Sivers function of sea quarks in the Light-Cone Model

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Outline

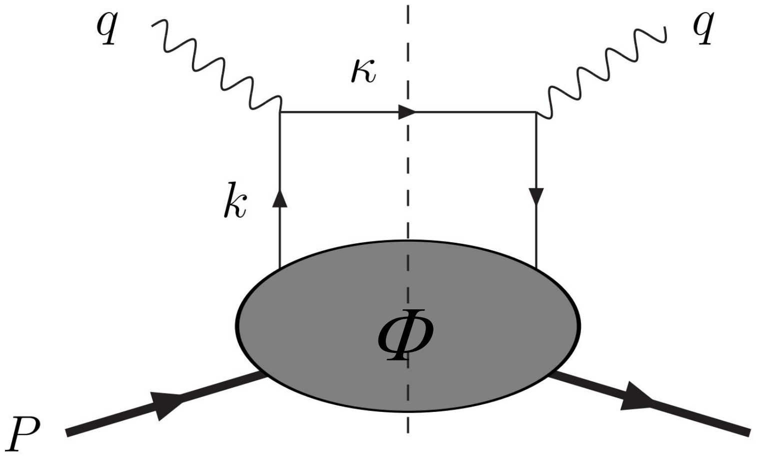
- Definition of the transverse momentum distributions (TMDs)
- Sea quarks
- The Sivers function of sea quarks
- Comparison with other model calculations
- Summary

Definition of TMDs

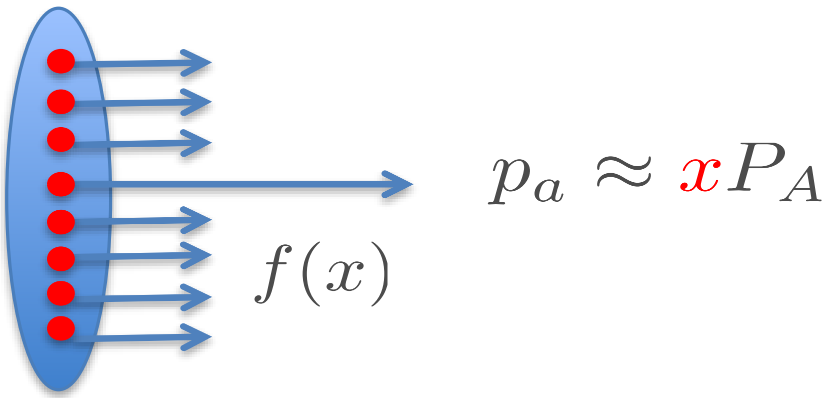
quark-quark correlation function

$$\Phi_{ij}(k;P,S) = \sum_X \int \frac{d^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4 \delta^4(P-k-P_X) \langle PS|\overline{\Psi}_j(0)|X\rangle\langle X|\Psi_i(0)|PS\rangle$$

$$= \int d^4\xi\, e^{ik\cdot\xi} \langle PS|\overline{\Psi}_j(0)\Psi_i(\xi)|PS\rangle$$



handbag diagram of the deep-inelastic scattering

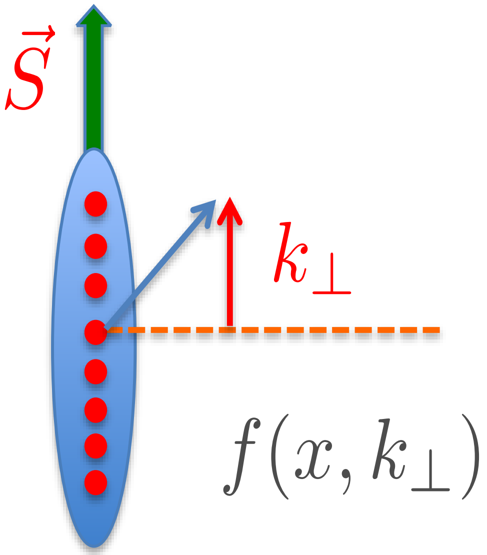


Longitudinal motion only

$$\Phi(x,S) = \frac{1}{2}\left[f_1(x)\,\not{n}_+ + S_L\, g_{1L}(x)\,\gamma^5\,\not{n}_+ + h_{1T}\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu \right]$$

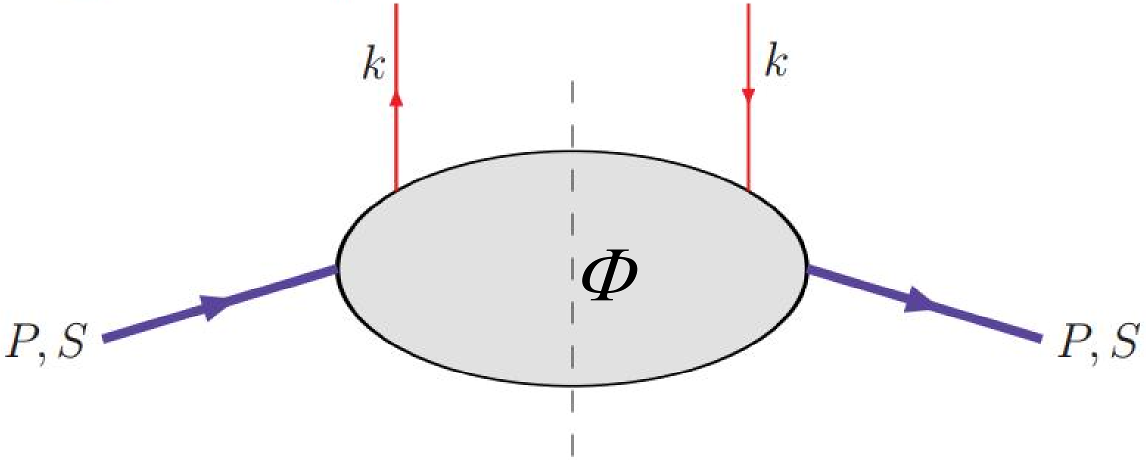
q Δq $\Delta_T q$

Definition of TMDs

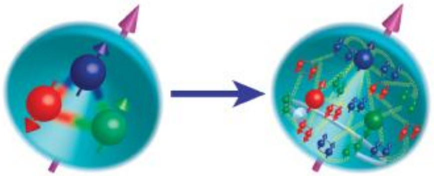


Longitudinal + transverse motion

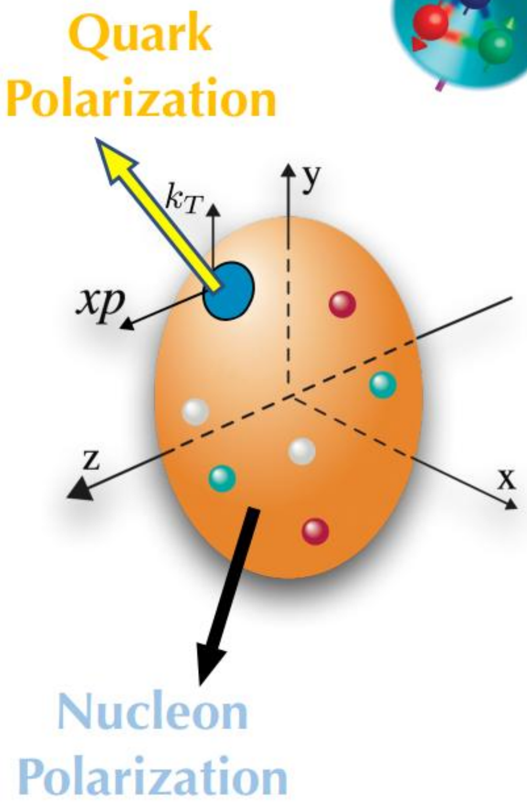
$$\Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \Big(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \Big) \gamma^5 \not{n}_+ + h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \Big(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \Big) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} + h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]$$



Leading-Twist TMDs



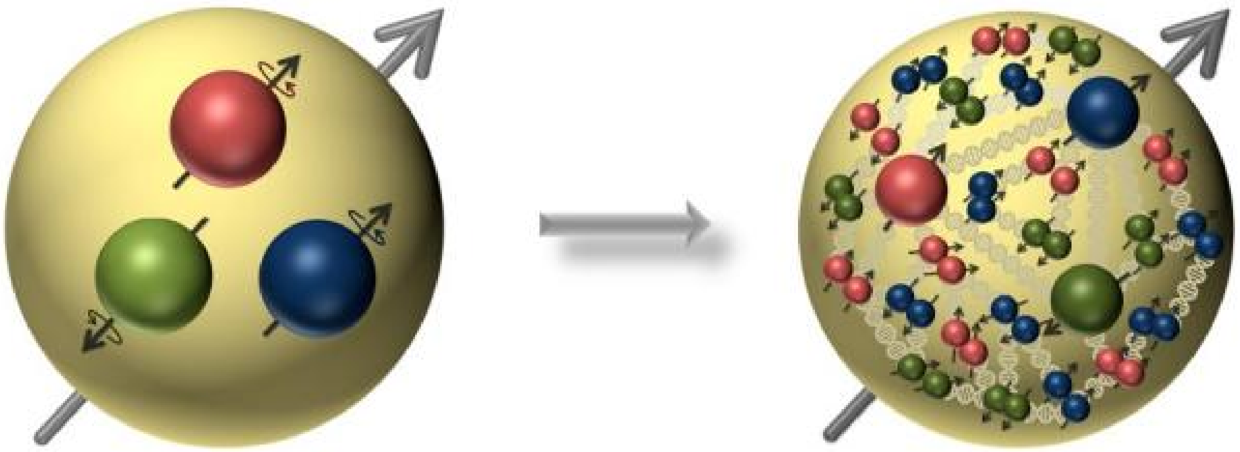
Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons



Nucleon Polarization		Quark Polarization: Unpolarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ Boer-Mulders
	L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^{\perp}(x,k_T^2)$
	T	$f_{1T}^{\perp}(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$	$h_1(x,k_T^2)$ Transversity; $h_{1T}^{\perp}(x,k_T^2)$ Pretzelosity

The **Sivers function** describes the distribution of unpolarized quark inside a transversely polarized nucleon.

Sea quarks



- Quark Model $|p\rangle = |uud\rangle$
- NLO $|uudg\rangle$ $|uudQ\overline{Q}\rangle$ ... $|uud\rangle \xrightarrow{boost} |uudg\rangle, |uudq\overline{q}\rangle$ ...
- meson-baryon fluctuation model $|p\rangle \to |MB\rangle \to |q\overline{q}B\rangle$

[S.J. Brodsky, B.-Q. Ma, Phys. Lett. B 381 (1996) 317.]

- for our model $|p\rangle \to |\pi^+ n\rangle \to |u\bar{d}n\rangle$ $|p\rangle \to |\pi^- \Delta^{++}\rangle \to |\bar{u}d\Delta^{++}\rangle$

The Sivers function of sea quarks

X. Luan and Z. Lu, Phys. Lett. B 833 (2022), 137299, doi:10.1016

1. Light-cone wave functions

For $|P\rangle \to |\pi B\rangle \to |q\bar{q}B\rangle$, the LCWFs can be defined as

$$\psi^{\lambda_N}_{\lambda_B\lambda_q\lambda_{\bar{q}}}(x,y,\boldsymbol{p}_T,\boldsymbol{r}_T) = \sqrt{\frac{r^+}{(P-r)^+}}\frac{1}{r^2-m_\pi^2}\overline{u}(P-r,\lambda_B)\gamma_5 U(P,\lambda_P)\sqrt{\frac{p^+}{(r-p)^+}}\frac{1}{p^2-m^2}\overline{u}(r-p,\lambda_q)\gamma_5 \nu(p,\lambda_{\bar{q}})$$
$$= \psi^{\lambda_N}_{\lambda_B}(y,\boldsymbol{r}_T)\psi_{\lambda_q\lambda_{\bar{q}}}(x,y,\boldsymbol{p}_T,\boldsymbol{r}_T) \quad (1)$$

we obtain $\psi^{\lambda_N}_{\lambda_B}(y,r_T)$: LCWFs of the nucleon in terms of πB

$$\psi^+_+(y,\boldsymbol{r}_T) = \frac{M_B-(1-y)M}{\sqrt{1-y}}\phi_1,$$
$$\psi^+_-(y,\boldsymbol{r}_T) = \frac{r_1+ir_2}{\sqrt{1-y}}\phi_1,$$
$$\psi^-_+(y,\boldsymbol{r}_T) = \frac{r_1-ir_2}{\sqrt{1-y}}\phi_1,$$
$$\psi^-_-(y,\boldsymbol{r}_T) = \frac{(1-y)M-M_B}{\sqrt{1-y}}\phi_1.$$

where

$$\phi_1(y,\boldsymbol{r}_T) = -\frac{g(r^2)\sqrt{y(1-y)}}{\boldsymbol{r}_T^2+L_1^2(m_\pi^2)}$$

$\psi_{\lambda_q \lambda_{\bar{q}}}(x, y, \boldsymbol{p}_T, \boldsymbol{r}_T)$: LCWFs of meson inside nucleon in terms of $q\bar{q}$

$$\psi_{++}(x, y, \boldsymbol{k}_T, \boldsymbol{r}_T) = \frac{my}{\sqrt{x(y-x)}}\phi_2,$$

$$\psi_{+-}(x, y, \boldsymbol{k}_T, \boldsymbol{r}_T) = \frac{y(k_1 - ik_2) - x(r_1 - ir_2)}{\sqrt{x(y-x)}}\phi_2,$$

$$\psi_{-+}(x, y, \boldsymbol{k}_T, \boldsymbol{r}_T) = \frac{y(k_1 + ik_2) - x(r_1 + ir_2)}{\sqrt{x(y-x)}}\phi_2,$$

$$\psi_{--}(x, y, \boldsymbol{k}_T, \boldsymbol{r}_T) = \frac{-my}{\sqrt{x(y-x)}}\phi_2,$$

where

$$\phi_2(x, y, \boldsymbol{k}_T, \boldsymbol{r}_T) = -\frac{g(k^2)\sqrt{\frac{x}{y}(1-\frac{x}{y})}}{(\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)^2 + L_2^2(m^2)}$$

For the form factor, we choose dipole regulator

$$g(p^2) = g^{dip}\frac{p^2 - m^2}{|p^2 - \Lambda_X^2|^2}$$

2. unpolarized distribution

The unpolarized sea quark TMD distribution $f_1^{\bar{q}/P}(x, \boldsymbol{p}_T)$ can be obtained by

$$f_1^{\bar{q}/P}(x, \boldsymbol{k}_T) = \int_x^1 \frac{dy}{y} \int d^2\boldsymbol{r}_T f_1^{\pi/P}(y, \boldsymbol{r}_T) f_1^{\bar{q}/\pi}(\frac{x}{y}, \boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)$$

$$f_1^{\bar{q}/P}(x, \boldsymbol{k}_T) = \int_x^1 \frac{dy}{y} \int d^2\boldsymbol{r}_T \frac{g_1^2}{16\pi^3} \frac{y(1-y)^2[\boldsymbol{r}_T^2 + (M_B - M(1-y))^2]}{[\boldsymbol{r}_T^2 + L_1^2(\Lambda_\pi^2)]^4} \frac{g_2^2}{8\pi^3} \frac{(1-\frac{x}{y})^2[m^2 + (\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)^2]}{[(\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)^2 + L_2^2(\Lambda_{\bar{q}}^2)]^4}$$

3. Sivers function

The parton Sivers function of proton $f_{1T}^{\perp}(x, y, \boldsymbol{p}_T^2, \boldsymbol{r}_T^2)$ can be obtained by

$$\frac{2(\widehat{\boldsymbol{s}}_T \times \boldsymbol{k}_T)\cdot\widehat{\boldsymbol{P}}}{M} f_{1T}^{\perp}(x, y, \boldsymbol{k}_T^2, \boldsymbol{r}_T^2) = \int \frac{d^2\boldsymbol{r}_T'}{16\pi^3} G(y, \boldsymbol{k}_T, \boldsymbol{k}_T')$$
$$\times \sum_{\{\lambda\}} [\psi^{\uparrow *}_{\lambda_B\lambda_q\lambda_{\bar{q}}}(x, y, \boldsymbol{k}_T, \boldsymbol{r}_T)\psi^{\uparrow}_{\lambda_B\lambda_q\lambda_{\bar{q}}}(x, y, \boldsymbol{k}_T', \boldsymbol{r}_T') - \psi^{\downarrow *}_{\lambda_B\lambda_q\lambda_{\bar{q}}}(x, y, \boldsymbol{k}_T, \boldsymbol{r}_T)\psi^{\downarrow}_{\lambda_B\lambda_q\lambda_{\bar{q}}}(x, y, \boldsymbol{k}_T', \boldsymbol{r}_T')]$$

with $$\boldsymbol{l}_T = \boldsymbol{k}_T - \boldsymbol{k}_T' = \boldsymbol{r}_T - \boldsymbol{r}_T'$$

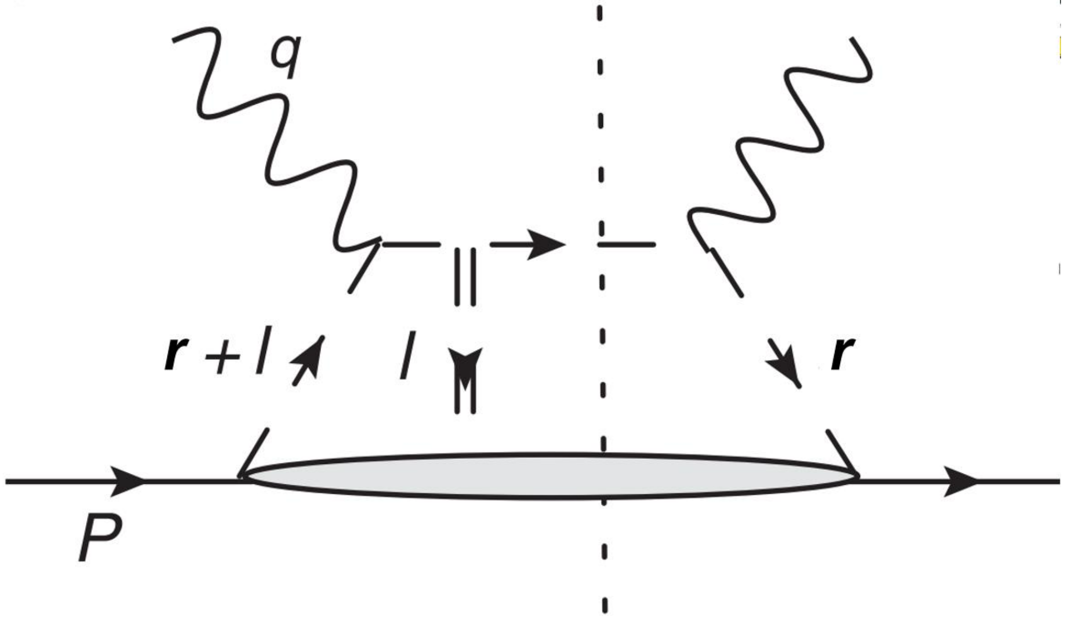
a. the final state interaction (in DIS) between the active antiquark and the spectator

$$f_{1T}^{\perp,\bar{q}/P}(x,\boldsymbol{k}_T^2) = -\int_x^1 \frac{dy}{y}\int d^2\boldsymbol{r}_T \int d^2\boldsymbol{l}_T \frac{g_1^2 C_F \alpha_s}{16\pi^4}\frac{\boldsymbol{l}_T\cdot\boldsymbol{k}_T}{\boldsymbol{k}_T^2}\frac{My(1-y)^2[M_B - M(1-y)]}{L_1^2(\Lambda_\pi^2)[\boldsymbol{r}_T^2 + L_1^2(\Lambda_\pi^2)]^2[(\boldsymbol{r}_T^2 - \boldsymbol{l}_T^2) + L_1^2(\Lambda_\pi^2)]^2}$$
$$\times \frac{g_2^2}{8\pi^3}\frac{(1-\frac{x}{y})^2[m^2 + (\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)^2 - (1-\frac{x}{y})\boldsymbol{l}_T\cdot(\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)]}{[(\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)^2 + L_2^2(\Lambda_{\bar{q}}^2)]^2[(\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T - (1-\frac{x}{y})\boldsymbol{l}_T)^2 + L_2^2(\Lambda_{\bar{q}}^2)]^2}.$$

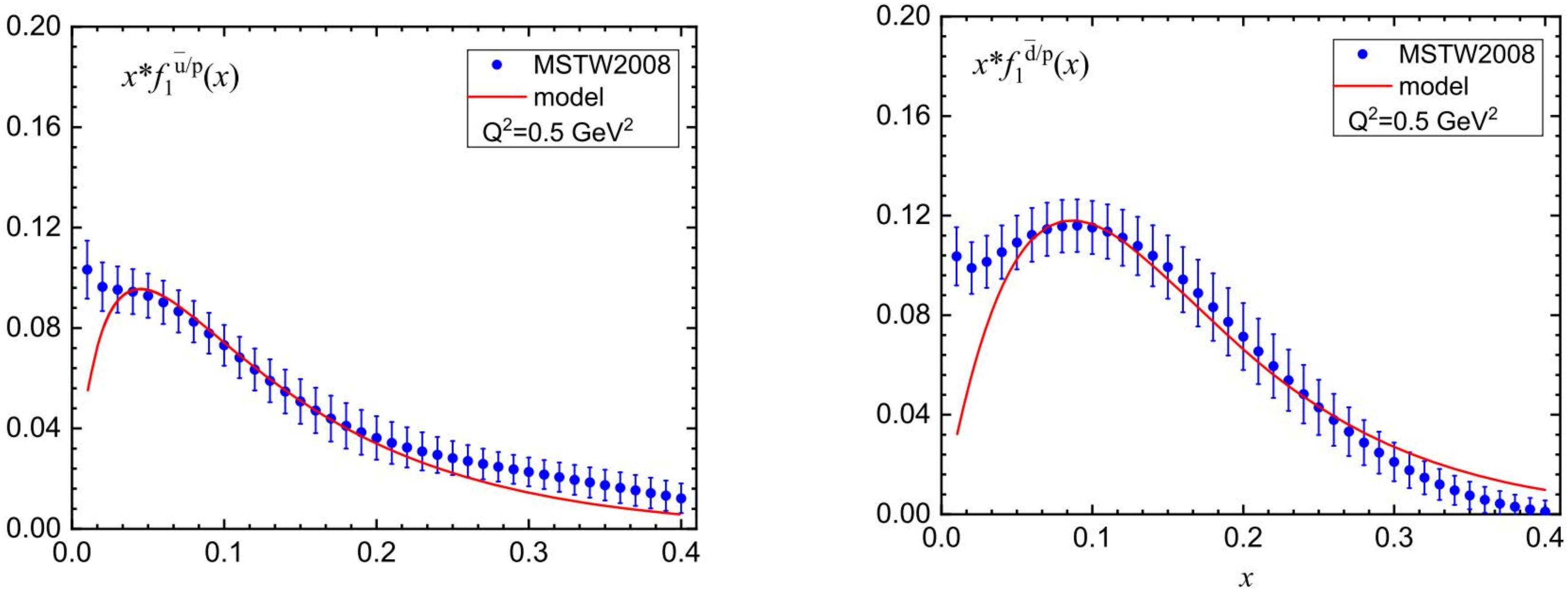
b. the final-state interaction occurs between the pion meson as a whole and the spectator baryon

$$k_T^i f_{1T}^{\perp}(x,y,\boldsymbol{k}_T^2,\boldsymbol{r}_T^2) = 16\pi^3 r_T^i f_{1T}^{\perp,\pi/P}(y,\boldsymbol{r}_T^2) f_1^{\bar{q}/\pi}(\frac{x}{y},\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)$$

$$f_{1T}^{\perp,\bar{q}/P}(x,\boldsymbol{k}_T^2) = -\int_x^1 \frac{dy}{y}\int d^2\boldsymbol{r}_T \frac{\boldsymbol{r}_T\cdot\boldsymbol{k}_T}{\boldsymbol{k}_T^2}$$
$$\times \frac{g_2^2}{8\pi^3}\frac{(1-\frac{x}{y})^2[m^2 + (\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)^2]}{[(\boldsymbol{k}_T - \frac{x}{y}\boldsymbol{r}_T)^2 + L_2^2(\Lambda_{\bar{q}}^2)]^4}$$
$$\times \frac{g_1^2 C_F \alpha_s}{16\pi^3}\frac{My(1-y)^2[M_B - M(1-y)]}{L_1^2(\Lambda_\pi^2)[\boldsymbol{r}_T^2 + L_1^2(\Lambda_\pi^2)]^3}.$$



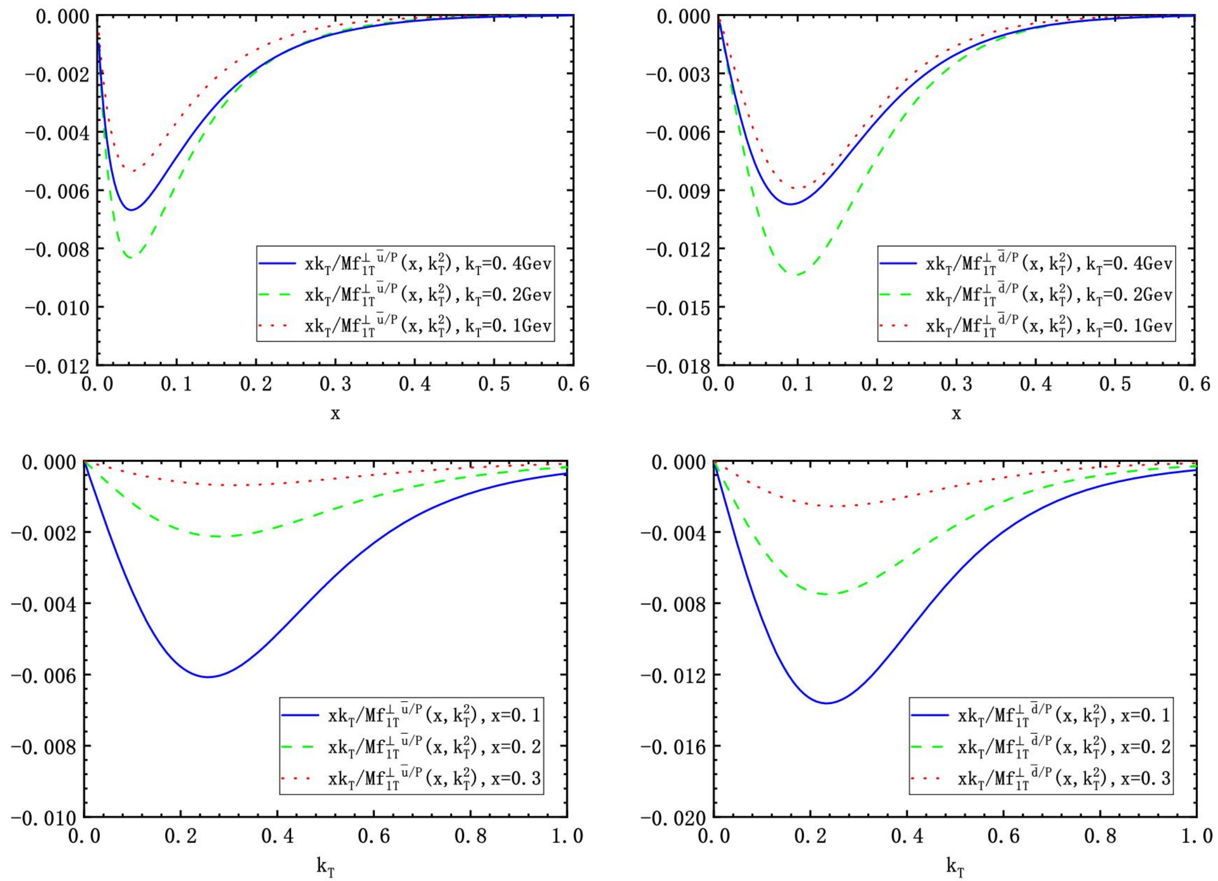
4. numerical results



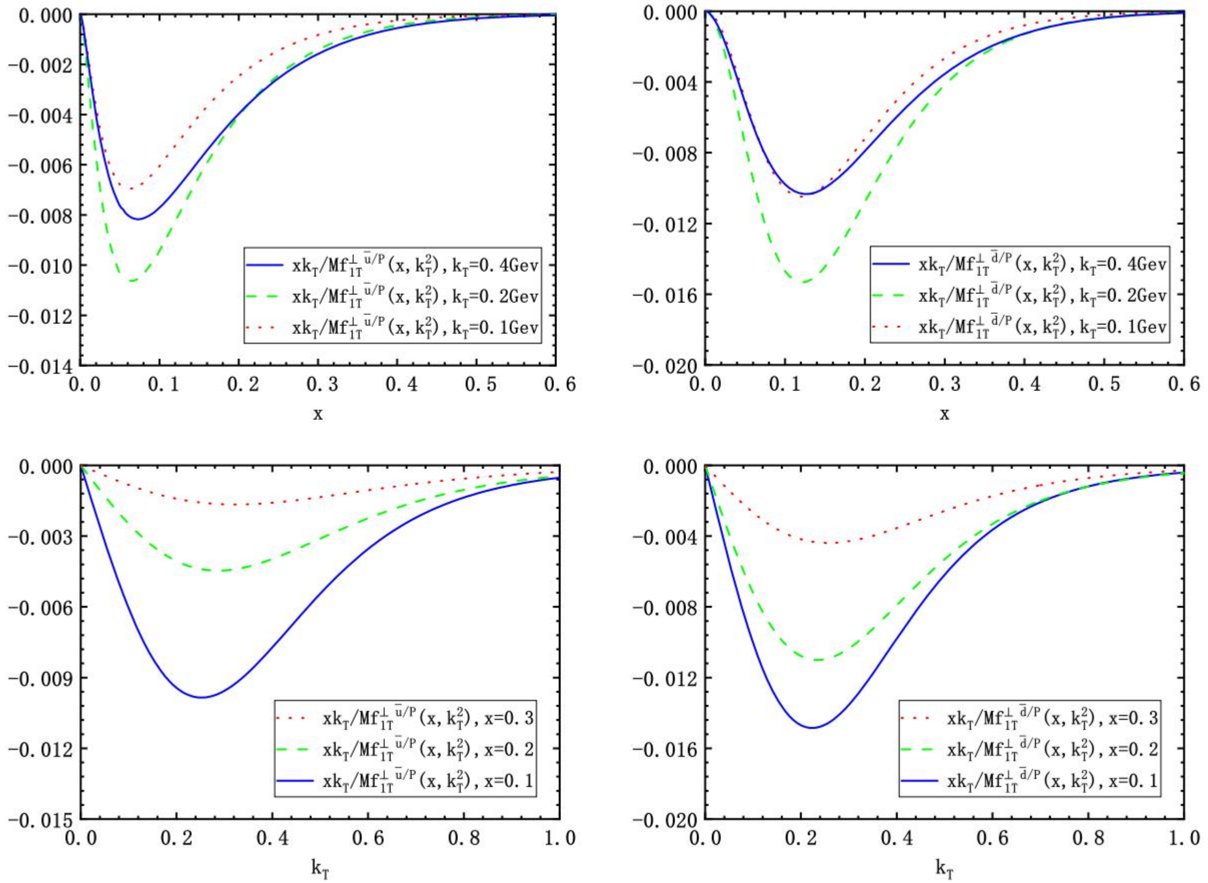
Parameters	$\bar{u}$	$\bar{d}$
g_1	9.33	5.79
g_2	4.46	4.46
Λ_π (GeV)	0.223	0.223
$\Lambda_{\bar{q}}$ (GeV)	0.510	0.510

Values of the parameters obtained from fitting the model to the **GRV LO** $f_1^{\bar{u}/\pi^-}(x)$ (**second and fourth row**) and **MSTW2008** $f_1^{\bar{q}/p}(x)$ (**first and third row**).

a.



b.

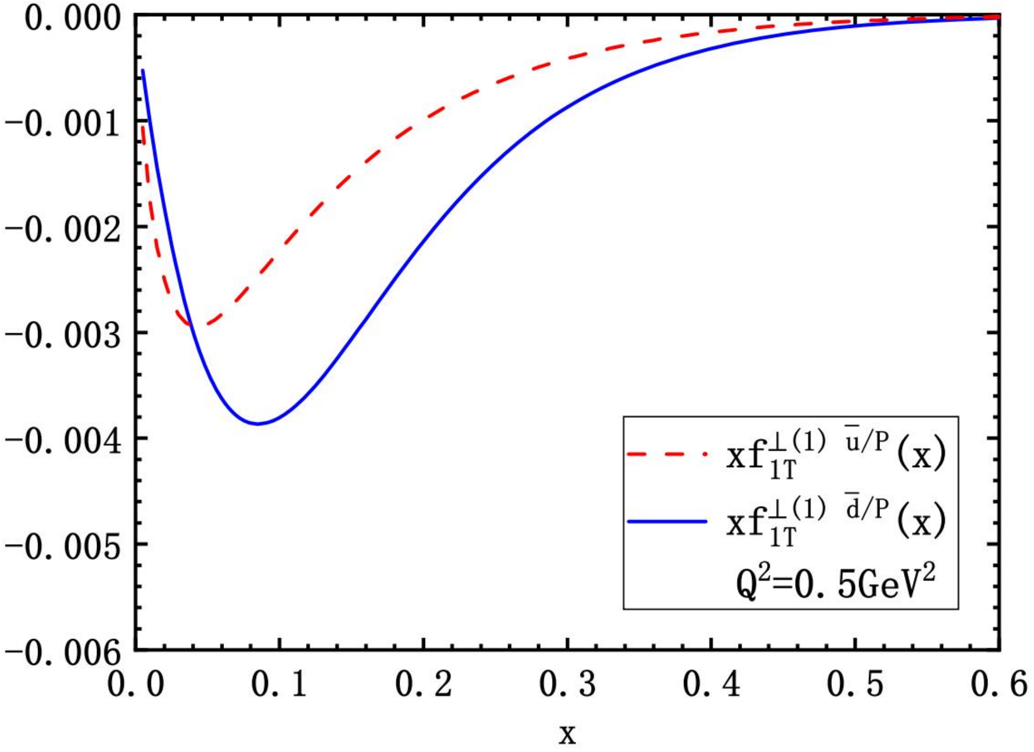


the x-dependence and the k_T-dependence of $x\frac{k_T}{M}f_{1T}^{\perp,\bar{u}/p}(x,\boldsymbol{k}_T^2)$ $x\frac{k_T}{M}f_{1T}^{\perp,\bar{d}/p}(x,\boldsymbol{k}_T^2)$

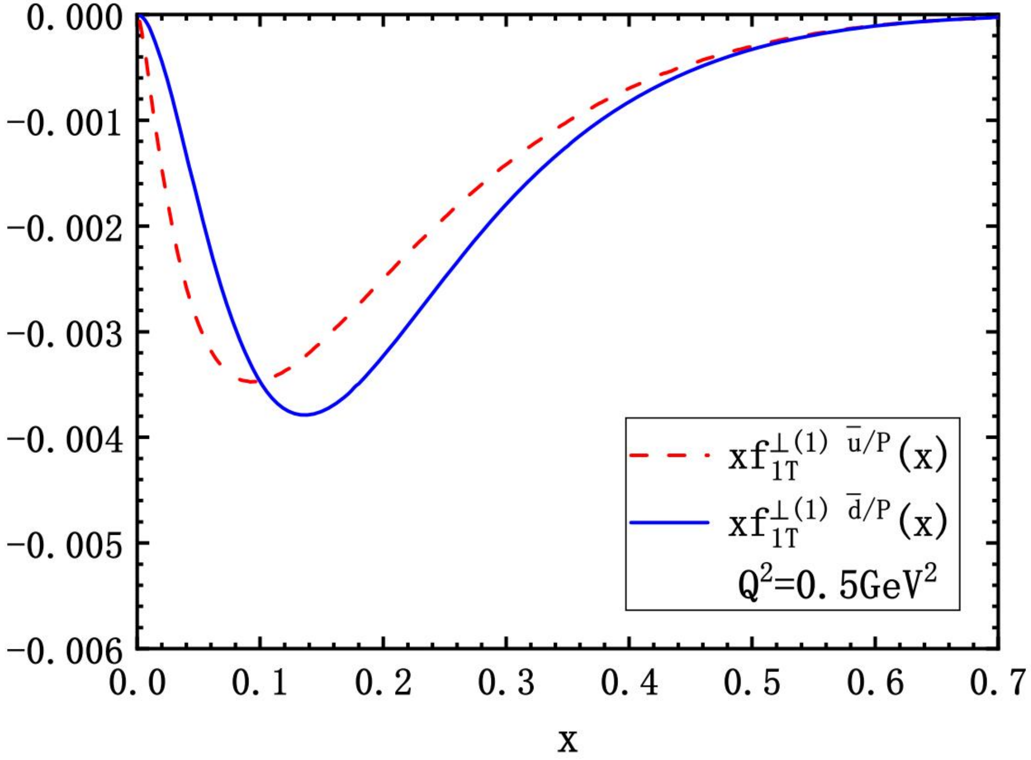
The first transverse moment of the sea quark Sivers function

$$f_{1T}^{\perp(1)\bar{q}/P}(x)=\int d^2\boldsymbol{p}_T\,\frac{\boldsymbol{p}_T^2}{2M^2}\,f_{1T}^{\perp\bar{q}/P}(x,\boldsymbol{p}_T^2)$$

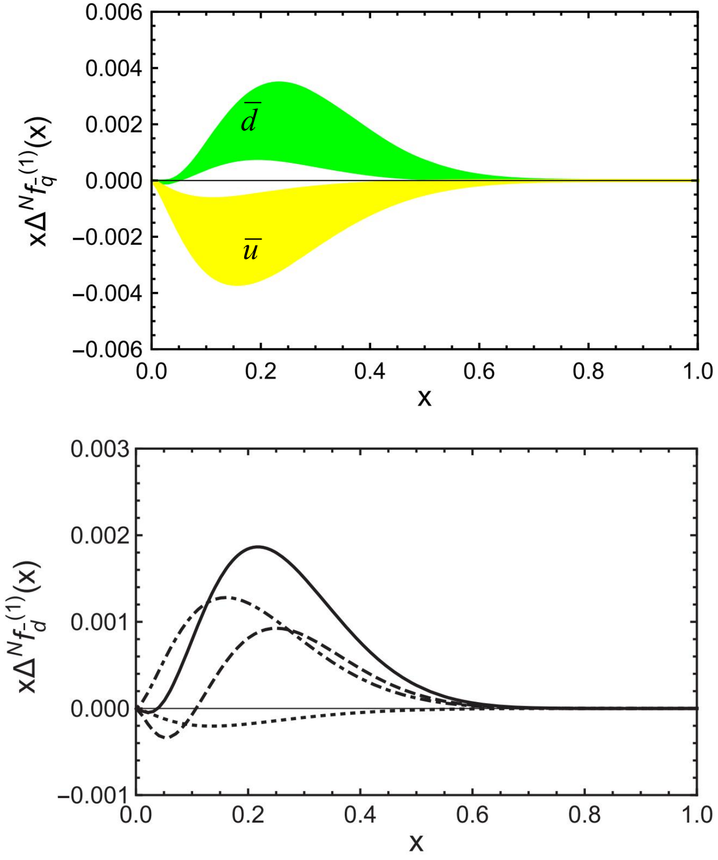
a.



b.

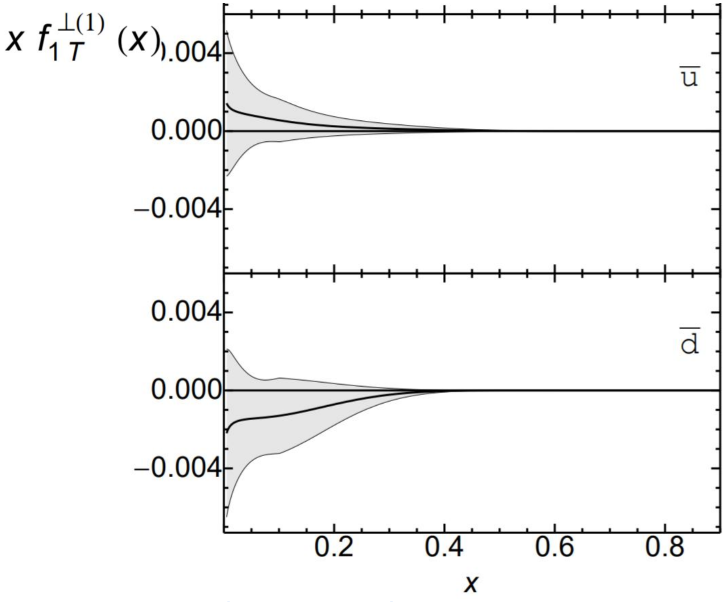


Comparison with other model calculations

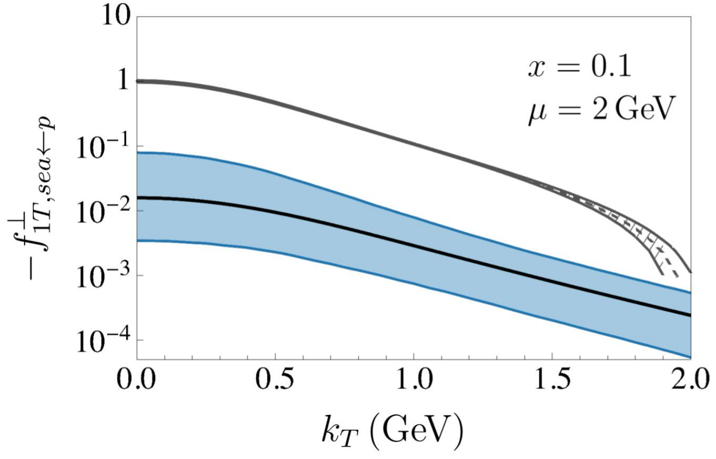


chiral model

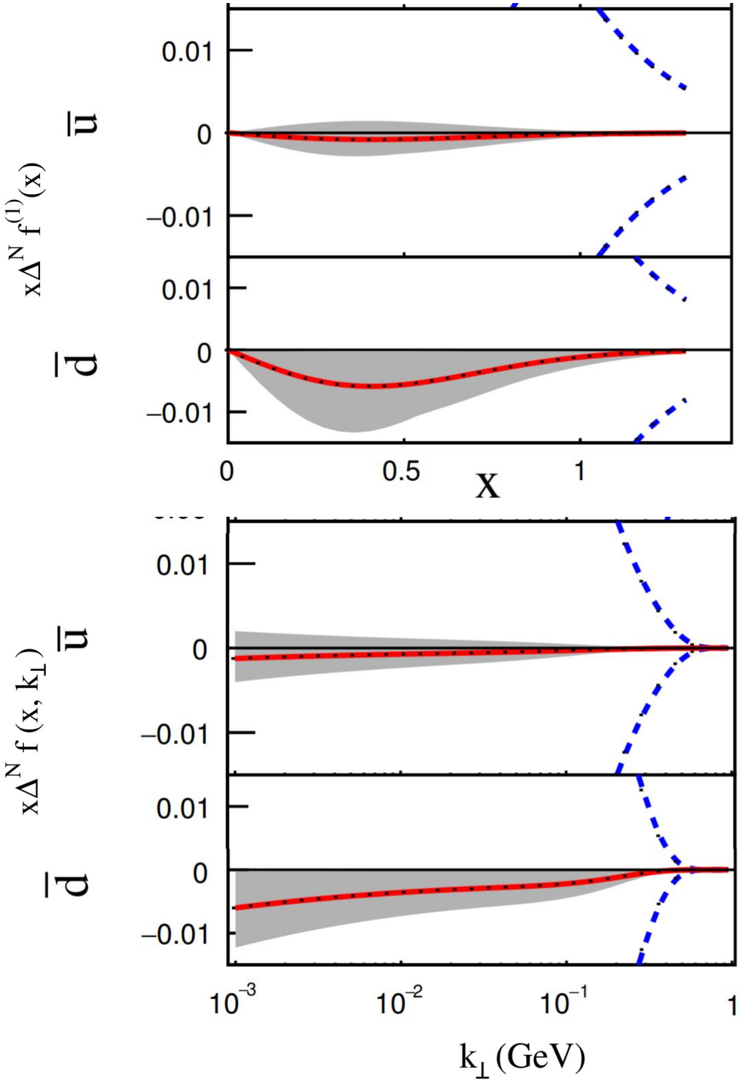
[F.c. He, P. Wang, Phys. Rev. D 100 (2019) 074032.]



[A. Bacchetta, M. Radici, PRL. 107 (2011) 212001.]



[M. Bury, A. Prokudin, A. Vladimirov, JHEP. 05 (2021) 151.]



[M. Anselmino, M. Boglione, U. D' Alesio, F. Murgia, A. Prokudin, JHEP. 2017 (2017) 046.]

Summary

- The sea quark Sivers function of the proton are studied using a light-cone model .
- The meson-baryon fluctuation model is used to generate the sea quark degrees of freedom.
- The LCWFs are derived for this composite system in terms of $q\bar{q}B$.
- Using the overlap representation of LCWFs, we calculated the unpolarized sea quark distribution function and the sea quark Sivers function.
- The values of the parameters in the model are fixed by fitting the model result to GRV LO parametrization and MSTW2008 parametrization.
- The numerical results show that the sign of the Sivers function of the $\bar{u}$ and $\bar{d}$ are negative in the entire x and k_T region. For the first transverse moment of the sea quark Sivers function, the size is 0.004 at most.
- We also compared our model results with others and find some similarities and differences between them which need to be further classified by future theoretical studies and experimental measurement.

THANK YOU!