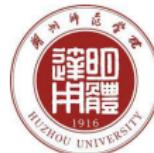


# Effective Field Theories for Hadron Spectroscopy

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R. Molina, T. Branz, F. Gil-Dominguez, L.R. Dai and E. Oset



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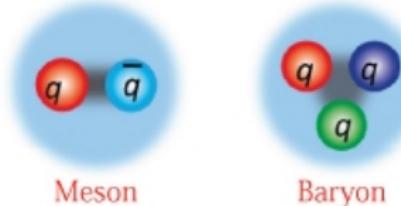
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# **Intro**

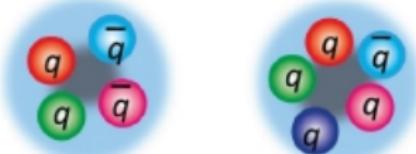
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# Hadrons

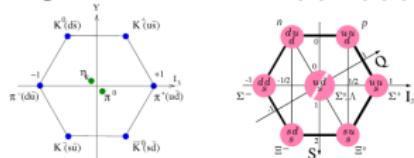
Standard Hadrons



Exotic Hadrons

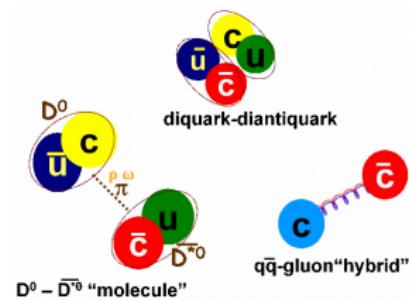


- 'Regular' hadrons:  $q\bar{q}$ ,  $qqq$

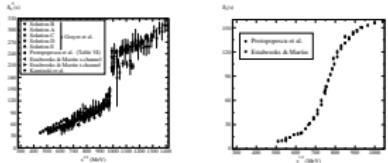


- Exotics:  $q\bar{q}q\bar{q}$ ,  $qqqq\bar{q}$ ,  $qqg$ , ...

Not  $q\bar{q}$ :  $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$



# Light hadrons



Close to ...

→ PP th.  $\pi\pi, K\bar{K} \dots$  PB  $K\Lambda, \pi\Sigma, \bar{K}N \dots$

→ PV th.  $K\bar{K}^*, \pi\rho, \pi\omega, \eta\omega \dots$  SB  $\sigma N \dots$

→ VV th.  $\rho\rho, K^*\bar{K}^* \dots$  VB  $\rho(\omega)N,$

$K^*N \dots$

→ Hybrid, Glueball candidate

| Particle       | $J^P$   | overall |
|----------------|---------|---------|
| $N$            | $1/2^+$ | ****    |
| $N(1440)$      | $1/2^+$ | ****    |
| $N(1520)$      | $3/2^-$ | ***     |
| $\chi(1535)$   | $1/2^+$ | ***     |
| $(\chi(1560))$ | $1/2^+$ | ***     |
| $N(1675)$      | $5/2^-$ | ***     |
| $N(1680)$      | $5/2^+$ | ***     |
| $\chi(1700)$   | $3/2^+$ | ***     |
| $N(1710)$      | $1/2^+$ | ***     |
| $N(1720)$      | $3/2^+$ | ***     |
| $N(1860)$      | $5/2^+$ | **      |
| $N(1875)$      | $3/2^-$ | **      |
| $N(1880)$      | $1/2^+$ | **      |

| Particle  | $J^P$   | Overall status | Status as seen in ... |             |  |
|-----------|---------|----------------|-----------------------|-------------|--|
|           |         |                | $NK$                  | $\Sigma\pi$ | Other channels                         |
| $A(1116)$ | $1/2^+$ | ****           |                       |             | $N\pi$ (weak decay)                    |
| $A(1380)$ | $1/2^-$ | **             | **                    | **          |  |
| $A(1405)$ | $1/2^+$ | ****           | ****                  | ****        |  |
| $A(1590)$ | $3/2^-$ | ****           | ****                  | ****        | $\Lambda\pi, \bar{\Lambda}\gamma$      |
| $A(1600)$ | $1/2^+$ | ****           | ***                   | ****        | $\Lambda\pi, \Sigma(1385)\pi$          |
| $A(1670)$ | $3/2^-$ | ****           | ****                  | ****        | $\Lambda\eta$                          |
| $A(1690)$ | $3/2^-$ | ****           | ****                  | ****        | $\Lambda\pi\pi, \Sigma(1385)\pi$       |
| $A(1710)$ | $1/2^+$ | *              | *                     | *           |  |
| $A(1810)$ | $1/2^+$ | ***            | ***                   | ***         | $\Lambda\pi\pi, \Sigma(1385)\pi, NK^*$ |
| $A(1830)$ | $1/2^+$ | ***            | ***                   | ***         | $NK^*$                                 |
| $A(1830)$ | $5/2^+$ | ****           | ****                  | ****        | $\Sigma(1385)\pi$                      |
| $A(1830)$ | $5/2^-$ | ****           | ****                  | ****        | $\Sigma(1385)\pi$                      |
| $A(1890)$ | $3/2^+$ | ****           | ****                  | **          | $\Sigma(1385)\pi, NK^*$                |
| $A(2000)$ | $1/2^-$ | *              | *                     | *           |  |

|                 |               |
|-----------------|---------------|
| $h_1(1415)$     | $0^-(1^{+-})$ |
| was $h_1(1380)$ |               |
| $a_1(1420)$     | $1^-(1^{++})$ |
| $f_1(1420)$     | $0^+(1^{++})$ |
| $a_1(1420)$     | $0^-(1^{--})$ |
| $f_2(1430)$     | $0^+(2^{++})$ |
| $a_0(1450)$     | $1^-(0^{++})$ |
| $\rho(1450)$    | $1^+(1^{--})$ |
| $\eta(1475)$    | $0^+(0^{-+})$ |
| $f_0(1500)$     | $0^+(0^{++})$ |

## LIGHT UNFLAVORED MESONS ( $S = C = B = 0$ )

For  $I = 1 (x, b, \rho, a): u \bar{d}, (u \bar{u} - d \bar{d})/\sqrt{2}, d \bar{u};$   
for  $I = 0 (\eta, \eta', h, h', a, \phi, f, f'): c_1(u \bar{u} + d \bar{d}) + c_2(s \bar{s})$

See related reviews:

Form Factors for Radiative Pion and Kaon Decays

Scalar Mesons below 2 GeV

$\rho(770)$

Pseudoscalar and Pseudovector Mesons in the 1400  
 $\rho(1450)$  and  $\rho(1700)$

- |  |               |                  |
|--|---------------|------------------|
| • $\pi^\pm$                                | $1^-(0^-)$    | $f_0(1510)$      |
| • $\pi^0$                                  | $1^-(0^{+-})$ | • $f_0(1525)$    |
| • $\eta$                                   | $0^+(0^{++})$ | $f_2(1565)$      |
| • $f_0(500)$<br>aka $\sigma, was f_0(600)$ | $0^+(0^{++})$ | $\rho(1570)$     |
| • $\rho(770)$                              | $1^+(1^{--})$ | $h_1(1595)$      |
| • $a(782)$                                 | $0^-(1^{--})$ | • $\pi(1600)$    |
| • $\eta'(958)$                             | $0^+(0^{-+})$ | $a_1(1640)$      |
| • $f_0(980)$                               | $0^+(0^{++})$ | $f_2(1640)$      |
| • $a_0(980)$                               | $1^-(0^{++})$ | • $\eta_2(1645)$ |
| • $\phi(1020)$                             | $0^-(1^{--})$ | • $a_0(1650)$    |
| • $b_1(1170)$                              | $0^-(1^{+-})$ | • $a_0(1670)$    |
| • $b_1(1235)$                              | $1^+(1^{+-})$ | • $\pi_0(1670)$  |
| • $a_1(1260)$                              | $1^-(1^{++})$ | • $\phi(1680)$   |
| • $f_2(1270)$                              | $0^+(2^{++})$ | • $\rho_1(1690)$ |
| • $f_2(1285)$                              | $0^+(1^{++})$ | • $\rho(1700)$   |
| • $\eta(1295)$                             | $0^+(0^{-+})$ | • $f_0(1700)$    |
| • $\pi(1300)$                              | $1^-(0^{++})$ | • $f_0(1760)$    |
| • $a_2(1320)$                              | $1^-(2^{++})$ | • $\pi(1800)$    |
| • $f_0(1370)$                              | $0^+(0^{++})$ | $f_2(1810)$      |
| • $\pi_1(1400)$                            | $1^-(1^{+-})$ | $X(1835)$        |
| • $\eta(1405)$                             | $0^+(0^{-+})$ | • $\phi_3(1850)$ |

## Methods: EFT's, Chiral symmetry, Unitarity

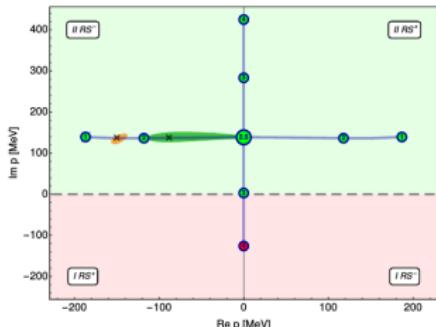
General properties of the scattering amplitudes: Analyticity, Unitarity, Crossing symmetry, applied often in combination with EFT (chiral symmetry)

- Unitarized Chiral Perturbation Theory. Oller, Oset, Pelaez (1997)
- Inverse Amplitude Method. Truong, Herrero, Dobado, Pelaez (1988)
- Roy-Steiner equations based on dispersion relations. Roy, Steiner, Hite (1971)
- N/D method, Oller (1998)
- Bethe-Salpeter ...

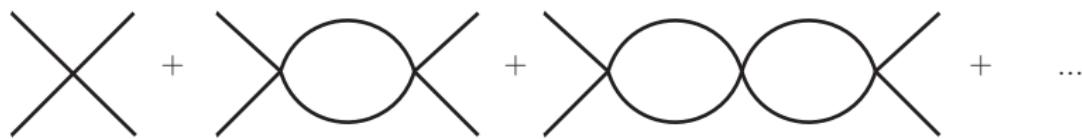
and LQCD!

Example of Application in combination with LQCD data

$\sigma$  meson. Guo, Alexandru, Molina, Mai, Döring, PRD98 (2018)



## Dynamically generated resonances



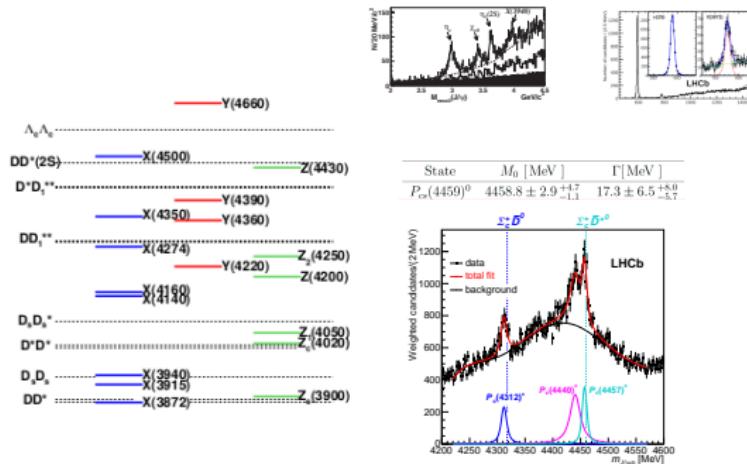
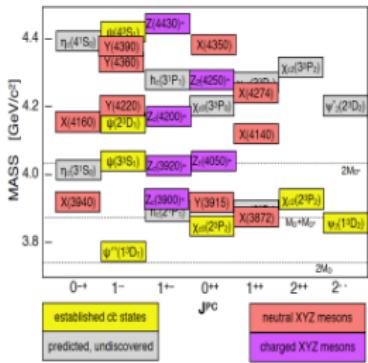
Many suggestions of **dynamically generated resonances**...

- Oller, Oset, Pelaez, Ramos, Hanhart, Krewald, Speth, Nieves, Inoue, Ruiz-Arriola, Meissner, Vicente-Vacas, Garcia-Recio, Molina, Roca, Geng, Alvarez-Ruso, Alarcon, Albaladejo, Nícola ...  
... and a **very long** list of authors! ...'

However, no clear statement for them in the light sector since ...

- Most of these exotic candidates can overlap with  $q\bar{q}/qqq$  except for those with non- $q\bar{q}$  quantum numbers like  $\pi_1(1400)$ ,  $\pi_1(1600)$  ...

## Heavy hadrons



## Clear evidence of exotic states!

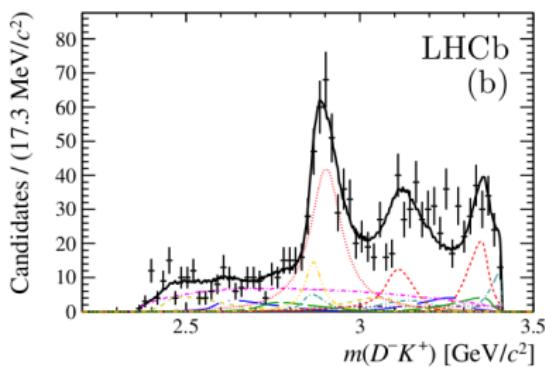
- **Hidden-charm** charged tetraquarks  $Z_c^+ \sim c\bar{d}u\bar{c}$  ( $D^{(*)}\bar{D}^{(*)}$ ).  
**Hidden-strange** candidate?  $a_0(980)$ ? ... more?
  - **Hidden-charm** (strange) pentaquarks  $P_{c(s)}^+ \sim c\bar{c}uud(s)$ , ( $\bar{D}^{(*)}\Sigma_c^{(*)}(\Xi_c^{(*)})$ ).  
**Hidden-strange** candidate?  $N^*(1535)$ , (strange)  $\Lambda(1405)$ , ...more?

# Flavor exotic tetraquark $T_{cs}(2900)$

## LHCb (2020)

Two states  $J^P = 0^+, 1^-$  decaying to  $\bar{D}K$ . First clear example of an heavy-flavor exotic tetraquark,  $\sim \bar{c}\bar{s}ud$ .

$$X_0(2866) : M = 2866 \pm 7 \quad \text{and} \quad \Gamma = 57.2 \pm 12.9 \text{ MeV},$$
$$X_1(2900) : M = 2904 \pm 5 \quad \text{and} \quad \Gamma = 110.3 \pm 11.5 \text{ MeV}.$$

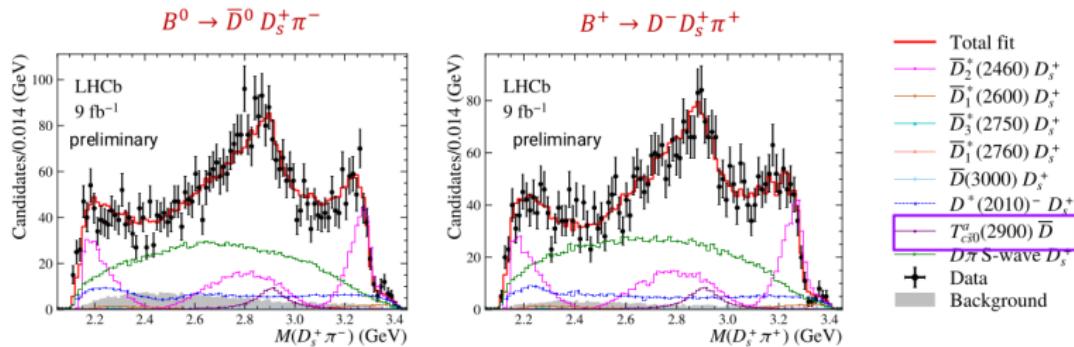


R. Aaij et al. (LHCb Collaboration), PRL125(2020), PRD102(2020)

# New exotic tetraquark seen in $D_s^+\pi^+$

LHCb (2022)

One state decaying  $T_{c\bar{s}}(2900)$  decaying to  $D_s^+\pi^-$  and  $D_s^+\pi^+$  has been observed  $\sim c\bar{s}ud$ .



- The analysis favors  $J^P = 0^+$  [arXiv:2212.02717](https://arxiv.org/abs/2212.02717)
- Mass,  $m = 2908 \pm 11 \pm 20$  MeV  $D^* K^*$  th.: 2903 MeV
- Width,  $\Gamma = 136 \pm 23 \pm 11$  MeV  $D_s^* \rho$  th.: 2890 MeV

# **Local-Hidden-Gauge Formalism**

---

# The hidden gauge formalism

Bando, Kugo, Yamawaki, PRL54,1215

## Lagrangian

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}_{III} \quad (1)$$

$$\mathcal{L}^{(2)} = \frac{1}{4} f^2 \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle \quad (2)$$

$$\mathcal{L}_{III} = -\frac{1}{4} \langle V_{\mu\nu} V^{\mu\nu} \rangle + \frac{1}{2} M_V^2 \langle [V_\mu - \frac{i}{g} \Gamma_\mu]^2 \rangle$$

$$D_\mu U = \partial_\mu U - ieQA_\mu U + ieUQA_\mu, \quad U = e^{i\sqrt{2}P/f}$$

Upon expansion of  $[V_\mu - \frac{i}{g} \Gamma_\mu]^2$ ,  **$\mathcal{L}'s$**

$$\mathcal{L}_{V\gamma} = -M_V^2 \frac{e}{g} A_\mu \langle V^\mu Q \rangle, \mathcal{L}_{VPP} = -ig \langle V^\mu [P, \partial_\mu P] \rangle, \mathcal{L}_{\gamma PP} = ieA_\mu \langle Q[P, \partial_\mu P] \rangle, \dots$$

$$\frac{F_V}{M_V} = \frac{1}{\sqrt{2}g}, \quad \frac{G_V}{M_V} = \frac{1}{2\sqrt{2}g}, \quad F_V = \sqrt{2}f, \quad G_V = \frac{f}{\sqrt{2}}, \quad g = \frac{M_V}{2f}$$

# Vector-vector scattering Bando,Kugo,Yamawaki

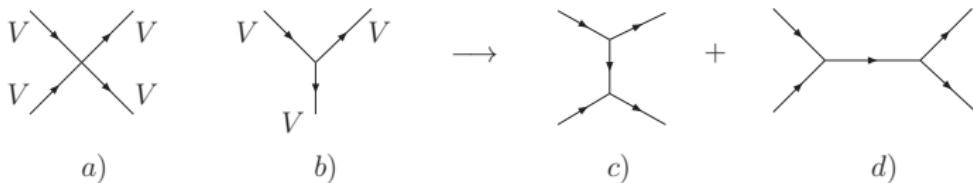
$$\mathcal{L}_{III} = -\frac{1}{4} \langle V_{\mu\nu} V^{\mu\nu} \rangle \rightarrow \mathcal{L}_{III}^{(3V)} = ig \langle (\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu \rangle$$

$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2} \langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle$$

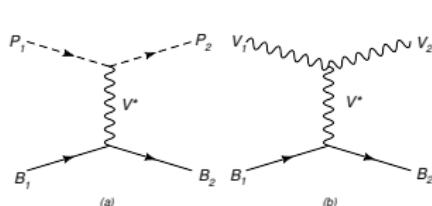
$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu]$$

$$g = \frac{M_V}{2f}$$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$



# PB and VB interaction



$$\begin{aligned}\mathcal{L}_{BBV} &= g(\langle \bar{B} \gamma_\mu [V^\mu, B] \rangle + \langle \bar{B} \gamma_\mu B \rangle \langle V^\mu \rangle) \\ t_{P_1 P_2 V} &= g_{15_F} C_{15_F} (15 \otimes 15) (q_1 + q_2)_\mu \epsilon^\mu \\ t_{B_1 \bar{B}_2 V} &= \{g_{15_1} C_{15_1} (20' \otimes \bar{20}') + g_{15_2} C_{15_2} (20' \otimes \bar{20}') \\ &\quad + g_1 C_1 (20' \otimes \bar{20}')\} \bar{u}_{r'}(p_2) \gamma \cdot \epsilon u_r(p_1)\end{aligned}$$

with  $g_{15_F} = -2\sqrt{2}g$ .  $g_{15_1}$ ,  $g_{15_2}$  and  $g_1$  are evaluated demanding

- 1) The coupling  $p\bar{p} \rightarrow J/\psi$ ,  $p\bar{p} \rightarrow \phi$  should be zero by OZI rules,
- 2) The coupling  $p\bar{p} \rightarrow \rho^0$  should be the one obtained in SU(3).

$$g_{15_1} = -g; \quad g_{15_2} = 2\sqrt{3}g; \quad g_1 = 3\sqrt{5}g.$$

Bethe-Salpeter equation

$$T = [I - VG]^{-1}V$$

## **New exotic hadrons**

---

# Pentaquark states

Wu, Molina, Oset, Zou (2010)

$$V_{ab(P_1B_1 \rightarrow P_2B_2)} = \frac{C_{ab}}{4f^2} (q_1^0 + q_2^0)$$

$$V_{ab(V_1B_1 \rightarrow V_2B_2)} = \frac{C_{ab}}{4f^2} (q_1^0 + q_2^0) \vec{\epsilon}_1 \cdot \vec{\epsilon}_2$$

|                        | $\bar{D}^*\Sigma_c$ | $\bar{D}^*\Lambda_c^+$ | $\rho N$      | $\omega N$   | $\phi N$ | $K^*\Sigma$ | $K^*\Lambda$ |
|------------------------|---------------------|------------------------|---------------|--------------|----------|-------------|--------------|
| $\bar{D}^*\Sigma_c$    | -1                  | 0                      | -1/2          | $\sqrt{3}/2$ | 0        | 1           | 0            |
| $\bar{D}^*\Lambda_c^+$ | 1                   | -3/2                   | $-\sqrt{3}/2$ | 0            | 0        | 1           |              |

$C_{ab}$  for the VB system in the sector  $I = 1/2, S = 0$ .

|                          | $\bar{D}_s^*\Lambda_c^+$ | $\bar{D}^*\Xi_c$ | $\bar{D}^*\Xi_c'$ | $\rho\Sigma$ | $\omega\Lambda$ | $\phi\Lambda$ | $\bar{K}^*N$ | $K^*\Xi$ |
|--------------------------|--------------------------|------------------|-------------------|--------------|-----------------|---------------|--------------|----------|
| $\bar{D}_s^*\Lambda_c^+$ | 0                        | $-\sqrt{2}$      | 0                 | 0            | 0               | -1            | $-\sqrt{3}$  | 0        |
| $\bar{D}^*\Xi_c$         | -1                       | 0                | $-3/2$            | $-1/2$       | 0               | 0             | $\sqrt{3}/2$ |          |
| $\bar{D}^*\Xi_c'$        |                          | -1               | $\sqrt{3}/2$      | $\sqrt{3}/2$ | 0               | 0             | $1/\sqrt{2}$ |          |

$C_{ab}$  for the VB system in the sector  $I = 0, S = -1$ .

| PB resonances (units in MeV) |      |          |            |             |               |                 |
|------------------------------|------|----------|------------|-------------|---------------|-----------------|
| $(I, S)$                     | $M$  | $\Gamma$ | $\Gamma_i$ |             |               |                 |
| $(1/2, 0)$                   |      |          | $\pi N$    | $\eta N$    | $\eta' N$     | $K\Sigma$       |
|                              | 4261 | 56.9     | 3.8        | 8.1         | 3.9           | 17.0            |
|                              |      |          | $\bar{K}N$ | $\pi\Sigma$ | $\eta\Lambda$ | $\eta'\Lambda$  |
| $(0, -1)$                    |      |          |            |             | $K\Sigma$     | $\eta_c\Lambda$ |
|                              | 4209 | 32.4     | 15.8       | 2.9         | 3.2           | 1.7             |
|                              |      |          |            |             | 2.4           | 5.8             |
|                              | 4394 | 43.3     | 0          | 10.6        | 7.1           | 3.3             |
|                              |      |          |            |             | 5.8           | 16.3            |

| VB resonances |      |          |              |              |                 |                 |
|---------------|------|----------|--------------|--------------|-----------------|-----------------|
| $(I, S)$      | $M$  | $\Gamma$ | $\Gamma_i$   |              |                 |                 |
| $(1/2, 0)$    |      |          | $\rho N$     | $\omega N$   | $K^*\Sigma$     |                 |
|               | 4412 | 47.3     | 3.2          | 10.4         | 13.7            | 19.2            |
|               |      |          | $\bar{K}^*N$ | $\rho\Sigma$ | $\omega\Lambda$ | $\phi\Lambda$   |
| $(0, -1)$     |      |          |              |              | $K^*\Xi$        | $J/\psi\Lambda$ |
|               | 4368 | 28.0     | 13.9         | 3.1          | 0.3             | 4.0             |
|               |      |          |              |              | 1.8             | 5.4             |
|               | 4544 | 36.6     | 0            | 8.8          | 9.1             | 0               |
|               |      |          |              |              | 5.0             | 13.8            |

$$\Lambda_b \rightarrow p + J/\psi + K^-$$

Experimental  $P_C$  states (LHCb, 2019)

| $M$ (MeV)                      | $\Gamma$ (MeV)                |
|--------------------------------|-------------------------------|
| $4311.9 \pm 0.7^{+6.8}_{-0.6}$ | $9.8 \pm 2.7^{+3.7}_{-4.5}$   |
| $4440.3 \pm 1.3^{+4.1}_{-4.7}$ | $20.6 \pm 4.9^{+8.7}_{-10.1}$ |
| $4457.3 \pm 0.6^{+4.1}_{-1.7}$ | $6.4 \pm 2^{+5.7}_{-1.9}$     |

$P_{Cs}$  states (LHCb, 2020)

|                                |                              |
|--------------------------------|------------------------------|
| $4458.8 \pm 2.9^{+4.7}_{-1.1}$ | $17.3 \pm 6.5^{+8.0}_{-5.7}$ |
|--------------------------------|------------------------------|

# Pentaquarks states

LHCb 2015.

$P_c^+(4380)$ ,  $M = (4380 \pm 8 \pm 29)$  MeV,  $\Gamma = (205 \pm 18 \pm 86)$  MeV, and

$P_c^+(4450)$ ,  $M = (4449.8 \pm 1.7 \pm 2.5)$  MeV,  $\Gamma = (39 \pm 5 \pm 19)$  MeV

Assignments:  $(3/2^+, 5/2^-)$  and  $(5/2^+, 3/2^-)$  are possible.

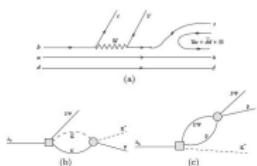


FIG. 1: Mechanisms for the  $\Lambda_b \rightarrow J/\psi K^- p$  reaction implementing the final state interaction

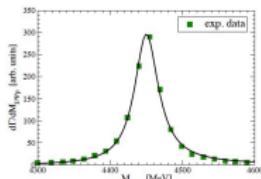
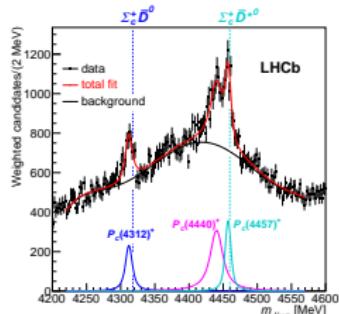


FIG. 2: Results for the  $K^- p$  and  $J/\psi p$  invariant mass distributions compared to the data of ref. [2].

The theoretical analysis of the  $\Lambda_b \rightarrow J/\psi K^- p$  Roca, Nieves, Oset, PRD92 (2015) supports the  $J^P = 3/2^-$  assignment of the pentaquark state, and its nature as  $\bar{D}^* \Sigma_c$ ,  $\bar{D}^* \Sigma_c^*$  molecule.



In 2019 the new experimental analysis shows one more pentaquark,  $P_c(4312)$  and  $P_c(4450)$  splits into two.

# Pentaquark states

- Generalized HGF formalism with HQSS [Xiao, Nieves, Oset \(2013,2019\)](#)

| Mass   | Width | Main channel        | $J^P$   | Experiment  |
|--------|-------|---------------------|---------|-------------|
| 4306.4 | 15.2  | $\bar{D}\Sigma_c$   | $1/2^-$ | $P_c(4312)$ |
| 4453.0 | 23.4  | $\bar{D}^*\Sigma_c$ | $1/2^-$ | $P_c(4440)$ |
| 4452.5 | 3.0   | $\bar{D}^*\Sigma_c$ | $3/2^-$ | $P_c(4457)$ |

Heavy quarks act as **spectators** if we exchange **light vectors**. Heavy quark spin symmetry is automatically fulfilled.

- OMEP, Interaction given by One-Meson-Exchange Potential, [Yamaguchi, Santopinto, PRD96 \(2017\)](#)
- Coupling meson-baryon with compact  $5q$  state, [Hosaka, Tacheuchi, Takizawa, Yamaguchi, Santopionto \(2017\)](#)
- Analysis of the  $J/\psi$  data with coupled-channel involving both one-pion exchange as well as short-range interaction, [Du, Baru, Guo, Hanhart, Meissner, Wang PRL124 \(2020\)](#)

# Flavour exotic states

- 2010. Prediction of several flavour exotic states

PHYSICAL REVIEW D 82, 014010 (2010)

## New interpretation for the $D_{s2}^*(2573)$ and the prediction of novel exotic charmed mesons

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(Received 4 May 2010; published 21 July 2010)

In this manuscript we study the vector-vector interaction within the hidden-gauge formalism in a coupled channel unitary approach. In the sector  $C = 1, S = 1, J = 2$  we get a pole in the  $T$  matrix around 2572 MeV that we identify with the  $D_{s2}^*(2573)$ , coupling strongly to the  $D^*K^*(D_s^*\phi(\omega))$  channels. In addition we obtain resonances in other exotic sectors which have not been studied before such as  $C = 1, S = -1, C = 2, S = 0$  and  $C = 2, S = 1$ . These “flavor-exotic” states are interpreted as  $D^*K^*$ ,  $D^*D^*$  and  $D^*D^*$  molecular states but have not been observed yet. In total we obtain nine states with different spin, isospin, charm, and strangeness of non- $C = 0, S = 0$  and  $C = 1, S = 0$  character, which have been reported before.

DOI: 10.1103/PhysRevD.82.014010

PACS numbers: 14.40.Rt, 12.40.Vv, 13.75.Lb, 14.40.Lb

- Free parameter fixed with  $D_{s2}(2573)$ ; couples to  $D^*K^*$ ,  $c\bar{q}q\bar{s}$
- Flavour exotic states with  $I = 0, J^P = \{0, 1, 2\}^+$  coupling to  $D^*\bar{K}^*$  are predicted,  $c\bar{q}s\bar{q}$
- Doubly charm states,  $I = 0; J^P = 1^+$ , close to  $D^*D^*$  are predicted,  $c\bar{q}c\bar{q}$ , and  $I = 1/2; J^P = 1^+$ , close to  $D^*D_s^*$   $c\bar{q}c\bar{s}$

# Flavour exotic states

Molina, Branz, Oset, PRD82(2010)

| $C, S$ | Channels                | $I[J^P]$   | $\sqrt{s}$                                  | $\Gamma_A(\Lambda = 1400)$ | $\Gamma_B(\Lambda = 1200)$ | State                         | $\sqrt{s}_{\text{exp}}$ | $\Gamma_{\text{exp}}$ |
|--------|-------------------------|------------|---|----------------------------|----------------------------|-------------------------------|-------------------------|-----------------------|
| 1, -1  | $D^* \bar{K}^*$         | $0[0^+]$   | 2848  |                            |                            | $X_0(2866)$ or $T_{cs}(2900)$ | 2866                    | 57                    |
|        |                         | $0[1^+]$   | 2839  | 23                         | 59                         |                               |                         |                       |
|        |                         | $0[2^+]$   | 2733  | 3                          | 3                          |                               |                         |                       |
| 1, 1   | $D^* K^*, D_s^* \omega$ | $0[0^+]$   | 2683  | 20                         | 71                         | $D_{s2}(2573)$                | 2572                    | 20                    |
|        |                         | $0[1^+]$   | 2707  | $4 \times 10^{-3}$         | $4 \times 10^{-3}$         |                               |                         |                       |
|        | $D_s^* \phi$            | $0[2^+]$   | 2572  | 7                          | 23                         |                               |                         |                       |
| 1, 1   | $D^* K^*, D_s^* \rho$   | $1[0^+]$   | Cusp structure around $D_s^* \rho, D^* K^*$ |                            |                            | new $T_{c\bar{s}}(2900)$      | 2908                    | 136                   |
| 1, 1   |                         | $1[1^+]$   | Cusp structure around $D_s^* \rho, D^* K^*$ |                            |                            |                               |                         |                       |
| 1, 1   |                         | $1[2^+]$   | 2786  | 8                          | 11                         |                               |                         |                       |
| 2, 0   | $D^* D^*$               | $0[1^+]$   | 3969  | 0                          | 0                          |                               |                         |                       |
| 2, 1   | $D^* D_s^*$             | $1/2[1^+]$ | 4101  | 0                          | 0                          |                               |                         |                       |

**Table 1:** All the quantities here are in MeV. Repulsion in  $C = 0, S = 1, I = 1/2$ ;  $C = 1, S = -1, I = 1$ ;  $C = 1, S = 2, I = 1/2$ ;  $C = 2, S = 0, I = 1$  and  $C = 2, S = 2, I = 0$  is found.

Form factors in the  $D^* D\pi$  vertex; Model A:  $F_1(q^2) = \frac{\Lambda_b^2 - m_\pi^2}{\Lambda_b^2 - q^2}$ , Titov, Kampfer EPJA7, PRC65 with  $\Lambda_b = 1.4, 1.5$  GeV and

$g = M_\rho / 2 f_\pi$ . Model B:  $F_2(q^2) = e^{q^2/\Lambda^2}$  Navarra, Nielsen, Bracco PRD65 (2002),  $\Lambda = 1, 1.2$  GeV and  $g_D = g_{D^* D\pi}^{\text{exp}} = 8.95$  (experimental value). Subtraction constant  $\alpha = -1.6$ .

**$D^*D^*$  and  $D^*D_s^*$  states**

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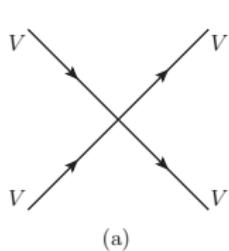
$DD^*$ :  $T_{cc}(3875)$ , LHCb, Nature(2022),  $\delta m = -360 \pm 40^{+4}_{-0}$  KeV,

$\Gamma = 48 \pm 2^{+0}_{-14}$  KeV

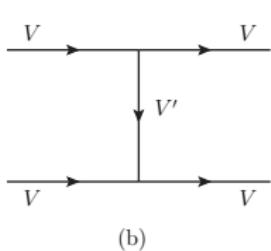
Signature in LQCD Virtual bound state,  $m_\pi \simeq 280$  MeV, Padmanath, Prelovsek PRL129(2022)

See Eulogio's talk on Thursday, 2.30pm

Feijoo, Liang, Oset, PRD104 (2021) Local Hidden-Gauge Approach  $D^0 D^{*+}$ ,  $D^+ D^{*0}$  correlation functions; Inverse problem, Vidana, Feijoo, Albaladejo, Oset, Nieves, 2303.06079, 2304.01870 (2023)



(a)



(b)

$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2} \langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle$$

$$\mathcal{L}_{III}^{(3V)} = ig \langle [V_\mu, \partial_\nu V_\mu] V^\nu \rangle$$

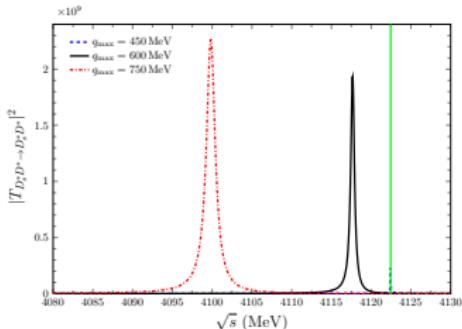
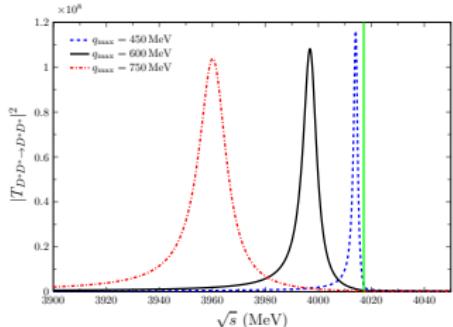
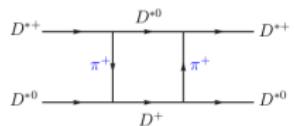
Repulsion for  $I = 1$

| $J$ | Amplitude                         | Contact | V-exchange   | $\sim$ Total |
|-----|-----------------------------------|---------|--|--------------|
| 1   | $D^* D^* \rightarrow D^* D^*$     | 0       | $\frac{g^2}{4} \left( \frac{2}{m_{J/\psi}^2} + \frac{1}{m_\omega^2} - \frac{3}{m_P^2} \right) \{(p_1 + p_3) \cdot (p_2 + p_4) + (p_1 + p_4) \cdot (p_2 + p_3)\}$ | $-25.4g^2$   |
| 1   | $D_s^* D^* \rightarrow D_s^* D^*$ | 0       | $-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{K^*}^2} + \frac{g^2(p_1+p_3)(p_2+p_4)}{m_{J/\psi}^2}$  | $-19.5g^2$   |

# $T_{cc}$ states from $D^*D^*/D^*D_s^*$

$$V \rightarrow V + i \operatorname{Im} V_{\text{box}}(DD^*)$$

$$T^{-1} = V^{-1} - G$$

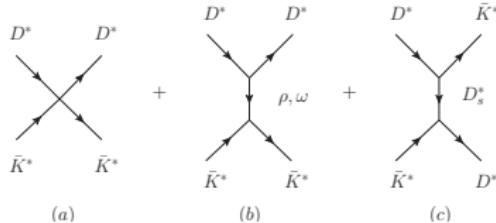


| $q_{\max} = 450$ MeV | $q_{\max} = 420$ MeV |
|----------------------|----------------------|
| $M_{D^*D^*}$         | 4014.08 MeV          |
| $B_{D^*D^*}$         | 3.23 MeV             |
| $\Gamma_{D^*D^*}$    | 2.3 MeV              |
| $M_{D_s^*D^*}$       | 4122.46 MeV (cusp)   |
| $\Gamma_{D_s^*D^*}$  | 70 – 100 KeV         |

**The  $X_0(2866)$  or  $T_{cs}(2900)$**

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## Local Hidden Gauge Approach



**Figure 1:**  $D^* \bar{K}^* \rightarrow D^* \bar{K}^*$  interaction

## Potential $V$ : contact + vector-meson exchange ( $\rho, \omega$ )

| $J$ | Amplitude                                 | Contact   | V-exchange | $\sim$ Total |
|-----|---|---|------------|--------------|
| 0   | $D^* \bar{K}^* \rightarrow D^* \bar{K}^*$ | $4g^2 - \frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{1}{2}g^2(\frac{1}{m_\omega^2} - \frac{3}{m_\rho^2})(p_1+p_3).(p_2+p_4)$  |            | $-9.9g^2$    |
| 1   | $D^* \bar{K}^* \rightarrow D^* \bar{K}^*$ | $0 \quad \frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{1}{2}g^2(\frac{1}{m_\omega^2} - \frac{3}{m_\rho^2})(p_1+p_3).(p_2+p_4)$ |            | $-10.2g^2$   |
| 2   | $D^* \bar{K}^* \rightarrow D^* \bar{K}^*$ | $-2g^2 - \frac{g^2(p_1+p_4).(p_2+p_3)}{m_{D_s^*}^2} + \frac{1}{2}g^2(\frac{1}{m_\omega^2} - \frac{3}{m_\rho^2})(p_1+p_3).(p_2+p_4)$ |            | $-15.9g^2$   |

**Table 2:** Tree level amplitudes for  $D^*\bar{K}^*$  in  $I = 0$ .

Attractive for  $I = 0$  and repulsive for  $I = 1$ .

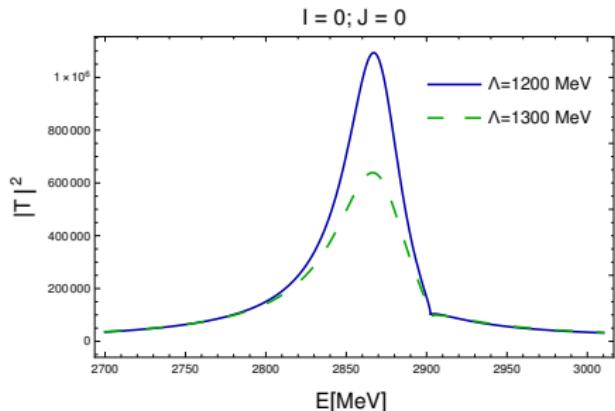
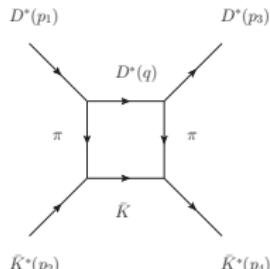
# Decay of the $T_{cs}(2900)$ to $D^* \bar{K}$

Molina, Oset PLB811 2020,  $\alpha = -1.474$ ,  $\Lambda = 1300$ .

| $I(J^P)$ | $M[\text{MeV}]$ | $\Gamma[\text{MeV}]$ | Coupled channels | state          |
|----------|-----------------|----------------------|------------------|----------------|
| $0(2^+)$ | 2775            | 38                   | $D^* \bar{K}^*$  | ?              |
| $0(1^+)$ | 2861            | 20                   | $D^* \bar{K}^*$  | ?              |
| $0(0^+)$ | 2866            | 57                   | $D^* \bar{K}^*$  | $T_{cs}(2900)$ |

**Table 3:** New results including the width of the  $D^* K$  channel.

$$T = [I - VG]^{-1} V$$



# How can we observe the $J^P = 1^+$ $T_{cs}(2900)$ state?

Amo Sanchez et al. (BABAR), PRD83(2011).

The  $\bar{B}^0 \rightarrow D^{*+} \bar{D}^{*0} K^-$  reaction:

- It proceeds via external emission (favoring the decay)
- It has the largest branching fraction (1.06%)
- It can produce the  $D^{*+} K^-$  in  $I = 0$  (decay mode of the  $1^+$  state).

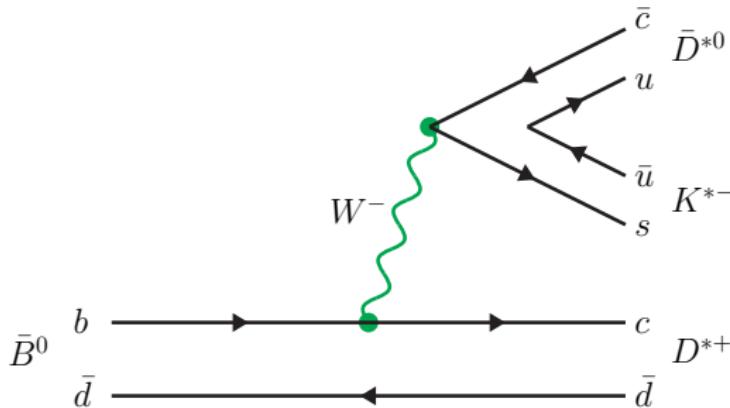
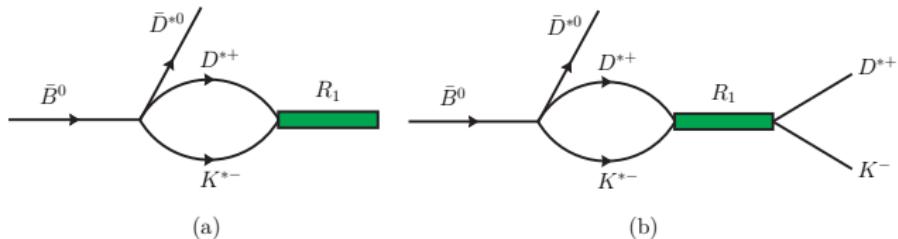


Figure 2: Diagrammatic decay of the  $\bar{B}^0 \rightarrow \bar{D}^{*0} D^{*+} K^{*-}$  at the quark level.

# How can we observe the $J^P = 1^+$ $T_{cs}(2900)$ state?

Hadronization + decay;  $\bar{B}^0 \rightarrow D^{*+} \bar{D}^{*0} K^-$



**Figure 3:** (a) Rescattering of  $D^{*+} K^{*-}$  to produce  $R_1$ ; (b) Decay of  $R_1$  to  $D^{*+} K^-$ . [Dai, Molina and Oset, PLB832 \(2022\)](#)

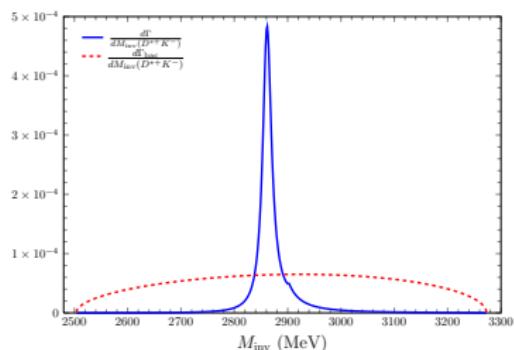
$$\frac{d\Gamma}{dM_{\text{inv}}(D^{*+} K^-)} = \frac{1}{(2\pi)^3} \frac{1}{4M_{\bar{B}^0}^2} \rho_{\bar{D}^{*0}} \tilde{\rho}_{K^-} - \sum |t'|^2$$

See also:

$\bar{B}^0 \rightarrow D^{*+} K^- \bar{K}^{*0}$ , PRD105(2022);

$2^+$ :  $B^+ \rightarrow D^+ D^- K^+$ ,

PLB833(2022), Bayar, Oset.



# The $T_{c\bar{s}}(2900)$

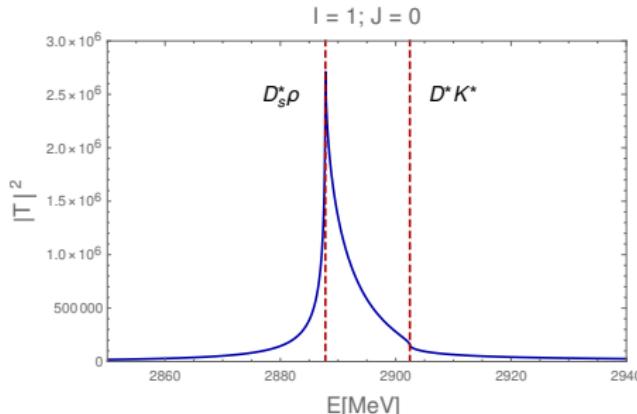
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# Exotic states

Phys. Rev. D 82 (2010), Molina, Branz, Oset

## 3.5 $C = 1; S = 1; I = 1$

In this sector the potential is attractive for the  $D^*K^* \rightarrow D_s^*\rho$  reaction. For  $J = 0$  and  $1$  this potential is around  $-7g^2$  whereas it is by a factor of two bigger  $-13g^2$  for  $J = 2$  (see Table 14). In fact, we only obtain a pole for  $J = 2$ . For  $J = 0$  and  $1$  we only observe a cusp in the  $D_s^*\rho$  threshold. In Table 5 we show the pole position and couplings to the different channels. Both channels,  $D^*K^*$  and  $D_s^*\rho$ , are equally important as one can deduce from the corresponding couplings. The broad width of the  $\rho$  meson has to be taken into account



$$\alpha = -1.6$$

$\rho$  width not included

$D^*K^* \rightarrow DK$  considered

Cusp around  $D_s^*\rho$ ,  $D^*K^*$  th.  
separated only by 14 MeV

# Local Hidden Gauge Approach

| $J$ | Amplitude                           | Contact | V-exchange   | $\sim$ Total |
|-----|-------------------------------------|---------|--|--------------|
| 0   | $D^* K^* \rightarrow D^* K^*$       | 0       | $\frac{g^2}{2} \left( \frac{1}{m_\rho^2} - \frac{1}{m_\omega^2} \right) (p_1 + p_3) \cdot (p_2 + p_4)$ | 0            |
| 0   | $D^* K^* \rightarrow D_s^* \rho$    | $4g^2$  | $-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2} - \frac{g^2(p_1+p_3)(p_2+p_4)}{m_{K^*}^2}$                   | $-6.8g^2$    |
| 0   | $D_s^* \rho \rightarrow D_s^* \rho$ | 0       | 0  | 0            |
| 1   | $D^* K^* \rightarrow D^* K^*$       | 0       | $\frac{g^2}{2} \left( \frac{1}{m_\rho^2} - \frac{1}{m_\omega^2} \right) (p_1 + p_3) \cdot (p_2 + p_4)$ | 0            |
| 1   | $D^* K^* \rightarrow D_s^* \rho$    | 0       | $\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2} - \frac{g^2(p_1+p_3)(p_2+p_4)}{m_{K^*}^2}$                    | $-6.6g^2$    |
| 1   | $D_s^* \rho \rightarrow D_s^* \rho$ | 0       | 0  | 0            |
| 2   | $D^* K^* \rightarrow D^* K^*$       | 0       | $\frac{g^2}{2} \left( \frac{1}{m_\rho^2} - \frac{1}{m_\omega^2} \right) (p_1 + p_3) \cdot (p_2 + p_4)$ | 0            |
| 2   | $D^* K^* \rightarrow D_s^* \rho$    | $-2g^2$ | $-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D^*}^2} - \frac{g^2(p_1+p_3)(p_2+p_4)}{m_{K^*}^2}$                   | $-12.8g^2$   |
| 2   | $D_s^* \rho \rightarrow D_s^* \rho$ | 0       | 0  | 0            |

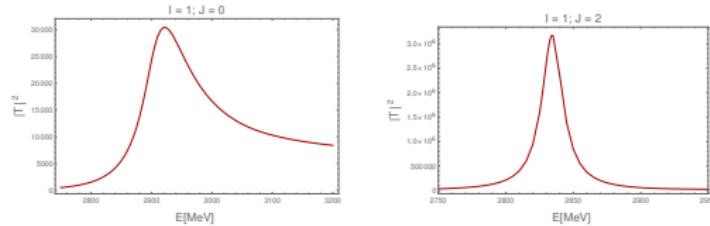
**Table 4:** Tree level amplitudes for  $D^* K^*$ ,  $D_s^* \rho$  in  $I = 1$ .  $C = 1; S = 1; I = 1$ .

The interaction is attractive for both  $I = 0$  and  $I = 1$ , favoring a  $J^P = 2^+$  state. (see PRD82 (2010) Molina, Branz, Oset, for  $I = 0$ )

New results,  $\alpha = -1.474$  to obtain the  $T_{cs}(2900)$  state in  $D^* \bar{K}^*$ .

### Convolution due to the vector meson mass distribution $\rho, K^*$

$$\tilde{G}(s) = \frac{1}{N} \int_{(M_1-4\Gamma_1)^2}^{(M_1+4\Gamma_1)^2} d\tilde{m}_1^2 \left(-\frac{1}{\pi}\right) \mathcal{I}m \frac{1}{\tilde{m}_1^2 - M_1^2 + i\Gamma(\tilde{m})\tilde{m}_1} G(s, \tilde{m}_1^2, M_2^2),$$

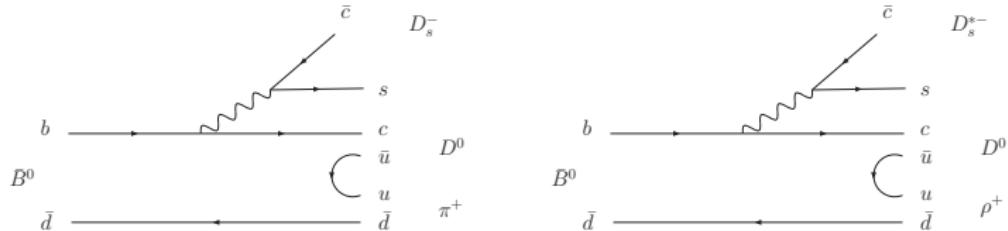


| $I(J^P)$ | $M[\text{MeV}]$ | $\Gamma[\text{MeV}]$ | Coupled channels    | state                |
|----------|-----------------|----------------------|---------------------|----------------------|
| $1(0^+)$ | 2920            | 130                  | $D^* K^*, D_s \rho$ | $T_{c\bar{s}}(2900)$ |
| $1(1^+)$ | 2922            | 145                  |                     | ?                    |
| $1(2^+)$ | 2835            | 20                   |                     | ?                    |

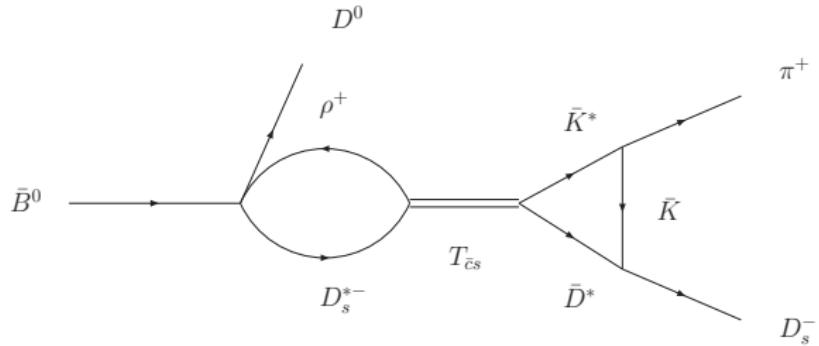
**Table 5:** PRD107(2023), Exp.  $(m, \Gamma) = (2908 \pm 11 \pm 20, 136 \pm 23 \pm 11)$  MeV

# Production of the $T_{\bar{c}s}(2900)$

$\bar{B}^0 \rightarrow D_s^- D^0 \pi^+$  in  $B$  decays



The  $T_{\bar{c}s}(2900)$  can be produced by means of **external emission**

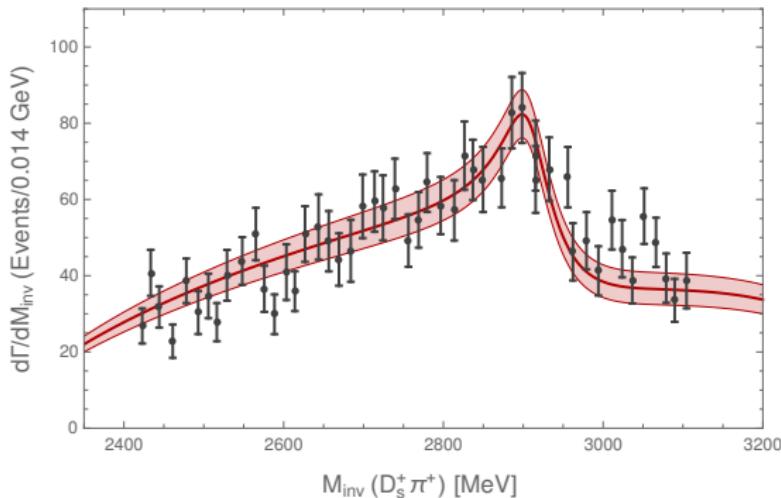


# Production of the $T_{\bar{c}s}(2900)$ in $B$ decays

$$T(E) = aG(E)_{D_s^* \rho} t_{D_s^* \rho \rightarrow \bar{D}^* \bar{K}^*}(E) t_L(E) + b \quad (3)$$

$E = M_{inv}(\pi^+ D_s^-)$ ;  $a, b$  parameters;  $t_L$  amplitude for the triangle loop.

$$\boxed{\frac{d\Gamma}{dM_{Inv}} = \frac{1}{(2\pi)^3} \frac{1}{4M_B^2} p_D \tilde{p}_\pi |T|^2}$$



## **$D_{s0}(2317)$ quark mass dependence**

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# Quark mass dependence of the $D(D^*)$ mesons

## Heavy Hadron Chiral Perturbation Theory ( $\text{HH}\chi\text{PT}$ )

E. Jenkins, NPB412 (1994)

$$\frac{1}{4}(D + 3D^*) = m_H + \alpha_a - \sum_{X=\pi,K,\eta} \beta_a^{(X)} \frac{M_X^3}{16\pi f^2} + \sum_{X=\pi,K,\eta} (\gamma_a^{(X)} - \lambda_a^{(X)} \alpha_a) \frac{M_X^2}{16\pi^2 f^2} \log(M_X^2/\mu^2) + c_a$$

$$(D^* - D) = \Delta + \sum_{X=\pi,K,\eta} (\gamma_a^{(X)} - \lambda_a^{(X)} \Delta) \frac{M_X^2}{16\pi^2 f^2} \log(M_X^2/\mu^2) + \delta c_a$$

$\mu = 770$  MeV;  $g^2 = 0.55$  MeV (Decay of the  $D^*$  meson)

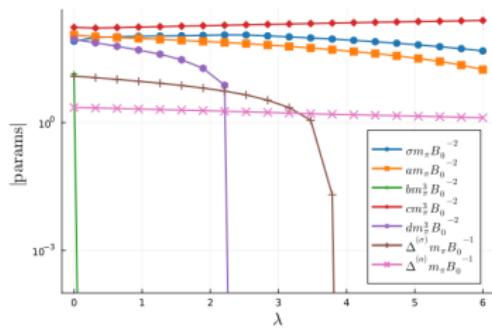
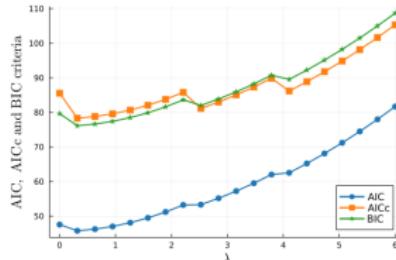
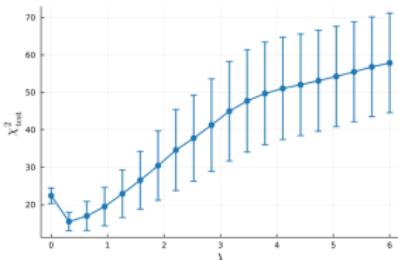
$$\left. \begin{aligned} \frac{1}{4}(D + 3D^*) &= m_H + f(\sigma, a, b, c, d) \\ (D^* - D) &= \Delta + g(\Delta^{(\sigma)}, \Delta^{(a)}) \end{aligned} \right\} \begin{array}{l} \text{9 parameters, but different collaborations/scale} \\ \text{settings, } 7 + 2 \times 7 = 21 \text{ parameters, } \sim 80 \text{ data} \\ \text{points} \end{array}$$

ETMC, PACS, HSC, CLS, RQCD, S.Prelovsek, MILC

# Quark mass dependence of the $D(D^*)$ mesons

LASSO + information criteria;

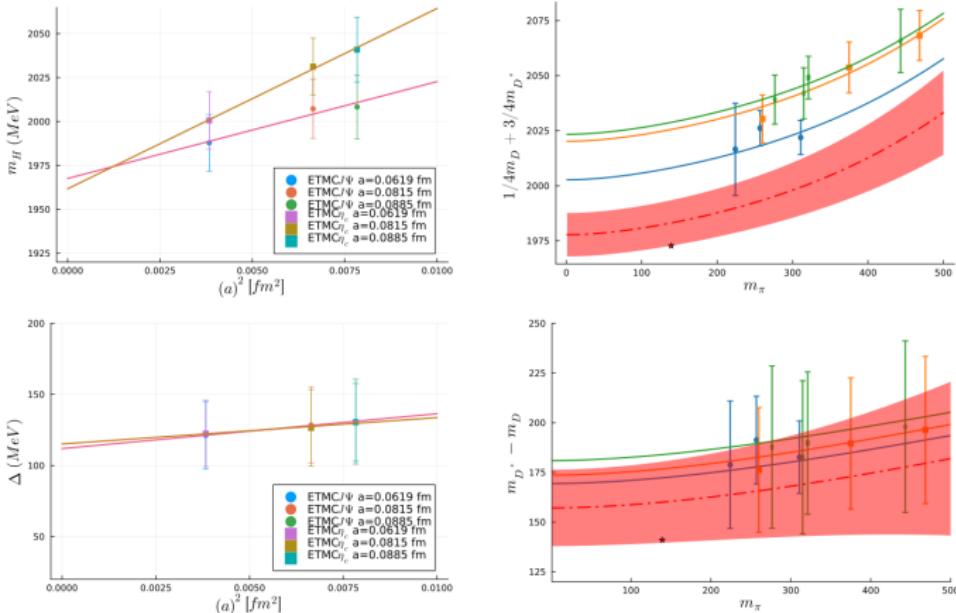
$$\chi_P^2 = \chi^2 + \lambda \sum_i^n |p_i|; \quad \text{Data} = \text{Training (70\%)} + \text{Test (30\%)} \quad (4)$$



$$\frac{1}{4}(m_D + 3m_{D^*}) = m_H + f(\sigma, a, \beta, c, d)$$

$$m_{D^*} - m_D = \Delta + g(\Delta^{(\sigma)}, \Delta^{(a)})$$

# $D(D^*)$ quark mass dependence



**Figure 5:** Extrapolation to the physical point of the ETMC data.

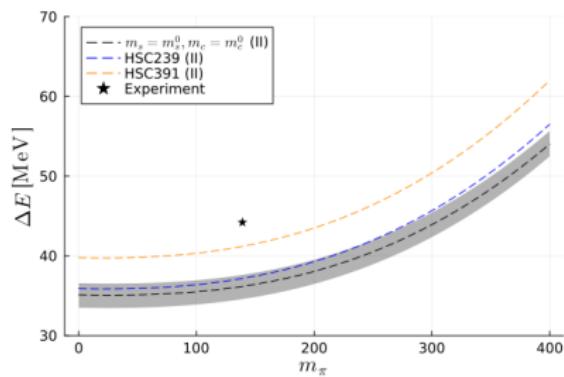
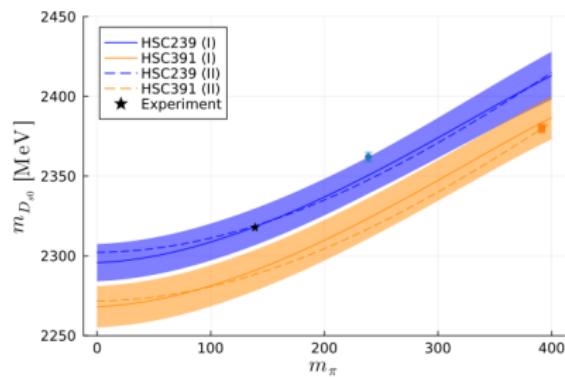
# Quark mass dependence of the $D_{s0}(2317)$ resonance in $DK$

Potential  $V(s)$  consistent with HQSS, See Fernando's talk at 2.30pm

See also L.S. Geng and Albaladejo's talk about  $D_{s0}(2317)$  (Femtoscopy)

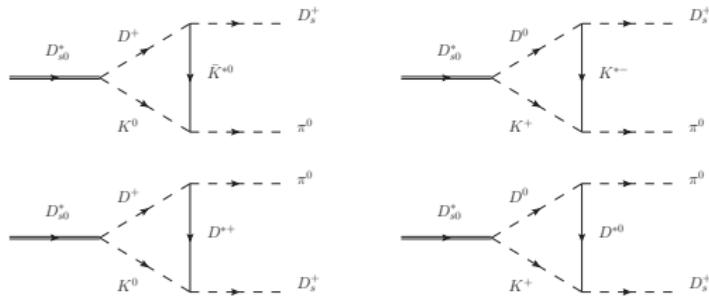
$$V(s) = V_{DK}(s) + V_{\text{ex}}(s); \quad 1 - Z \simeq 0.7 - 0.8$$

$$2306.01848 \quad V_{DK} = -\frac{s-u}{2f^2} \quad ; \quad V_{\text{ex}} = \frac{V_{c\bar{s}}^2}{s-m_{c\bar{s}}^2} \quad (5)$$



$$\begin{aligned} m_\pi &= 236 \text{ MeV}; a_t^{-1} = 5.667 \text{ GeV}; a_t M_{\eta_c} = 0.2412, M_{\eta_c} = 2986 \text{ MeV}; \\ m_\pi &= 391 \text{ MeV}; a_t^{-1} = 6.079 \text{ GeV}; a_t M_{\eta_c} = 0.2735; M_{\eta_c} = 2963 \text{ MeV}; \end{aligned}$$

# Decay width of the $D_{s0}^*(2317)$ to $D_s^+ \pi^0$



**Figure 6:** Feynman diagrams of the  $D_s^{*0} \rightarrow D_s^+ \pi^0$ .

$\pi^0 - \eta$  mixing

$$\tilde{\pi}^0 = \pi^0 \cos \tilde{\epsilon} + \eta_8 \sin \tilde{\epsilon}$$

$$\tilde{\eta} = -\pi^0 \sin \tilde{\epsilon} + \eta_8 \cos \tilde{\epsilon}$$

$$\text{with } \eta_8 = \frac{2\sqrt{2}}{3}\eta - \frac{1}{3}\eta'$$

$$g_X = \frac{g_{DK}^{(I=0)}}{\sqrt{2}}$$

$$\Gamma_X = |\vec{p}_f| \frac{|t|^2}{8\pi m_X^2} \quad (6)$$

$$\boxed{\Gamma_{D_{s0}^*} = 100 \pm 30 \text{ KeV}}$$

H. L. Fu et al.,  $120^{+18}_{-4}$  MeV,  
EPJA58(2022), M. Cleven,

$c\bar{s}$  state:  $\Gamma = 7.83^{+1.97}_{-1.55}$  KeV

M. Han et al., 2305.04250 (2023)

## **Conclusions**

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## Conclusions

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- The HGF has predicted many exotic states. Some of them discovered. A new table of exotic particles is coming ...
- The  $X_0(2866)$  or  $T_{c\bar{s}}(2900)$  is compatible with a  $D^*\bar{K}^*$  resonance decaying to  $D\bar{K}$ . Proposed reactions to observe the  $1^+$  state:  
 $\bar{B}^0 \rightarrow D^{*+}\bar{D}^{*0}K^-$ , PLB832 (2022), Dai, Molina, Oset,  
 $\bar{B}^0 \rightarrow D^{*+}K^-\bar{K}^{*0}$ , PRD105 (2022); and the  $2^+$  state:  
 $B^+ \rightarrow D^+D^-K^+$ , PLB833 (2022), Bayar and Oset.
- The  $T_{c\bar{s}}(2900)$  is more likely to be a failed bound state, or cusp structure around the  $D^*K^*$ ,  $D_s^*\rho$  thresholds.
- The combination of LQCD with EFT's is a useful tool to extract the properties of resonances with high accuracy
- The study of the pion mass dependence of the  $D_{s0}(2317)$  supports the  $DK$  molecular picture