Low-lying Baryon Resonances from Lattice QCD

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Outline

- present recent results for Δ, $\Lambda(1405)$ resonances from lattice QCD
- excited-states energies in lattice QCD
- finite-volume energies $\Rightarrow$ scattering phase shifts
- hadron resonance properties: masses, decay widths
- extra: scalar glueball in full QCD

Collaborators

- this work done in collaboration with
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- computing resource:
  - Frontera at Texas Advanced Computing Center

Masses/widths of resonances from lattice QCD

- evaluate finite-volume energies of stationary states corresponding to decay products of resonance for variety of total momenta
- such energies obtained from Markov-chain Monte Carlo estimates of appropriate temporal correlation functions
- parametrize either the K-matrix or its inverse for the relevant scattering processes
- Lüscher quantization condition determines finite-volume spectrum from the K matrix
- determine best fit values of the parameters in the K-matrix by matching the spectrum from quantization condition to spectrum obtained from lattice QCD

Excited states from correlation matrices

- energies from temporal correlations $C_{ij}(t) = \langle 0|\overline{O}_i(t) O_j(0)|0\rangle$
- in finite volume, energies are discrete (neglect wrap-around)

$$C_{ij}(t) = \sum_n Z_i^{(n)} Z_j^{(n)*}\ e^{-E_n t}, \qquad Z_j^{(n)} = \langle 0|\ O_j\ |n\rangle$$

- not practical to do fits using above form
- define new correlation matrix $\widetilde{C}(t)$ using a single rotation

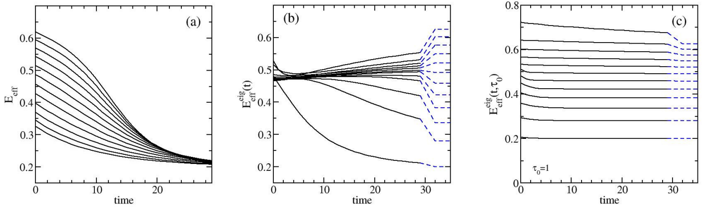
$$\widetilde{C}(t) = U^\dagger\ C(\tau_0)^{-1/2}\ C(t)\ C(\tau_0)^{-1/2}\ U$$

- columns of U are eigenvectors of $C(\tau_0)^{-1/2}\, C(\tau_D)\, C(\tau_0)^{-1/2}$
- choose τ_0 and τ_D large enough so $\widetilde{C}(t)$ diagonal for $t > \tau_D$
- 2-exponential fits to $\widetilde{C}_{\alpha\alpha}(t)$ yield energies E_α and overlaps $Z_j^{(n)}$
- energy shifts from non-interacting using 1-exp fits to ratio of correlators

Correlator matrix toy model

- example: 12×12 correlator matrix with $N_e = 200$ eigenstates

$$E_0 = 0.20, \qquad E_n = E_{n-1} + \frac{0.08}{\sqrt{n}}, \qquad Z_j^{(n)} = \frac{(-1)^{j+n}}{1 + 0.05(j-n)^2}.$$



- left: effective energies of diagonal elements of correlator matrix
- middle: effective energies of eigenvalues of $C(t)$
- right: effective energies of eigenvalues of $C(\tau_0)^{-1/2}\, C(t)\, C(\tau_0)^{-1/2}$ for $\tau_0 = 1$

Building blocks for single-hadron operators

- important to use good operators to see signal before noise growth
- building blocks: covariantly-displaced LapH-smeared quark fields
- stout links $\widetilde{U}_j(x)$
- Laplacian-Heaviside (LapH) smeared quark fields

$$\widetilde{\psi}_{a\alpha}(x) = \mathcal{S}_{ab}(x,y)\,\psi_{b\alpha}(y), \qquad \mathcal{S} = \Theta\left(\sigma_s^2 + \widetilde{\Delta}\right)$$

- 3d gauge-covariant Laplacian $\widetilde{\Delta}$ in terms of $\widetilde{U}$
- displaced quark fields:

$$q^A_{a\alpha j} = D^{(j)}\widetilde{\psi}^{(A)}_{a\alpha}, \qquad \overline{q}^A_{a\alpha j} = \widetilde{\overline{\psi}}^{(A)}_{a\alpha}\,\gamma_4\,D^{(j)\dagger}$$

- displacement $D^{(j)}$ is product of smeared links:

$$D^{(j)}(x,x') = \widetilde{U}_{j_1}(x)\,\widetilde{U}_{j_2}(x+d_2)\,\widetilde{U}_{j_3}(x+d_3)\ldots\widetilde{U}_{j_p}(x+d_p)\delta_{x',\,x+d_{p+1}}$$

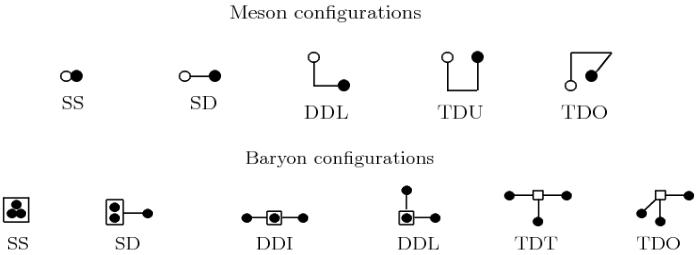
- to good approximation, LapH smearing operator is

$$\mathcal{S} = V_s V_s^\dagger$$

  - columns of matrix V_s are eigenvectors of $\widetilde{\Delta}$

Extended operators for single hadrons

- quark displacements build up orbital, radial structure



$$\overline{\Phi}^{AB}_{\alpha\beta}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot(\mathbf{x}+\frac{1}{2}(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta))}\delta_{ab}\; \overline{q}^{B}_{b\beta}(\boldsymbol{x},t)\; q^{A}_{a\alpha}(\boldsymbol{x},t)$$

$$\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\mathbf{x}}\varepsilon_{abc}\; \overline{q}^{C}_{c\gamma}(\boldsymbol{x},t)\; \overline{q}^{B}_{b\beta}(\boldsymbol{x},t)\; \overline{q}^{A}_{a\alpha}(\boldsymbol{x},t)$$

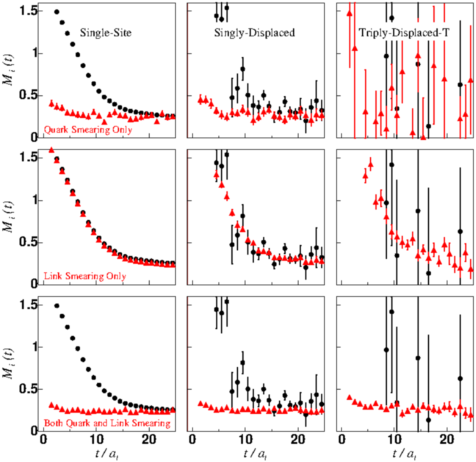
- group-theory projections onto irreps of lattice symmetry group

$$\overline{M}_l(t) = c^{(l)*}_{\alpha\beta}\; \overline{\Phi}^{AB}_{\alpha\beta}(t) \qquad \overline{B}_l(t) = c^{(l)*}_{\alpha\beta\gamma}\; \overline{\Phi}^{ABC}_{\alpha\beta\gamma}(t)$$

- definite momentum $\boldsymbol{p}$, irreps of little group of $\boldsymbol{p}$

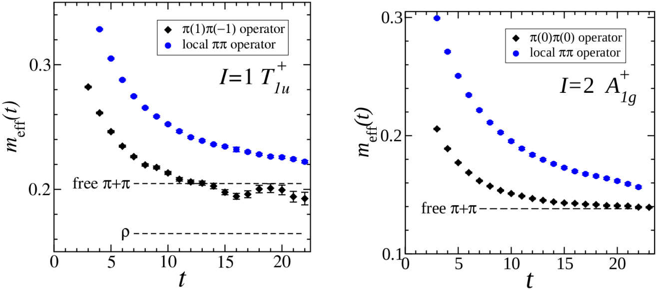
Importance of smeared fields

- effective masses of 3 selected nucleon operators shown
- noise reduction of displaced-operators from link smearing
  $n_\rho\rho = 2.5, n_\rho = 16$
- quark-field smearing
  $\sigma_s = 4.0, n_\sigma = 32$
  reduces excited-state contamination



Two-hadron operators

- comparison of $\pi(\boldsymbol{k})\pi(-\boldsymbol{k})$ and localized $\sum_{\boldsymbol{x}} \pi(\boldsymbol{x})\pi(\boldsymbol{x})$ operators



- important to use superposition of products of single-hadron operators of definite momenta
- efficient construction, generalizes to three or more hadrons

Quark propagation

- quark propagator Q is inverse D^{-1} of Dirac matrix
  - rows/columns involve lattice site, spin, color
  - very large $N_{\rm tot} \times N_{\rm tot}$ matrix for each flavor

$$N_{\rm tot} = N_{\rm site} N_{\rm spin} N_{\rm color}$$

  - for $64^3 \times 128$ lattice, $N_{\rm tot} \sim 400$ million
- not feasible to compute (or store) all elements of D^{-1}
- point-to-all trick for local operators: use translation invariance

$$\sum_{\mathbf{y}} \sum_{\mathbf{x}} Q^{(a)}(\mathbf{y}, t_f|\mathbf{x}, t_i)\, Q^{(b)}(\mathbf{y}, t_f|\mathbf{x}, t_i) \cdots \longrightarrow \sum_{\mathbf{y}} Q^{(a)}(\mathbf{y}, t_f|\mathbf{x}_0, t_i)\, Q^{(b)}(\mathbf{y}, t_f|\mathbf{x}_0, t_i) \dots$$

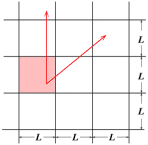
- cannot use this trick for good multi-hadron operators

$$\sum_{\mathbf{y}_1, \mathbf{y}_2, \dots} \sum_{\mathbf{x}_1, \mathbf{x}_2, \dots} Q^{(a)}(\mathbf{y}_1, t_f|\mathbf{x}_1, t_i)\, Q^{(b)}(\mathbf{y}_2, t_f|\mathbf{x}_2, t_i) \dots$$

- our solution: the stochastic LapH method!
- Monte Carlo estimates of D^{-1} with clever variance reduction

Quantum numbers in toroidal box

- periodic boundary conditions in cubic box
  - not all directions equivalent $\Rightarrow$ using J^{PC} is wrong!!



- label stationary states of QCD in a periodic box using irreps of cubic space group even in continuum limit
  - zero momentum states: little group O_h

$$A_{1a}, A_{2ga}, E_a, T_{1a}, T_{2a}, \quad G_{1a}, G_{2a}, H_a, \qquad a = g, u$$

  - on-axis momenta: little group C_{4v}

$$A_1, A_2, B_1, B_2, E, \quad G_1, G_2$$

  - planar-diagonal momenta: little group C_{2v}

$$A_1, A_2, B_1, B_2, \quad G_1, G_2$$

  - cubic-diagonal momenta: little group C_{3v}

$$A_1, A_2, E, \quad F_1, F_2, G$$

- include G parity in some meson sectors (superscript $+$ or $-$)

Spin content of cubic box irreps

- numbers of occurrences of Λ irreps in J subduced

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1
5	0	0	1	2	1
6	1	1	1	1	2
7	0	1	1	2	2

J	G_1	G_2	H	J	G_1	G_2	H
$\frac{1}{2}$	1	0	0	$\frac{9}{2}$	1	0	2
$\frac{3}{2}$	0	0	1	$\frac{11}{2}$	1	1	2
$\frac{5}{2}$	0	1	1	$\frac{13}{2}$	1	2	2
$\frac{7}{2}$	1	1	1	$\frac{15}{2}$	1	1	3

Scattering phase shifts from lattice QCD

- each finite-volume energy E related to S matrix (and phase shifts) by the quantization condition
$$\det[1 + F^{(\boldsymbol{P})}(S-1)] = 0$$
- F matrix in $JLSa$ basis states given by
$$\langle J'm_{J'}L'S'a'|F^{(\boldsymbol{P})}|Jm_JLSa\rangle = \delta_{a'a}\delta_{S'S}\ \frac{1}{2}\Big\{\delta_{J'J}\delta_{m_{J'}m_J}\delta_{L'L} + \langle J'm_{J'}|L'm_{L'}Sm_S\rangle\langle Lm_LSm_S|Jm_J\rangle W^{(\boldsymbol{P}a)}_{L'm_{L'};\ Lm_L}\Big\}$$
- total ang mom J, J', orbital L, L', spin S, S', channels a, a'
- W given by
$$-iW^{(\boldsymbol{P}a)}_{L'm_{L'};\ Lm_L} = \sum_{l=|L'-L|}^{L'+L}\sum_{m=-l}^{l}\frac{\mathcal{Z}_{lm}(\boldsymbol{s}_a,\gamma,u_a^2)}{\pi^{3/2}\gamma u_a^{l+1}}\sqrt{\frac{(2L'+1)(2l+1)}{(2L+1)}} \times\langle L'0, l0|L0\rangle\langle L'm_{L'}, lm|Lm_L\rangle.$$
- compute Rummukainen-Gottlieb-Lüscher (RGL) shifted zeta functions $\mathcal{Z}_{lm}$

Kinematics

- work in spatial L^3 volume with periodic b.c.
- total momentum $\boldsymbol{P} = (2\pi/L)\boldsymbol{d}$, where $\boldsymbol{d}$ vector of integers
- calculate lab-frame energy E of two-particle interacting state in lattice QCD
- boost to center-of-mass frame by defining:

$$E_{\rm cm} = \sqrt{E^2 - \boldsymbol{P}^2}, \qquad \gamma = \frac{E}{E_{\rm cm}},$$

- assume N_d channels
- particle masses m_{1a}, m_{2a} and spins s_{1a}, s_{2a} of particle 1 and 2
- for each channel, can calculate

$$\begin{aligned} \boldsymbol{q}^2_{{\rm cm},a} &= \frac{1}{4}E^2_{\rm cm} - \frac{1}{2}(m^2_{1a} + m^2_{2a}) + \frac{(m^2_{1a} - m^2_{2a})^2}{4E^2_{\rm cm}}, \\ u^2_a &= \frac{L^2\boldsymbol{q}^2_{{\rm cm},a}}{(2\pi)^2}, \qquad \boldsymbol{s}_a = \left(1 + \frac{(m^2_{1a} - m^2_{2a})}{E^2_{\rm cm}}\right)\boldsymbol{d} \end{aligned}$$

K matrix

- quantization condition relates single energy E to entire S-matrix
- cannot solve for S-matrix (except single channel, single wave)
- approximate S-matrix with functions depending on handful of fit parameters
- obtain estimates of fit parameters using many energies
- easier to parametrize Hermitian matrix than unitary matrix
- introduce K-matrix (Wigner 1946)

$$S = (1 + iK)(1 - iK)^{-1} = (1 - iK)^{-1}(1 + iK)$$

- Hermiticity of K-matrix ensures unitarity of S-matrix
- with time reversal invariance, K-matrix must be real and symmetric
- multichannel effective range expansion (Ross 1961)

$$K^{-1}_{L'S'a';\,LSa}(E) = q_{a'}^{-L'-\frac{1}{2}}\ \widetilde{K}^{-1}_{L'S'a';\,LSa}(E_{\rm cm})\ q_a^{-L-\frac{1}{2}},$$

Quantization condition

- quantization condition can be written

$$\det(1 - B^{(\boldsymbol{P})}\widetilde{K}) = \det(1 - \widetilde{K}B^{(\boldsymbol{P})}) = 0$$

- we define the box matrix by

$$\langle J'm_{J'}L'S'a'|\; B^{(\boldsymbol{P})}\; |Jm_J LSa\rangle = -i\delta_{a'a}\delta_{S'S}\; u_a^{L'+L+1}\; W^{(\boldsymbol{P}a)}_{L'm_{L'};\; Lm_L}$$
$$\times\langle J'm_{J'}|L'm_{L'}, Sm_S\rangle\langle Lm_L, Sm_S|Jm_J\rangle$$

- box matrix is Hermitian for u_a^2 real
- quantization condition can also be expressed as

$$\det(\widetilde{K}^{-1} - B^{(\boldsymbol{P})}) = 0$$

- these determinants are real

Block diagonalization

- quantization condition involves determinant of infinite matrix
- make practical by (a) transforming to a block-diagonal basis and (b) truncating in orbital angular momentum
- block-diagonal basis

$$|\Lambda\lambda nJLSa\rangle = \sum_{m_J} c_{m_J}^{J(-1)^L;\,\Lambda\lambda n} |Jm_J LSa\rangle$$

- little group irrep Λ, irrep row λ, occurrence index n
- transformation coefficients depend on J and $(-1)^L$, not on S, a
- replaces m_J by (Λ, λ, n)
- group theoretical projections with Gram-Schmidt used to obtain coefficients
- use notation and irrep matrices from PRD 88, 014511 (2013)
- box matrix elements computed using C++ software available on github: TwoHadronsInBox
- reference: NPB924, 477 (2017)

Current ensemble

- currently using CLS D200 ensemble
- size: $64^3 \times 128$ lattice, $a \sim 0.066$ fm
- open boundary conditions in time
- number of configs = 2000
- quark masses: $m_\pi \sim 200$ MeV, $m_K \sim 480$ MeV
- smearing: $N_{\mathrm{ev}} = 448$
- sources:
  $t_0 = 35$ forward,
  $t_0 = 64$ forward and backward,
  $t_0 = 92$ backward
- software: common subexpression elimination with tensor contractions (Ben Hörz)
- heavy use of batched BLAS routines

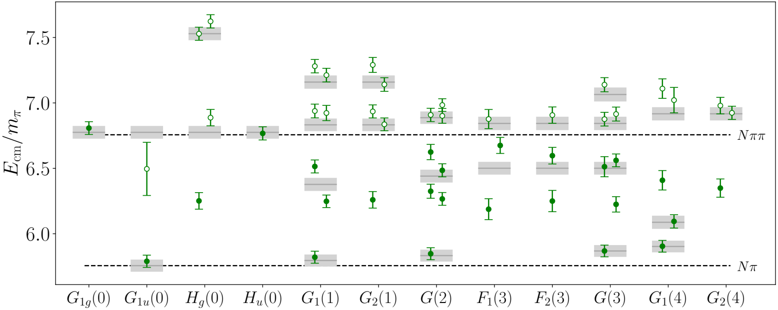
Flavor channels

Isospin channel	D200 Number of Correlators
$I = 0,\ S = 0,\ NN$	8357
$I = 0,\ S = -1,\ \Lambda, N\overline{K}, \Sigma\pi$ (45 SH)	8143
$I = \frac{1}{2},\ S = 0,\ N\pi$	696
$I = \frac{1}{2},\ S = -1,\ N\Lambda, N\Sigma$	17816
$I = 1,\ S = 0,\ NN$ (66 SH)	7945
$I = \frac{3}{2},\ S = 0,\ \Delta, N\pi$	3218
$I = \frac{3}{2},\ S = -1,\ N\Sigma$	23748
$I = 0,\ S = -2,\ \Lambda\Lambda, N\Xi, \Sigma\Sigma$ (66 SH)	16086
$I = 2,\ S = -2,\ \Sigma\Sigma$ (66 SH)	4589
Single hadrons	33

Motivation

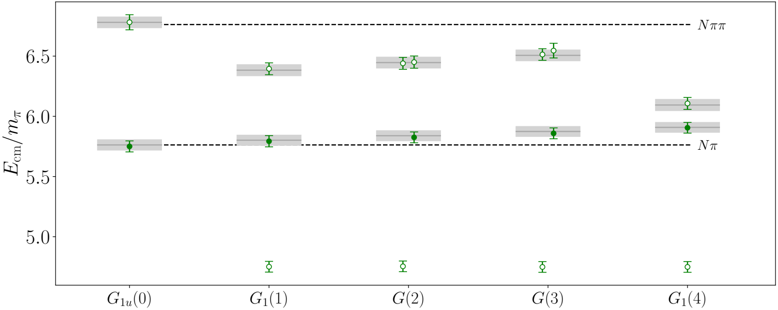
- meson-baryon amplitudes useful for pheno. at $m_\pi^{\rm phys}$ and for chiral EFT's at varying $m_\pi^{\rm phys}$.
- $\Delta(1232) \rightarrow N\pi$ used as a d.o.f. in some EFT's
- scattering lengths $a_{N\pi}^{I=3/2}$ and $a_{N\pi}^{I=1/2}$ impact lattice-pheno. discrepancy for $\sigma_{\pi N}$, relevant for dark matter direct detection. (see arxiv:1602.07688)
- recent Δ-resonance study in Nucl. Phys. B987, 116105 (2023)
- lattice QCD is good laboratory to study $\Lambda(1405)$ by varying quark masses.

$I = 3/2\ N\pi$ spectrum determination



- irreps with leading $(2J, L) = (3, 1)$ wave: $H_g(0)$, $G_2(1)$, $F_1(3)$, $G_2(4)$.
- irrep with leading $(1, 0)$ wave: $G_{1u}(0)$.
- irrep with leading $(1, 1)$ wave: $G_{1g}(0)$ not included because ground state is inelastic.
- irreps with s- and p-wave mixing: $G_1(1)$, $G(2)$, $G_1(4)$.

$I = 1/2$ spectrum determination



- isodoublet $N\pi$ spectrum

Parametrization of K-matrix

- each partial wave parametrized using effective range expansion
- remember $\sqrt{s} = E_{\rm cm} = \sqrt{m_\pi^2 + q_{\rm cm}^2} + \sqrt{m_N^2 + q_{\rm cm}^2}$
- for $I = 3/2$, $J^P = 3/2^+$ wave

$$\frac{q_{\rm cm}^3}{m_\pi^3} \cot \delta_{3/2^+} = \frac{6\pi\sqrt{s}}{m_\pi^3 g_{\Delta,\rm BW}^2}(m_\Delta^2 - s),$$

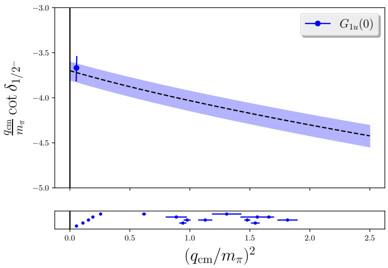
- other waves, used

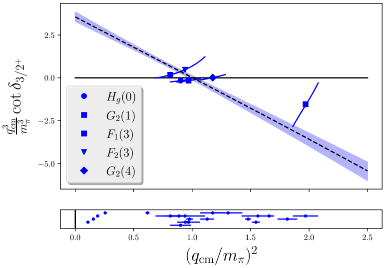
$$\frac{q_{\rm cm}^{2\ell+1}}{m_\pi^{2\ell+1}} \cot \delta_{J^P}^I = \frac{\sqrt{s}}{m_\pi A_{J^P}^I},$$

- fit parameter $A_{J^P}^I$ related to scattering length by

$$m_\pi^{2\ell+1} a_{J^P}^I = \frac{m_\pi}{m_\pi + m_N} A_{J^P}^I.$$

Isoquartet scattering amplitudes

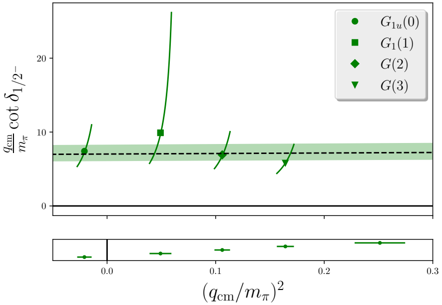




- $I = 3/2$ s- and p-wave scattering amplitudes
- mass and width parameter of Δ-resonance

$$\frac{m_\Delta}{m_\pi} = 6.257(35), \qquad g_{\Delta,\mathrm{BW}} = 14.41(53),$$

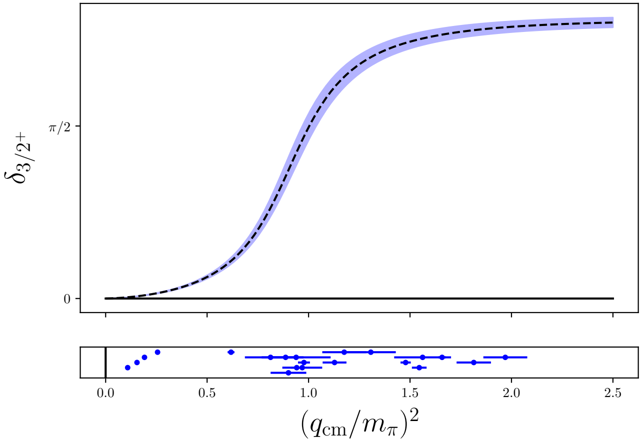
$I = 1/2$ scattering amplitudes



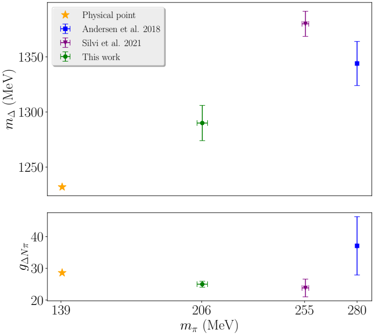
- scattering lengths

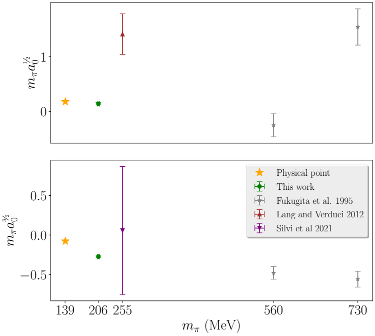
$$m_\pi a_0^{3/2} = -0.2735(81)\,, \qquad m_\pi a_0^{1/2} = 0.142(22),$$

Δ resonance



Comparison to previous works



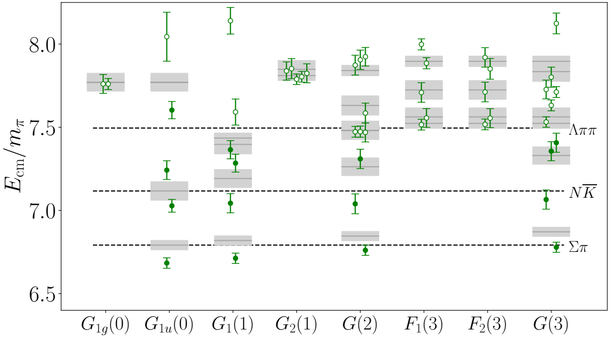


- above, $g_{\Delta N\pi}$ is defined in terms of the decay width in leading-order chiral effective theory

$$\Gamma^{\rm LO}_{\rm EFT} = \frac{g^2_{\Delta N\pi}}{48\pi}\,\frac{E_N + m_N}{E_N + E_\pi}\,\frac{q^3}{m_N^2}$$

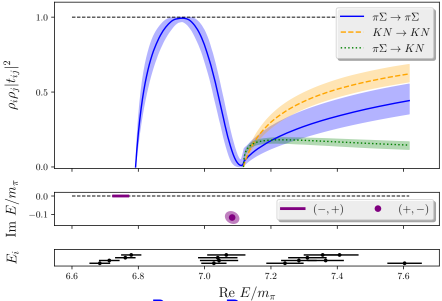
Study of $\Lambda(1405)$ resonance

- Study of elusive $\Lambda(1405)$ nearly completed
- CLS D200 ensemble with $m_\pi \approx 200$ MeV
- Finite volume spectrum of $\Sigma\pi$ and $N\overline{K}$ states below



Study of $\Lambda(1405)$ resonance

- PDG lists $\Lambda(1405)$ as single $I=0,\ J^P=\frac{1}{2}^-$ resonance strangeness -1
- Recent models based on chiral effective theory and unitary suggest two nearby overlapping poles
- Our study supports two-pole structure (preliminary)
- Virtual bound state below $\Sigma\pi$ threshold, resonance pole below $N\overline{K}$ threshold
- First lattice QCD study of this coupled-channel system using full operator set

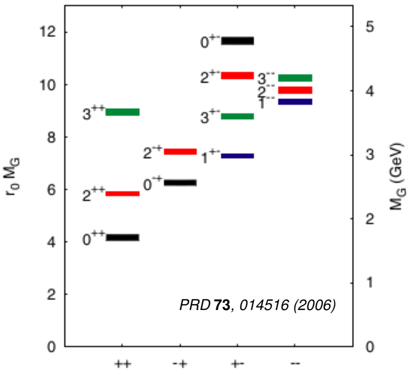


K matrix parametrization

- For best parametrization, used $\ell_{\max} = 0$ in ERE
$$\frac{E_{\rm cm}}{M_\pi}\tilde{K}_{ij} = A_{ij} + B_{ij}\Delta_{\pi\Sigma}$$
- where A_{ij} and B_{ij} are symmetric and real coefficients with i and j denoting either of the two scattering channels, and
$$\Delta_{\pi\Sigma} = (E_{\rm cm}^2 - (M_\pi + M_\Sigma)^2)/(M_\pi + M_\Sigma)^2$$
- other parametrizations also used:
    - an ERE for $\tilde{K}^{-1}$
    - removing factor of $E_{\rm cm}$ above
    - Blatt-Biedenharn form
- forms with one pole strongly disfavored

The scalar glueball

- glueball: hypothetical bound state of gluons
- experimental evidence elusive, light scalar candidates:
  - $f_0(1370), f_0(1500), f_0(1710)$
- lattice studies to date:
  - light scalar $\sim 1600 - 1700$ MeV
  - most in pure $SU(3)$ or quenched approx. (no quark/meson mixing!)



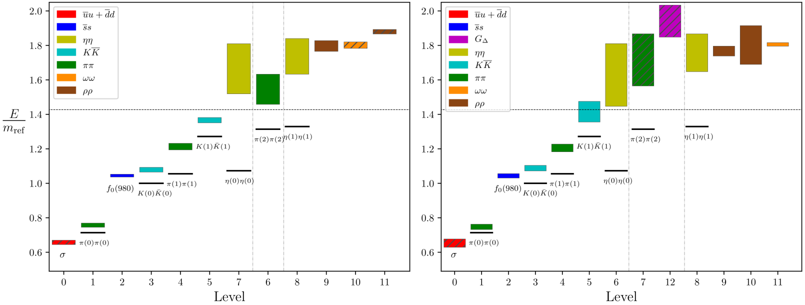
- here: extract low-lying A_{1g}^+ spectrum with $q\overline{q}$, meson-meson, & glueball operators
  - first look (from the lattice) at mixing between glueball, $q\overline{q}$, and two-hadron states

Why are glueballs with quarks so hard in lattice QCD?

- must extract all levels lying below glueballs of interest
- many 2-meson, 3-meson, 4-meson levels expected below
- 2-meson correlators require timeslice-to-timeslice propagators
- glueballs expected to be resonances
- glueballs require high statistics: difficult with quarks
- scalar sector requires large VEV subtraction

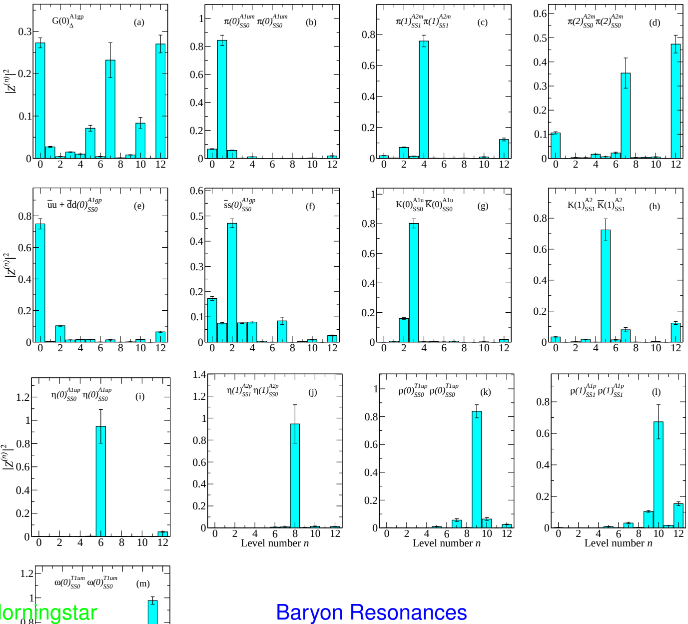
A_{1g}^{+} spectrum

- $24^3 \times 128$ anisotropic lattice, $m_\pi \sim 390$ MeV



- bad news for the scalar glueball?

A_{1g}^{+} overlaps



Summary

- stochastic LapH method works very well
  - allows evaluation of all needed quark-line diagrams
- large numbers of excited-state energy levels can be estimated
- scattering phase shifts can be computed
- infinite-volume resonance parameters from finite-volume energies below 2 particle thresholds
- hadron resonance properties: masses, decay widths
- presented recent results for Δ, $\Lambda(1405)$ resonances
- scalar glueball in full QCD
- next study: the Roper resonance (need for three-particle states)