Gravitational structure of nucleon and Δ resonances

Herzallah Alharazin
Ruhr University Bochum

Gravitational form factors

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For spin 1/2

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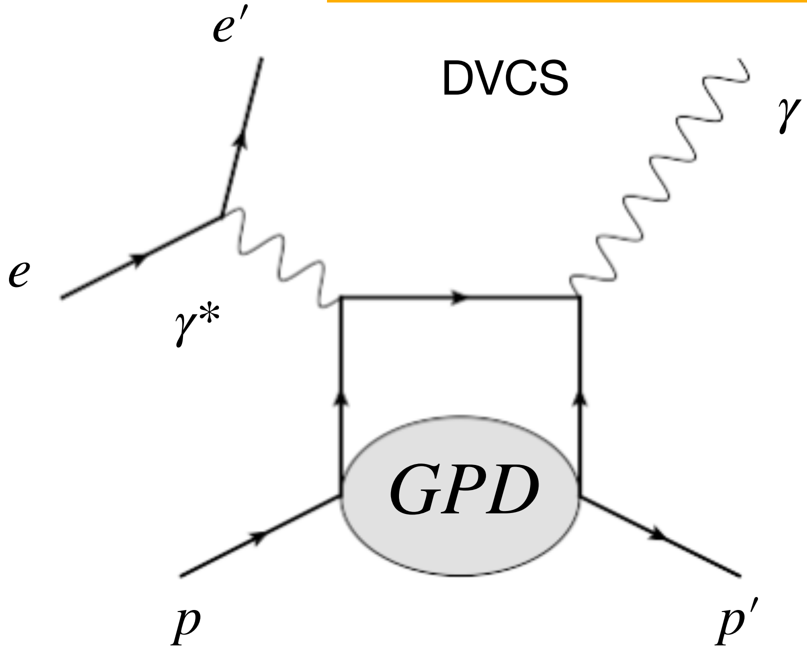

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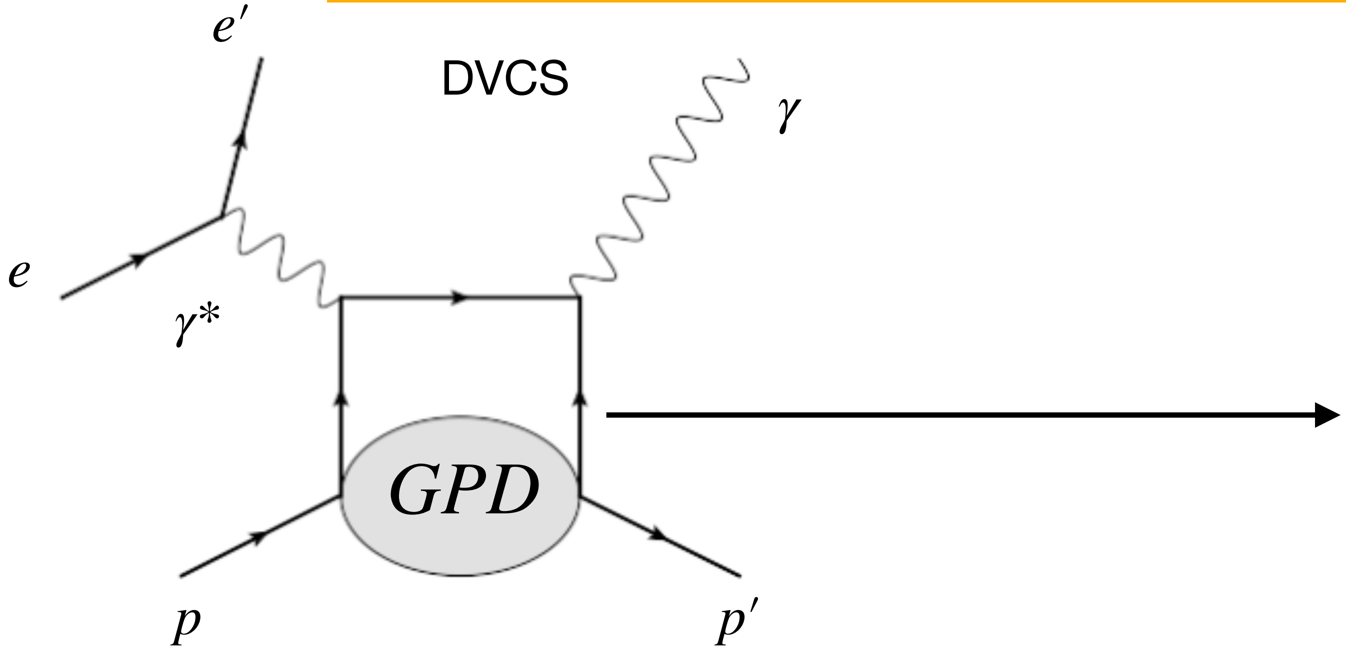

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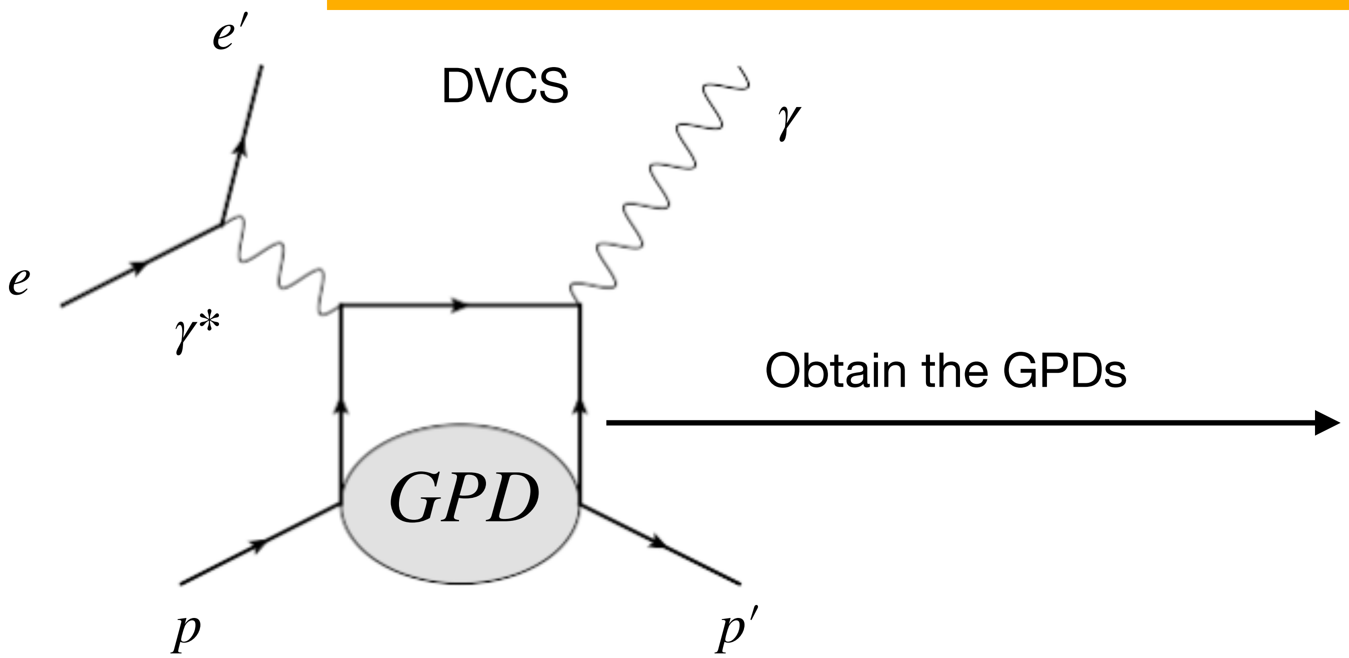

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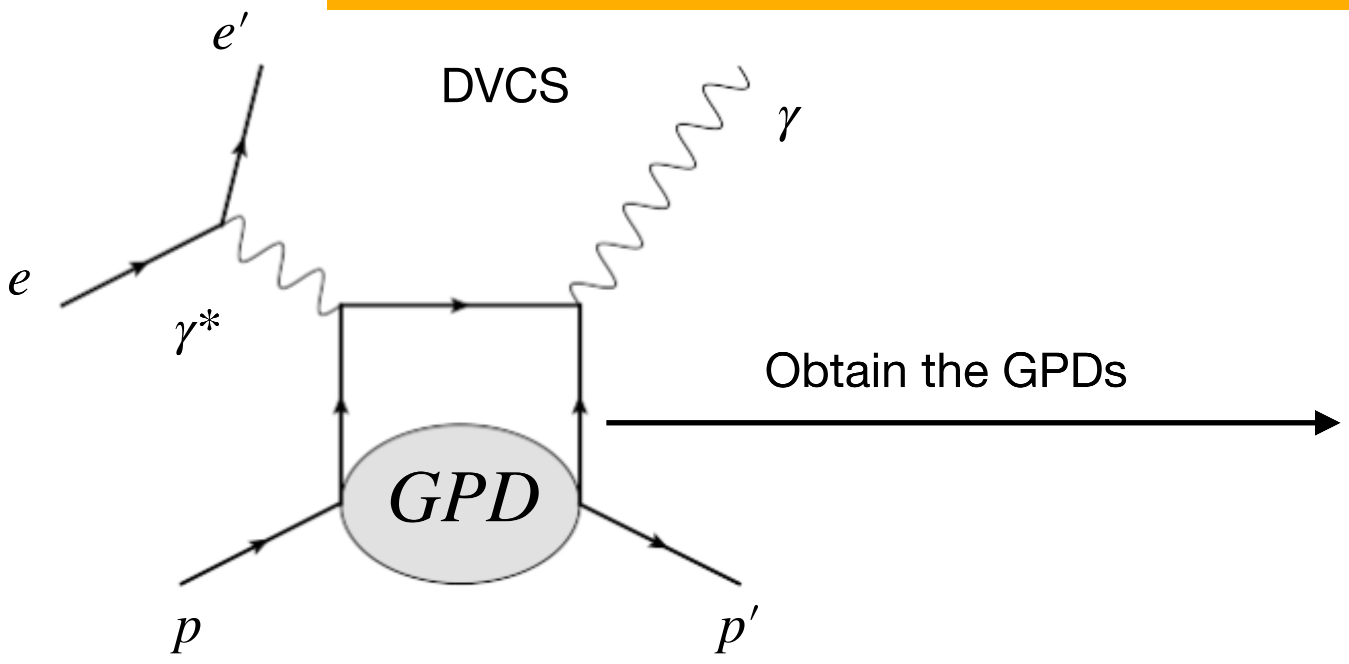

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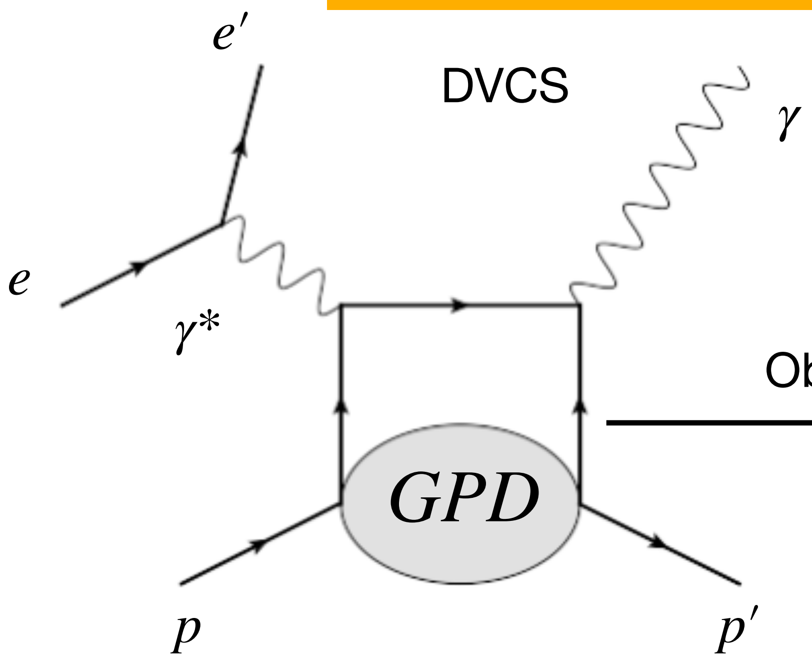

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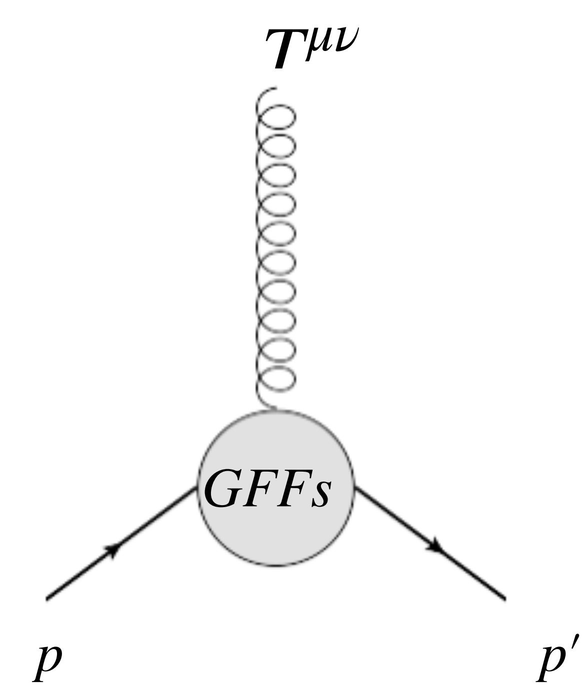

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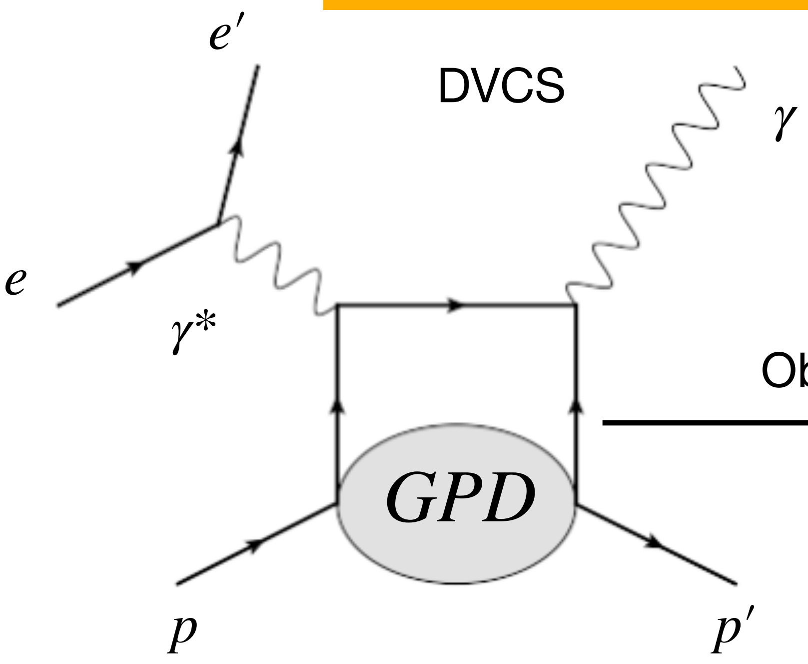

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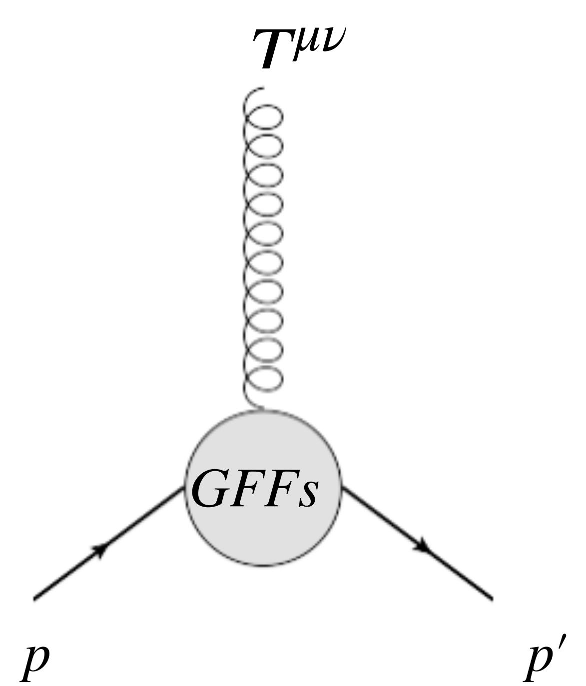

The observables in DVCS are the Compton form factors CFFs. From them one can obtain the GPDs, e.g.

$$Re\mathscr{H}(\xi,t)+i\ Im\mathscr{H}(\xi,t) = \sum_q e_q^2 \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right] H_q(x,\xi,t)$$

More details see, e.g. Burkert, et al., [Colloquium: Gravitational Form Factors of the Proton]

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For that we need the effective chiral Lagrangian up to second chiral order in curved spacetime

H. Alharazin, D. Djukanovic, J. Gegelia, M.V. Polyakov, ``Chiral theory of nucleons and pions in the presence of an external gravitational field, ``Phys.Rev.D 102 (2020) 7, 076023

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$$\int d^4x \sqrt{-g}\Big[h_1 R \ g^{\alpha\beta}\bar\Psi^i_\alpha \Psi^i_\beta + h_2 R \ \bar\Psi^i_\alpha \gamma^\alpha\gamma^\beta\Psi^i_\beta + ih_3 R\left(g^{\alpha\lambda}\bar\Psi^i_\alpha\gamma^\beta \overrightarrow{\nabla}_\lambda \Psi^i_\beta - g^{\beta\lambda}\bar\Psi^i_\alpha\gamma^\alpha \overleftarrow{\nabla}_\lambda \Psi^i_\beta\right) + \cdots$$

GFFs for of nucleon

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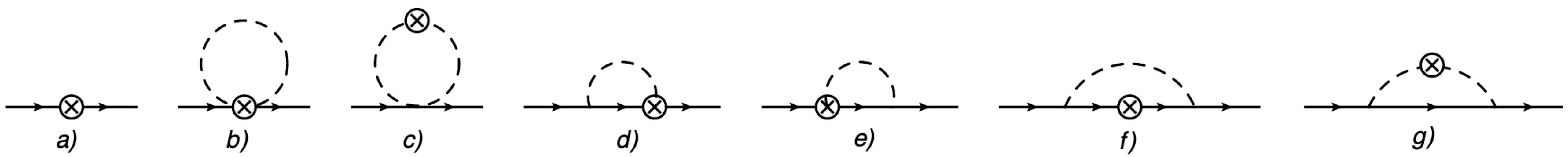

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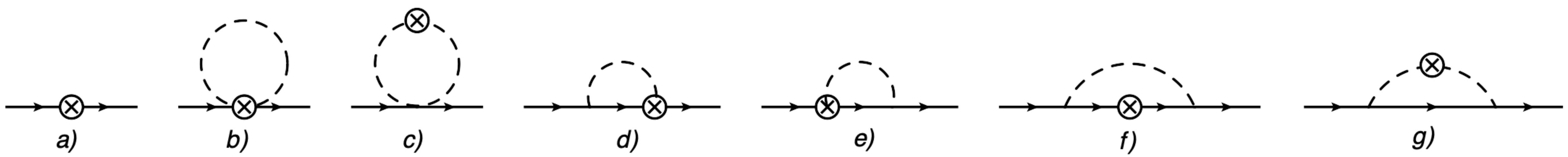

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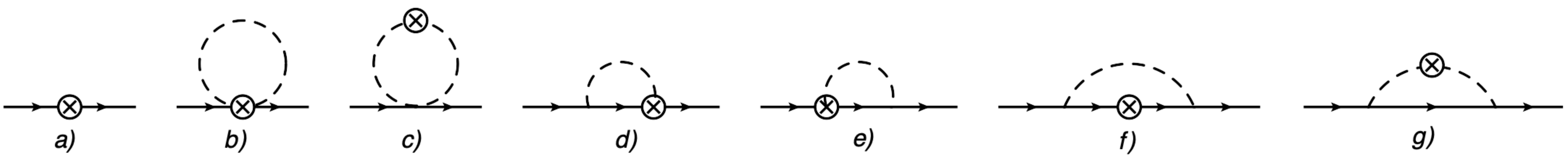

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We expand the results in powers of small quantities and define the slopes s_F of the GFFs as

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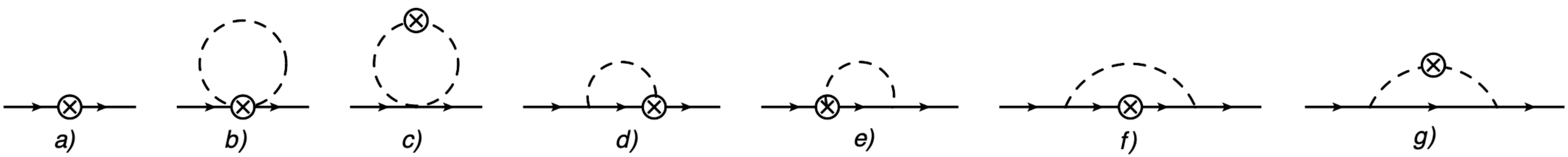

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$$s_J = \frac{g_A^2 \left(4 c_9 m_N - 5\right)}{64 \pi^2 F^2} - \frac{g_A^2 \ln \frac{M_\pi}{m_N}}{32 \pi^2 F^2} + \frac{7 M_\pi g_A^2}{128 \pi F^2 m_N} + \mathcal{O}(M_\pi^2)$$

$$s_D = -\frac{g_A^2 m_N}{40 \pi F^2 M_\pi} - \frac{\ln \frac{M_\pi}{m_N} \left(5 g_A^2 + 4 \left(c_2 + 5 c_3\right) m_N\right)}{80 \pi^2 F^2} + \frac{g_A^2 \left(3 + (15 c_8 + 5 c_9) m_N\right)}{60 \pi^2 F^2} + \frac{\left(4 c_1 - c_2 - 7 c_3\right) m_N}{40 \pi^2 F^2} + \mathcal{O}(M_\pi)$$

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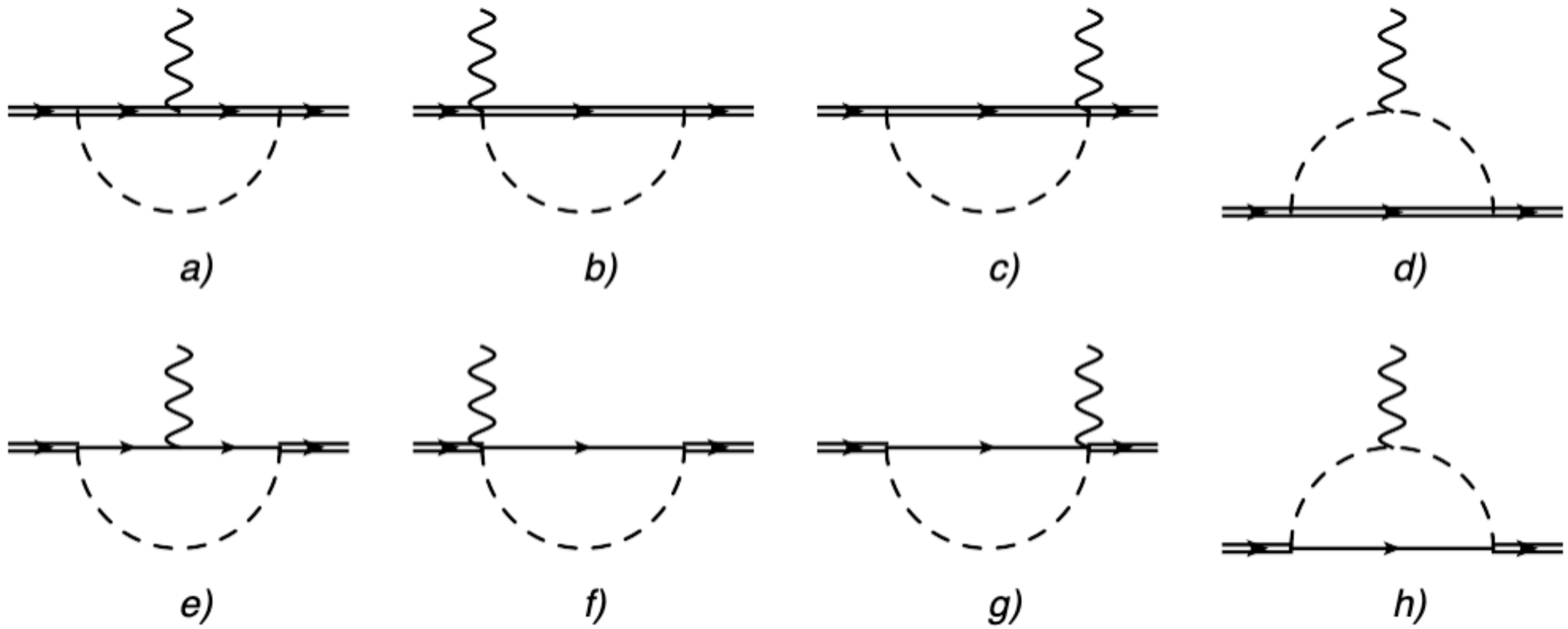

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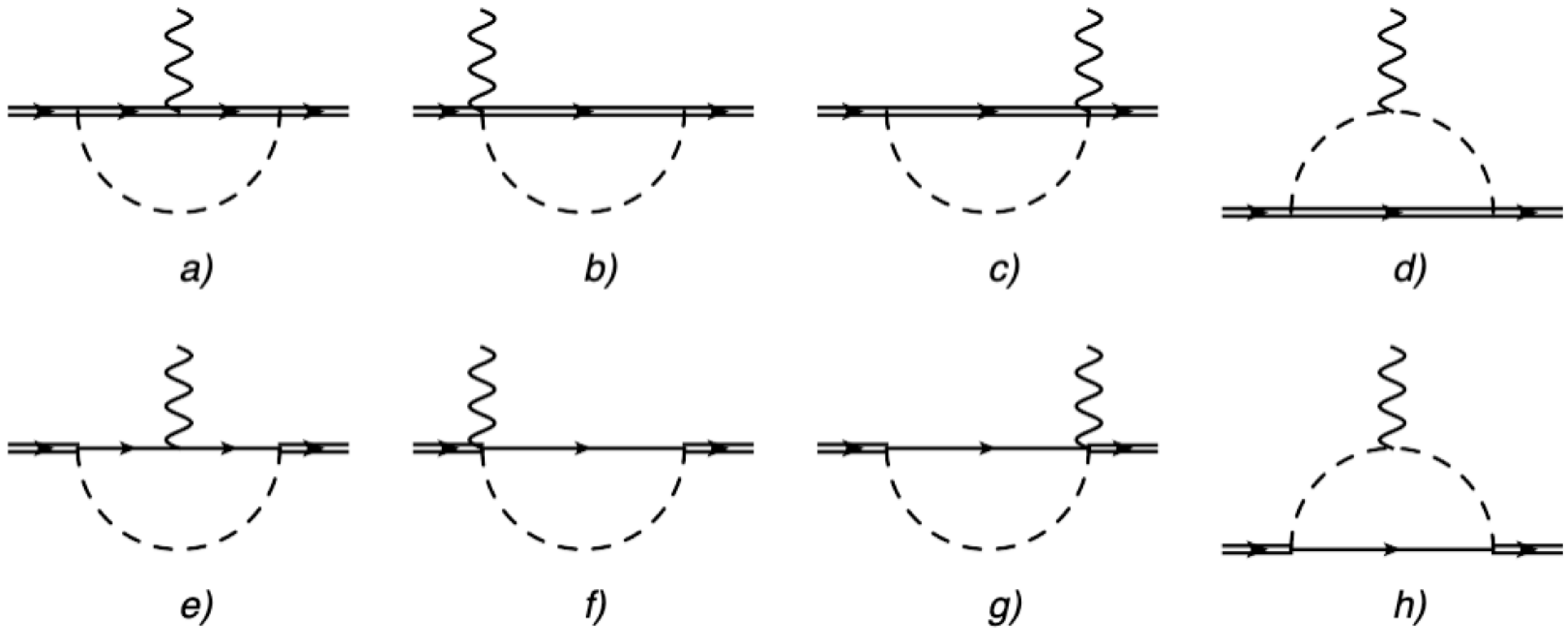

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$$\langle p_f, s_f | T^{\mu\nu} | p_i, s_i \rangle = -\bar{u}_{\alpha'}(p_f, s_f) \left[\frac{P^\mu P^\nu}{m_\Delta} \left(\eta^{\alpha'\alpha} F_{1,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2} F_{1,1}(t) \right) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{4m_\Delta} \left(\eta^{\alpha'\alpha} F_{2,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2} F_{2,1}(t) \right) \right.$$

$$\left. + \frac{i}{2m_\Delta} P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho \left(\eta^{\alpha'\alpha} F_{4,0}(t) - \frac{\Delta^{\alpha'}\Delta^\alpha}{2m_\Delta^2} F_{4,1}(t) \right) - \frac{1}{m_\Delta} \left(\eta^{\alpha(\mu}\Delta^{\nu)}\Delta^{\alpha'} + \eta^{\alpha'(\mu}\Delta^{\nu)}\Delta^\alpha - 2\eta^{\mu\nu}\Delta^\alpha\Delta^{\alpha'} - \Delta^2 \eta^{\alpha(\mu}\eta^{\nu)\alpha'} \right) F_{5,0}(t) \right] u_\alpha(p_i, s_i)$$

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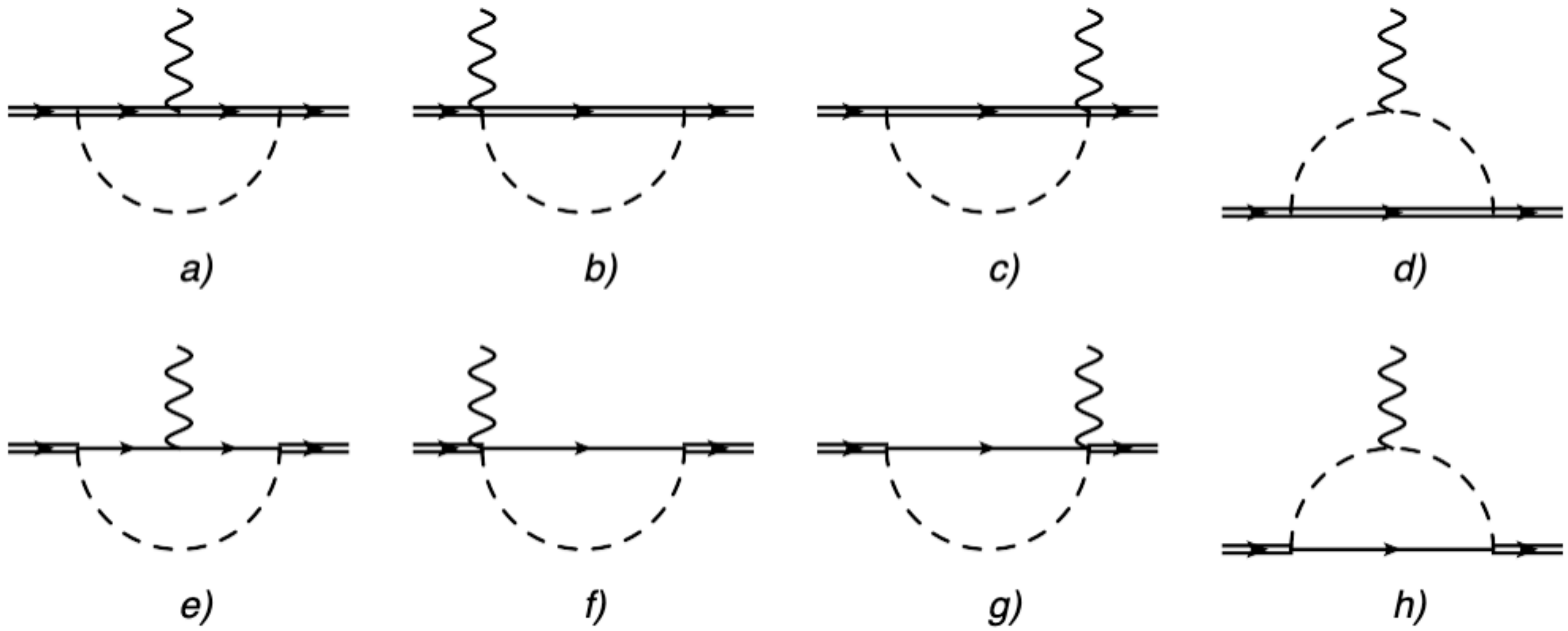

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We expand the results in powers of small quantities and define the slopes $s_{F_{i,j}}$ of the GFFs as

$$F_{i,j}(t) = F_{i,j}(0) + s_{F_{i,j}}t + \mathcal{O}(t^2)$$

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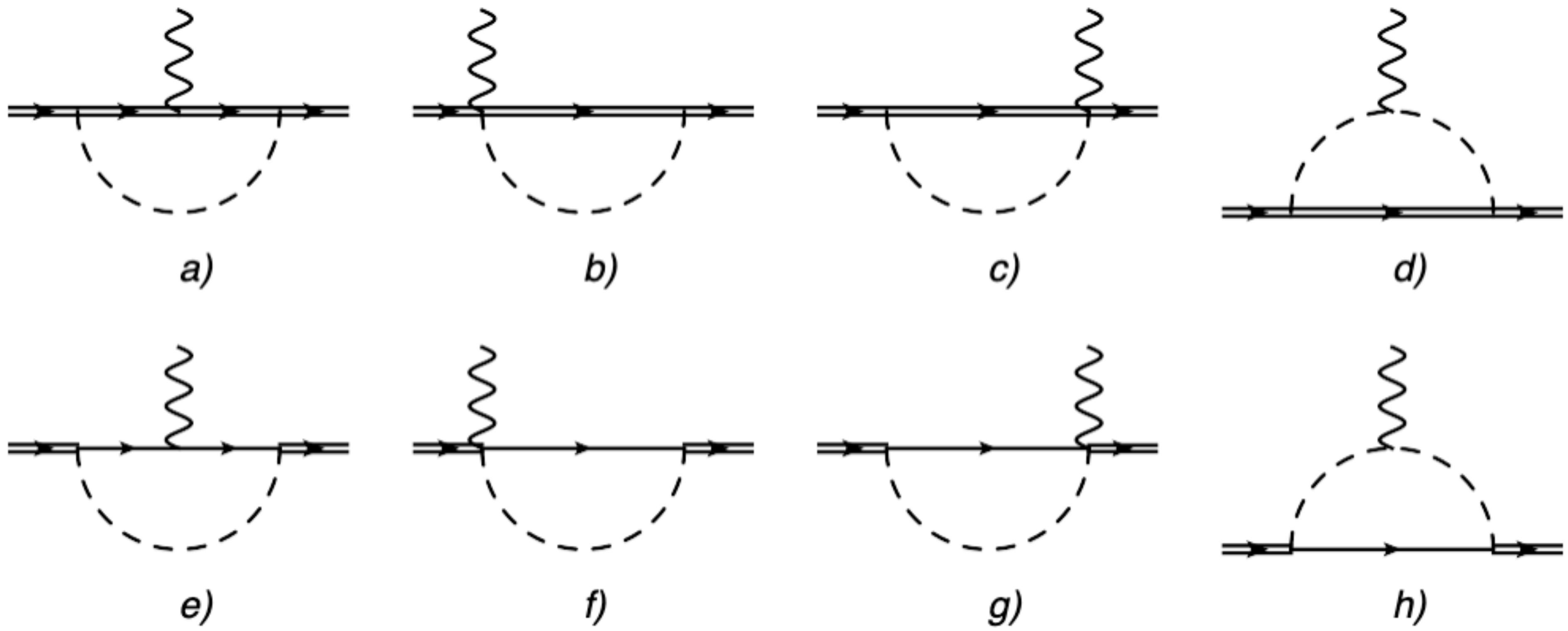

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We applied the EOMS scheme with renormalization scale $\mu = m_N$.

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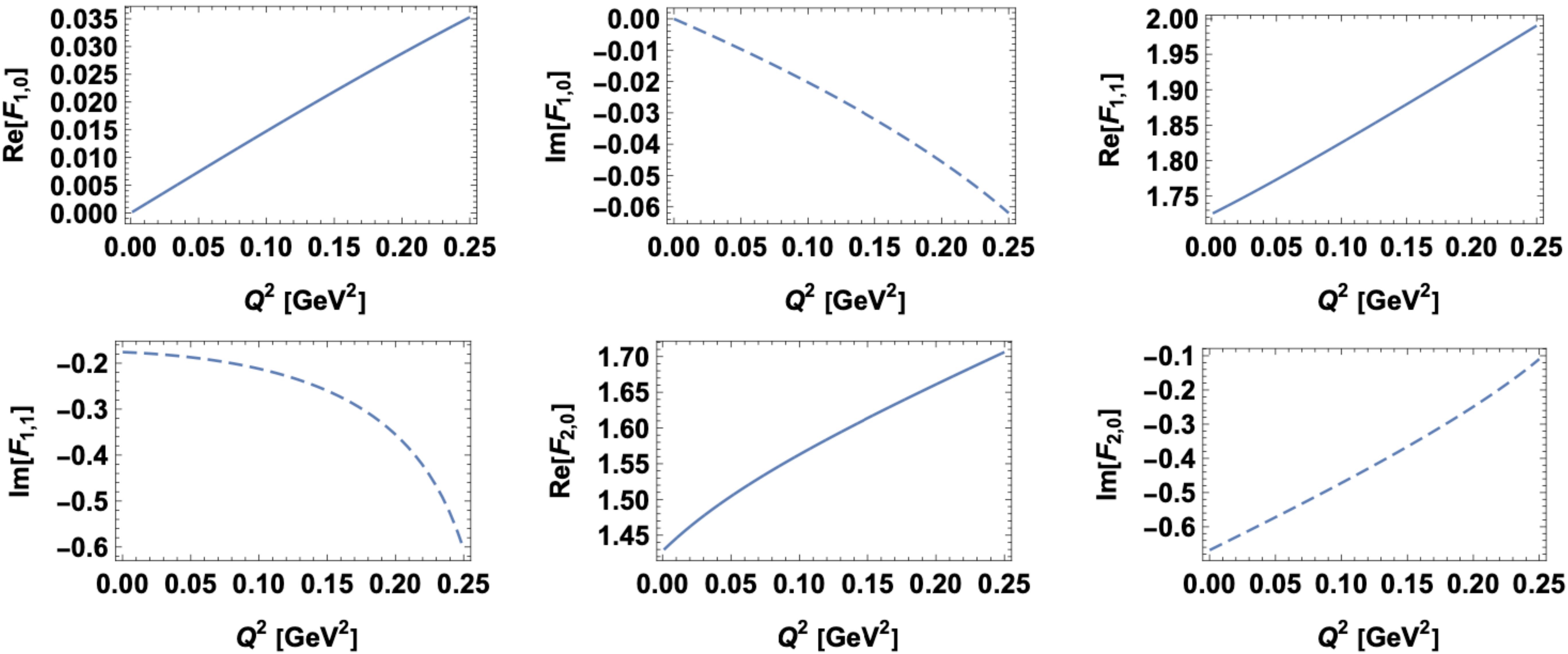

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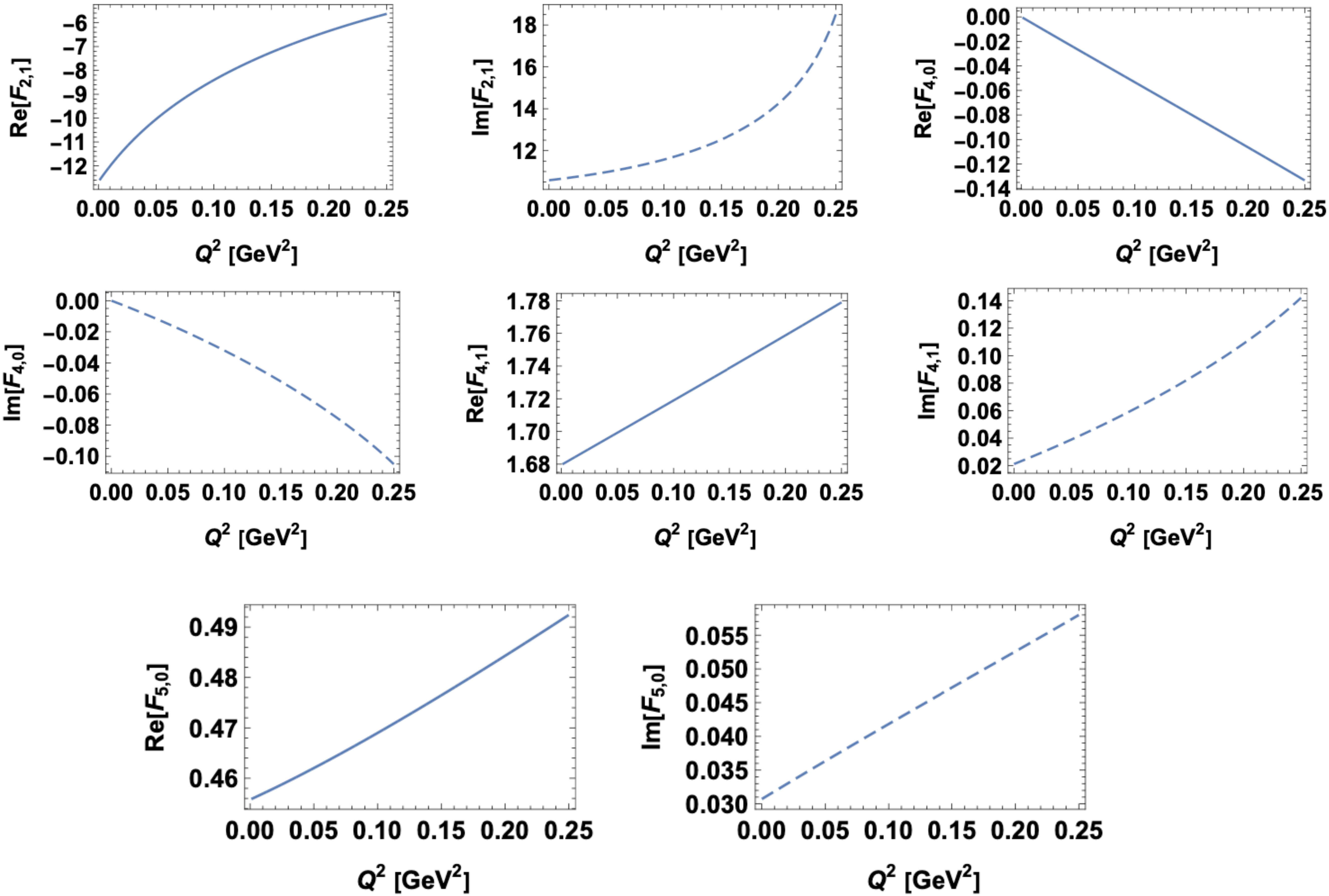

Novel spatial densities

[Zero average momentum frame (ZAMF)]

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We consider $T^{\mu\nu}_{\phi}(\mathbf{r}) = \langle \Phi, \mathbf{X}, s' \,|\, \hat{T}^{\mu\nu}(\mathbf{r},0) \,|\, \Phi, \mathbf{X}, s \rangle$, where $|\, \Phi, \mathbf{X}, s \rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\, \phi(s, \mathbf{p})\, e^{-i\mathbf{p}\cdot\mathbf{X}} \,|\, p, s \rangle$

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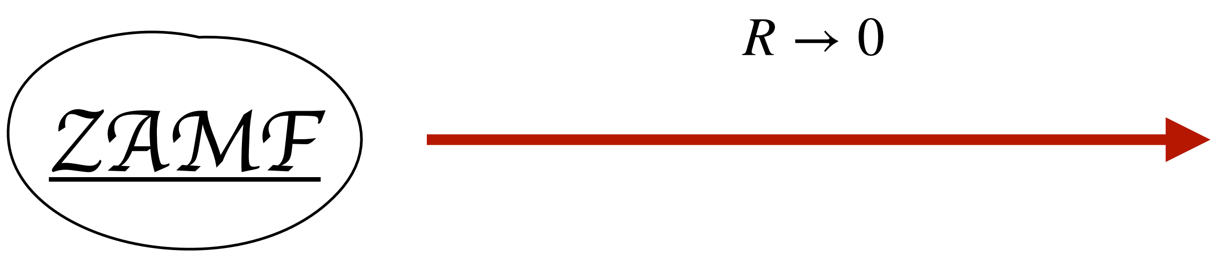

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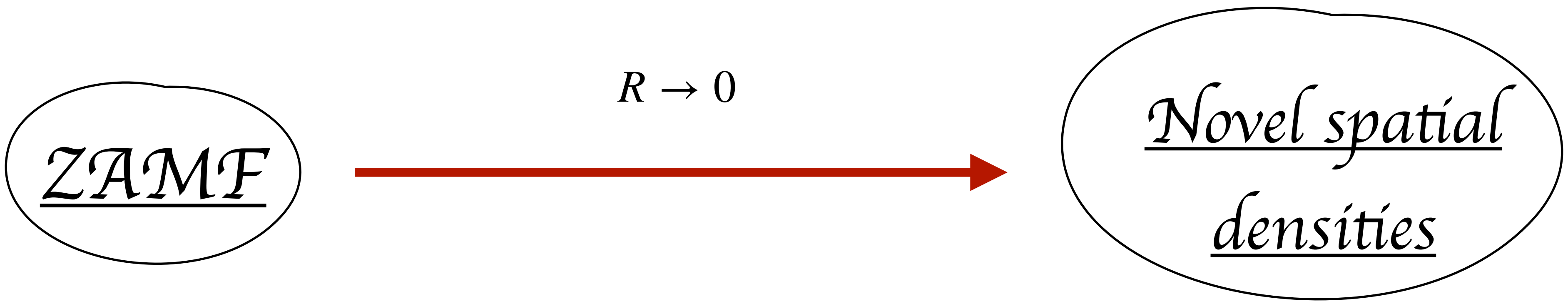

Gravitational structure of the nucleon

Large distance asymptotics of the energy distribution in ZAMF

Large distance asymptotics of the pressure and shear forces distributions in ZAMF

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From the non-analytic contributions of ChPT results in the chiral limit we can obtain the long range ($1/\Lambda_{\text{strong}} \ll r \ll 1/M_\pi$) behavior of the distributions

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$$T_\phi^{00}(s', s, r) = \rho\,(r)\,\delta_{s's}$$

J. Y. Panteleeva, et al., Definition of gravitational local spatial densities for spin-0 and spin-1/2 systems [arXiv:2211.09596 [hep-ph]]

$$\rho(r) = N_{\phi,\infty}\left(\frac{27g_A^2}{512F^2 m_N}\frac{1}{r^6} + \frac{2(c_2 m_N - 10g_A^2)}{5\pi^2 F^2 m_N^2}\frac{1}{r^7}\right)$$

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$$N_{\phi,\infty} = \frac{1}{R}\int d\tilde{P}\tilde{P}^3\,|\,\tilde{\phi}(|\,\tilde{\mathbf{P}}\,|)\,|^2$$

$$N_{\phi,0} = \frac{R}{2}\int d\tilde{P}\tilde{P}\,|\,\tilde{\phi}(|\,\tilde{\mathbf{P}}\,|)\,|^2$$

Only overall normalization of densities depends on the wave packet.

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Monopole energy distribution:

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Monopole spin distribution:

Gravitational structure of Δ resonances

From the non-analytic contributions of ChPT results in the chiral limit we can obtain the long range ($1/\Lambda_{\text{strong}} \ll r \ll 1/M_\pi$) behavior of the distributions

Large distance asymptotics of the energy distribution in ZAMF

$$T^{00}_\phi(s', s, r) = N_{\phi,\infty} \left\{ \rho_0^E(r)\, \delta_{s's} + \rho_2^E(r)\, Y_2^{kl}\left(\Omega_r\right) \hat{Q}^{kl}_{s's} \right\}$$

Monopole energy distribution: $\rho_0^E(r) = \frac{25 g_1^2}{1536 F^2 m_\Delta} \frac{1}{r^6} - \frac{10 g_1^2}{81 \pi^2 F^2 m_\Delta^2} \frac{1}{r^7}$ $\longrightarrow$ $\rho_0^E(r) > 0$

Large distance asymptotics of the angular momentum distribution in ZAMF

$$T^{0i}_\phi(s', s, r) = N_{\phi,\infty} \left[\epsilon^{ikn} \hat{S}^k_{s's} Y_1^n \frac{1}{r} \rho_1^J(r) \; + \; \epsilon^{ikn} \hat{O}^{ktz}_{s's} Y_3^{ntz} \frac{1}{r} \rho_3^J(r) \right]$$

$$J^i(r, s', s) = \epsilon^{ijk} r^j t^{0k}_\phi(s', s, r) = N_{\phi,R} \left\{ \left(\frac{2}{3} \delta^{il} Y_0 - Y_2^{il} \right) \rho_1^J(r)\, \hat{S}^l_{s's} + \left[-Y_4^{iltz} + \frac{2}{35} \left(8 \delta^{il} Y_2^{tz} + \delta^{it} Y_2^{lz} + \delta^{iz} Y_2^{lt} \right) \right] \rho_3^J(r)\, \hat{O}^{ltz}_{s's} \right\}$$

Monopole spin distribution: $\rho_1^J(r) = \frac{5 g_1^2}{162 \pi^2 F^2 m_\Delta} \frac{1}{r^5} - \frac{125 g_1^2}{3072 F^2 m_\Delta^2} \frac{1}{r^6}$

Large distance asymptotics of the pressure and shear forces distributions in ZAMF

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$\rightarrow$

$$\frac{2}{3}s(r) + p(r) > 0$$

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↓

I.A. Perevalova, M. V. Polyakov, and P. Schweitzer. [Phys. Rev. D 94, 054024.]

Summary

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We generalized the effective chiral Lagrangian of nucleons, pions and Δ resonances to curved spacetime up to second chiral order and calculated the corresponding GFFs

We applied the ZAMF approach and obtained the long range behavior of the local spatial densities of the nucleons and delta resonances