# Relativistic Cosmology on the linearized past light-cone

Based on JCAP02(2021)014

In collaboration with G. Fanizza, G. Marozzi, G. Schiaffino


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  - Gauge issue
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- Light-cone perturbation theory
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- Scalar-Vector-Tensor/Scalar-Pseudo-Scalar relation
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# Introduction

# Standard Cosmological Perturbation Theory

Standard Cosmological Perturbation Theory

The Cosmological Perturbation Theory is a ubiquitous tool in Cosmology.

Cosmological perturbations originate from random fields sourced by quantum fluctuations in the early universe.

The understanding of the statistics of the perturbations in late-time cosmological observables as the angular/luminosity distance and the cosmological redshift may allow us to probe the physics of the primordial universe.

$$\frac{T_{\mu\nu} - \bar{T}_{\mu\nu}}{max\,(\bar{T})} \ll 1\,, \frac{g_{\mu\nu} - \bar{g}_{\mu\nu}}{max\,(\bar{g})} \ll 1$$

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Metric Perturbations

In the standard perturbation theory in FLRW geometry

$$g_{\mu\nu}\left(\eta, x^{i}\right) = \bar{g}_{\mu\nu}\left(\eta\right) + \delta g_{\mu\nu}\left(\eta, x^{i}\right)$$

$$\delta g_{\mu\nu}\left(\eta, x^{i}\right) = a^{2}\left(\bar{\eta}\right)\begin{pmatrix} -2\phi & -\mathcal{B}_{i} \\ -\mathcal{B}_{j} & C_{ij} \end{pmatrix}$$

$$\bar{g}_{\mu\nu}\left(\bar{\eta}\right) = a^{2}\left(-1, \bar{\gamma}_{ij}\right)$$

# Metric Perturbations

Due to the SO(3) symmetries of $\bar{\gamma}_{ij}$ , we may decompose the perturbations as Scalars Vectors and Tensors (SVT)

$$\mathcal{B}_i = B_i + \partial_i B \qquad \nabla_i B^i = 0$$

$$C_{ij} = -2\psi\bar{\gamma}_{ij} + 2\bar{D}_{ij}E + 2\nabla_{(i}F_{j)} + 2h_{ij}$$

$$D_{ij} = \nabla_{(i}\partial_{j)} - \frac{1}{3}\bar{\gamma}_{ij}\Delta_3$$

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$$\nabla_i F^i = 0 \qquad \nabla^i h_{ij} = 0 \qquad \bar{\gamma}^{ij} h_{ij} = 0$$

Metric Perturbations

$$\psi^{(n)} = -\frac{1}{6}\bar{\gamma}^{ij}C_{ij}^{(n)}$$

$$(\Delta_3)^2 E^{(n)} = \frac{3}{4}D^{ij}C_{ij}^{(n)}$$

$$\Delta_3 F_i^{(n)} = \nabla^j C_{ji}^{(n)} + 6\nabla_i\psi^{(n)} - 4\nabla_i\left(\psi^{(n)} + \frac{1}{3}\Delta_3 E^{(n)}\right)$$

$$(\Delta_3)^2 h_{ij}^{(n)} = \frac{1}{2}\left[\Pi_i^l\Pi_j^k - \frac{1}{2}\Pi_{ij}\Pi^{lk}\right]C_{lk}^{(n)}$$

$$\Pi_{ij} = \bar{\gamma}_{ij}\Delta_3 - \nabla_i\nabla_j$$

$$h_{ij}^{(n)} = \frac{1}{2}C_{ij}^{(n)} - \frac{3\nabla_i\nabla_j}{\Delta_3}\psi^{(n)} + \left[\bar{\gamma}_{ij} + \frac{\nabla_i\nabla_j}{\Delta_3}\right]\left(\psi^{(n)} + \frac{1}{3}\Delta_3 E^{(n)}\right) - \frac{1}{\Delta_3}\left[\nabla^l\nabla_{(i}C_{j)l}^{(n)}\right]$$

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## Gauge Issue

General Relativity is a covariant theory with diffeomorphism invariance. Hence, it is built in terms of coordinate independent quantities.

$$\tilde{ds}^2 = \tilde{g}_{\mu\nu} d\tilde{x}^{\mu} d\tilde{x}^{\nu} = g_{\mu\nu} dx^{\mu} dx^{\nu} = ds^2$$

$$\tilde{\rho}\left(\tilde{x}\right) = \rho\left(x\right)$$

Cosmological observables also needs to be coordinate independent quantities.



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Cosmological observables also needs to be coordinate independent quantities.

However, breaking the quantities into background/perturbations is not a covariant procedure and introduces a spurious coordinate dependence.

Hence, one of the challenges in theoretical cosmology, is to provide gauge invariant expressions for cosmological observables, which guarantees the sanity of the mathematical expressions.

Gauge Issue

However, breaking the quantities into background/perturbations is not a covariant procedure and introduces spurious coordinate dependence.

$$\tilde{x}^{\mu} = x^{\mu} + \xi^{\mu}$$

$$\tilde{\rho}\left(x^{\mu} - \xi^{\mu}\right) = \bar{\rho}\left(x^{\mu}\right) + \tilde{\delta\rho}\left(x\right) + \xi^{\mu}\partial_{\mu}\bar{\rho} = \bar{\rho}\left(x^{\mu}\right) + \delta\rho\left(x\right)$$

$$\tilde{\delta\rho} = \delta\rho - \xi^{\mu}\partial_{\mu}\bar{\rho}$$

This is known as the passive gauge transformation approach. Although, in this presentation we will adopt the active gauge transformation approach.

$$\tilde{\mathbf{T}} = exp\left(\mathcal{L}_{\xi}\right)\mathbf{T}$$

$$\delta^{(1)}\tilde{\mathbf{T}} = \delta^{(1)}\mathbf{T} - \mathcal{L}_{\xi}\bar{\mathbf{T}}$$

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## Metric Perturbations

- Due to diffeomorphism invariance of GR, we have freedom in choosing $\epsilon^\eta$ and $\epsilon^i$. Also, the physics should not depend on this parameters.

$$\eta \to \tilde{\eta} = \eta + \epsilon^\eta$$

$$x^i \to \tilde{x}^i = x^i + \epsilon^i \quad \epsilon^i = e^i + \partial^i \epsilon \qquad \nabla_i e^i = 0$$

- Under a coordinate transformation.

$$\tilde{\delta g}_{\mu\nu} = \delta g_{\mu\nu} + \mathcal{L}_\epsilon g_{\mu\nu}$$

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- Under a coordinate transformation.

$$\tilde{\delta g}_{\mu\nu} = \delta g_{\mu\nu} + \mathcal{L}_\epsilon g_{\mu\nu}$$

$$\tilde{\psi} = \psi + \mathcal{H}\epsilon^\eta + \frac{1}{3}\Delta_3 \epsilon$$

$$\tilde{\phi} = \phi - \mathcal{H}\epsilon^\eta - \partial_\eta \epsilon^\eta$$

$$\tilde{E} = E - \epsilon$$

$$\tilde{B} = B - \epsilon^\eta + \partial_\eta \epsilon$$

$$\tilde{F}_i = F_i - e_i$$

$$\tilde{B}_i = B_i + \partial_\eta \left( \frac{e_i}{a^2} \right)$$

$$\tilde{h}_{ij} = h_{ij}$$

## Gauge Issue

The gauge issue is at the forefront of providing theoretical predictions to cosmological observables.

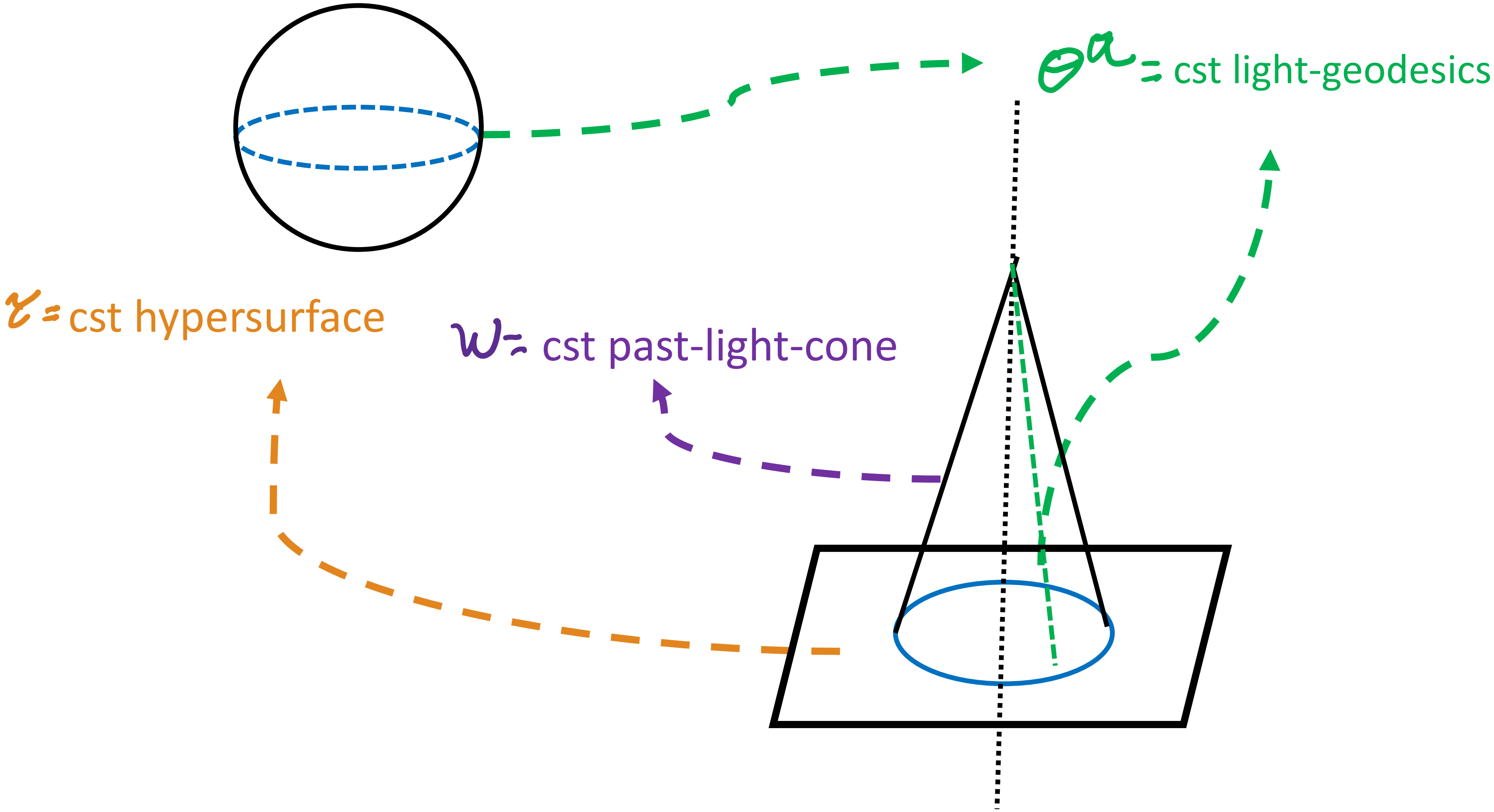
1. Averaging prescriptions.
2. Primordial gravitational waves.
3. Late-time cosmological observables.

Regarding the third point, the approach of the pioneering works (see Bonvin, Durrer and Gasperini PRD 2006) in providing the luminosity distance, was based on solving the geodesic equation order by order.

Hence, having a covariant approach might be a smoking gun in the gauge issue!!

# GLC Gauge

# The non-linear GLC gauge



The non-linear GLC gauge

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}\left(d\theta^a - U^a dw\right)\left(d\theta^b - U^b dw\right)$$

- Simplify the light-like geodesics

$$u_\mu = -\partial_\mu \tau = -\delta^\tau_\mu \qquad k^\mu = \omega\left(u^\mu + d^\mu\right)$$

$$k^\mu = \omega \delta^\mu_\tau \qquad \omega = k^\mu u_\mu$$

- M. Gasperini, G. Marozzi, F. Nugier, G. Veneziano, JCAP07(2011)008

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$$u_\mu = -\partial_\mu \tau = -\delta^\tau_\mu$$

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$$d_A = \frac{\sqrt{\gamma}}{\left(\frac{det \partial_\tau \gamma_{ab}}{4\sqrt{\gamma}}\right)_o}$$

- G. Fanizza, M. Gasperini, G. Marozzi, G. Veneziano, JCAP **11** (2013)

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## Gauge Issue

Hence the GLC gauge provide us non-perturbative and covariant expressions for cosmological observables.

In this presentation we will show how to obtain perturbative gauge invariant observables in terms of light-cone perturbations. Additionally, we will investigate gauge invariant quantities and how different gauge fixings might be implemented as perturbations on the past light-cone.

# Light-cone Perturbations

## LC perturbation theory

- The GLC gauge provides a non-perturbative expression for the Angular Distance.
- For what concerns the first order angular distance-redshift relation, the expressions can become really involved already at first order.
- The GLC gauge can provide non-perturbative results without refer to any background geometry.

$$f_{\mu\nu}^{GLC}\left(\tau, w, \theta^a\right) = \begin{pmatrix} 0 & -\Upsilon & 0 \\ -\Upsilon & \Upsilon^2 + U^2 & -U_a \\ 0 & -U_a & \gamma_{ab} \end{pmatrix}$$

## LC perturbation theory

How the GLC metric entries relates with the SVT decomposition?

In order to answer this question, we need to refer to a background with SO(3) symmetries.

$$\bar{f}_{\mu\nu}^{GLC}\left(\bar{\tau},\bar{w},\bar{\theta}^a\right) = \begin{pmatrix} 0 & -a & 0 \\ -a & a^2 & 0 \\ 0 & 0 & \bar{\gamma}_{ab} \end{pmatrix}$$

$$\bar{\tau} = \int \frac{d\eta}{a}$$

$$\bar{w} = r + \eta$$

$$\bar{\gamma}_{ab}^{GLC} = \bar{\gamma}_{ab}^{FLRW}$$

$$\theta^a_{GLC} = \theta^a_{FLRW}$$

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LC perturbation theory

- Adding general perturbations to the FLRW geometry

$$\delta f_{\mu\nu}\left(\tau, w, \theta^a\right) = a^2 \begin{pmatrix} L & M & V_a \\ M & N & \mathcal{U}_a \\ V_a & \mathcal{U}_a & \gamma_{ab} \end{pmatrix}$$

- Scalar/Pseudo-Scalar (SPS) decomposition

$$V_a = r^2\left(D_a v + \tilde{D}_a \hat{v}\right)$$

$$U_a = r^2\left(D_a u + \tilde{D}_a \hat{u}\right)$$

$$\delta\gamma_{ab} = 2\left[\bar{\gamma}_{ab}\nu + r^2\left(D_{ab}\mu + \tilde{D}_{ab}\hat{\mu}\right)\right]$$

LC perturbation theory

- An infinitesimal change in the coordinates

$$\tilde{x}^\mu = x^\mu + \xi^\mu \qquad \delta \tilde{f}_{\mu\nu} = \delta f_{\mu\nu} - 2\nabla_{(\mu}\xi_{\nu)}$$

- Leads to gauge transformations

$$\tilde{L} = L + \frac{2}{a}\partial_\tau \xi^w$$

$$\tilde{M} = M + \partial_\tau\left(\frac{\xi^\tau}{a} - \xi^w\right) + \frac{1}{a}\partial_w \xi^w$$

$$\tilde{N} = N - 2H\xi^\tau + 2\partial_w\left(\frac{\xi^\tau}{a} - \xi^w\right)$$

LC perturbation theory

- The gauge transformations are given by

$$\tilde{V}_a = V_a + \frac{1}{a}\partial_a \xi^w - \bar{\gamma}_{ab}\partial_\tau \xi^b$$

$$\tilde{U}_a = U_a + \partial_a\left(\frac{\xi^\tau}{a} - \xi^w\right) - \bar{\gamma}_{ab}\partial_w \xi^b$$

$$\tilde{\delta\gamma}_{ab} = \delta\gamma_{ab} - 2\bar{\gamma}_{ab} H\xi^\tau + \frac{2\bar{\gamma}_{ab}}{r}\left(\frac{\xi^\tau}{a} - \xi^w\right) - \left(\bar{\gamma}_{ac}D_b + \bar{\gamma}_{bc}D_a\right)\xi^c$$

## LC perturbation theory

- Adding general perturbations to the FLRW geometry

$$\longrightarrow \qquad \delta f_{\mu\nu}\left(\tau, w, \theta^a\right) = a^2 \begin{pmatrix} L & M & V_a \\ M & N & \mathcal{U}_a \\ V_a & \mathcal{U}_a & \gamma_{ab} \end{pmatrix}$$

- Fixing the GLC gauge

$$f^{GLC}_{\mu\nu} \overset{!}{=} \bar{f}_{\mu\nu} + \delta f_{\mu\nu} \longrightarrow \begin{cases} L = 0 \\ V_a = 0 \\ N + 2aM = 0 \end{cases}$$

LC perturbation theory

- With the GLC gauge fixed we can obtain the redshift and the angular distance redshift-relation.

$$1+z=\frac{a_o}{a_s}\left(1+\frac{1}{2}N|_s^o\right) \qquad d_A = ar\frac{\left[1+\nu-\frac{1}{2}\left(1-\frac{1}{arH}\right)N|_o^z\right]}{\left(1+\nu-ar\partial_\tau\nu\right)_o}$$

- We note absence of integral terms along the geodesics.
- Therefore, we can interpret the GLC gauge as the gauge where integral effects on the angular distance-redshift relation vanishes.

More details in, G. Fanizza, G. Marozzi, MM, G. Schiaffino, *The Cosmological Perturbation Theory on the Geodesic Light-Cone background*, **JCAP** 02 (2021)

# SVT/GLC Relation

SVT/GLC relation

Acting with the operators $\left(\bar{\gamma}^{ij},\, \nabla^i,\, \bar{D}^{ij}\right)$

we may extract the SVT d.o.f. from the standard perturbations.

$$\left(\bar{\gamma}^{ij},\, \nabla^i,\, \bar{D}^{ij}\right) C_{ij} \propto \left(\psi,\, F_j + \partial_j \Delta_3 E + \partial_j \psi,\, \Delta_3^2 E + \Delta_3 \psi\right)$$

Re-writing it in terms of GLC, we obtain a relation between SVT/GLC perturbations, decomposing the GLC perturbations in SPS we obtain a SVT/SPS relation.

$$\bar{\gamma}^{ij} C_{ij} = -6\psi \rightarrow C_{rr} + 4\bar{\gamma}^{ab} C_{ab} = N + 4\nu$$

$$\psi = -\frac{1}{6}\left(N + 4\nu\right)$$

$$C_{ij} \rightarrow \psi \rightarrow E \rightarrow F_i \rightarrow h_{ij}$$

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$$C_{ij} \to \psi \to E \to F_i \to h_{ij}$$

SVT/GLC relation

- From the divergence of $\mathcal{B}_i$

$$\nabla^i \mathcal{B}_i \to \Delta_3 B = -\left(\partial_w + \frac{2}{r}\right)(N + aM) - D^a\left(U_a + aV_a\right)$$

$$\Delta_3 B \to B_r = -(N + aM) - \partial_r B$$

$$B_a = -(U_a + aV_a) - \partial_a B$$

SVT/GLC relation

- Analogously for $C_{ij}$

$$\bar{\gamma}^{ij} C_{ij} \qquad\qquad \psi = -\frac{1}{6}(N + 4\nu)$$

$$\bar{D}^{ij} C_{ij}$$

$$\Delta_3\left(\psi + \frac{1}{3}\Delta_3 E\right) = \frac{1}{2}\left(r^{-1}\partial_w + r^{-2} - \frac{r^{-2}}{2}D^2\right)N + \frac{1}{2r^2}\left(D^2\right)^2\mu + \frac{1}{4r^2}D^2\mu$$
$$\frac{r^{-2}}{2}\left(\partial_w + \frac{3}{r}\right)r^2 D^2 u - \left(\partial_w^2 + \frac{3}{r}\partial_w + r^{-2} + \frac{r^{-2}}{2}D^2\right)\nu$$

SVT/GLC relation

- Analogously for $C_{ij}$

$\bar{\gamma}^{ij}C_{ij}$ → $\psi = -\frac{1}{6}(N + 4\nu)$

$\bar{D}^{ij}C_{ij}$

↓

$$\Delta_3\left(\psi + \frac{1}{3}\Delta_3 E\right) = \frac{1}{2}\left(r^{-1}\partial_w + r^{-2} - \frac{r^{-2}}{2}D^2\right)N + \frac{1}{2r^2}\left(D^2\right)^2\mu + \frac{1}{4r^2}D^2\mu$$

$$\frac{r^{-2}}{2}\left(\partial_w + \frac{3}{r}\right)r^2 D^2 u - \left(\partial_w^2 + \frac{3}{r}\partial_w + r^{-2} + \frac{r^{-2}}{2}D^2\right)\nu$$

SVT/GLC relation

- Analogously for $C_{ij}$

$$\nabla^i C_{ij} \dashrightarrow \Delta_3 F_r = \left(\partial_w + \frac{2}{r}\right) N + D^2 u - \frac{4}{r}\nu + 6\partial_w \psi - 4\partial_w \left(\psi + \frac{1}{3}\Delta_3 E\right)$$

$$\nabla^i C_{ij} \dashrightarrow \Delta_3 F_a = \left(\partial_w + \frac{2}{r}\right) U_a + r^{-2} D^b \delta\gamma_{ba} + 6\partial_a \psi - 4\partial_a \left(\psi + \frac{1}{3}\Delta_3 E\right)$$

## SVT/GLC relation

- Using the previous results

$$h_{rr} = \frac{1}{2}N + \psi - D_{ww}E - \nabla_w F_r$$

$$h_{ar} = \frac{1}{2}U_a - D_{wa}E - \nabla_{(a}F_{r)}$$

$$h_{ab} = \frac{1}{2}\delta\gamma_{ab} + \bar{\gamma}_{ab}\psi - D_{ab}E - \nabla_{(a}F_{b)}$$

# A coordinate independent approach

Projected perturbations

We use the background orthogonal basis

$$\bar{u}_\mu \bar{u}^\mu = -1\,, \qquad \bar{n}^\mu \bar{n}_\mu = 1\,, \qquad \bar{n}^\mu \bar{u}_\mu = 0\,,$$

$$\bar{e}^\mu_A \bar{e}_{\mu B} = \delta_{AB}\,, \qquad \bar{n}^\mu \bar{e}_{\mu A} = 0\,,$$

$$\bar{s}^\mu_\pm \bar{s}_{\mu\pm} = 0\,, \qquad \bar{s}^\mu_\pm \bar{s}_{\mu\mp} = 1\,,$$

which allows a coordinate independent decomposition of the perturbations into spin-0, -1 and -2.

$$\mathcal{S} \equiv \bar{u}^\mu \bar{u}^\nu \delta g_{\mu\nu}\,, \qquad \mathcal{T}_{||} \equiv \bar{n}^\mu \bar{n}^\nu \delta g_{\mu\nu}\,, \qquad \mathcal{V}_{||} \equiv \bar{n}^\mu \bar{u}^\mu \delta g_{\mu\nu}\,, \qquad \mathcal{T} \equiv r^{-2} \bar{s}^\mu_\pm \bar{s}^\nu_\mp \delta g_{\mu\nu}$$

$$\mathcal{V}_\pm \equiv r^{-1} \bar{u}^\mu \bar{s}^\nu_\pm \delta g_{\mu\nu}\,, \qquad \mathcal{T}_{||\pm} \equiv r^{-1} \bar{n}^\mu \bar{e}^\nu_\pm \delta g_{\mu\nu}\,,$$

$$\mathcal{T}_{\pm\pm} \equiv r^{-2} \bar{s}^\mu_\pm \bar{s}^\nu_\pm \delta g_{\mu\nu}$$

Projected perturbations

Which provide the relation between standard and GLC perturbations

| | | $\nu$ | $N$ | $M$ | $L$ | $u$ | $v$ | $\hat{u}$ | $\hat{v}$ | $\mu$ | $\hat{\mu}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank 0 | $\mathcal{T} \to \mathcal{C}$ | ✓ | | | | | | | | | |
| | $\mathcal{T}_{\|\|} \to \mathcal{C}_{rr}$ | | ✓ | | | | | | | | |
| | $\mathcal{V}_{\|\|} \to \mathcal{B}_r$ | | ✓ | ✓ | | | | | | | |
| | $\mathcal{S} \to \phi$ | | ✓ | ✓ | ✓ | | | | | | |
| Rank 1 | $\mathcal{T}^{\mathbf{E}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{E}}_r$ | | | | | ✓ | | | | | |
| | $\mathcal{V}^{\mathbf{E}}_{\pm} \to \mathcal{B}^{\mathbf{E}}$ | | | | | ✓ | ✓ | | | | |
| | $\mathcal{T}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{B}}_r$ | | | | | | | ✓ | | | |
| | $\mathcal{V}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{B}^{\mathbf{B}}$ | | | | | | | ✓ | ✓ | | |
| Rank 2 | $\mathcal{T}^{\mathbf{E}}_{\pm\pm} \to \mathcal{C}^{\mathbf{E}}$ | | | | | | | | | ✓ | |
| | $\mathcal{T}^{\mathbf{B}}_{\pm\pm} \to \mathcal{C}^{\mathbf{B}}$ | | | | | | | | | | ✓ |

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| | | $\nu$ | $N$ | $M$ | $L$ | $u$ | $v$ | $\hat{u}$ | $\hat{v}$ | $\mu$ | $\hat{\mu}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank 0 | $\mathcal{T} \to \mathcal{C}$ | ✓ | | | | | | | | | |
| | $\mathcal{T}_{\|\|} \to \mathcal{C}_{rr}$ | | ✓ | | | | | | | | |
| | $\mathcal{V}_{\|\|} \to \mathcal{B}_{r}$ | | ✓ | ✓ | | | | | | | |
| | $\mathcal{S} \to \phi$ | | ✓ | ✓ | ✓ | | | | | | |
| Rank 1 | $\mathcal{T}^{\mathbf{E}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{E}}_{r}$ | | | | | ✓ | | | | | |
| | $\mathcal{V}^{\mathbf{E}}_{\pm} \to \mathcal{B}^{\mathbf{E}}$ | | | | | ✓ | ✓ | | | | |
| | $\mathcal{T}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{B}}_{r}$ | | | | | | | ✓ | | | |
| | $\mathcal{V}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{B}^{\mathbf{B}}$ | | | | | | | ✓ | ✓ | | |
| Rank 2 | $\mathcal{T}^{\mathbf{E}}_{\pm\pm} \to \mathcal{C}^{\mathbf{E}}$ | | | | | | | | | ✓ | |
| | $\mathcal{T}^{\mathbf{B}}_{\pm\pm} \to \mathcal{C}^{\mathbf{B}}$ | | | | | | | | | | ✓ |

## Projected perturbations

Which provide the relation between standard and GLC perturbations

| | | $\nu$ | $N$ | $M$ | $L$ | $u$ | $v$ | $\hat{u}$ | $\hat{v}$ | $\mu$ | $\hat{\mu}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank 0 | $\mathcal{T} \to \mathcal{C}$ | ✓ | | | | | | | | | |
| | $\mathcal{T}_{\|\|} \to \mathcal{C}_{rr}$ | | ✓ | | | | | | | | |
| | $\mathcal{V}_{\|\|} \to \mathcal{B}_r$ | | ✓ | ✓ | | | | | | | |
| | $\mathcal{S} \to \phi$ | | ✓ | ✓ | ✓ | | | | | | |
| Rank 1 | $\mathcal{T}^{\mathbf{E}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{E}}_r$ | | | | | ✓ | | | | | |
| | $\mathcal{V}^{\mathbf{E}}_{\pm} \to \mathcal{B}^{\mathbf{E}}$ | | | | | ✓ | ✓ | | | | |
| | $\mathcal{T}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{B}}_r$ | | | | | | | ✓ | | | |
| | $\mathcal{V}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{B}^{\mathbf{B}}$ | | | | | | | ✓ | ✓ | | |
| Rank 2 | $\mathcal{T}^{\mathbf{E}}_{\pm\pm} \to \mathcal{C}^{\mathbf{E}}$ | | | | | | | | | ✓ | |
| | $\mathcal{T}^{\mathbf{B}}_{\pm\pm} \to \mathcal{C}^{\mathbf{B}}$ | | | | | | | | | | ✓ |

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Projected perturbations

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| | | $\nu$ | $N$ | $M$ | $L$ | $u$ | $v$ | $\hat{u}$ | $\hat{v}$ | $\mu$ | $\hat{\mu}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank 0 | $\mathcal{T} \to \mathcal{C}$ | ✓ | | | | | | | | | |
| | $\mathcal{T}_{\|\|} \to \mathcal{C}_{rr}$ | | ✓ | | | | | | | | |
| | $\mathcal{V}_{\|\|} \to \mathcal{B}_{r}$ | | ✓ | ✓ | | | | | | | |
| | $\mathcal{S} \to \phi$ | | ✓ | ✓ | ✓ | | | | | | |
| Rank 1 | $\mathcal{T}^{\mathbf{E}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{E}}_{r}$ | | | | | ✓ | | | | | |
| | $\mathcal{V}^{\mathbf{E}}_{\pm} \to \mathcal{B}^{\mathbf{E}}$ | | | | | ✓ | ✓ | | | | |
| | $\mathcal{T}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{C}^{\mathbf{B}}_{r}$ | | | | | | | ✓ | | | |
| | $\mathcal{V}^{\mathbf{B}}_{\|\|\pm} \to \mathcal{B}^{\mathbf{B}}$ | | | | | | | ✓ | ✓ | | |
| Rank 2 | $\mathcal{T}^{\mathbf{E}}_{\pm\pm} \to \mathcal{C}^{\mathbf{E}}$ | | | | | | | | | ✓ | |
| | $\mathcal{T}^{\mathbf{B}}_{\pm\pm} \to \mathcal{C}^{\mathbf{B}}$ | | | | | | | | | | ✓ |

Tensor Modes (Spin-2)

- With the helicity decomposition, we may project the tensor modes

$$h_\pm \equiv s^a_\pm s^b_\pm h_{ab} = s^a_\pm s^b_\pm \left( \frac{\delta\gamma_{ab}}{2} - \nabla_{(a}\chi_{b)} \right)$$

$$h_\pm = \frac{r^2}{2} \eth^2_\pm \left(\mu \mp i\hat{\mu}\right) + \frac{1}{\sqrt{2}} \eth_\pm \chi_\pm$$

## Tensor Modes (Spin-2)

- With the helicity decomposition, we may project the tensor modes

$$\tilde{\chi}_a = \chi_a - \epsilon_a = \chi_a - r^2 \left( D_a \chi + \tilde{D}_a \hat{\chi} \right)$$

$$\tilde{\mu} = \mu - \chi$$

$$\tilde{\hat{\mu}} = \hat{\mu} - \hat{\chi}$$

$$\tilde{h}_\pm = \frac{r^2}{2} \eth_\pm^2 \left[ \mu - \chi \mp i \left( \hat{\mu} - \hat{\chi} \right) \right] + \frac{1}{\sqrt{2}} \eth_\pm \chi_\pm + \frac{r^2}{2} \eth_\pm^2 \left( \chi \mp i \hat{\chi} \right) = h_\pm$$

Tensor Modes (Spin-2)

- The advantage is that the **B**-modes are simple

$$h^{\mathbf{B}} \equiv -\frac{i}{2}\left(\bar{\eth}^2 h_{++} - \eth^2 h_{--}\right) = \left[D^2\left(D^2+2\right)\right] r^2 \hat{\mu} - \underbrace{\frac{1}{\Delta_3}\left[f\left(\hat{u},\hat{\mu}\right)\right]}_{\eth_{\pm} F^{\mathbf{B}}_{\pm}}$$

- In the early-universe fixing the GLC gauge and neglecting vector modes

$$h^{\mathbf{B}} \sim \left[D^2\left(D^2+2\right)\right] r^2 \hat{\mu} + \ldots$$

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## Tensor Modes (Spin-2)

- In the UCG

$$\tilde{\chi}_a^{UCG} = \chi_a - \epsilon_a = \chi_a - r^2 \left( D_a \chi + \tilde{D}_a \hat{\chi} \right) = 0$$

$$\frac{1}{\sqrt{2}} \eth_\pm \tilde{\chi}_\pm |_{UCG} = \frac{1}{\sqrt{2}} \eth_\pm \chi_\pm + \frac{r^2}{2} \eth_\pm^2 \left( \chi \mp i \hat{\chi} \right) = 0$$

$$\tilde{h}_\pm = \frac{r^2}{2} \eth_\pm^2 \left[ \mu \mp i \hat{\mu} \right]_{UCG}$$

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## General perspectives

- The gauge invariance is a key feature in cosmological observables.
- Although G.R. is a coordinate independent theory, perturbation theory is not a covariant procedure and introduce spurious coordinate dependence.
- The GLC gauge offers a covariant procedure to cosmological observables.
- The LC perturbations are compatible offers a recipe to gauge invariant cosmological observables.
- We provided the expression for the gauge invariant tensor modes in terms of LC perturbations.
- We also saw that the SPS decomposition offers an easy interpretation of E and B modes.

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# Thank you!!!



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