Metric-Affine Gravity: general view and some recent results

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DarkCosmoGrav: New Frontiers
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R.P. and E. Sezgin, “New class of ghost- and tachyon-free metric affine gravities” Phys. Rev. D101, (2020) 084040, e-Print: 1912.01023 [hep-th]

A. Baldazzi, O. Melichev and R.P., “Metric-Affine Gravity as an effective field theory” Ann.Phys. 438 (2022) 168757, e-Print: 2112.10193 [gr-qc]

Work in progress with E. Sezgin and O. Melichev

Status quo

SM: QFT of em, weak, strong interactions.
Spacetime is fixed Minkowski.

GR: classical field theory of gravity.
Geometry of spacetime is dynamical.

Widely different conceptual foundations and mathematical tools.

Reduce the gap between these descriptions.

Uniformization vs. unification.

Analogy with strong interactions

Chiral field $U \in (SU(2)_L \times SU(2)_R)/SU(2)_V$

$$U = exp(\pi/f_\pi)$$

π=pion, f_π=pion decay constant

Metric in $\mathbb{R}^4$ = $g \in GL(4)/O(1,3)$

$$g = exp(h/m_P)$$

h=graviton, $m_P = \sqrt{8\pi G}$=Planck mass

Chiral vs. gravitational dynamics

Chiral action

$$S = \int dx \left[\frac{f_\pi^2}{4} tr(U^{-1}\partial U)^2 + \ell_1 tr((U^{-1}\partial U)^2)^2 + \ell_2 tr((U^{-1}\partial U)^2)^2 + O(\partial^6) \right]$$

Gravitational action

$$S = \int dx \sqrt{g} \left[2m_P^2 \Lambda + m_P^2 R + \ell_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ell_2 R_{\mu\nu} R^{\mu\nu} + \ell_3 R^2 + O(\partial^6) \right]$$

$$R \sim \Gamma\Gamma \sim (g^{-1}\partial g)^2$$

Differences disappear for unimodular gravity.

General EFT recipe

Use the ratio $E/4\pi f_\pi$, resp. $E/4\pi m_P$, as expansion parameter.

By power counting, at a given order in the expansion, only a finite number of diagrams contribute.

Even though chiral theory is non-renormalizable, quantum (loop) corrections to pion cross sections can be calculated unambiguously.

The same is true for gravity
Donoghue: quantum corrections to Newtonian potential

Lessons

- this is a quantum theory of gravity
- consistent and predictive below m_P
- it agrees with all experimental data
- open issues in the UV, IR, strong field...
- not a quantum theory of spacetime
- like pions, the metric may not be a fundamental field

MAG - notation

	coefficients	cov. der.	curvature
Independent conn.	${A_\mu}^a{}_b$	D_μ	${F_{\mu\nu}}^a{}_b$
LC conn.	${\Gamma_\mu}^a{}_b$	∇_μ	${R_{\mu\nu}}^a{}_b$

What is the gauge group?

Not a physically meaningful question.
It is related to the type of frames we use.

Normally the gauge transformations are the diffeomorphisms.
This corresponds to using coordinate frames.

For some purposes it may be advantageous to use the smallest gauge group still compatible with locality: the volume–preserving diffeomorphisms (unimodular gravity).

Can enlarge the gauge group introducing local frame rotations.
Since in general the holonomy of the connection is $GL(4)$, it makes sense to present the theory in a general $GL(4)$ gauge.
This corresponds to working with arbitrary frames in the tangent bundle.

Gravity

In GR we have a frame field (local linear bases) $\theta^a{}_\mu$ and a metric g_{ab}. They are nonlinear objects

- metric $g_{ab} \in GL(4)/O(1,3)$, (signature $-,+,+,+$),
- frame field $\theta^a{}_\mu \in GL(4)$, ($\det\theta \neq 0$),

They carry nonlinear realizations of $GL(4)$.
Think of them as Goldstone bosons.

$$g_{\mu\nu}(x) = \theta^a{}_\mu(x)\,\theta^b{}_\nu(x)\,g_{ab}(x)$$

Transformations under $GL(4)$

Linear changes of basis=$GL(4)$ gauge transformations

$$
\begin{aligned}
\theta &\mapsto \Lambda^{-1}\theta \\
g &\mapsto \Lambda^T g \Lambda
\end{aligned}
$$

Two "unitary gauges" for $GL(4)$:

- $\theta_\mu^a = \delta_\mu^a$: coordinate frames - breaks $GL(4)$ to $\{\mathbf{1}\}$
  metric formulation - breaks $GL(4)$ to $\{\mathbf{1}\}$
- $g_{ab} = \eta_{ab}$: orthonormal frames - breaks $GL(4)$ to $O(1,3)$
  vierbein formulation

not enough freedom to fix both.

Torsion, Nonmetricity, Curvature

Let A_{abc} be an independent connection

$$
\begin{aligned}
T_\mu{}^a{}_\nu &= \partial_\mu \theta^a{}_\nu - \partial_\nu \theta^a{}_\mu + A_\mu{}^a{}_b \, \theta^b{}_\nu - A_\nu{}^a{}_b \, \theta^b{}_\mu \\
Q_{\lambda ab} &= -\partial_\lambda g_{ab} + A_\lambda{}^c{}_a \, g_{cb} + A_\lambda{}^c{}_b \, g_{ac} \\
F_{\mu\nu}{}^a{}_b &= \partial_\mu A_\nu{}^a{}_b - \partial_\nu A_\mu{}^a{}_b + A_\mu{}^a{}_c A_\nu{}^c{}_b - A_\nu{}^a{}_c A_\mu{}^c{}_b
\end{aligned}
$$

T, Q are the covariant derivatives of the Goldstone bosons

Cartan view of MAG

MAG as a theory of frame, metric, connection.

$$S(\theta^a{}_\mu, g_{ab}, A_\mu{}^a{}_b)$$

Powers of T, Q, F and their D-covariant derivatives.

The Levi-Civita connection

Unique connection with $T = Q = 0$

$$\Gamma_{abc} = \frac{1}{2}\left(E_{cab} + E_{abc} - E_{bac}\right) + \frac{1}{2}\left(C_{abc} + C_{bac} - C_{cab}\right)$$

where $E_{cab} = {\theta^{-1}}_c{}^{\lambda}\,\partial_\lambda g_{ab}$
$C_{abc} = g_{ad}\,\theta^d{}_\lambda\left({\theta^{-1}}_b{}^{\mu}\,\partial_\mu {\theta^{-1}}_c{}^{\lambda} - {\theta^{-1}}_c{}^{\mu}\,\partial_\mu {\theta^{-1}}_b{}^{\lambda}\right)$

Its curvature is the Riemann tensor

$$R_{\mu\nu}{}^a{}_b = \partial_\mu \Gamma_\nu{}^a{}_b - \partial_\nu \Gamma_\mu{}^a{}_b + \Gamma_\mu{}^a{}_c \Gamma_\nu{}^c{}_b - \Gamma_\nu{}^a{}_c \Gamma_\mu{}^c{}_b$$

Distortion, disformation, contortion

Any connection A can be split uniquely in

$$A = \Gamma + \Phi$$

("post–Riemannian decomposition") where

$$\phi_{\alpha\beta\gamma} = L_{\alpha\beta\gamma} + K_{\alpha\beta\gamma} \ ,$$

$$\begin{aligned} L_{\alpha\beta\gamma} &= \frac{1}{2}\left(Q_{\alpha\beta\gamma} + Q_{\gamma\beta\alpha} - Q_{\beta\alpha\gamma}\right) \ , \\ K_{\alpha\beta\gamma} &= \frac{1}{2}\left(T_{\alpha\beta\gamma} + T_{\beta\alpha\gamma} - T_{\alpha\gamma\beta}\right) \ . \end{aligned}$$

conversely

$$T_{\alpha\beta\gamma} = \phi_{\alpha\beta\gamma} - \phi_{\gamma\beta\alpha} \ , \qquad Q_{\alpha\beta\gamma} = \phi_{\alpha\beta\gamma} + \phi_{\alpha\gamma\beta}$$

Einstein view of MAG

Consequently

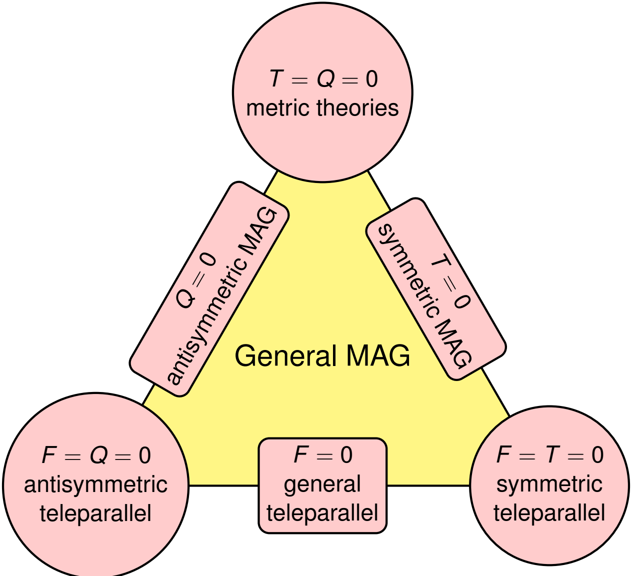
$$S(\theta^a{}_\mu, g_{ab}, A_\mu{}^a{}_b) = S(\theta^a{}_\mu, g_{ab}, \Gamma_\mu{}^a{}_b + \phi_\mu{}^a{}_b) = S'(\theta^a{}_\mu, g_{ab}, \Phi_\mu{}^a{}_b)$$

Powers of T, Q, R and their ∇-covariant derivatives.
Now T, Q are independent fields.

Any MAG is equivalent to a metric theory of gravity coupled to some “matter” fields.

Calculations easier in Einstein-variables.

Main kinematical sublasses of MAGs



Dynamical equivalences: GR

Up to total derivatives, R is equivalent to

$$\mathbb{T} = \frac{1}{4} T_{abc} T^{abc} + \frac{1}{2} T_{abc} T^{acb} - T^{b}{}_{ba} T_{c}{}^{ca}$$

for the antisymmetric MAG,

$$\mathbb{Q} = \frac{1}{4} Q_{abc} Q^{abc} - \frac{1}{2} Q_{abc} Q^{bac} - \frac{1}{4} Q^{b}_{ab} Q^{ac}{}_{c} + \frac{1}{2} Q^{b}_{ab} Q_{c}{}^{ca}$$

for the symmetric MAG and

$$\mathbb{G} = \mathbb{T} + \mathbb{Q} - Q_{abc} T^{abc} - Q_{ab}{}^{b} T_{c}{}^{ca} + Q^{b}{}_{ba} T_{c}{}^{ca} \ .$$

for the general MAG.

Every metric theory has teleparallel equivalents.

Some actions for MAG

Poincaré Gauge Theory (PGT) view $A_{\mu[ab]}$ and $\theta^a{}_\mu$ as gauge fields
and $F_{\mu\nu[ab]}$ and $T_\mu{}^a{}_\nu$ as field strength.
this motivates

$$\mathcal{L} = a^{TT} TT + c^{FF} FF$$

In teleparallel case $\mathcal{L} = f(\mathbb{T})$ or $f(\mathbb{Q})$.

Let us be systematic and order Lagrangians according to their dimension. (T and Q have dimension one, F and R have dimension 2).

General action of MAG

In Cartan-variables and including terms of dimension 2 and 4:

$$
\begin{aligned}
\mathcal{L} = \; & a^F F + a^{TT} TT + a^{TQ} TQ + a^{QQ} QQ \\
& + c^{FF} FF \\
& \quad + c^{FT} FDT + c^{FQ} FDQ + c^{TT} (DT)^2 + c^{TQ} DTDQ + c^{QQ} (DQ)^2 \\
& + c^{FTT} FTT + c^{FTQ} FTQ + c^{FQQ} FQQ \\
& + c^{TTT} TTDT + \ldots + c^{QQQ} QQDQ \\
& + c^{TTTT} TTTT + \ldots + c^{QQQQ} QQQQ \, ,
\end{aligned}
$$

Similar classification in Einstein variables. Large number of invariants.

Counting independent invariants of dimension 4

Antisymmetric (Christensen 1979)

R^2	$(\nabla T)^2$	$R\nabla T$	$R T^2$	$T^2\nabla T$	T^4	Total
3	9	2	14	31	33	92

Symmetric

R^2	$(\nabla Q)^2$	$R\nabla Q$	$R Q^2$	$Q^2\nabla Q$	Q^4	Total
3	16	4	22	59	69	173

General

R^2	$(\nabla\phi)^2$	$R\nabla\phi$	$R\phi^2$	$\phi^2\nabla\phi$	ϕ^4	Total
3	38	6	56	315	504	922

Further kinematical restrictions: Weyl theory as MAG

In a symmetric MAG with

$$Q_{\mu\alpha\beta} = b_\mu g_{\alpha\beta}$$

the quadratic MAG action collapses to

$$S(g) = \int d^4x\sqrt{g}\left[\frac{g_1 + 12g_4}{32\pi G g_4} b_\mu b^\mu + \frac{g_3}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{16\pi G}(2\Lambda - R)\right]$$

which, introducing a Stückelberg field χ can be rewritten

$$S = \int d^4x\sqrt{g}\left[\frac{g_1 + 12g_4}{2} D_\mu\chi D^\mu\chi + g_2\chi^4 + \frac{g_3}{4} F_{\mu\nu}F^{\mu\nu} - g_4\chi^2\mathcal{R}^{(s)}\right] .$$

Weyl gauge field+dilaton

or equivalently

$$S = \int d^4x \sqrt{g} \left[\frac{g_1}{2} D_\mu \chi D^\mu \chi + g_2 \chi^4 + \frac{g_3}{4} F_{\mu\nu} F^{\mu\nu} - g_4 \chi^2 \mathcal{R}^{(b)} \right] .$$

$$D_\mu \chi = (\partial_\mu + b_\mu) \chi .$$

D.M. Ghilencea, JHEP 03 (2019) 049, arXiv: 1812.08613 [hep-th]
D. Sauro, O. Zanusso, Class.Quant.Grav. 39 (2022) 18, 185001, arXiv: 2203.08692 [hep-th]

Analogy with electroweak interactions

Central question in the physics is status of symmetries.

Basic properties of many systems determined by Higgs mechanism.
The electroweak interactions at energies below the Fermi scale are described by the electroweak chiral model.

EWSB

Goldstone bosons $\sigma \in (SU(2)_L \times U(1)_Y)/U(1)_Q$ coupled to $SU(2)_L \times U(1)_Y$-YM fields A_μ.

$$\mathcal{L} = -\frac{f^2}{2} D\sigma^2 \qquad \text{where} \qquad D\sigma = \partial\sigma + A^a K_a(\sigma)$$

In unitary gauge $\sigma = \sigma_0$

$D\sigma_0 = \sum_{a \in (SU(2)_L \times U(1)_Y)/U(1)_Q} A^a K_a(\sigma_0)$

$$\mathcal{L} = -\frac{1}{2} f^2 \sum_{a \in (SU(2)_L \times U(1)_Y)/U(1)_Q} A^a A^a$$

Low Energy EFT

- for $p \ll m_\rho$, $\rho = \rho_0$
- for $p \ll m_A$, $D\sigma = 0$
  or $F^a_{\mu\nu}|_{a \in (SU(2)_L \times U(1)_Y)/U(1)_Q} = 0$

analog of Meissner effect

Gravitational Higgs mechanism in Cartan view

In Cartan-variables

$$S_G(\theta, g, A) \;=\; m^2 \int d^4x \sqrt{|g|} \left[T_{\dots} T^{\cdots} + Q_{\dots} Q^{\cdots} + T_{\dots} Q^{\cdots} \right]$$

expanding around flat background: $A = 0$, $\theta = \mathbf{1}$, $g = \eta$

$$\begin{aligned} T_{\mu}{}^{a}{}_{\nu} &= A_{\mu}{}^{a}{}_{\nu} - A_{\nu}{}^{a}{}_{\mu} \\ Q_{\mu ab} &= A_{\mu ab} + A_{\mu ba} \end{aligned}$$

kinetic term of Goldstone bosons becomes

$$S_G = m^2 \int d^4x \, \sqrt{|g|} \, A_{\dots} A^{\cdots}$$

In general a non-degenerate quadratic form

The Palatini term

$$S_P(A,g,\theta) = \frac{m_P^2}{2}\int d^4x\,\sqrt{|g|}\;F(A)_{ab}{}^{ab}$$

also gives

$$\frac{m_P^2}{2}\int d^4x\,\sqrt{|g|}\;\left(A_a{}^{ac}A_{bc}{}^{b} - A_{bac}A^{acb}\right)$$

(a degenerate quadratic form)

Gravitational Higgs mechanism in Einstein view

Using the post-Riemannian expansion
$A = \Gamma + \Phi$

$$
\begin{aligned}
T_{\mu}{}^{a}{}_{\nu} &= \Phi_{\mu}{}^{a}{}_{\nu} - \Phi_{\nu}{}^{a}{}_{\mu} \\
Q_{\mu ab} &= \Phi_{\mu ab} + \Phi_{\mu ba}
\end{aligned}
$$

therefore (in any gauge and without assumptions on the background)

$$
S_G = \int d^4x \sqrt{|g|}\, \Phi_{\dots} \Phi^{\dots}
$$

Macroscopic gravity

If mass matrix is nondegenerate, assuming all masses are $O(m)$, at $p \ll m$

$$\Phi = 0 \iff T = Q = 0 \iff A = \Gamma(\theta, g)$$

independent of the detailed form of the action.

To summarize

MAG below the Planck scale looks like a low energy EFT where Higgs phenomenon occurs at Planck scale.
Metric is the order parameter.

The conditions $T = 0$ and $Q = 0$ are the same as $D\varphi = 0$ (Meissner effect).

Core questions for quantum theory of spacetime:

- what is the dynamical origin of the Planck scale?
- why is there a nondegenerate metric?
- is the metric fundamental or “emergent”?

Scenarios

Natural to assume that all masses are $m \sim m_P$. No new d.o.f.s in the domain of validity of the EFT. Then MAG as an EFT is essentially indistinguishable from GR as an EFT.

Still has somewhat increased explanatory power: explains dynamically the conditions $T = Q = 0$.

If there are eigenvalues $m \ll m_P$ there may be new propagating d.o.f.s in the domain of validity of the EFT. What kind of states?

Spin$^{\text{parity}}$ states

$\delta g_{\alpha\beta}$ or $\delta\theta_{\alpha\beta}$

	s	a
TT	2^+_4, 0^+_5	1^+_4
TL	1^-_7	1^-_8
LL	0^+_6	-

$\delta A_{\alpha\beta\gamma}$ or $\Phi_{\alpha\beta\gamma}$

	ts	hs	ha	ta
TTT	3^-, 1^-_1	2^-_1, 1^-_2	2^-_2, 1^-_3	0^-
$TTL + TLT + LTT$	2^+_1, 0^+_1	-	-	1^+_3
$\frac{3}{2}LTT$	-	2^+_2, 0^+_2	1^+_2,	-
$TTL + TLT - \frac{1}{2}LTT$	-	1^+_1	2^+_3, 0^+_3	-
$TLL + LTL + LLT$	1^-_4	1^-_5	1^-_6	-
LLL	0^+_4	-	-	-

Spin projector formalism

The quadratic action for the variables A, h

$$\begin{aligned} S^{(2)} &= \frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\Big(A^{\lambda\mu\nu}\,\mathcal{O}^{(AA)}_{(C)}{}_{\lambda\mu\nu}{}^{\tau\rho\sigma}\,A_{\tau\rho\sigma} \\ &\quad + 2A^{\lambda\mu\nu}\,\mathcal{O}^{(Ah)}_{(C)}{}_{\lambda\mu\nu}{}^{\rho\sigma}\,h_{\rho\sigma} + h^{\mu\nu}\,\mathcal{O}^{(hh)}_{(C)}{}_{\mu\nu}{}^{\rho\sigma}\,h_{\rho\sigma}\Big) \end{aligned}$$

can be rewritten

$$\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\sum_{JPij} a_{ij}(J^{\mathcal{P}})\,\Psi P_{ij}(J^{\mathcal{P}})\Psi\ ,$$

where $\Psi = (A, h)$, e.g in the A-A sector

$$\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\sum_{JPij} a_{ij}(J^{\mathcal{P}})\,A^{\lambda\mu\nu}P_{ij}(J^{\mathcal{P}})_{\lambda\mu\nu}{}^{\tau\rho\sigma}A_{\tau\rho\sigma}\ ,$$

Ghost- and tachyon-free MAGs

Antisymmetric MAG:

E. Sezgin and P. van Nieuwenhuizen, Phys. Rev. D21, (1980) 3269

Y.C. Lin, M.P. Hobson and A.N. Lasenby, arXiv:1812.02675

Symmetric MAG:

R.P. and E. Sezgin, Phys. Rev. D101, (2020) 084040

C. Marzo, Phys.Rev.D 105 (2022) 6, 065017

DIY MAG:

A. Baldazzi, O. Melichev and R.P. Ann.Phys. 438 (2022) 168757

- measure zero in theory space
- consistency only checked at level of free propagators

A six-parameter family of symmetric MAGs

Projective invariance, no spin 3^-, only one (massless) spin 2^+

$$
\begin{aligned}
S(g,A) = -\frac{1}{2}\int d^nx\,\sqrt{|g|}\Big[& -\frac{1}{4}(10c_7+3c_{11})F^{\mu\nu\rho\sigma}\Big(F_{\mu\nu\rho\sigma}-2F_{\mu\rho\nu\sigma}\Big) \\
& +2F^{(13)}_{[\mu\nu]}\Big(c_7F^{(13)\mu\nu}+c_{11}F^{(14)\mu\nu}\Big)-\frac{2}{3}(c_7+4c_{11})\,F^{(14)}_{[\mu\nu]}F^{(14)\mu\nu} \\
& -a_0F-\frac{1}{16}(15a_0+120a_6+28a_7-22a_8)\,Q^{\rho\mu\nu}Q_{\rho\mu\nu} \\
& +\frac{1}{8}(9a_0+40a_6-28a_7-6a_8)\,Q^{\rho\mu\nu}Q_{\nu\mu\rho} \\
& +a_6\,Q_\mu Q^\mu+a_7\,\widetilde{Q}_\mu\widetilde{Q}^\mu+\frac{1}{72}(a_0+8a_6-4a_7+74a_8)\,Q_\mu\widetilde{Q}^\mu\Big]
\end{aligned}
$$

New ghost and tachyon-free theories

No ghosts and tachyons provided

$$a_0 > 0 \,, \quad 4\,a_0 + 20\,a_6 - 7\,a_7 + 2\,a_8 < 0 \,,$$

$$(a_0 + 8a_6 - 4a_7 + 2a_8)(89a_0 + 520a_6 - 212a_7 + 82a_8) > 0 \,,$$

$$c_{11} < -2c_7 \ \text{ for } \ c_7 \le 0 \,, \quad \text{or} \quad c_{11} < -\frac{10}{3}c_7 \ \text{ for } \ c_7 > 0 \,.$$

Contain: one massless spin 2^+ one massive spin 2^-, 1^+ and 1^- each

In Einstein form, contain no R^2 terms. Non-renormalizable.

Probably radiatively unstable.

Spin 2^-

Antisymmetric MAG

$$\mathcal{L} = b_1^{TT}\Big[-\frac{1}{3}\nabla_\mu K_{\nu\rho\sigma}\nabla^\mu K^{\nu\rho\sigma} - \frac{1}{3}\nabla_\mu K_{\nu\rho\sigma}\nabla^\mu K^{\rho\nu\sigma} + \nabla_\mu K^\alpha{}_{\alpha\nu}\nabla^\mu K_\beta{}^{\beta\nu}$$
$$+\frac{1}{3}\nabla_\mu K^\mu{}_{\rho\sigma}\nabla_\nu K^{\nu\rho\sigma} + \frac{2}{3}\nabla_\mu K^\mu{}_{\rho\sigma}\nabla_\nu K^{\rho\nu\sigma} + \frac{2}{3}\nabla_\mu K_\rho{}^\mu{}_\sigma\nabla_\nu K^{\rho\nu\sigma}$$
$$+\frac{1}{3}\nabla_\mu K_\rho{}^\mu{}_\sigma\nabla_\nu K^{\sigma\nu\rho} - \nabla_\mu K^\alpha{}_\alpha{}^\mu\nabla_\nu K^\beta{}_\beta{}^\nu + 2\nabla_\mu K^{\mu\nu}{}_\rho\nabla_\nu K^\alpha{}_\alpha{}^\rho\Big]$$

The Fourier transform of the linearized Lagrangian is

$$\mathcal{L} \;=\; -\frac{1}{2}K\cdot q^2\left[P_{22}\left(2^-\right) - P_{33}\left(1^-\right)\right]\cdot K$$

Spin 3^-

General MAG

$$\begin{aligned}\mathcal{L} \;=\; & -2b_1^{QQ}\Big[\tfrac{1}{4}\nabla_\mu Q_{\alpha\beta\gamma}\nabla^\mu Q^{\alpha\beta\gamma} + \tfrac{1}{2}\nabla_\mu Q_{\alpha\beta\gamma}\nabla^\mu Q^{\beta\alpha\gamma} \\ & -\nabla_\mu \mathrm{tr}_{(12)}Q_\alpha \nabla^\mu \mathrm{tr}_{(12)}Q^\alpha - \nabla_\mu \mathrm{tr}_{(12)}Q_\alpha \nabla^\mu \mathrm{tr}_{(23)}Q^\alpha - \tfrac{1}{4}\nabla_\mu \mathrm{tr}_{(23)}Q_\alpha \nabla^\mu \mathrm{tr}_{(23)}Q^\alpha \\ & -\tfrac{1}{4}\mathrm{div}_{(1)}Q_{\alpha\beta}\mathrm{div}_{(1)}Q^{\alpha\beta} - \mathrm{div}_{(1)}Q_{\alpha\beta}\mathrm{div}_{(2)}Q^{\alpha\beta} - \mathrm{div}_{(2)}Q_{(\alpha\beta)}\mathrm{div}_{(2)}Q^{(\alpha\beta)} \\ & -\tfrac{1}{8}\mathrm{trdiv}_{(1)}Q_{\alpha\beta}\mathrm{trdiv}_{(1)}Q^{\alpha\beta} - \tfrac{1}{2}\mathrm{trdiv}_{(1)}Q_{\alpha\beta}\mathrm{trdiv}_{(2)}Q^{\alpha\beta} - \tfrac{1}{2}\mathrm{trdiv}_{(2)}Q_{\alpha\beta}\mathrm{trdiv}_{(2)}Q^{\alpha\beta} \\ & +\mathrm{div}_{(12)}Q_\alpha \mathrm{tr}_{(23)}Q^\alpha + \tfrac{1}{2}\mathrm{div}_{(23)}Q_\alpha \mathrm{tr}_{(23)}Q^\alpha \\ & +2\mathrm{div}_{(12)}Q_\alpha \mathrm{tr}_{(12)}Q^\alpha + \mathrm{div}_{(23)}Q_\alpha \mathrm{tr}_{(12)}Q^\alpha\Big]\,,\end{aligned}$$

where $\mathrm{div}_{(1)}Q_{\alpha\beta} = \nabla^\lambda Q_{\lambda\alpha\beta}$ etc.. when linearized, propagates only a massless spin-3 state.

Indeed setting $S_{\alpha\beta\gamma} = Q_{(\alpha\beta\gamma)}$, and $b_1^{QQ} = 1/3$ it becomes the Fronsdal Lagrangian

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\Big[-q^2 S_{\alpha\beta\gamma} S^{\alpha\beta\gamma} + 3q^2 \mathrm{tr}_{(12)} S_\alpha \mathrm{tr}_{(12)} S^\alpha \\ &\quad + 3\mathrm{div}_{(1)} S_{\alpha\beta} \mathrm{div}_{(1)} S^{\alpha\beta} + \frac{3}{2}(\mathrm{trdiv}_{(1)} S)^2 - 6\mathrm{div}_{(12)} S_\alpha \mathrm{tr}_{(12)} S^\alpha \Big]\end{aligned}$$

The “higher spin symmetry” $\delta S_{\alpha\beta\gamma} = \partial_{(\alpha}\Lambda_{\beta\gamma)}$ is an accidental symmetry.

Massive case under investigation

(restrictions as an EFT - see e.g. B. Bellazzini, F. Riva, J. Serra and F. Sgarlata, JHEP **10** (2019), 189 [arXiv:1903.08664 [hep-th]])

Extension to the UV

In Landau-Ginzburg theory and in the SM the Goldstone bosons are embedded in a linear representation.
One more field is needed.

In gravity no new field needed. Simplest linear realization would keep the same fields but relax constraints.

In linear realization, unbroken phase can be realized,
but difficult to write actions that make sense in both phases.

“UV complete” theories

In general, the continuum limit can be taken at a free or interacting fixed point.

Scenario I: 4DG a.k.a. Stelle gravity

At fundamental level a (scale-invariant) curvature squared theory.

Perturbative calculation (Avramidi and Barvinsky 1985)
Asymptotically free for appropriate choices of signs.

Issues with ghosts/tachyons.
They may be confined (Holdom, Donoghue...) or otherwise made harmless (Mannheim, Strumia, Anselmi...)

Similar to QCD: renormalizable, asymptotically free and unitary theory; Planck scale arises by dimensional transmutation.
Never need to do anything else than perturbative QFT.

Scenario II: Asymptotic safety

- Start from the EFT, picture, with all possible Lagrangian terms allowed by the symmetries.
- "Theory space" is parametrized by the coefficients of these monomials, made dimensionless by powers of the RG scale, e.g. $\tilde{G} = G\mu^2$.
- AS happens if the EFT automatically mends itself: instead of diverging, all these dimensionless parameters reach finite values, i.e. a fixed point, in the UV limit.

Calculations

The RG for the general dimension-2 Lagrangian

C. Pagani and R.P. Class.Quant.Grav. 32 (2015) 19, 195019, e-Print: 1506.02882 [gr-qc]

The RG for non-integrable Weyl theory

C. Pagani and R.P. Class.Quant.Grav. 31 (2014) 115005, e-Print: 1312.7767 [hep-th]

The beta functions generated by the Lagrangian

$$c_1 F_{abcd} F^{abcd} + a_1 T_{abc} T^{abc}$$

demonstrate that PGT is non-renormalizable (O. Melichev, paper in preparation).

Summary

- the theory of gravitons is well understood as an EFT
- it may be extended to MAG treated as an EFT;
  – similarities to strong and ew interactions
  – Higgs phenomenon explains why we have the Levi-Civita connection at low energy
  – a prelude to a possible unification of forces
- there may be UV completion of theory at a free or interacting FP, in terms of the same field variables, with enhanced predictivity
- beyond QFT: in the symmetric phase there is no metric - dynamical origin of the VEV related to origin of spacetime