# II (Fully) non-thermal production

How light can DM be?

- $m \lesssim$ few MeV : excluded by BBN for fully thermalized DM
  (too large $\Delta N_{eff}$)

- $m \lesssim$ few keV : " " structure formation for DM
  [directly] produced from SM thermal bath ($\rightsquigarrow$ FIMPs ..)

- $m \lesssim 1\,(0.1)$ keV : " " phase-space degeneracy in dSphs
  for fermionic DM (Tremaine-Gunn, completely independent of production)

$\rightsquigarrow$ bosonic light DM ~ unconstrained for non-thermal production

NB: very high number density $\rightsquigarrow$ more "field" than "particle"!

e.g. $n_\gamma^{CMB} \sim T_\gamma^3 \sim 400\ \mathrm{cm}^{-3}$

$$\Omega h^2 \sim 0.1 \;\hat{=}\; n_{DM} \sim \left(\frac{m}{\mathrm{keV}}\right)^{-1} \mathrm{cm}^{-3}$$

[$\times \sim 10^6$ @ Galactic scales / $r \sim R_\odot$]

## II.1 Misalignment mechanism

Scalar field $\phi$: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$

$$\Rightarrow 0 = \Box\phi + V' = \ddot{\phi} + 3H\dot{\phi} \underbrace{- a^{-2}\nabla^2\phi}_{\to \frac{k^2}{a^2}\phi} + V'$$

→ redshifts quickly away → neglect

[but important to study, e.g., axion miniclusters]

consider $V \simeq \frac{1}{2}m^2\phi^2$ ; $m = m(T)$

1. "slow roll" at early times ($T \gtrsim T_c$): [like inflation; difference: $\rho_\phi \ll \rho$!]

$$3H\dot{\phi} \gg m^2\phi \simeq 0 \Rightarrow 0 \simeq \ddot{\phi} + 3H\dot{\phi} \Rightarrow \dot{\phi} \propto a^{-3}$$

$$\Rightarrow \text{(very soon)} \quad \phi(t) \longrightarrow \phi = \phi_I = \text{const.}$$

2. late times ($T \lesssim T_c$): $\ddot{\phi} + 3H\dot{\phi} + \underline{m^2(T)}\phi = 0$

[e.g. phase transitions]

assumption: $m(T)$ changes 'adiabatically' (= slow w.r.t. other timescales)

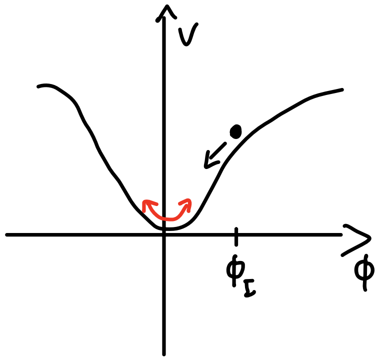
$\rightsquigarrow$ WKB ansatz: $\underline{\phi} = \underline{A}(t)\, e^{i\underline{\theta}(t)}$

[A = slow, $\theta$ = fast] all real $\Rightarrow$ two equations

$$\Rightarrow \text{i) } 0 = \underbrace{\frac{\ddot{A}}{A} + 3H\frac{\dot{A}}{A}}_{\ll \dot{\theta}^2 \text{ (WKB!)}} - \dot{\theta}^2 + m^2 \Rightarrow \dot{\theta} \sim \pm m \Leftrightarrow \theta = \pm\int dt\, m + \text{const.}$$

$$\text{ii) } 0 = \dot{A} + \frac{A}{2}\left(3H + \frac{\ddot{\theta}}{\dot{\theta}}\right) \overset{\text{i)}}{=} \dot{A} + \frac{A}{2}\left(3H + \frac{\dot{m}}{m}\right)$$

$$\Rightarrow A(t) \propto a^{-3/2}\, m^{-1/2}$$




↓ keep only real part

$$\boxed{\phi(t) = \underbrace{C\cdot a(t)^{-3/2}\, m(t)^{-1/2}}_{\text{slowly varying envelope}}\;\underbrace{\cos\int_{t_c}^{t} dt'\, m(t')}_{\text{rapidly oscillating } [H\sim\frac{1}{2t}\gg m]}}$$

energy density? $\rho = T^{00} = \dot\phi^2 - g^{00}\mathcal{L} = \frac{1}{2}\dot\phi^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi) \simeq \frac{1}{2}\dot\phi^2 + \frac{1}{2}m^2\phi^2$

$\rightsquigarrow H \gtrsim m: \rho = \frac{1}{2}m^2\phi_I^2$ (= const., as during inflation)

$H \lesssim m: \rho \approx \frac{1}{2}C^2 a^{-3} m^{-1}\underbrace{\{m^2\sin^2(\int m\,dt) + m^2\cos^2(\int m\,dt)\}}_{=m^2}$

↑ NB: $\partial_t(a^{-3/2}m^{-1/2})$ = 'small'
$\partial_t \cos$ = 'large'!

$$\Rightarrow \boxed{\rho(t) = \frac{1}{2}m(t_c)\,m(t)\left(\frac{a_c}{a}\right)^3\phi_I^2}$$

$\big|\, m(t_c) \stackrel{!}{=} H_c \sim \frac{T_c^2}{M_{pl}} \propto a_c^{-2}$

$\rightsquigarrow \rho_0 \sim \frac{keV}{cm^3}\sqrt{\frac{m_0}{eV}}\sqrt{\frac{m_0}{m_c}}\left(\frac{\phi_I}{10^{11}\,GeV}\right)^2$

↑ $\sim \rho_{DM,obs}$

$\Rightarrow$ need large $\phi_I$ or $\frac{m_0}{m_c} \lll 1$

pressure? $p = T^{ii} \simeq \frac{1}{2}\dot\phi^2 - \frac{1}{2}m^2\phi^2$

$\rightsquigarrow H \gtrsim m: p = -\rho$ 'Λ'

$H \lesssim m: p \approx \frac{1}{2}C^2 a^{-3} m^{-1}\{m^2\sin^2(\int m\,dt) - m^2\cos^2(\int m\,dt)\}$

$\Rightarrow \boxed{\langle p\rangle \approx 0}$ $\underset{(n=\frac{\rho}{m}\propto a^{-3})}{\Longrightarrow}$ "CDM"

if $t_c \ll t_{eq}$ $\Leftrightarrow m(t_c) \gtrsim 10^{-27}\,eV$
$\sim 7H(t_{eq})$

⌈NB: need to show this also for perturbations
$\rightsquigarrow$ roughly the same ✓ ⌋

Two possibilities ...

1. $t_c < t_{inflation}$ : $\phi_I$ = arbitrary initial condition = unknown, free parameter

   [small patch w/ $\phi = \phi_I$ will be blown up to entire universe today]

2. $t_c > t_{inflation}$ : $\phi_I^2 \to \langle \phi_I^2 \rangle = \frac{\pi^2}{3} f_a^{-2}$ for periodic potentials

   [many different $\phi_I$ within each patch of size $H_c^{-1}$]

   $[f_a] = \text{mass}^{-1}$

   $\Rightarrow \Omega h^2 = \text{const.} \frac{m_0}{f_a^2}$

   ⊕ very predictive

   [⊖] large isocurvature fluctuations (↝ neglected $\nabla\phi$ terms)

## model classes

- ALPs ↝ typically pseudo-Goldstones, only derivative couplings
  → shift symmetry
  → often periodic potentials
  $V \simeq m^2 \phi_c^2 \left(1 - \cos \frac{\phi}{\phi_c}\right)$ ← SSB scale

  ↝ $\mathcal{L} \supset -\frac{1}{4} g \phi \underbrace{F_{\mu\nu} \tilde{F}^{\mu\nu}}_{\propto \vec{E}\cdot\vec{B}}$

  → axion-photon conversion

  → astro-/lab constraints

  e.g. QCD axion : $\mathcal{L} \sim \underbrace{\frac{\alpha_s}{8\pi} \frac{a}{f_a} G^{i\,\mu\nu} \tilde{G}^i_{\mu\nu}}_{\text{to explain small CP value dynamically}} + \underbrace{\frac{(\partial_\mu a)}{f_a}}_{\to g} \bar{q} \gamma^\mu \gamma^5 q$

  $f_a \to$ SSB, production

  → g (→ constraints) closely related to $f_a$ ⇒ predictive !

  $\Rightarrow m_a \sim 10^{-5}\,\text{eV} \ll E_{EW} \sim \text{TeV}$ = original hope

  ⟷ ALPs: $\sim 10^{-6} - 10^{-3}$

- hidden photons, $U(1)'$

  ↝ $\mathcal{L} \supset \frac{\chi}{2} F_{\mu\nu} F'^{\mu\nu}$ → observational constraints

  + similar $T^{\mu\nu}$ ↝ $\rho, p$ → misalignment mechanism

# II.2 Gravitational production

$\rightsquigarrow$ In particular at end of inflation. One option: inflaton decay $\rightsquigarrow$ Boltzmann
Here: what happens if there are "no" couplings? Well, always gravity...

## QFT in curved space

recall minimal coupling : $\partial_\mu \to \nabla_\mu$ ; $\eta_{\mu\nu} \to g_{\mu\nu}$

QFT : "particles" are representations of the Poincaré group

$\Rightarrow$ ?

e.g. free scalar field : $(\Box_\eta + m^2)\phi = 0$

$$\Rightarrow \phi \sim e^{-ik\cdot x} a_{\vec{k}} + e^{ik\cdot x} a^+_{\vec{k}}$$

= operator-valued solution of KG equation, "particle-wave duality"

$\rightsquigarrow$ Def. vacuum : $a_k |0\rangle = 0$ $\rightsquigarrow$ Lorentz invariant!

1-particle states : $\sim a_k^+ |0\rangle$

NB: need time-like Killing vector to distinguish positive/negative frequency modes!

$$\partial_t u_{\vec{k}} = \mp i\omega f_{\vec{k}} \quad ; \omega > 0$$

in general : $(\Box_g + m^2)\phi = 0$ $\qquad | \ \phi(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\vec{x}} \phi_{\vec{k}}(t)$

$$\rightsquigarrow \phi_{\vec{k}}^{(t)} = u_{\vec{k}}^{(t)} a_{\vec{k}} + u_{\vec{k}}^{*(t)} a^+_{-\vec{k}}$$

[for real $\phi(x)$]

would-be positive frequency mode

$$= v_{\vec{k}} b_{\vec{k}} + v_{\vec{k}}^* b^+_{-\vec{k}}$$

$u, v$ = $\underbrace{\text{complete \& orthonormal}}_{(*)}$ set of solutions to KG eq.

$[(u_i, u_j) = -i\int_\Sigma d\Sigma^\mu \sqrt{-g_\Sigma}\; u_i \overleftrightarrow{\partial_\mu} u_j^* = \delta_{ij}]$

⇝ choice no longer unique, even if locally of (flat space =) harmonic oscillator type!

$$(*) \Rightarrow v_k = \alpha_k^{(t)} u_k + \beta_k^{(t)} u_k^* \quad \Leftrightarrow \quad \begin{pmatrix} b_k \\ b_k^+ \end{pmatrix} = \begin{pmatrix} \alpha_k^* & -\beta_k^* \\ -\beta_k & \alpha_k \end{pmatrix} \begin{pmatrix} a_k \\ a_k^+ \end{pmatrix}$$

"Bogoliubov coefficients"

NB: mix of "positive" and "negative" frequencies! (wrt $u_k$)

e.g. vacuum w.r.t. $u_k$ : $a_k|0\rangle_u = 0$

⇒ particle number w.r.t. $v_k$:

$$n_v = {}_u\langle 0| b_k^+ b_k |0\rangle_u = |\beta_k|^2 \neq 0 \qquad \text{"particle production"}$$

In general: $u_k$ = some initial mode choice at $t = t_0$, typically in asymptotically flat region / free-fall coordinates

⇝ $u_k \sim \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t} [e^{i\vec{k}\vec{x}}]$

$v_k$ = general solution to KG eq. at $t > t_0$

⇒ $\beta_k(t)$ ⇒ continuous particle production

⇝ e.g. inflation