Particle dark matter

(and how to produce it)

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In some sense, all we (really) know about DM:

$$\boxed{\Omega_{DM} h^2 \approx 0.12}$$

$\rightsquigarrow$ one "measured" ΛCDM parameter to fit many observations

- $\Omega_{DM} \neq 0$ @ many σ $\rightsquigarrow$ no doubt about existence
- $\frac{\Delta \Omega_{DM}}{\Omega_{DM}} \lesssim 1\%$ $\rightsquigarrow$ this is a precision "measurement"

$\sim 10\%$ for much more general case (7ΛCDM), allowing DM to invisibly disappear to dark radiation [1803.03644]

+ many constraints on

- how cold DM must be
  (structure formation, BBN)
- interactions with visible matter
  (direct & indirect searches, early universe production)
- interactions with itself or other "dark sector" particles
  (structure formation, production)

$\Rightarrow$ vanilla CDM / "dust" does a depressingly good job!

$\Rightarrow$ any theory of DM should at least allow to predict $\Omega_{DM} h^2$, ideally with matching accuracy!

outline

I. Production from the thermal bath $\gtrsim$ 3 lectures

I.1 : Freeze-out mechanism

("standard", [co-annihilation, aDM?, dark sectors]
[impact of] kinetic decoupling)

I.2 : Freeze-in mechanism

(following an approach that stresses analogy w/ I.1)

I.3 : Alternatives

(SIMP, Pandemic, ...)

II. (Fully) non-thermal production $\lesssim$ 2 lectures

II.1 : Misalignment

(axions, dark photons)

II.2 : Gravitational production

(Bogoliubov vs. decay !?)

III.3 : Primordial black holes

⇝ quite a few other options [e.g. DM from decay];
personal selection, with focus on what is most commonly discussed
+ only sporadic bibliographic references
⇝ contact/ask me for more pointers to literature!

I.1 : Freeze-out mechanism

Boltzmann equation : $L[f_i] = C[f_i]$

⇝ formally describes evolution of $f_i(\vec{x}, \vec{p}, t)$
for any particle species i

Liouville operator : ⇝ evolution in absence of collisions

[cf. Liouville's theorem in classical mechanics
$\frac{\partial f}{\partial t} + \{f, H\} = \frac{df}{dt} = 0$]

$p^0 = p^0(p^i)$

$$L[f] = \frac{df}{d\lambda} = \frac{\partial x^\mu}{\partial \lambda}\frac{\partial f}{\partial x^\mu} + \frac{dp^i}{\partial \lambda}\frac{\partial f}{\partial p^i}$$

standard choice:
p^i = "physical momentum" = $m \frac{dx^i}{d\tau}$ ← free-fall coordinates
= measured by freely-falling observer

$\Leftrightarrow \bar{p}^i \equiv m \frac{d\bar{x}^i}{d\tau} = \frac{\partial \bar{x}^i}{\partial x^\nu} p^\nu$ = standard momentum in GR (= 4-vector)
→ geodesic eqn. in free fall

$$g^{\mu\nu} = \eta^{\rho\sigma} \frac{\partial \bar{x}^\mu}{\partial x^\rho}\frac{\partial \bar{x}^\nu}{\partial x^\sigma} \overset{\text{FRW}}{\Rightarrow} \frac{\partial \bar{x}^0}{\partial x^0} = 1, \quad \frac{\partial \bar{x}^i}{\partial x^j} = a^{-1}\delta^i_j$$

$\Rightarrow \boxed{\bar{p}^i = a^{-1} p^i}$ NB: not comoving $= a p^i$!

$(\bar{p}^0 = p^0)$

$$\Rightarrow L[f] \overset{\lambda = \tau}{=} p^0 \partial_t f + \frac{\partial f}{\partial p^i}\Big\{ \underbrace{\frac{da}{d\tau}}_{} \bar{p}^i + a \underbrace{\frac{d\bar{p}^i}{d\tau}}_{} \Big\}$$

$f = f(t, \vec{p})$ for FRW

$= \dot{a} p^0 \bar{p}^i$
$= H p^0 p^i$

$= -\Gamma^i_{\mu\nu}\bar{p}^\mu \bar{p}^\nu$ | $\Gamma^i_{0j} = \Gamma^i_{j0} = H\delta^i_j$
$= -2H\bar{p}^i\bar{p}^0$

$$\Rightarrow \boxed{L = p^0(\partial_t - H\,\vec{p}\cdot\nabla_p)\, f(t, \underset{\text{physical momentum}}{\vec{p}})}$$

$$= p^0 \partial_t f(t, \underbrace{\vec{p}\cdot a}_{\text{comoving momentum}})$$

$\Rightarrow$ for <u>number density</u> $n(t)$:

$$C_n[f] \equiv g \int \frac{d^3p}{(2\pi)^3} \frac{C[f]}{E}$$

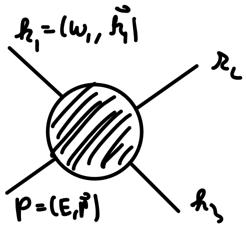
$$\stackrel{!}{=} \partial_t \Big[\underbrace{g \int \frac{d^3p}{(2\pi)^3} f}_{\equiv n(t)} \Big] - H g \int \frac{d^3p}{(2\pi)^3} \underbrace{\vec{p}\cdot\nabla_p f}_{\rightarrow -f(\nabla_p \vec{p}) = -3}$$

$$= \dot{n} + 3Hn \qquad \Big|\ H = \frac{\dot{a}}{a}$$

$$= a^{-3}\partial_t (a^3 n)$$

$\Rightarrow$ " C_n = change of comoving number density "

Collision operator for $2 \leftrightarrow 2$ processes:



$$C[f] = \frac{1}{2g} \int \prod_i \left(\frac{d^3k_i}{(2\pi)^3 2\omega_i} \right) (2\pi)^4 \delta^{(4)}(p + k_1 - k_2 - k_3)$$

$$\times \Big\{ |M|^2_{\leftarrow} f_2 f_3 (1 \mp f_1)(1 \mp f) \qquad \text{Fermi suppression / Bose enhancement}$$

summed over all d.o.f. g_i!

$$- |M|^2_{\rightarrow} f f_1 (1 \mp f_2)(1 \mp f_3) \Big\}$$

most important (NB: $Z_2 \Rightarrow$ DM is stable)

a) DM scattering (3 = DM; 1,2 = SM / heat bath)

$\Rightarrow C_n[f] = 0$ ⇝ exercise!

b) DM pair annihilation / creation

1 = DM ("χ") ; 2,3 = heat bath ("ψ") $\Rightarrow f_\psi = \frac{1}{e^{E/T} \pm 1}$ ($\mu \simeq 0$ in SM!)

- $|M|^2_{\leftarrow} = |M|^2_{\rightarrow}$ (CP invariance)
- detailed balance ("$\leftarrow = \rightarrow$ in EQ")
- $f_\chi \propto e^{-E/T}$ (NR DM in kinetic EQ)

⇝ exercise!

$$\boxed{\dot{n} + 3Hn = -\langle\sigma v\rangle\,(\underbrace{n^2}_{\text{"annihilation"}} - \underbrace{n_{eq}^2}_{\text{"production"}})} = C_n[f]$$

$$\underline{\underline{\langle\sigma v\rangle}} \equiv \frac{\int d^3p \int d^3\tilde{p}\, f_\chi(E)\, f_\chi(\tilde{E})\, \sigma v_{Mol}}{\int d^3p \int d^3\tilde{p}\, f_\chi(E)\, f_\chi(\tilde{E})} = \ldots = \underline{\underline{\int_1^\infty d\tilde{s}\, \frac{4x\sqrt{\tilde{s}}\,(\tilde{s}-1)\, K_1(2\sqrt{\tilde{s}}\,x)}{K_2^2(x)}\,\sigma}}$$

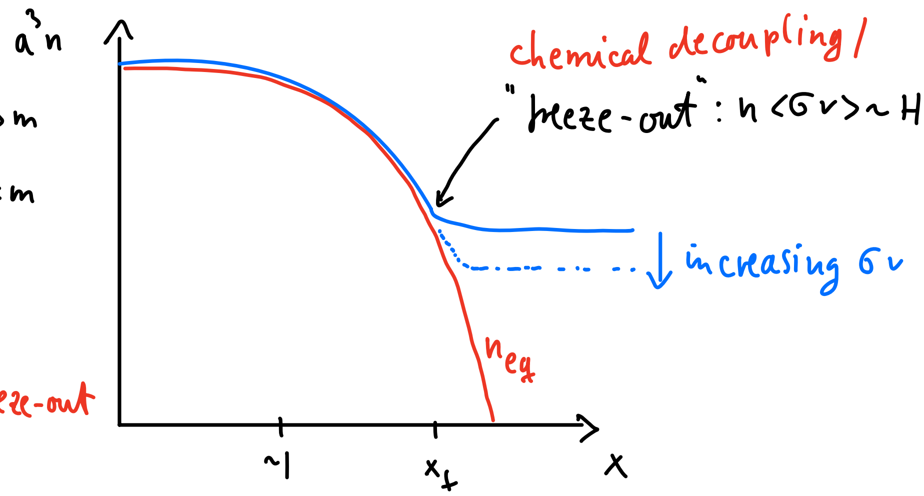
Gondolo & Gelmini '91

$\left[x \equiv \frac{m}{T}\ ;\ \tilde{s} \equiv \frac{s}{4m^2}\right]$

recall:

$$n_{eq} \propto \begin{cases} T^3 & \text{for } T \gg m \\ T^{3/2}\, e^{-m/T} & \text{for } T \ll m \end{cases}$$

⇝ relevant for $\left\{ \begin{matrix} \text{HDM}, \gamma, \ldots \\ \text{CDM} \end{matrix} \right\}$ freeze-out



rough estimate: • $n_f \cdot \langle \sigma v \rangle \overset{!}{\sim} H_f \sim \frac{T_f^2}{m_{pl}}$

• $n_f \sim \frac{\rho_\chi^{eq}}{m_\chi} \frac{T_f^3}{T_{eq}^3} \overset{!}{\sim} T_{eq}\, T_f^2 \underbrace{\frac{T_f}{m_\chi}}_{\text{"}\mathcal{O}(1)\text{"}}$

$\rho_\chi^{eq} \sim \rho_\gamma^{eq} \sim T_{eq}^4$

$$\boxed{\langle \sigma v \rangle \sim \frac{1}{T_{eq} \cdot m_{pl}}} \overset{!}{=} \frac{\alpha^2}{m_\chi^2}$$

for correct relic density

$T_{eq} \sim 3\,eV \quad m_{pl} \sim 2 \cdot 10^{18}\,GeV$

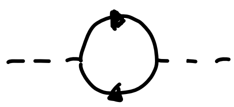
numerically: $\Omega_\chi h^2 \sim \frac{3 \cdot 10^{-27}\, cm^3/s}{\langle \sigma v \rangle} \sim \frac{3 \cdot 10^{-10}\, GeV^{-2}}{\langle \sigma v \rangle \sim \alpha^2 m_\chi^{-2}}$

~ 0.1 e.g. for • $m_\chi \sim TeV$

• $\alpha \sim 10^{-2}$

=> **W**eakly **I**nteracting **M**assive **P**articles are generically produced with correct relic density!

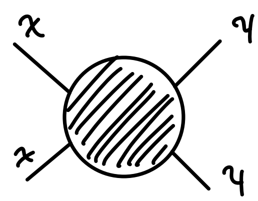
⤳ a "miracle"? • cf. SM hierarchy problem

 => $\delta m_h^2 \propto \Lambda^2$

• many good candidate BSM theories, e.g. SUSY

= independent motivation from particle physics (+ correct RD)

- WIMP DM is highly predictive:



direct detection

RD + indirect detection

(RD+) collider production

$\Rightarrow$ vanilla WIMPs are ruled out

(:$\Leftrightarrow$ ⊘ ~ 1 number)

- "exceptions" (very common!)

A) $q_{RD} \neq q_{detection}$

B) $\langle \sigma v \rangle_{RD} \gg \langle \sigma v \rangle_0$

examples for B:

i) $\sigma v = a + b\, v_{rel}^2 + \ldots$

$\Rightarrow \langle \sigma v \rangle \overset{v = v_{lab}}{=} a + 6 b\, x^{-1} + \ldots$

NB: s-wave $= a + a_2 v^2 + \ldots$

p-wave $= b_2 v^2 + b_4 v^4 + \ldots$

$\uparrow b$

today: $v \sim 10^{-3}$

freez-out: $T \sim \frac{1}{3}\frac{p^2}{m} \sim \frac{m}{x_f}$

$\Rightarrow v \sim \sqrt{\frac{3}{20}} \sim 0.4$

$\Rightarrow$ can be solved analytically:

Kolb & Turner

$$\boxed{\Omega h^2 \simeq 8.77 \cdot 10^{-11}\,\mathrm{GeV}^{-2}\, \frac{x_f}{g_{eff}^{1/2}} \times \times \left[a + 3 b\, x_f^{-1} + \ldots \right]^{-1}}$$

ii) strong v- dependence (other than i)

- (kinematic) thresholds

  $\chi\chi \to \psi\psi$; $m_\psi \gtrsim m_\chi$

- (s-channel) resonances

  $\chi\chi \to \phi^* \to \psi\psi$; $m_\phi \sim 2m_\chi$

- non-perturbative effects

  "Sommerfeld enhancement",
  bound states, ...

iii) co-annihilations

$\rightarrow$ network of "dark" particles
w/ $m_{\chi_n} > \ldots m_{\chi_1} \equiv m_\chi$, charged
under same Z_2 (e.g. R parity)

$\rightsquigarrow$ will eventually decay to χ

$\Rightarrow$ $n \equiv n_{\chi_1} + n_{\chi_2} + \ldots + n_\chi$ satisfies
same Boltzmann eq., with

$$\langle \sigma v \rangle_{\text{eff.}} = \sum_{ij} c_{ij} \langle \sigma v \rangle_{ij}$$

[hep-ph/9704361]

iv) asymmetric DM : $n_\chi \neq n_{\bar\chi}$

$\rightarrow$ cf. standard model...
motivation : $\Omega_\chi h^2 \sim \Omega_b h^2$
$\rightsquigarrow$ ~ no indirect detection

v) freeze-out in secluded / "dark" sector (= A & B)

⋮

numerical treatment needed !

$\rightsquigarrow$ highly accurate public tools :

DarkSUSY $\rightsquigarrow$ exercises!

micrOMEGAs

madDM

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