



# Survey of hadronic molecules

Feng-Kun Guo

Institute of Theoretical Physics, Chinese Academy of Sciences

HADRON2023



June 5-9, 2023  
Genova, Italy

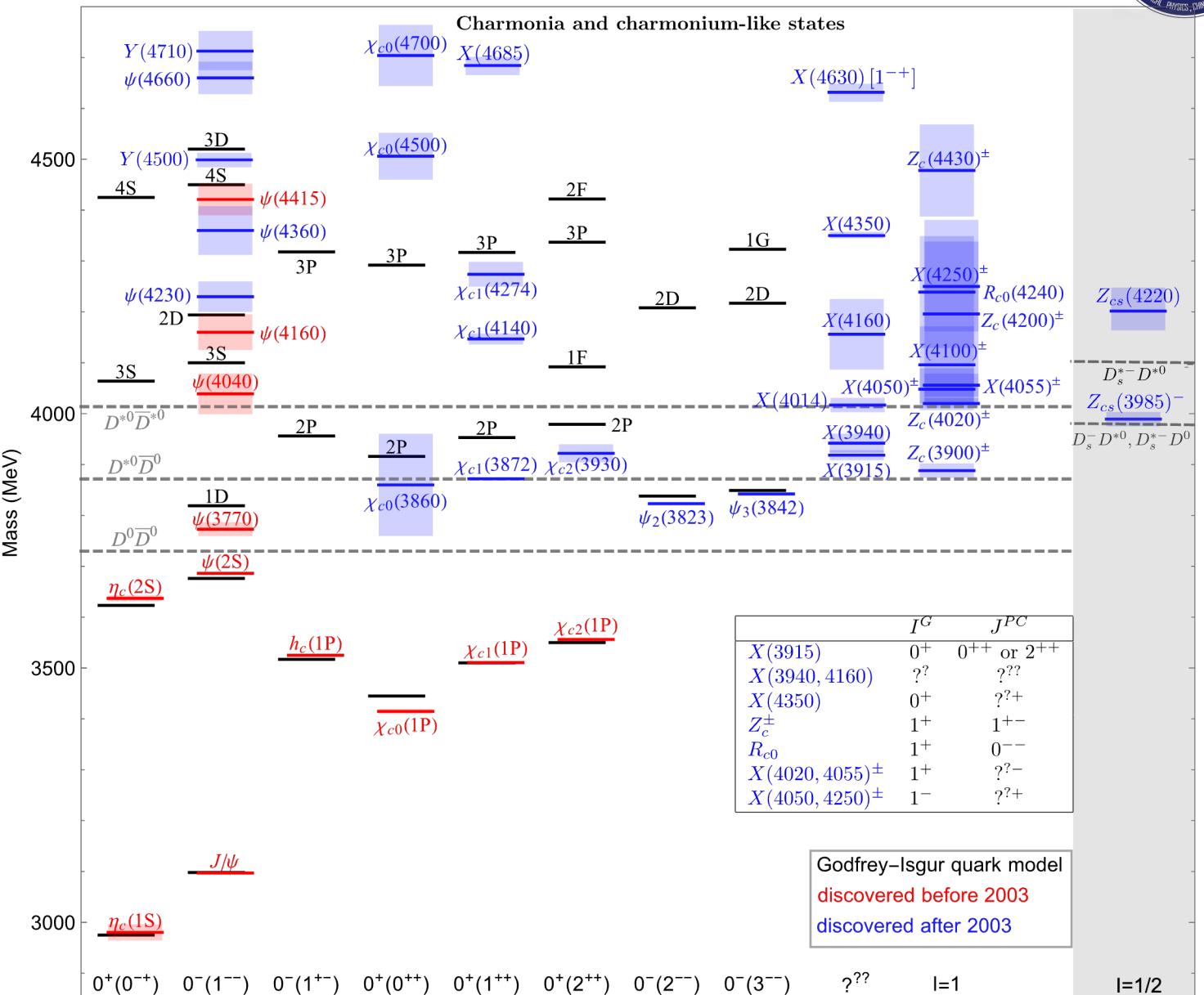
# Charmonia and charmonium-like structures

- Abundance of new states from peak hunting

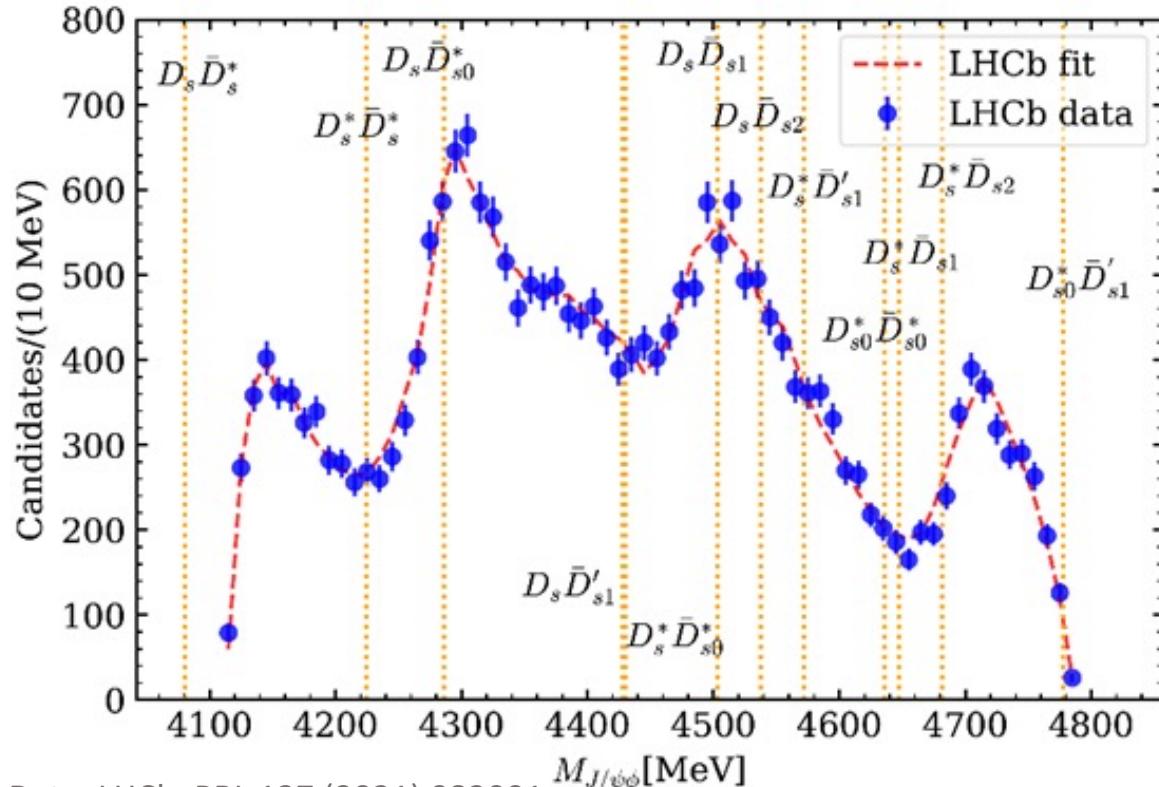
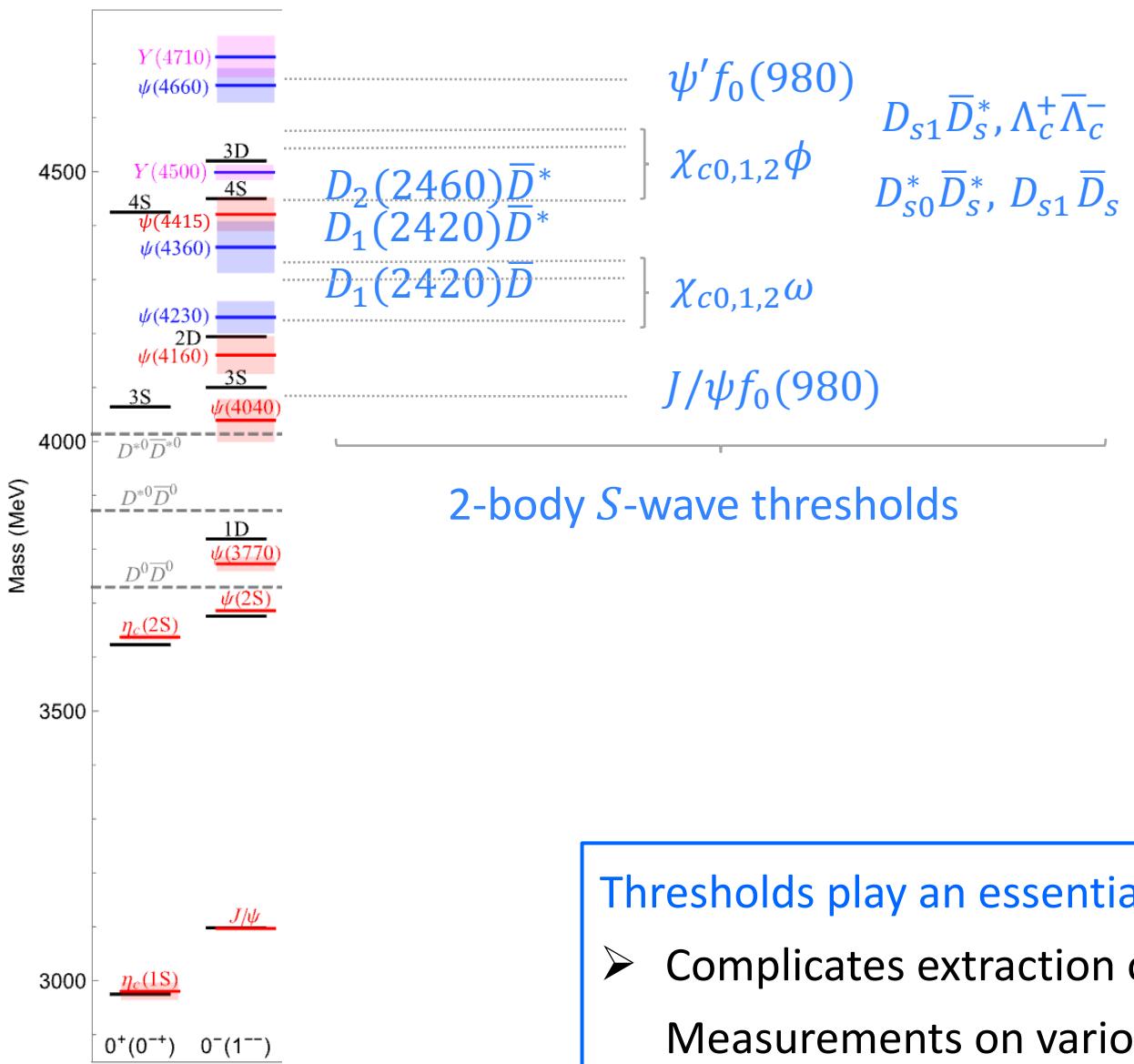
- $b$ -hadron ( $B, \Lambda_b$ ) decays
- Hadron/heavy-ion collisions
- $\gamma\gamma$  processes
- $e^+e^-$  collisions

- What are they?

- Nonperturbative QCD  $\Rightarrow$  difficult!



# Many thresholds above 4 GeV



Data: LHCb, PRL 127 (2021) 082001

Plot: X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]

## Thresholds play an essential role

- Complicates extraction of resonance properties!
- Measurements on various final states are important
- Hadronic molecules?

# Hadronic molecules

- Composite systems of hadrons

- analogues of the deuteron ( $\approx pn$  bound state)
- bound by the residual strong force, more extended than  $1/\Lambda_{\text{QCD}}$

- Compositeness 1 –  $Z$

S. Weinberg (1965); V. Baru et al. (2004); T. Hyodo et al. (2012); F. Aceti, E. Oset (2012); Z.-H. Guo, J. Oller (2016); I. Matuschek et al. (2021); J. Song et al. (2022); M. Albaladejo, J. Nieves (2022) ; .... for reviews, see T. Hyodo, IJMPA 28 (2013) 1330045; FKG, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, RMP 90 (2018) 015004

- probability of finding the physical state in two-hadron component (S-wave loosely bound)
- can be expressed in terms of low-energy observables

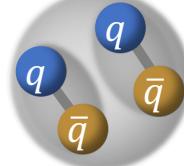
➤ coupling constant       $g_{\text{NR}}^2 \approx (1 - Z) \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}$        $E_B$ : binding energy;  $\mu$ : reduced mass

➤ ERE parameters (scattering length, effective range)  $a \approx -\frac{2(1 - Z)}{(2 - Z)\sqrt{2\mu E_B}}, r_e \approx -\frac{Z}{(1 - Z)\sqrt{2\mu E_B}}$  (for  $r_e \leq 0$ )

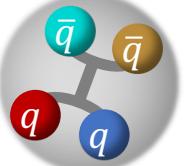
➤ phase shift       $1 - Z = 1 - \exp\left(\frac{1}{\pi} \int_0^\infty dE \frac{\delta(E)}{E - E_B}\right)$       Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

- ✓ derived with separable  $T$ -matrix & pole-dominance approximation of  $\delta(E)$
- ✓ valid independent of the sign of  $r_e$

## Different confinement pictures



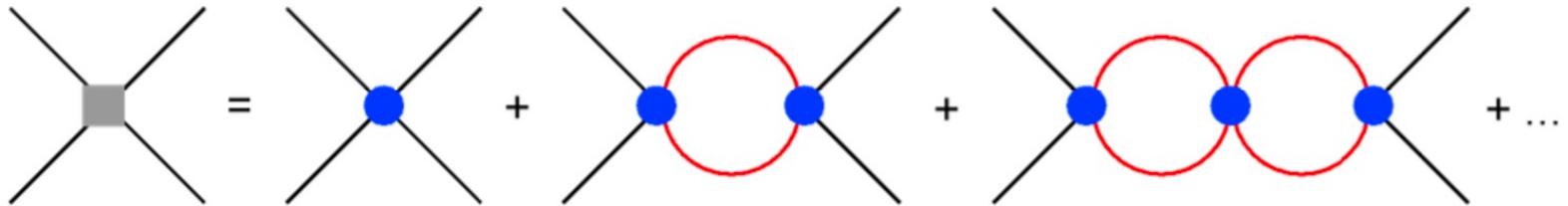
hadronic molecule



compact tetraquark

# Hadronic molecules in a NREFT at leading order

- Consider two hadrons in  $S$ -wave, near-threshold region  $\Rightarrow$  nonrelativistic EFT



$$\begin{aligned}
 T_{\text{NR}}(E) &= C_0 + C_0 G_{\text{NR}}(E) C_0 + C_0 G_{\text{NR}}(E) C_0 G_{\text{NR}}(E) C_0 + \dots \\
 &= \frac{1}{C_0^{-1} - G_{\text{NR}}(E)} = \frac{2\pi/\mu}{2\pi/(\mu C_0^r) - \sqrt{-2\mu E - i\epsilon}}
 \end{aligned}$$

□ Effective coupling:  $g_{\text{NR}}^2 = \lim_{E \rightarrow -E_B} (E + E_B) T_{\text{NR}}(E) = \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}$

□ Recall  $g_{\text{NR}}^2 \approx (1 - Z) \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}$ , the pole obtained at LO NREFT with a constant contact term is **purely composite**

➤ Range corrections: other components at shorter distances

✧ coupling to additional states/channels

✧ energy/momentum-dependent interactions: higher order

# Molecular line shapes at LO

- Poles at LO NREFT: bound or virtual state

- Bound and virtual state can hardly be distinguished above threshold ( $E > 0$ )

$$|T_{\text{NR}}(E)|^2 \propto \left| \frac{1}{\pm\kappa + i\sqrt{2\mu E}} \right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

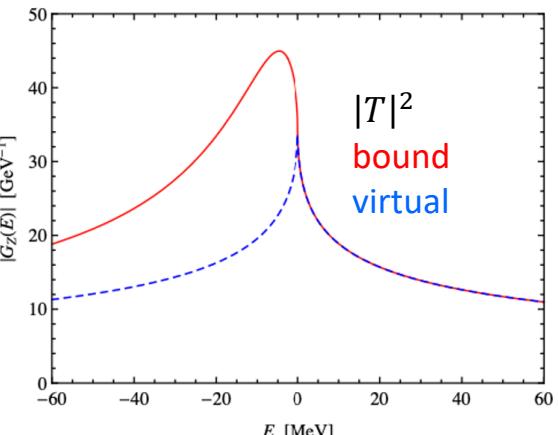
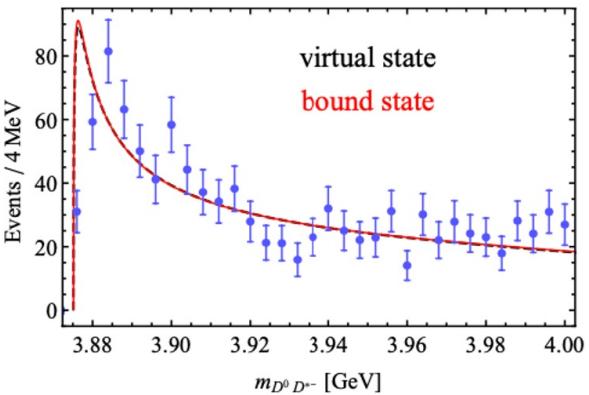
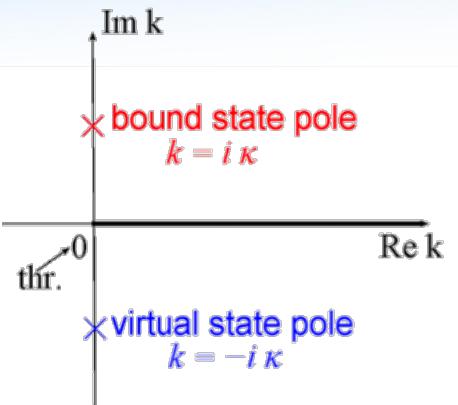
- Different below threshold ( $E < 0$ )

- bound state: peaked below threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

- virtual state: sharp cusp at threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa + \sqrt{-2\mu E})^2}$$



# Molecular line shapes at LO

- Poles at LO NREFT: bound or virtual state

- Bound and virtual state can hardly be distinguished above threshold ( $E > 0$ )

$$|T_{\text{NR}}(E)|^2 \propto \left| \frac{1}{\pm\kappa + i\sqrt{2\mu E}} \right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

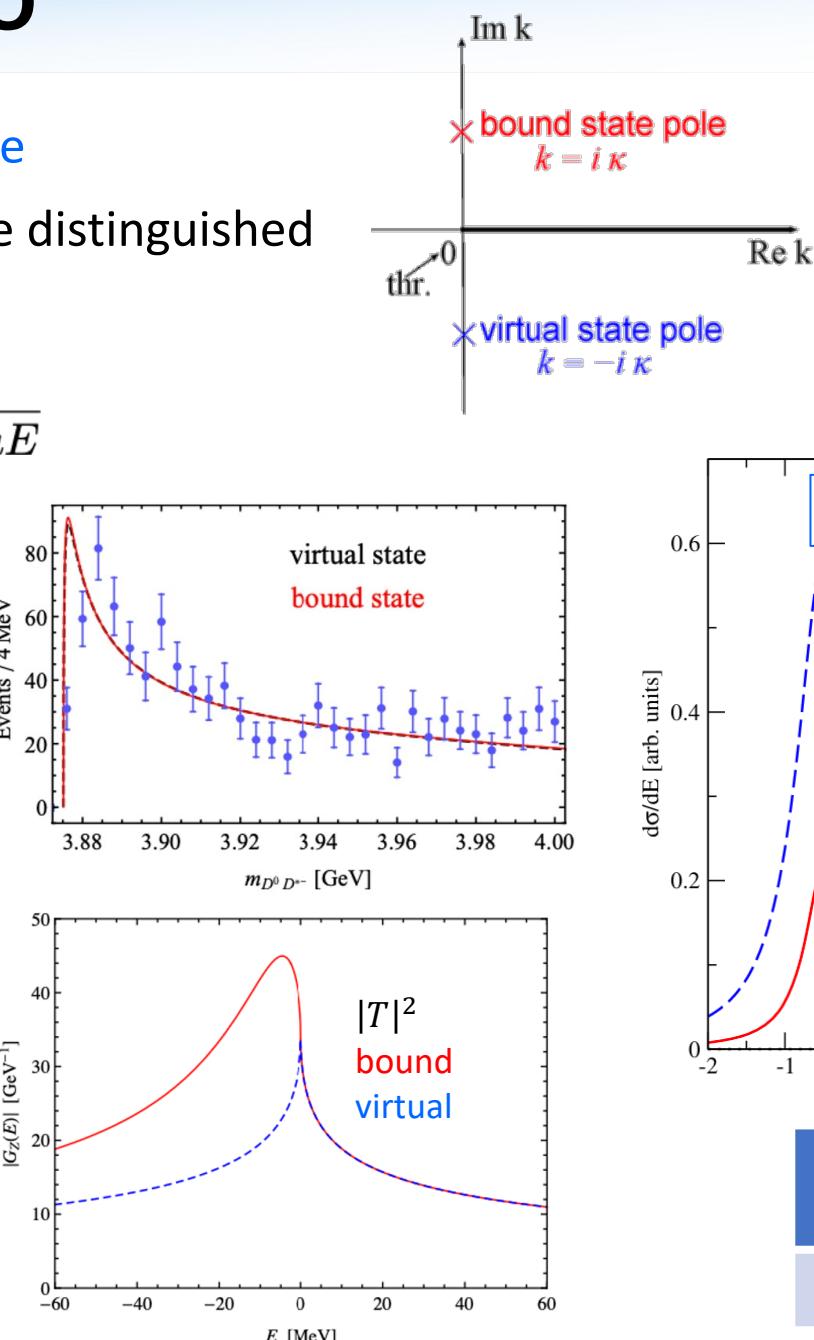
- Different below threshold ( $E < 0$ )

- bound state: peaked below threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

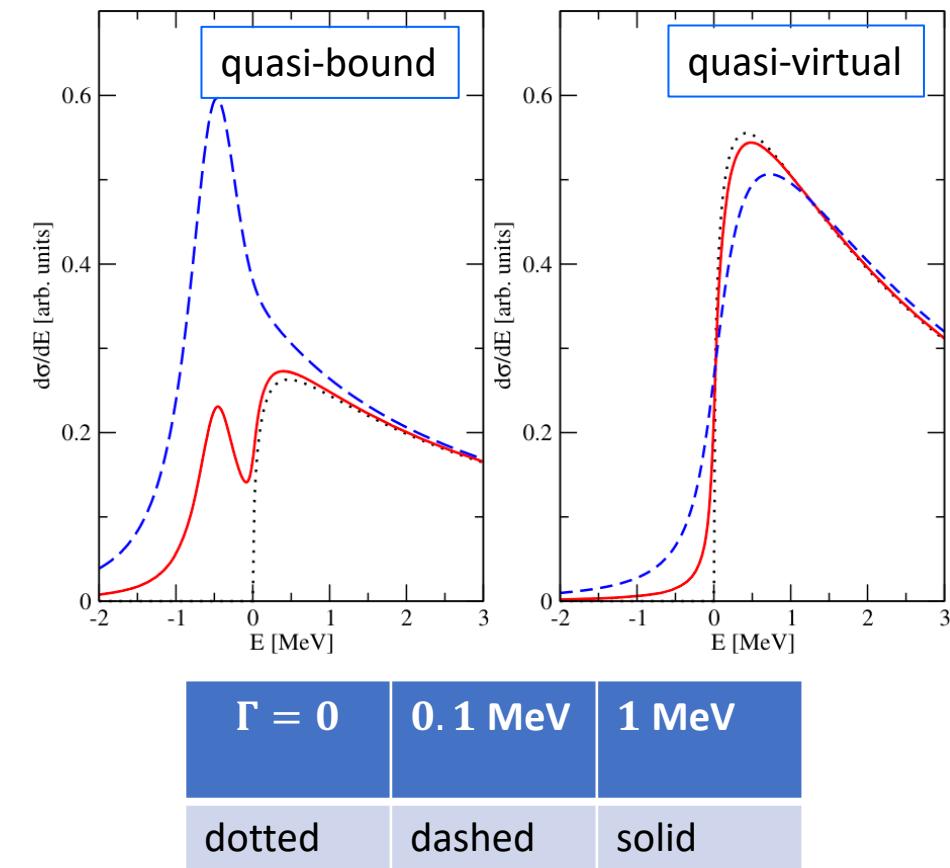
- virtual state: sharp cusp at threshold

$$|T_{\text{NR}}(E)|^2 \propto \frac{1}{(\kappa + \sqrt{-2\mu E})^2}$$



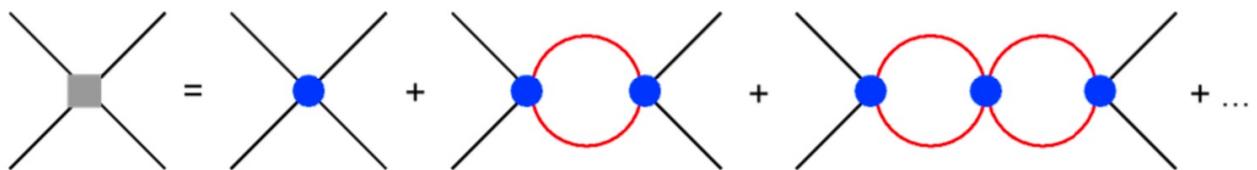
FKG, et al., RMP 90 (2018) 015004;  
N. Brambilla et al., PRt 873 (2020) 1

line shapes w/ phase space;  
one unstable constituent:



# NREFT at LO for coupled channels

- Full threshold structure needs to be measured in a lower channel (ch-1)  $\Rightarrow$  coupled channels
- Consider a two-channel system, construct a “nonrelativistic” effective field theory (NREFT)
  - Energy region around the higher threshold (ch-2),  $\Sigma_2$
  - Expansion in powers of  $E = \sqrt{s} - \Sigma_2$
  - Momentum in the lower channel can also be expanded



$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1} = -\frac{8\pi\Sigma_2}{\det} \begin{pmatrix} \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{1}{a_{11}} - ik_1 \end{pmatrix}$$

$$\det = \left( \frac{1}{a_{11}} - ik_1 \right) \left( \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} \right) - \frac{1}{a_{12}^2}$$

Effective scattering length with open-channel effects becomes complex,  $\text{Im} \frac{1}{a_{22,\text{eff}}} \leq 0$

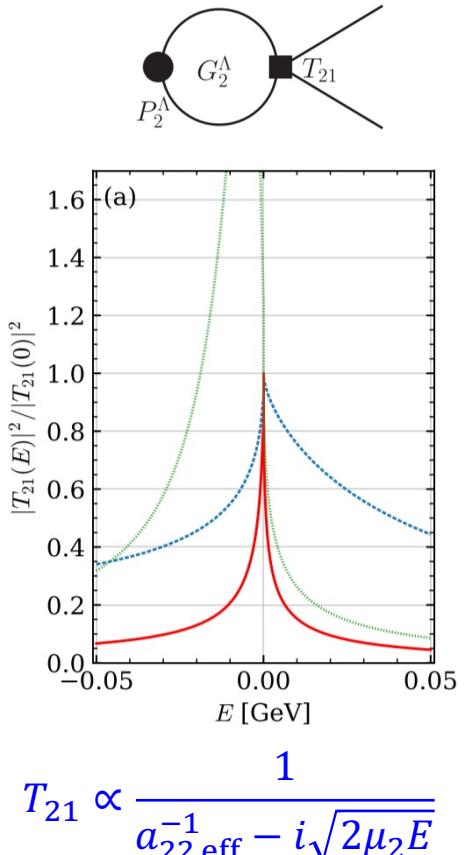
$$T_{22}(E) = -\frac{8\pi}{\Sigma_2} \left[ \frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E) \right]^{-1} \quad \frac{1}{a_{22,\text{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2(1 + a_{11}^2 k_1^2)} - i \frac{a_{11}^2 k_1}{a_{12}^2(1 + a_{11}^2 k_1^2)}.$$

# Distinct line shapes of the same pole

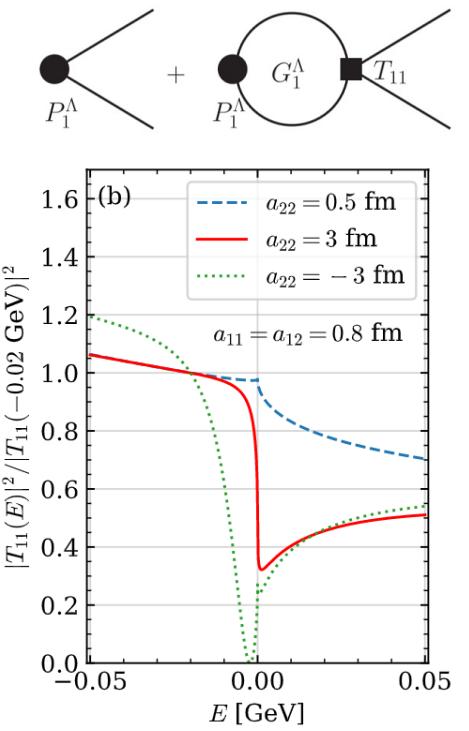
X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

Line shapes of the same pole depend on the production mechanism. Consider production of particles in ch-1

- Dominated by ch-2
  - Maximal at threshold for positive  $\text{Re}(a_{22,\text{eff}})$  (attraction),  $\text{FWHM} \propto 1/\mu$ 
    - more pronounced for heavier hadrons and stronger interactions
  - Peaking at pole for negative  $\text{Re}(a_{22,\text{eff}})$



- Dominated by ch-1
  - One pole and one zero
  - Universality for large scattering length: for large  $|a_{22}|$ , there must be a dip around threshold (zero close to threshold)

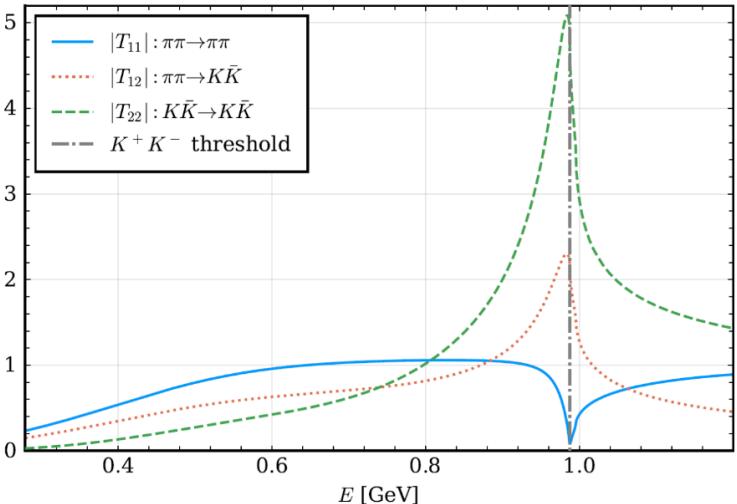


# Distinct line shapes of the same pole

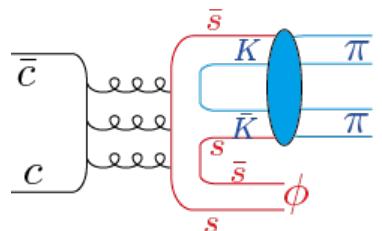
- Example:  $f_0(980)$

- $T$ -matrix for  $\pi\pi$  and  $K\bar{K}$  coupled channels

with the T-matrix from  
L.-Y. Dai, M. R. Pennington, PRD 90 (2014) 036004

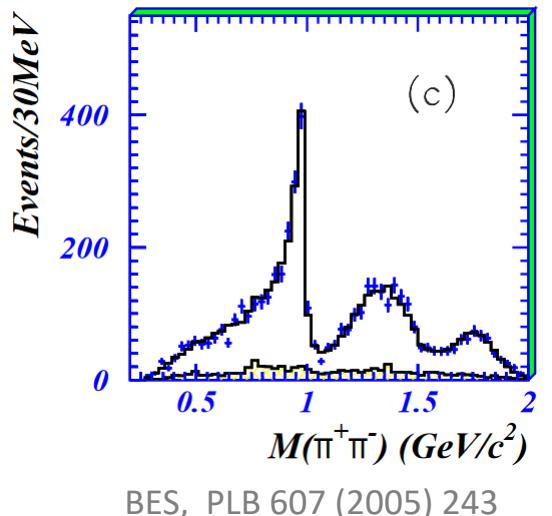


- $J/\psi \rightarrow \phi \pi^+ \pi^-$

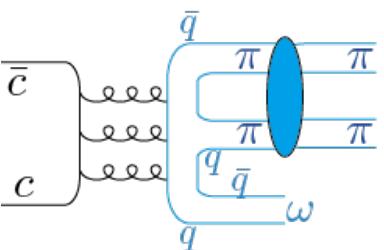


Driving channel:  $K\bar{K}$

$J/\psi \rightarrow \phi K\bar{K} \rightarrow \phi \pi^+ \pi^-$

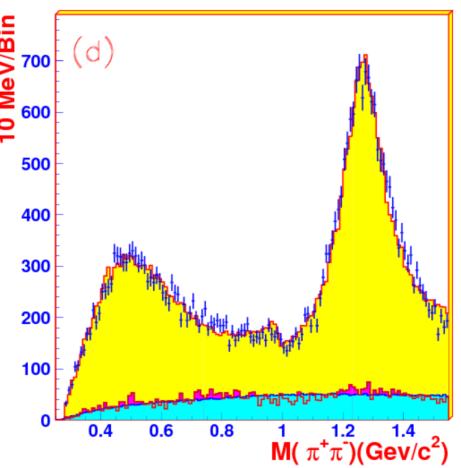


- $J/\psi \rightarrow \omega \pi^+ \pi^-$



Driving channel:  $\pi\pi$

$J/\psi \rightarrow \omega \pi\pi \rightarrow \omega \pi^+ \pi^-$



# Binding mechanism

- One-boson exchange

- One-pion exchange

N.A. Tönqvist, ZPC 61 (1994) 525; ...

➤ systems like  $D\bar{D}$ ,  $\Sigma_c\bar{D}$  unbound

Composite	$J^{PC}$	Deuson
$D\bar{D}^*$	$0^{-+}$	$\eta_c(\approx 3870)$
$D\bar{D}^*$	$1^{++}$	$\chi_{c1}(\approx 3870)$
$D^*\bar{D}^*$	$0^{++}$	$\chi_{c0}(\approx 4015)$
$D^*\bar{D}^*$	$0^{-+}$	$\eta_c(\approx 4015)$
$D^*\bar{D}^*$	$1^{+-}$	$h_{c0}(\approx 4015)$
$D^*\bar{D}^*$	$2^{++}$	$\chi_{c2}(\approx 4015)$
$B\bar{B}^*$	$0^{-+}$	$\eta_b(\approx 10545)$
$B\bar{B}^*$	$1^{++}$	$\chi_{b1}(\approx 10562)$
$B^*\bar{B}^*$	$0^{++}$	$\chi_{b0}(\approx 10582)$
$B^*\bar{B}^*$	$0^{++}$	$\eta_b(\approx 10590)$
$B^*\bar{B}^*$	$1^{+-}$	$h_b(\approx 10608)$
$B^*\bar{B}^*$	$2^{++}$	$\chi_{b2}(\approx 10602)$

- One-vector exchange

S. Krewald, R. Lemmer, F. Sassen, PRD 69 (2004) 016003; ...

➤  $D\bar{D}$  bound state predicted

Y.-J. Zhang, H.-C. Chiang, P.-N. Shen, B.-S. Zou, PRD 74 (2006) 014013;  
D. Gammermann et al., PRD 76 (2007) 074016; ...

❖ Lattice QCD

S. Prelovsek et al., JHEP06 (2021) 035; talk by Prelovsek, June 7, 11:00

➤ Hidden-charm pentaquarks above 4 GeV (including  $\Sigma_c\bar{D}$ ) predicted

J.-J. Wu, R. Molina, E. Oset, B.-S. Zou, PRL 105 (2010) 232001; ...  
talk by R. Molina, June 6, 11:00

- Soft-gluon exchanges talk by A. Nefediev, June 7, 14:30, DAD - Room 4H

- ☞ Survey of the molecular spectrum in a simple model

❖ light-vector-meson exchanges

❖ single channel

❖ neglecting mixing

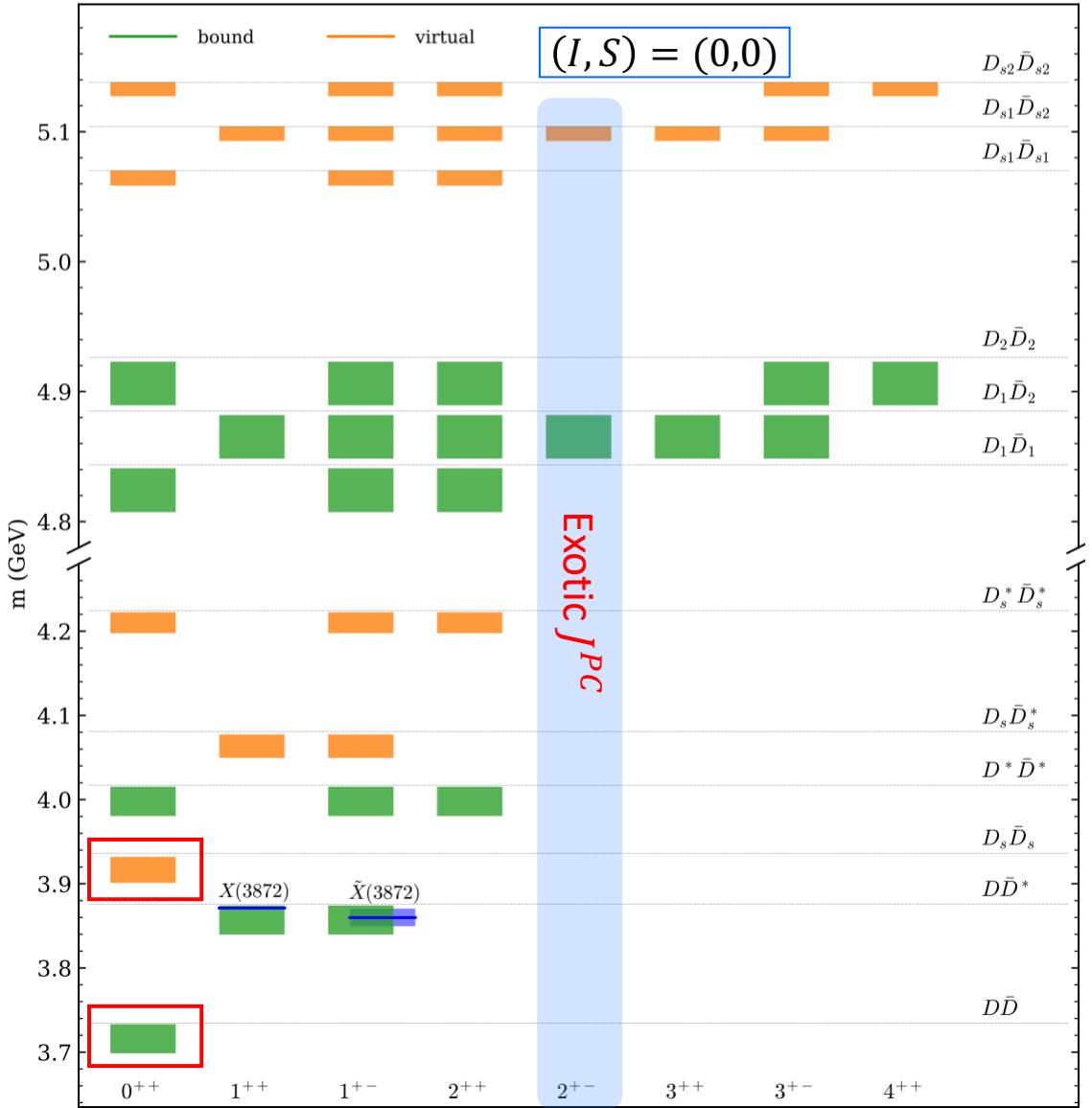
X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65;  
CTP 73 (2021) 015201

Extension of the survey including more meson exchanges:

F.-Z. Peng, M. Sanchez-Sanchez, M.-J. Yan, M. Pavon Valderrama, PRD 105 (2022) 034028;  
M.-J. Yan, F.-Z. Peng, M. Pavon Valderrama, arXiv:2304.14855;  
talk by M. Pavon Valderrama, June 9, 09:30, DAD - Room 4H

# Survey of hadronic molecules: hidden-charm mesons w/ $P = +$

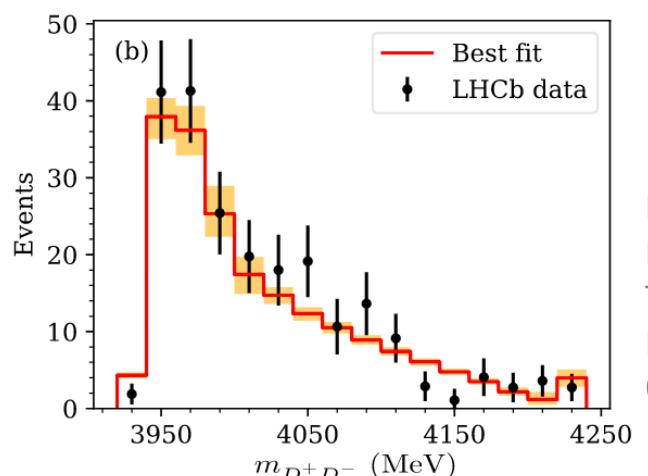
X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65



- ✓ > 200 hidden-charm hadronic molecules
- ✓  $X(3872)$  as a  $\bar{D}D^*$  bound state
- ✓  $\tilde{X}(3872)$  COMPASS, PLB 783 (2018) 334
- ✓  $\bar{D}D$  bound state from lattice S. Prelovsek et al., JHEP06 (2021) 035

& other models C.-Y. Wong, PRC 69 (2004) 055202; Y.-J. Zhang et al., PRD 74 (2006) 014013; D. Gamermann et al., PRD 76 (2007) 074016; J. Nieves et al., PRD 86 (2012) 056004; ...

- ✓  $X(3960)$  in  $B^+ \rightarrow D_s^+ D_s^- K^+$



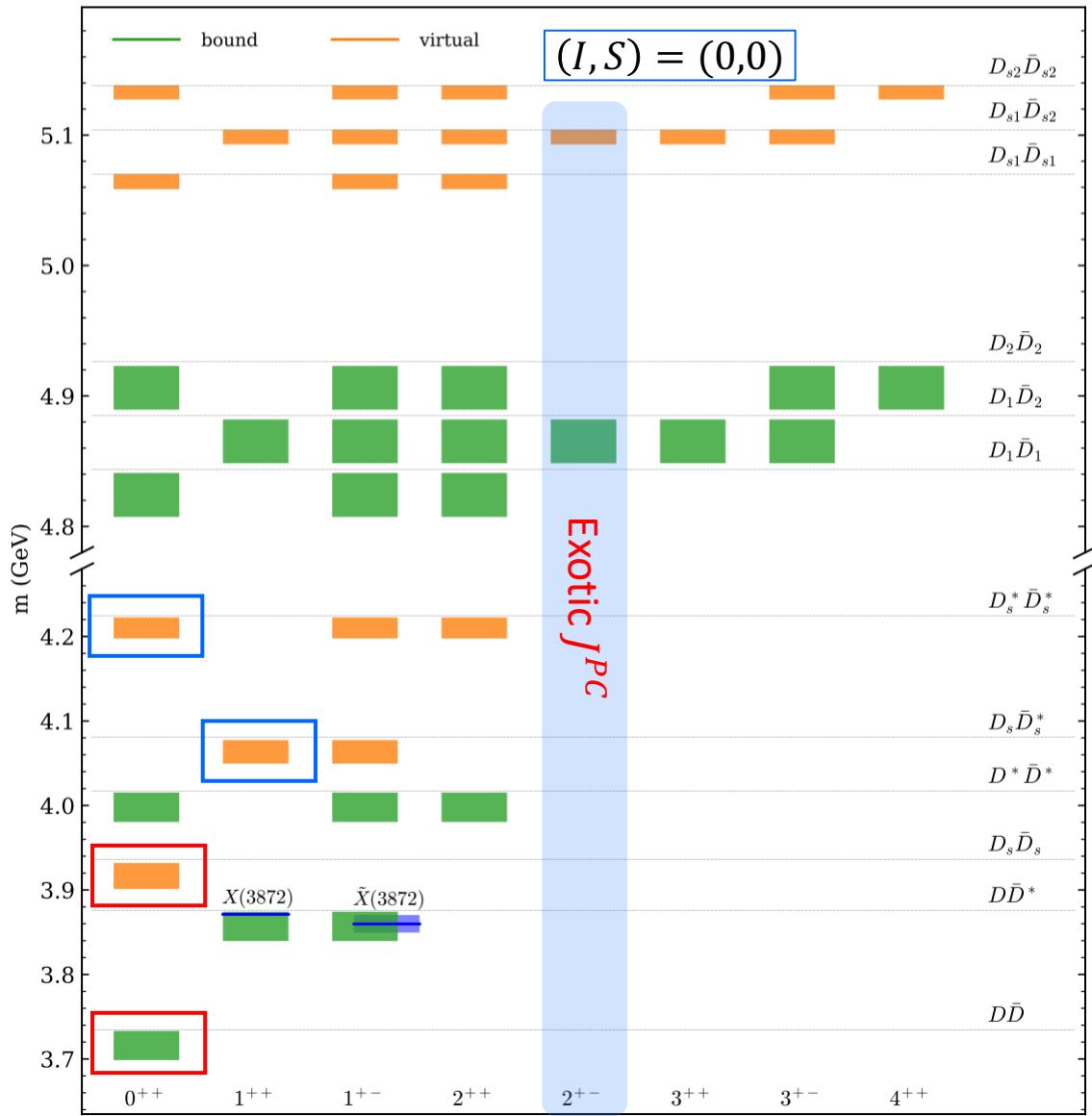
Data from LHCb, arXiv:2210.15153  
Fit in  
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, FKG, J. Nieves, B.-S. Zou, Sci. Bull. 68 (2023) 2056

pole at  $3936.5^{+0.4}_{-0.9} + i (16.1^{+4.2}_{-2.2})$  MeV

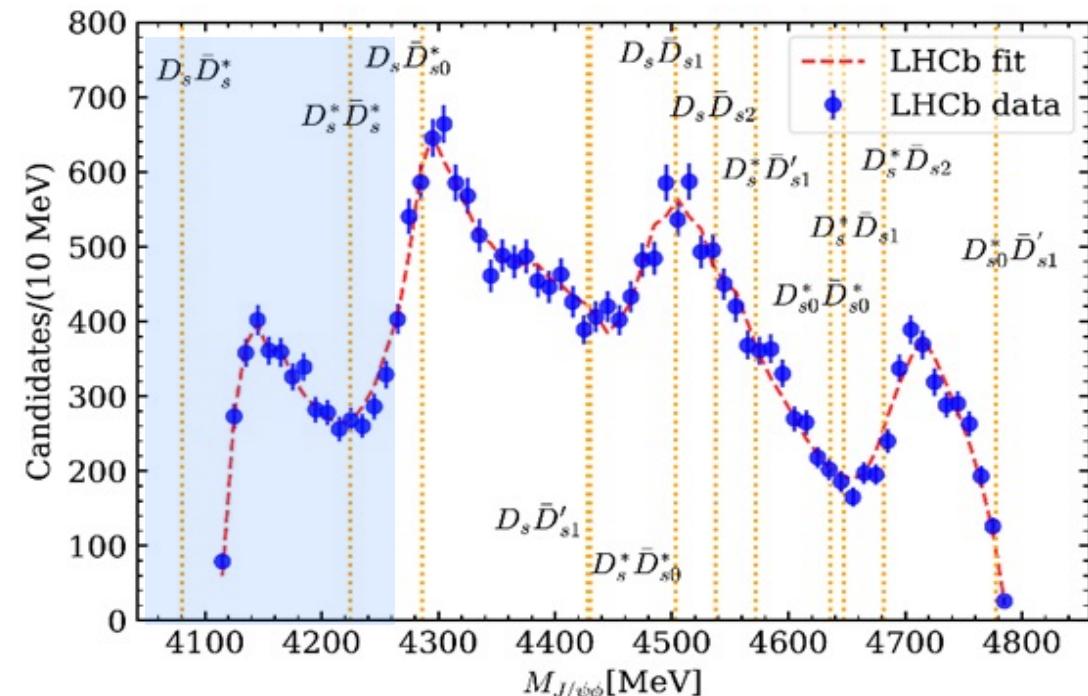
# Survey of hadronic molecules: hidden-charm mesons w/ $P = +$



X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65



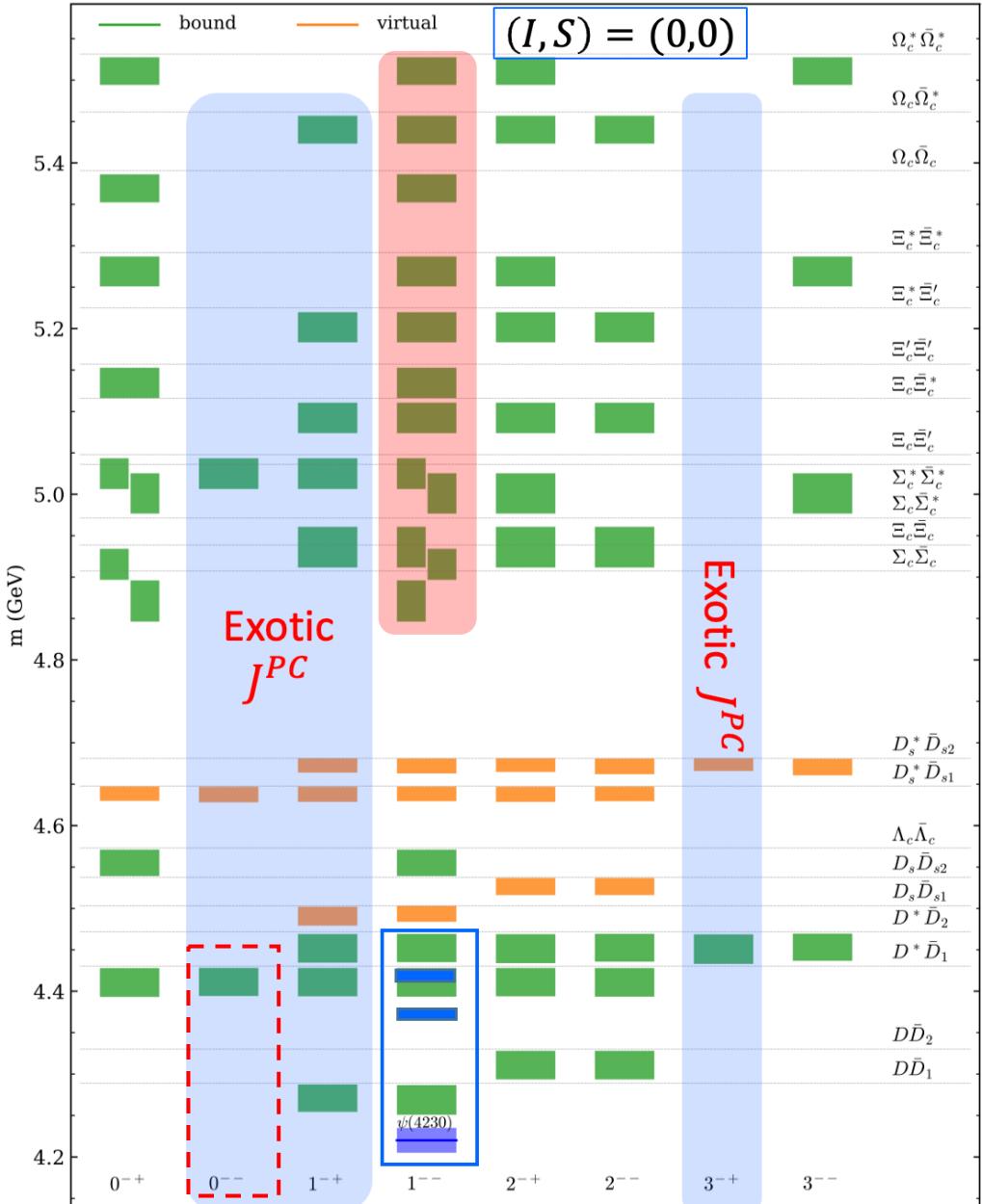
✓  $D_s \bar{D}_s^*$ ,  $D_s^* \bar{D}_s^*$  virtual states?



Data: LHCb, PRL 127 (2021) 082001

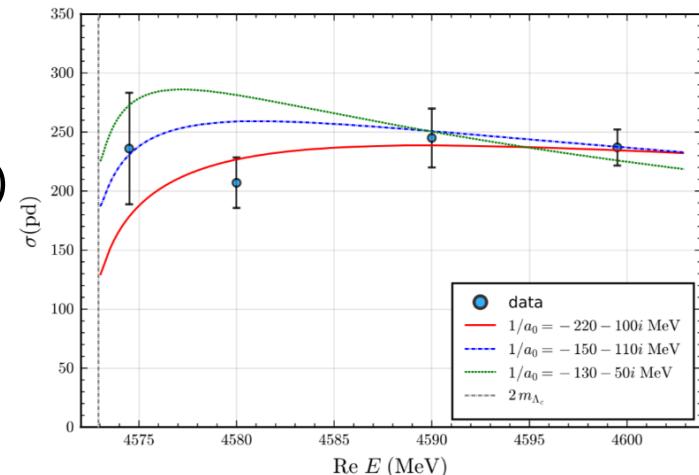
Virtual poles found from the fit in X. Luo, S.X. Nakamura, PRD 107 (2023) L011504

# Hidden-charm mesons w/ $P = -$



- ✓  $Y(4260)/\psi(4230)$  as a  $\bar{D}D_1$  bound state
- ✓  $\psi(4360), \psi(4415)$ :  $D^*\bar{D}_1, D^*\bar{D}_2$ ?
- ✓ Evidence for  $1^{--} \Lambda_c \bar{\Lambda}_c$  molecular state in BESIII data
  - Sommerfeld factor
  - near-threshold pole
  - different from  $Y(4630)$

Data from BESIII, PRL 120 (2018)  
132001;  
see also Q.-F. Cao et al., PRD 100  
(2019) 054040



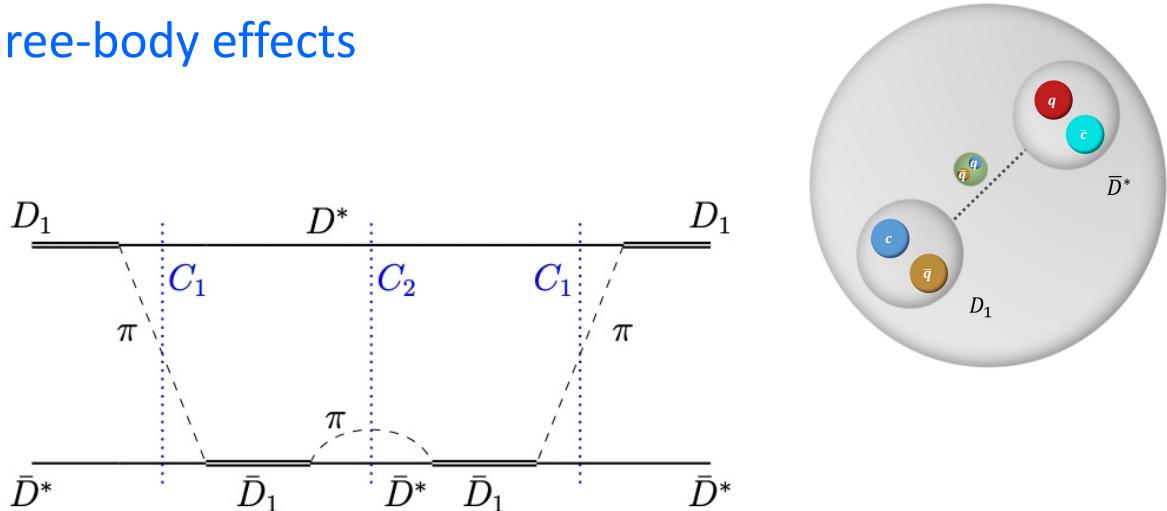
- ✓ Numerous states with exotic quantum numbers  $0^{--} [\psi_0], 1^{-+} [\eta_{c1}], 3^{-+} [\eta_{c3}]$   
e.g.,  $e^+ e^- \rightarrow \gamma \eta_{c1,3}, \omega \eta_{c1,3}; \eta_{c1,3} \rightarrow D \bar{D}^* \pi, J/\psi \omega, \dots$
- ✓ Many  $1^{--}$  states in [4.8, 5.6] GeV: BEPC-II-Upgrade, Belle-II, LHCb, STCF, PANDA, ...

# Closer look at the $0^{--}$ state

T. Ji, X.-K. Dong, FKG, B.-S. Zou, PRL 129 (2022) 102002

- $0^{--}$  spin partner  $\psi_0(4360)$  [ $D^*\bar{D}_1$ ] of  $\psi(4230), \psi(4360), \psi(4415)$  as  $D\bar{D}_1, D^*\bar{D}_1, D^*\bar{D}_2$  hadronic molecules
- Robust against the inclusion of coupled channels and three-body effects

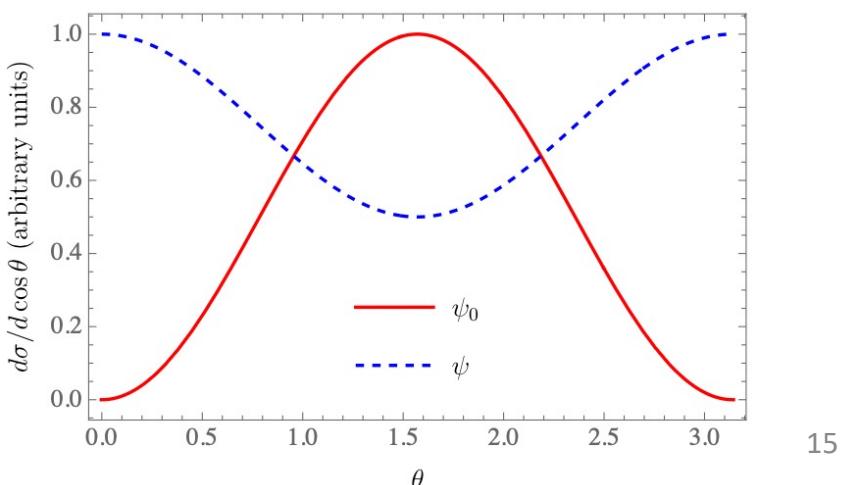
Molecule	Components	$J^{PC}$	Threshold	$E_B$
$\psi(4230)$	$\frac{1}{\sqrt{2}}(D\bar{D}_1 - \bar{D}D_1)$	$1^{--}$	4287	$67 \pm 15$
$\psi(4360)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 - \bar{D}^*D_1)$	$1^{--}$	4429	$62 \pm 14$
$\psi(4415)$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_2 - \bar{D}^*D_2)$	$1^{--}$	4472	$49 \pm 4$
$\psi_0$	$\frac{1}{\sqrt{2}}(D^*\bar{D}_1 + \bar{D}^*D_1)$	$0^{--}$	4429	<b><math>63 \pm 18</math></b>



- May be searched for using  $e^+e^- \rightarrow \psi_0\eta, \psi_0 \rightarrow J/\psi\eta, D\bar{D}^*, D^*\bar{D}^*\pi, \dots$

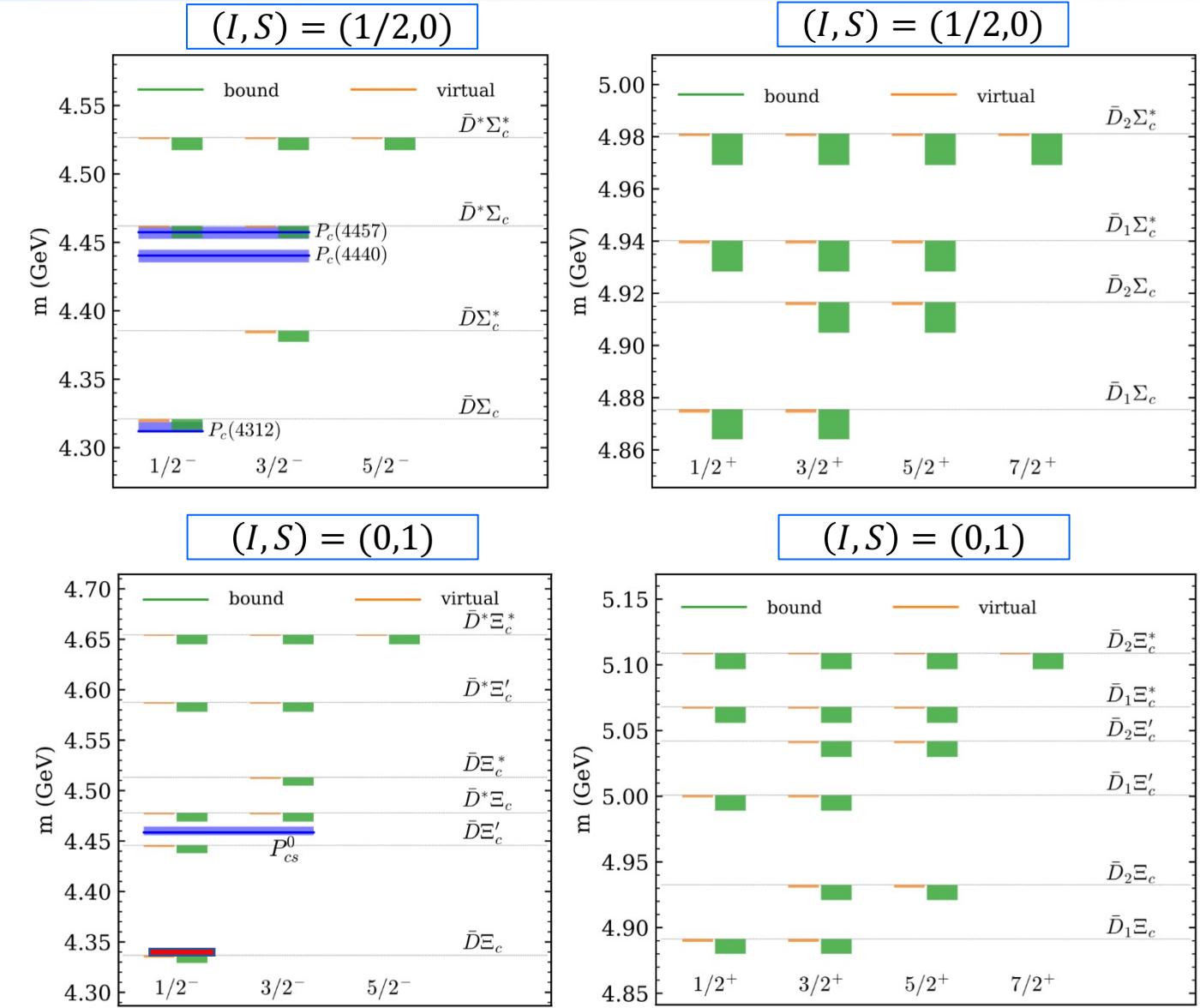
$M = (4366 \pm 18) \text{ MeV}$ ,

$\Gamma < 10 \text{ MeV}$

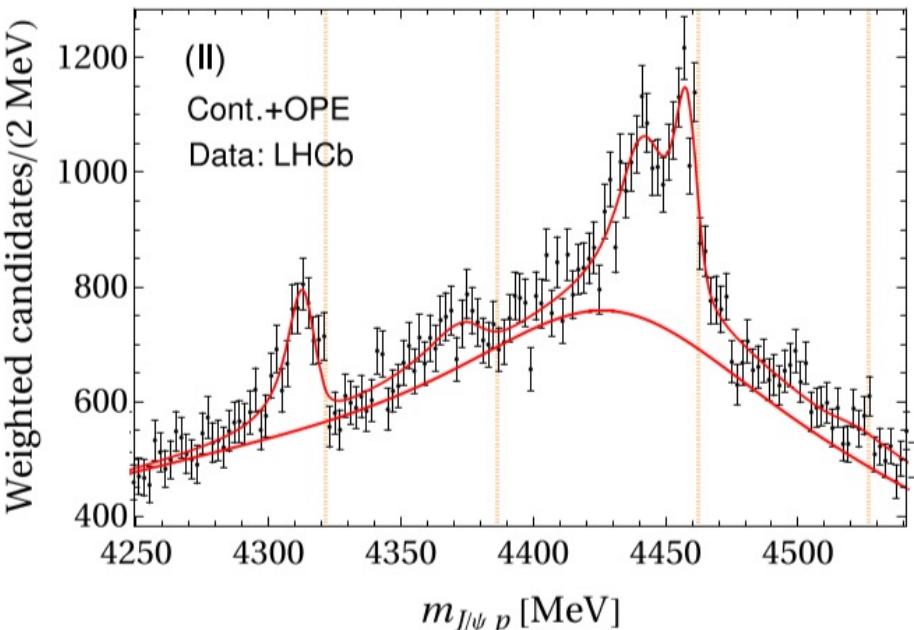


# Hidden-charm pentaquarks

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65



- ✓  $P_c$  states as  $\bar{D}^{(*)}\Sigma_c^{(*)}$  molecules
- ✓ The LHCb data can be well described in a pionful EFT

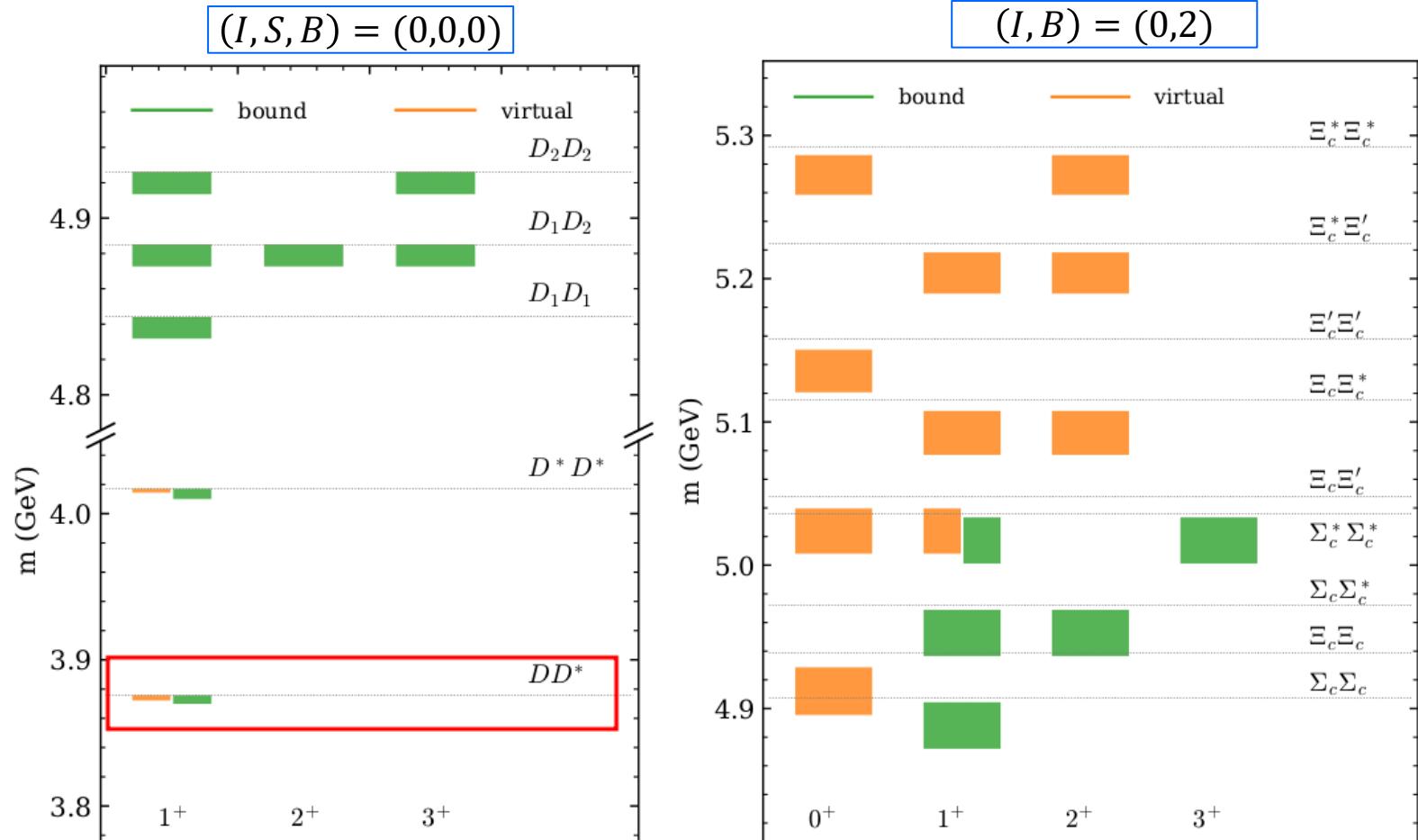


M.-L. Du et al., PRL 124 (2020) 072001; JHEP 08 (2021) 157

- ✓  $P_{cs}(4459)$ : 2  $\bar{D}^*\Xi_c$  molecular states
- ✓  $P_{cs}(4338)$ :  $\bar{D}\Xi_c$  molecular state

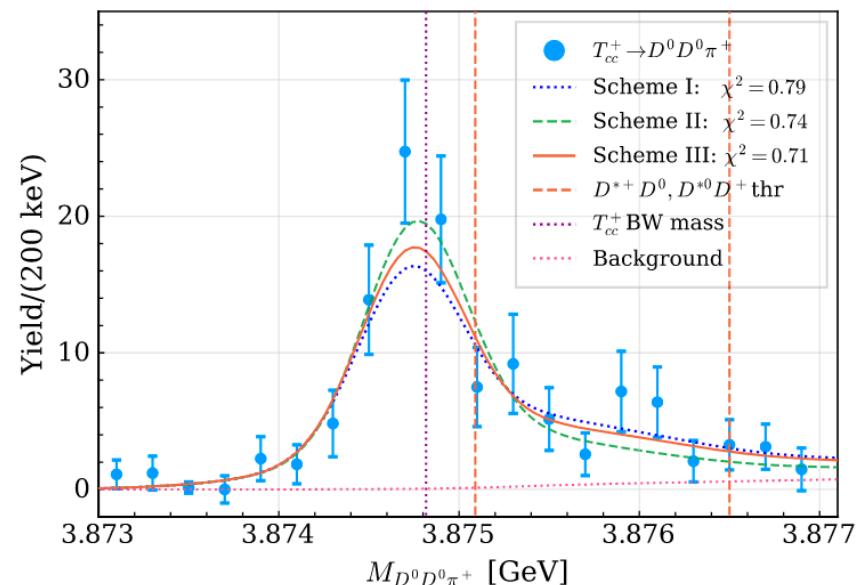
# Double-charm tetraquarks and dibaryons

X.-K. Dong, FKG, B.-S. Zou, CTP 73 (2021) 125201



- ✓ isoscalar  $DD^*$  molecular state
- ✓ It has a spin partner  $1^+ D^*D^*$  state
- ✓ Many ( $> 100$ ) other similar double-charm molecular states

- ✓  $T_{cc}(3875)$  as  $D^*D$  molecule
- ✓ The LHCb data can be well described in a pionful EFT w/ 3-body effects



M.-L. Du et al., PRD 105 (2022) 014024;  
 Talk by V. Baru, June 8, 15:00, DAD - Room 4H

# Summary

- A rich spectrum of hadronic molecules is expected
- General rule for (near-)threshold structures: S-wave attraction, more prominent for heavier particles and stronger attraction
- Pole behavior: distinct line shapes depending on reaction mechanism
- Universality: a dip (for large  $|a_{22}|$ ) at the higher channel threshold in  $T_{11}$

More talks on hadronic molecules:

Plenary	DAD - Room 4H	DAD - Room Benvenuto	DAD - Room 5L
R. Molina, June 6, 11:00	L. Dai, June 5, 14:30	F. Gil, June 6, 14:30	L.-R. Dai, June 7, 17:10
M. Mai, June 6, 11:30	A. Feijoo, June 5, 15:30	L.-S. Geng, June 6, 14:50	C. Fernandez-Ramirez, June 7, 17:50
Y. Yamaguchi, June 7, 11:30	M.-J. Yan, June 5, 17:25	M. Albaladejo, June 6, 15:10	
	A. Nefediev, June 7, 14:30; 16:55	L.-P. He, June 6, 15:30	
	E. Oset, June 8, 14:30	N. Ikeno, June 6, 16:30	
	V. Baru, June 8, 15:00	A. Feijoo, June 6, 17:10	
	L.-R. Dai, June 8, 15:30	J. Nieves, June 7, 14:30	
	A. Ramos, June 8, 17:00	S. Yasui, June 7, 17:45	
	N. Ikeno, June 8, 18:10	L.-R. Dai, June 8, 17:50	
	M. Pavon Valderrama, June 9, 09:30	A. Asokan, June 8, 18:10	

## Thank you for your attention!

# Reviews in the last few years

## ● >>10 review articles:

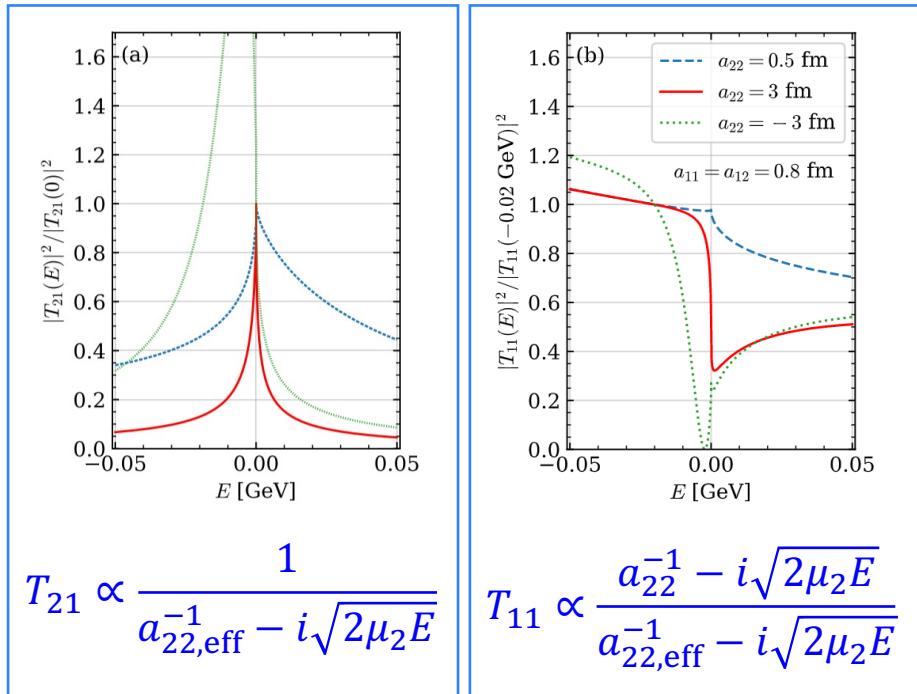
- H.-X. Chen et al., *The hidden-charm pentaquark and tetraquark states*, Phys. Rept. 639 (2016) 1
- A. Hosaka et al., *Exotic hadrons with heavy flavors: X, Y, Z, and related states*, PTEP 2016 (2016) 062C01
- J.-M. Richard, *Exotic hadrons: review and perspectives*, Few Body Syst. 57 (2016) 1185
- R. F. Lebed, R. E. Mitchell, E. Swanson, *Heavy-quark QCD exotica*, PPNP 93 (2017) 143
- A. Esposito, A. Pilloni, A. D. Polosa, *Multiquark resonances*, Phys. Rept. 668 (2017) 1
- FKG, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, RMP 90 (2018) 015004
- A. Ali, J. S. Lange, S. Stone, *Exotics: Heavy pentaquarks and tetraquarks*, PPNP 97 (2017) 123
- S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, RMP 90 (2018) 015003
- Y.-R. Liu et al., *Pentaquark and tetraquark states*, PPNP107 (2019) 237
- N. Brambilla et al., *The XYZ states: experimental and theoretical status and perspectives*, Phys. Rept. 873 (2020) 154
- Y. Yamaguchi et al., *Heavy hadronic molecules with pion exchange and quark core couplings: a guide for practitioners*, JPG 47 (2020) 053001
- FKG, X.-H. Liu, S. Sakai, *Threshold cusps and triangle singularities in hadronic reactions*, PPNP 112 (2020) 103757
- G. Yang, J. Ping, J. Segovia, *Tetra- and penta-quark structures in the constituent quark model*, Symmetry 12 (2020) 1869
- C.-Z. Yuan, Charmonium and charmoniumlike states at the BESIII experiment, Natl. Sci. Rev. 8 (2021) nwab182
- H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, S.-L. Zhu, *An updated review of the new hadron states*, RPP 86 (2023) 026201
- L. Meng, B. Wang, G.-J. Wang, S.-L. Zhu, *Chiral perturbation theory for heavy hadrons and chiral effective field theory for heavy hadronic molecules*, Phys. Rept. 1019 (2023) 2266;
- .....

## ● + a book:

- A. Ali, L. Maiani, A. D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)

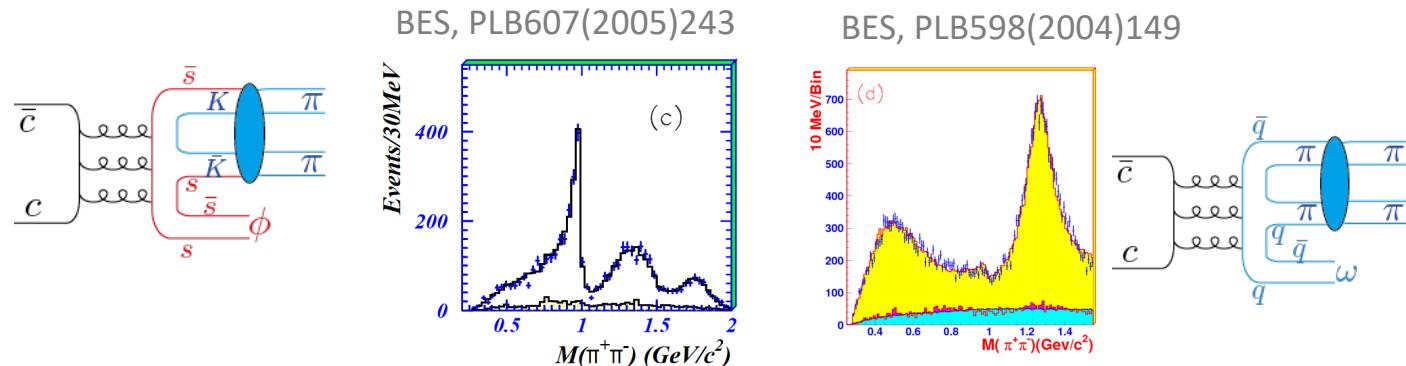
# $Z_{cs}(3985)$ and $Z_{cs}(4000)$ : different or not?

- Line shapes of the same state can strongly depend on reactions



X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

- E.g.,  $f_0(980)$ : peak in  $J/\psi \rightarrow \phi \pi^+ \pi^-$  dip in  $J/\psi \rightarrow \omega \pi^+ \pi^-$



- Description of BESIII  $Z_{cs}(3985)$  and LHCb  $Z_{cs}(4000)$  w/ the same state

Ortega, Entem, Fernandez, PLB 818 (2021) 136382

