Search for Gamma-Ray Counterparts of IceCube Neutrino Events in the AGILE Public Archive

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NEUTEL 2023, Venice, Italy, 23-27 October 2023

High Energy Astrophysical Neutrinos

High energy neutrinos production processes (E > TeV):

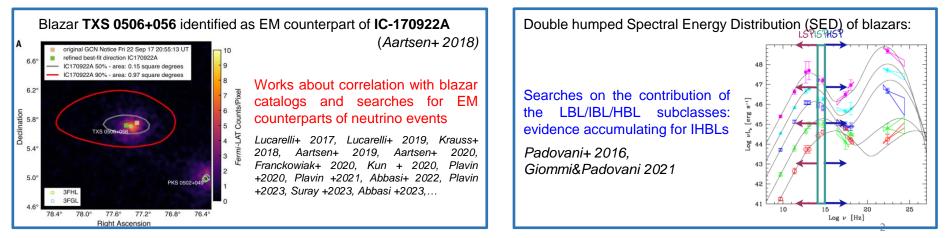
- pp interactions $p+N \rightarrow X + \max \times (\pi^+ + \pi^- + \pi^0)$
- py interaction (photo-pion reaction, threshold process)

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p + \gamma \rightarrow \Delta^+ \rightarrow n + \pi^+
\rightarrow p + \pi^0
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Neutrino sources also gamma ray sources - Lepto-Hadronic Scenario vs Purely Leptonic Scenario (SSC or EIC)

Cosmic ray accelerators as producers of high energy neutrinos \rightarrow Blazars (BL Lacertae + FSRQs) as best candidates

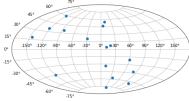
Eichler 1979, Stecker+ 1992, Mannheim 1995, Halzen&Zas 1997, Padovani&Resconi 2014, Ahlers&Halzen 2015, Tavecchio&Ghisellini 2015, Murase+ 2018, Giommi+ 2020, Giommi&Padovani 2021, Buson+ 2022, Bellenghi+ 2023, Buson+ 2023,...



Presentation Outline

- Search for candidate gamma-ray counterparts of a sample of IceCube neutrino events using the public data archive of the AGILE Mission
- Building of the light curves centered on the neutrino position using the AGILE-LV3 tool at the Space Science Data Center (SSDC) and multifrequency archival data
- Analysis of light curves over different time intervals around event T₀ (1 month, 1 year and full mission)
- Identification and classification of candidate blazar counterparts using the VOU-Blazars tool and ASI SED builder
- Estimation of contribution of different blazar subclasses

The IceCube neutrino sample (16 events) (September 2018 – March 2020)



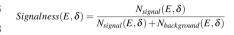
Sample analized in my Master thesis (2022) *Advisors:* R. Poggiani, C. Pittori

IceCube Name	MJD	Event Type	RA	DEC	Error Radius*	Energy**	Signalness	1	b
			(deg)	(deg)	(arcmin)	$(\times 10^2 \text{ TeV})$		(deg)	(deg)
IC-180908A	58369	EHE	$144.58^{+1.55}_{-1.45}$	$-2.13\substack{+0.9\\-1.2}$	20.36	1.1998	0.34364	237.2	35.1
IC-181023A	58414	EHE	$270.18\substack{+2.00 \\ -1.70}$	$-8.57\substack{+1.25\\-1.30}$	17.50	1.1998	0.28016	19.4	7.2
IC-190503A	58606	EHE	$120.28\substack{+0.57\\-0.77}$	$6.35\substack{+0.76 \\ -0.70}$	14.99	1.0000	0.36266	215.2	18.4
IC-181014A	58405	HESE	$225.15\substack{+1.40 \\ -2.85}$	$-34.80\substack{+1.15\\-1.85}$	73.79	6037.47[pe]	0.10	331.0	20.9
IC-190104A	58487	HESE	$357.98\substack{+2.3\\-2.1}$	$-26.65\substack{+2.2\\-2.5}$	73.79	8584.32[pe]	0.35	31.6	-76.7
IC-190124A	58504	HESE	$307.40\substack{+0.8\\-0.9}$	$-32.18\substack{+0.7 \\ -0.7}$	73.79	13555.75[pe]	0.91	10.8	-33.8
IC-190221A	58535	HESE	$268.81^{+1.2}_{-1.8}$	$-17.04\substack{+1.3\\-0.5}$	73.79	12264.54[pe]	0.37	11.4	4.2
IC-190331A	58573	HESE	$337.68\substack{+0.23 \\ -0.34}$	$-20.70\substack{+0.30\\-0.48}$	534.00	198736.44[pe]	0.57	36.6	-57.3
IC-190504A	58607	HESE	65.79	-37.44	73.79	7328.35[pe]	0.63	239.9	-44.7
IC-190619A	58653	GOLD	$343.26\substack{+4.08\\-2.63}$	$10.73\substack{+1.51 \\ -2.61}$	162.59	1.9870	0.54551	81.8	-42.5
IC-190730A	58694	GOLD	$225.79\substack{+1.28 \\ -1.43}$	$10.47\substack{+1.14\\-0.89}$	71.10	2.9881	0.67158	11.1	54.8
IC-190922A	58748	GOLD	$167.43\substack{+3.40\\-2.63}$	$-22.39\substack{+2.88\\-2.89}$	177.00	31.139	0.20165	274.1	34.7
IC-190922B	58748	GOLD	$5.76^{+1.19}_{-1.37}$	$-1.57\substack{+0.93\\-0.82}$	64.80	1.8737	0.50501	106.8	-63.6
IC-191001A	58757	GOLD	$314.08\substack{+6.56 \\ -2.26}$	$12.94\substack{+1.50\\-1.47}$	177.00	2.1742	0.58898	60.2	20.3
IC-191119A	58806	GOLD	$230.10\substack{+4.76 \\ -6.48}$	$3.17\substack{+3.36 \\ -2.09}$	220.19	1.7648	0.44999	5.5	47.1
IC-200109A	58857	GOLD	$164.49^{+4.94}_{-4.19}$	$11.87\substack{+1.16 \\ -1.36}$	174.60	3.7523	0.76931	237.2	59.3



IceCube Alert System (> 2016) EHE $E = (500 \text{ TeV} \div 10 \text{ PeV})$ HESE $E = (100 \text{ TeV} \div 1 \text{ PeV})$

GOLD (>2019) signalness > 50%



*: 50% CL for EHE neutrinos, 90% CL for HESE, GOLD neutrinos

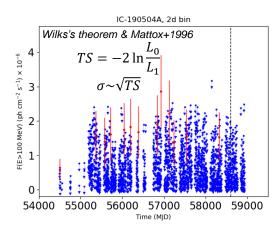
**: energy in photoelectrons for HESE neutrinos

Search for Candidate Gamma-ray Counterparts in the AGILE-GRID public archive (SSDC AGILE-LV3 Analysis Tool)

The AGILE Mission Tavani+ 2008



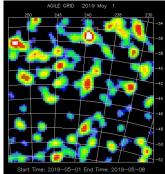
Light curve of 2AGL J0429-3755 (PKS 0426-380)



Full-mission (16 yrs) AGILE light curves using:

- Public data (AGILE-LV3 data archive, Energy > 100 MeV)
- AGILE-LV3 ASI SSDC tool: official on-line Maximum Likelihood analysis





Analysis Output

Light curves with bin duration: 2, 4, 7, 28 days

Significance (\sqrt{TS}) of a **Candidate Detections** (Maximum Likelihood results with $\sqrt{TS} > 3$) Modified Julian Date **(MJD)**, **Flux** (ph/cm^2/s), Spectral Energy Distribution **(SED)** data

Example: the HESE neutrino event IC-190504A

MJD of neutrino event: T_0 (MJD)= 58607

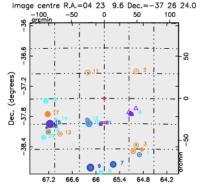
Multiple candidate detections in all light curves

Closest candidate detection ($\Delta T \sim -9$ months) in light curve with bin duration of 2 days:

- $\sqrt{TS} = 3.1$
- MJD of the Candidate Detection = 58324.5 ± 1.0
- Flux = (1.53 ± 0.72)e-6 ph/cm^2/s

Identification and Analysis of the Candidate EM Counterparts

VOU-BLAZARS Tool (Chang+ 2019)



۲	HBL	candidate
۲	IBL (candidate

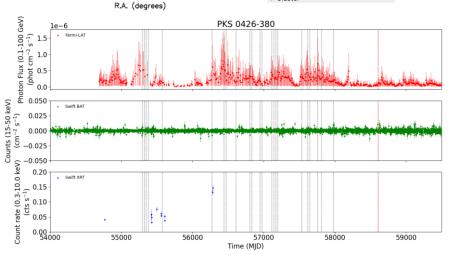
- EBL candidate
- Non-jetted AGN
- Onknown type radio/X−ray source
 Onknown type radio/X−
- ☆ 3HSP source
- 5BZCat source
 CRATES source
- Radio source
- X-ray source
- △ Gamma-ray source
- ▲ Unassociated Gamma—ray source





II GRB

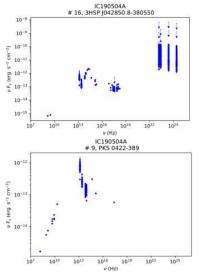


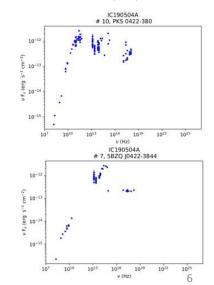


New candidate sources for all neutrino events e.g.: IC-190504A (HESE)

N _{Candidate}	Source name	Туре	RA	DEC	Distance(arcmin)	γ-ray Counterpart
15	PKS 0426-380	LBL/IBL	67.17	-37.94	71.95	4FGL J0428.6-375
16	3HSP J042850.8-380550	IBL	67.21	-38.10	78.12	0
6	WISEA J042025.10-374445.0	IBL	65.11	-37.65	37.4	4LAC J0420.3-374
10	PKS 0422-380	LBL/IBL	66.17	-37.94	35.1	0
9	PKS 0422-389	LBL	66.14	-38.81	83.95	0
7	5BZQ J0422-3844	LBL/IBL	65.56	-38.75	79.2	0

Previously reported candidates (*Giommi*+ 2020, *GCN* 24401): PKS 0426-380 (LBL) and 4LAC J0420.3-374 (IBL)

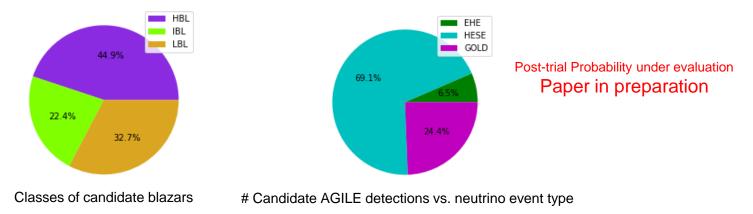




Conclusions

Analysis of 16 IceCube neutrino events from September 2018 to March 2020

- Full-mission (16 yrs) AGILE light curves using public data and AGILE-LV3 SSDC tool
- SED of identified candidates with VOU-Blazars tool: new candidate EM counterparts for all neutrino events
- 2/16 light curves with association to 2AGL catalog blazars (IC-190504A and IC-190730A)
- 8/16 light curves show significant detections ($\sqrt{TS} > 3$) within T₀ ± 1 year:
 - 2/3 EHE neutrinos (IC-180908A and IC-190503A)
 - 3/6 HESE neutrinos (IC-190104A, IC-190221A, IC-190504A)
 - 3/7 GOLD neutrinos (IC-190619A, IC-190922A, IC-191001A)



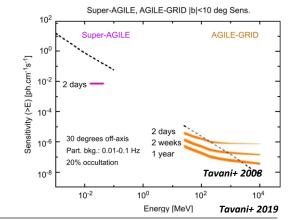
EXTRA SLIDES



The AGILE Mission

- Launch: 23/04/2007, more than 16 years activity
- Low Equatorial Orbit with 2.5° inclination
- Average altitudine: ~ 500 km
- Since 2009: "Spinning" observing mode
- Revolution period ~ 7 min: 80% sky coverage every day

Tavani+ 2008



AGILE-GRID:

- Silicon Tracker (ST): gamma-ray conversion into e+/e- pairs, charged particles track reconstruction
- Minicalorimeter (MCAL): measurement of charged particle energy (for estimation of incident photons energy); 30 CsI(TI) bars with PIN photodiodes

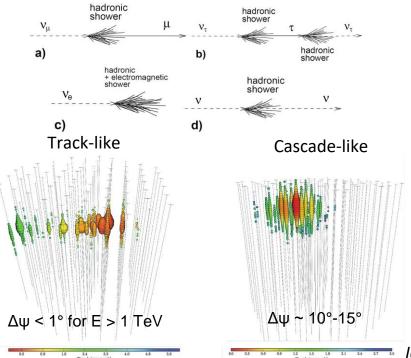
FOV ~ 2.5 sr

Angular resolution ~1.2° for E > 100 MeV (up to 30° off-axis) Sensitivity $(10-20)x10^{-8}$ ph/cm²/s (depending on sky region and exposure time)

Gamma-ray imaging detector (GRID)	
Energy range	30 MeV-50 GeV
Field of view	$\sim 2.5 \text{ sr}$
Flux sensitivity ($E > 100$ MeV, 5σ in 10^6 s)	$3 \times 10^{-7} (\text{ph cm}^{-2} \text{ s}^{-1})$
Angular resolution at 100 MeV (68% cont. radius)	3.5°
Angular resolution at 400 MeV (68% cont. radius)	1.2°
Source location accuracy (high Gal. lat., 90% C.L.))	$\sim 15 \operatorname{arcmin}$
Energy resolution (at 400 MeV)	$\Delta E/E \sim 1$
Absolute time resolution	$\sim 2 \mu s$
Dead time	~ 100–200 µs
Hard X-ray imaging detector (Super-AGILE)	
Energy range	18–60 keV
Single (1-dim.) detector FOV (FW at zero sens.)	$107^{\circ} \times 68^{\circ}$
Combined (2-dim.) detector FOV (FW at zero sens.)	$68^{\circ} \times 68^{\circ}$
Sensitivity (18–60 keV, 5σ in 1 day)	~15 mCrab
Angular resolution (pixel size)	6 arcmin
Source location accuracy (S/N~10)	~1-2 arcmin
Energy resolution (FWHM)	$\Delta E \sim 8 \text{ keV}$
Absolute time resolution	$\sim 2 \mu s$
Mini-calorimeter	
Energy range	0.35-50 MeV
Energy resolution (at 1.3 MeV)	13% FWHM
Absolute time resolution	$\sim 3 \mu s$
Dead time (for each of the 30 CsI bars)	$\sim 20 \mu s$

IceCube

- Neutrino detection via Charged Current (CC) and Neutral Current (NC) (σ ~ 10⁻³⁸ cm² at 1 GeV):
 - Charged Current (CC) $v_l + N \rightarrow l + X$
 - Neutral Current (NC) $v_l + N \rightarrow v_l + X$



IceCube Alert System (> 2016)



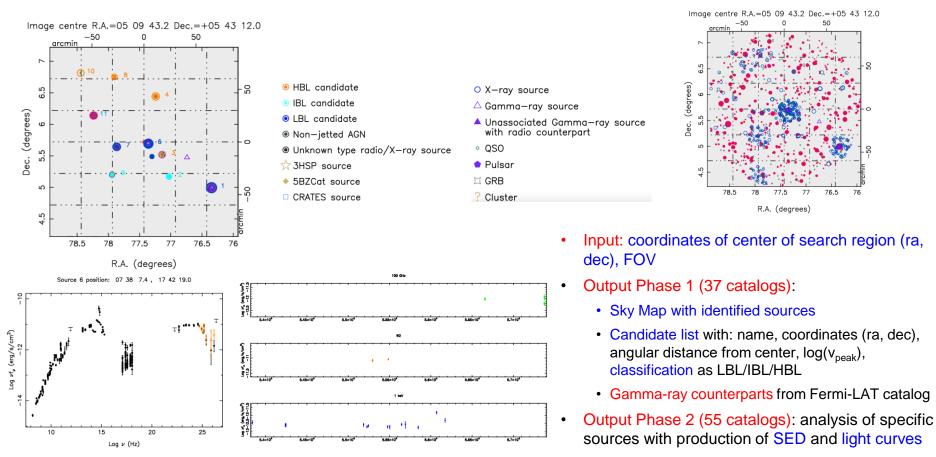
- **EHE** (Extremely High Energy event): E = 500 TeV 10 PeV
- **HESE** (High Energy Starting Event): track-like events with interaction vertex inside detector, E = 100 TeV 1PeV
- GOLD (>2019): signalness > 50%
- BRONZE (>2019): signalness > 30%

$$Signalness(E, \delta) = \frac{N_{signal}(E, \delta)}{N_{signal}(E, \delta) + N_{background}(E, \delta)}$$

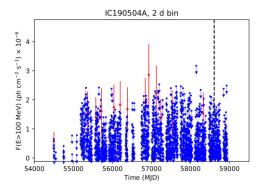
δ =declination

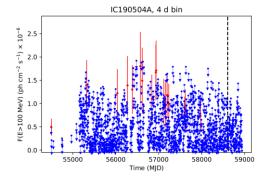
²⁴ ²⁷ ³⁰ IceCube Collaboration

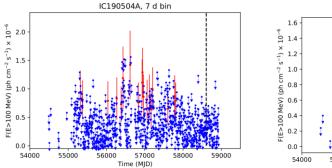
VOU-Blazars Tool (Chang+ 2019)

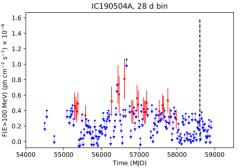


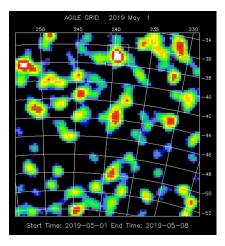
IC-190504A (HESE)

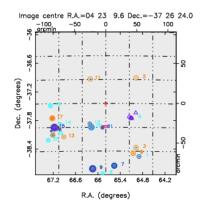








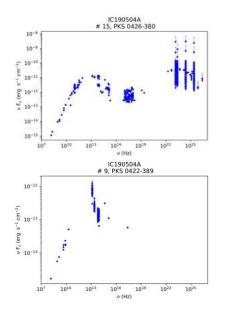


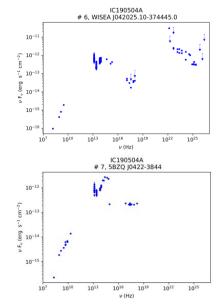


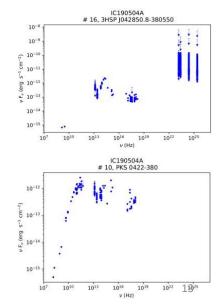
IC-190504A (HESE)

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16	3HSP J042850.8-380550	IBL	67.21	-38.10	78.12	0
6	WISEA J042025.10-374445.0	IBL	65.11	-37.65	37.4	4LAC J0420.3-374
10	PKS 0422-380	LBL/IBL	66.17	-37.94	35.1	0
9	PKS 0422-389	LBL	66.14	-38.81	83.95	0
7	5BZQ J0422-3844	LBL/IBL	65.56	-38.75	79.2	0

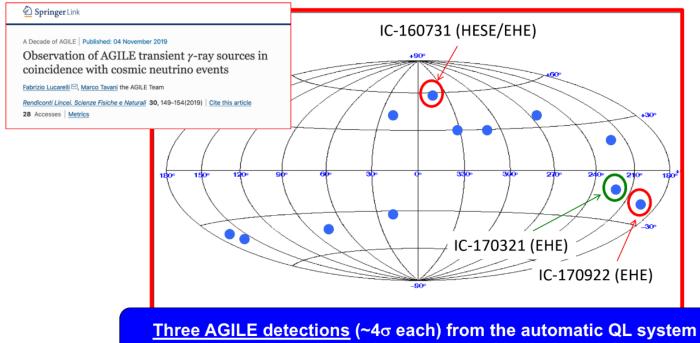
Previously reported candidates (Giommi+ 2020, GCN 24401): PKS 0426-380 (LBL) and 4LAC J0420.3-374 (IBL)





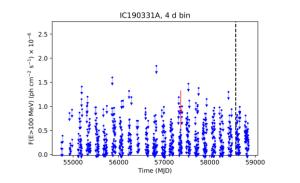


AGILE detections of IceCube Neutrino Events (F. Lucarelli et al, ApJ 870, 2019)



consistent with time/position of 3 IC events out of 10!

C. Pittori talk @20MAGIC

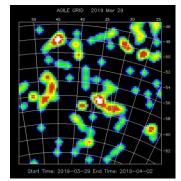


Lightcurve centered on arrival direction of neutrino: $T_0 = 58573 \text{ MJD}$ Bin duration: 4 days $\Delta T = -3.3 \text{ years}$ MJD Detection = 57365.5 ± 2.0 (09-12-2015) Flux = (0.91 ± 0.42)e-6 ph/cm^2/s (sqrt(TS)=3.0)

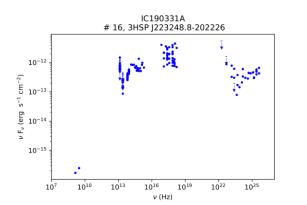
IC-190331A (HESE)

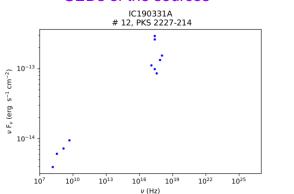
ATel #12623, Lucarelli+ (2019): new identified source AGL J2233-2212 spatially and temporally coincident with the neutrino event

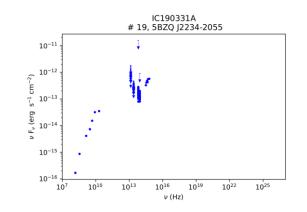
Some candidates not previously reported



NCandidate	Source name	Туре	RA	DEC	Distance(arcmin)	γ -ray Counterpart
	AGL J2233-2212					
16	3HSP J223248.8-202226	HBL	338.20	-20.37	35.3	4FGL J2232.6-2023
12	PKS 2227-214	LBL	337.65	-21.21	30.8	-
19	5BZQ J2234-2055	LBL	338.73	-20.92	60.83	-



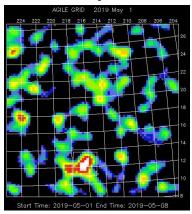


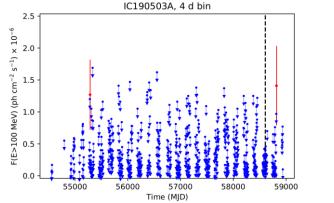


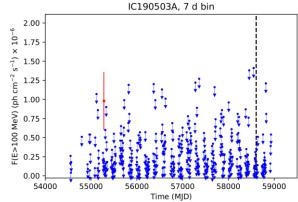
SEDs of the sources

IC-190503A (EHE)

Map at time T_0 and time bin 7 days







Candidate Associated sources

N _{Candidate}	Source name	Туре	RA	DEC	Distance(arcmin)	γ-ray Counterpart
1	3HSP J080056.5+073235	HBL	124.24	7.54	71.1	4FGL J0800.9+0733
6	MRK 1210	Sy2	121.02	5.11	86.37	-
7	WISEA J080406.36+064843.3	HBL	121.03	6.81	51.23	-
4	SDSS J080352.97+061704.3	IBL	120.97	6.28	40.27	-

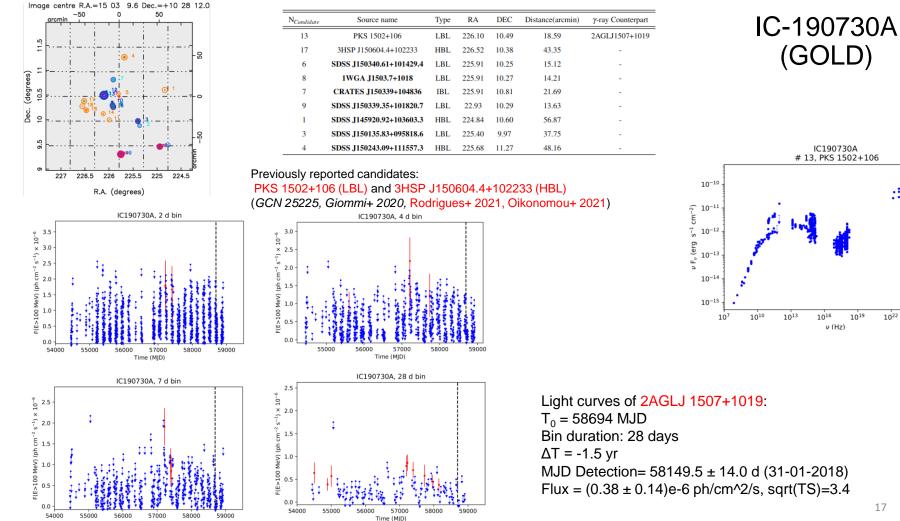
One previously reported candidate (*GCN 24378*): 3HSP J080056.5+073235 (HBL, 4FGL J0800.9+0733)

Lightcurve centered on arrival direction of neutrino:

 $T_0 = 58606 \text{ MJD}$

Detections for 4, 7 days binning

Closest Detection: sqrt(TS) = 3.4 $MJD = 58817.5 \pm 2.0 (30-11-2019)$ $\Delta T = +7$ months $Flux = (1.41\pm0.62)e-6$ ph/cm^2/s



Time (MJD)



IC190730A

13, PKS 1502+106

1013

1016

v (Hz)

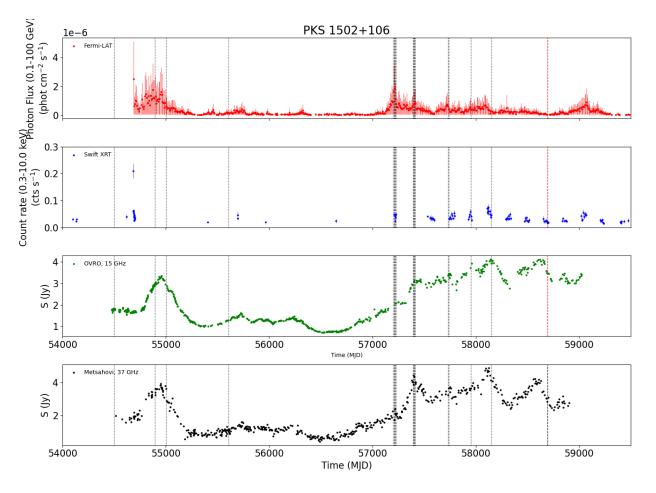
1019

1022

1025

MJD Detection= 58149.5 ± 14.0 d (31-01-2018)

IC-190730A (GOLD)



Results: EHE neutrinos

IceCube Name	T ₀	Event Type	$F_{\gamma}(E > 100 \text{ MeV})$	Δt	\sqrt{TS}	Bin	Associated Sources
	(MJD)		$\times 10^{-6} ({\rm ph}~{\rm cm}^{-2}~{\rm s}^{-1})$	(yr)		(days)	
IC-180908A	58369	EHE	2.18 ± 0.95	+0.66	3.5	2	3HSP J093938.5-031503 (HBL)
							MO J093826.55-004244.5 (LBL)
IC-181023A	58414	EHE	-	-	-	-	4FGL J1804.5-0852 (BCU)
							4FGL J1805.8-0803 (UNK)
IC-190503A	58606	EHE	1.41 ± 0.62	+0.66	3.4	4	3HSP J080056.5+073235 (HBL)
			1.27 ± 0.55	-9.2	3.4	4	Mrk 1210 (Sy2)
			0.98 ± 0.38	-9.2	3.5	7	WISEA J080406.36+064843.3 (HBL)
							SDSS J080352.97+061704.3 (IBL)

2 events out of 3 show a candidate detection within \pm 1 year around T₀ of neutrino event

New candidate counterparts (not previously reported in literature) in bold

2AGL associated source

Multiple detections in all light curves, only those closest to T_0 reported here

Results: HESE neutrinos

IceCube Name	T ₀	Event Type	$F_{\gamma}(E > 100 \text{ MeV})$	Δt	\sqrt{TS}	Bin	Associated Sources
	(MJD)		$\times 10^{-6} ({\rm ph}~{\rm cm}^{-2}~{\rm s}^{-1})$	(yr)		(days)	
IC-181014A	58405	HESE	1.07 ± 0.48	-7.3	3.1	4	2AGL J1459-3542/PKS 1454-354 (LBL)
			0.32 ± 0.11	-9.7	3.4	28	PMN J1505-3432 (LBL)
IC-190104A	58487	HESE	1.60 ± 0.71	0.84	3.0	4	IC 5362 (BCG)
			1.43 ± 0.61	-2.7	3.3	4	VOU J235815-285341 (HBL)
			0.72 ± 0.31	-3.0	3.3	7	3HSP J235034.3-300604 (HBL)
			0.33 ± 0.13	-3.0	3.2	28	3HSP J235023.3-243602 (HBL)
							1WGA J2350.5-2620 (Sy2)
							5BZQ J2353-2743 (IBL)
							MCXC J2351.6-2605 (CoG)
IC-190124A	58504	HESE	1.26 ± 0.56	-2.6	3.2	7	5BZQ J2024-3253/PKS 2021-330 (LBL)
			0.94 ± 0.40	-10.9	3.3	2	WISEA J202151.77-324420.6 (LBL)
			0.85 ± 0.39	-5.3	3.0	7	Abell 3681 (CoG)
			0.61 ± 0.19	-5.3	4.2	28	PMN J2028-3154 (LBL)
IC-190221A	58535	HESE	1.12 ± 0.45	+0.07	3.1	7	4FGL J1758.6-1622 (Radio source)
							4FGL J1750.4-1723 (Gamma source)
							4FGL J1751.6-1750 (BCU)
IC-190331A	58573	HESE	0.91 ± 0.42	-3.3	3.0	4	AGL J2233-2212 (HBL)
							3HSP J223248.8-202226 (HBL)
							PKS 2227-214 (LBL)
							5BZO J2234-2055 (LBL)
IC-190504A	58607	HESE	1.53 ± 0.72	-0.78	3.1	2	2AGL J0429-3755/PKS 0426-380 (LBL/IE
			0.73 ± 0.35	-1.7	3.2	4	4LAC J0420.3-3745 (IBL)
			0.79 ± 0.32	-2.2	3.4	7	3HSP J042850.8-380550 (IBL)
			0.29 ± 0.12	-1.7	3.0	28	PKS 0422-380 (LBL)
							PKS 0422-389 (LBL)
							5BZQ J0422-3844 (LBL/IBL)

3 events out of 6 show a candidate detection within \pm 1 year around T₀ of neutrino event

Results: GOLD neutrinos

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	IceCube Name	T ₀ (MJD)	Event Type	$F_{\gamma}(E > 100 \text{ MeV})$ $\times 10^{-6} (\text{ph cm}^{-2} \text{ s}^{-1})$	Δt (yr)	\sqrt{TS}	Bin (days)	Associated Sources
	IC-190619A	58653	GOLD	1.61±0.75	-0.47	3.2		4FGL J2252.6+1245 (Gamma-ray source)
	IC-190019A	58055	GOLD	1.01 ± 0.73 1.18 ± 0.49	-0.47	3.2		PKS 2251+11 (BLRG)
				1.14 ± 0.52	-5.1	3.3	4	WISEA J224854.73+111427.3 (HBL)
								UGC 12237 (Sy2)
								UGC 12243 (Sy1)
								MCXC J2250.3+1054 (CoG)
								3C 454.3 (FSRQ, within a few deg)
	IC-190730A	58694	GOLD	1.33 ± 0.65	-3.5	3.1	2	2AGL J1507+1019/PKS 1502+106 (LBL)
				1.23 ± 0.60	-2.6	3.1	4	3HSP J150604.4+102233 (HBL)
2AGL associated				0.86 ± 0.34	-3.5	3.3	7	SDSS J150340.61+101429.4 (LBL)
COURCO				0.38 ± 0.14	-1.5	3.4	28	1WGA J1503.7+1018 (LBL)
source								CRATES J150339+104836 (IBL)
								SDSS J150339.35+101820.7 (LBL)
								SDSS J145920.92+103603.3 (HBL)
								SDSS J150135.83+095818.6 (LBL)
								SDSS J150243.09+111557.3 (HBL)
	IC-190922A	58748	GOLD	0.83 ± 0.33	-0.25	3.5	7	4FGL J1103.6-2329 (BL Lac)
				2.01 ± 0.84	-3.8	3.2	2	4FGL J1120.0-2204 (Gamma ray source)
								4FGL J1100.0-2044 (BCU)
· · · ·								HE 1106-2321 (HBL)
events out of 7 show a								MQ J111552.48-215640.7 (IBL)
a Black Lack de la 10 ta 🖌	IC-190922B	58748	GOLD	1.53 ± 0.68	-3.8	3.0	4	4FGL J0022.0+0006 (BL Lac, within a few deg)
andidate detection within ± 1				0.79 ± 0.36	-4.8	3.0		LBQS 0020-0202 (HBL)
oor around T of noutring avant				0.41 ± 0.16	-4.8	3.0	28	Mrk 947 (Sy2)
ear around T ₀ of neutrino event								PGC 1077256 (HBL)
								MQ J002002.35-002122.9 (HBL)
			0.01 P					SDSS J001920.37-003532.9 (LBL)
	IC-191001A	58757	GOLD	2.10 ± 0.90	-0.82	3.7	2	4FGL J2115.2+1218 (BL Lac)
			0.01 5					4FGL J2052.7+1218 (Millisecond pulsar)
	IC-191119A	58806	GOLD	1.81 ± 0.86	-7.2	3.1		MQ J151757.77+021544.2 (HBL)
				1.39 ± 0.70	-9.8	3.0	2	ZW 20733 (IBL)
								WISEA J152506.63+022425.1 (HBL)
								CRATES J152122+042021 (Radio galaxy)
	IC-200109A	58857	GOLD	1.18 ± 0.53	1.6	2.0	4	4FGL J1518.8+0203 (Pulsar)
	IC-200109A	28827	GOLD		-1.6	3.0	4	TXS 1100+122 (IBL)
				1.19 ± 0.53	-4.6	3.0	4	PGC 1403648 (HBL) ZW 11570 (HBL)
								SDSS J110108.25+111635.3 (IBL)
								PKS 1058+110 (Sy1)
								Mkn 728 (Sy1)
								Abell 1142 (CoG)
								DRCG 22-46 (CoG)

Multiple detections in all light curves, only those closest to T₀ reported here

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Post-trial significance of identified detections

For each candidate detection we estimated (Bulgarelli+2012, Lucarelli+2017):

- The *p-value* (or *pre-trial probability, p*): the probability for a candidate detection to be a false detection;
- The *Post-trial probability* (*P*) to have *k* or more random detections in non-overlapping time intervals out of *N* trials (i.e., the number of generates maps with exposure above 80 cm^2/s/sr centered on arrival direction neutrinos coordinates); $P(N|X > k) = 1 \quad \left(\sum_{i=1}^{k-1} {N \choose i} p_{i}^{j}(1-p)^{N-j}\right)$

$$P(N, X \ge k) = 1 - \left(\sum_{j=0}^{k-1} \binom{N}{j} p^j (1-p)^{N-j}\right)$$

- The *Combined post-trial probability* (*P_{cpt}*) to have a spatial coincidence of an AGILE-GRID detection with the IceCube error region inside the 10 deg *Region of Interest* (ROI);

$$P_{cpt} = P \times P_{ROI}$$
 where $P_{ROI} = 10^{-2}$

- The Joint post-trial probability (P_{joint}) is the probability to have a number *n* of neutrinos with a false candidate detection over a year from T₀ out of *N* trials given by the product of the total number of the neutrino events analysed and the number of optimization of search radius (equal 1 in our work).

$$P_{joint}(\text{post-trial}) = 1 - \left(1 - \prod_{i=1}^{n} P_i\right)^N$$