The D_s^+ decay into $\pi^+ K_S^0 K_S^0$ reaction and the I=1 partner of the $f_0(1710)$ state

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Dai, Oset & Geng, EPJC82 (2022) 225 ✓

Oset, Dai, & Geng, "Repercussion of the a0(1710) [a0(1817)] resonance and future developments", Sci. Bull. 68 (2023) 243

Outline

1. Motivation

BESIII experiments

$\Longrightarrow$ **surprising large ratio of R_1** $\Longrightarrow$ **why** $\Longrightarrow$ **prediction of R_2**

2. Formalism

chiral unitary approach

External emission		$f_0(1710)$ $(I=0)$ $\mathbf{a_0(1710)}$ $(I=1)$
Internal emission	$\Longrightarrow$	obtaining amplitudes for different mechanisms
Hadronization		interference (constructive or destructive)

3. Results

4. Summary

1. Motivation

• **An isospin $I = 0, f_0(1710)$ resonance has, however, been known for quite some time** [Particle Data Group, Prog Theor Exp Phys 2022]

• **Recent BESIII experiments**

It was found the branching fraction [PRD104 (2021) 012016]

$$\mathrm{Br}[D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”};\ \text{“}f_0(1710)\text{”} \to K^+K^-] = (1.0 \pm 0.2 \pm 0.3) \times 10^{-3}$$

and in another work it was found that [PRD105 (2022) L051103]

$$\mathrm{Br}[D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”};\ \text{“}f_0(1710)\text{”} \to K_S^0 K_S^0] = (3.1 \pm 0.3 \pm 0.1) \times 10^{-3}$$

where “$f_0(1710)$” was supposed to be the $f_0(1710)$ resonance. Thus one finds

$$R_1 = \frac{\Gamma(D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”} \to \pi^+ K^0 \bar{K}^0)}{\Gamma(D_s^+ \to \pi^+ \text{“}f_0(1710)\text{”} \to \pi^+ K^+ K^-)} = 6.20 \pm 0.67$$

• **If “$f_0(1710)$” was the $f_0(1710)$ resonance this latter ratio should be 1**

$\Longrightarrow$ **hidden below, or around the $f_0(1710)$, there should be an $I = 1$ resonance responsible for this surprising large ratio**

• **A mixture of the two resonances** and **their interference** would be responsible for a different K^+K^- or $K^0\bar{K}^0$ production?

$$|K\bar{K}, I=0\rangle = -\frac{1}{\sqrt{2}}\left(K^0\bar{K}^0 + K^+K^-\right)$$

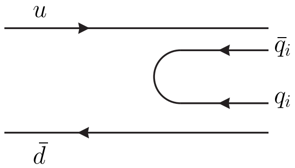
$$|K\bar{K}, I=1, I_3=0\rangle = \frac{1}{\sqrt{2}}\left(K^0\bar{K}^0 - K^+K^-\right)$$

• **let us recall to the $a_0(980)$ case**

standard $q\bar{q}$ quark model $a_0^+(980) \Longrightarrow$ would be $u\bar{d}$

it decays to $K\bar{K} \Longrightarrow$ might be surprising with no strange quarks

The answer to this lies in the **hadronization** of the $u\bar{d}$ which gets attached to a $\bar{q}q$ state with the **quantum numbers of the vacuum** via



Hadronization of a $u\bar{d}$ component into two mesons

$$u\bar{d} \to \sum_i u\,\bar{q}_i q_i\,\bar{d} = u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d}$$

$u\bar{d} \to \sum_i P_{1i}P_{i2} = (P^2)_{12}$ and finally

$$u\bar{d} \to \frac{2}{\sqrt{3}}\eta\pi^+ + K^+\bar{K}^0$$

chiral unitary approach $\Longrightarrow a_0(980)$ **generated as the interaction of the coupled channels $\pi\eta$ and $K\bar{K}$** [Oller,Oset,NPA620(1997)438]

The extension of these ideas to the interaction of vector mesons

in the work of [Geng, Oset, Vector meson-vector meson interaction in a hidden gauge unitary approach, PRD79 (2009) 074009], using as a source of interaction of the vector mesons the local hidden gauge approach [Bando, Kugo, Yamawaki, Phys Rept 164 (1988) 217]

$\Longrightarrow$ **Interestingly, two resonances were found in the region of energies discussed**

Couplings of $f_0(1710)$ and $a_0(1710)$ to VV channels, in units of MeV

	$K^*\bar{K}^*$	$\rho\rho$	$\omega\omega$	$\omega\phi$	$\phi\phi$
$f_0(1710)$	$(7124, i96)$	$(-1030, i1086)$	$(-1763, i108)$	$(3010, -i210)$	$(-2493, -i204)$
	$K^*\bar{K}^*$	$\rho\rho$	$\rho\omega$	$\rho\phi$	
$a_0(1710)$	$(7525, -i1529)$	0	$(-4042, i1391)$	$(4998, -i1872)$	

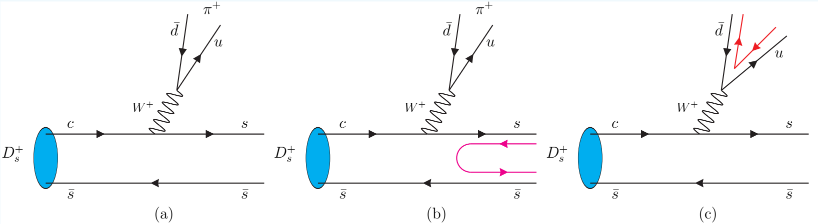
there was no experimental information at the time of its prediction makes the two resonances qualify roughly as $K^*\bar{K}^*$ molecules in analogy to the $K\bar{K}$ approximate nature of the $a_0(980)$ [Oller,Oset,NPA620(1997)438]

Similar conclusions have been reached more recently in [Du, Gülmez, Guo, Meißner, Wang. Interactions between vector mesons and dynamically generated resonances. Eur Phys J C 78 (2018) 988]

The smaller binding of the a_0 **comes as a natural consequence** of a weaker potential in $I = 1$ than in $I = 0$

2. Formalism

External emission with π^+ production and hadronization



(a) Cabibbo-favored decay mode of D_s^+ at the quark level $\Longrightarrow H_1$

(b) Hadronization of the $s\bar{s}$ component $\Longrightarrow H_2$

(c) Hadronization of the $u\bar{d}$ component $\Longrightarrow H_3$

- three mesons in the final state $\Longrightarrow$ must hadronize a pair of quarks $\Longrightarrow$ introducing an extra $\bar{q}q$ with the vacuum quantum numbers ($\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s$)
- the hadronization $\Longrightarrow$ must produce a pair of vector mesons $\Longrightarrow$ produce the $f_0(1710)$ and $a_0(1710)$ resonances

Thus, hadronizing $s\bar{s} \rightarrow \sum_i s\,\bar{q}_i q_i\,\bar{s} = V_{3i}V_{i3} = (V^2)_{33}$

where V is the $q_i\bar{q}_j$ matrix red(vector meson)

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

$\Longrightarrow$ the hadronic state

$$H_1 = (V^2)_{33}\pi^+ = \left(K^{*-}K^{*+} + \bar{K}^{*0}K^{*0} + \phi\phi\right)\pi^+$$

Another possibility $\Longrightarrow$ hadronize the $u\bar{d}$ component with VP or PV

pseudoscalar $\Longrightarrow$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} \end{pmatrix}$$

[Dai, Oset & Geng, EPJC82 (2022) 225]

$u\bar{d} \rightarrow \sum_i u\,\bar{q}_i q_i\,\bar{d}$

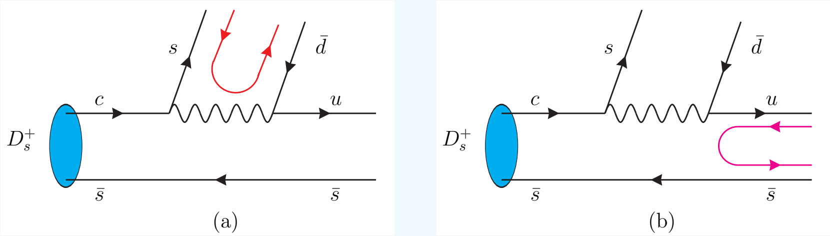
$\rightarrow$ obtaining $(VP)_{12}$ and $(PV)_{12}$

We aim at getting $\pi^+ f_0(1710)$ and $\pi^+ a_0(1710)$ which have G-parity negative and positive respectively

$$H_2 = \phi[(VP)_{12} + (PV)_{12}]$$
$$H_3 = \phi[(VP)_{12} - (PV)_{12}]$$

Suppressed by a color factor $\frac{1}{N_c}$

Internal emission and hadronization



(a) hadronization of the $s\bar{d}$ pair (b) hadronization of the $u\bar{s}$ pair

- We must hadronize with VP and PV combinations $(VP)_{32}$, $(PV)_{32}$, $(VP)_{13}$, $(PV)_{13}$
- form the **good G-parity** combinations

$$H_4 = K^{*+}(VP)_{32} + \bar{K}^{*0}(PV)_{13} \quad \text{negative}$$
$$H_5 = K^{*+}(VP)_{32} - \bar{K}^{*0}(PV)_{13} \quad \text{positive}$$

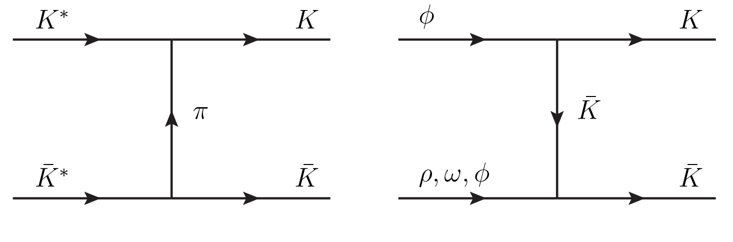
- The different mechanisms have different weights

$$H_1 : A \qquad H_2 : A\alpha \qquad H_3 : A\beta \qquad H_4 : A\gamma \qquad H_5 : A\delta \,.$$

Transitions

hadronic states H_i do not have $K\bar{K}$ in the final state $\Longrightarrow$ must produce the $f_0(1710)$ and $a_0(1710)$ $\Longrightarrow$ then let them decay into $K\bar{K}$.

interference of amplitudes $\Longrightarrow$ look explicitly

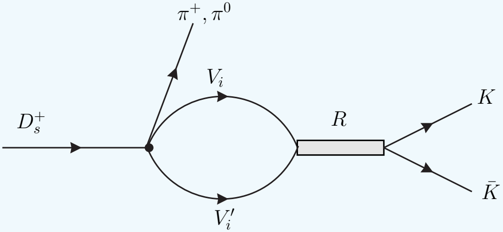


(a) (b)

(a) $K^*\bar{K}^* \to K\bar{K}$ transitions driven by π exchange;

(b) $\phi(\rho, \omega, \phi) \to K\bar{K}$ transitions driven by K exchange

Mechanisms for $D_s^+ \to \pi^+ K^+ K^- (K^0 \bar{K}^0)$ and $D_s^+ \to \pi^0 K^+ \bar{K}^0$



The mechanism for $f_0(1710)$ and $a_0(1710)$ production and $K\bar{K}$ final state

$$
\begin{cases}
H_1: & \pi^+ f_0(1710) \text{ with } \pi^+ K^* \bar{K}^* \text{ and } \pi^+ \phi\phi \text{ terms.} \\
H_2: & \pi^+ f_0(1710) \text{ with } \omega\phi\pi^+ \text{ term.} \\
H_3: & \pi^+ a_0(1710)\ (I_3 = 0) \text{ with } \pi^+ \rho^0 \phi \text{ term;} \\
 & \pi^+ a_0(1710)\ (I_3 = 1) \text{ with } \pi^0 \rho^+ \phi \text{ term.} \\
H_4: & \pi^+ f_0(1710) \text{ with } \pi^+ K^* \bar{K}^* \text{ term.} \\
H_5: & \pi^+ a_0(1710)\ (I_3 = 0) \text{ with } \pi^+ K^* \bar{K}^* \text{ term;} \\
 & \pi^+ a_0(1710)\ (I_3 = 1) \text{ with } \pi^0 K^* \bar{K}^* \text{ term.}
\end{cases}
$$

- write $\tilde{t}_{f_0}$ and $\tilde{t}_{a_0}$ [$\tilde{t}_{a_0}(I_3=0)$, $\tilde{t}_{a_0}(I_3=1)$ are the same]
- due to $G_{\omega\phi}$ and $G_{\rho\phi}$ loop functions are remarkably similar to $G_{K^*\bar{K}^*}$

$$\begin{aligned}
\tilde{t}_{f_0} &= A\{-\sqrt{2}\,G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{f_0,K^*\bar{K}^*} + G_{\phi\phi}(M_{\rm inv})\sqrt{2}\,g_{f_0,\phi\phi} \\
&- \sqrt{2}\left(\gamma - \alpha\,\frac{g_{f_0,\omega\phi}}{g_{f_0,K^*\bar{K}^*}}\right) G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{f_0,K^*\bar{K}^*}\} \\
&= A\{-\sqrt{2}\,G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{f_0,K^*\bar{K}^*} + G_{\phi\phi}(M_{\rm inv})\sqrt{2}\,g_{f_0,\phi\phi} \\
&- \sqrt{2}\,\gamma'\,G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{f_0,K^*\bar{K}^*}\} \\
\tilde{t}_{a_0} &= -A\sqrt{2}\;G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{a_0,K^*\bar{K}^*}\left(\delta - \beta\,\frac{g_{f_0,\rho\phi}}{g_{a_0,K^*\bar{K}^*}}\right) \\
&= -A\sqrt{2}\,\delta'\,G_{K^*\bar{K}^*}(M_{\rm inv})\,g_{a_0,K^*\bar{K}^*}
\end{aligned}$$

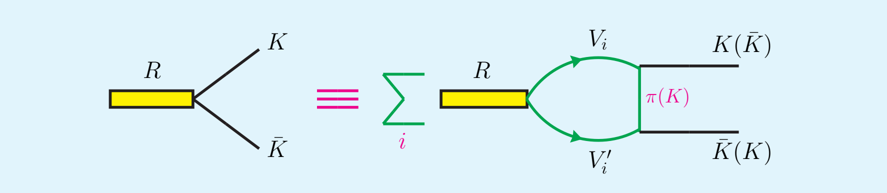
thus we have only two effective parameters

$$\gamma' = \gamma - \alpha\,\frac{g_{f_0,\omega\phi}}{g_{f_0,K^*\bar{K}^*}}\,, \qquad \delta' = \delta - \beta\,\frac{g_{f_0,\rho\phi}}{g_{a_0,K^*\bar{K}^*}}$$

Amplitude for $R \to K\bar{K}$

Next we need to see how the resonances f_0, a_0 decay into $K\bar{K}$ [the dynamics employed in Geng, Oset, PRD79, 074009 (2009) applied to the transitions]
We need the Lagrangian

$$\mathcal{L}_{VPP} = -ig\,\langle [P, \partial_\mu P] V^\mu \rangle$$



Following [Oller,Meissner, PLB500(2001) 263] we can proceed factorizing the $VV' \to K\bar{K}$ transition on shell and the π or K propagators are

$$D_\pi = \frac{1}{-M_{K^*}^2 + m_K^2 - m_\pi^2}\,, \quad D_K = \frac{1}{-M_{K^*}^2}$$

$K^*\bar{K}^* \to K\bar{K}$ transitions driven by π exchange;
$\phi(\rho,\omega,\phi) \to K\bar{K}$ transitions driven by K exchange

from $\mathcal{L}_{VPP}$ to obtain the weights $\widetilde{W}_i$

$$I=0 \quad \begin{cases} K^*\bar{K}^* \to K\bar{K}\,, & \widetilde{W} = \frac{3}{2}\,D_\pi \\ \phi\omega \to K\bar{K}\,, & \widetilde{W} = 2\,D_K \\ \phi\phi \to K\bar{K}\,, & \widetilde{W} = -2\,D_K \\ \rho\rho \to K\bar{K}\,, & \widetilde{W} = \sqrt{3}\,D_K \\ \omega\omega \to K\bar{K}\,, & \widetilde{W} = -D_K \end{cases} \tag{0.1}$$

$$I=1 \quad \begin{cases} K^*\bar{K}^* \to K\bar{K}\,, & \widetilde{W} = -\frac{1}{2}\,D_\pi \\ \phi\rho \to K\bar{K}\,, & \widetilde{W} = 2\,D_K \\ \omega\rho \to K\bar{K}\,, & \widetilde{W} = -\sqrt{2}\,D_K \end{cases}$$

Then the weights for f_0 or a_0 production are given by

$$\begin{aligned} W_{f_0} &= \sum_i g_{f_0,i}\,\widetilde{W}_i\,G_i(M_{\rm inv})\,, \\ W_{a_0} &= \sum_i g_{a_0,i}\,\widetilde{W}_i\,G_i(M_{\rm inv}) \end{aligned}$$

where the sum over i goes over the channels of $I=0$ and $I=1$ respectively.

$$t_{K^+K^-} = -\tilde{t}_{f_0}\frac{1}{M_{\rm inv}^2 - M_{f_0}^2 + iM_{f_0}\Gamma_{f_0}} W_{f_0}\frac{1}{\sqrt{2}} g_{K\bar{K}} - \tilde{t}_{a_0}\frac{1}{M_{\rm inv}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} W_{a_0}\frac{1}{\sqrt{2}} g_{K\bar{K}}$$

$$t_{K^0\bar{K}^0} = -\tilde{t}_{f_0}\frac{1}{M_{\rm inv}^2 - M_{f_0}^2 + iM_{f_0}\Gamma_{f_0}} W_{f_0}\frac{1}{\sqrt{2}} g_{K\bar{K}} + \tilde{t}_{a_0}\frac{1}{M_{\rm inv}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} W_{a_0}\frac{1}{\sqrt{2}} g_{K\bar{K}}$$

$$\mathbf{t}_{\mathbf{K^+\bar{K}^0}} = \tilde{t}_{a_0}\frac{1}{M_{\rm inv}^2 - M_{a_0}^2 + iM_{a_0}\Gamma_{a_0}} W_{a_0} g_{K\bar{K}}$$

$$t_{K^+K_S^0} = -\frac{1}{\sqrt{2}} t_{K^+\bar{K}^0}$$

The differential decay width

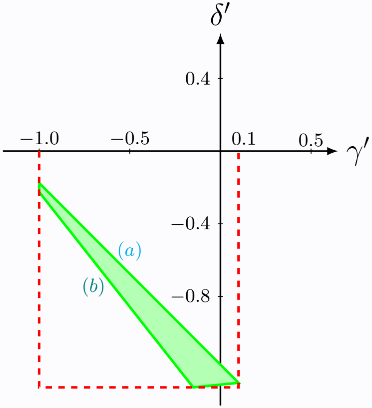
$$\frac{d\Gamma_i}{dM_{\rm inv}(K\bar{K})} = \frac{1}{(2\pi)^3}\frac{1}{4M_{D_s}^2} p_\pi \tilde{p}_k |t_i|^2$$

$$R_1 = \frac{\Gamma(D_s^+ \to \pi^+ K^0 \bar{K}^0)}{\Gamma(D_s^+ \to \pi^+ K^+ K^-)}, \quad R_2 = \frac{\Gamma(D_s^+ \to \pi^0 K^+ K_S^0)}{\Gamma(D_s^+ \to \pi^+ K^+ K^-)}$$

3. Results

we have only two effective parameters

$$\gamma' = \gamma - \alpha \frac{g_{f_0,\omega\phi}}{g_{f_0,K^*\bar{K}^*}}, \qquad \delta' = \delta - \beta \frac{g_{f_0,\rho\phi}}{g_{a_0,K^*\bar{K}^*}}$$



a narrow region of the parameters

$\gamma' \in [-1, 1]$,
$\delta' \in [-1.3, 1.3]$

consistent with the large N_c limit within uncertainties

we evaluate $R_1 = 6.20 \pm 0.67$ in agreement with BESIII experiment [PRD104 (2021) 012016; PRD105 (2022) L051103]

The challenge of the approach is to make prediction of R_2 and **we obtained** $\Longrightarrow R_2^{\rm theo} \simeq 1.31 \pm 0.12$

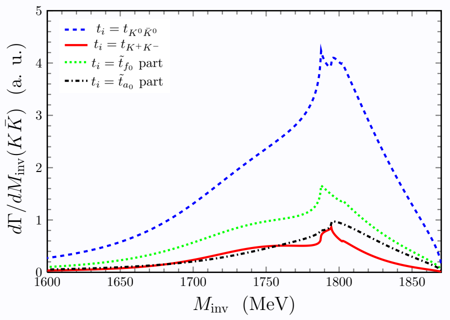
Our prediction

From $R_2^{\text{theo}} \simeq 1.31 \pm 0.12$, we obtained

$$\text{Br}[D_s^+ \to \pi^0 a_0(1710)^+; a_0(1710)^+ \to K^+ K_S^0] \simeq (1.3 \pm 0.4) \times 10^{-3}$$

which was a prediction before this ratio was measured.

Further analysis



$\Longleftarrow$ $\gamma' = -0.5$, $\delta' = -0.75$ (middle of the allowed region)

- in the $K^0\bar{K}^0$ mass distribution there has been a constructive interference of the f_0 and a_0 resonances
- while in the K^+K^- mass distribution the interference has been destructive

This is exactly the reason suggested in the experimental analysis
$D_s^+ \to \pi^+K^+K^-$ [PRD104 (2021) 012016]
$D_s^+ \to \pi^+K_S^0K_S^0$ reactions [PRD105 (2022) L051103]

$\Longrightarrow$ **to justify the existence of the $a_0(1710)$ resonance**
which should give the same K^+K^- or $K^0\bar{K}^0$ mass distributions should there be only the $f_0(1710)$ state

- **our prediction $\Longrightarrow$ based on our approach**

  which is only tied to the theoretical couplings of the $f_0(1710)$ and $a_0(1710)$ resonances to the different coupled channels that build up the resonance and their decay amplitudes to $K\bar{K}$
  [Geng and Oset, PRD79(2009)074009]

- **our prediction $\Longrightarrow$ based on BESIII experiments**
  branching ratios of $D_s^+ \to \pi^+ K_S^0 K_S^0$ and $D_s^+ \to \pi^0 K^+ K^-$
  [PRD104(2021)012016; PRD105(2022)L051103]

$\Longrightarrow$ **a boost to the molecular interpretation on the nature of these two $f_0(1710)$ and $a_0(1710)$ resonances**

further development $\Longrightarrow$

further developments of the idea in EPJC82(2022) 225
PRD105(2022)116010; arXiv:2210.12992] showing the relevance of a_0 state in the process

Summary

1) at first we investigate the two $D_s^+ \to \pi^+ K^+ K^-, \pi^+ K_S^0 K_S^0$ reactions

based on the prediction of $f_0(1710)$ and $a_0(1710)$ as a molecular states of $K^* \bar{K}^*$ and other vector-vector coupled channels

details in [Geng, Oset, PRD79(2009)074009]

2)based two parameters related to external and internal emission

$\Longrightarrow$ determine a narrow region of the parameters consistent with the large N_c limit within uncertainties

$\Longrightarrow$ evaluate and explain the surprising large ratio R_1, which is in agreement with BESIII experiments

3)we made a prediction [Dai, Oset, Geng, EPJC82 (2022) 225]

$$\mathrm{Br}[D_s^+ \to \pi^0 a_0(1710)^+; a_0(1710)^+ \to K^+ K_S^0] \simeq (1.3 \pm 0.4) \times 10^{-3}$$

4) Now we are happy to see a fair prediction with the coming data of the branching fraction

[BESIII Collaboration, PRL129 (2022) 182001] $\Longrightarrow$ **new** $a_0(1817)$ **resonance**

$$\mathrm{Br}[D_s^+ \to \pi^0 a_0(1710)^+; a_0(1710)^+ \to K^+ K_S^0] \simeq (3.44 \pm 0.52 \pm 0.32) \times 10^{-3}$$

Ending this talk by copying one sentence form our paper that " this new a_0 resonance as an important state will shed light into the structure of scalar mesons in the light quark sector and other relevant issues currently under debate in hadron physics"

see more valuale and interesting discussions in

Oset, Dai, & Geng, "Repercussion of the $a_0(1710)$ [$a_0(1817)$] resonance and future developments", Sci. Bull. 68 (2023) 243

THANK YOU