

Transverse-momentum dependent distribution functions for a pion with Minkowskian dynamics



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Overview

The *elective* space where QCD is investigated is the Euclidean one, not the physical space, since the indefinite metric of the Minkowski space generates many problems, which could be avoided by replacing the time t by it or the energy E by iE .

The complete equivalence of the quantum field theories played in the two spaces is rooted in the Osterwalder and Schrader theorems, stating necessary and sufficient conditions to be fulfilled by correlation functions in 4D Euclidean space (Schwinger functions) for univocally defining the Wightman correlation functions in Minkowski spacetime.

Shortly, the O-S theorems show under which conditions the Wick rotation is a well defined isomorphism relating quantum field theories in Euclidean space and in Minkowski one.

The primary tool for investigating QCD in Euclidean space is the lattice (IQCD), but relevant advancements have been achieved with a continuum approach (cQCD), also through a complexification of the Euclidean space.

BUT the physical observations are obtained in Minkowski space, and therefore, to improve our confidence in the approximations applied in the Euclidean space, it could be helpful to attempt to replicate in Minkowski space a program analog to the cQCD, already played in the Euclidean space: i.e. combining BSE and a truncated tower of DSEs.

⇒ Hadron Tomography is challenging from both experimental and theoretical side !

The Nakanishi-Integral-Representation way

⇒ The approach we are pursuing is based on in-depth analysis of the N-leg amplitudes carried out in the 60' by Noburo Nakanishi, within the Feynman-diagrams framework.

Nakanishi formally obtained a compact and elegant expression of the full N-leg amplitude, written by means of the Feynman parametrization,

$$f_N(\tilde{s}) = \sum_{\mathcal{G}} f_{\mathcal{G}}(\tilde{s}) \propto \prod_h \int_0^1 dz_h \int_0^\infty d\gamma \frac{\delta(1 - \sum_h z_h) \phi_N(\vec{z}, \gamma)}{(\gamma - \sum_h z_h s_h)}$$

$\mathcal{G} \equiv$ infinite set of graphs contributing to N-leg amplitude

$\tilde{s} \equiv \{s_1, s_2, \dots, s_N\} \Rightarrow N$ independent scalar products, from external momenta

$\phi_N(\vec{z}, \gamma) \equiv$ Nakanishi weight function. It is a combination of distributions, $\delta(\dots)$ and $\Theta(\dots)$ (N.B.: γ is non compact, while \vec{z} is compact).

One can get rid of the perturbation stigma by using unknown real weight functions (with the dependence upon the same variables), on place of distributions.

In spite of its apparent simplicity, to determine the Nakanishi weight functions, $g_i(z, \gamma)$, becomes highly non trivial, when the fermionic dof are taken into account in the BSE.

To anticipate: all the scalar functions entering the relevant vertex function describing a bound-system and dependent upon the four-momenta at disposal, are expressed through the Nakanishi Integral Representation.

In the Pion, our bed-test, we will have

$$\phi_i(k; P) = \int_{-1}^1 dz' \int_0^\infty d\gamma' \frac{g_i(\gamma', z'; \kappa^2)}{[k^2 + z'(P \cdot k) - \gamma' - \kappa^2 + i\epsilon]^3}$$

$P \equiv$ pion 4-momentum ($P^2 = M_\pi^2$)

$k \equiv$ relative 4-momentum

$\kappa^2 \equiv m^2 - M_\pi^2/4$ and m is a fermionic effective mass (to be defined in what follows...)

N.B. The power of the denominator depends on the smoothness we need to implement. This freedom is already in the original Nakanishi analysis: the trade-off was between derivatives of the distributions in the numerator and the power in the denominator.

Pion as a quark-antiquark bound-system

The Bethe-Salpeter Equation for a 0^- system reads

$$\begin{aligned}\Phi(k; P) &= S(k + \frac{P}{2}) \int \frac{d^4 k'}{(2\pi)^4} S^{\mu\nu}(q) \Gamma_\mu(q) \Phi(k'; P) \hat{\Gamma}_\nu(q) S(k - \frac{P}{2}) \\ \hat{\Gamma}_\nu(q) &= C \Gamma_\nu(q) C^{-1}\end{aligned}$$

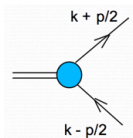
where we use (for now): i) bare propagators for quarks and gluons; ii) ladder approximation with massive gluons, iii) an extended quark-gluon vertex

$$S(P) = \frac{i}{\not{P} - m + i\epsilon}, \quad S^{\mu\nu}(q) = -i \frac{g^{\mu\nu}}{q^2 - \mu^2 + i\epsilon}, \quad \Gamma^\mu = ig \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon} \gamma^\mu,$$

We set the value of the scale parameter (300 MeV) from the combined analysis of Lattice simulations, the Quark-Gap Equation and Slanov-Taylor identity.
[Oliveira, WP, Frederico, de Melo EPJC 78(7), 553 (2018) & EPJC 79 (2019) 116 & EPJC 80 (2020) 484]

NIR for fermion-antifermion 0^- Bound State

BS amplitude for a quark-antiquark 0^- bound state:



$$\Phi(k; P) = \sum_{i=1}^4 S_i(k; P) \phi_i(k; P)$$

Dirac structures for a pseudoscalar system is given by

$$S_1(k; P) = \gamma_5, \quad S_2(k; P) = \frac{\not{P}}{M} \gamma_5, \quad S_3(k; P) = \frac{k \cdot P}{M^3} \not{P} \gamma_5 - \frac{\not{k}}{M} \gamma_5, \quad S_4(k; P) = \frac{i}{M^2} \sigma^{\mu\nu} P_\mu k_\nu \gamma_5$$

Using the NIR for each scalar functions ϕ_i and the light-front projection (i.e. the integration over $k^- = k^0 - k^3$, that allows to project the dynamics onto the hyperplane $x^+ = 0$)

BSE \Rightarrow System of coupled integral equations

The coupled integral-equation system (see also NIR+covariant LF, Carbonell and Karmanov JPA 2010) in ladder approximation, reads (cf. de Paula, et al, PRD **94**, 071901 (2016) & EPJC **77**, 764 (2017))

$$\int_0^\infty \frac{d\gamma' g_i(\gamma', z; \kappa^2)}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2) \kappa^2]^2} = i M g^2 \sum_j \int_0^\infty d\gamma' \int_{-1}^1 dz' \mathcal{L}_{ij}(\gamma, z; \gamma', z') g_j(\gamma', z'; \kappa^2)$$

In ladder approximation, the Nakanishi Kernel, \mathcal{L}_{ij} , has an analytical expression and contains **singular contributions** that can be regularized 'a la Yan (Chang and Yan, Quantum field theories in the infinite momentum frame. II. PRD **7**, 1147 (1973)).

Numerical solutions are obtained by discretizing the system using **a polynomial basis**, given by the Cartesian product of Laguerre(γ) \times Gegenbauer(z). One remains with a Generalized eigenvalue problem, where a non-symmetric matrix and a symmetric one are present

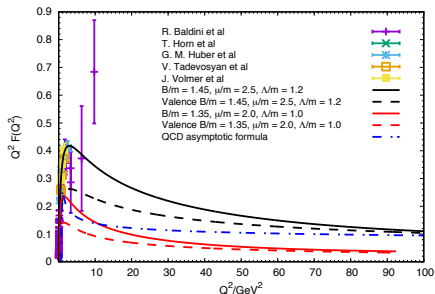
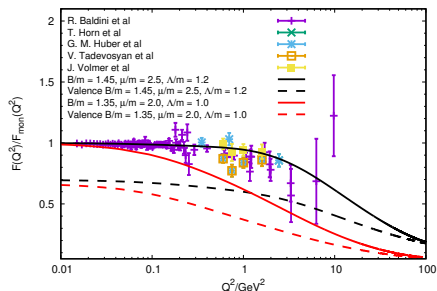
$$A \vec{c} = \lambda B \vec{c}$$

N.B. the eigenvector \vec{c} contains the coefficients of the expansion of the Nakanishi weight functions $g_i(z, \gamma; \kappa^2)$.

If one gets **real solutions of the GEVP**, then one can **validate** the NIR approach.

Pion em form factor in ladder approximation

From E. Ydrefors et al., PLB **820** (2021) 136494.



Black solid curve: pion FF, obtained from the solution of the BSE in ladder approximation, with $m_q = 255$ MeV, $m_g = 637.5$ MeV and $\Lambda = 306$ MeV, which controls the extended quark-gluon vertex (all in the range of LQCD calculations). Then the exp. decay constant $f_\pi^{PDG} = 130.50(1)(3)(13)$ MeV is reproduced.

Dashed line: LF-valence contribution (LF-valence probability = 0.70, self-consistently obtained)

Right Panel: Dash-dotted line; asymptotic expression from Brodsky-Lepage PRD 22 (1980): $Q^2 F_{asy}(Q^2) = 8\pi\alpha_s(Q^2)f_\pi^2$.

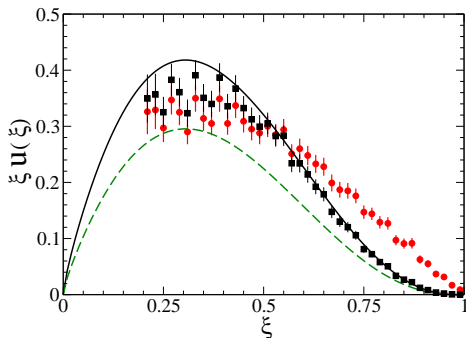
Parton distribution function

W. de Paula et al., PRD **105**, L071505 (2022).

From the charge-symmetric (anti-symmetric) expression of the leading-twist TMD $f_1^{S(AS)}(\gamma, \xi)$, one gets the PDF at the initial scale $u(\xi)$

$$f_1^{S(AS)}(\gamma, \xi) = \frac{f_1^q(\gamma, \xi) \pm f_1^{\bar{q}}(\gamma, 1 - \xi)}{2} \Rightarrow u(\xi) = \int_0^\infty d\gamma f_1^S(\gamma, \xi).$$

Comparison with the data at 5.2 GeV scale



Solid line: full calculation of the BSE evolved from the initial scale $Q_0 = 0.360$ GeV to $Q = 5.2$ GeV

Full dots: experimental data from E615

Full squares: reanalyzed experimental data from Aicher et al PRL 105, 252003 (2010). evolved to $Q = 5.2$ GeV

Dashed line: The evolved LF valence contribution

Parton distribution function II

Low order Mellin moments at scales $Q = 2.0$ GeV and $Q = 5.2$ GeV.

| | BSE ₂ | LQCD ₂ | BSE ₅ | LQCD ₅ |
|-----------------------|------------------|-------------------|------------------|-------------------|
| $\langle x \rangle$ | 0.259 | 0.261 ± 0.007 | 0.221 | 0.229 ± 0.008 |
| $\langle x^2 \rangle$ | 0.105 | 0.110 ± 0.014 | 0.082 | 0.087 ± 0.009 |
| $\langle x^3 \rangle$ | 0.052 | 0.024 ± 0.018 | 0.039 | 0.042 ± 0.010 |
| $\langle x^4 \rangle$ | 0.029 | | 0.021 | 0.023 ± 0.009 |
| $\langle x^5 \rangle$ | 0.018 | | 0.012 | 0.014 ± 0.007 |
| $\langle x^6 \rangle$ | 0.012 | | 0.008 | 0.009 ± 0.005 |

LQCD, $Q = 2.0$ GeV: $\langle x \rangle$ - Alexandrou et al PRD 103, 014508 (2021)
 $\langle x^2 \rangle$ and $\langle x^3 \rangle$ - Alexandrou et al PRD 104, 054504 (2021)

LQCD, $Q = 5.0$ GeV: $\langle x \rangle$ - Alexandrou et al PRD 103, 014508 (2021)

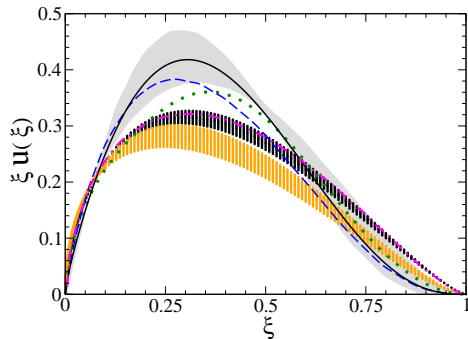
N.B. following Cui et al EPJC **80**, 1064 (2020), we used for the evolution: lowest order DGLAP equation + an effective strong charge. . One needs:

the hadronic scale $Q_0 = 0.360$ GeV to fit the first Mellin moment.

not too far from the Cui et al estimate $Q_0 = 0.330 \pm 0.030$ GeV

Parton distribution function III

Comparison with other theoretical calculations at 5.2 GeV scale



Solid line: full calculation of the BSE evolved from the initial scale $Q_0 = 0.360$ GeV to $Q = 5.2$ GeV

Dashed line: DSE calculation from Cui et al, Eur. Phys. J. A 58, 10 (2022)

Dash-dotted line: DSE calculation with dressed quark-photon vertex from Bednar et al PRL 124, 042002 (2020)

Dotted line: BLFQ collaboration, PLB 825, 136890 (2022)

Gray area: LQCD results from C. Alexandrou et al (2021)

Black and Orange vertical lines from JAM collaboration, private communication.

For the evolved $\xi u(\xi)$, the exponent of $(1 - \xi)^{\eta_5}$ is $\eta_5 = 2.94$, when $\xi \rightarrow 1$,

LQCD: Alexandrou et al PRD 104, 054504 (2021) obtained 2.20 ± 0.64

Cui et al EPJA 58, 10 (2022) obtained 2.81 ± 0.08

Pion Transverse Momentum-Dependent Distributions

One can define the T-even subleading quark uTMDs, starting from the decomposition of the pion correlator [Mulders and Tangerman, Nucl. Phys. B 461, 197 (1996)].

twist -2 uTMD:

$$f_1^q(\gamma, \xi) = \frac{N_c}{4} \int d\phi_{\mathbf{k}_\perp} \int_{-\infty}^{\infty} \frac{dy^- d\mathbf{y}_\perp}{2(2\pi)^3} e^{i[\tilde{\mathbf{k}} \cdot \tilde{\mathbf{y}}]} \langle P | \bar{\psi}_q(-\frac{y}{2}) \hat{1} \psi_q(\frac{y}{2}) | P \rangle \Big|_{y^+=0}$$

twist-3 uTMD (in LC gauge, $A^+ = 0$):

$$\frac{M}{P^+} e^q(\gamma, \xi) = \frac{N_c}{4} \int d\phi_{\mathbf{k}_\perp} \int_{-\infty}^{\infty} \frac{dy^- d\mathbf{y}_\perp}{2(2\pi)^3} e^{i[\tilde{\mathbf{k}} \cdot \tilde{\mathbf{y}}]} \langle P | \bar{\psi}_q(-\frac{y}{2}) \gamma^+ \psi_q(\frac{y}{2}) | P \rangle \Big|_{y^+=0}$$

and

$$\frac{M}{P^+} f^{\perp q}(\gamma, \xi) = \frac{N_c M}{4|\mathbf{k}_\perp|^2} \int d\phi_{\mathbf{k}_\perp} \int_{-\infty}^{\infty} \frac{dy^- d\mathbf{y}_\perp}{2(2\pi)^3} e^{i[\tilde{\mathbf{k}} \cdot \tilde{\mathbf{y}}]} \langle P | \bar{\psi}_q(-\frac{y}{2}) \mathbf{k}_\perp \cdot \boldsymbol{\gamma}_\perp \psi_q(\frac{y}{2}) | P \rangle \Big|_{y^+=0}$$

with $\tilde{\mathbf{k}} \cdot \tilde{\mathbf{y}} = \xi P^+ y^- / 2 - \mathbf{k}_\perp \cdot \mathbf{y}_\perp$.

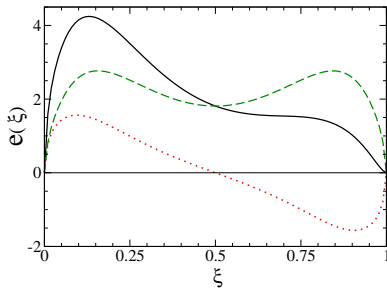
The corresponding symmetric and antisymmetric subleading collinear PDFs are:

$$e^{S(AS)}(\xi) = \int_0^\infty d\gamma e^{S(AS)}(\gamma, \xi), \quad f^{\perp S(AS)}(\xi) = \int_0^\infty d\gamma f^{\perp S(AS)}(\gamma, \xi)$$

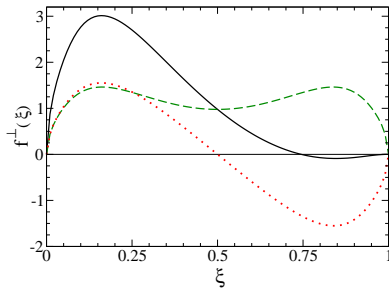
For the quark ones: $e^q(\xi) = e^S(\xi) + e^{AS}(\xi)$ and $f^{\perp q}(\xi) = f^{\perp S}(\xi) + f^{\perp AS}(\xi)$

Transverse Momentum-Dependent Distributions II

Subleading Collinear PDFs



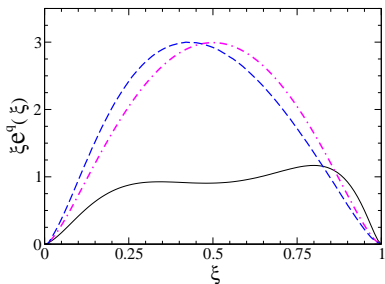
Solid line: quark twist-3 uTMD $e(\xi)$
Dashed line: Symmetric twist-3 uTMD $e^S(\xi)$
Dotted: Anti-symmetric twist-3 uTMD $e^{AS}(\xi)$



Solid line: quark twist-3 uTMD $f^\perp(\xi)$
Dashed line: Symmetric twist-3 uTMD $f^{\perp S}(\xi)$
Dotted: Anti-symmetric twist-3 uTMD $f^{\perp AS}(\xi)$

Transverse Momentum-Dependent Distributions II

Comparison with a LF Constituent Quark Model

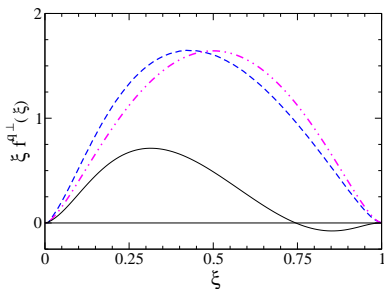


Quark unpolarized collinear PDF, $\xi e^q(\xi)$.

Solid line: full calculation.

Dashed line: $m/M u^q(\xi)$, as suggested by Lorcé et al, EPJC 76, 415 (2016), but with our PDF.

Double-dot-dashed line: the same as the dashed line, but using the valence approximation of $u^q(\xi)$ with norm = 1 and not 0.7.

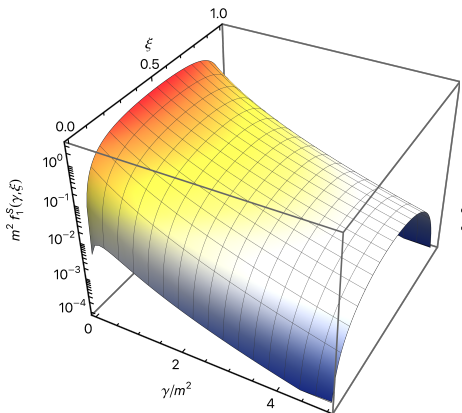


Quark unpolarized collinear PDF, $\xi f^{q\perp}(\xi)$.

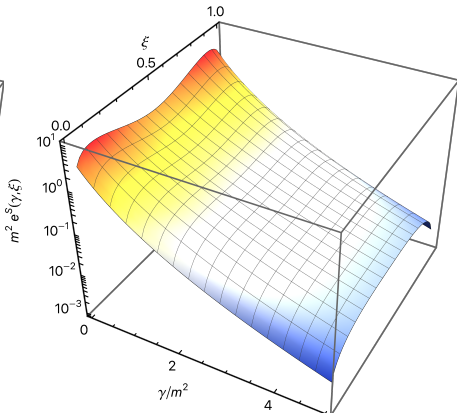
Solid line: full calculation.

Dashed line: $\xi f^{q\perp}(\xi) = u^q(\xi)$, as suggested by Lorcé et al, EPJC 76, 415 (2016), but with our PDF.

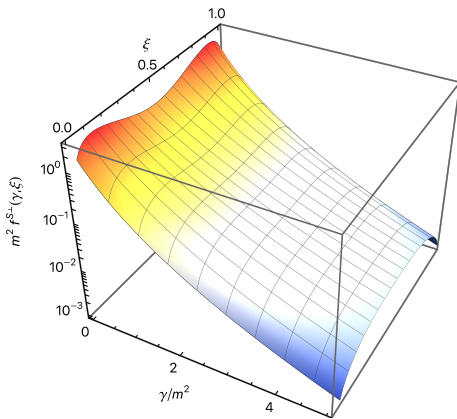
Double-dot-dashed line: the same as the dashed line, but using the valence approximation of $u^q(\xi)$ with norm=1 and not 0.7



Twist-2 uTMD $f_1^S(\gamma, \xi)$



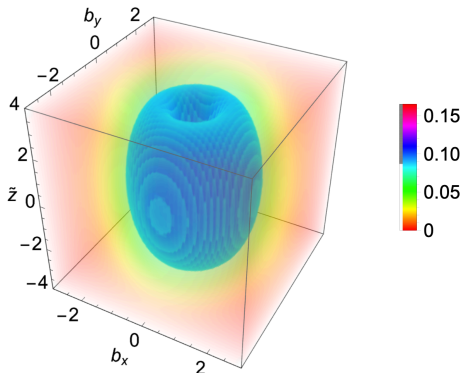
Twist-3 uTMD $e^S(\gamma, \xi)$



Twist-3 uTMD $f^{\perp S}(\gamma, \xi)$

A 3D iconic view of the Pion

Density plot of $|\mathbf{b}_\perp|^2 |\psi(\tilde{z}, b_x, b_y)|^2$, with $\psi(\tilde{z}, b_x, b_y)$ obtained from our solutions of the ladder Bethe-Salpeter equation [W. de Paula et al PRD 103, (2021) 014002]

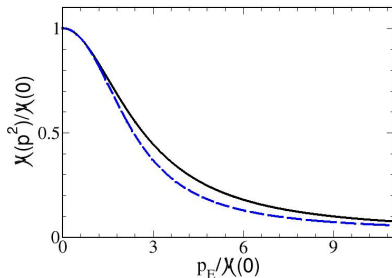


$$\tilde{z} \equiv \text{loffe-time} = \tilde{z} = x \cdot P_{\text{target}} \Rightarrow \tilde{z} = x^- P_{\text{target}}^+ / 2 \text{ on the hyperplane } x^+ = 0$$
$$\{b_x, b_y\} \equiv \text{transverse coordinates}$$

What next?

After completing the investigation of the pion BSE with fixed-mass quark, i.e. a $q\bar{q}$ bound system, we are addressing the running-mass case, pointing to the other face of the medal \Rightarrow Dynamically Chiral-Symmetry Breaking. First phenomenologically

Wave-function renorm. constant $Z(p^2) = 1$ and a running-mass, $\mathcal{M}(p_E^2) = m_0 - m^3/(p_E^2 - \lambda^2)$, with $m_0 = 0.008$ GeV, $m = 0.648$ GeV and $\lambda = 0.9$ GeV adjusted to LQCD calculations by O. Oliveira, et al, PRD **99** (2019) 094506. First results in A. Castro et al, arXiv:2305.12536



The quark running-mass, $\mathcal{M}(p^2)$, as a function of the Euclidean momentum $p_E = \sqrt{-p^2}$, in units of the IR mass $\mathcal{M}(0) = 0.344$ GeV. Solid line: our model. Dashed line: accurate fit of the LQCD calculations

Then, formal and numerical results of the fermion gap-equation in Minkowski space will be exploited, following, e.g., i) D. Duarte, et al PRD **105**, 114055 (2022), and ii) C. Mezrag and GS, EPJC **81** (2021) 34.

Summary

- The near future will offer an innovative view of the dynamics inside the hadrons, thanks to the experimental activity planned at the Electron-ion colliders, and plenty of measurements pointing to the 3D tomography of hadrons will become available.
- For the pion, many results, em form factor, PDF, TMDs, loffe-time \times transverse plane distribution, have been obtained by using the ladder-approximation of the $q\bar{q}$ -BSE.
- The pion has an important role, given its dual nature: $q\bar{q}$ bound-system and Goldstone boson, i.e. the golden gate to address the DCSB and the emergent-mass phenomena. Our aim is to implement a framework analogous to the one already developed in Euclidean space.
- Phenomenological investigations in Minkowski-space, once the approach composed by BSE and gap-equations will be fully available, could offer fresh insights in hadron dynamics and possibly implement an interplay with well-established lattice and continuous QCD communities.

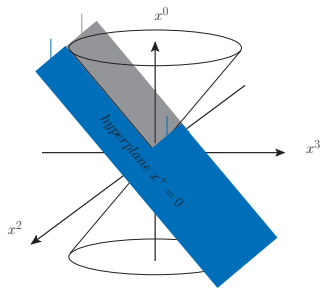
Thank you for the attention!!!

Back-up slides

An iconic view of the pion from the light-cone

W. de Paula, et al, PRD 103, 014002 (2021) The probability distribution of the quarks inside the pion, sitting on the the hyperplane $x^+ = 0$, tangent to the light-cone, is evaluated in the space given by the Cartesian product of the *loffe-time* and the plane spanned by the transverse coordinates \mathbf{b}_\perp .

Why? In addition to the usual the infinite-momentum frame one can study the deep-inelastic scattering processes in the target frame, adopting the configuration space, so that a more detailed investigation of the space-time structure of the hadrons can be performed. The *loffe-time* is useful for studying the relative importance of short and long light-like distances.



The covariant definition of the loffe-time is $\tilde{z} = x \cdot P_{target}$, and it becomes $\tilde{z} = x^- P_{target}^+ / 2$ on the hyperplane $x^+ = 0$

Fermion-scalar bound-system in the chiral limit

Aline Noronha et al, PRD **107**, 096019 (2023)

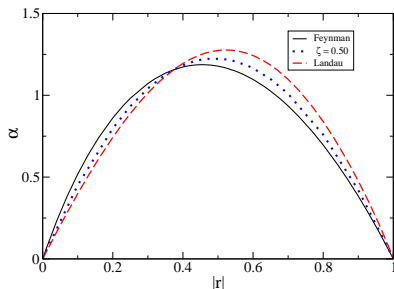
The homogeneous BSE of a $(1/2)^+$ bound-system, with both fermionic and bosonic degrees of freedom (dubbed a mock nucleon), is studied in Minkowski space, in order to analyze the chiral limit in covariant gauges.

The chiral limit induces a scale invariance of the model and consequently generates a wealth of striking features:

- it reduces the number of nontrivial Nakanishi weight functions, from two to only one;
- the form of the surviving weight function has a factorized dependence on the two relevant variables, compact and non-compact;
- the coupling constant becomes an explicit function of the real exponent governing the power-law falloff of (the nontrivial Nakanishi weight function, and in turn) the LF valence wave function.

Fermion-scalar bound system in the chiral limit

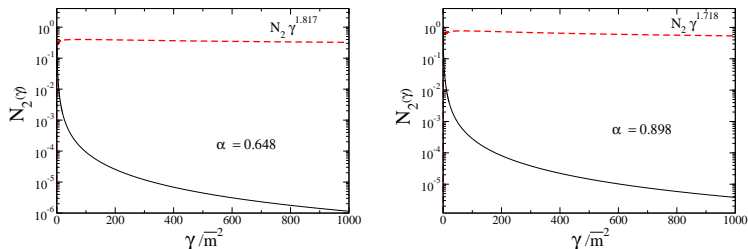
Aline Noronha et al, PRD **107**, 096019 (2023)



Coupling constant of the fermion-boson system in the chiral limit vs. the power r ($g_i(\gamma, z) = \gamma^r f_{i,r}(z)$). Solid line: Feynman gauge ($\zeta = 1$). Dotted line: $\zeta = 0.5$ gauge. Dashed line: Landau gauge ($\zeta = 0$).

Fermion-scalar bound system in the chiral limit

Aline Noronha et al, PRD **107**, 096019 (2023)



The normalized LF amplitude $N_2(\gamma) = \psi_2(\gamma, \xi_0 = 0.5)/\psi_2(0, \xi_0 = 0.5)$ as a function of the transverse momentum square $\gamma = |\vec{k}_\perp|^2$ in the Feynman gauge, with $m_s/m_f = 2$ and binding energy ratio $B/\bar{m} = 0.1$ (i.e. $M/\bar{m} = 1.9$). Left panel: $\mu/\bar{m} = 0.15$ and $\alpha = 0.648$. Solid line: $N_2(\gamma)$. Dashed line: $N_2(\gamma) \times \gamma^{1.817}$. Right panel: $\mu/\bar{m} = 0.5$ and $\alpha = 0.898$. Solid line: $N_2(\gamma)$. Dashed line: $N_2(\gamma) \times \gamma^{1.718}$.