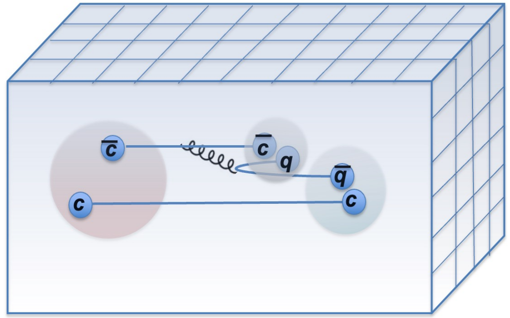
Spectroscopy of hadrons with heavy quarks from lattice QCD

Sasa Prelovsek

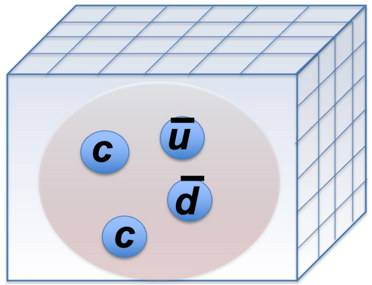
University of Ljubljana, Slovenia

Jozef Stefan Institute, Ljubljana , Slovenia



HADRON 2023

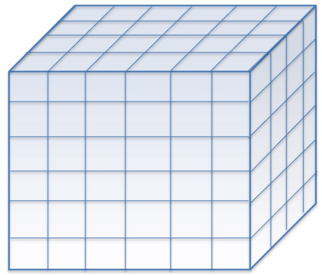
June, 2023, Genoa



QCD: $\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$ $\quad g_s \not\ll 1$ at hadronic energy scale

Lattice QCD

$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\; C\; e^{-S_{QCD}/\hbar}$$



zero temperature
(in this talk)

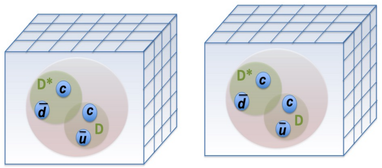
often "non-precision" studies:

single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

Main quantity extracted for spectroscopy: eigen-energies E_n

$\sum_n |n\rangle\langle n|$ $\qquad$ Eucledian time $\quad e^{-iE_n t_M}$

$$C_{ij}^{2pt}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\, e^{-E_n t_E} \langle n | \mathcal{O}_j^+ | 0 \rangle$$



contributions of excited states supressed

E_n allow to extract masses stable hadrons and decaying resonances (as I'll explain later)

Hadron structure

$$C \to \langle H | J | H \rangle \;,\;\; \langle 0 | J | H \rangle$$

studied mostly for strongly stable hadrons H

recent reviews:

- N. Brambilla et al. 1907.07583, Phys. Rept
- M. Mai, U. Meissner, C. Urbach, 2206.01477
- N. Brambilla, 2111.10788
- P. Bicudo, 2212.07793

Strongly-stable conventional hadrons : masses, structure

low-lying quarkonia are storngly stable when QQ annihilation is omitted

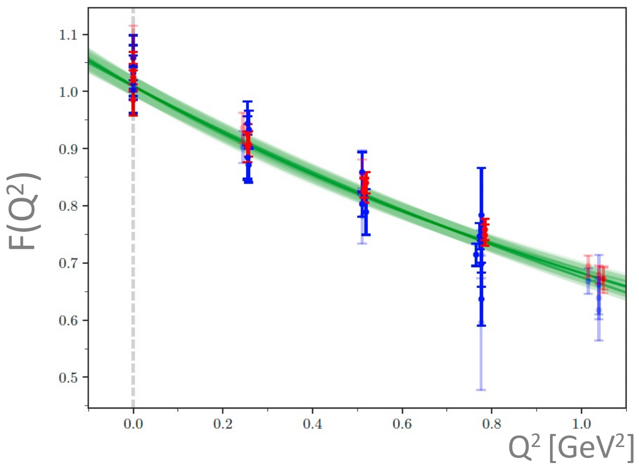
quarkonium spatial profiles:

Knechtli, Peardon et al 2205.11564

$\bar{c}c$

charge distribution

$$\langle \chi_{c0} | J^{\mu}_{em}(Q) | \chi_{c0} \rangle$$



$$\rho_{el}(\vec{r}) = F.T.\ F(\vec{Q})$$

$$\langle r^2 \rangle = -6 \frac{dF}{dQ^2}_{Q^2=0}$$

$$\langle r^2_{\eta_c} \rangle^{\frac{1}{2}} < \langle r^2_{\chi_{c0}} \rangle^{\frac{1}{2}} < \langle r^2_{\eta'_c} \rangle^{\frac{1}{2}}$$

in agreement with expectation from quark model

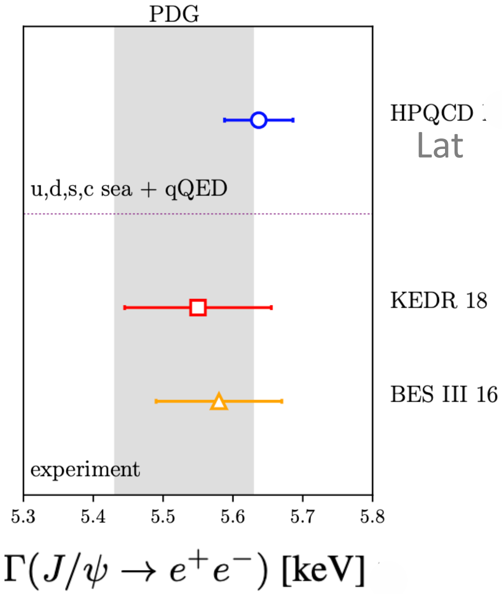
Delaney et al, HadSpec: 2301.08213

agree with well exp

$\bar{c}c$

info on wave function at the origin

$$\langle 0 | \bar{c}\gamma^{\mu} c | J/\psi \rangle$$



+ many other properties of cc and bb

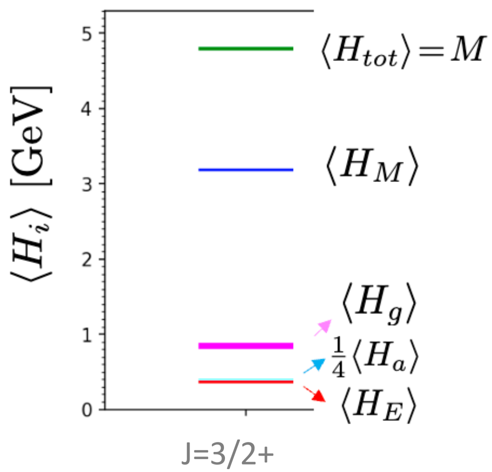
HPQCD: 2204.02137, 2005.01845, 2101.08103

QCD+QED

ccc

mass decomposition

$$\langle \text{baryon} | H_i | \text{baryon} \rangle$$



$$H_{\mathrm{QCD}} = H_E^{(\mu)} + H_M + H_g^{(\mu)} + \frac{1}{4} H_a$$

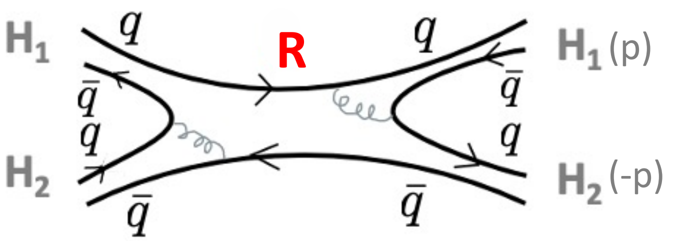
$$H_E^{(\mu)} = \sum_f \int d^3x \bar{\psi}^{(f)} i(\vec{D} \cdot \vec{\gamma}) \psi^{(f)},$$

$$H_M = \sum_f \int d^3x \bar{\psi}^{(f)} m_f \psi^{(f)},$$

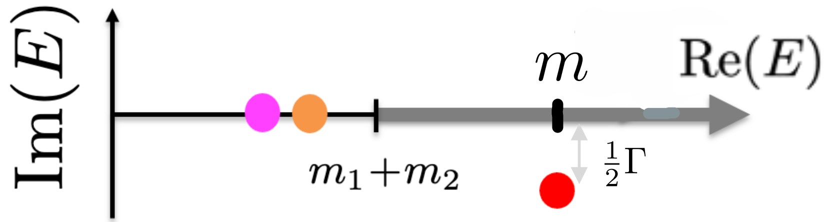
$$H_g^{(\mu)} = \int d^3x \frac{1}{2}(E^2 + B^2),$$

Yin-Bo Li et al, 2211.04713

Resonances R→ H_1 H_2 , bound states near threshold



scattering amplitude T(E)



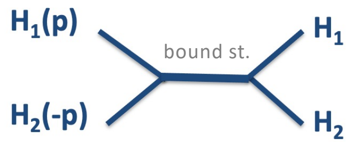
Virtual bound st. — $p = -i\,|p|$

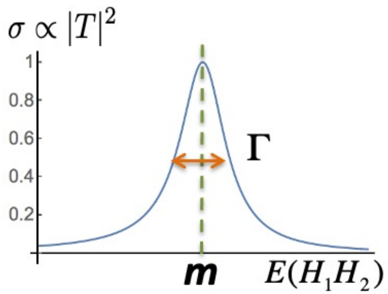
Bound st. — $p = i\,|p|$

Resonance

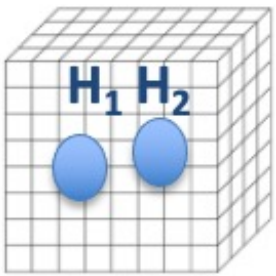
$$T(E) \propto \frac{1}{E^2 - m^2}$$

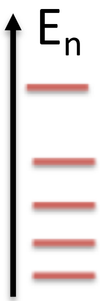
$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$





Scattering amplitude T(E) from lattice QCD





many resonances and bound states extracted in this way by now

real E — for real E — for complex E

$$E \to T(E) \to T(E^c)$$

analytic relation:
Luscher 1991
generalizations by many authors

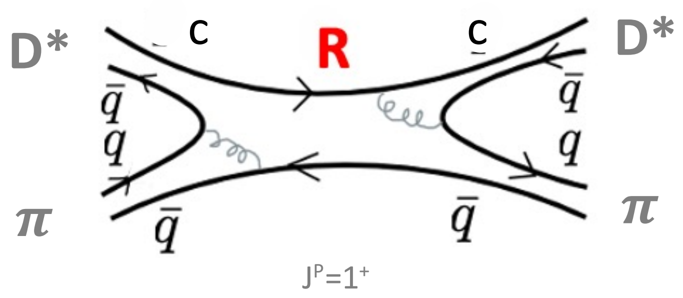
analytic contin. to complex E

rediscovery via neutral networks:

Y. Lu et al 2210.02184

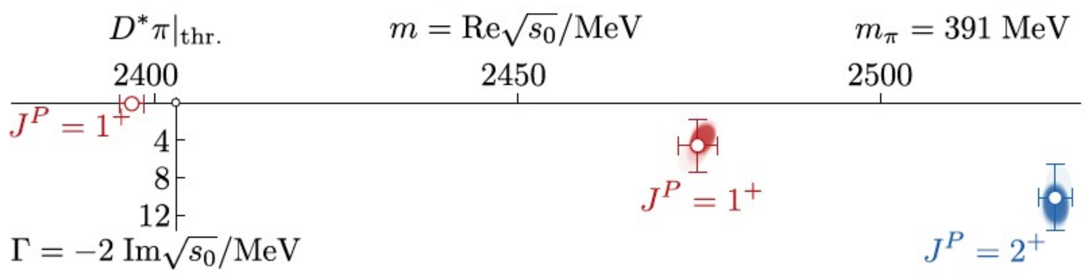
Example: Charmed mesons in D*π scattering

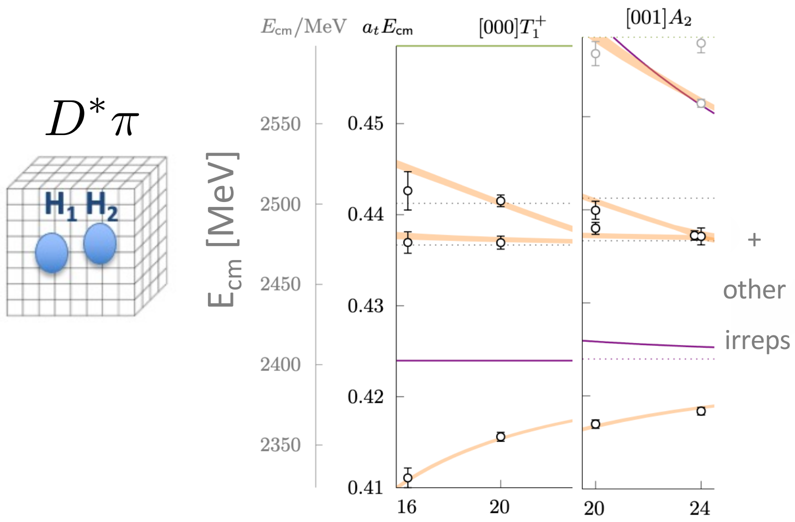
N. Lang & D. Wilson, HadSpec, 2205.05026, PRL : $m_\pi \approx 391$ MeV

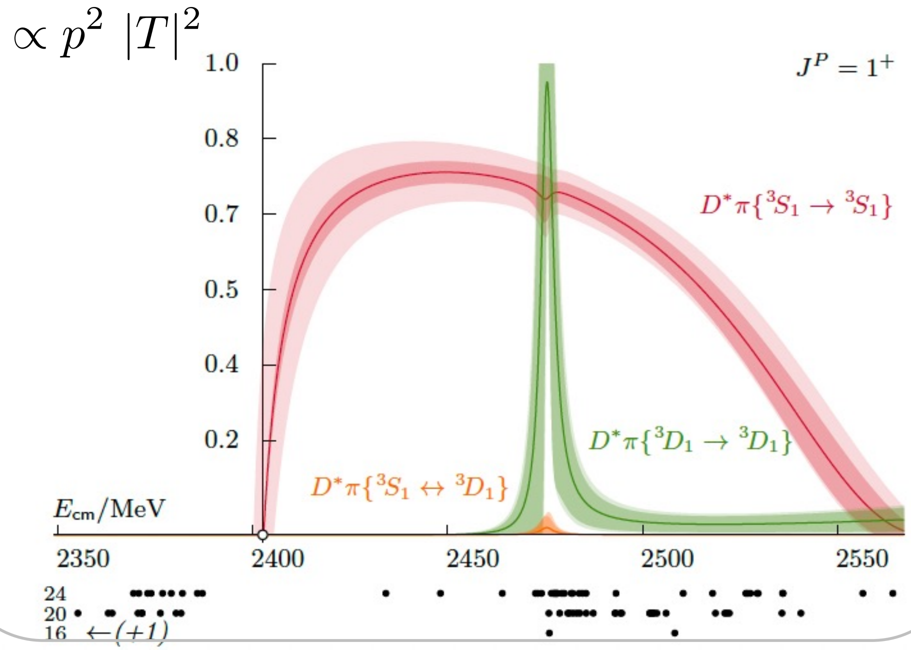


partial waves: S, D

pole positions







physics interpretation:

later in the slides

Current challenges and endeavours

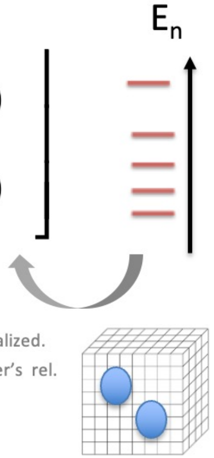
Resonances from coupled-channel scattering

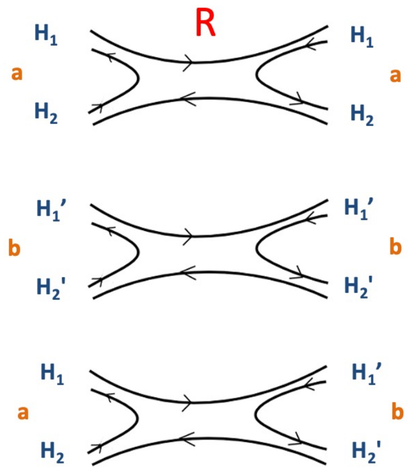
$$R \to H_1 H_2,\ H_1' H_2', \ldots$$

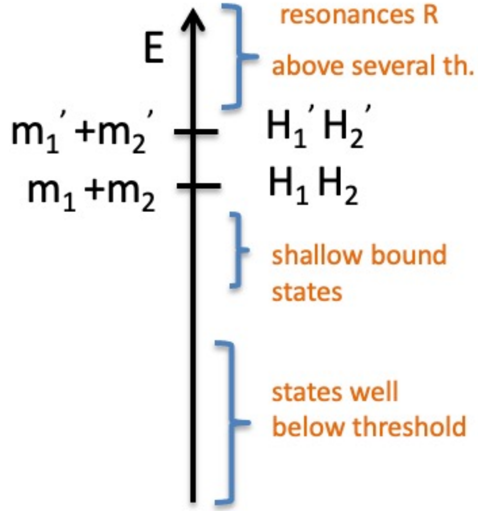
channel a : $H_1 H_2$
channel b : $H_1' H_2'$

$$T(E) = \begin{bmatrix} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{bmatrix}$$

a -> a, a -> b, b->a, b -> b







Status:

many studies in light sector (mosly HadSpec): reviewed by D. Wilson @ Hadron 2021

heavy sector: charmed mesons: HadSpec 2016, 607.07093

charmonium-like mesons: SP et al, 2011.02541

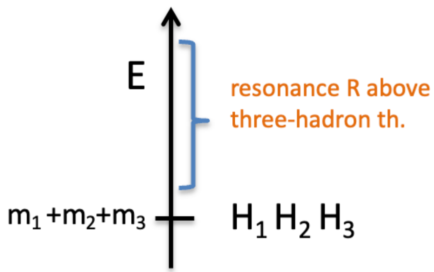
Zc: HALQCD method, 1602.03465, PRL

$H_1 H_2 H_3$ scattering , R→ $H_1 H_2\ H_3$

$\pi\pi\pi,\quad KK\pi,\quad \ldots$ $\quad a1 \to \pi\pi\pi$

all in light sector — no other resonace for now

reviewed by Fernando Ramirez Lopez @ Hadron 2021



journey to various hadron sectors

(with heavy quarks)

exciting experimental discoveries by:

LHCb, Belle, BesIII, Babar, ...

most of discovered exotic hadrons contain heavy quarks:

(these are more likely to be quasi-bound due to small kinetic energy of Q)

Q=c,b   *q=u,d,s*

$\bar{c}c\ ,\quad \bar{c}c\ \bar{q}q$

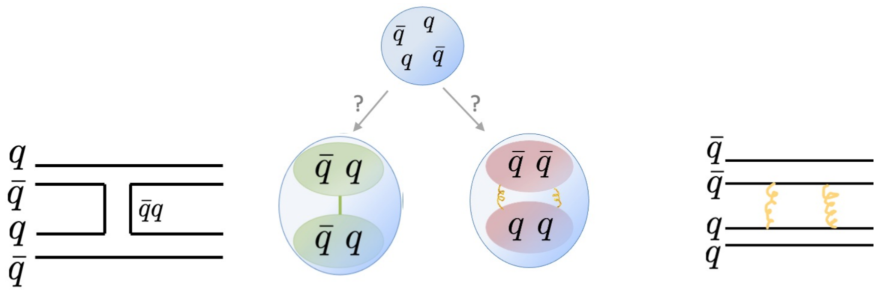
$\bar{b}b\ ,\quad \bar{b}b\ \bar{q}q$

$QQ\ \bar{q}\bar{q}$

$\bar{Q}q\ ,\quad \bar{Q}q\ \bar{q}q$

dibaryons

possible
binding mechanisms



"molecule"

How challenging is a given state for ab-initio study? General rule:

more strong decay channels -> more challenging charmonium(like) sector

$\bar{c}c$, $\bar{c}q\bar{q}c$, $\bar{c}cqqq$

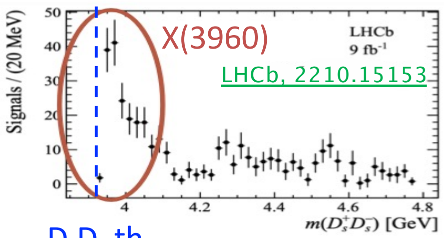
Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

$$\frac{|c^2_{\bar{D}D}|}{|c^2_{\bar{D}_s D_s}|} = 0.02\ ^{+0.02}_{-0.01}$$

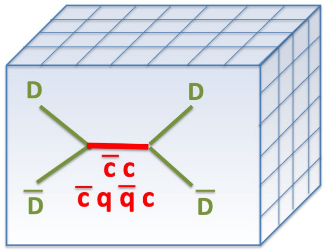
$$T_{ij}(E_{cm}) \sim \frac{c_i\ c_j}{E^2_{cm} - m^2}$$

$\bar{D}_s D_s$ $J^P=0^+$ state



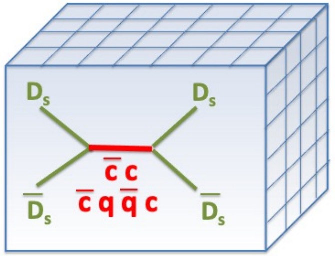
D_sD_s th.

$\bar{D}D$ $J^P=0^+$ state

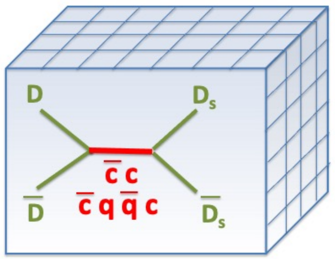


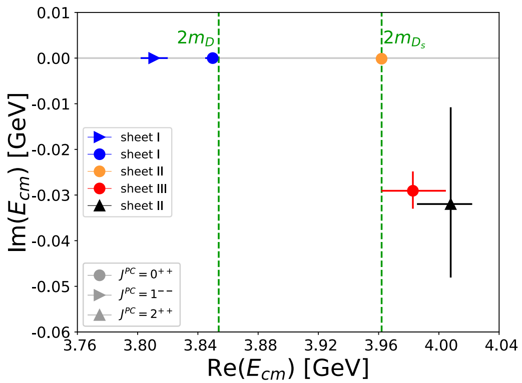
predicted in models [Oset et al, 0612179 PRD, Guo et al 2101.01021]

seen in re-analysis of exp. [Danilkin et al 2111.15033, Ji, F.K. Guo et al., 2212.00613]



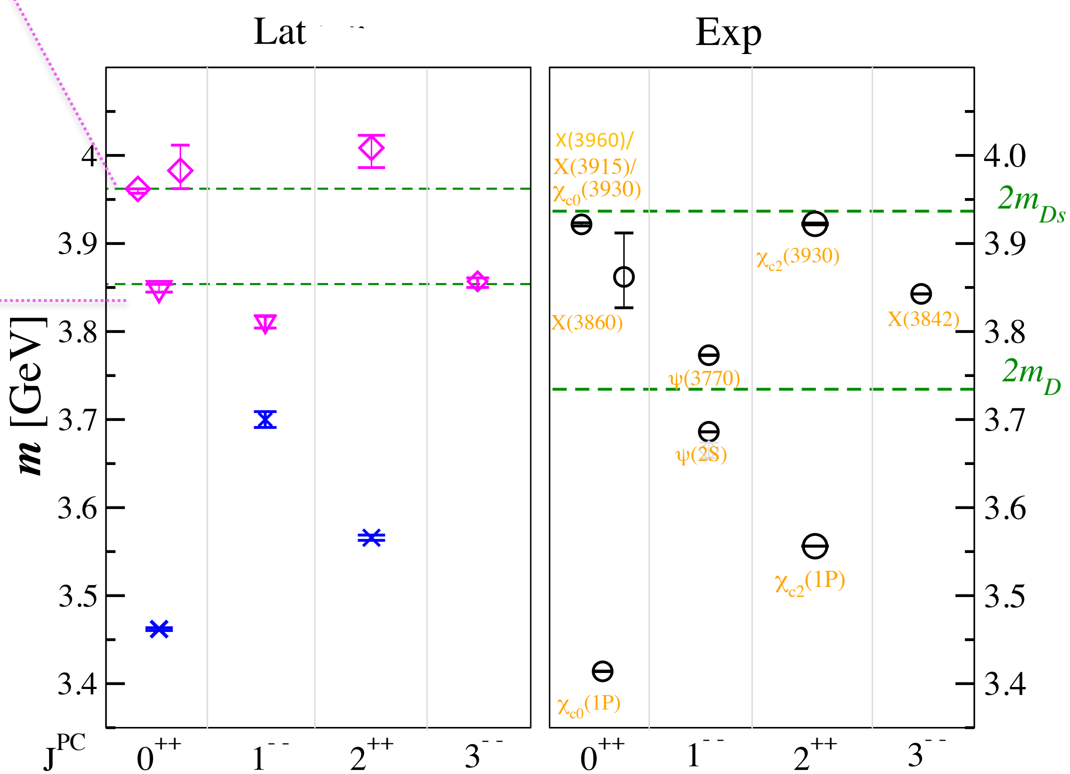
+ expected conventional charmonia





$\bar{D}D - \bar{D}_s D_s$

$m_\pi \simeq 280$ MeV



S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP,
1905.03506 PRD

CLS ensembles

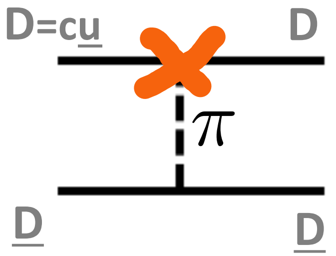
Likely interpretation of some near-threshold states: "molecules" attracted by V exchange

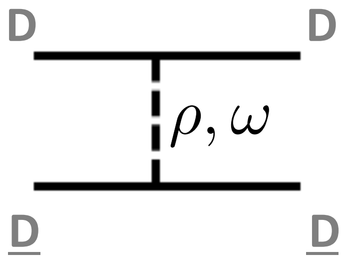
a number of pheno studies
Oset et al, 0612179 PRD,
Wu, Molina, Oset, Zou, 1007.0573, PRL
Guo et al, 2101.01021,...

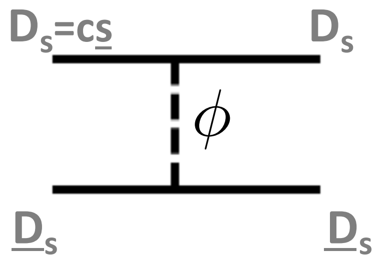
$\bar{c}q\bar{q}c$ $I=0$ $J^P=0^+$

now support also from lattice

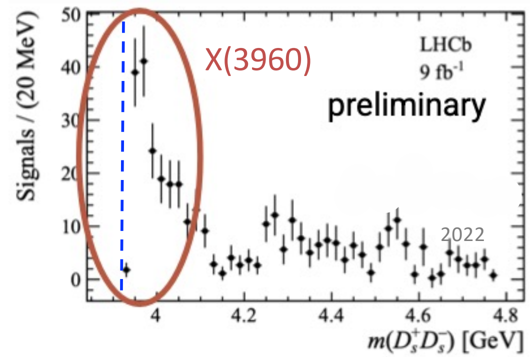
$\bar{D}D$ $\bar{D}_s D_s$





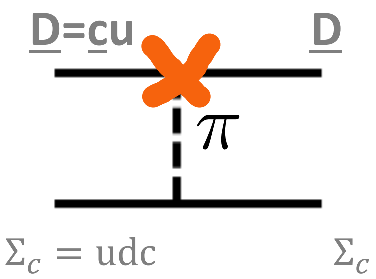


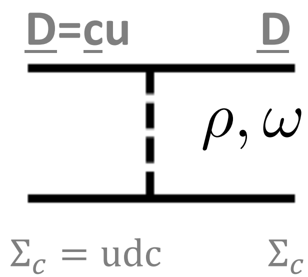
D̲sDs th.



$c\bar{c}uud$

$\bar{D}\Sigma_c$

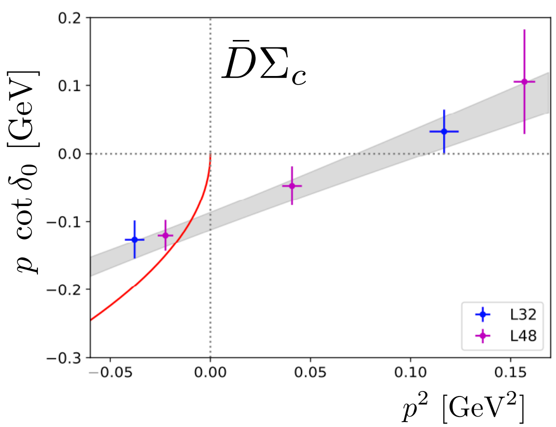




D̲Σ_c , D̲*Σ_c s-wave scattering from lat.

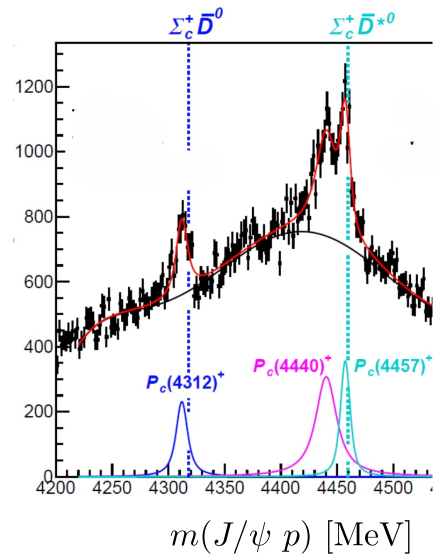
H. Xiang et al., 2210.08555 $m_\pi \approx 294$ MeV

caution: coupling to charmonium+proton neglected



D̲ Σ_c : $m_{Pc}-(m_D+m_{\Sigma c})$ = - 6 (3) MeV
D̲* Σ_c : $m'_{Pc}-(m_{D*}+m_{\Sigma c})$ = - 7 (3) MeV
$J^P=1/2^-$

LHCb 2019



Doubly heavy tetraquarks

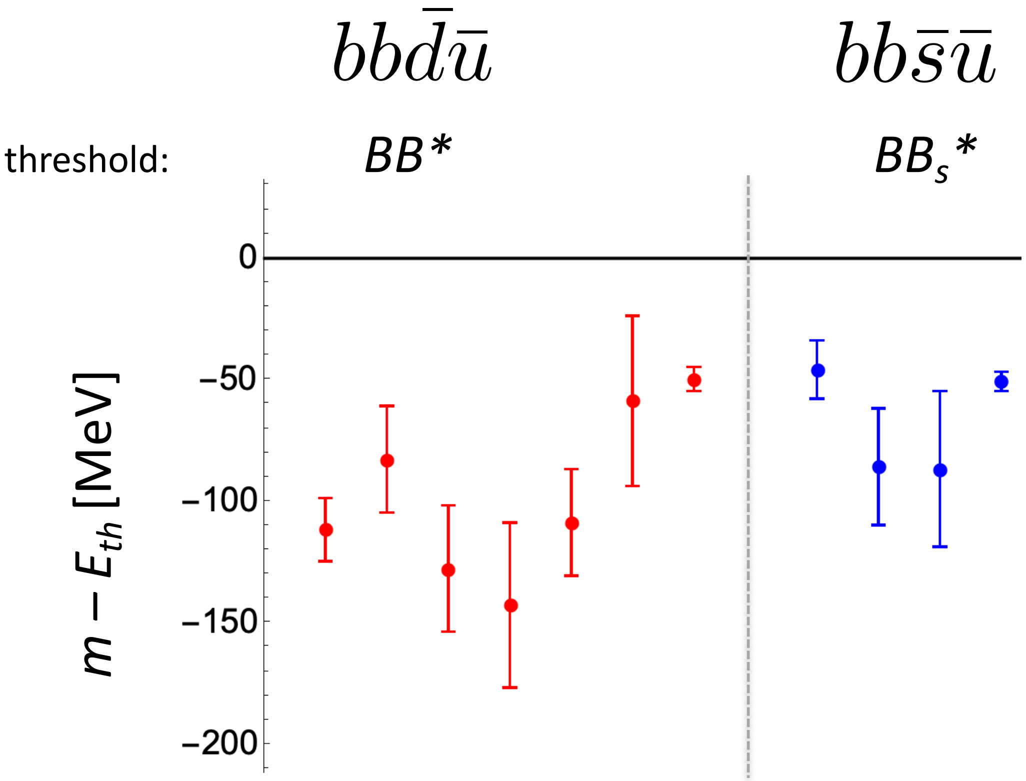
$QQ'\bar{q}\bar{q}'$

Q=c,b   q=u,d,s

Doubly bottom tetraquarks

not found in exp, difficult to find

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

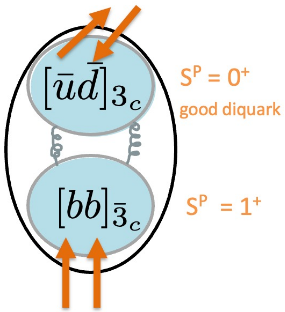


lattice QCD studies
from left to right (lattice QCD)

Hudspith, Mohler, 2303.17295
HALQCD, 2306.03565 (cosidering coupling with B*B*)
Leskovec, Meinel, Pflaumer, Wagner, 1904.04197
Junnarkar, Mathur, Padmanth, 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021 PosLat)
Bicudo, Wagner et al. 1612.02758, static potentials
Brown, Orginost, 1210.1953, static potentials

Hudspith, Mohler, 2303.17295
Meinel, Pflaumer, Wagner, 2205.13982
Junnarkar, Mathur, Padmanth 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021, PosLat)

likely dominant
(B and B* to close
in BB* molecule
with binding ~0.1 GeV)



supported also by almost all model studes
Karliner and Rosner (2017), Janc and Rosina (2004), ...

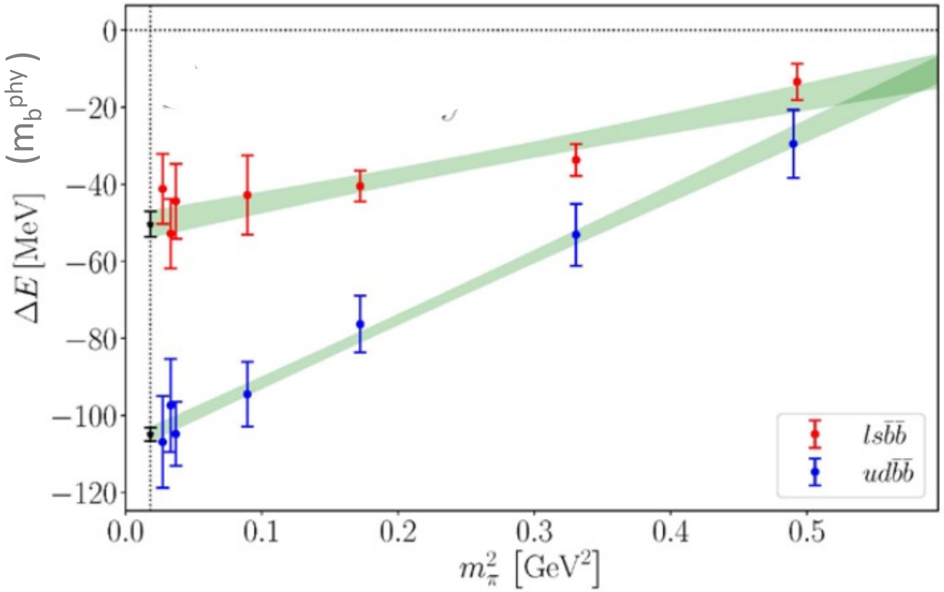
these are the only tetraquarks where lattice
finds support for significant [qq][qq]

good and bad diquark properties:
Francis et al, 2201.03332

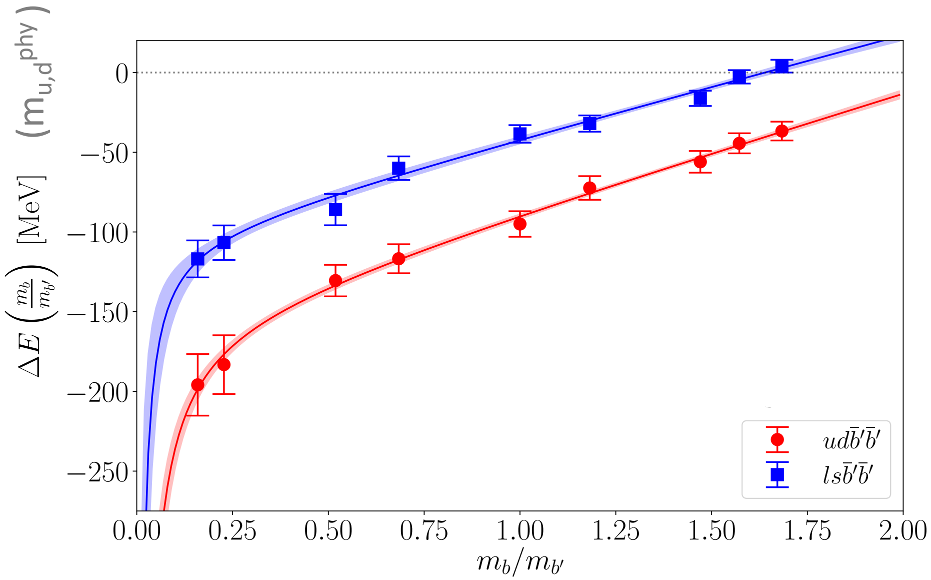
Doubly bottom tetraquarks $bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$

Colquhoun, Francis, Hudspith, Maltman, Lewis
1810.10550 , *PoS* LATTICE2021 (2022) 144



$m_{u,d}$ increases →



$m_{b'}$ decreases →

Other $QQ'\bar{q}\bar{q}'$ and J^P : $bc\bar{q}\bar{q}'$, $cc\bar{q}\bar{q}'$

Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering T(E): more challenging

Doubly charm tetraquark

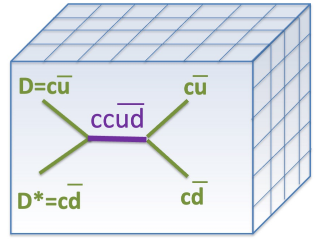
dependence on $m_{u/d}$

$cc\bar{d}\bar{u}$

I=0, J^P=1+

$$T \propto \frac{1}{p\cot\delta - ip}$$

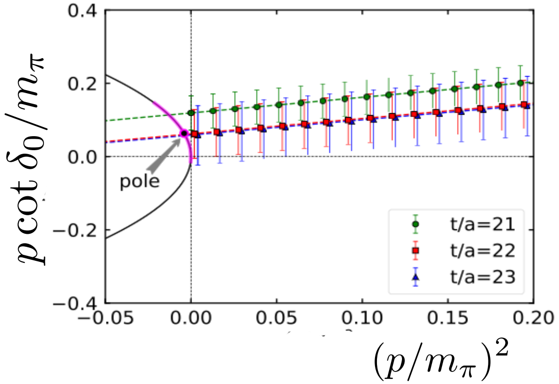
D* is stable at these m_π



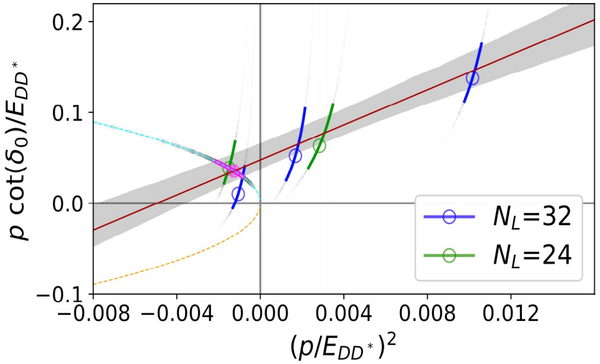
$T(E) \propto \frac{1}{E^2 - m^2}$ for E~m

LHCb

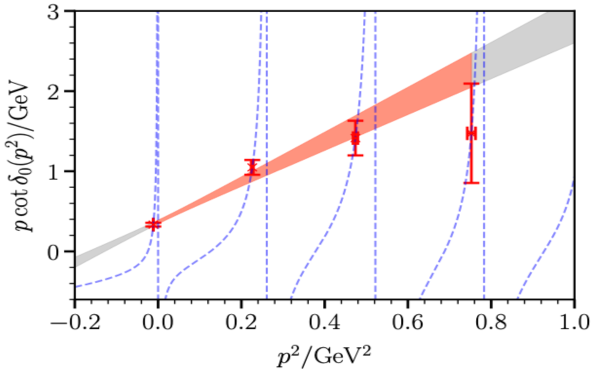
HALQCD, 2302.04505, m_π ≈146 MeV



Padmanath, SP: 2202.10110, PRL, m_π ≈ 280 MeV



CLQCD 2206.06185, PLB, m_π ≈ 348 MeV

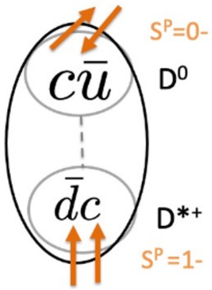


- 0.36(4) MeV   - 0.045(77) MeV $-9.9(^{+3.6}_{-7.2})$ MeV : binding energy δm

bound st.   virtual bound st. pole

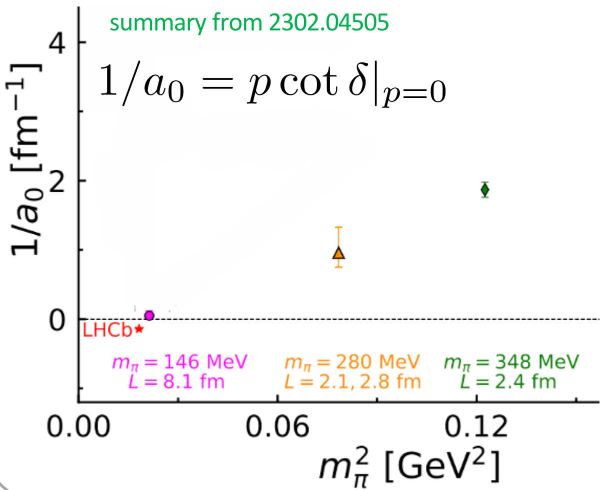
$\pi, \rho, \pi\pi$?

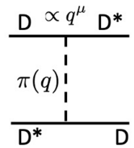
$m_{u/d}$ $V(r) \propto -\frac{e^{-m_{ex} r}}{r}$



- consistent with molecular picture
- no claim it is inconsistent with other interpretations

summary from 2302.04505





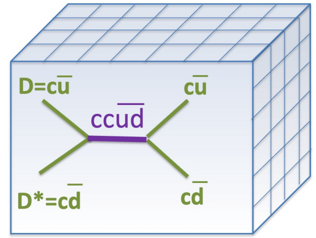
Disclaimer:

- the extaction of pole omits possible effect from the left hand cut
- investigated by Du, F.K. Guo, Nefedievet al. 2303.09441
- under ongoing investigation, keep tuned

Doubly charm tetraquark

$cc\bar{d}\bar{u}$

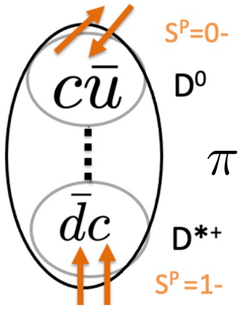
I=0, $J^P=1^+$



D* is stable at these m_π

dependence on $m_{u/d}$

Exchange of which particles drives the attraction within molecular picture?

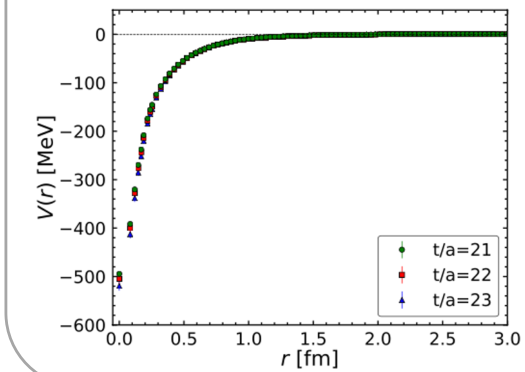


$\pi, \rho, \pi\pi$?

HALQCD, 2302.04505, m_π ≈146 MeV

$\pi, \rho, \pi\pi$?

V(r) $\simeq -\frac{e^{-2m_\pi r}}{r^2}$ r > 1 fm

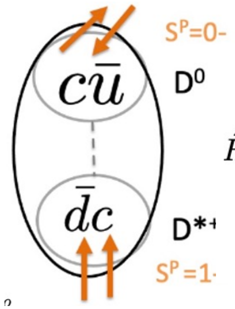


CLQCD 2206.06185, PLB, m_π ≈ 348 MeV

$\pi, \rho, \pi\pi$?

not excluded

dependence on m_c

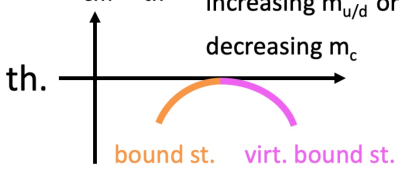


$$\hat{H}_{kin} = \frac{\hat{p}^2}{2\, m_{red}}$$

$$m_r \simeq \frac{m_D m_{D^*}}{m_D + m_{D^*}}$$

$\delta m = E^p_{cm} - E_{th}$

increasing $m_{u/d}$ or decreasing m_c



Padmanath, SP: 2202.10110, PRL, m_π ≈ 280 MeV

- trend verified for two charm quark masses

Heavy-light mesons

$$Q\bar{q}, \quad Q\bar{q}\ q\bar{q}$$

Scalar heavy-light mesons $J^P = 0^+$

Scattering on the lattice

~~$SU(3)_F$~~

S=1 Mohler et al, 1308.3175, PRL
Lang et al, 1403.8103, PRD
RQCD, 1706.01247, PRD
HadSpec 2008.06432, JHEP

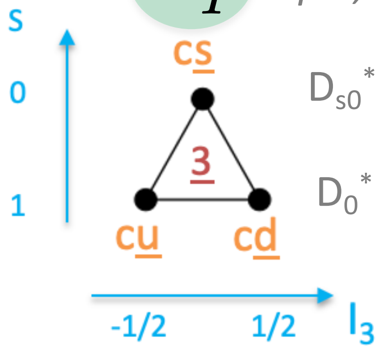
S=0 Mohler et al. 1208.4059, PRD
HadSpec, 1607.07093, JHEP
HadSpec 2102.04973, JHEP

S=-1 HadSpec, 2008.06432, JHEP

$SU(3)_F$: Gregory et al, 2106.15391
attraction in 6, repulsion in 15

Conventional quark model

$c\bar{q}$ q=u,d,s



new paradigm supported by:

- lattice
- ChPT+HQET, UChPT
- reanalysis of exp data
- states circled by blue feature in the spectrum

New paradigm

Lutz et al, 2003 PLB, 2209.10601
Du et al, 1712.07957, PRD
Albaladejo et al, 1610.06727,
Hanhart, Guo et al, 2212.07856,......
Beveren, Rupp; Dmitrasinovic

$c\bar{q}$ + $c\bar{q}\ q\bar{q}$ q=u,d,s n=u,d

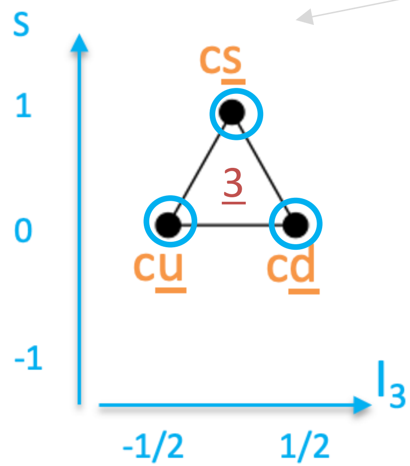
$\underline{3} \otimes 8 = \underline{3} \oplus 6 \oplus 15$ $SU(3)_F$

most attractive — attractive — repulsive

$D_{s0}(2317)$: 70-100% DK molecule — 2.3 GeV

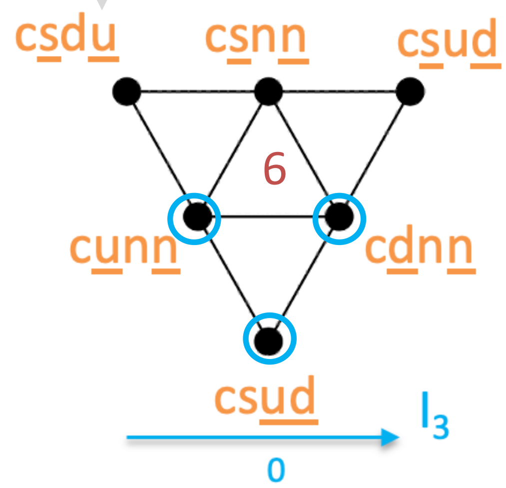
lat: 2.1-2.2 GeV (pole)

PDG: 2.3 GeV (BW)



mix

~~$SU(3)_F$~~



mixes with 15

2.4-2.5 GeV
reanalysis of lat 1607.07093
by Albaladejo 1610.06727;
Hanhart et al. 2212.07856

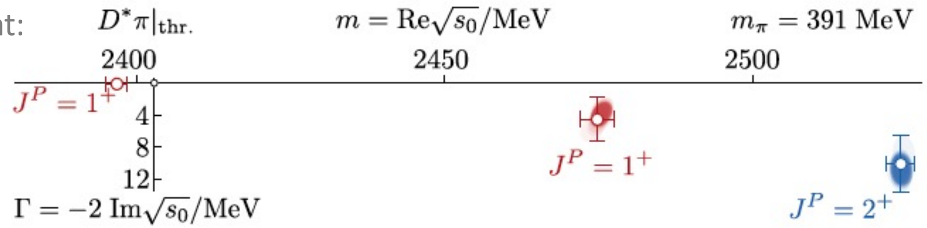
virtual bound state
predicted UChPT 1610.06727
lat HadSpec 2008.06432
partner of X(2900)
[LHCb 2009.00025] ?

Charm-light: $J^P=1^+$ $c\bar{u}$, $c\bar{q}q\bar{u}$

PDG:

$D_1(2430)$	$D_1(2420)$
m ≈ 2412 MeV	Γ ≈ 30 MeV
Γ ≈ 315 MeV	

Lat:

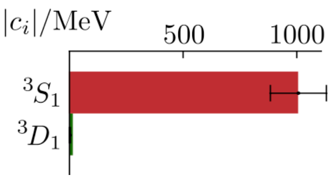


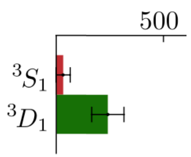
$D_1 \to D^*\pi$

S-wave

$D_1 \to D^*\pi$

D-wave





$$T_{ij}(E_{cm}) \sim \frac{c_i\, c_j}{E_{cm}^2 - m^2}$$

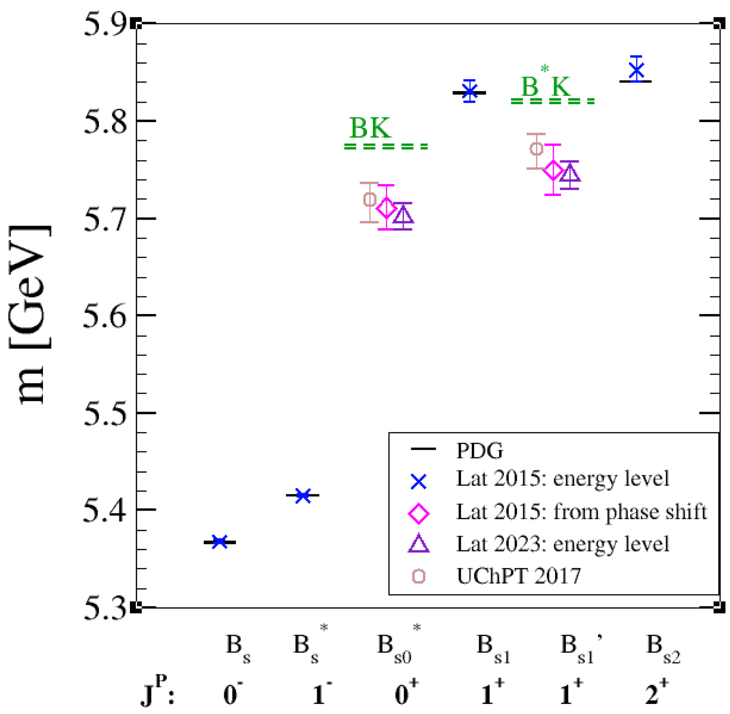
HQET expectations confirmed

HQET: Isgur & Wise, PRL 1991, ...

Lat: N. Lang & D. Wilson, HadSpec, 2205.05026, PRL

Bottom-strange: $J^P=0^+, 1^+$ $b\bar{s}$, $b\bar{q}q\bar{s}$

predictions for missing states



Lat 2015: Lang, Mohler, SP, Woloshyn, 1501.01646

Lat 2023: Hudspith, Mohler, 2303.17295

UChPT 2017: Du et al, 1712.07957

quark model (bs + BK): Yang et al, 2207.07320

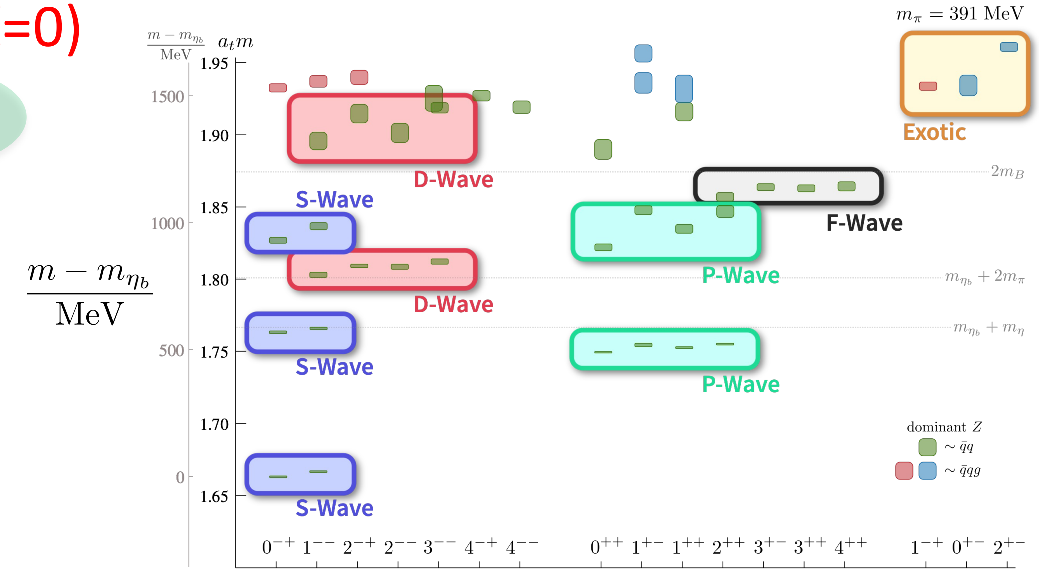
quark model (bs): Bs0 and Bs1 at or above threshod bottomonium(like) sector

$\bar{b}b$ $\bar{b}Gb$ $\bar{b}b\bar{q}q$

Bottomonia (I=0)

$\bar{b}\,b$ $\bar{b}Gb$

non-static b quarks



relativistic b quarks

Ryan & Wilson (HadSpec) 2008.02656, JHEP

omitting effects from strong decays and thresholds

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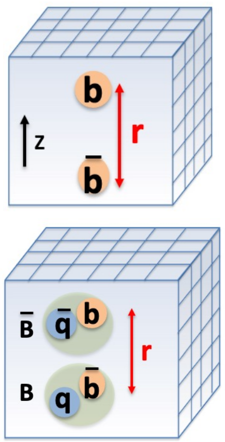
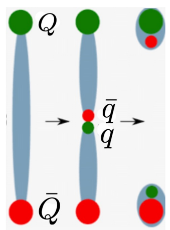
static potentials



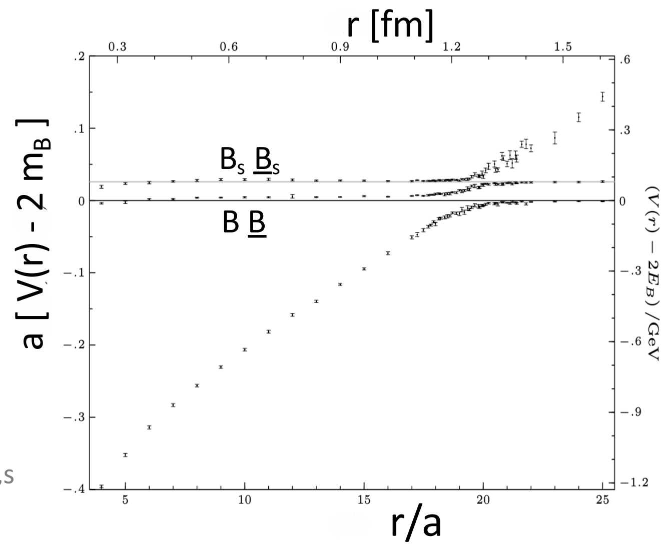


q=u,d,s

string breaking

SESAM, Bali et al, 2005, 0505012 →

Bulava et al, 1902.04006, PLB

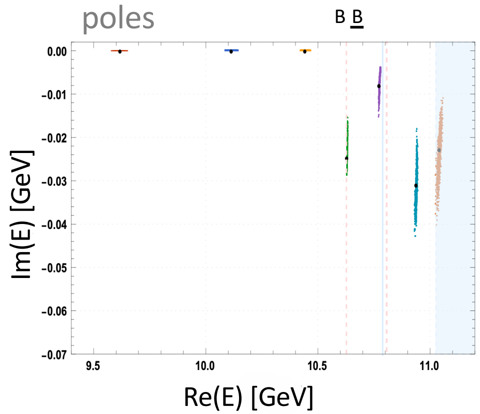


Bicudo, Cardoso, Mueller, Wagner, 2205.11475,

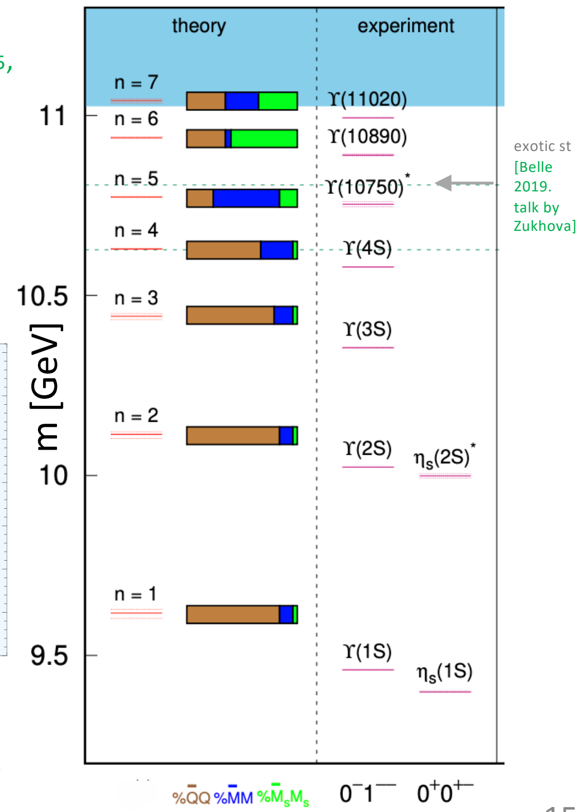
$\tilde{J}$=0: here; $\tilde{J}$=1,2,3 in the paper;

$\tilde{J}$=total ang. mom. without heavy quark spins

$\bar{b}b$ $\bar{b}b\bar{q}q$



see also: Castella, 2207.09365, 2207.09365

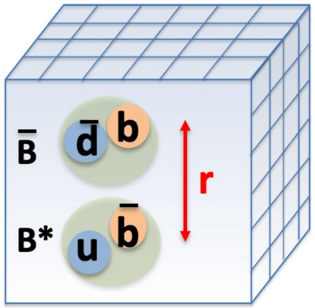


Bottomonium-like states

static potentials

$I=1,\ J^P=1^+$

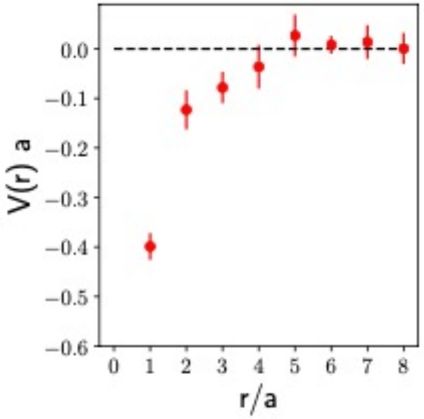
$\bar b \bar b d \bar u$



$\bar b \bar b d \bar u \to B\bar B^*$

$\Upsilon\pi$

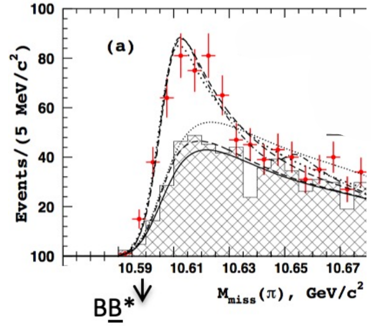
Peters, Wagner, Bicudo
SP, Bahtiyar, Petkovic, Sadl 2019, 2020



bound state just below BB̲* th.

likely related to

Z_b [Belle, 2011, 2016]



Brambilla, 2111.10788;

Soto & Castella, 2005.00552

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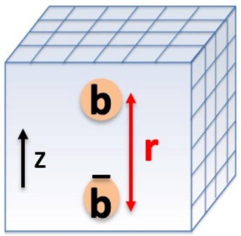
static potentials

I=0, varius J^P

$\bar b\ b$

$\bar b G b$

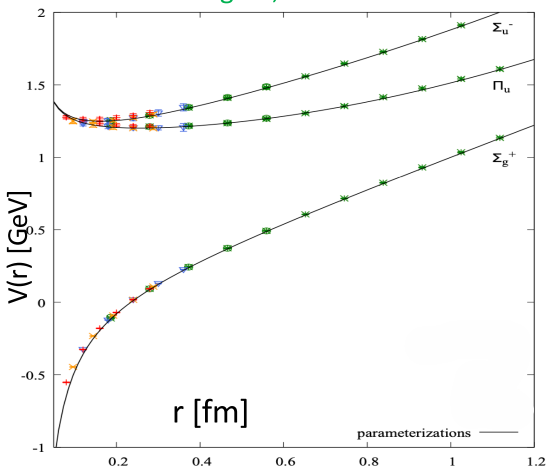


omit strong decays

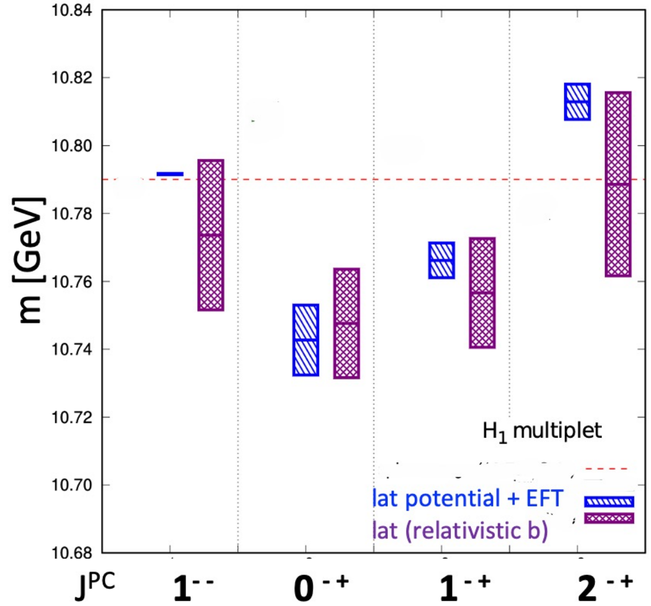
quenched

Juge, Kuti, Morningstar, 1997, 1998

Schlosser & Wagner, 2111.00741



$\bar b G b$ Segovia, Tarrus; Brambilla @ MITP 2022 (with 1/mb spin effects)



Ryan & Willson 2020

see also: Brambilla et al, 1805.07713, 1908.11699, 2212.09187

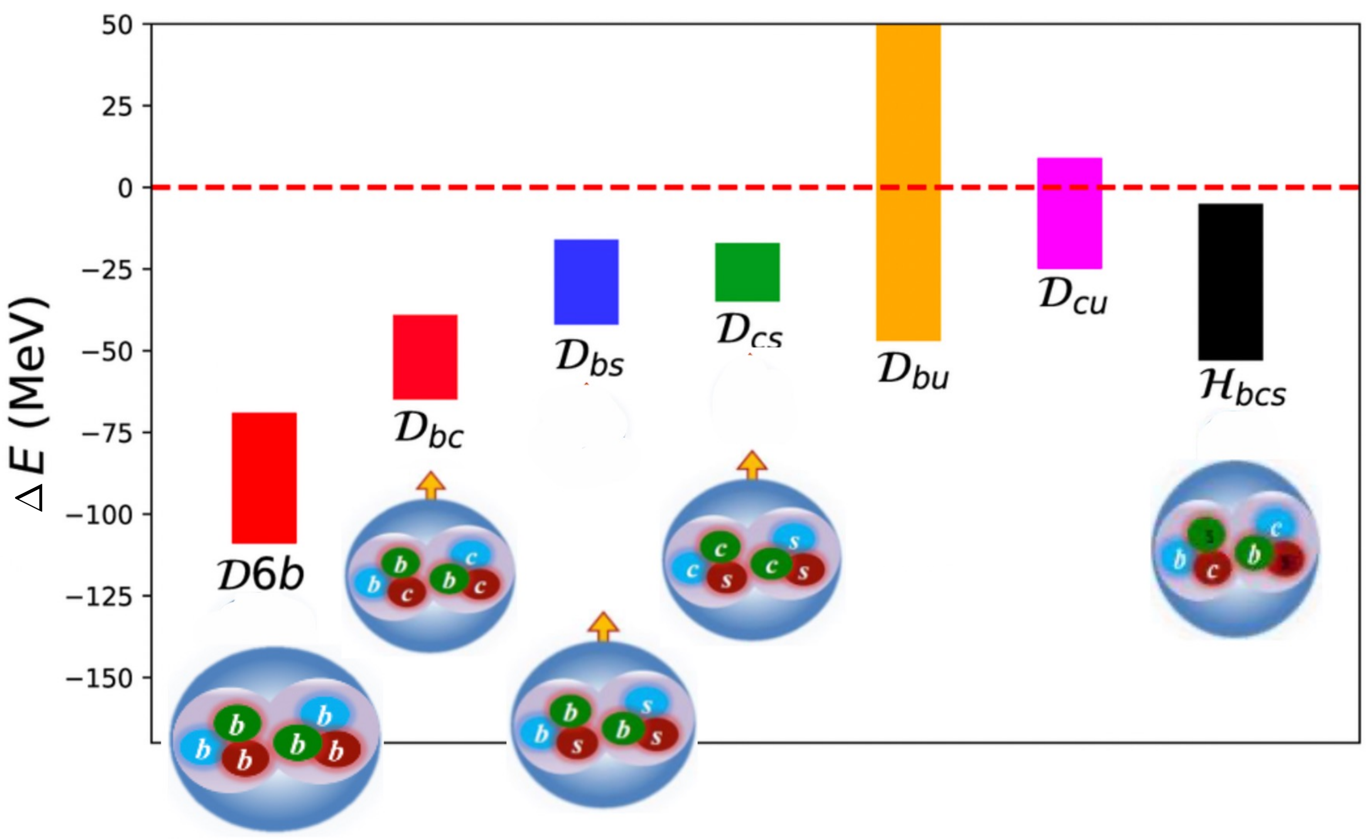
Di-baryons with heavy quarks

binding energy $\Delta E = m - m_{B1} - m_{B2}$

Junnarkar Mathur
1906.06054, PRL

Mathur, Padmanath, Chakraborty
2205.02862

Junnarkar, Mathur, 2206.02942, PRL



Looking ahead

now: Evolution in Eucledian time

$t_E = -i\, t_M$

$$C(t_E) = \sum_n A_n\, e^{-E_n t_E}$$

drawbacks:

- contribution of excited En supressed
- all En below the energy of interest must be extracted

quantum computers: evolution in real Minkowski time

$$C(t_M) = \sum_n B_n\, e^{-iE_n t_M}$$

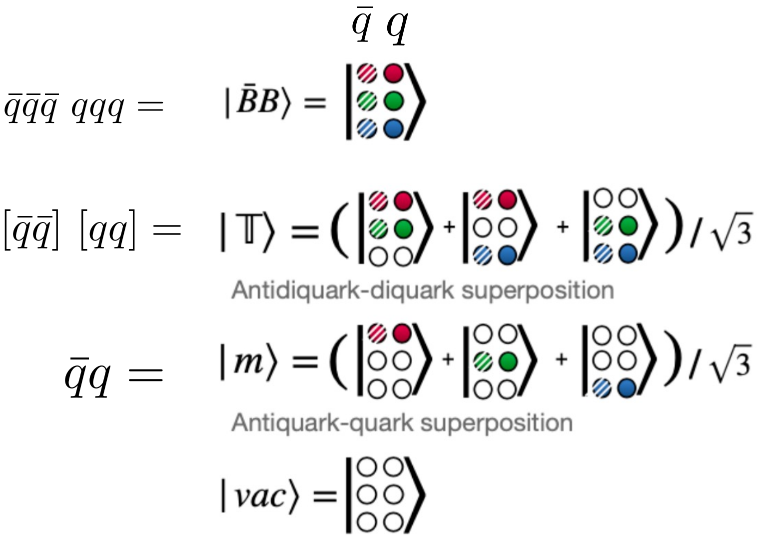
not ready to render conclusions concerning QCD yet, hopefuly it will some day

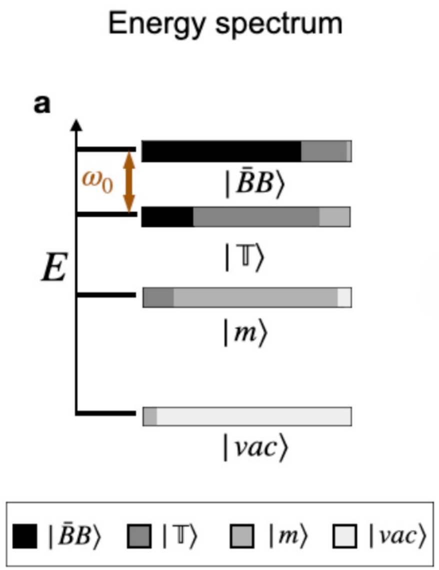
Y. Atas et al, 2207.03473

Simulating one-dimensional quantum chromodynamics on a quantum computer: Real-time evolutions of tetra- and pentaquarks

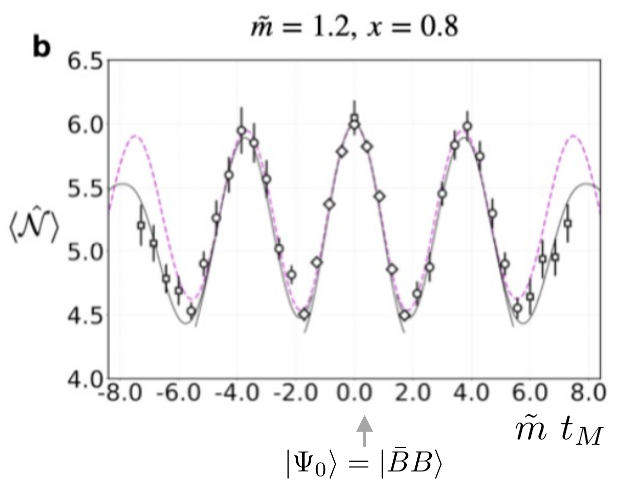
- 1D: one discretized spatial dimension, only one spatial site
- time not discretized
- $SU(3)_c$ Hamiltonian
- computed on quantum IBM computer, up to 4 qubits

Example: mixing of states with one flavor q (with rather heavy mass)





real-time evolution of the particle number

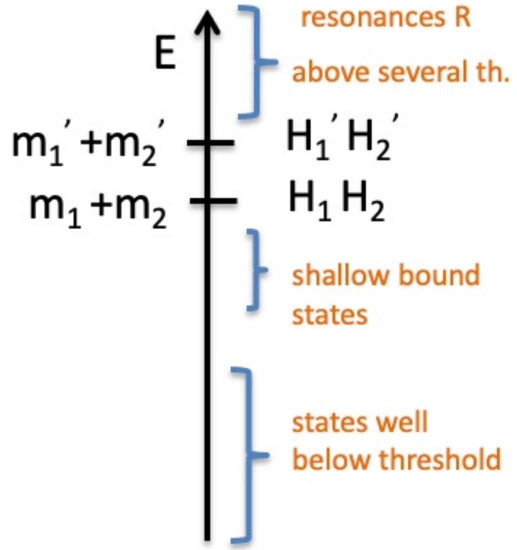


$$\langle\hat{\mathcal{N}}(t)\rangle = \langle\Psi_0|\, e^{it\hat{H}^{(3)}}\, \hat{H}^{(3)}_m\, e^{-it\hat{H}^{(3)}}\, |\Psi_0\rangle$$

$t = t_M$

$$\bar{B}B \leftrightarrow T \quad \text{mixing}$$

Conclusions



Compliments to experimental colleagues for great results

Status on hadrons from Lattice :

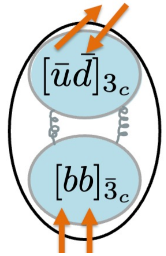
- hadrons that are not resolved (yet)

  strongly decay via many decay channels: $Z_c(4430)$, X(6900),...

- available: valuable results on conventional and exotic hadrons

  strongly stable ; strongly decaying to 1,2,3 channels

  support for specific binding mechanisms



certain charmonium-like states

  one picture can not explain all exotic hadrons

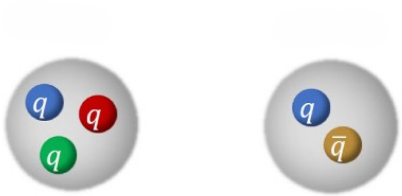
  for each exotic hadron there is at least one viable picture

A direction that might lead to a valuable insight into dynamics of hadrons:

identify and inspect states that can be rigorously studied by theory and well explored by experiments

quark-mass dependence of these states in theory ... could lead to further clues about their nature backup

Conventional hadrons



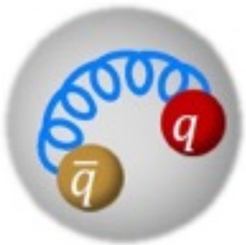
Exotic hadrons

minimal quark content

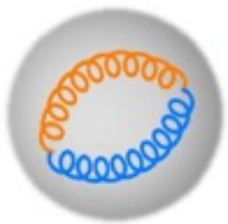
tetraquark

pentaquark

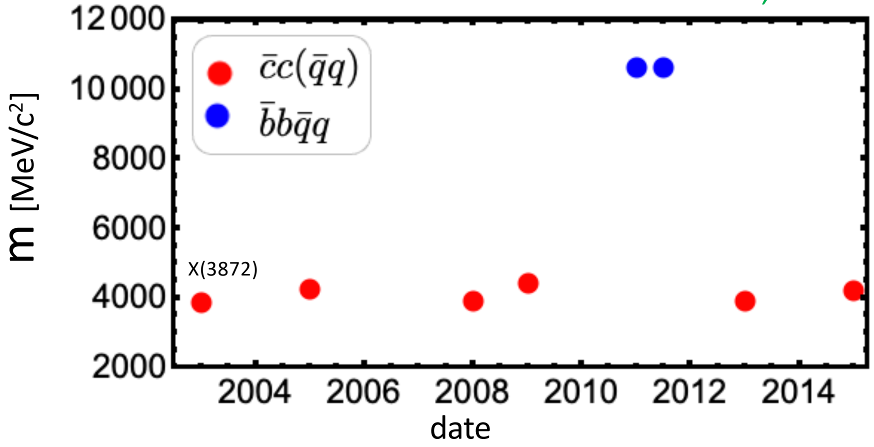
hybrid meson



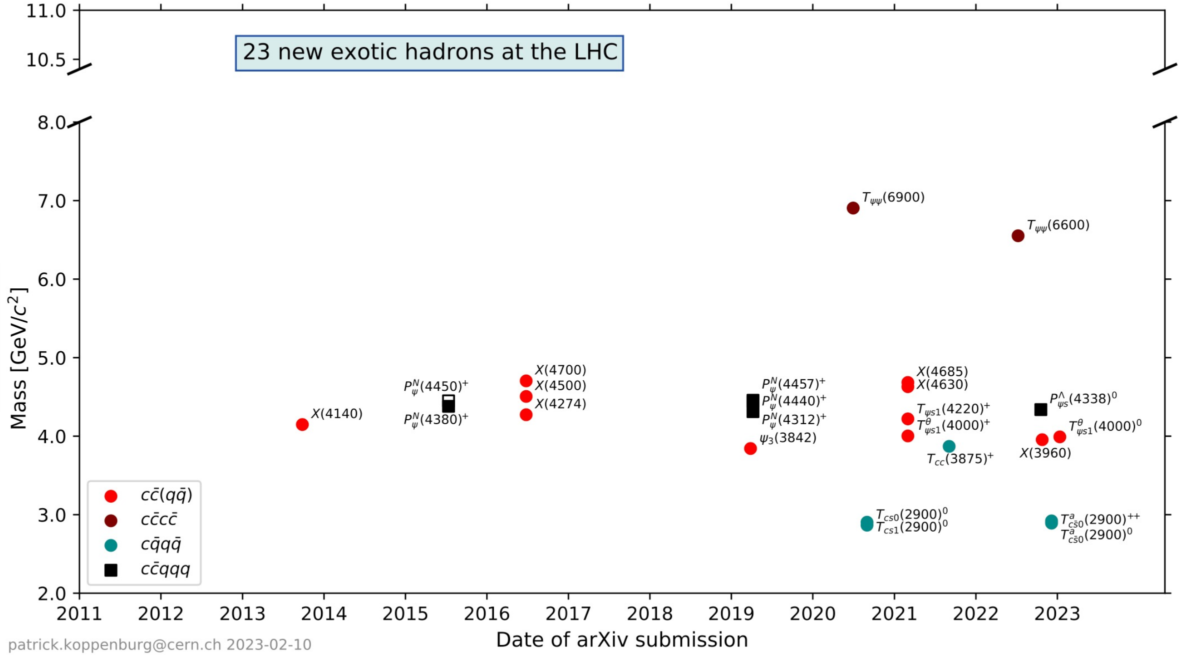
glueball



Belle, BaBar

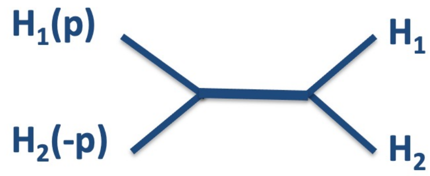


LHCb, CMS, ATLAS



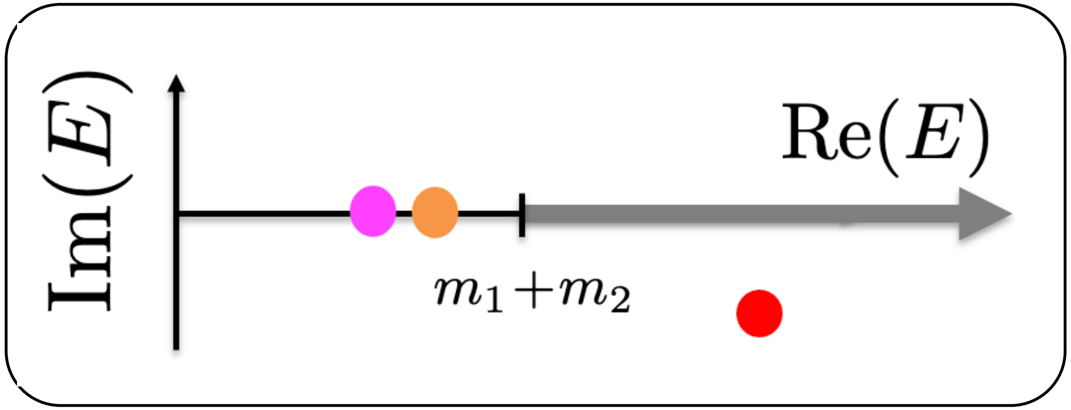
slike FK Guo

Definitions: bound state, virtual bound state & resonance



$$T(E) \propto \frac{1}{E^2 - m^2} \qquad T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$

Poles of *T(E)*, *E=E* *cm*



Virtual bound st.   Bound st.   Resonance

p = - i |p| p = i |p|

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + (-p)^2} \; < \; m_1 + m_2$$