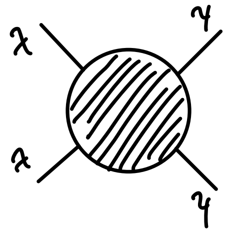
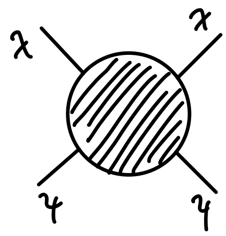
Kinetic decoupling

review: 0903.0189



$\Rightarrow$ chemical EQ until $x_f \sim 20$ $\rightsquigarrow$ $\boxed{\Omega_\chi h^2}$



$\Rightarrow$ kinetic EQ until $x_{kd} \gg x_f$ [generically, since $n_\psi \gg n_\chi$]

$\rightsquigarrow$ minimal protohalo mass $\boxed{M_{cut}}$

two independent effects:

a) no growth of density perturbations

on scales $\lambda < r_s = \int_0^{t_{kd}} dt \, \frac{c_s}{a}$ $\leftarrow$ $= \sqrt{\frac{dp}{d\rho}}$ = sound speed

around decoupling: (dark) acoustic oscillations
$\rightsquigarrow$ additional cutoff, may imprint features on $P(k)$

b) wash-out of density perturbations

on scales $\lambda < \lambda_{FS} = \int_{t_{kd}}^{t_{eq}} dt \, \frac{\langle v \rangle}{a}$

sufficient to consider temperature parameter: / "velocity dispersion"
/ "2nd moment of f"

$$T_\chi \equiv \left\langle \frac{p^2}{3E} \right\rangle = \frac{g}{n} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(E) = \begin{cases} T & \text{for } T \gg T_{kd} \\ \frac{c}{a^2} & \text{for } T \ll T_{kd} \; [\ll m] \end{cases}$$

$\downarrow$ transition typically very fast

$$\simeq \begin{cases} T & \text{for } T \gtrsim T_{kd} \\ T_{kd} \left(\frac{a_{kd}}{a}\right)^2 & \text{for } T \lesssim T_{kd} \end{cases}$$

Boltzmann equation:

$$C_T \equiv g\int\frac{d^3p}{(2\pi)^3}\,\frac{C[f]}{E}\,\frac{p^2}{3E}$$

$$\stackrel{!}{=} g\int\frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3E}\left(\partial_t - H\,p\cdot\nabla_p\right)f$$

$E=\sqrt{p^2+m^2}$

$$= \partial_t(nT_\chi) + H g\int\frac{d^3p}{(2\pi)^3}\, f\,\underbrace{\frac{\partial}{\partial p^i}\left(p^i\,\frac{p^2}{3E}\right)}_{=3\frac{p^2}{3E}+2\frac{p^2}{3E}-\frac{1}{3}\frac{p^4}{E^2}}$$

$$= n\Big\{\dot{T}_\chi + \underbrace{\frac{\dot{n}}{n}}_{=-3H\text{ if } C_n=0}T_\chi + 5HT_\chi\left[+\frac{1}{3}H\left\langle\frac{p^4}{E^2}\right\rangle\right]\Big\}$$

$p^2/E^2 \ll 1$

$2\to2$ elastic scattering:

$$C = \frac{1}{2g}\int\frac{d^3k}{(2\pi)^3 2\omega}\int\frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}\int\frac{d^3\tilde{p}}{(2\pi)^3\tilde{E}}(2\pi)^4\,\delta^{(4)}(\tilde{p}+\tilde{k}-p-k)$$

$$|\mathcal{M}|^2_{\chi\gamma\leftrightarrow\chi\gamma}\left\{[1\mp f_\gamma(\omega)]\,f_\gamma(\tilde{\omega})\,f(\tilde{E}) - [1\mp f_\gamma(\tilde{\omega})]\,f_\gamma(\omega)\,f(E)\right\}$$

- $t \ll m^2$
- $T \lesssim m$

...

$$\simeq \frac{E}{2}\,\gamma(T)\left[TE\,\partial_p^2 + \left(p + 2T\frac{E}{p} + T\frac{p}{E}\right)\partial_p + 3\right]f(E)$$

momentum transfer rate $= \dfrac{1}{48\pi^3 g\, m^3}\displaystyle\int d\omega\, f_\gamma\,\partial_\omega\left(k^4\langle|\mathcal{M}|^2\rangle_t\right)$

$\neq$ scattering rate $\sim \frac{m}{T}\cdot\gamma$

[reason: "random walk" in momentum space]

$$\langle|\mathcal{M}|^2\rangle_t \equiv \frac{1}{8k^4}\int_{-4k^2_{cm}}^{0}dt\,(-t)\,|\mathcal{M}|^2$$

$$= 16\pi\, m_\chi^2\,\sigma_T$$

"transfer cross section"

$$\Rightarrow C_T = g\int \frac{d^3p}{(2\pi)^3}\,\frac{C}{E}\,\frac{p^2}{3E}$$

$$= \frac{1}{6}\gamma g \int \frac{d^3p}{(2\pi)^3}\left\{ T\,\underbrace{p^2\partial_p^2}_{\to\,\partial_p^2 p^2 = 2} + \Big(\underbrace{\frac{p^3}{E}}_{-g T_\chi} + \underbrace{2Tp}_{-2T} + T\frac{p^3}{E^2}\Big)\partial_p + \underbrace{3\frac{p^2}{E}}_{\to\, g T_\chi} \right\} f$$

$$= \frac{3}{2}\gamma(T)\, n\,(T_\chi - T) \times \left\{ 1 + \mathcal{O}\left(\frac{p^2}{E^2}\right)\right\}$$

$$\Rightarrow \boxed{\dot{T}_\chi + 2HT_\chi = \frac{3}{2}\gamma\,(T_\chi - T)}$$

$\rightsquigarrow$ independent eq. for T_χ

$\rightsquigarrow T_{kd} \rightsquigarrow M_{cut}$

typical scales?

model-dependent!

$\rightsquigarrow M_{cut} \sim (10^{-11} - 10^{-3})M_\odot$ for neutralinos ("WIMPs")

$\neq$ "earth mass" $\simeq 10^{-6} M_\odot$!

$\rightarrow$ difficult to observe directly

$\lesssim 10^{10} M_\odot$ e.g. for $\begin{cases} \sim\text{TeV DM w/} \sim \text{MeV mediators} \\ \sim \text{keV (warm) DM} \end{cases}$

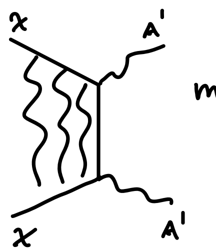
$\rightarrow$ constrained by Ly-α, dwarf galaxy count exceptions : $x_{kd} \cancel{\gg} x_{cd}$ ⇒ direct impact on $\Omega_\chi h^2$

⤳ can not use standard $\dot{n} + 3Hn = -\langle\sigma v\rangle (n^2 - n_{eq}^2)$!

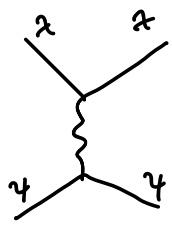
⤳ must consider coupled eqs. for n, T_χ

(or solve BE at phase-space level)

e.g. : • Sommerfeld enhancement (near parametric resonances)



$m_{A'} \lesssim m_\chi$

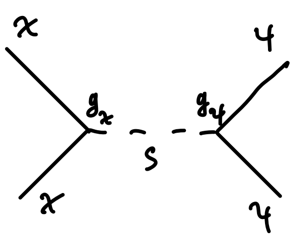


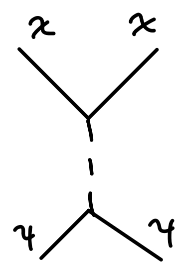
$\sigma v \propto a + \frac{b}{v^2}$

⤳ second era of annihilations

⤳ late KD

• narrow resonances





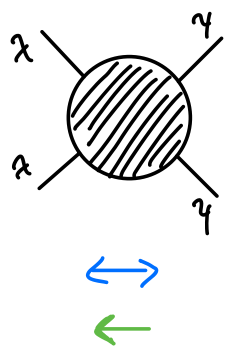
⇒ need very small $g_\chi g_\psi$ for $m_\chi \sim m_S/2$!

(for correct $\Omega_\chi h^2$)

⇒ early KD : $x_{kd} \sim x_{cd}$

⤳ fully implemented in DarkSUSY...

I.2 Freeze-in mechanism



- freeze-out : initial eq. w/ heat bath

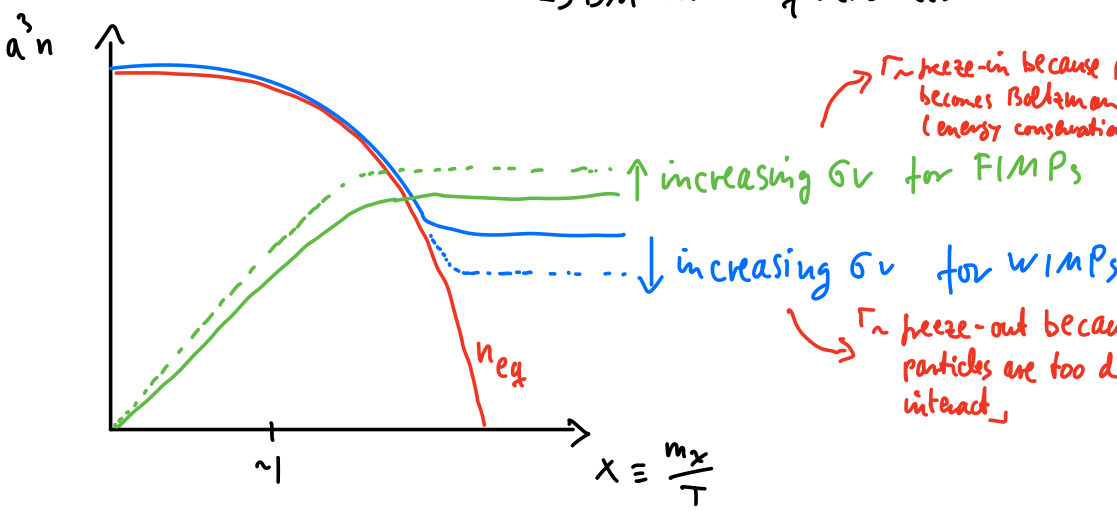
  $\Leftrightarrow \langle \sigma v \rangle n_{eq} > H$ "WIMPs"

  $\Rightarrow g > g_{min}$

- freeze-in : $g \ll g_{min}$ "FIMPs"

  (partially motivated by absence of DM signals)

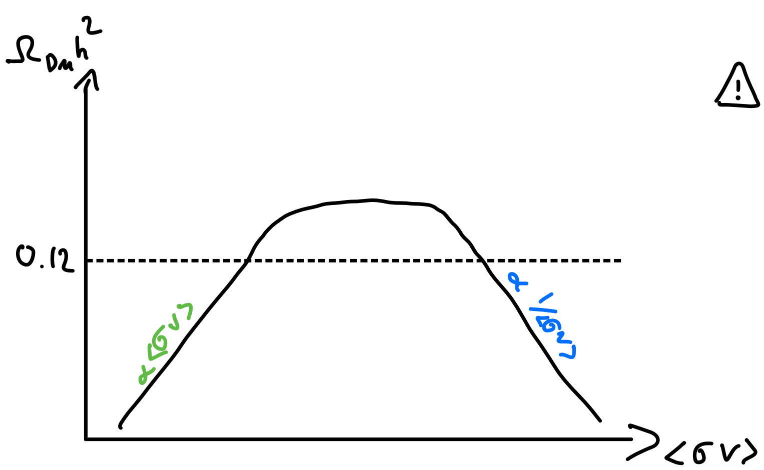
  $\Rightarrow$ DM never equilibrizes



⌐ freeze-in because production becomes Boltzmann suppressed (energy conservation!) ⌐

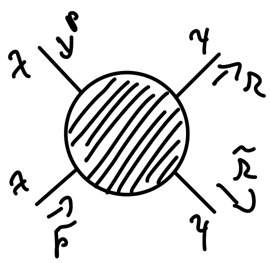
⌐ freeze-out because particles are too dilute to interact ⌐

$\Rightarrow$ always two possibilities to obtain correct relic density!



⚠ FIMPs are sensitive to initial conditions, WIMPs are not

Collision operator



$$C_n = \int \frac{d^3p}{(2\pi)^3 2E} \int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}} \int \frac{d^3\Omega}{(2\pi)^3 2\omega} \int \frac{d^3\tilde{\Omega}}{(2\pi)^3 2\tilde{\omega}} (2\pi)^4 \delta^{(4)}(p+\tilde{p}-\Omega-\tilde{\Omega})$$

$$\times |M|^2 \left\{ f_\gamma(\omega) f_\gamma(\tilde{\omega}) - f_\chi(E) f_\chi(\tilde{E}) \underbrace{[1 \pm f_\gamma(\omega)]}_{\equiv \bar{f}_\gamma(\omega)} [1 \mp f_\gamma(\tilde{\omega})] \right\}$$

CP invariance

$f_\chi \ll 1 \Rightarrow$ • $\bar{f}_\chi \simeq 1$ [BUT $\bar{f}_\gamma \neq 1$!]

⌈relativistic production
⟺ WIMPs : cons. of $E \sim m \gg T \Rightarrow f_\chi \ll 1$⌋

• [still:] only production ("$\longrightarrow$" $= 0$)
("def. of freeze-in")
[⇝ "non-thermal production"]

NB: $f_\gamma(\omega) = \dfrac{1}{e^{\omega/T} \pm 1}$ only in cosmic (plasma) frame!
⌈⟺ WIMPs : all (NR) frames equivalent⌋

math identity : $f_\gamma(\omega) f_\gamma(\tilde{\omega}) \overset{\omega+\tilde{\omega}=E+\tilde{E}}{=} f_\gamma(\omega) f_\gamma(\tilde{\omega})\, e^{(\omega+\tilde{\omega})/T} \underbrace{e^{-E/T}}_{\equiv f_\chi^{MB}(E)} e^{-\tilde{E}/T}$

$$= f_\chi^{MB}(E)\, f_\chi^{MB}(\tilde{E})\, \bar{f}_\gamma(\omega)\, \bar{f}_\gamma(\tilde{\omega})$$

$\Rightarrow$ production can be described by the annihilation of a would-be population of χ with MB distribution!
⌈⟺ WIMPs actually follow MB in eq. !⌋

$$\Rightarrow \boxed{\dot{n}_\chi + 3Hn = \langle \sigma v \rangle_{\chi\chi\to\psi\psi} \left(n_\chi^{eq}\right)^2}$$ 2111.14871

- formal analogy to WIMP case (w/ annihilation cross section)
  - ⇝ highly useful for
    - numerical implementation
    - resonances w/ off-shell bath particles

- allows direct integration over t (or T)
  [r.h.s. no longer depends on n]

- quantum statistics / finite-T effects : → are taken into account, despite appearance

$\frac{1}{2}$ for self-conjugate

$$\sigma v_{Mol} = \underbrace{\frac{1}{N_\psi} \frac{1}{4E\tilde{E}} \int \frac{d^3k}{(2\pi)^3 2\omega} \int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}} (2\pi)^4 \delta^{(4)}(p+\tilde{p}-k-\tilde{k}) |M|^2}_{\text{standard def., in vacuum}} \underline{\bar{f}_\psi(\omega)\, \bar{f}_\psi(\tilde{\omega})}$$

$$= \sigma(s, \gamma)\, v$$

↖ Lorentz factor between CMS and plasma frame

[must be possible to write in this form : f_ψ singles out frame, but still $O(3)$ symmetric!]

⋮

$$\boxed{\langle \sigma v \rangle_{\chi\chi\to\psi\psi} = \frac{8x^2}{K_2^2(x)} \int_1^\infty d\tilde{s}\, \tilde{s}(\tilde{s}-1) \int_1^\infty d\gamma \sqrt{\gamma^2-1}\, e^{-2\sqrt{\tilde{s}}\, x\gamma}\, \sigma(s,\gamma)}$$

$$\xrightarrow[\sigma(s,\gamma)\to\sigma(s)]{} \int_1^\infty d\tilde{s}\, \frac{4x\sqrt{\tilde{s}}(\tilde{s}-1) K_1(2\sqrt{\tilde{s}}\, x)}{K_2^2(x)} \sigma(s) \quad ✓$$ = Gondolo - Gelmini