F_2 \rightarrow$ we add the $\mathcal{O}(p^4)$ $\not\Delta$ terms



- Disp+χPT$= F_{\rm EM}^{\rm disp} +$ diagrams without 2π cut from χPT
  - $F_{\rm EM}^{\rm disp}$



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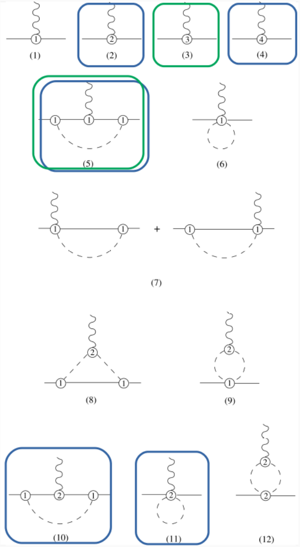
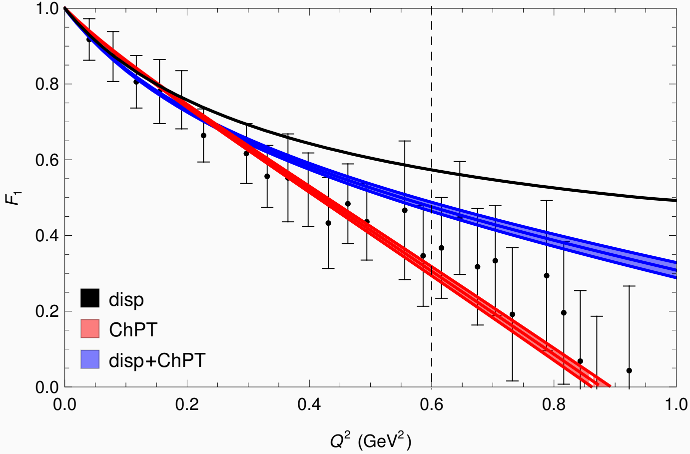
    - green: $F_1 - F_1(0)$ (the charge is trivial)
    - blue: $F_2 \to$ we add the $\mathcal{O}(p^4)$ $\not\Delta$ terms
  - Disp and χPT differ in the renormalization (UV)
    - At $\mathcal{O}(p^3)$ disp and χPT agree on the M_π nonanalyticities
    - Example: the dispersive contribution from $T^{\rm point} \sim \frac{1}{F^2}$ agrees with χPT $F_1^{\rm point} \sim F_1^{(9)} \sim q^2 \log M_\pi$
    - differences absorbed in LECs



- **Dirac f.f.,** F_1
  - $\langle N|V^\mu|N\rangle = \bar{u}'\left[\gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F_2^N(q^2)\right]u$, $F_1 = F_E + \frac{Q^2}{4m^2}\frac{(F_M - F_E)}{(1+Q^2/(4m^2))}$, $Q^2 = -q^2$.
  - $F_1 = 1 + \frac{q^2}{6}\left[-12d_6 + \langle r_1^{2(\log M_\pi)}\rangle \log M_\pi\right] + \mathscr{O}(p^4)$
  - Comparison with LQCD data [Djukanovic PRD 103(2021)] $\longleftarrow$ controlled FV and discret. effects
  - In the χPT and disp+χPT F_1, d_6 is fitted to LQCD
  - Disp $\longrightarrow$ q^2 curvature



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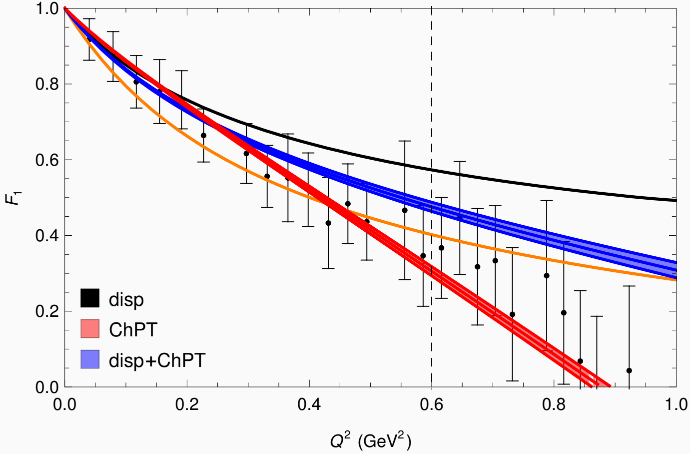
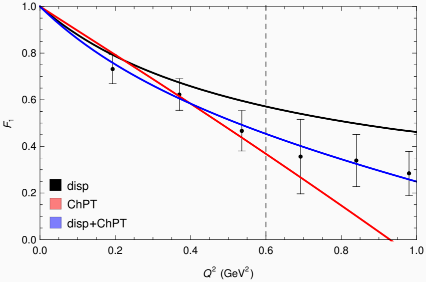
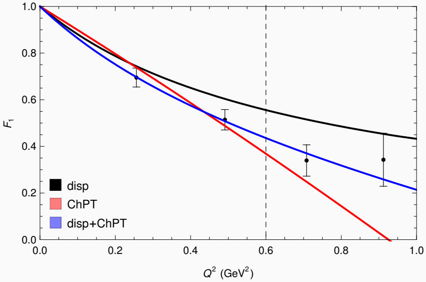




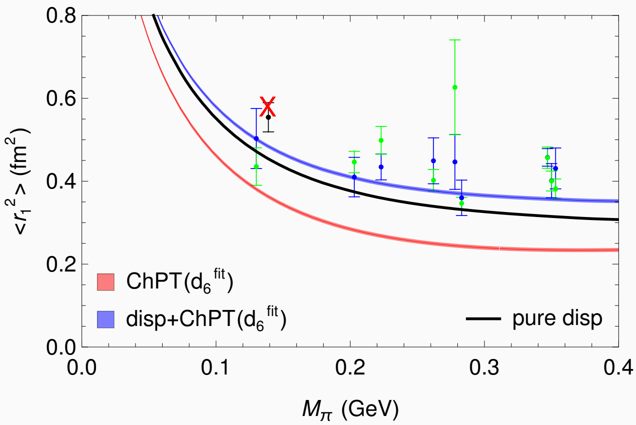
(h) H105 $M_\pi = 0.278$ GeV



(i) N302 $M_\pi = 0.353$ GeV

- **Disp+χPT describes well the M_π dep.**
  - good d_6 fit for $Q^2 < 0.6$ GeV^2 and $M_\pi \lesssim 350$ MeV
  - outperforms the pure dispersive and plain χPT descriptions

	Disp (prediction)	χPT	**Disp+χPT**
$d_6(\mu = m_\rho)$ (GeV^{-2})	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV^{-2})	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47-1) = 1.61$	$24.6/(47-1) = 0.53$
$\langle r_1^2 \rangle_{\mathrm{phys}}$ (fm^2)	0.4541	0.3626 ± 0.0047	$\mathbf{0.4838 \pm 0.0047}$



- $F_1^{(v)} = 1 + \frac{1}{6}\langle r_1^2\rangle^{(v)} q^2 + \mathscr{O}(q^4)$, $\langle r_1^2\rangle^{\mathrm{PDG}} = 0.577$ fm^2
- Heavy baryon fit to LQCD from [Djukanovic PRD 103(2021)]: $\langle r_1^2\rangle^{\mathrm{HB}} = 0.554 \pm 0.035$ fm^2

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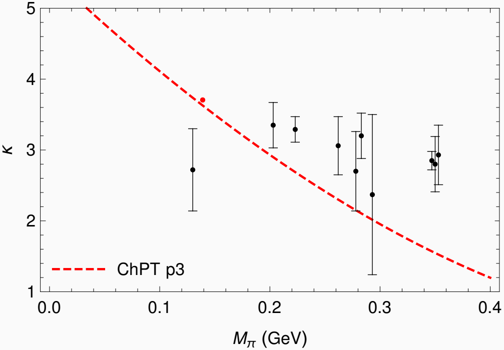
$\langle r_1^2\rangle_{\mathrm{phys}}$ (fm^2)	0.4541	0.3626 ± 0.0047	$\mathbf{0.4838 \pm 0.0047}$

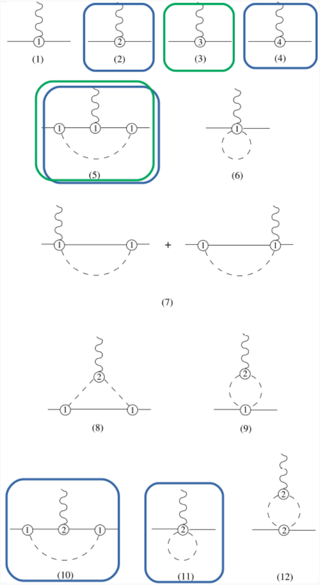
- Pauli f.f., F_2
  - $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1^N(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2^N(q^2) \right] u$, $F_2 = \frac{F_M - F_E}{(1+Q^2/(4m^2))}$
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(v)} q^2 + \mathcal{O}(q^4) \right]$
  - $\mathcal{O}(p^3)$ χPT is not enough (and Δ subtleties)

    $F_2(0)^{o(3)} = c_6 - \left(\frac{\pi m_N g_A^2}{4\pi^2 F^2} \right) M_\pi$

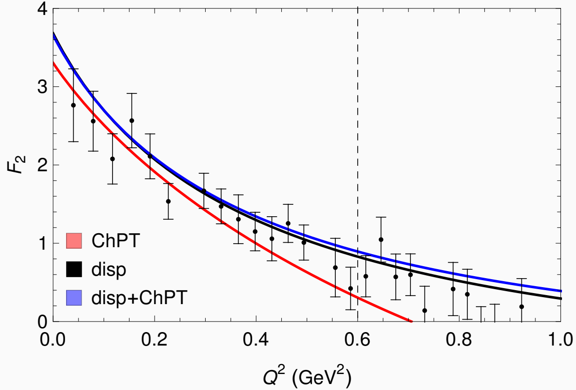


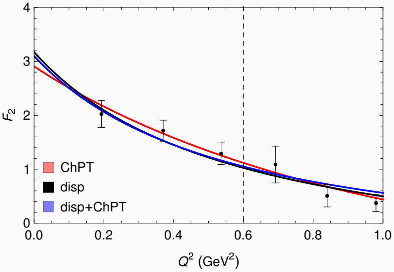
  - $\Longrightarrow$ include $\not\Delta$ $\mathcal{O}(p^4)$
  - disp+χPT $\mathcal{O}(p^4)$: $F_2 = F_2^{\rm disp} + F_2^{\rm tree} + F_2^{\chi \rm PTloop}$,

    $F_2^{\rm tree} = c_6 - 16 e_{106} m_N M_\pi^2 + 2q^2 (d_6 + 2e_{74} m_N)$

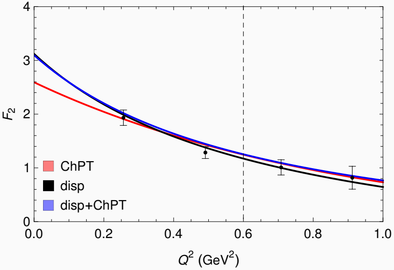


- F_2 **fit to LQCD**
  - In F_2^{disp} free c_6
  - In $F_2^{\chi\mathrm{PT}}$ and $F_2^{disp+\chi\mathrm{PT}}$, free c_6, e_{106}, e_74
  - χPT $\mathcal{O}(p^4)$ and disp separately are good enough to describe the data



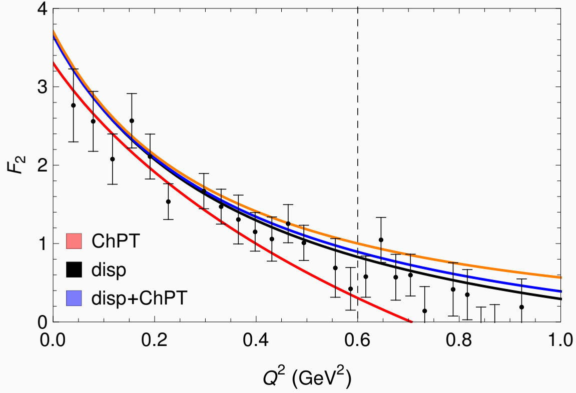


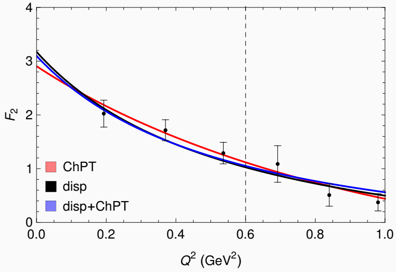
(a) H105 $M_\pi = 0.278$ GeV



(b) N302 $m_\pi = 0.353$ GeV

- F_2 **fit to LQCD**
  - In F_2^{disp} free c_6
  - In $F_2^{\chi\mathrm{PT}}$ and $F_2^{disp+\chi\mathrm{PT}}$, free c_6, e_{106}, e_74
  - χPT $\mathscr{O}(p^4)$ and disp separately are good enough to describe the data





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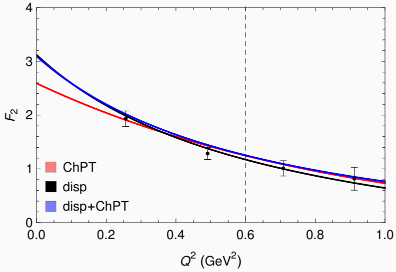

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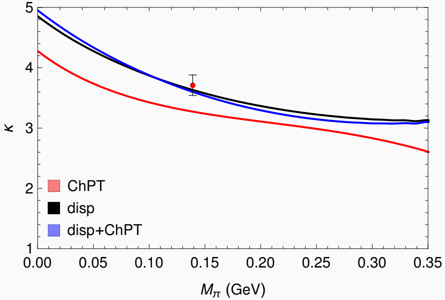
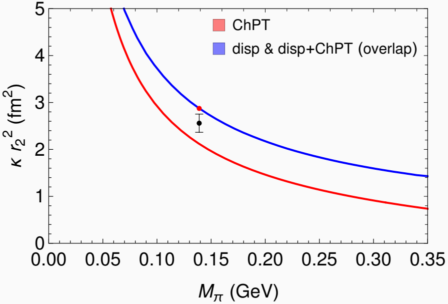




- $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(v)} q^2 + \mathcal{O}(q^4) \right]$
- . $\kappa_{\mathrm{PDG}} = 3.706$,

  $\kappa_{\mathrm{HB}} = 3.71 \pm 0.17$,

  $\langle r_2^2 \rangle_{\mathrm{PDG}} = 0.7754\ \mathrm{fm}^2$,

  $\langle r_2^2 \rangle_{\mathrm{HB}} = 0.690 \pm 0.042\ \mathrm{fm}^2$.

	Disp+c_6	χPT	**Disp+χPT**
χ^2/dof	$\frac{49.95}{47-2} = 1.11$	$\frac{44.18}{47-4} = 1.027$	$\frac{56.08}{47-4} = 1.304$
χ_0^2/dof	1.09	1.027	1.283
κ_{phys}	3.64	3.42	**3.61**
$\langle r_2^2 \rangle_{\mathrm{phys}}$ (fm^2)	0.673	0.619	**0.668**

Axial form factor

- $F_A \rightarrow$ spin distribution
  Weak interaction is $V - A$
  - $A^{i\mu} = \bar{q}\gamma^\mu\gamma_5\frac{\tau^i}{2}q$, $\langle N|A^\mu|N\rangle = \bar{u}'\left[\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N}G_P(q^2)\right]\gamma_5\frac{\tau^i}{2}u$

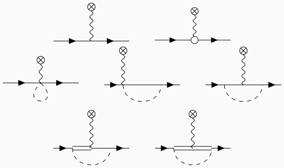


Figure: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

- **Nucleon axial isovector form factor**
  - $F_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
  - g_A and F_A dependence in q^2 are necessary in ν oscillations experiments
  - μ capture, β-decay
  - χPT calculation of F_A
    $\Longrightarrow$ extract $\langle r_A^2\rangle$ from lattice QCD without ad-hoc parametrization
  - $g_A(M_\pi)$: test of πN scatt. in χPT Alvarado & Alvarez-Ruso PRD 105 (2021)
- **$\mathcal{O}(p^4)$ F_A in relativistic χPT**
  - $F_A = \mathring{g}_A + 4d_{16}M_\pi^2 + d_{22}q^2 + \text{loops}(M_\pi, q^2)$
  - **Difference between orders $\simeq$ theoretical uncertainty**

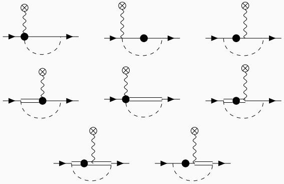
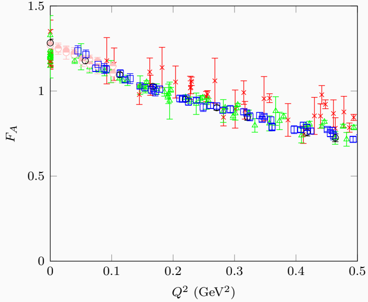
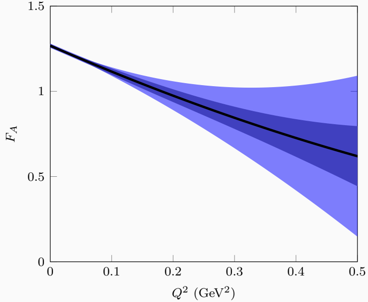


Figure: $\mathcal{O}(p^4)$

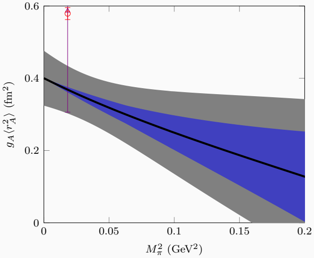
- F_A: **Meta-analysis of large set of recent LQCD results**
  - Many recent works ⇒ substantial improvements
  - RQCD[1] + PNDME[2] + ”Mainz”[3] + PACS[4] + ETMC[5]
  - Reasonably good fit: χ^2 plateau
    $\Longrightarrow M_\pi^{\mathrm{cut}} \simeq 400$ MeV, $Q^2_{\mathrm{cut}} = 0.36$ GeV^2

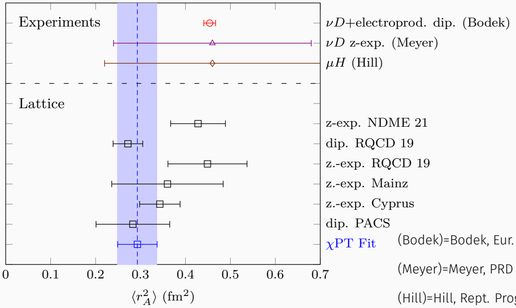




[1] Bali et al. JHEP 05 (2020)
[2] Park et al. 2103.05599
[3] Meyer et al. Modern Phys. A 34 (2019)
[4] Shintani et al. PRD 102 (2020)
[5] Alexandrou et al. PRD 103 (2021)

$F_A = g_A\left(1+\frac{1}{6}\boxed{\langle r_A^2\rangle}\, q^2\right)$ AXIAL RADIUS





(Bodek)=Bodek, Eur. Phys. J. C 53, 349 (2008)

(Meyer)=Meyer, PRD 93, 113015 (2016)

(Hill)=Hill, Rept. Prog. Phys. 81 (2018)

- **Our $\mathcal{O}(p^4)$ χPT extraction:**
  - M_π slope driven by loops with Δ
  - $d_{22} = 0.29 \pm 1.69$ GeV^{-2} (no assumptions on $\Delta\Delta\pi$ coupling enlarges error)
    - d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. Guerrero et al. PRD, 102 (2020)
  - $\boxed{\langle r_A^2\rangle(M_{\rm phys}) = 0.293 \pm 0.044\ \text{fm}^2}$
  - Empirical determinations (model dependent) are in tension with ours and with most of LQCD extractions
  - Tipically the extracted $\langle r_A^2\rangle^{\rm phys}$ value varies depending on the parametrisation
  - **Our QCD based parametrisation leads to a value in line with most of the individual LQCD extractions**

Conclusions

- F_{EM}
  - **Dirac f.f.,** F_1
    - The dispersive calculation supplemented with χPT contributions outperforms the pure dispersive and plain χPT descriptions
    - it fits well the LQCD F_1 at least for $Q^2 < 0.6$ GeV^2 and $M_\pi \lesssim 350$ MeV
    - $\langle r_1^2 \rangle_{\mathbf{phys}} = 0.4838 \pm 0.0047$ fm^2
    - value close to the LQCD HB [Djukanovic PRD 103(2021)] extraction and to the experimental one
  - **Pauli f.f.,** F_2
    - Disp, $\mathscr{O}(p^4)$ χPT and disp+χPT describe the data well
    - $\kappa_{\mathbf{phys}} = 3.61$ and $\langle r_2^2 \rangle_{\mathbf{phys}} = 0.668$ fm^2
    - values in line with the HB and the experimental ones

- F_A
  - Succesful description of LQCD $F_A(q^2)$ using $\mathscr{O}(p^4)$ relativistic χPT
    - Fit describes data in $M_\pi^{\mathbf{cut}} \simeq 400$ MeV, $Q^2_{\mathbf{cut}} = 0.36$ GeV^2
  - There is tension between the experimental and lattice extraction of $\langle r_A^2 \rangle$
  - We extract $\langle r_A^2 \rangle^{\mathrm{phys}} = 0.291 \pm 0.052\ \mathrm{fm}^2$ without ad hoc parametrisations

- (Useful LEC values extracted from both calculations)

	Disp (prediction)	χPT	**Disp+χPT**
$d_6(\mu = m_\rho)$ (GeV^{-2})	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV^{-2})	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47-1) = 1.61$	$24.6/(47-1) = 0.53$
$\langle r_1^2 \rangle_{\mathrm{phys}}$ (fm^2)	0.4541	0.3626 ± 0.0047	$\mathbf{0.4838 \pm 0.0047}$

Extra

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^4)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
d_{16} (GeV^{-2}) (free)	-1.021 ± 0.048		0.983 ± 0.062	-1.46 ± 1.00
d_{22} (GeV^{-2}) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV^{-1})	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV^{-1})	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV^{-1})	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV^{-1})	-	3.31 ± 0.13	-	2.61 ± 0.10
a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV^{-2}) (free)	-	-	-	2.27 ± 2.28
$\widetilde{b}_4$ (GeV^{-2}) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm^{-2}) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm^{-2}) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm^{-1}) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm^{-2} GeV^{-2}) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm^{-1} GeV^{-2}) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\mathring{m}$ (GeV)	0.874	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	-	1.166	1.166

Extra 2

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^4)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
d_{16} (GeV^{-2}) (free)	-1.021 ± 0.048		0.983 ± 0.062	-1.46 ± 1.00
d_{22} (GeV^{-2}) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV^{-1})	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV^{-1})	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV^{-1})	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV^{-1})	-	3.31 ± 0.13	-	2.61 ± 0.10
a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV^{-2}) (free)	-	-	-	2.27 ± 2.28
$\widetilde{b}_4$ (GeV^{-2}) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm^{-2}) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm^{-2}) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm^{-1}) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm^{-2} GeV^{-2}) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm^{-1} GeV^{-2}) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\mathring{m}$ (GeV)	0.874	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	-	1.166	1.166
χ_0^2/dof	$857.31/(127-9) = 7.27$		$533.87/(127-10) = 4.45$	$196.58/(127-13) = 1.724$