The gluon distribution functions and angular momentum in the proton

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OUTLINE

1. Introduction

GPD(Generalized parton distribution) $f(x,\xi,\Delta^2)$:

- The mass decomposition and spin decomposition of hadrons can be expressed as the sum rules of parton GPDs.
- Fourier transforming GPDs with respect to $\Delta_\perp$ yields the IPDs $f(x,\mathbf{b}_\perp^2)$.

Wigner distribution $W(x,\mathbf{k}_\perp,\mathbf{b}_\perp)$:

- a quantum mechanical analogue of the classical phase-space density operator
- The mixed distribution has a probabilistic interpretation.
- It can be used to calculate the expectation value of any single-particle physical observable.

Experimental measurement:

- The GPDs are experimentally accessible through the hard exclusive reactions, such as the deep virtual Compton scattering(DVCS) and deep virtual meson production(DVMP).
- The Wigner distributions can not be observed directly.

Light-cone spectator model[arXiv:1611.00125]:

- It provides a method to generate the gluonic degrees of freedom from the proton target.

$$|p; S\rangle \rightarrow |g_{s_g} X_{s_X}(uud)\rangle$$

2. Generalized gluon-gluon correlator for a proton

Fully-unintegrated gluon-gluon correlator[arXiv:1307.4497]:

$$W^{g[\mu\nu;\rho\sigma]}_{\lambda'\lambda}(k,P,\Delta)=\frac{1}{k\cdot n}\int\frac{d^4z}{(2\pi)^4}\,e^{ik\cdot z}\,\langle p',\Lambda'|2\mathrm{Tr}\Big[G^{\mu\nu}\Big(-\frac{z}{2}\Big)\mathcal{W}\,G^{\rho\sigma}\Big(\frac{z}{2}\Big)\mathcal{W}'\Big]\,|p,\Lambda\rangle \qquad (1)$$

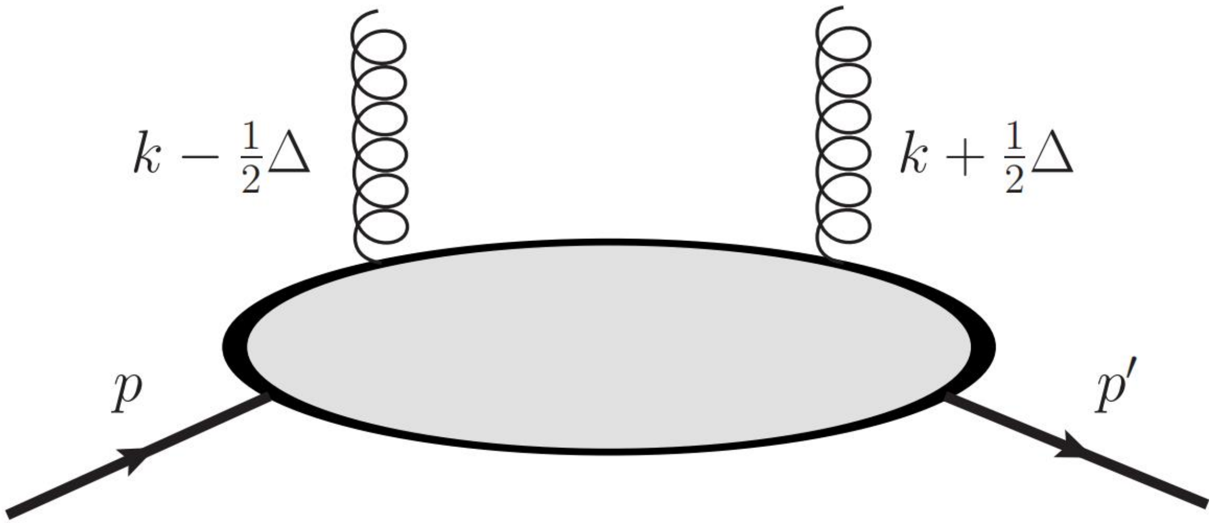


FIG. 1: Kinematics for the fully-unintegrated gluon-gluon correlator.

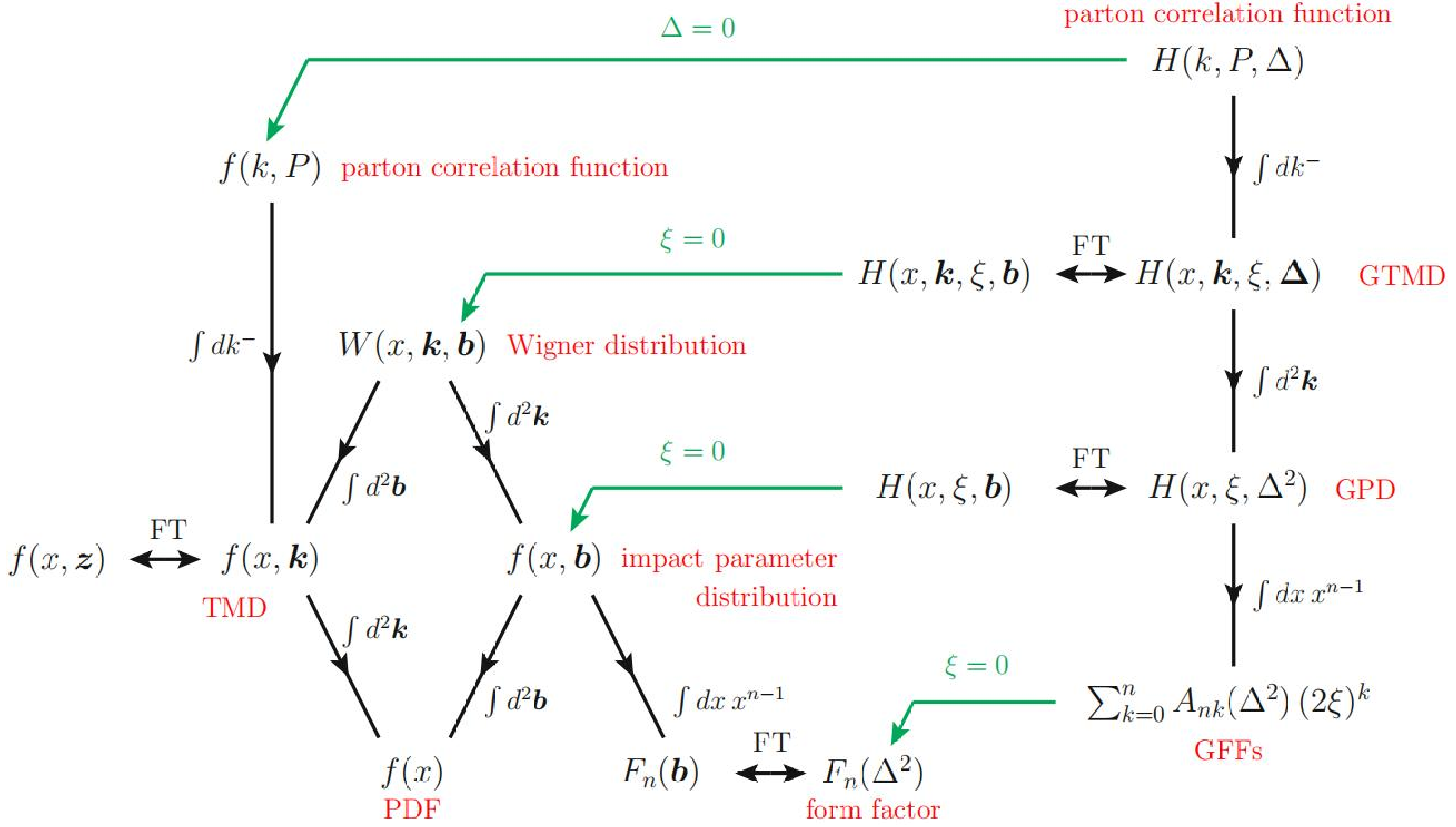


FIG. 2: Selected quantities that can be derived from the fully-unintegrated two-parton correlation function H(k,P,Δ)[arXiv:1512.01328].

Chiral-even gluon GPDs:

$$F^g(x,\xi,\Delta^2) = \delta_T^{ij} F^{g[ij]}(x,\xi,\Delta^2) = \frac{1}{2P^+}\bar{u}(p',\lambda')\left(\gamma^+ H^g(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E^g(x,\xi,t)\right)u(p,\lambda) \qquad (2)$$

$$\tilde{F}^g(x,\xi,\Delta^2) = i\epsilon_T^{ij} F^{g[ij]}(x,\xi,\Delta^2) = \frac{1}{2P^+}\bar{u}(p',\lambda')\left(\gamma^+\gamma_5 \tilde{H}^g(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M}\tilde{E}^g(x,\xi,t)\right)u(p,\lambda) \qquad (3)$$

Chiral-odd gluon GPDs:

$$\begin{aligned} F_T^{g,ij}(x,\xi,\Delta^2) =& -\hat{\boldsymbol{S}} F^{g[ij]}(x,\xi,\Delta^2) \\ =& \frac{\hat{\boldsymbol{S}}}{2P^+}\frac{P^+\Delta_T^i - \Delta^+ P_T^i}{2MP^+}\bar{u}(p',\lambda')\left(i\sigma^{+j}H_T^g(x,\xi,t) + \frac{\gamma^+\Delta_T^j - \Delta^+\gamma_T^j}{2M}E_T^g(x,\xi,t)\right. \\ &\left. + \frac{P^+\Delta_T^j - \Delta^+ P_T^j}{M^2}\tilde{H}_T^g(x,\xi,t) + \frac{\gamma^+ P_T^j - P^+\gamma_T^j}{M}\tilde{E}_T^g(x,\xi,t)\right)u(p,\lambda) \end{aligned} \qquad (4)$$

F-type gluon GTMDs(Generalized transverse momentum distributions):

$$
\begin{aligned}
\mathcal{W}^g_{\lambda',\lambda} =& \delta^{ij}_\perp W^{g[ij]}_{\lambda',\lambda} \\
=& \frac{1}{2M}\bar{u}(p',\lambda')\left[F^g_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P+}F^g_{1,2} + \frac{i\sigma^{i+}\Delta^i_\perp}{P+}F^g_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}F^g_{1,4}\right]u(p,\lambda) \\
=& \frac{1}{M\sqrt{1-\xi^2}}\left\{\left[M\delta_{\lambda',\lambda} - \frac{1}{2}(\lambda\Delta^1_\perp + i\Delta^2_\perp)\delta_{-\lambda',\lambda}\right]F^g_{1,1} + (1-\xi^2)(\lambda k^1_\perp + ik^2_\perp)\delta_{-\lambda',\lambda}F^g_{1,2}\right. \\
&\left. + (1-\xi^2)(\lambda\Delta^1_\perp + i\Delta^2_\perp)\delta_{-\lambda',\lambda}F^g_{1,3} + \frac{i\epsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}\left[\lambda M\delta_{\lambda',\lambda} - \frac{\xi}{2}(\Delta^1_\perp + i\lambda\Delta^2_\perp)\delta_{-\lambda',\lambda}\right]F^g_{1,4}\right\}
\end{aligned}
\quad (5)
$$

G-type gluon GTMDs:

$$
\begin{aligned}
\widetilde{\mathcal{W}}^g_{\lambda',\lambda} =& -i\epsilon^{ij}_\perp W^{g[ij]}_{\lambda',\lambda} \\
=& \frac{1}{2M}\bar{u}(p',\lambda')\left[-\frac{i\epsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}G^g_{1,1} + \frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P+}G^g_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P+}G^g_{1,3} + i\sigma^{+-}\gamma_5 G^g_{1,4}\right]u(p,\lambda) \\
=& \frac{1}{M\sqrt{1-\xi^2}}\left\{-\frac{i\epsilon^{ij}_\perp k^i_\perp\Delta^j_\perp}{M^2}\left[M\delta_{\lambda',\lambda} - \frac{1}{2}(\lambda\Delta^1_\perp + i\Delta^2_\perp)\delta_{-\lambda',\lambda}\right]G^g_{1,1} + (1-\xi^2)(k^1_\perp + i\lambda k^2_\perp)\delta_{-\lambda',\lambda}G^g_{1,2}\right. \\
&\left. + (1-\xi^2)(\Delta^1_\perp + i\lambda\Delta^2_\perp)\delta_{-\lambda',\lambda}G^g_{1,3} + \left[\lambda M\delta_{\lambda',\lambda} - \frac{\xi}{2}(\Delta^1_\perp + i\lambda\Delta^2_\perp)\delta_{-\lambda',\lambda}\right]G^g_{1,4}\right\}
\end{aligned}
\quad (6)
$$

The Wigner distributions of the unpolarized/longitudinally polarized gluon in the unpolarized/longitudinally polarized proton:

$$W_{UU}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\frac{1}{2}(W^g_{\uparrow,\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)+W^g_{\downarrow,\downarrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)) \tag{7}$$

$$W_{LU}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\frac{1}{2}(W^g_{\uparrow,\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)-W^g_{\downarrow,\downarrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)) \tag{8}$$

$$W_{UL}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\frac{1}{2}(\widetilde{W}^g_{\uparrow,\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)+\widetilde{W}^g_{\downarrow,\downarrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)) \tag{9}$$

$$W_{LL}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\frac{1}{2}(\widetilde{W}^g_{\uparrow,\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)-\widetilde{W}^g_{\downarrow,\downarrow}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)) \tag{10}$$

Fourier transform:

$$W^g_{\lambda',\lambda}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\int\frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\mathcal{W}^g_{\lambda',\lambda}(x,\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) \tag{11}$$

Relationships between GTMDs and Wigner distributions:

$$W_{UU}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\mathcal{F}^g_{1,1}(x,0,\boldsymbol{k}^2_\perp,\boldsymbol{k}_\perp\cdot\boldsymbol{b}_\perp,\boldsymbol{b}^2_\perp) \tag{12}$$

$$W_{LU}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=-\frac{1}{M^2}\epsilon^{ij}_\perp k^i_\perp\frac{\partial}{\partial b^j_\perp}\mathcal{F}^g_{1,4}(x,0,\boldsymbol{k}^2_\perp,\boldsymbol{k}_\perp\cdot\boldsymbol{b}_\perp,\boldsymbol{b}^2_\perp) \tag{13}$$

$$W_{UL}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\frac{1}{M^2}\epsilon^{ij}_\perp k^i_\perp\frac{\partial}{\partial b^j_\perp}\mathcal{G}^g_{1,1}(x,0,\boldsymbol{k}^2_\perp,\boldsymbol{k}_\perp\cdot\boldsymbol{b}_\perp,\boldsymbol{b}^2_\perp) \tag{14}$$

$$W_{LL}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\mathcal{G}^g_{1,4}(x,0,\boldsymbol{k}^2_\perp,\boldsymbol{k}_\perp\cdot\boldsymbol{b}_\perp,\boldsymbol{b}^2_\perp) \tag{15}$$

Relationships between GTMDs and GPDs:

$$H^g(x,\xi,\boldsymbol{\Delta}^2_\perp)=\int d^2\boldsymbol{k}_\perp\left[F^g_{1,1}+2\xi^2\left(\frac{\boldsymbol{\Delta}_\perp\cdot\boldsymbol{k}_\perp}{\boldsymbol{\Delta}^2_\perp}F^g_{1,2}+F^g_{1,3}\right)\right] \tag{16}$$

$$E^g(x,\xi,\boldsymbol{\Delta}^2_\perp)=\int d^2\boldsymbol{k}_\perp\left[-F^g_{1,1}+2(1-\xi^2)\left(\frac{\boldsymbol{\Delta}_\perp\cdot\boldsymbol{k}_\perp}{\boldsymbol{\Delta}^2_\perp}F^g_{1,2}+F^g_{1,3}\right)\right] \tag{17}$$

$$\widetilde{H}^g(x,\xi,\boldsymbol{\Delta}^2_\perp)=\int d^2\boldsymbol{k}_\perp\left[2\xi\left(\frac{\boldsymbol{\Delta}_\perp\cdot\boldsymbol{k}_\perp}{\boldsymbol{\Delta}^2_\perp}G^g_{1,2}+G^g_{1,3}\right)+G^g_{1,4}\right] \tag{18}$$

$$\widetilde{E}^g(x,\xi,\boldsymbol{\Delta}^2_\perp)=\int d^2\boldsymbol{k}_\perp\left[\frac{2(1-\xi^2)}{\xi}\left(\frac{\boldsymbol{\Delta}_\perp\cdot\boldsymbol{k}_\perp}{\boldsymbol{\Delta}^2_\perp}G^g_{1,2}+G^g_{1,3}\right)-G^g_{1,4}\right] \tag{19}$$

Kinetic gluon OAM(orbital angular momentum):

$$\begin{aligned} L_z^g &= \int dx L_z^g(x) \\ &= \frac{1}{2}\int dx\{x[H^g(x,0,0)+E^g(x,0,0)] - \tilde{H}^g(x,0,0)\} \end{aligned} \tag{20}$$

Canonical gluon OAM:

$$\begin{aligned} l_z^g &= \int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z W_{LU}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp) \\ &= -\int dx d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{M^2} F_{1,4}^g(x,0,\boldsymbol{k}_\perp^2,0,0) \end{aligned} \tag{21}$$

Gluon spin-orbit correlations:

$$\begin{aligned} C_z^g &= \int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z W_{UL}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp) \\ &= \int dx d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{M^2} G_{1,1}^g(x,0,\boldsymbol{k}_\perp^2,0,0) \end{aligned} \tag{22}$$

3. Gluon GPDs and Wigner distributions

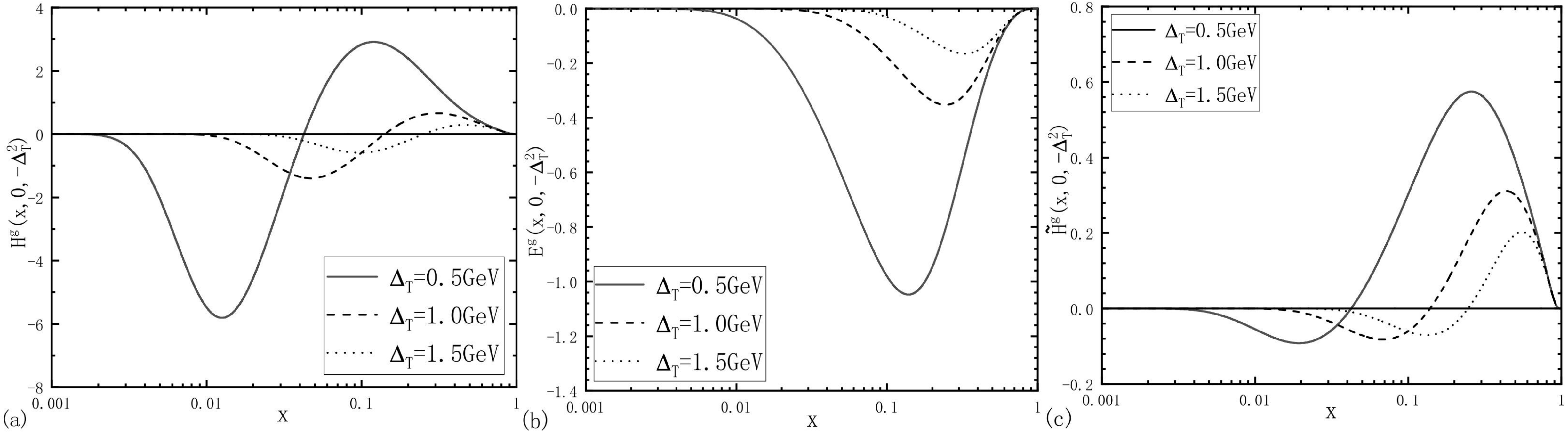


FIG. 3: The dependence of the chiral-even gluon GPDs H^g, E^g and $\tilde{H}^g$ on x at $\xi = 0$ when Δ_T = 0.5 GeV, 1.0 GeV, 1.5 GeV, respectively.

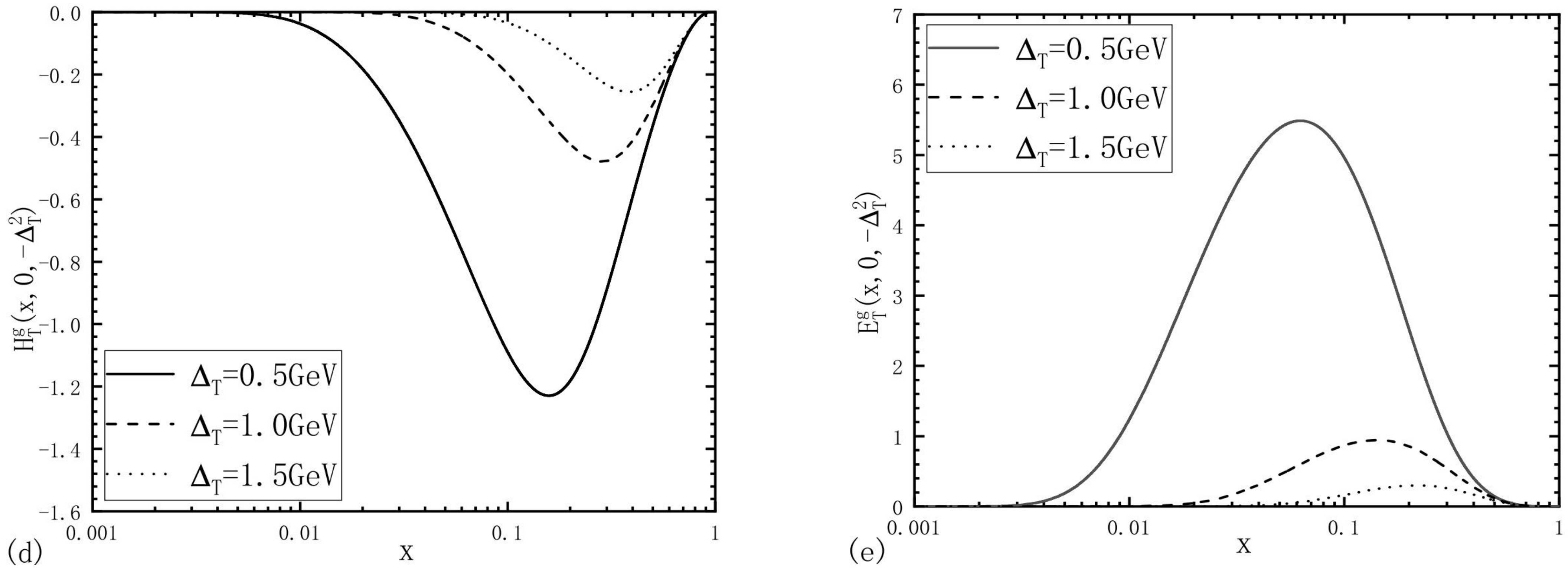
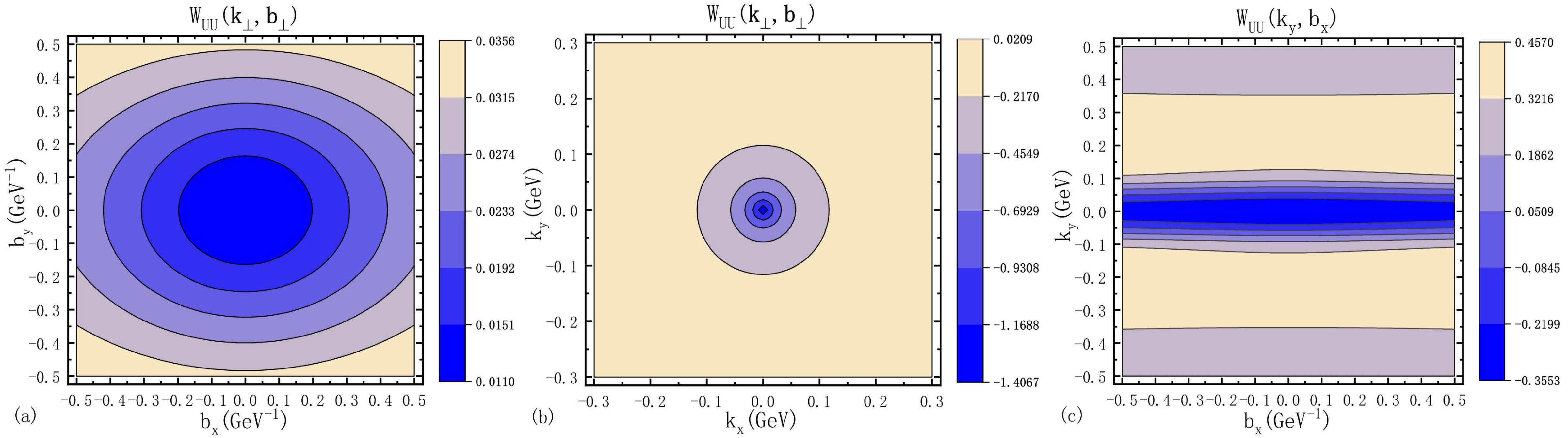
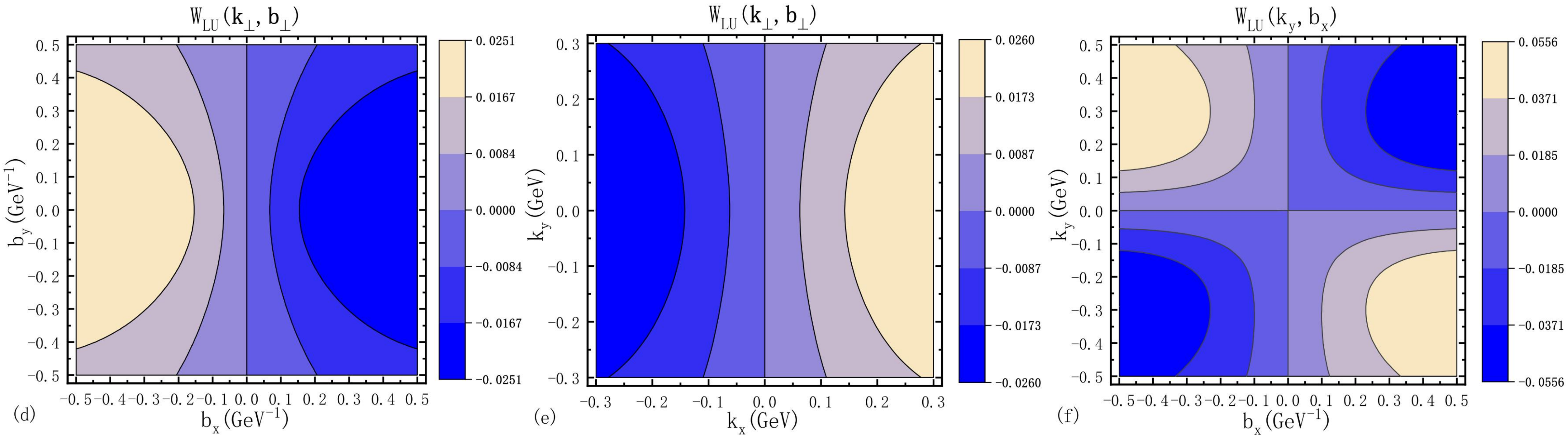


FIG. 4: The dependence of the chiral-odd gluon GPDs H_T^g and E_T^g on x at $\xi = 0$.



$$\int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z W_{UU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = 0 \tag{23}$$

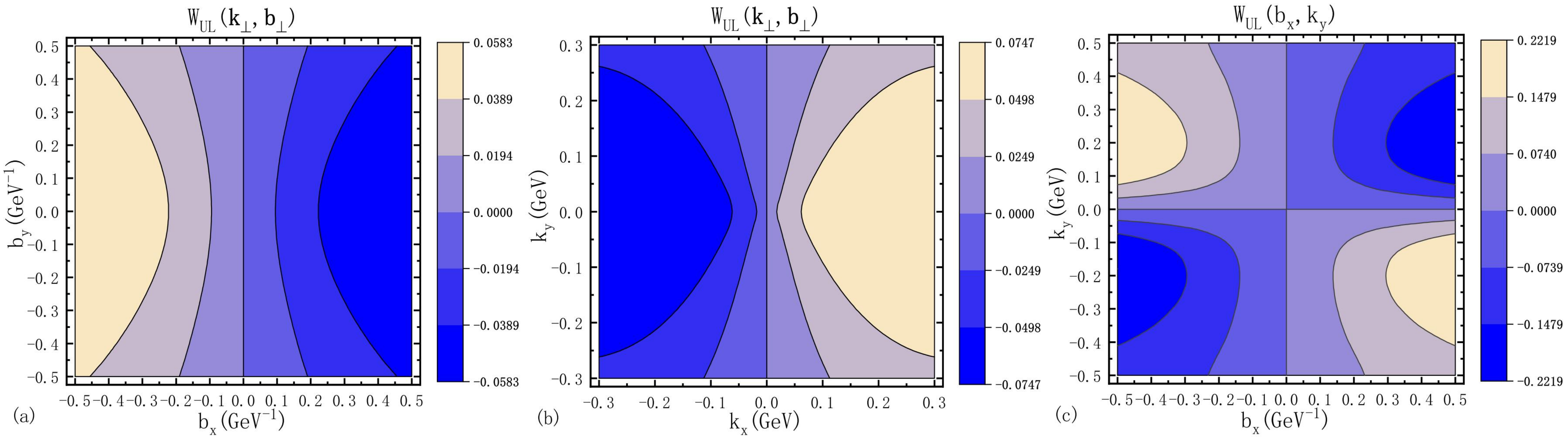
FIG. 5: The contour plots of the gluon Wigner distribution $W_{UU}(\mathbf{k}_\perp, \mathbf{b}_\perp)$. The first plot displays the distribution in $\mathbf{b}_\perp$ space with $\mathbf{k}_\perp = k_\perp \hat{\mathbf{j}} = 0.5$ GeV $\hat{\mathbf{j}}$. The second plot displays the distribution in $\mathbf{k}_\perp$ space with $\mathbf{b}_\perp = b_\perp \hat{\mathbf{j}} = 0.5\ \mathrm{GeV}^{-1}\ \hat{\mathbf{j}}$. The third plot displays the distribution in the mixed space of b_x and k_y.



$$W_{LU}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp) = -\frac{1}{M^2}\epsilon_\perp^{ij}k_\perp^i\frac{\partial}{\partial b_\perp^j}\mathcal{F}_{1,4}^g(x,0,\boldsymbol{k}_\perp^2,\boldsymbol{k}_\perp\cdot\boldsymbol{b}_\perp,\boldsymbol{b}_\perp^2) \quad (13)$$

$$l_z^g = \int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp(\boldsymbol{b}_\perp\times\boldsymbol{k}_\perp)_z W_{LU}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp) \quad (21)$$

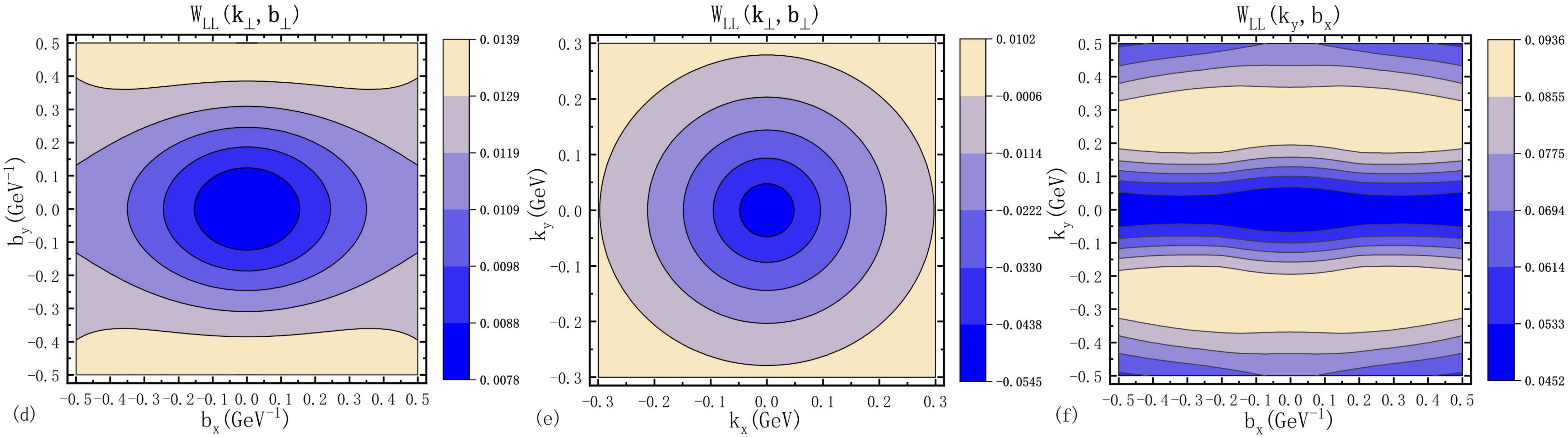
FIG. 6: The contour plots of the gluon Wigner distribution W_{LU} ($\mathbf{k}_\perp$, $\mathbf{b}_\perp$).



$$W_{UL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = \frac{1}{M^2} \epsilon_\perp^{ij} k_\perp^i \frac{\partial}{\partial b_\perp^j} \mathcal{G}_{1,1}^g(x, 0, \boldsymbol{k}_\perp^2, \boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp, \boldsymbol{b}_\perp^2) \tag{14}$$

$$C_z^g = \int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z W_{UL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) \tag{22}$$

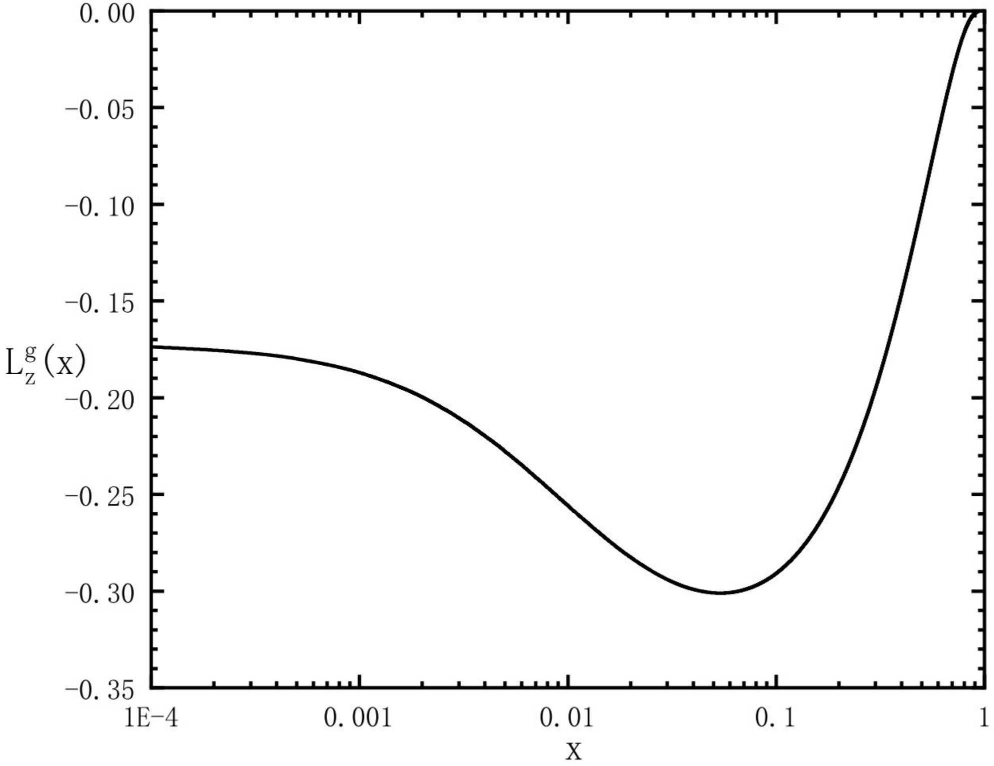
FIG. 7: The contour plots of the gluon Wigner distribution $W_{UL}(\mathbf{k}_\perp, \mathbf{b}_\perp)$.



$$\int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z W_{LL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp) = 0 \tag{24}$$

FIG. 8: The contour plots of the gluon Wigner distribution W_{LL} ($\mathbf{k}_\perp$, $\mathbf{b}_\perp$).

4. Gluon orbital angular momentum and spin-orbit correlations



$$L_z^g = \frac{1}{2}\int dx\{x[H^g(x,0,0) + E^g(x,0,0)] - \tilde{H}^g(x,0,0)\} \quad (20)$$

FIG. 9: The dependence of the kinetic gluon OAM $L_z^g(x)$ on x.

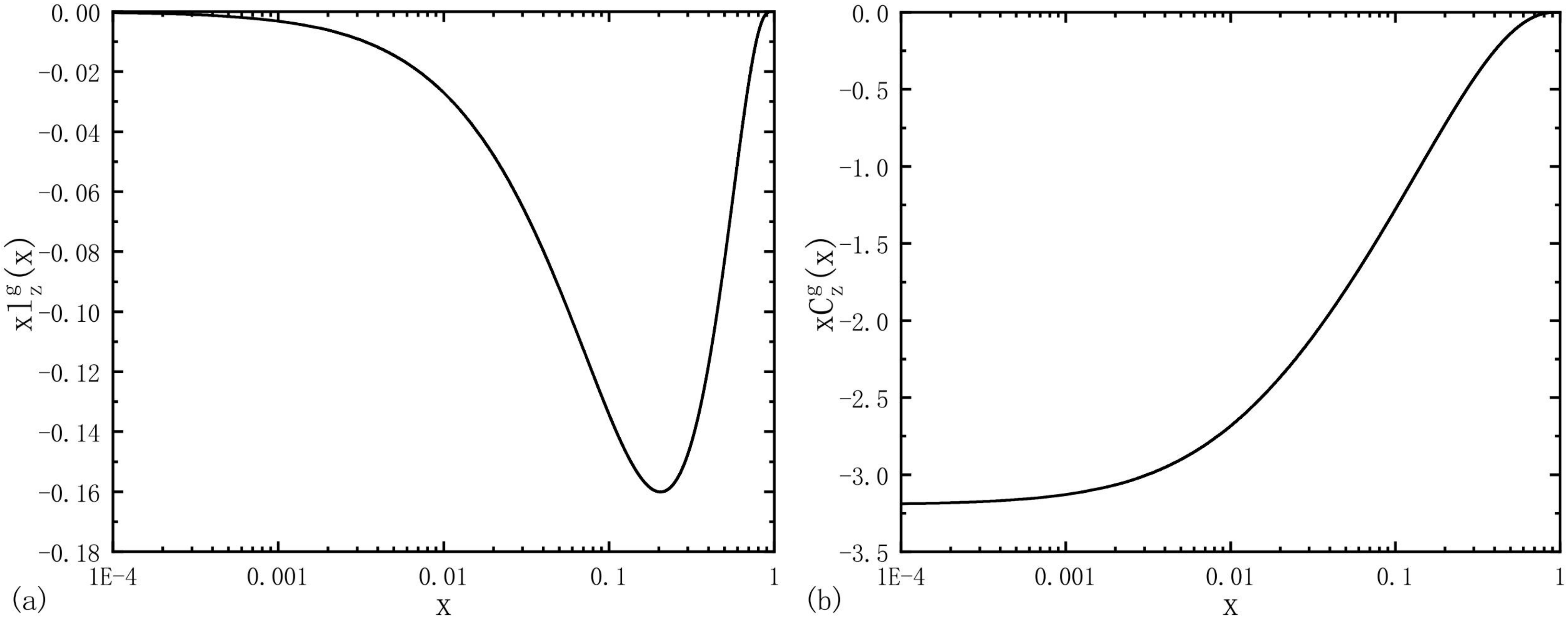


FIG. 10: The left plot displays the dependence of the canonical gluon OAM $l_z^g(x)$ (timed with x) on x. The right plot displays the dependence of the gluon spin-orbit correlations $C_z^g(x)$ (timed with x) on x.

5. Conclusions

- Among the eight leading twist gluon GPDs, only H^g, E^g, $\widetilde{H}^g$, H_T^g, E_T^g and $\widetilde{H}_T^g$ survive at $\xi = 0$, and $\widetilde{H}_T^g=0$ in this model.
- The behaviors of $H^g(x,0,\Delta_T^2)$ and $\widetilde{H}^g(x,0,\Delta_T^2)$ are different from their respective forward limits($\Delta=0$) $f_1^g(x)$ and $g_1^g(x)$.
- $E^g(x,0,\Delta_T^2)$ and $H_T^g(x,0,\Delta_T^2)$ share similar shape since $E^g=xH_T^g$.
- $L_z^g(x)$ is almost determined by $\frac{1}{2}xE^g(x,0,0)$ since $xH^g(x,0,0)-\widetilde{H}^g(x,0,0)\approx 0$.
- The model result of the total gluon angular momentum $J^g=0.190$ agrees with recent lattice result($J^g=0.187$)[arXiv:2003.08486] within uncertainty.

- The corresponding plots of W_{UU} and W_{LL} are similar. There is no net OAM in both cases, that is, the space is isotropic.
- The corresponding plots of W_{LU} and W_{UL} are similar, where the multipole structures imply the existence of the canonical OAM and spin-orbit correlations, respectively.
- In this model, $l_z^g(x) \neq C_z^g(x)$.
- The negative $l_z^g(x)$ implies that the total gluon OAM will reduce the total angular momentum contribution of the gluon to the proton spin, while the negative $C_z^g(x)$ indicates that the gluon spin and OAM are antialigned.

THANK YOU