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Mass Spectra of Ξ and Ω Baryons using Hypercentral Constituent Quark Model

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Outline of the talk

- Hadron Spectroscopy
- Light Baryons
- The Model
- Results for non-strange and strange baryons
- Regge Trajectories, Magnetic Moments
- Summary

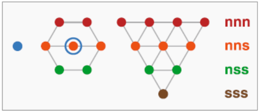
Hadron Spectroscopy

Hadron spectroscopy continues to be a cornerstone of strong interaction studies and a highly active area of research. A systematic description of baryon resonances will likely require combined efforts from experiment, lattice QCD, amplitude analyses, phenomenology, and functional methods.
The experimental investigation of the excited baryon spectrum has been a long-standing element of the hadronic-physics program. An important goal has been the search for so called "missing resonances" baryonic states predicted by the quark model based on three constituent quarks but which have not yet been observed experimentally; it may indicate to model the baryon spectrum with a different effective degrees of freedom.

Light Baryons

The representation of a few known states of all the light, strange baryons :

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{7}{2}^-$	$\frac{9}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$
N(939)	N(1535)	N(1720)	N(1520)	N(1680)	N(1675)	N(1990)	N(2190)	N(2220)	N(2250)	
N(1440)	N(1650)	N(1900)	N(1700)	N(1860)	N(2060)					
N(1710)	N(1895)		N(1875)	N(2000)	N(2570)					
N(1880)			N(2120)							
N(2100)										
N(2300)										
Δ(1910)	Δ(1620)	Δ(1232)	Δ(1700)	Δ(1905)	Δ(1930)	Δ(1950)	Δ(2200)	Δ(2300)	Δ(2400)	Δ(2420)
	Δ(1900)	Δ(1600)	Δ(1940)	Δ(2000)						
		Δ(1920)								
Λ(1116)	Λ(1380)	Λ(1890)	Λ(1520)	Λ(1820)	Λ(1830)	Λ(2085)	Λ(2100)	Λ(2350)		
Λ(1600)	Λ(1405)		Λ(1690)	Λ(2110)						
Λ(1810)	Λ(1670)									
	Λ(1800)									
Σ(1189)	Σ(1750)	Σ(1385)	Σ(1670)	Σ(1915)	Σ(1775)	Σ(2030)				
Σ(1660)	Σ(1900)		Σ(1910)							
Σ(1880)										
Ξ(1315)		Ξ(1530)	Ξ(1820)							
		Ω(1672)								



Hypercentral Constituent Quark Model

In a CQM all the effects which go beyond the description of a baryon as a system of three confined particles (as for example the effects of the gluons, sea quarks), are parametrized in an effective way with the introduction of a large constituent quark mass. The three quark wavefunction can be written as:

$$\psi = \phi_{flavour}\chi_{spin}\xi_{colour}\eta_{space}$$

The spatial part is expressed as Jacobi co-ordinates for three body system as

$$\rho = \frac{1}{\sqrt{2}}(\mathbf{r_1} - \mathbf{r_2}); \;\; \lambda = \frac{1}{\sqrt{6}}(\mathbf{r_1} + \mathbf{r_2} - 2\mathbf{r_3}) \tag{1}$$

Hyperspherical coordinates as hyperradius x and hyperangle ξ from Jacobi coordinates as

$$x = \sqrt{\rho^2 + \lambda^2}; \;\; \xi = arctan(\frac{\rho}{\lambda}) \tag{2}$$

The model itself suggets that the potential to be chosen should be hypercentral i.e. depending only on hyperradius x. The hyperradius x depends at the same time on all the three constituent coordinates, therefore an hypercentral potential is not a pure two body interaction but can also contain three body terms.The deduced choice for the hypercentral potential is hypercoulomb type. However, it is not confining the quarks within the hadrons so a linear term [2] αx is added.

$$V(x) = -\frac{\tau}{x} + \alpha x \tag{3}$$

Also, this potential form cannot account for splittings of multiplet levels. So, an additional hyperfine splitting term is to be incorporated.

$$V_{SD}(x) = V_{LS}(x)(\mathbf{L}\cdot\mathbf{S}) + V_{SS}(x)\left[S(S+1) - \frac{3}{2}\right] \tag{4}$$

$$+V_T(x)\left[S(S+1) - \frac{3(\mathbf{S}\cdot\mathbf{x})(\mathbf{S}\cdot\mathbf{x})}{x^2}\right]$$

Here, a correction term with $\frac{1}{m}$ dependence is also considered as a refinement to the potential.

$$V^1(x) = -C_F C_A \frac{\alpha_s^2}{4x^2} \tag{5}$$

The far observed drawback of hCQM of lower mass for higher spin state for a given angular momentum quantum number l has been attempted to resolve through a second-order correction of mass i.e. $\frac{1}{m^2}$. The idea is based on the pNRQCD using Lattice QCD simulations as studied by Y. Koma *et al.* which has been applied to heavy quarkonia only till now. In the non-relativistic approaches, the Cornell potential has been applied to mesons as similar to linear confining potential for heavy and light baryon studies, an additional correction term to the mass may be helpful to reproduce the experimental spectra of light baryons phenomenologically.

The $O(1/m^2)$ consists of spin-orbit and spin-tensor corrections described as,

$$V^2_{ls}(x) = (\frac{c_s}{2x}\frac{dV}{dx} + \frac{c_F}{x}(V_1 + V_2))l \cdot s \quad (6)$$

$$V^2_{s12}(x) = \frac{c_F^2}{3} V_3 s_{12} \quad (7)$$

$$V_1 = -(1-\epsilon)\alpha; \quad V_2 = \frac{\alpha^{'}}{x^2} + \epsilon\alpha; \quad V_3 = \frac{3\sigma}{x^3} \quad (8)$$

The Hamiltonian is

$$H = \frac{P^2}{2m} + V(x) + V_{SD}(x) + \frac{1}{m}V^1(x) + \frac{1}{m^2}V^2(x) \quad (9)$$

The Non-Strange Baryons: N and Δ

The first and foremost members of octet and decuplet families, nucleon N and Δ baryon have always been of interest. Various decay of strange as well as other heavy baryons are ultimately reaching to these light baryons. The isospin partners u and d quarks have generally been treated at the same footings however, it leads to 2 and 4 respective isospin states for N and Δ.

The study of Δ is not only limited to high energy realm but also in the field of astrophysics wherein Δ isobars are studied under quark meson coupling model for the possibility of appearance in neutron star.Historically, the presence of $\Delta(1232)$ has been an essential step towards color degree of freedom. Incorporating the recent additions, 8 four star, 4 three star and many other experimental status have been explored with the values ranging from $J = \frac{1}{2}$ to $J = \frac{15}{2}$ and still many states are awaited of confirmation of existence as listed by Particle Data Group (PDG).

N Baryon

State	J^P	P	N	[2]	PDG[1]	Status	[22]	[24]	[25]	[7]	[33]	[8]	[35]	[40]
1S	$\frac{1}{2}^+$	938	948	939	938	****	939	939	960	938	939	938	938	1000 ± 18
2S	$\frac{1}{2}^+$	1419	1427	1425	1410-1470	****	1511	1450.8	1430	1444	1462	1492	1440	
3S	$\frac{1}{2}^+$	1683	1691	1721	1680-1740	***	1776	1699	1710	1832	1748	1763	1710	
4S	$\frac{1}{2}^+$	2028	2035	2089	2050-2150	***								
5S	$\frac{1}{2}^+$	2436	2441	2515										
1P	$\frac{1}{2}^-$	1520	1523	1565	1515-1545	****	1537	1536.1	1501	1567	1497	1511	1538	1539 ± 69
1P	$\frac{3}{2}^-$	1494	1502	1535	1510-1520	****	1537	1550.3	1517	1567	1548	1511	1523	1634 ± 44
1P	$\frac{5}{2}^-$	1461	1469	1495	1665-1680	****					1655		1678	
2P	$\frac{1}{2}^-$	1838	1842	1898	1880-1910	**	1888		1895				2090	1895 ± 128
2P	$\frac{3}{2}^-$	1810	1816	1865	1850-1920	***			1880					1982 ± 128
2P	$\frac{5}{2}^-$	1773	1779	1820										
1D	$\frac{1}{2}^+$	1794	1800	1849	1830-1930	***	1890		1870	1887				1848 ± 120
1D	$\frac{3}{2}^+$	1763	1768	1815	1660-1750	****	1648	1682.7	1690		1734	1725	1700	1773 ± 91
1D	$\frac{5}{2}^+$	1722	1729	1769	1680-1690	****	1799	1704.2	1689	1689.8	1738	1735	1683	
1D	$\frac{7}{2}^+$	1671	1678	1712							1943		1990	
2D	$\frac{1}{2}^+$	2211	2213	2244							2060			1998 ± 59
2D	$\frac{3}{2}^+$	2174	2178	2204	1890-1950	****					2058		1900	2298 ± 191
2D	$\frac{5}{2}^+$	2123	2128	2150	1950-2150				2090		2007		2000	
2D	$\frac{7}{2}^+$	2061	2067	2083	1950-2100				2060				1990	
1F	$\frac{3}{2}^-$	2145	2139	2167	2060-2160	***							2080	2296 ± 129
1F	$\frac{5}{2}^-$	2090	2089	2112	2030-2200	***							2220	
1F	$\frac{7}{2}^-$	2022	2027	2045	2140-2220	****		2135	2180				2150	
1F	$\frac{9}{2}^-$	1940	1951	1963	2250-2320	****		2270	2280	2232.4		2240		
1G	$\frac{9}{2}^+$	2371	2376		2200-2300	****		2273	2200	2174.3			2245	
1H	$\frac{11}{2}^-$	2766	2769		2550-2750	***		2620		2534.5			2650	

Δ Baryon

State	J^P	Δ^0	[3]	PDG[1]	Status	[24]	[7]	[21]	[22]	[8]	[23]	[35]	[4]	[32]	[40]
1S	$\frac{3}{2}^+$	1232	1232	1230-1234	****	1245	1232	1235	1247	1231	1232	1232	1232	1232	1344 ± 27
2S	$\frac{3}{2}^+$	1610	1611	1500-1640	****	1609	1659.1	1714	1689	1658	1727	1625		1600	
3S	$\frac{3}{2}^+$	1932	1934	1870-1970	***		2090.2	1930	2042	1914	1921	1935		1920	
4S	$\frac{3}{2}^+$	2257	2256	-	-										
5S	$\frac{3}{2}^+$	2584	2579	-	-										
1P	$\frac{1}{2}^-$	1601	1625	1590-1630	****	1711	1667.2	1673	1830	1737	1573	1645	1685		1454 ± 140
1P	$\frac{3}{2}^-$	1623	1593	1690-1730	****	1709	1667.2	1673	1830	1737	1573	1720	1685		1570 ± 67
1P	$\frac{5}{2}^-$	1629	1550	-	-										
2P	$\frac{1}{2}^-$	2024	1956	1840-1920	***			2003	1910		1910	1900			1914 ± 322
2P	$\frac{3}{2}^-$	2043	1919	1940-2060	**				1910			1940			
2P	$\frac{5}{2}^-$	2051	1871	1900-2000	***			2003	1910	1908		1945			
1D	$\frac{1}{2}^+$	1910	1905	1850-1950	****	1851	1873.5	1930	1827	1891	1953	1895		1910	1751 ± 190
1D	$\frac{3}{2}^+$	1922	1868	1870-1970	***	1936	2090.2	1930	2042	1914	1921	1935		1920	
1D	$\frac{5}{2}^+$	1935	1818	1855-1910	****	1934	1873.5	1930	2042	1891	1901	1895		1905	
1D	$\frac{7}{2}^+$	1944	1756	1915-1950	****	1932	1873.5	1930	2042	1891	1955	1950		1950	
2D	$\frac{1}{2}^+$	2347	2227	-	-										2211 ± 126
2D	$\frac{3}{2}^+$	2360	2190	-	-										2204 ± 82
2D	$\frac{5}{2}^+$	2374	2140	2015	**							2200			
2D	$\frac{7}{2}^+$	2385	2078	-	-										
1F	$\frac{7}{2}^-$	2259	2037	2150-2250	***						2200				
1F	$\frac{9}{2}^-$	2273	1952	-	-										
1G	$\frac{11}{2}^+$	2611		2300-2600	****										
1H	$\frac{13}{2}^-$	2958		2794 ± 80	**										
1I	$\frac{15}{2}^+$	3313		2990 ± 100	**										

The Strange Baryons: Λ, Σ, Ξ and Ω

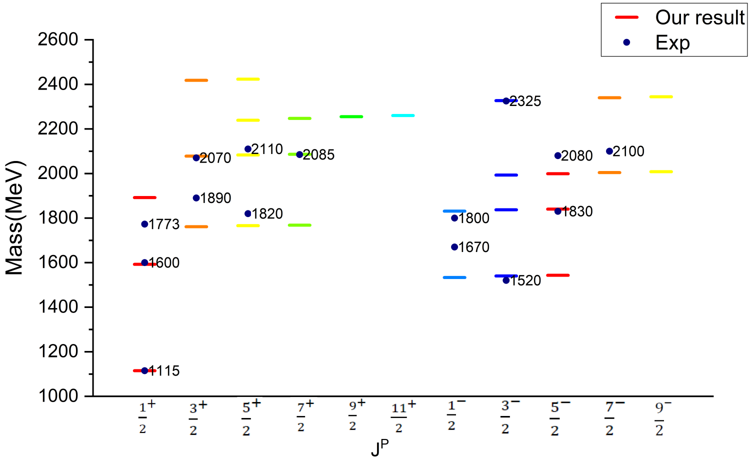
The presence of strange quark in a baryon draws attention because of the fact that it is a bit heavier compared to u and d quarks whereas considerably light compared to c and b quarks. Particularly the strangeness S=-2,-3 Ξ baryons have not been observed experimentally like other light sector baryons. In case of Σ and Λ baryons, all the properties are not known completely. Most of the data for strange baryons have been based from earlier studies from bubble chamber for K^- reactions. The $\Lambda(1405)$ with $J^P = \frac{1}{2}^-$ is still a mysterious state in the lambda spectrum. This state is lower than the non-strange counterpart $N^*(1535)$. Recent studies have attempted to understand this state as some hadronic molecular. In terms of multi-strangeness, Ω baryon is in a similar position like Ξ as both are not easily observed in experiments and not more information has been readily available since bubble chamber data.

Λ Baryon

State	J^P	M1	M2	M_{exp}[1]	
1S	$\frac{1}{2}^+$	1115	1115	1115	
2S	$\frac{1}{2}^+$	1592	1589	1600	
3S	$\frac{1}{2}^+$	1885	1892	1810	
4S	$\frac{1}{2}^+$	2202	2220		
5S	$\frac{1}{2}^+$	2540	2571		
State	J^P	M1	M2	M	M_{exp}[1]
$1^2P_{1/2}$	$\frac{1}{2}^-$	1546	1558	1535	1670
$1^2P_{3/2}$	$\frac{3}{2}^-$	1534	1544	1540	1520
$1^4P_{1/2}$	$\frac{1}{2}^-$	1553	1564	1533	
$1^4P_{3/2}$	$\frac{3}{2}^-$	1540	1551	1542	
$1^4P_{5/2}$	$\frac{5}{2}^-$	1524	1533	1543	
$2^2P_{1/2}$	$\frac{1}{2}^-$	1834	1858	1831	1800
$2^2P_{3/2}$	$\frac{3}{2}^-$	1819	1841	1837	1690
$2^4P_{1/2}$	$\frac{1}{2}^-$	1841	1867	1829	
$2^4P_{3/2}$	$\frac{3}{2}^-$	1827	1850	1837	
$2^4P_{5/2}$	$\frac{5}{2}^-$	1807	1827	1840	1830
$1^2D_{3/2}$	$\frac{3}{2}^+$	1769	1789	1762	1890
$1^2D_{5/2}$	$\frac{5}{2}^+$	1746	1767	1766	1820
$1^4D_{1/2}$	$\frac{1}{2}^+$	1794	1814	1756	
$1^4D_{3/2}$	$\frac{3}{2}^+$	1777	1798	1761	
$1^4D_{5/2}$	$\frac{5}{2}^+$	1755	1776	1766	
$1^4D_{7/2}$	$\frac{7}{2}^+$	1727	1748	1768	

State	J^P	M1	M2	M	M_{exp}[1]
$2^2D_{3/2}$	$\frac{3}{2}^+$	2076	2113	2079	2070
$2^2D_{5/2}$	$\frac{5}{2}^+$	2051	2085	2083	2110
$2^4D_{1/2}$	$\frac{1}{2}^+$	2105	2144	2072	
$2^4D_{3/2}$	$\frac{3}{2}^+$	2086	2123	2078	
$2^4D_{5/2}$	$\frac{5}{2}^+$	2060	2096	2083	
$2^4D_{7/2}$	$\frac{7}{2}^+$	2029	2061	2086	2085
$1^2F_{5/2}$	$\frac{5}{2}^-$	2005	2039	1999	
$1^2F_{7/2}$	$\frac{7}{2}^-$	1970	2002	2004	2100
$1^4F_{3/2}$	$\frac{3}{2}^-$	2043	2079	1993	
$1^4F_{5/2}$	$\frac{5}{2}^-$	2015	2050	1999	
$1^4F_{7/2}$	$\frac{7}{2}^-$	1980	2013	2004	
$1^4F_{9/2}$	$\frac{9}{2}^-$	1939	1969	2008	
$1^2G_{7/2}$	$\frac{7}{2}^+$	2253	2302	2248	
$1^2G_{9/2}$	$\frac{9}{2}^+$	2204	2246	2255	2350
$1^4G_{5/2}$	$\frac{5}{2}^+$	2305	2363	2239	
$1^4G_{7/2}$	$\frac{7}{2}^+$	2265	2316	2247	
$1^4G_{9/2}$	$\frac{9}{2}^+$	2216	2260	2254	
$1^4G_{11/2}$	$\frac{11}{2}^+$	2159	2195	2260	

Λ Baryon



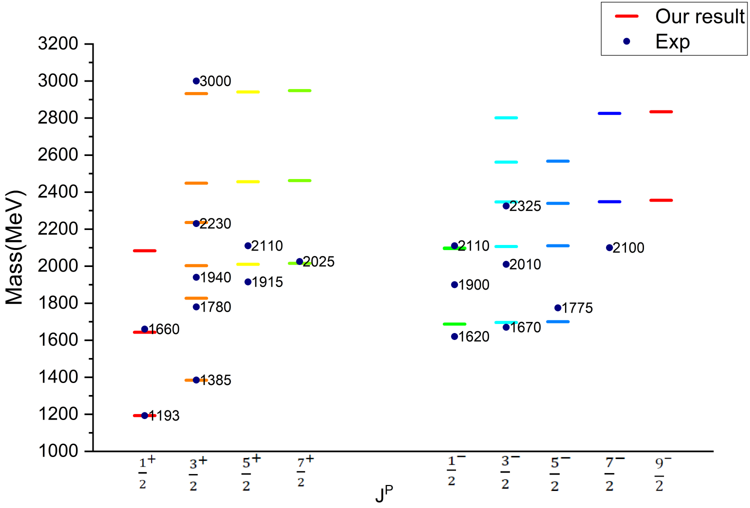
Our result
Exp
2600
2400
2200
2000
1800
1600
1400
1200
1000
Mass(MeV)
2325
2070
2110
2085
2080
2100
1890
1820
1800
1830
1773
1670
1600
1520
1115
$\frac{1}{2}^+$
$\frac{3}{2}^+$
$\frac{5}{2}^+$
$\frac{7}{2}^+$
$\frac{9}{2}^+$
$\frac{11}{2}^+$
$\frac{1}{2}^-$
$\frac{3}{2}^-$
$\frac{5}{2}^-$
$\frac{7}{2}^-$
$\frac{9}{2}^-$
J^P

Σ Baryon

State	J^P	M1	M2	M_{exp}[1]	
1S	$\frac{1}{2}^+$	1193	1193	1193	
	$\frac{3}{2}^+$	1384	1384	1385	
2S	$\frac{1}{2}^+$	1643	1643	1660	
	$\frac{3}{2}^+$	1827	1827		
3S	$\frac{1}{2}^+$	2083	2099		
	$\frac{3}{2}^+$	2229	2236		
4S	$\frac{1}{2}^+$	2560	2589		
	$\frac{3}{2}^+$	2675	2693		
5S	$\frac{1}{2}^+$	3067	3108		
	$\frac{3}{2}^+$	3159	3189		
State	J^P	M1	M2	M	M_{exp}[1]
$1^2P_{1/2}$	$\frac{1}{2}^-$	1720	1725	1687	1620
$1^2P_{3/2}$	$\frac{3}{2}^-$	1698	1702	1696	1670
$1^4P_{1/2}$	$\frac{1}{2}^-$	1731	1736	1684	1750
$1^4P_{3/2}$	$\frac{3}{2}^-$	1709	1713	1698	
$1^4P_{5/2}$	$\frac{5}{2}^-$	1680	1683	1700	1775
$2^2P_{1/2}$	$\frac{1}{2}^-$	2128	2145	2098	1900
$2^2P_{3/2}$	$\frac{3}{2}^-$	2099	2114	2106	1910
$2^4P_{1/2}$	$\frac{1}{2}^-$	2142	2159	2095	
$2^4P_{3/2}$	$\frac{3}{2}^-$	2114	2129	2106	
$2^4P_{5/2}$	$\frac{5}{2}^-$	2076	2087	2110	

State	J^P	M1	M2	M	M_{exp}[1]
$1^2D_{3/2}$	$\frac{3}{2}^+$	2040	2057	2003	1940
$1^2D_{5/2}$	$\frac{5}{2}^+$	1998	2013	2010	1915
$1^4D_{1/2}$	$\frac{1}{2}^+$	2086	2107	1992	
$1^4D_{3/2}$	$\frac{3}{2}^+$	2055	2074	2002	
$1^4D_{5/2}$	$\frac{5}{2}^+$	2014	2029	2010	
$1^4D_{7/2}$	$\frac{7}{2}^+$	1962	1974	2015	2025
$2^2D_{3/2}$	$\frac{3}{2}^+$	2481	2510	2448	
$2^2D_{5/2}$	$\frac{5}{2}^+$	2432	2459	2456	
$2^4D_{1/2}$	$\frac{1}{2}^+$	2536	2568	2438	
$2^4D_{3/2}$	$\frac{3}{2}^+$	2499	2529	2446	
$2^4D_{5/2}$	$\frac{5}{2}^+$	2451	2478	2455	
$2^4D_{7/2}$	$\frac{7}{2}^+$	2390	2414	2462	
$1^2F_{5/2}$	$\frac{5}{2}^-$	2386	2416	2339	
$1^2F_{7/2}$	$\frac{7}{2}^-$	2318	2343	2349	
$1^4F_{3/2}$	$\frac{3}{2}^-$	2461	2495	2327	
$1^4F_{5/2}$	$\frac{5}{2}^-$	2406	2437	2337	
$1^4F_{7/2}$	$\frac{7}{2}^-$	2338	2365	2348	
$1^4F_{9/2}$	$\frac{9}{2}^-$	2257	2278	2356	

Σ Baryon



Our result
Exp
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
Mass(MeV)
3000
2230
2110
2025
1940
1915
1780
1660
1385
1193
2325
2110
2100
2010
1900
1775
1670
1620
$\frac{1}{2}^+$
$\frac{3}{2}^+$
$\frac{5}{2}^+$
$\frac{7}{2}^+$
J^P
$\frac{1}{2}^-$
$\frac{3}{2}^-$
$\frac{5}{2}^-$
$\frac{7}{2}^-$
$\frac{9}{2}^-$

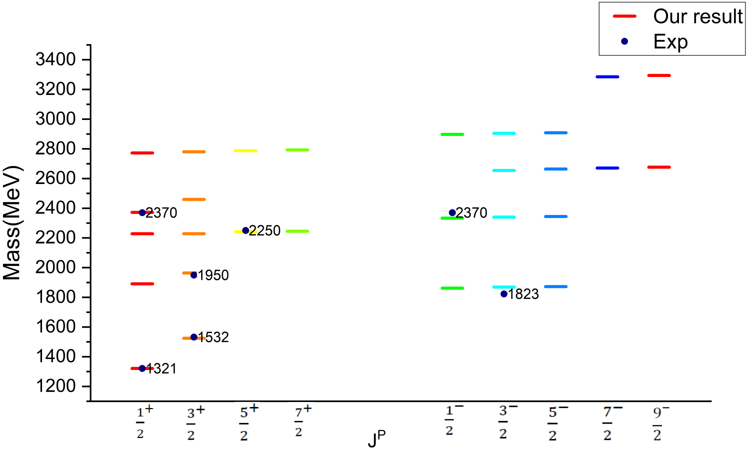
Ξ Baryon

State	J^P	M1	M2	M_{exp}[1]
1S	$\frac{1}{2}^+$	1322	1321	1321
	$\frac{3}{2}^+$	1531	1524	1532
2S	$\frac{1}{2}^+$	1884	1891	
	$\frac{3}{2}^+$	1971	1964	
3S	$\frac{1}{2}^+$	2361	2372	
	$\frac{3}{2}^+$	2457	2459	
4S	$\frac{1}{2}^+$	2935	2954	
	$\frac{3}{2}^+$	3029	3041	
5S	$\frac{1}{2}^+$	3591	3620	
	$\frac{3}{2}^+$	3679	3702	

State	J^P	M1	M2	M	M_{exp}[1]
$1^2P_{1/2}$	$\frac{1}{2}^-$	1886	1889	1862	
$1^2P_{3/2}$	$\frac{3}{2}^-$	1871	1873	1869	1823
$1^4P_{1/2}$	$\frac{1}{2}^-$	1894	1897	1860	
$1^4P_{3/2}$	$\frac{3}{2}^-$	1879	1881	1870	1823
$1^4P_{5/2}$	$\frac{5}{2}^-$	1859	1859	1872	
$2^2P_{1/2}$	$\frac{1}{2}^-$	2361	2373	2333	
$2^2P_{3/2}$	$\frac{3}{2}^-$	2337	2347	2340	
$2^4P_{1/2}$	$\frac{1}{2}^-$	2373	2386	2330	
$2^4P_{3/2}$	$\frac{3}{2}^-$	2349	2360	2341	
$2^4P_{5/2}$	$\frac{5}{2}^-$	2318	2325	2344	

State	J^P	M1	M2	M	M_{exp}[1]
$1^2D_{3/2}$	$\frac{3}{2}^+$	2270	2281	2236	
$1^2D_{5/2}$	$\frac{5}{2}^+$	2234	2244	2241	
$1^4D_{1/2}$	$\frac{1}{2}^+$	2310	2322	2228	
$1^4D_{3/2}$	$\frac{3}{2}^+$	2283	2295	2235	
$1^4D_{5/2}$	$\frac{5}{2}^+$	2247	2257	2242	
$1^4D_{7/2}$	$\frac{7}{2}^+$	2203	2211	2245	
$2^2D_{3/2}$	$\frac{3}{2}^+$	2819	2842	2782	
$2^2D_{5/2}$	$\frac{5}{2}^+$	2771	2791	2788	
$2^4D_{1/2}$	$\frac{1}{2}^+$	2874	2899	2772	
$2^4D_{3/2}$	$\frac{3}{2}^+$	2838	2861	2780	
$2^4D_{5/2}$	$\frac{5}{2}^+$	2790	2810	2788	
$2^4D_{7/2}$	$\frac{7}{2}^+$	2729	2747	2793	
$1^2F_{5/2}$	$\frac{5}{2}^-$	2713	2736	2664	
$1^2F_{7/2}$	$\frac{7}{2}^-$	2647	2666	2671	
$1^4F_{3/2}$	$\frac{3}{2}^-$	2786	2813	2654	
$1^4F_{5/2}$	$\frac{5}{2}^-$	2733	2757	2662	
$1^4F_{7/2}$	$\frac{7}{2}^-$	2667	2687	2670	
$1^4F_{9/2}$	$\frac{9}{2}^-$	2588	2603	2676	

Ξ Baryon



Our result
Exp
3400
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
Mass(MeV)
2370
2250
1950
1532
1321
2370
1823
$\frac{1}{2}^+$ $\frac{3}{2}^+$ $\frac{5}{2}^+$ $\frac{7}{2}^+$
J^P
$\frac{1}{2}^-$ $\frac{3}{2}^-$ $\frac{5}{2}^-$ $\frac{7}{2}^-$ $\frac{9}{2}^-$

Comparison of Results with Other Approaches

J^P	$Mass_{cal}1$	$Mass_{cal}2$	[36]	[13]	[5]	[18]	[37]	[22]	[39]	[34]	[32]	[40]
$\frac{1}{2}^+$	1322	1321	1330	1310	1305	1334	1348	1317	1318	1325	1317	1303 ± 13
	1884	1891	1886	1876	1840	1727	1805	1772	1932	1891	1750	2178 ± 48
	2310	2322	1993	2062	2040	1932	1868				1980	2231 ± 44
	2361	2372	2012	2131	2100			1874			2054	2408 ± 45
	2874	2899	2091	2176	2130						2107	
	2935	2954	2142	2215	2150						2149	
	3527	3562	2367	2249	2230						2254	
	3591	3620			2345							
$\frac{3}{2}^+$	1531	1524	1518	1539	1505	1524	1528	1552	1539	1520	1526	1553 ± 18
	1971	1964	1966	1988	2045	1878		1653	2120	1934	1952	2228 ± 40
	2270	2281	2100	2076	2065	1979					1970	2398 ± 52
	2283	2295	2121	2128	2115						2065	2574 ± 52
	2457	2459	2122	2170	2165						2114	
	2819	2842	2144	2175	2170						2174	
	2838	2861	2149	2219	2210						2184	
	3029	3041	2421	2257	2230						2218	
	3455	3489		2279	2275						2252	
$\frac{5}{2}^+$	2234	2242	2108	2013	2045					1936	1959	
	2247	2295	2147	2141	2165					2025	2102	
	2771	2791	2213	2197	2230						2170	
	2790	2810		2231	2230						2205	
	3391	3423		2279	2240						2239	
	3415	3448										
$\frac{7}{2}^+$	2203	2211	2189	2169	2180					2035	2074	
	2729	2747		2289	2240						2189	
	3336	3366										

Comparison of Results with Other Approaches

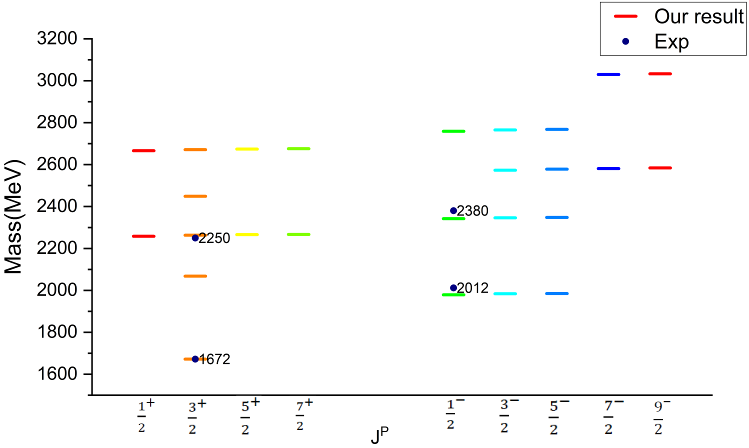
J^P	$Mass_{cal}1$	$Mass_{cal}2$	[36]	[13]	[5]	[18]	[37]	[22]	[39]	[34]	[32]	[40]
$\frac{1}{2}^-$	1886	1889	1682	1770	1755	1869			1658	1725	1772	1716 ± 43
	1894	1897	1758	1922	1810	1932				1811	1894	1837 ± 28
	2361	2373	1839	1938	1835	2076					1926	1844 ± 43
	2373	2386	2160	2241	2225							2758 ± 78
	2929	2948	2210	2266	2285							
	2946	2966	2233	2387	2300							
$\frac{3}{2}^-$	1871	1873	1764	1780	1785	1828	1792	1861	1820	1759	1801	1906 ± 29
	1879	1881	1798	1873	1880	1869		1971		1826	1918	1894 ± 38
	2337	2347	1904	1924	1895	1932					1976	2497 ± 61
	2349	2360	2245	2246	2240							2426 ± 73
	2786	2813	2252	2284	2305							
	2894	2913	2350	2353	2330							
	2912	2931	2352	2384	2340							
	3426	3465		2416	2385							
$\frac{5}{2}^-$	1859	1859	1853	1955	1900	1881				1883	1917	
	2318	2325	2333	2292	2345							
	2713	2736	2411	2409	2350							
	2733	2757		2425	2385							
	2865	2884		2438								
$\frac{7}{2}^-$	2647	2666	2460	2320	2355							
	2667	2687	2474		2425							
	3249	3280		2464								
	3274	3306		2481								
$\frac{9}{2}^-$	2588	2603	2502	2505								
	3173	3201		2570								

Ω Baryon

State	J^P	M1	M2	M_{exp}[1]	
1S	$\frac{3}{2}^+$	1672	1672		
2S	$\frac{3}{2}^+$	2057	2068		
3S	$\frac{3}{2}^+$	2429	2449		
4S	$\frac{3}{2}^+$	2852	2885		
State	J^P	M1	M2	M	M_{exp}[1]
$1^2P_{1/2}$	$\frac{1}{2}^-$	1987	1996	1979	
$1^2P_{3/2}$	$\frac{3}{2}^-$	1978	1985	1983	
$1^4P_{1/2}$	$\frac{1}{2}^-$	1992	2001	1977	
$1^4P_{3/2}$	$\frac{3}{2}^-$	1983	1991	1984	
$1^4P_{5/2}$	$\frac{5}{2}^-$	1970	1997	1985	
$2^2P_{1/2}$	$\frac{1}{2}^-$	2345	2363	2342	
$2^2P_{3/2}$	$\frac{3}{2}^-$	2332	2349	2346	
$2^4P_{1/2}$	$\frac{1}{2}^-$	2352	2370	2340	
$2^4P_{3/2}$	$\frac{3}{2}^-$	2339	2356	2346	
$2^4P_{5/2}$	$\frac{5}{2}^-$	2321	2338	2348	

State	J^P	M1	M2	M	M_{exp}[1]
$1^2D_{3/2}$	$\frac{3}{2}^+$	2269	2288	2263	
$1^2D_{5/2}$	$\frac{5}{2}^+$	2250	2267	2266	
$1^4D_{1/2}$	$\frac{1}{2}^+$	2291	2311	2258	
$1^4D_{3/2}$	$\frac{3}{2}^+$	2276	2295	2263	
$1^4D_{5/2}$	$\frac{5}{2}^+$	2257	2275	2266	
$1^4D_{7/2}$	$\frac{7}{2}^+$	2233	2249	2267	
$2^2D_{3/2}$	$\frac{3}{2}^+$	2671	2703	2671	
$2^2D_{5/2}$	$\frac{5}{2}^+$	2646	2676	2674	
$2^4D_{1/2}$	$\frac{1}{2}^+$	2699	2733	2666	
$2^4D_{3/2}$	$\frac{3}{2}^+$	2681	2713	2670	
$2^4D_{5/2}$	$\frac{5}{2}^+$	2656	2686	2674	
$2^4D_{7/2}$	$\frac{7}{2}^+$	2623	2652	2676	
$1^2F_{5/2}$	$\frac{5}{2}^-$	2585	2614	2578	
$1^2F_{7/2}$	$\frac{7}{2}^-$	2552	2579	2581	
$1^4F_{3/2}$	$\frac{3}{2}^-$	2622	2653	2573	
$1^4F_{5/2}$	$\frac{5}{2}^-$	2595	2625	2577	
$1^4F_{7/2}$	$\frac{7}{2}^-$	2562	2590	2582	
$1^4F_{9/2}$	$\frac{9}{2}^-$	2521	2548	2584	

Ω Baryon



Our result
Exp
3200
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2400
2200
2000
1800
1600
Mass(MeV)
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2012
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$\frac{3}{2}^+$
$\frac{5}{2}^+$
$\frac{7}{2}^+$
J^P
$\frac{1}{2}^-$
$\frac{3}{2}^-$
$\frac{5}{2}^-$
$\frac{7}{2}^-$
$\frac{9}{2}^-$

The Baryon Properties: Regge Trajectories, Magnetic Moment and Decay Width

An important property concluded from baryon spectrum is the plot of J, total angular momentum against M^2 as well as principle quantum number n against M^2. These lines are so far observed to be linear and non-intersecting for light baryon spectrum. These plots provide a confirmation between experimental and theoretical predicted masses of excited state with their respective quantum numbers. This holds true for positive and negative parity states as well. The equations are as follows;

$$J = aM^2 + c; \quad n = bM^2 + c_1 \tag{10}$$

Figure 1: Regge trajectory Δ $J \to M^2$

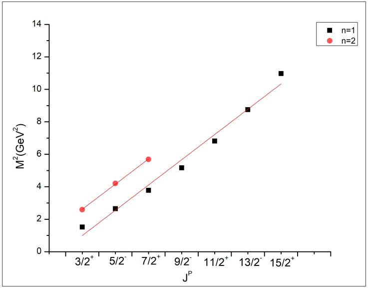


Figure 2: Regge trajectory Λ for $J \to M^2$

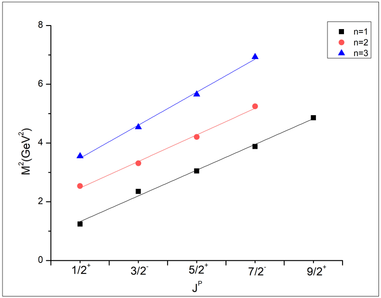


Figure 3: Regge trajectory Σ for $J \to M^2$

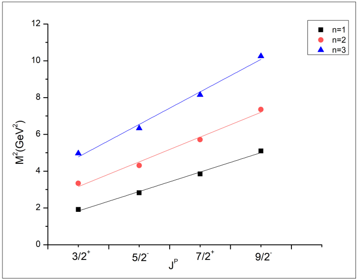
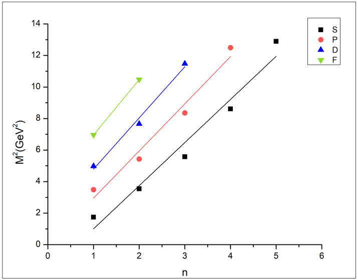


Figure 4: Regge trajectory Ξ for $n \to M^2$



Magnetic Moment

Baryon magnetic moment is expressed as

$$\mu_B = \sum_q \langle \phi_{sf} | \mu_{qz} | \phi_{sf} \rangle \tag{11}$$

Baryon	σ_{qz}	μ (μ_N)
Δ^{++}(uuu)	$3\mu_u$	4.568
Δ^{+}(uud)	$2\mu_u + \mu_d$	2.28
Δ^{0}(udd)	$2\mu_d + \mu_u$	0
Δ^{-}(ddd)	$3\mu_d$	-2.28
Ξ^{0}(uss)	$\frac{1}{3}(4\mu_s - \mu_u)$	-1.50
Ξ^{-}(dss)	$\frac{1}{3}(4\mu_s - \mu_d)$	-0.46
Ξ^{*0}(uss)	$(2\mu_s + \mu_u)$	0.766
Ξ^{*-}(dss)	$(2\mu_s + \mu_d)$	-1.962
Σ^{+}(uus)	$\frac{1}{3}(4\mu_u - \mu_s)$	2.79
Σ^{0}(uds)	$\frac{1}{3}(2\mu_u + 2\mu_d - \mu_s)$	0.839
Σ^{-}(dds)	$\frac{1}{3}(4\mu_d - \mu_s)$	-1.113
Σ^{*+}(uus)	$(2\mu_u + \mu_s)$	2.877
Σ^{*0}(uds)	$(\mu_u + \mu_d + \mu_s)$	0.353
Σ^{*-}(dds)	$(2\mu_d + \mu_s)$	-2.171
Λ^{0}(uds)	μ_s	-0.606

Radiative Decay Width

Transition magnetic moment as well as radiative decay width are also important in the understanding of internal structure of baryon as well as magnetic and electric transitions.

$$\mu(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}) = \langle B_{\frac{1}{2}^+}, S_z = \frac{1}{2} | \mu_z | B_{\frac{3}{2}^+}, S_z = \frac{1}{2} \rangle \tag{12}$$

$$\frac{2\sqrt{2}}{3}(\mu_u^{eff} - \mu_s^{eff}) \tag{13}$$

The effective mass here is a geometric mean of those for spin $\frac{1}{2}$ and $\frac{3}{2}$. Our result comes out to be $2.378\mu_N$ which is in good agreement with various models implemented. The radiative decay width is obtained as,

$$\Gamma_R = \frac{q^3}{m_p^2} \frac{2}{2J+1} \frac{e^2}{4\pi} |\mu_{\frac{3}{2}^+ \to \frac{1}{2}^+}|^2 \tag{14}$$

where q is the photon energy, m_p is the proton mass and J is the initial angular momentum giving $\Gamma_R = 0.214$ MeV. The total decay width available from experiment is 9.1 MeV. Thus, the branching ratio $\frac{\Gamma_R}{\Gamma_{total}}$ is 2.35% where the PDG data suggests $< 3.7\%$. Thus, it is in accordance with other results

Radiative Decay Width

Decay	Wave-function	Transition moment(in μ_N)	Γ_R(in MeV)
$\Delta^+ \to P\gamma$	$\frac{2\sqrt{2}}{3}(\mu_u - \mu_d)$	2.47	0.63
$\Delta^0 \to N\gamma$	$\frac{2\sqrt{2}}{3}(\mu_d - \mu_u)$	-2.48	0.59
$\Sigma^{*0} \to \Lambda^0\gamma$	$\frac{\sqrt{2}}{\sqrt{3}}(\mu_u - \mu_d)$	2.296	0.4256
$\Sigma^{*0} \to \Sigma^0\gamma$	$\frac{\sqrt{2}}{3}(\mu_u + \mu_d - 2\mu_s)$	0.923	0.0246
$\Sigma^{*+} \to \Sigma^+\gamma$	$\frac{2\sqrt{2}}{3}(\mu_u - \mu_s)$	2.204	0.1404
$\Sigma^{*-} \to \Sigma^-\gamma$	$\frac{2\sqrt{2}}{3}(\mu_d - \mu_s)$	-0.359	0.0037

Summary and Conclusion

- The non-relativistic constituent quark model with potential depending on hypercentral one has been used with the aim of obtaining all possible spin-parity states of light, strange baryons of octet and decuplet family.
- The earlier study lead with first order correction in mass gave reasonable results but the ordering of spin-parity could not be corrected. However, with $1/m^2$ correction the hierarchy of the splitting has been obtained which has been one of the important aspect of this study.
- The study has obtained a number of resonance states upto higher J values which is expected to be of great help in upcoming as well as ongoing experiments like PANDA, J-PARC, and so on.
- The comparison is not just with the experimental data but also with a wide range of theoretical and phenomenological approaches. This shall allow to reach the possible modifications of any approach to better interpret the internal quark dynamics.

Summary and Conclusion

- The higher strangeness baryons with least information with some puzzling states are expected to get a new insight to be looked for in experiments through this spectra.
- The properties derived based on the observed spectrum i.e. Regge trajectory follows the linear nature.
- The magnetic moment and radiative decay widths are quite comparable with other known results leading to the fact that the spectrum shall serve the purpose towards the search of yet unobserved resonances which in turn will lead to the improvement of theoretical understanding of the light, strange baryons.

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thank you

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