

Common tools: statistical methods for combination of experimental results

**JENNIFER2 meeting
(November 2022, Prague)**

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Introduction

- The main deliverable will be a **document** detailing recipes on **how to properly combine results from different experiments**, in presence of multi-parameter analysis:

emphasis on **combination of likelihoods** as a function of the parameter of interests (~5) and the nuisance parameters (~hundreds)

(complete likelihood at their highest possible level of dimensionality to preserve coherence of information for further manipulation: profiling/marginalization...)

- Second optional deliverable (if personpower): **software tool** for storing and combination of user-provided likelihoods

A conceptual, technical (and sociological) challenge

October 2018: J2 Consortium General Meeting <https://agenda.infn.it/event/16350/timetable/#20181030.detailed>

September 2019: J2 Kick-off meeting <https://agenda.infn.it/event/19571/timetable/#20190912.detailed>

... and then COVID: 2 years suspension of travels...

But we learned how to work differently: a lot of new developments !

- T2K-NOVA combined analysis
- T2K-Superkamiokande combined analysis
- Belle2 – Belle combinations

Analyzers and conveners from J2!
Real implementations on-going!

T2K-NOVA

- **Combination of full likelihoods** (including all oscillation and systematics parameters) **into dedicated software containers**

$$L(\bar{o}, \bar{f}, \bar{f}', \bar{\alpha}) = L_{\text{T2K}}(\bar{o}, \bar{f}, \bar{\alpha}) \times L_{\text{NOVA}}(\bar{o}, \bar{f}', \bar{\alpha}) \times L_{\text{other}}(\bar{o})$$

\bar{o} = oscillation parameters

\bar{f}, \bar{f}' = uncorrelated systematics

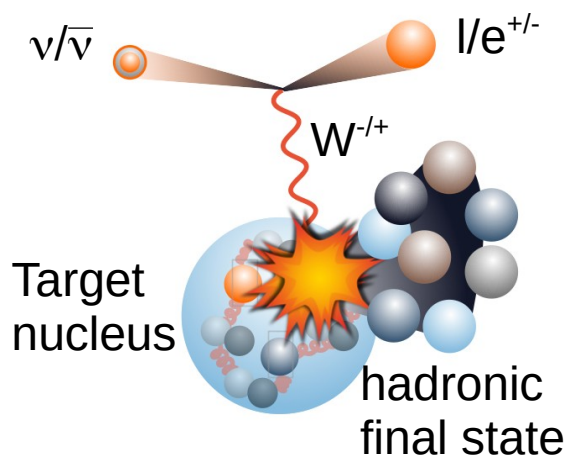
$\bar{\alpha}$ = correlated systematics

Alternative of combining reduced (pre-marginalised/pre-profiled) likelihoods not feasible because too computationally expensive (required in at least 4D space of oscillation parameters + correlated systematics)

- **Correlations?** Starting from widely different models of n flux and n-N interactions.

- unpractical to produce new MC with unified models

- big physics challenge!



- T2K only focus on leptonic final state: dominated by 'simple' charge current at lower energy (600 MeV)

- NOvA exploits the whole final state to reconstruct the neutrino energy (~ 2 GeV)

- No model is available to cover the whole energy and describes leptonic-hadronic correlations

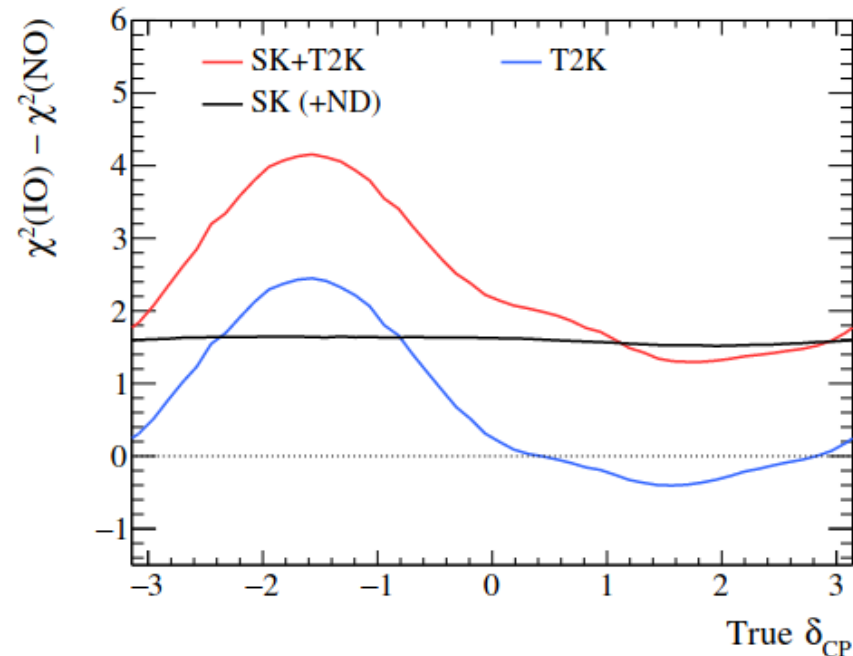
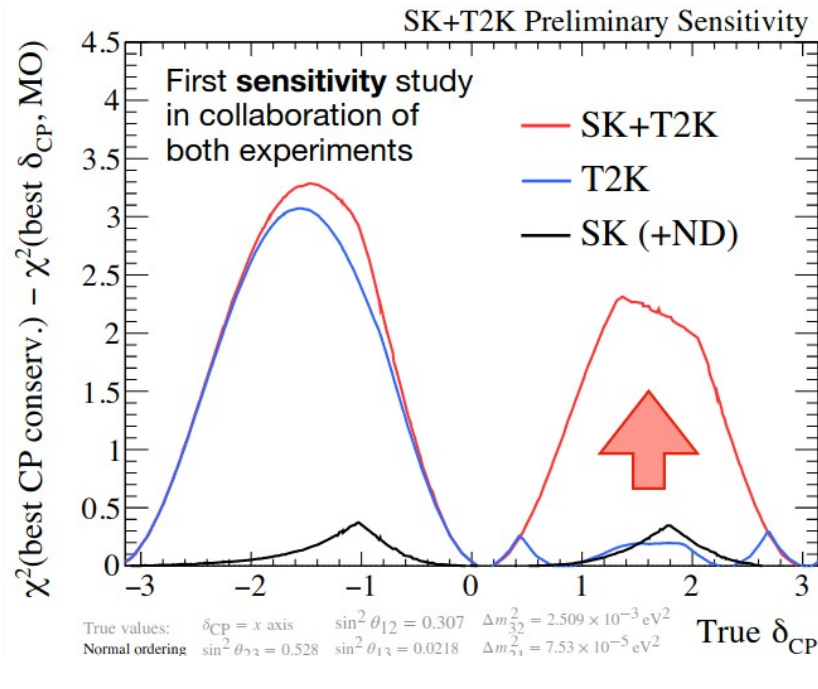
- Detailed studies on-going to assess the impact of correlations and expose possible biases

- **Timeline ~ 2023**

T2K-SuperKamiokande

- **Combination of data: SK atmospheric neutrinos + T2K beam with same far detector**

Correlations of flux, n-N interactions and detector systematics evaluated and included when relevant



- **Timeline ~ 2023**

A step into the future

The T2K/NOVA/SK joint fits are opening the road to a much more challenging future:
HyperKamiokande and DUNE

Statistics will be a factor ~ 20 larger thanks to more powerful beams and larger/better detectors \rightarrow oscillation measurements will not anymore be limited by statistics but by systematics

CPV discovery and MH determination are 'low hanging fruits' to be harvested.
After that: precision oscillation measurements and go beyond PMNS standard paradigm
Combination between experiments allows to

- highly enhance the credibility of our systematics models
- strongly enlarge the BSM reach

Jennifer 2 people are strongly involved in present generation of joint fits \rightarrow producing results of paramount importance for HEP and paving the road to the future

Collider side: combined Belle + Belle II analyses

- Similar collisions/initial states
- Different detectors
- Full data re-analysis including relevant correlations between Belle and Belle II
- Consistent uniformization of analysis models and nuisance parameters
- Same software (same analysts)

Combined analysis of Belle and Belle II data to determine the CKM angle ϕ_3 using $B^+ \rightarrow D(K_S^0 h^+ h^-) h^+$ decays



The Belle and Belle II collaborations

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ABSTRACT: We present a measurement of the Cabibbo-Kobayashi-Maskawa unitarity triangle angle ϕ_3 (also known as γ) using a model-independent Dalitz plot analysis of $B^+ \rightarrow D(K_S^0 h^+ h^-) h^+$, where D is either a D^0 or \bar{D}^0 meson and h is either a π or K . This is the **first measurement that simultaneously uses Belle and Belle II data**, combining samples corresponding to integrated luminosities of 711 fb^{-1} and 128 fb^{-1} , respectively. All data were accumulated from energy-asymmetric e^+e^- collisions at a centre-of-mass energy corresponding to the mass of the $\Upsilon(4S)$ resonance. We measure $\phi_3 = (78.4 \pm 11.4 \pm 0.5 \pm 1.0)^\circ$, where the first uncertainty is statistical, the second is the experimental systematic uncertainty and the third is from the uncertainties on external measurements of the D -decay strong-phase parameters.

KEYWORDS: B Physics, CKM Angle Gamma, e^+e^- Experiments

ARXIV EPRINT: [2110.12125](https://arxiv.org/abs/2110.12125)

JHEP02(2022)063

Belle + Belle II : results

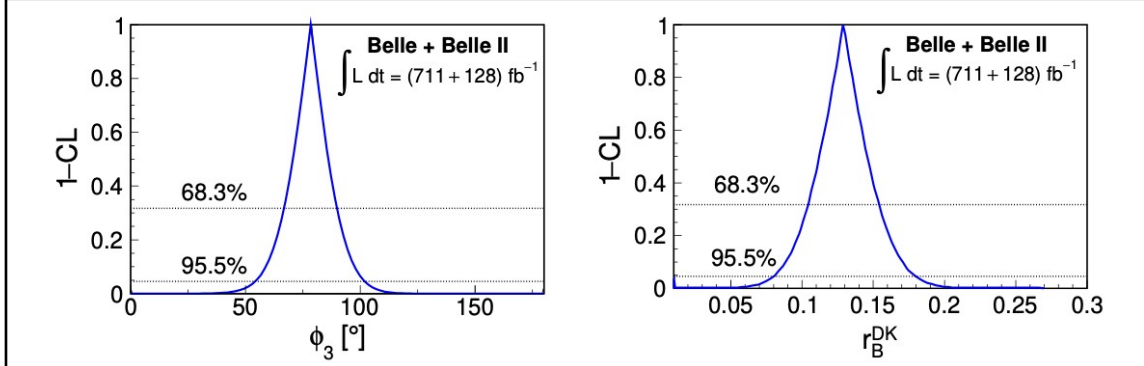


Figure 10. p -value as a function of (left) ϕ_3 and (right) r_B^{DK} calculated using the methods described in ref. [53].

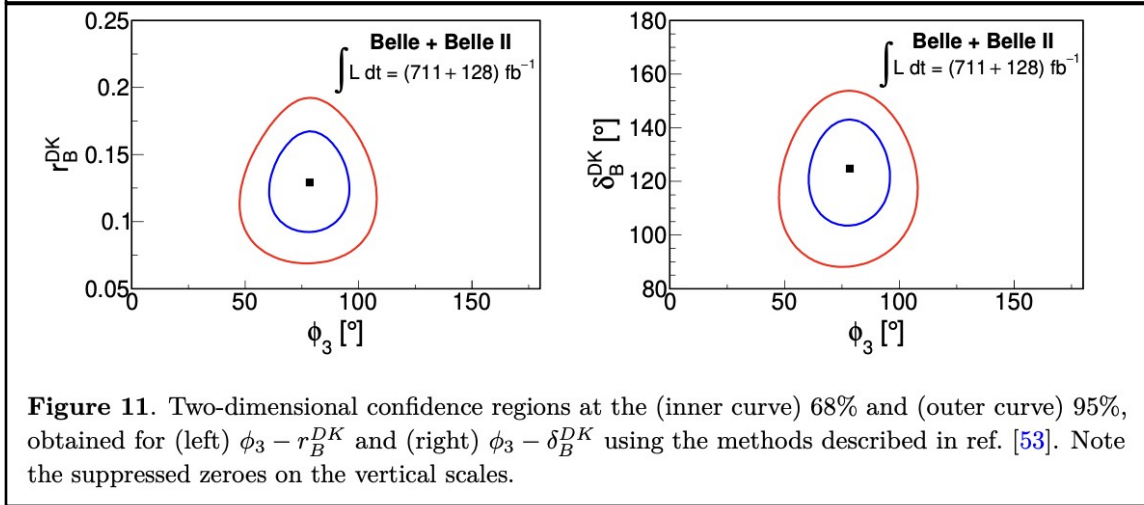


Figure 11. Two-dimensional confidence regions at the (inner curve) 68% and (outer curve) 95%, obtained for (left) $\phi_3 - r_B^{DK}$ and (right) $\phi_3 - \delta_B^{DK}$ using the methods described in ref. [53]. Note the suppressed zeroes on the vertical scales.

$$A_{B^+}(m_-^2, m_+^2) \propto A_{\bar{D}}(m_-^2, m_+^2) + r_B^{DK} e^{i(\delta_B^{DK} - \phi_3)} A_D(m_-^2, m_+^2), \quad (2.1)$$

where $A_{\bar{D}}(m_-^2, m_+^2)$ [$A_D(m_-^2, m_+^2)$] is the $\bar{D}^0 \rightarrow K_S^0 h^+ h^-$ [$D^0 \rightarrow K_S^0 h^+ h^-$] decay amplitude at a point in the Dalitz plot described by m_-^2 and m_+^2 , which are the squared invariant masses of the $K_S^0 h^-$ and $K_S^0 h^+$ particle combinations, respectively. Here r_B^{DK} and δ_B^{DK} are the ratio of the magnitudes of the suppressed to favoured $B^+ \rightarrow DK^+$ amplitudes and the relative strong-phase difference between them, respectively. The world-average value of r_B^{DK} is 0.0996 ± 0.0026 [5],² which means that the direct CP -violating effects are of the order 10%.

Comments


- Provides an **internal and detailed view of how relevant/critical is to keep consistency** in the models assumed.
 - eg, In principle could use it as a test bench to compare results from a fully combined analysis with the results of combining only likelihoods (or reductions thereof) with approximations/limitations
- Such Belle+Belle II combined analyses are happening for various other results
- A different scenario/perspective is surfacing for the option of **Belle II + LHCb** combinations: slow luminosity start-up for SKEB/Belle II makes it unlikely the emerging of a compelling case for topics to combine on the Jennifer 2 timescale

Finally: data sharing!


Belle II has started a systematic push toward data sharing

HEPData Record for $B^+ \rightarrow K^+ \nu \bar{\nu}$

First HEPData Record for Belle II is online since 28.9.2022!



https://www.hepdata.net/record/ins1860766



HEPData @HEPData · 29. 8.

Added #Belle-II data on "Search for $B^+ \rightarrow K^+ \nu \bar{\nu}$ Decays Using an Inclusive Tagging Method at Belle II" to [hepdata.net/record/ins1860...](https://www.hepdata.net/record/ins1860766)

🗨↻♥↑

Search for $B^+ \rightarrow K^+ \nu \bar{\nu}$ Decays Using an Inclusive Tagging Method at Belle II

The Belle-II collaboration

Abudinén, F., Adachi, I., Adamczyk, K., Ahlburg, P., Aihara, H., Akopov, N., Aloisio, A., Ky, N. Anh, Asner, D.M., Atmacan, H.

Phys.Rev.Lett. 127 (2021) 181802, 2021.

<https://doi.org/10.17182/hepdata.130199>

JournalINSPIREResources

Abstract (data abstract)


SuperKEKB Belle II. Measurement of the branching fraction of $B^+ \rightarrow K^+ \nu \bar{\nu}$ at the Belle II experiment at the SuperKEKB. The analysed data sample corresponds to an integrated luminosity of 63 fb^{-1} collected at the $\Upsilon(4S)$ resonance and a sample of 9 fb^{-1} collected at an energy 60 MeV below the resonance between 2019-2021. Since no significant signal was observed, limit of 4.1×10^{-3} was set using CL_s method.

Based on publication [\[PRL 127, 181802 \(2021\)\]](#)

git repository for the this record:

git clone ssh://git@stash.desy.de:7999/~sstefkov/hep_data_b_knunu.git

Slavomira Stefkova, Belle II Data Preservation Workshop, 07.10.2022

 **KIT**
Karlsruher Institut für Technologie

There are a number of **commonalities** between the technical needs of combination and those of data sharing (standard format, documentation, etc): this **complementary** effort will assist the ultimate mission of our group.

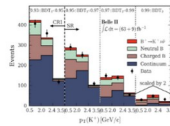
Data sharing: existing example

Post-fit yields in $Y(4S)$

Postfit yields $Y(4S)$ [10.11266/hepdata.130193.v1.02](https://hepdata.ligo.org/hepdata/hepdata.130193.v1.02)

Figure 3 in <https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.127.181802>

Yields in on-resonance data and as predicted by the simultaneous fit to the on- and off-resonance data, corresponding to an integrated luminosity of 63 and 9 fb^{-1} , respectively. The predicted yields are shown individually for charged and neutral B -meson decays and the five continuum background categories. The leftmost three bins belong to the first control region (CR1) with $\text{BDT}_2 \in [0.93; 0.95]$ and the other nine bins correspond to the signal region (SR), three for each range of $\text{BDT}_2 \in [0.95; 0.97; 0.99; 1.0]$. Each set of three bins is defined by $p_T(K^+) \in [0.5; 2.0; 2.4; 3.5] \text{ GeV}/c^2$.



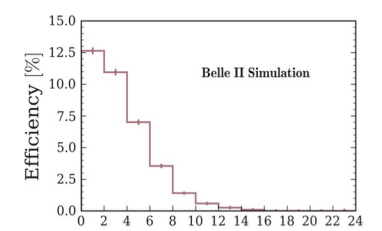
- observables**
 - signal strength μ
- phrases**
 - FCNC
 - $b \rightarrow s$ transition
 - electroweak penguin decay
 - missing energy
- reactions**
 - $B^+ \rightarrow K^+ \nu \bar{\nu}$

$p_T \times \text{BDT}_2$ bins	yield	Observed data	Number of signal events $B^+ \rightarrow K^+ \nu \bar{\nu}$	Number of events from charged B backgrounds	Number of events from neutral B backgrounds	Number of events from $c\bar{c}$ backgrounds
$[0.5; 2.0] \times [0.93; 0.95]$	407.0 ± 20.2	9.6767794	123.073966	62.1993461	144.517922	
$[2.0; 2.4] \times [0.93; 0.95]$	359.0 ± 18.9	6.32168858	47.9504222	32.7334206	150.685436	
$[2.4; 3.5] \times [0.93; 0.95]$	118.0 ± 10.9	1.6782059	6.20851002	3.56989512	63.613553	
$[0.5; 2.0] \times [0.95; 0.97]$	291.0 ± 17.1	10.2102786	93.8582929	47.179443	82.5892068	

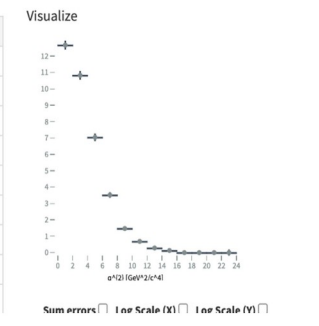


[PRL 127, 181802 (2021)]

independent variable and 1 dependent variable



Luminosity	$63+9 \text{ fb}^{-1}$
$q^2 [\text{GeV}^2/c^4]$	Efficiency
0.0 - 2.0	12.66745696 ± 0.27207299
2.0 - 4.0	10.82571692 ± 0.26443688
4.0 - 6.0	7.0488885 ± 0.2279063
6.0 - 8.0	3.51769225 ± 0.1713566
8.0 - 10.0	1.46683133 ± 0.1183359
10.0 - 12.0	0.68175914 ± 0.0879258
12.0 - 14.0	0.27954132 ± 0.06074916
14.0 - 16.0	0.12747088 ± 0.04873342



Limits:

- Curves with observed central values, expected central values, 1 and 2 sigma expected values
- In HEPData encoded with 1 independent variable, 2 dependent variables

```

independent_variables:
- header: {name: '$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$'}
values:
- {value: 0.0}
- {value: 9.292929292929293e-07}
- {value: 1.858585858585858e-06}

```

```

dependent_variables:
- header: {name: 'CL_{s} value'}
qualifier:
{name: 'Limit', value: 'Observed'}
{name: 'Luminosity', units: 'fb^{-1}', value: 63+9}
values:
- {value: 1.1}
- {value: 0.953245962}
- {value: 0.910805129}
- {value: 0.872237598}

```

```

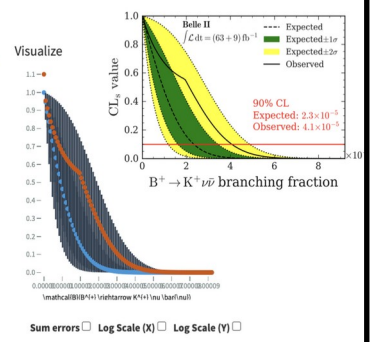
- header: {name: 'CL_{s} value'}
qualifier:
{name: 'Limit', value: 'Expected'}
{name: 'Luminosity', units: 'fb^{-1}', value: 63+9}
values:
- value: 0.99590567
errors:
{assymerror: plus: 0.999502538, minus: 0.999904355}, label: '1 sigma'}
{assymerror: plus: 0.997366525, minus: 0.998621089}, label: '2 sigma'}
- value: 0.847729031
errors:
{assymerror: plus: 0.979556031, minus: 0.995933079}, label: '1 sigma'}
{assymerror: plus: 0.898783907, minus: 0.945208474}, label: '2 sigma'}
- value: 0.715606801

```

[PRL 127, 181802 (2021)]

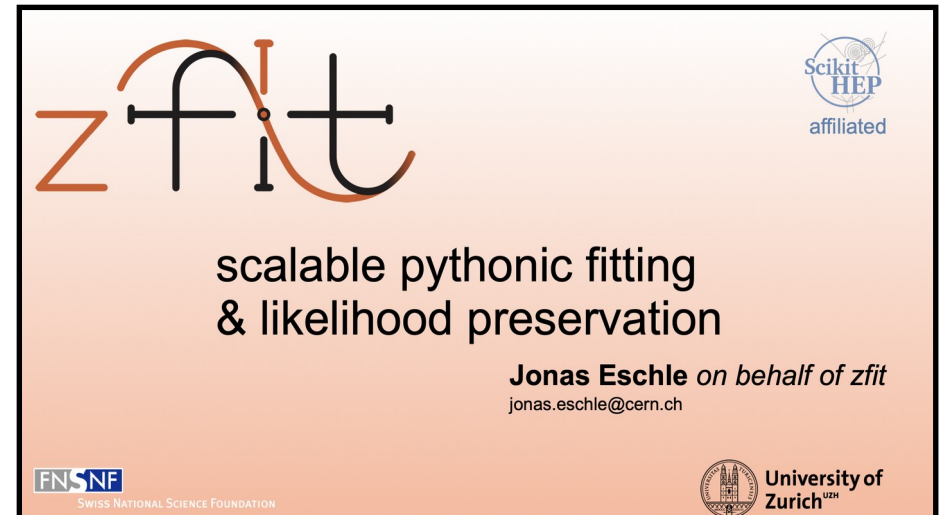
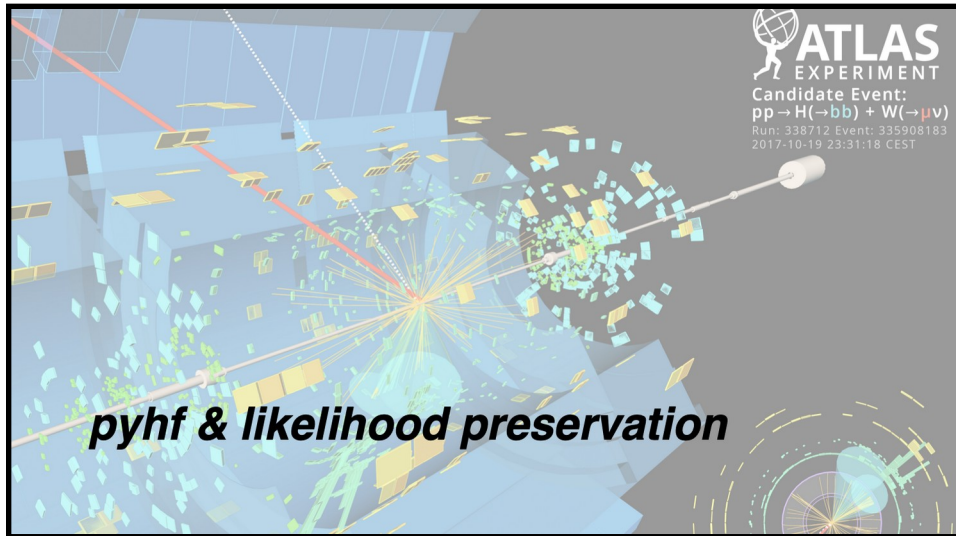
Showing 50 of 100 values

Limit	Observed	Expected
Luminosity	$63+9 \text{ fb}^{-1}$	
$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$	CL value	
0.0	1.1	0.998621089 ± 0.008144000000000000 1 sigma
9.292929292929293e-07	0.953245962	0.945208474 ± 0.008144000000000000 1 sigma
1.858585858585858e-06	0.910805129	0.890664314 ± 0.008144000000000000 1 sigma
2.787878787878788e-06	0.872237598	0.836616884 ± 0.008144000000000000 1 sigma
3.711717171717173e-06	0.837177085	0.783342012 ± 0.008144000000000000 1 sigma
4.646464646464647e-06	0.805284872	0.730882399 ± 0.008144000000000000 1 sigma
5.575757575757576e-06	0.776264559	0.683497347 ± 0.008144000000000000 1 sigma
6.505050505050505e-06	0.749850745	0.636447247 ± 0.008144000000000000 1 sigma
7.434343434343435e-06	0.725805403	0.590765945 ± 0.008144000000000000 1 sigma
8.363636363636363e-06	0.703914061	0.54663455 ± 0.008144000000000000 1 sigma
9.292929292929294e-06	0.683984266	0.504201266 ± 0.008144000000000000 1 sigma



So far limited to higher level information: final plots, limits, efficiencies, but can and will be extended to lower-level info such as likelihoods

For instance...



From Belle II data preservation workshop:
<https://indico.belle2.org/event/7653/>

Significant progress, in the past few years toward the goal of “likelihood preservation”

Mature tools exist that allow preserving the likelihood information in a way that is documented and re-usable for fits, within the tool itself.

A large and growing community is using and developing these.

→ our J2 task can profit/contribute of these efforts!

Summary

- **J2 collaborators are working on joint analysis thus pursuing / implementing in real life the WP5 Common tools subtask !**

- T2K+SuperKamiokande, T2K+NOVA on-going → results expected in 2023
Exploring very different approach for combination

- First Belle + Belle II combination published!
Very useful tool to test internally the different combination approach on the same joint analysis

- **Data sharing efforts** are also developing tools for likelihood preservation: really pointing to the importance of our Common tools subtask