

ELECTROWEAK STRUCTURE OF THE NUCLEON

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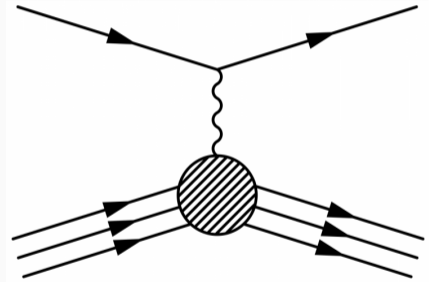
ELECTROWEAK STRUCTURE OF THE NUCLEON

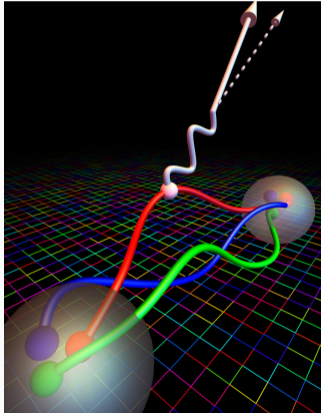
• $\langle B | \dots | \rangle$ / charge distribution and magnetism

• / spin distribution

Weak interaction is

$$\begin{aligned} \cdot \quad \langle m | \dots | \rangle &= \langle h^1(0) j^m(0) j^1(0) | \dots | \rangle = \langle \frac{h}{\Omega} g^m \alpha_1^i(0^2) + \frac{s^{mn} n}{2} \alpha_2^i(0^2) | \dots | \rangle, \\ \cdot \quad \langle m | \dots | \rangle &= \langle g^m g_5, h^1(0) j^m(0) j^1(0) | \dots | \rangle = \langle g_m \alpha(0^2) + \frac{m}{2} r_{\bar{n}}(0^2) g_5 | \dots | \rangle; \end{aligned}$$



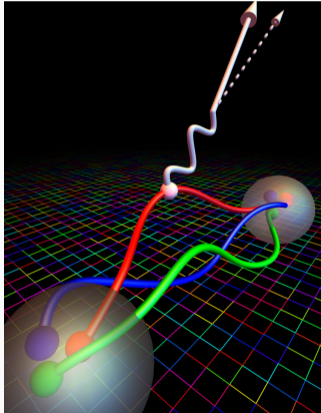


- **Experimental determinations**

- Proton radius puzzle [Pohl et al., Nature 466, 213 (2010)]

- **Lattice QCD**

- Nucleon f.f. is a benchmark for LQCD
- Uncertainties reduced for unphysical large μ_p
- Technical difficulties ! recent progress (cite)
- Experimental and lattice ² parametrisation:
 - dipole ansatz = Θ
 - z-expansion ; \Rightarrow different $h_{\gamma}^2(i)$, and $\alpha_{\mathbb{B}[}$ in general
 - ...



- **Experimental determinations**

- Proton radius puzzle [Pohl et al., Nature 466, 213 (2010)]

- **Lattice QCD**

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- Uncertainties reduced for unphysical large μ_p
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- Experimental and lattice 2 parametrisation:

- dipole ansatz = Θ
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- ...

- **Chiral Perturbation Theory (cPT)**

- EFT for QCD at low energy
- QCD based parametrization of 2 and μ_p dependencies \Rightarrow extrapolate lattice results to the phys. point and extract h^2_i and \mathbf{k} from the lattice simulations
- Account for finite volume, lattice spacing and excited states
- Determining cPT LECs from the lattice \Rightarrow predicting other observables

- **Dispersion theory**

- Enlarge the 2 range of plain cPT

- **Goal:** Disp+cPT = good 2 and μ_p description

DISPERSION THEORY

· Enlarge the 2 range of cPT (r dynamics)

1. Disp. rel. (Cauchy)

$$\alpha(p^2) = \frac{1}{4\mu_p^2} \frac{\text{Im} \alpha(p^2)}{p^2 - e}$$

2. Unitarity) $\text{Im} \alpha = \frac{1}{2} \sum_{i,j} g_{ij}^\dagger g_{ij} = p^+ p^- ; \dots$

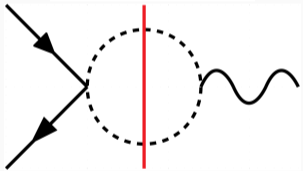
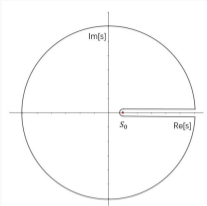
· $\eta = 1$, pp must be iso-vector

$$\alpha(p^2) = \frac{1}{2} \alpha^{(1)} + \alpha^{(2)} \frac{s^3}{2}$$

3. Using full $^1 1 pp$ and $g pp$ vertices with μ_p dep.

$$\alpha(p^2) = \frac{1}{12\mu_p^2} \frac{1}{p^2 - 1/2} \left(\frac{3}{2} \frac{O_p}{e} \right) ;$$

[Granados et al, EPJ A 53 (2017)]



DISPERSION THEORY

- Our two vertices, \mathcal{V} and \mathcal{O}_p , include nonperturbatively the pp scattering amplitude,

$$A(\theta) = \frac{8}{p} \hat{a}_{p-} (2\ell + 1) \tilde{n} \cdot (\cos q) \cdot (\theta)$$

$$\ell = 1, \quad \frac{1}{2} \frac{1}{8\pi} \sin d \quad d$$

$$(\theta) = \exp \left[\frac{1}{4\mu_p^2} \frac{d(\theta)}{p} \frac{d(\theta)}{e} \right]$$

- We fit $\mathbf{R}[\ell]$ to d from [Garcia-Martin PRD 83(2011)]

- We check that the μ_p dependence is realistic

$$\mathcal{O}_p(\theta) = [1 + a] (\theta)$$

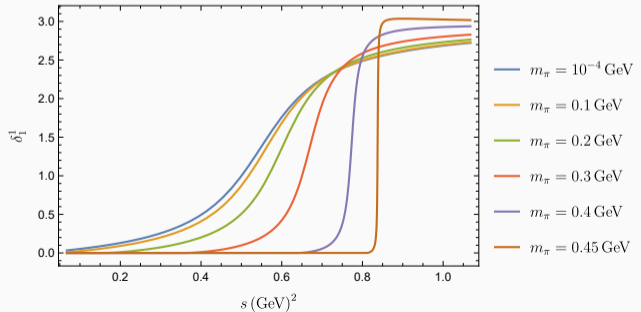
$$(\theta) = \mathcal{V}(\theta) + (\theta) \tilde{n} + \mathcal{I}(\theta)$$

$$\mathcal{I}(\theta) = (\theta) \frac{1}{4\mu_p^2} \frac{d(\theta)}{p} \frac{\mathcal{V}(\theta) \sin d(\theta)}{j(\theta) j(\theta) e}$$

- \mathcal{V} and \tilde{n} from pp in cPT

$$M = \frac{1}{2} 4 = \mathcal{V}, \tilde{n}$$

$$\alpha(\theta) = \frac{1}{12p} \frac{\mathcal{V}(\theta)}{4\mu_p^2} \frac{3}{p} \frac{\mathcal{O}_p(\theta)}{e}$$



DISPERSION THEORY

- Our two vertices, \mathcal{O}_p and \mathcal{O}_p , include nonperturbatively the pp scattering amplitude,

$$A(\theta) = \frac{8}{p} \hat{a}_{p-} \cdot (2\ell + 1) \tilde{n} \cdot (\cos q) \cdot (\theta)$$

$$\ell = 1, \quad \frac{1}{2} \frac{\sin d(\theta)}{\sin \theta} \quad \theta$$

$$(\theta) = \exp \left[-\frac{\theta}{4\mu_p^2} \frac{d(\theta)}{e} \right];$$

- We fit $\mathbf{R}[\ell]$ to d from [Garcia-Martin PRD 83(2011)]

- We check that the μ_p dependence is realistic

$$\mathcal{O}_p(\theta) = [1 + a] (\theta)$$

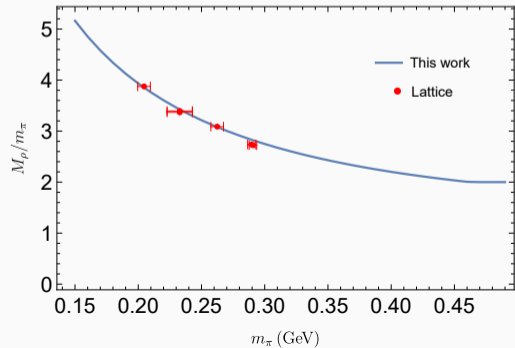
$$(\theta) = \mathcal{Y}(\theta) + (\theta) \tilde{n} + \mathcal{Z}(\theta)$$

$$\mathcal{Z}(\theta) = (\theta) \frac{\theta}{4\mu_p^2} \frac{\mathcal{Y}(\theta) \sin d(\theta)}{p \int (\theta) j(\theta) \frac{d(\theta)}{e}}$$

- \mathcal{Y} and \tilde{n} from pp in cPT

$$M = \frac{1}{2} 4 = \mathcal{Y}, \tilde{n}$$

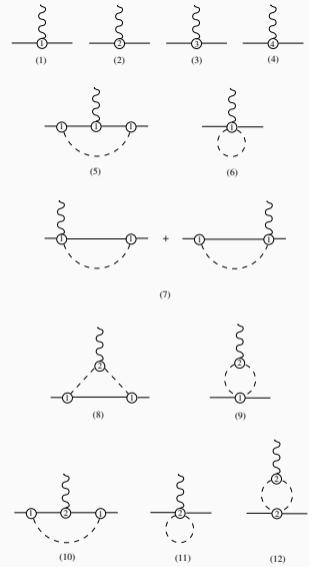
$$\alpha(\theta) = \frac{1}{12p} \frac{\mathcal{Z}(\theta)}{4\mu_p^2} \frac{3}{p} \frac{\mathcal{O}_p}{1-2(\frac{3}{2} \frac{\mathcal{O}_p}{e})};$$



· Disp+cPT = $\mathcal{O}_B^{\text{Se}}$ + diagrams without $2p$ cut from cPT

· $\mathcal{O}_B^{\text{Se}}$

· + cPT diagrams



- Disp+cPT = $\alpha_{\text{B}}^{\text{Se}}$ + diagrams without $2p$ cut from cPT
- $\alpha_{\text{B}}^{\text{Se}}$

- + cPT diagrams

- Relativistic and with explicit γ (1232) [Bauer et al., PRC 86 (2012)]
- green: $\alpha_1 = \alpha_1(0)$ (the charge is trivial)
- blue: $\alpha_2 \neq 0$ we add the $O(\epsilon^4)$ = terms

- Disp+cPT = $\alpha_{\text{B}}^{\text{Se}}$ + diagrams without $2p$ cut from cPT

- $\alpha_{\text{B}}^{\text{Se}}$

- + cPT diagrams

- Relativistic and with explicit ϵ (1232) [Bauer et al., PRC 86 (2012)]

- green: $\alpha_1 \rightarrow \alpha_1(0)$ (the charge is trivial)

- blue: $\alpha_2 \neq$ we add the $O(\epsilon^4) =$ terms

- Disp and cPT differ in the renormalization (UV)

- At $O(\epsilon^3)$ disp and cPT agree on the μp nonanalyticities

- Example: the dispersive contribution from $\alpha_1^{\text{eBS}^* z} \frac{1}{\sigma^2}$ agrees with cPT

$$\alpha_1^{\text{eBS}^* z} \alpha_1^{(9)} \log^2 \mu p$$

- differences absorbed in LECs

Dirac f.f., α_1

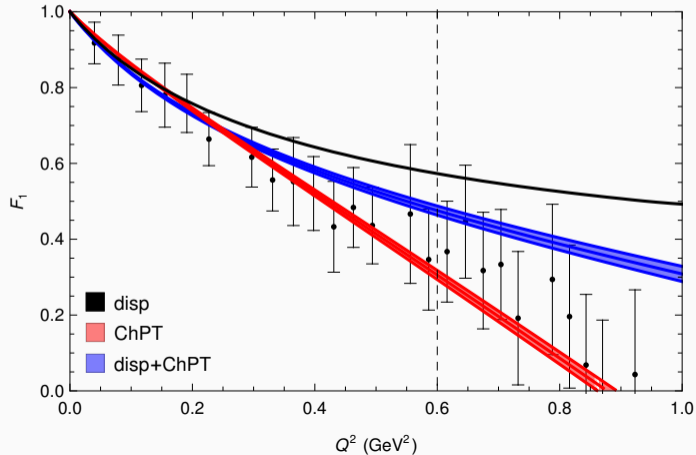
$$h^1 j^m j^1 i = \theta^0 g^m \alpha_1^i (Q^2) + \frac{s^{mn} n}{2} \alpha_2^i (Q^2), \quad \alpha_1 = \alpha_S + \frac{\delta^2}{4} \frac{(\alpha_\mu - \alpha_S)}{(1 + \delta^2 = (4 - 2))}, \quad \delta^2 = \dots$$

$$\alpha_1 = 1 + \frac{2}{6} \dots + h_1^{2(\log \mu p)} i \log \mu p + O(Q^{-4})$$

Comparison with LQCD data [Djukanovic PRD 103(2021)] controlled FV and discret. effects

In the cPT and disp+cPT α_1 , α_6 is fitted to LQCD

Disp / Q^2 curvature



• Dirac f.f., α_1

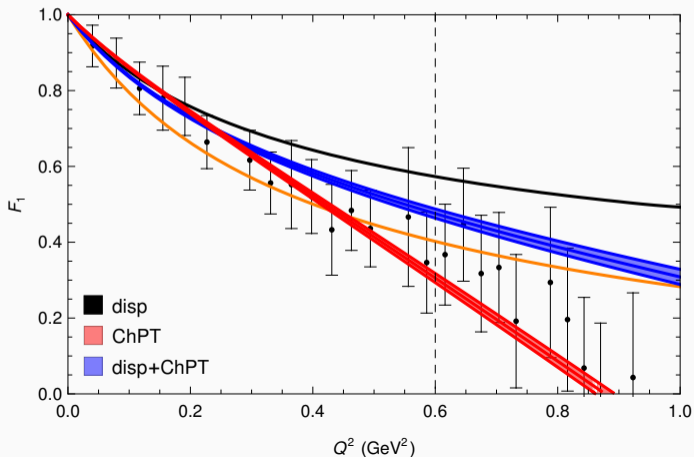
$$\cdot h^1 j^m j^1 i = \theta^0 g^m \alpha_1^i (Q^2) + \frac{s^{mn} n}{2} \alpha_2^i (Q^2), \quad \alpha_1 = \alpha_S + \frac{\delta^2}{4} \frac{(\alpha_\mu - \alpha_S)}{(1 + \delta^2 = (4 - 2))}, \quad \delta^2 = \dots$$

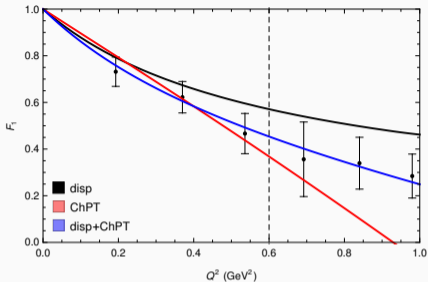
$$\cdot \alpha_1 = 1 + \frac{2}{6} h^2_{12} + h^2_{12} (\log \mu p) i \log \mu p + O(Q^{-4})$$

• Comparison with LQCD data [Djukanovic PRD 103(2021)] controlled FV and discret. effects

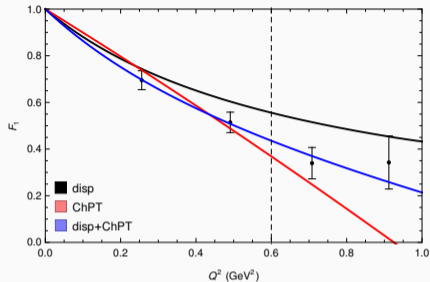
• In the cPT and disp+cPT α_1 , α_6 is fitted to LQCD

• Disp / Q^2 curvature





(h) H105 $\mu_p = 0.278$ GeV

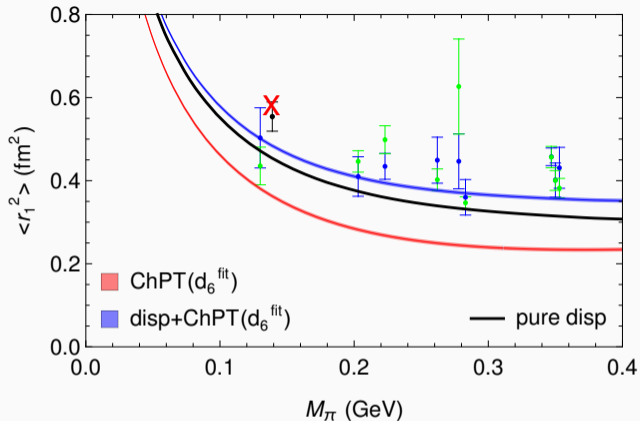


(i) N302 $\mu_p = 0.353$ GeV

· **Disp+cPT describes well the μ_p dep.**

- good χ^2 fit for $Q^2 < 0.6$ GeV² and $\mu_p \sim 350$ MeV
- outperforms the pure dispersive and plain cPT descriptions

	Disp (prediction)	cPT	Disp+cPT
$\epsilon(m = r) (\text{GeV}^{-2})$	-	0.074 0.010	0.416 0.010
$\epsilon(m = i) (\text{GeV}^{-2})$	-	0.422 0.010	0.155 0.010
$\chi^2 = \chi^2_{\text{dof}}$	108.9=47 = 2.32	73.9=(47 1) = 1.61	24.6=(47 1) = 0.53
$h^2_{\text{ep}} (\text{fm}^2)$	0.4541	0.3626 0.0047	0.4838 0.0047



$$\cdot \alpha_1^{(d)} = 1 + \frac{1}{6} h_1^2 i^{(d)} + O(\epsilon^4),$$

$$h_1^2 i^{(d)} \text{K} = 0.577 \text{ fm}^2$$

· Heavy baryon fit to LQCD from [Djukanovic PRD 103(2021)]: $h_1^2 i^{(d)} \text{O}^3 = 0.554 \pm 0.035 \text{ fm}^2$

	Disp (prediction)	cPT	Disp+cPT
$\alpha_1(m = m_\pi) \text{ (GeV}^{-2}\text{)}$	-	0.074 ± 0.010	0.416 ± 0.010
$\alpha_1(m = m_\rho) \text{ (GeV}^{-2}\text{)}$	-	0.422 ± 0.010	0.155 ± 0.010
$c^2 = \alpha_1 \text{BH}$	108.9 ± 47 = 2.32	73.9 ± (47 ± 1) = 1.61	24.6 ± (47 ± 1) = 0.53
$h_1^2 i^{(d)} \text{K} \text{ (fm}^2\text{)}$	0.4541	0.3626 ± 0.0047	0.4838 ± 0.0047

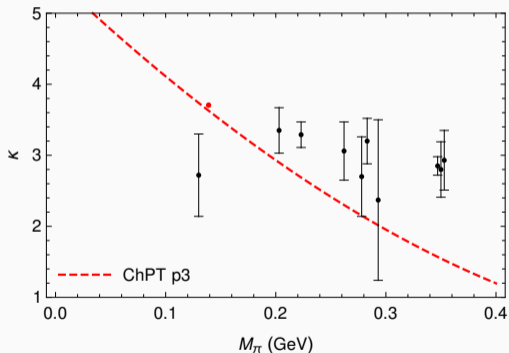
· Pauli f.f., \mathcal{O}_2

$$\cdot h^i j^m j^i i = \int d^4x g^m \mathcal{O}_1(x) + \frac{s^{mn} n}{2} \mathcal{O}_2(x) \quad , \quad \mathcal{O}_2 = \frac{q_\mu \sigma^{\mu\nu}}{(1 + \delta^2 = (4 - 2))}$$

$$\cdot \mathcal{O}_2^{(1)} = k^{(1)} \left(1 + \frac{1}{6} h^2 i^{(1)} \right)^2 + \mathcal{O}(4)$$

· $\mathcal{O}(3)$ cPT is not enough (and subtleties)

$$\mathcal{O}_2(0)^{(3)} = \frac{p_i^2}{4p^2 \sigma^2} \mu p$$

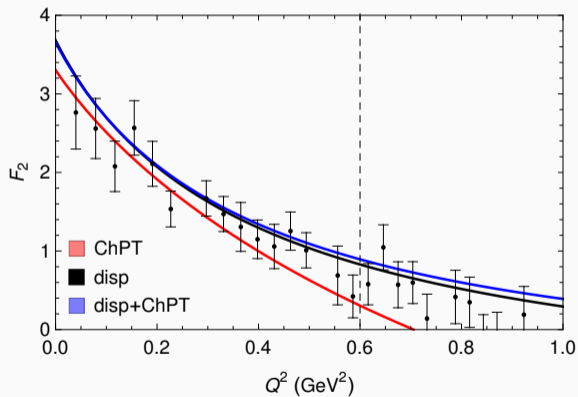


· => include $\mathcal{O}(4)$

$$\cdot \text{disp+cPT } \mathcal{O}(4): \mathcal{O}_2 = \mathcal{O}_2^{\text{@Se}} + \mathcal{O}_2^{\text{zqCC}} + \mathcal{O}_2^{\text{cdy Ybbe}}$$

$$\mathcal{O}_2^{\text{zqCC}} = \frac{1}{6} \frac{p_i^2}{16 \cdot 106} \mu_p^2 + 2 \cdot \left(\frac{1}{6} + 2 \cdot \frac{1}{74} \right)$$

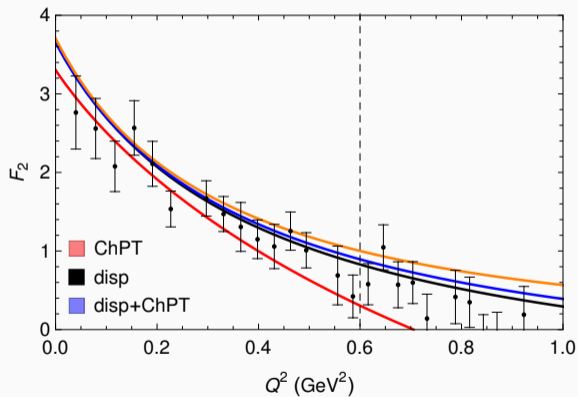
- o_2 fit to LQCD
 - In o_2 free 6
 - In o_2^{cdy} and o_2^{+cdy} , free $6, 106, 74$
 - cPT $O(4)$ and disp separately are good enough to describe the data



(a) H105 $\mu_p = 0.278$ GeV

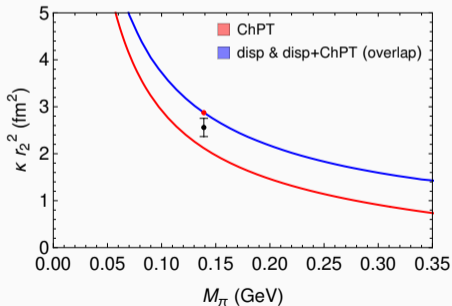
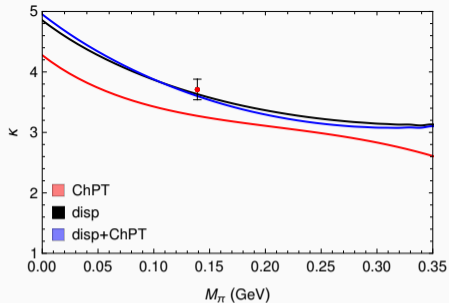
(b) N302 $p = 0.353$ GeV

- o_2 fit to LQCD
 - In o_2^{Se} free $_6$
 - In o_2^{cdy} and $o_2^{\text{+cdy}}$, free $_6, 106, 74$
 - cPT $O(4)$ and disp separately are good enough to describe the data



(a) H105 $\mu_p = 0.278 \text{ GeV}$

(b) N302 $p = 0.353 \text{ GeV}$



$$\cdot o_2^{(i)} = k^{(i)} \left[1 + \frac{1}{6} h^2 i^{(i)} \right]^2 + O(\epsilon^4)$$

$$\cdot k_d \kappa = 3.706,$$

$$k_{O3} = 3.71 \pm 0.17,$$

$$h^2 i_d \kappa = 0.7754 \text{ fm}^2,$$

$$h^2 i_{O3} = 0.690 \pm 0.042 \text{ fm}^2.$$

	Disp+ ϵ	cPT	Disp+cPT
$c^2_{\text{@bH}}$	$\frac{49.95}{47.2} = 1.11$	$\frac{44.18}{47.4} = 1.027$	$\frac{56.08}{47.4} = 1.304$
$c^2_{0\text{@bH}}$	1.09	1.027	1.283
$k_{eP\%}$	3.64	3.42	3.61
$h^2 i_{eP\%}$ (fm ²)	0.673	0.619	0.668

AXIAL FORM FACTOR

- $O(1)$ spin distribution

Weak interaction is

$$L = \bar{\psi} \gamma^\mu (g_V + g_A \gamma_5) \psi \bar{\psi} \gamma_\mu (g_V + g_A \gamma_5) \psi$$

- **Nucleon axial isovector form factor**

$$G_A(Q^2) = 1 + \frac{1}{6} h^2 i^2 + O(Q^4)$$

- h^2 and Q^2 dependence in $G_A(Q^2)$ are necessary in n oscillations experiments
- m capture, b -decay
- cPT calculation of G_A
 - => extract $h^2 i$ from lattice QCD without ad-hoc parametrization
- (μ_p) : test of p^1 scatt. in cPT [Alvarado & Alvarez-Ruso PRD 105 \(2021\)](#)
- $O(4)$ G_A in relativistic cPT
 - $G_A = 1 + 4 \mu_p^2 + 22 Q^2 + \dots$
 - Difference between orders Q^2 theoretical uncertainty

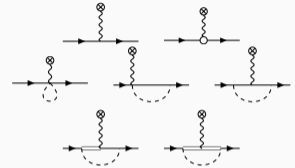


Figure: $O(2)$ and $O(3)$ (w. f. renormalisation not shown)

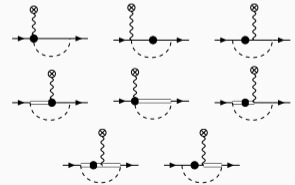
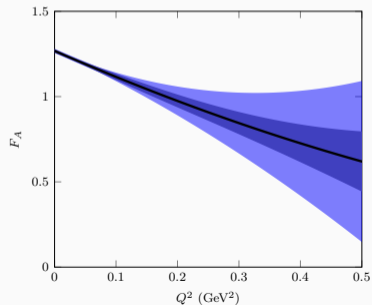
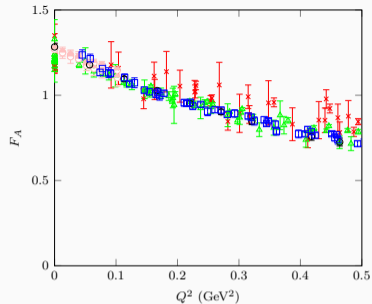


Figure: $O(4)$

• \circ : Meta-analysis of large set of recent LQCD results

- Many recent works) substantial improvements
- RQCD^[1] + PNDME^[2] + "Mainz"^[3] + PACS^[4] + ETMC^[5]
- Reasonably good fit: c^2 plateau
=> $\mu_p^{<-z}$ ' 400 MeV, $\delta_{<-z}^2 = 0.36 \text{ GeV}^2$



[1] [Bali et al. JHEP 05 \(2020\)](#)

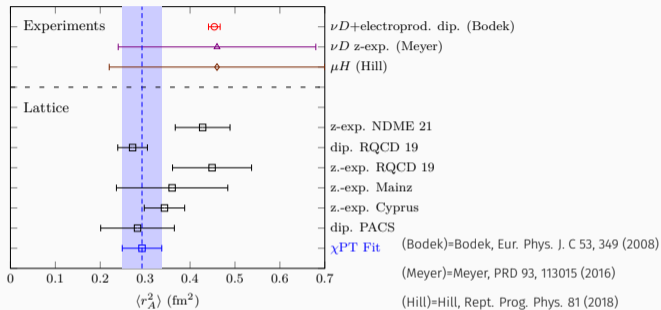
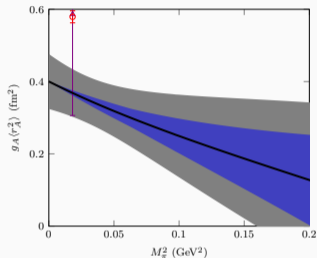
[2] [Park et al. 2103.05599](#)

[3] [Meyer et al. Modern Phys. A 34 \(2019\)](#)

[4] [Shintani et al. PRD 102 \(2020\)](#)

[5] [Alexandrou et al. PRD 103 \(2021\)](#)

$$O = 1 + \frac{1}{6} \boxed{h^2 i}^2 \quad \text{AXIAL RADIUS}$$



• Our $O(4)$ cPT extraction:

- μ_p slope driven by loops with
- $_{22} = 0.29 \pm 1.69 \text{ GeV}^{-2}$ (no assumptions on p coupling enlarges error)
- $_{22}$ compatible with $O(3)$ p electroprod. [Guerrero et al. PRD, 102 \(2020\)](#)

$$\boxed{h^2 i(\mu_{eP\%}) = 0.293 \pm 0.044 \text{ H}^2}$$

- Empirical determinations (model dependent) are in **tension** with ours and with most of LQCD extractions
- Typically the extracted $h^2 i(\mu_{eP\%})$ value varies depending on the parametrisation
- Our QCD based parametrisation leads to a value in line with most of the individual LQCD extractions

- $O(3)$
 - Dirac f.f., o_1
 - The dispersive calculation supplemented with cPT contributions outperforms the pure dispersive and plain cPT descriptions
 - it fits well the LQCD o_1 at least for $Q^2 < 0.6 \text{ GeV}^2$ and $\mu_p \sim 350 \text{ MeV}$
 - $h^2_{1i} eP_{\text{obs}} = 0.4838 \pm 0.0047 \text{ fm}^2$
 - value close to the LQCD HB [Djukanovic PRD 103(2021)] extraction and to the experimental one
 - Pauli f.f., o_2
 - Disp, $O(4)$ cPT and disp+cPT describe the data well
 - $k_{eP_{\text{obs}}} = 3.61$ and $h^2_{2i} eP_{\text{obs}} = 0.668 \text{ fm}^2$
 - values in line with the HB and the experimental ones
- $O(4)$
 - Successful description of LQCD $o(2)$ using $O(4)$ relativistic cPT
 - Fit describes data in $\mu_p^{\leq Z} \sim 400 \text{ MeV}$, $Q_{\leq Z}^2 = 0.36 \text{ GeV}^2$
 - There is tension between the experimental and lattice extraction of h^2_{2i}
 - We extract $h^2_{2i} eP_{\text{obs}} = 0.291 \pm 0.052 \text{ fm}^2$ without ad hoc parametrisations
- (Useful LEC values extracted from both calculations)

	Disp (prediction)	cPT	Disp+cPT
m_r (GeV ²)	-	0:074 0:010	0:416 0:010
m_i (GeV ²)	-	0:422 0:010	0:155 0:010
$c^2 = \frac{h^2}{4\pi} \frac{e^2}{m^2}$	108:9=47 = 2:32	73:9=(47 1) = 1:61	24:6=(47 1) = 0:53
$h^2 \frac{e^2}{4\pi m^2}$ (fm ²)	0.4541	0.3626 0:0047	0:4838 0:0047

	$O(3) =$	$O(4) =$	$O(3)$	$O(4)$
(free)	1.1782 0.0073		1.2041 0.0074	1.274 0.041
$_{16}(\text{GeV } ^2)(\text{free})$	1.021 0.048		0.983 0.062	1.46 1.00
$_{22}(\text{GeV } ^2)(\text{free})$	1.275 0.086		3.77 1.96	0.29 1.69 (free $_1$)
$_1(\text{free})$	-	-	1.35	1.35
$_1(\text{GeV } ^1)$	-	-	0.69 0.69	0.66 0.56
$_1(\text{GeV } ^1)$	-	0.89 0.06	-	1.15 0.05
$_2(\text{GeV } ^1)$	-	3.38 0.15	-	1.57 0.10
$_3(\text{GeV } ^1)$	-	4.59 0.09	-	2.54 0.05
$_4(\text{GeV } ^1)$	-	3.31 0.13	-	2.61 0.10
$_1(\text{GeV } ^1)$	-	-	-	0.90
$_1(\text{GeV } ^2)(\text{free})$	-	-	-	0.27 4.96
$_2(\text{GeV } ^2)(\text{free})$	-	-	-	2.27 2.28
$e_4(\text{GeV } ^2)(\text{free})$	-	-	-	12.48 1.28
$_1(\text{fm } ^2)(\text{free})$	8.4 5.8	-	5.6 5.9	0.25 16.5 (consistent)
$_2(\text{fm } ^2)(\text{free})$	8.6 2.6	-	7.1 2.6	6.36 4.20
$_3(\text{fm } ^1)(\text{free})$	0.25 0.21	-	0.08 0.22	0.36 0.47
$_1(\text{fm } ^2 \text{ GeV } ^2)(\text{free})$	100 40	-	76 44	64 121
$_2(\text{fm } ^2 \text{ GeV } ^2)(\text{free})$	31 21	-	21 22	15 46
$_3(\text{fm } ^1 \text{ GeV } ^2)(\text{free})$	0.63 1.49	-	0.36 1.63	2.54 3.98
(GeV)	0.874	0.874	0.855	0.855
(GeV)	-	-	1.166	1.166

	$O(3) =$	$O(4) =$	$O(3)$	$O(4)$
(free)	1:1782 0:0073		1:2041 0:0074	1:274 0:041
$_{16}(\text{GeV}^2)(\text{free})$	1:021 0:048		0:983 0:062	1:46 1:00
$_{22}(\text{GeV}^2)(\text{free})$	1:275 0:086		3:77 1:96	0:29 1:69 (free $_1$)
-	-	-	1:35	1:35
$_1(\text{free})$	-	-	0:69 0:69	0:66 0:56
$_1(\text{GeV}^1)$	-	0:89 0:06	-	1:15 0:05
$_2(\text{GeV}^1)$	-	3:38 0:15	-	1:57 0:10
$_3(\text{GeV}^1)$	-	4:59 0:09	-	2:54 0:05
$_4(\text{GeV}^1)$	-	3:31 0:13	-	2:61 0:10
$_1(\text{GeV}^1)$	-	-	-	0:90
$_1(\text{GeV}^2)(\text{free})$	-	-	-	0:27 4:96
$_2(\text{GeV}^2)(\text{free})$	-	-	-	2:27 2:28
$e_4(\text{GeV}^2)(\text{free})$	-	-	-	12:48 1:28
$_1(\text{fm}^2)(\text{free})$	8:4 5:8	-	5:6 5:9	0:25 16:5 (consistent)
$_2(\text{fm}^2)(\text{free})$	8:6 2:6	-	7:1 2:6	6:36 4:20
$_3(\text{fm}^1)(\text{free})$	0:25 0:21	-	0:08 0:22	0:36 0:47
$_1(\text{fm}^2 \text{GeV}^2)(\text{free})$	100 40	-	76 44	64 121
$_2(\text{fm}^2 \text{GeV}^2)(\text{free})$	31 21	-	21 22	15 46
$_3(\text{fm}^1 \text{GeV}^2)(\text{free})$	0:63 1:49	-	0:36 1:63	2:54 3:98
(GeV)	0.874	0.874	0.855	0.855
(GeV)	-	-	1.166	1.166
$c_0^2 @ \mathbf{bH}$	857:31=(127 9) = 7:27		533:87=(127 10) = 4:45	196:58=(127 13) = 1:724