The D_s^+ decay into $\pi^+ K_S^0 K_S^0$ reaction and the $I = 1$ partner of the f_0(1710) state

L. R. Dai([1])(*), E. Oset ([2]) and L. S. Geng([3])

([1]) *School of science, Huzhou University, Huzhou, 313000, Zhejiang, China*
([2]) *Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC Institutos de Investigación de Paterna, Aptdo.22085, 46071 Valencia, Spain*
([3]) *School of Physics, Beihang University, Beijing, 102206, China*

Summary. — Two identified decay modes of the $D_s^+ \to \pi^+ K^{*+} K^{*-}, \pi^+ K^{*0} \bar{K}^{*0}$ reactions producing a pion and two vector mesons are discussed in this talk. The posterior vector-vector interaction generates two resonances that we associate to the $f_0(1710)$ and the $a_0(1710)$ recently claimed, and they decay to the observed K^+K^- or $K_S^0 K_S^0$ pair, leading to the reactions $D_s^+ \to \pi^+ K^+ K^-, \pi^+ K_S^0 K_S^0$. The results depend on two parameters related to external and internal emission. We determine a narrow region of the parameters consistent with the large N_c limit within uncertainties which gives rise to decay widths in agreement with experiment. With this scenario we make predictions for the branching ratio of the $a_0(1710)$ contribution to the $D_s^+ \to \pi^0 K^+ K_S^0$ reaction, finding values within the range of $(1.3 \pm 0.4) \times 10^{-3}$. We are happy to see that we have a fair prediction with BESIII experiments, confirming the new $a_0(1710)$ [$a_0(1817)$] resonance. This is an important state and will shed light into the structure of scalar mesons in light quark sector and other relevant issues currently under debate in hadron physics.

1. – Motivation

An isospin $I = 0$, $f_0(1710)$ resonance has been known for quite some time [1]. It was found from the recent BESIII experiments, the branching fraction [2]

$$\mathrm{Br}[D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”};\ \text{“}f_0(1710)\text{”} \to K^+K^-] = (1.0 \pm 0.2 \pm 0.3) \times 10^{-3}\,,$$

and in another work it was found that [3]

$$\mathrm{Br}[D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”};\ \text{“}f_0(1710)\text{”} \to K_S^0 K_S^0] = (3.1 \pm 0.3 \pm 0.1) \times 10^{-3}\,,$$

(*) dailianrong@zjhu.edu.cn

where "$f_0(1710)$" was supposed to be the $f_0(1710)$ resonance. Thus one finds

$$R_1 = \frac{\Gamma(D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”} \to \pi^+ K^0 \bar{K}^0)}{\Gamma(D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”} \to \pi^+ K^+ K^-)} = 6.20 \pm 0.67\,. \tag{1}$$

But, it is easy to proof that if "$f_0(1710)$" was the $f_0(1710)$ resonance, this latter ratio should be 1. Therefore, hidden below or around the $f_0(1710)$, there should be an $I = 1$ resonance responsible for this surprising large ratio. We think a mixture of the two resonances and their interference would be responsible for a different K^+K^- or $K^0\bar{K}^0$ production, due to

$$\begin{aligned} |K\bar{K}, I=0, I_3=0\rangle &= -\frac{1}{\sqrt{2}}\big(K^0\bar{K}^0 + K^+K^-\big)\,, \\ |K\bar{K}, I=1, I_3=0\rangle &= \frac{1}{\sqrt{2}}\big(K^0\bar{K}^0 - K^+K^-\big)\,. \end{aligned} \tag{2}$$

As we know in the chiral unitary approach, $a_0(980)$ is dynamically generated as the interaction of the coupled channels $\pi\eta$ and $K\bar{K}$. Then an extension of these ideas to the interaction of vector mesons was done, and interestingly two resonances of $f_0(1710)$ and $a_0(1710)$ were predicted qualifying roughly as $K^*\bar{K}^*$ molecules [4, 5].

2. – Formalism

Fig. 1 shows the Cabibbo-favored decay mode of D_s^+ at the quark level and the hadronization with the vacuum quantum numbers ($\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s$). Fig. 2 shows the internal emission and hadronization, which is suppressed by a color factor $1/N_c$. The external and internal emissions will produce the $f_0(1710)$ and $a_0(1710)$ resonances [6].

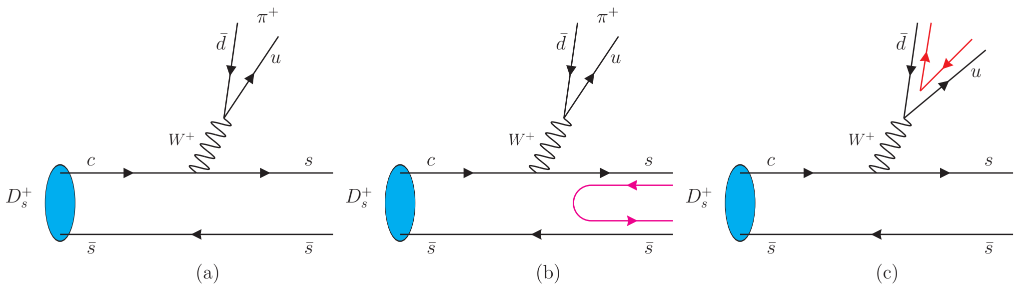

Fig. 1. – External emission of D_s^+ decay with π^+ production at the quark level (a) and hadronization of the $s\bar{s}$ component (b) and the $u\bar{d}$ component (c) with the vacuum quantum numbers.

From Figs. 1 and 2 and because the $G_{\omega\phi}$ and $G_{\rho\phi}$ loop functions are remarkably similar to $G_{K^*\bar{K}^*}$, finally the amplitudes $\tilde{t}_{f_0}$ and $\tilde{t}_{a_0}$ can be written

$$\begin{aligned} \tilde{t}_{f_0} &= A\{-\sqrt{2}\,G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{f_0,K^*\bar{K}^*} + G_{\phi\phi}(M_{\rm inv})\sqrt{2}\,g_{f_0,\phi\phi} \\ &\quad - \sqrt{2}\,\gamma'\,G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{f_0,K^*\bar{K}^*}\}\,, \\ \tilde{t}_{a_0} &= -A\sqrt{2}\,\delta'\,G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{a_0,K^*\bar{K}^*}\,. \end{aligned} \tag{3}$$

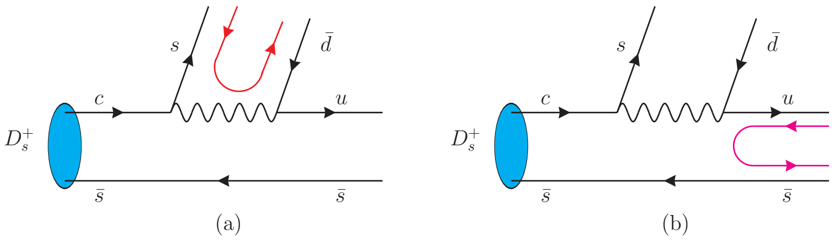

Fig. 2. – Internal emission of D_s^+ decay and hadronization of the $s\bar{d}$ pair (a) and the $u\bar{s}$ pair (b).

with the two effective parameters

$$\gamma' = \gamma - \alpha \, \frac{g_{f_0,\omega\phi}}{g_{f_0,K^*\bar{K}^*}} \,, \qquad \delta' = \delta - \beta \frac{g_{f_0,\rho\phi}}{g_{a_0,K^*\bar{K}^*}} \,.$$

and the global factor A will disappear when we evaluate the ratios of production. We also

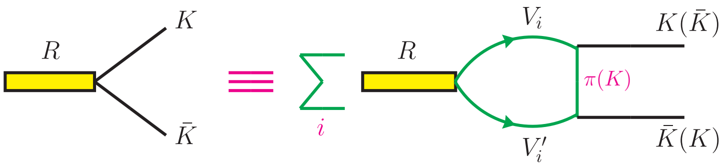

Fig. 3. – Amplitude for $R \to K\bar{K}$ for a resonance build up from the V_i, V_i' channels. Diagrams with $\bar{K}K$ instead of $K\bar{K}$ in the final state appear with ρ, ω, ϕ vector mesons but not for the $V_i, V_i' \equiv K^*\bar{K}^*$.

need amplitudes of the two resonances decay into $K\bar{K}$, shown in Fig. 3, with $K^*\bar{K}^* \to K\bar{K}$ transitions driven by π exchange, and $\phi(\rho,\omega,\phi) \to K\bar{K}$ transitions driven by K exchange, the weights are given by

$$(4) \qquad W_{f_0} = \sum_i g_{f_0,i} \, \widetilde{W}_i \, G_i(M_{\rm inv}) \,, \quad W_{a_0} = \sum_i g_{a_0,i} \, \widetilde{W}_i \, G_i(M_{\rm inv}) \,.$$

where $g_{f_0,i}$ and $g_{a_0,i}$ are the couplings of the $f_0(1710)$ and $a_0(1710)$ resonances to the different coupled channels that build up the resonance, the $\widetilde{W}_i$ coefficients can be evaluated from Lagrangian for $V \to PP$, and the sum over i goes over the channels of $I = 0$ and $I = 1$ respectively. Finally we obtain the t_i amplitudes in the following

$$\begin{aligned}
t_{K^+K^-} &= -\tilde{t}_{f_0} \frac{1}{M_{\rm inv}^2 - M_{f_0}^2 + iM_{f_0}\Gamma_{f_0}} W_{f_0} \frac{1}{\sqrt{2}} g_{K\bar{K}} - \tilde{t}_{a_0} \frac{1}{M_{\rm inv}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} W_{a_0} \frac{1}{\sqrt{2}} g_{K\bar{K}} \,, \\
t_{K^0\bar{K}^0} &= -\tilde{t}_{f_0} \frac{1}{M_{\rm inv}^2 - M_{f_0}^2 + iM_{f_0}\Gamma_{f_0}} W_{f_0} \frac{1}{\sqrt{2}} g_{K\bar{K}} + \tilde{t}_{a_0} \frac{1}{M_{\rm inv}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} W_{a_0} \frac{1}{\sqrt{2}} g_{K\bar{K}} \,, \\
t_{K^+\bar{K}^0} &= \tilde{t}_{a_0} \frac{1}{M_{\rm inv}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} W_{a_0} g_{K\bar{K}} \,, \\
t_{K^+K_S^0} &= -\frac{1}{\sqrt{2}} t_{K^+\bar{K}^0} \,.
\end{aligned}$$

The differential decay width

$$\frac{d\Gamma_i}{dM_{\rm inv}(K\bar{K})} = \frac{1}{(2\pi)^3}\,\frac{1}{4M_{D_s}^2}\,p_\pi\,\tilde{p}_k\,|t_i|^2\,. \tag{5}$$

The ratios are defined

$$R_1 = \frac{\Gamma(D_s^+ \to \pi^+ K^0 \bar{K}^0)}{\Gamma(D_s^+ \to \pi^+ K^+ K^-)}\,, \quad R_2 = \frac{\Gamma(D_s^+ \to \pi^0 K^+ K_S^0)}{\Gamma(D_s^+ \to \pi^+ K^+ K^-)}\,. \tag{6}$$

3. – Results

For the two effective parameters, a narrow region of the parameters $\gamma' \in [-1, 0.1]$, $\delta' \in [-1.3, 1.3]$, are obtained and shown in Fig. 4, which are consistent with the large N_c limit within uncertainties.

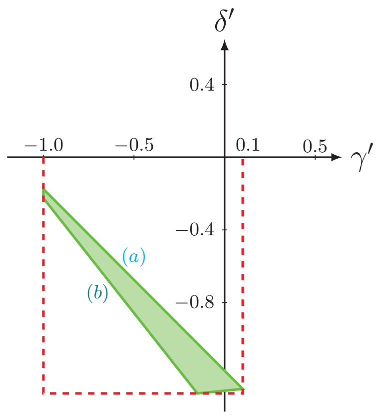

Fig. 4. – The range of two effective parameters

Using the above parameters, we obtain the ratio of $R_1 = 6.20 \pm 0.67$, which is in good agreement with BESIII experimental data [2, 3].

Next, the big challenge of the approach is to make prediction of R_2. In [6], $R_2^{\rm theo} \simeq 1.31 \pm 0.12$ was obtained and from this ratio we have evaluated

$${\rm Br}[D_s^+ \to \pi^0 a_0(1710)^+; a_0(1710)^+ \to K^+ K_S^0] \simeq (1.3 \pm 0.4) \times 10^{-3}\,, \tag{7}$$

which was a prediction before this ratio was measured.

We make a further analysis by taking $\gamma' = -0.5$, $\delta' = -0.75$ (middle of the allowed region) in Fig. 5, finding that in the $K^0\bar{K}^0$ mass distribution there has been a constructive interference from the two resonances of $I = 0$ and $I = 1$, while in the K^+K^- mass distribution the interference has been destructive. This is exactly the reason suggested in the experimental analysis to justify the existence of the $a_0(1710)$ resonance [2, 3], because it should give the same K^+K^- or $K^0\bar{K}^0$ mass distributions should there be only the $f_0(1710)$ state. Hence, we give a boost to the molecular interpretation on the nature of these two $f_0(1710)$ and $a_0(1710)$ resonances.

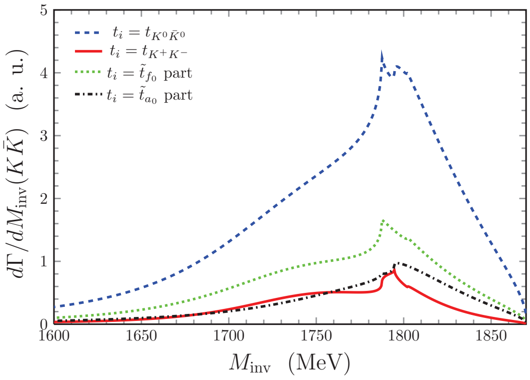

Fig. 5. – Mass distributions $d\Gamma/dM_{\rm inv}$ for the cases of Eq. (5).

4. – Summary

Based on the prediction of $f_0(1710)$ and $a_0(1710)$ as a molecular states of $K^*\bar{K}^*$ and other vector-vector coupled channels, we investigate the two $D_s^+ \to \pi^+ K^+ K^-$ and $D_s^+ \to \pi^+ K_S^0 K_S^0$ reactions. Two effective parameters related to external and internal emission are obtained with a narrow region, which is consistent with the large N_c limit within uncertainties. Using the allowed parameters, we can reasonably explain the surprising large ratio of R_1, in good agreement with recent BESIII experiments. We further made a prediction of $\text{Br}[D_s^+ \to \pi^0 a_0(1710)^+; a_0(1710)^+ \to K^+ K_S^0] \simeq (1.3 \pm 0.4) \times 10^{-3}$.

Now we are happy to see a fair prediction with the coming data of the branching fraction $\text{Br}[D_s^+ \to \pi^0 a_0(1710)^+; a_0(1710)^+ \to K^+ K_S^0] \simeq (3.44 \pm 0.52 \pm 0.32) \times 10^{-3}$ [7], confirming the existence of new $a_0(1817)$ resonance. Our predicted state of $a_0(1710)$ [new $a_0(1817)$], as an important state, will shed light into the structure of scalar mesons in the light quark sector and other relevant issues currently under debate in hadron physics.

* * *

The author acknowledge supports partly from the National Natural Science Foundation of China under Grants Nos. 12175066 and 11975009.

REFERENCES

[1] Zyla P.A. et al. (Particle Data Group), *Prog. Theor. Exp. Phys.*, **2020** (083C01) .
[2] Ablikim M. et al. (BESIII Collaboration), *Phys. Rev. D*, **104** (2021) 012016.
[3] Ablikim M. et al. (BESIII Collaboration), *Phys. Rev. D*, **105** (2022) L051103.
[4] Oller Y. A. and Oset E., *Nucl. Phys. A*, **620** (1997) 438.
[5] Geng L. S. and Oset E., *Phys. Rev. D*, **79** (2009) 074009.
[6] Dai L. R., Oset E. and Geng L. S., *Eur. Phys. J. C*, **82** (2022) 225.
[7] Ablikim M. et al. (BESIII Collaboration), *Phys. Rev. lett.*, **129** (2022) 182001