Fermion soliton stars

arXiv:2301.08709

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Outline of the presentation

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- Brief introduction to self-gravitating solitons

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- How to evade no-go theorems: **fermion soliton stars**

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- **Astrophysical implications**
- Self-gravitating solitons in the **Standard Model**?

What is a (gravitational) soliton?

Solution of the equations of motion

- Regular
- Asymptotically flat
- Finite energy
- Static

Gravitational solitons in **vacuum** GR?

$$S = \int d^4x \sqrt{-g}\frac{R}{16\pi G}$$

- The answer is **NO**
- The only globally regular, asymptotically flat, static vacuum solution of Einstein's equations with finite energy is **Minkowski spacetime** [Einstein, Pauli, 1943]

Einstein-Klein-Gordon theory?

$$S = \int d^4x\,\sqrt{-g}\Big(\frac{R}{16\pi G} - \frac{1}{2}\partial^\mu\phi^*\partial_\mu\phi - U(|\phi|^2)\Big)$$

- **Complex** scalar field
- The answer is **YES** ⟶ **BOSON STARS**

[Kaup, 1968; Ruffini, Bonazzola, 1969; Colpi, et. al., 1986]

What if the scalar field is real?

No-go theorems **prevent the existence** of solitonic solutions for very generic classes of scalar potential [Derrick, 1964; Herdeiro, Oliveira, 2019]

What to do?

Let’s add more ingredients…

$$S = \int d^4x \sqrt{-g} \Big(\frac{R}{16\pi G} - \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \underbrace{\frac{1}{2}\mu^2\phi^2 \Big(1 - \frac{\phi}{\phi_0}\Big)^2}_{= U(\phi)} +$$

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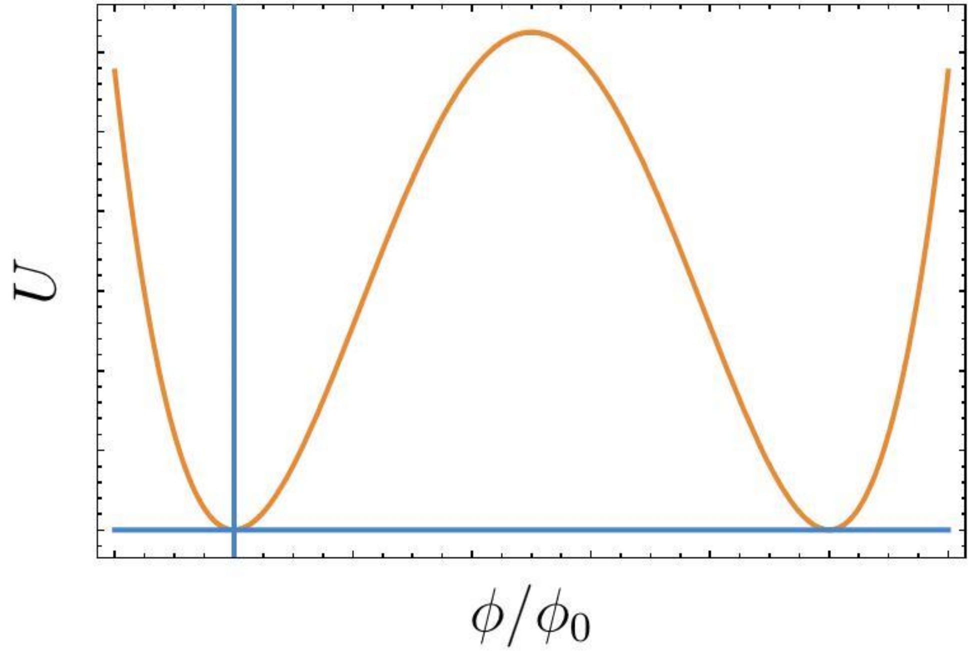

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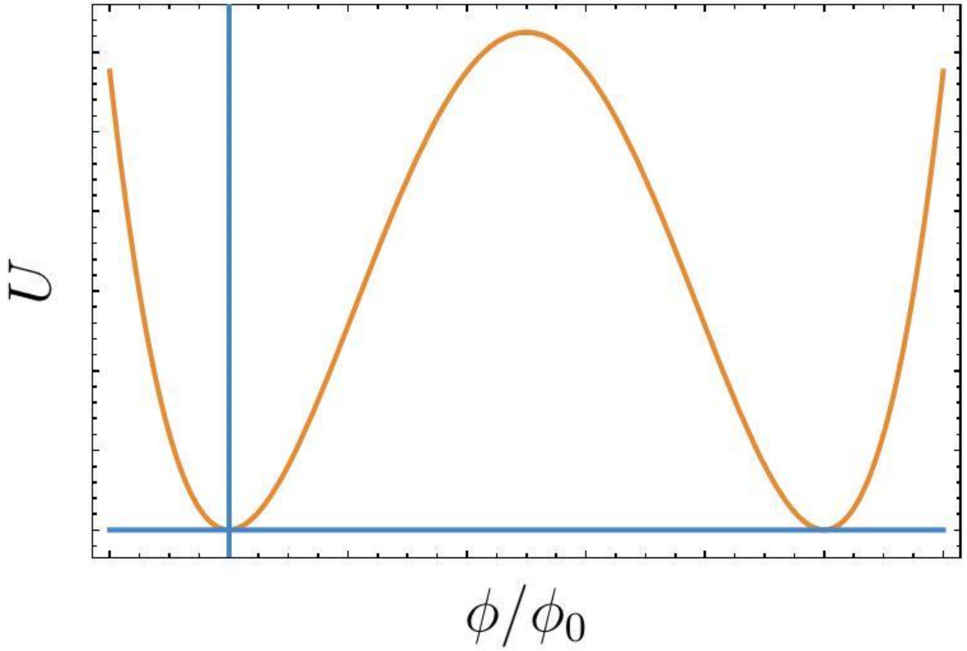

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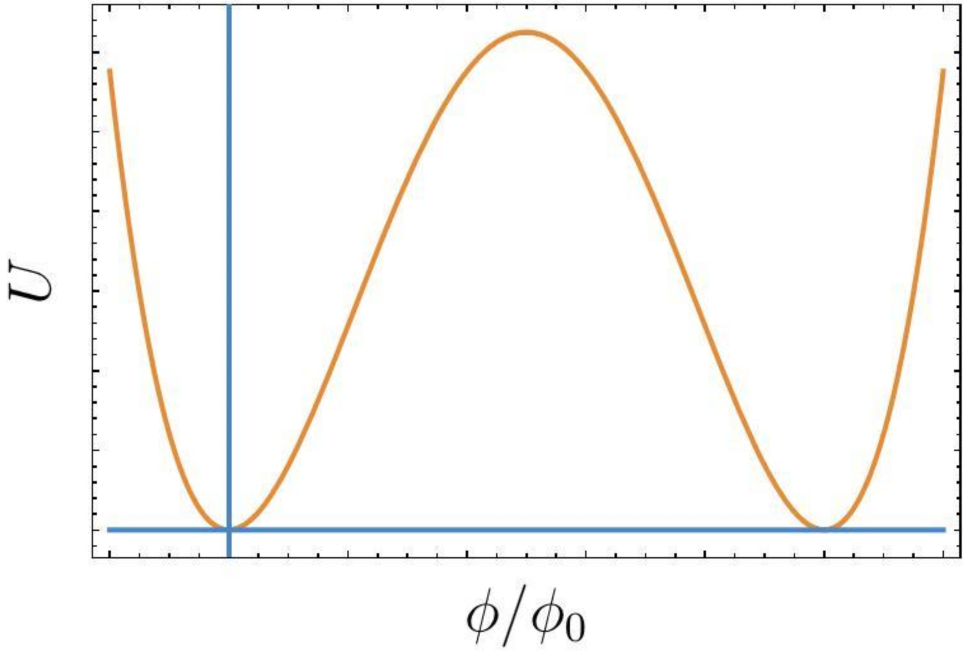
$$m_{\mathrm{eff}} = m_f - f\phi$$



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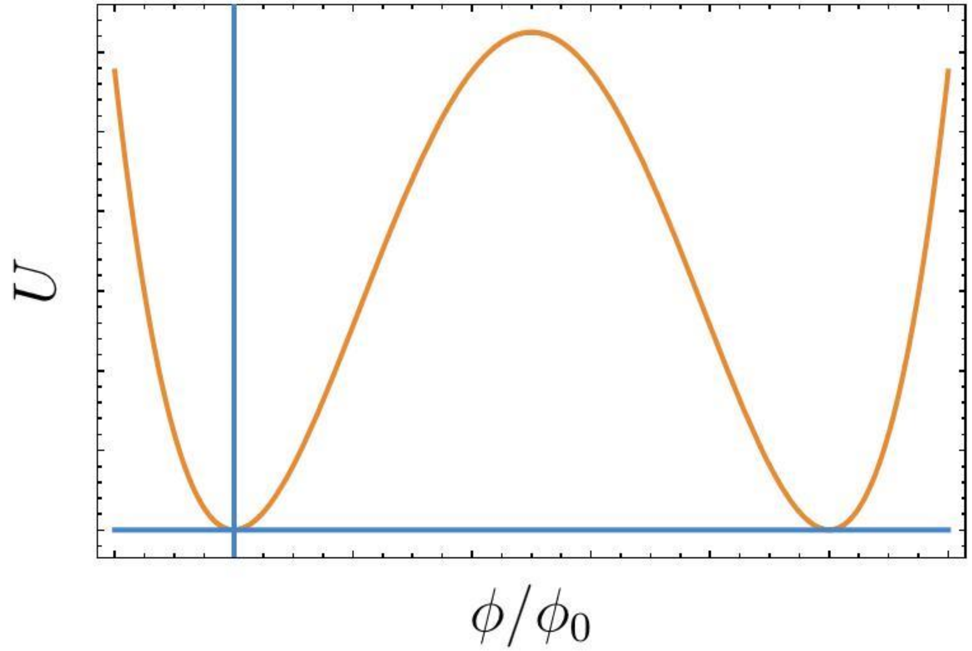
$$\frac{1}{2}\mu^2\phi^2\Big(1-\frac{\phi}{\phi_0}\Big)^2 = U(\phi)$$

$$m_{\text{eff}} = m_f - f\phi = m_f\Big(1-\frac{\phi}{\phi_0}\Big)$$

$$f = \frac{m_f}{\phi_0}$$



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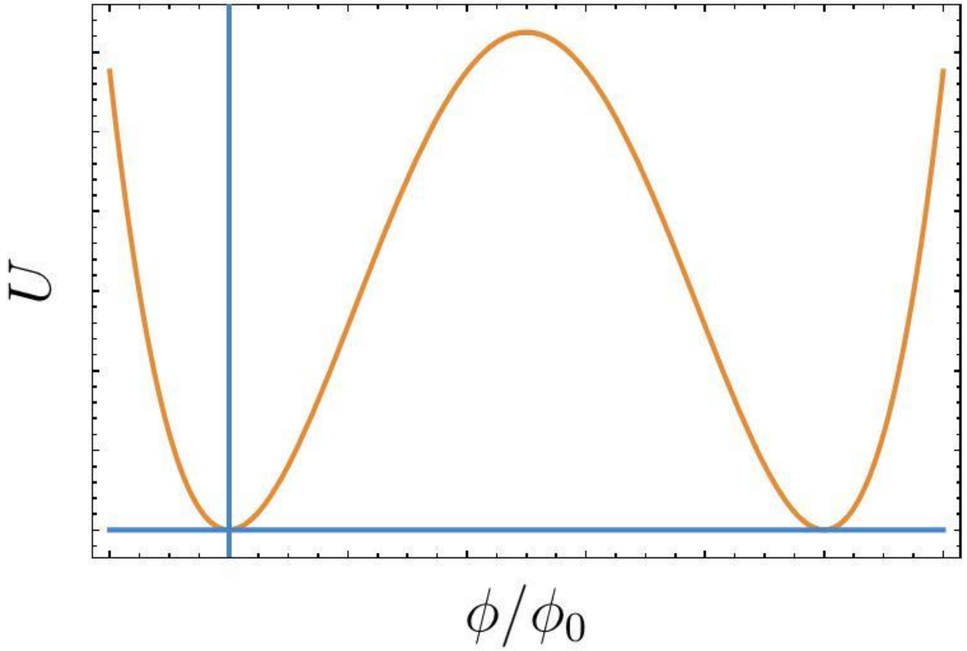
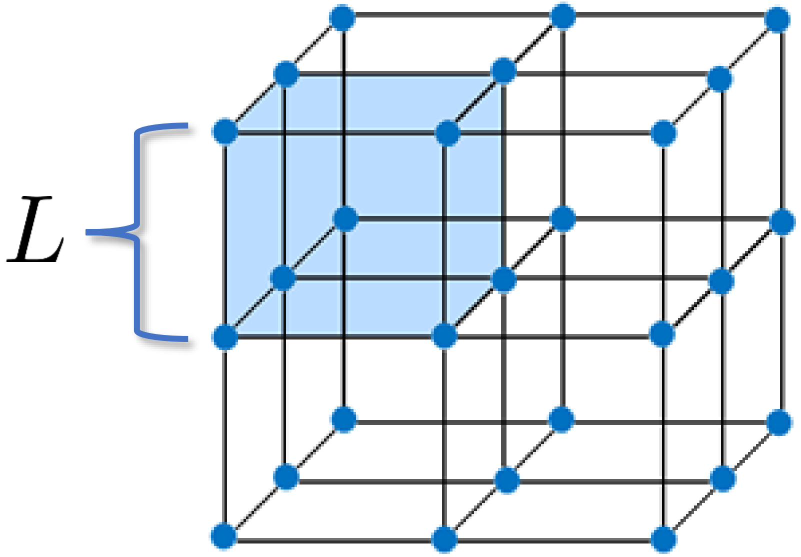
[Lee, Pang, 1987]



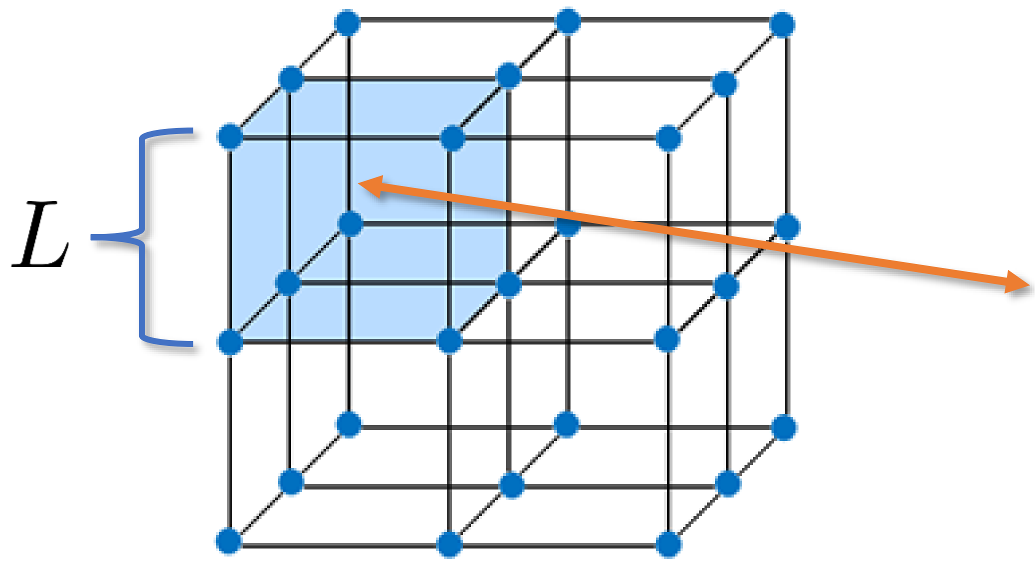
Thomas-Fermi approximation

- Devide the three-space into small cubes $L_{g_{\mu\nu},\phi} \gg L \gg \frac{1}{m_f}$



Thomas-Fermi approximation

- Devide the three-space into small cubes $L_{g_{\mu\nu},\phi} \gg L \gg \frac{1}{m_f}$
- Fill each cube with a degenerate Fermi gas of Fermi momentum k_F



- $W[k_F]$ (Energy density)
- $P[k_F]$ (Pressure)
- $S[k_F] = <\bar{\psi}\psi>$ (Scalar density)

Equations of motion

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$$= T_{\mu\nu}^{[\phi]}$$

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3. $$F(k_F, g_{\mu\nu}, \phi) = 0$$

We are looking for solitons…

- Static and spherically symmetric background

$$ds^2 = -e^{2u(\rho)}dt^2 + e^{2v(\rho)}d\rho^2 + \rho^2 d\Omega^2$$

- Scalar field in the ground state

$$\phi = \phi(\rho)$$

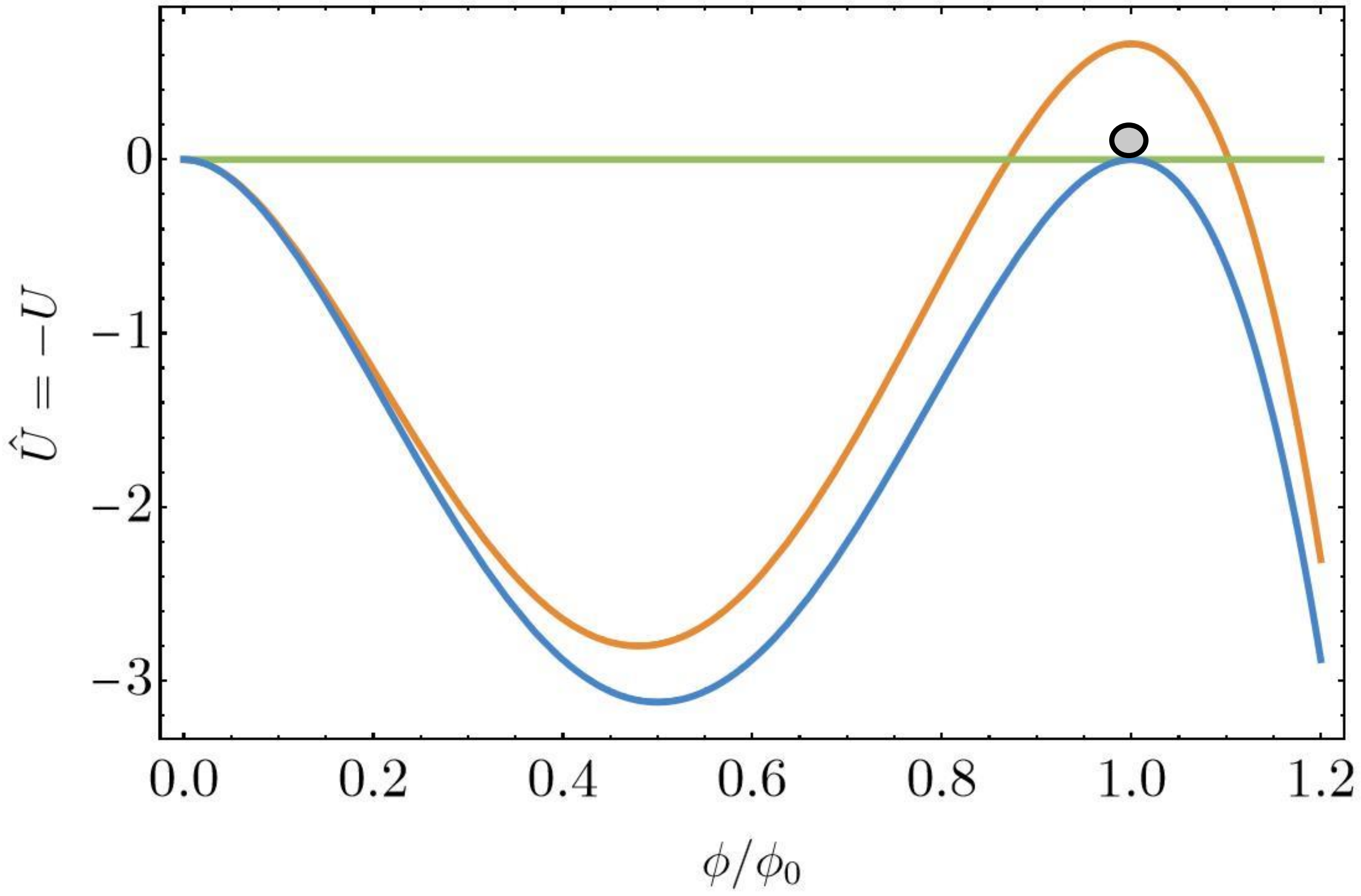
Physical picture

$$\Box\phi = \frac{\partial U}{\partial \phi} - fS$$

- Work in flat spacetime
- $\rho \to t, \quad \hat{U} := -U$

$$\partial_t^2 \phi = -\frac{2}{t}\partial_t \phi - \frac{\partial \hat{U}}{\partial \phi} - fS$$

1D motion in the inverted potential



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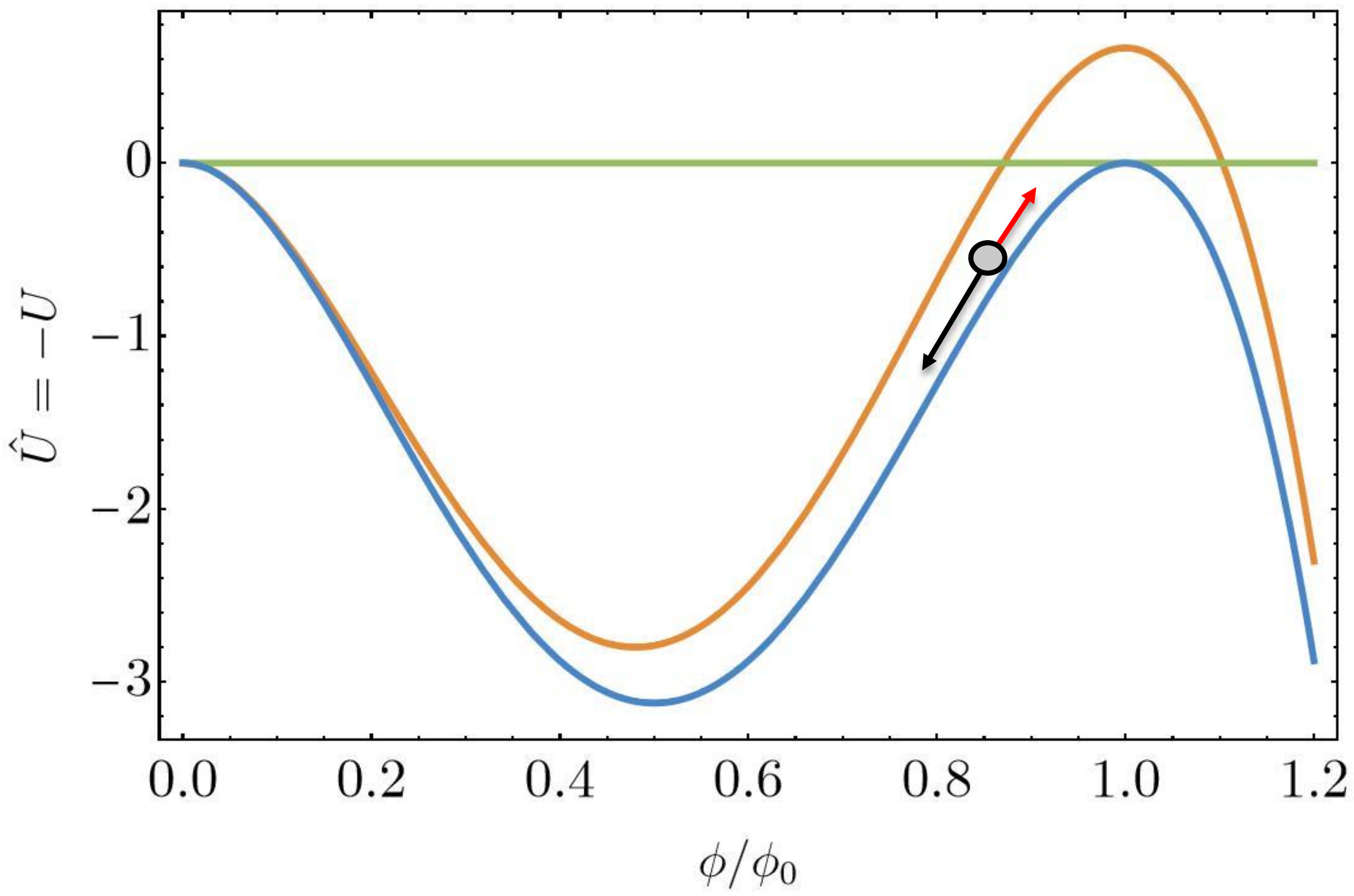

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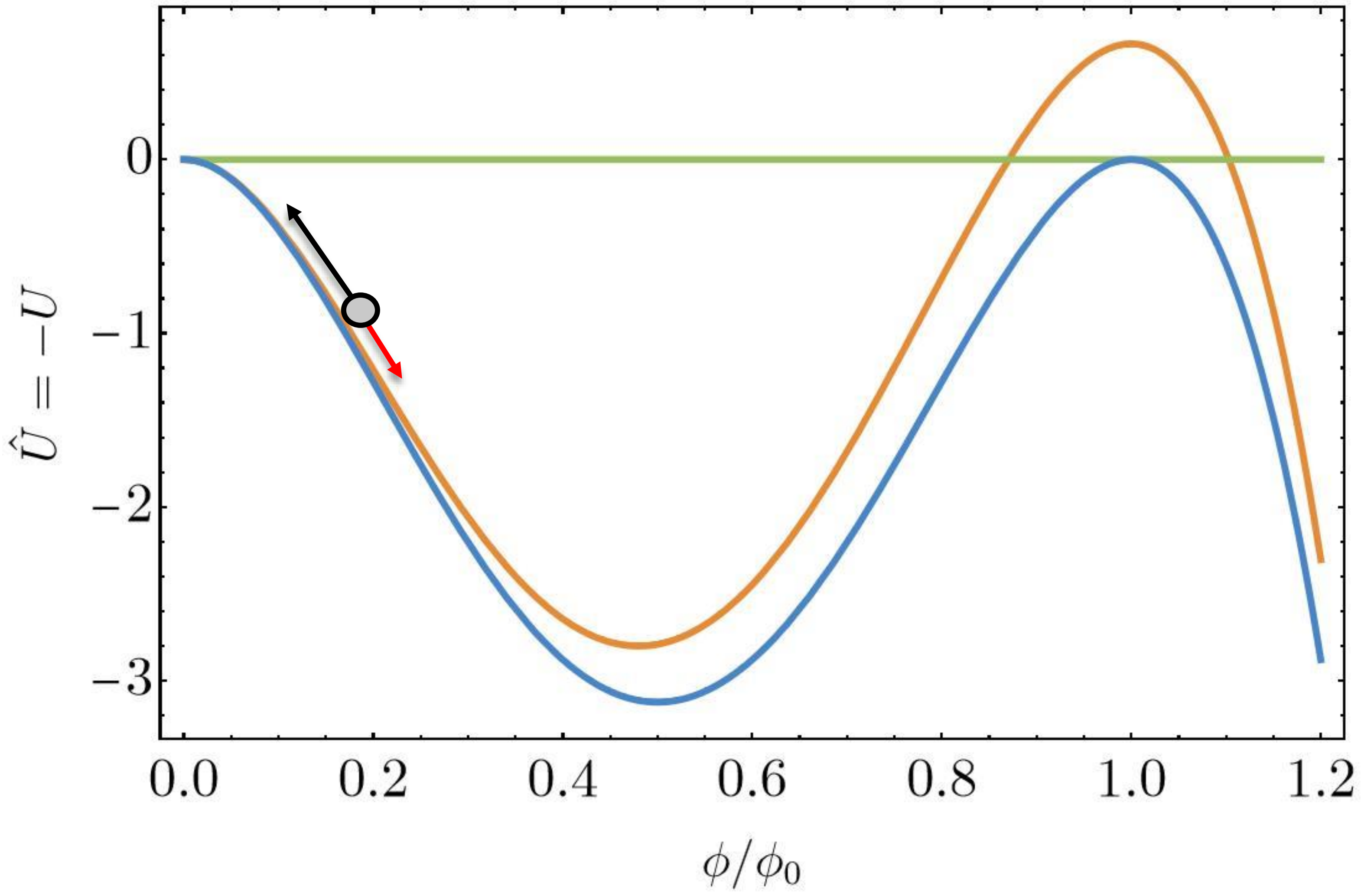

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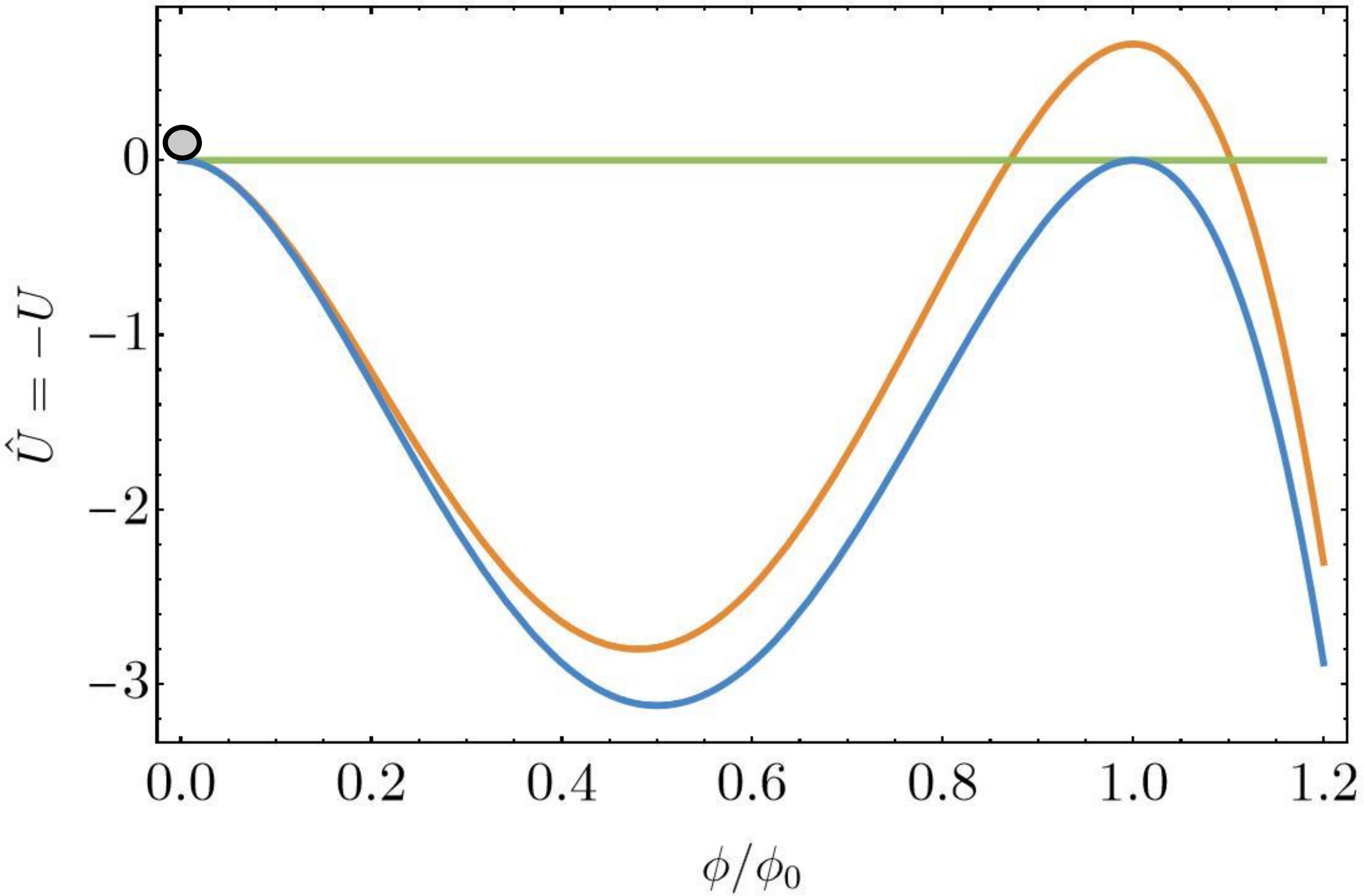
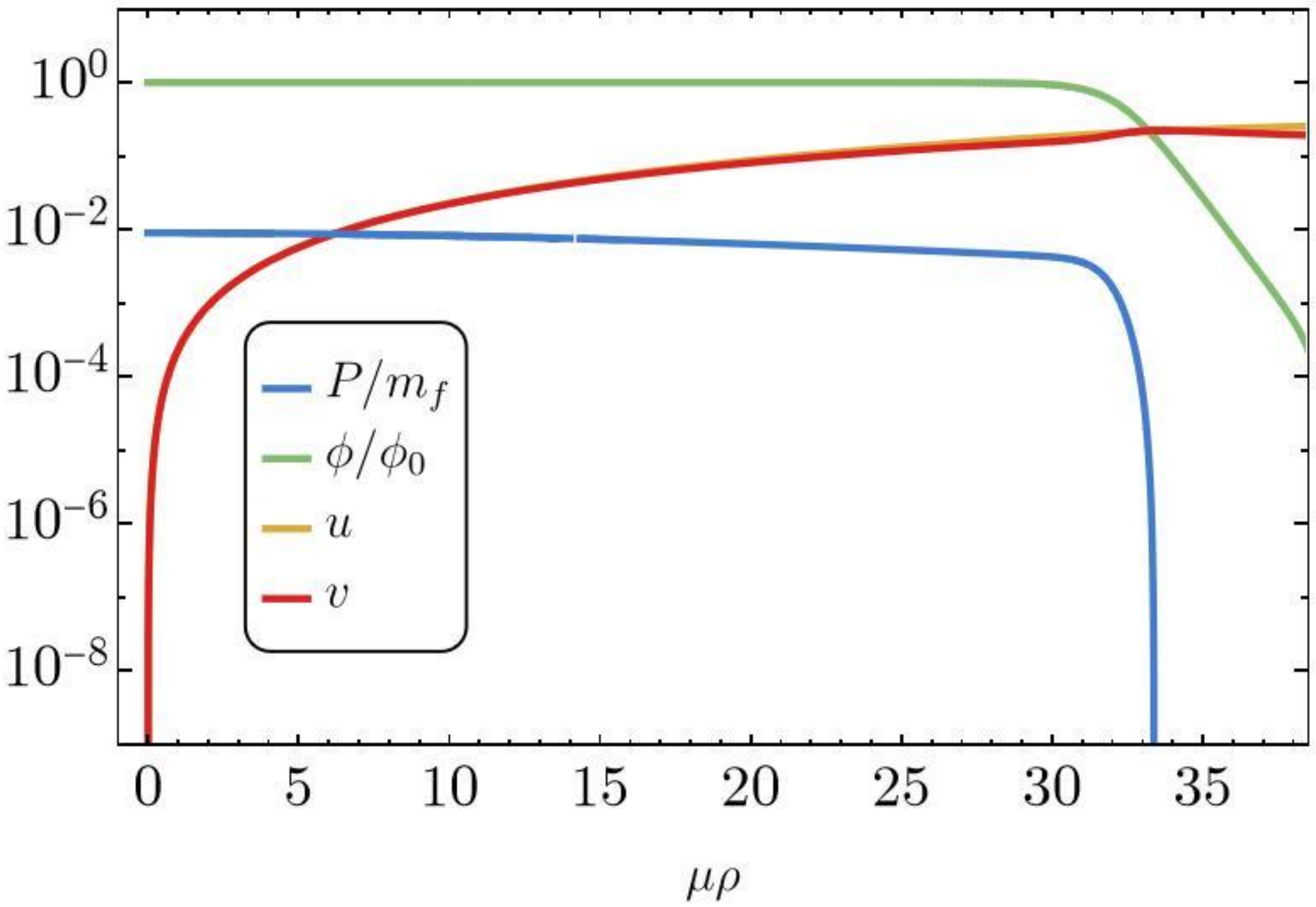


Example of solution



Scaling in the $\mu R \gg 1$ regime

- Fundamental parameters

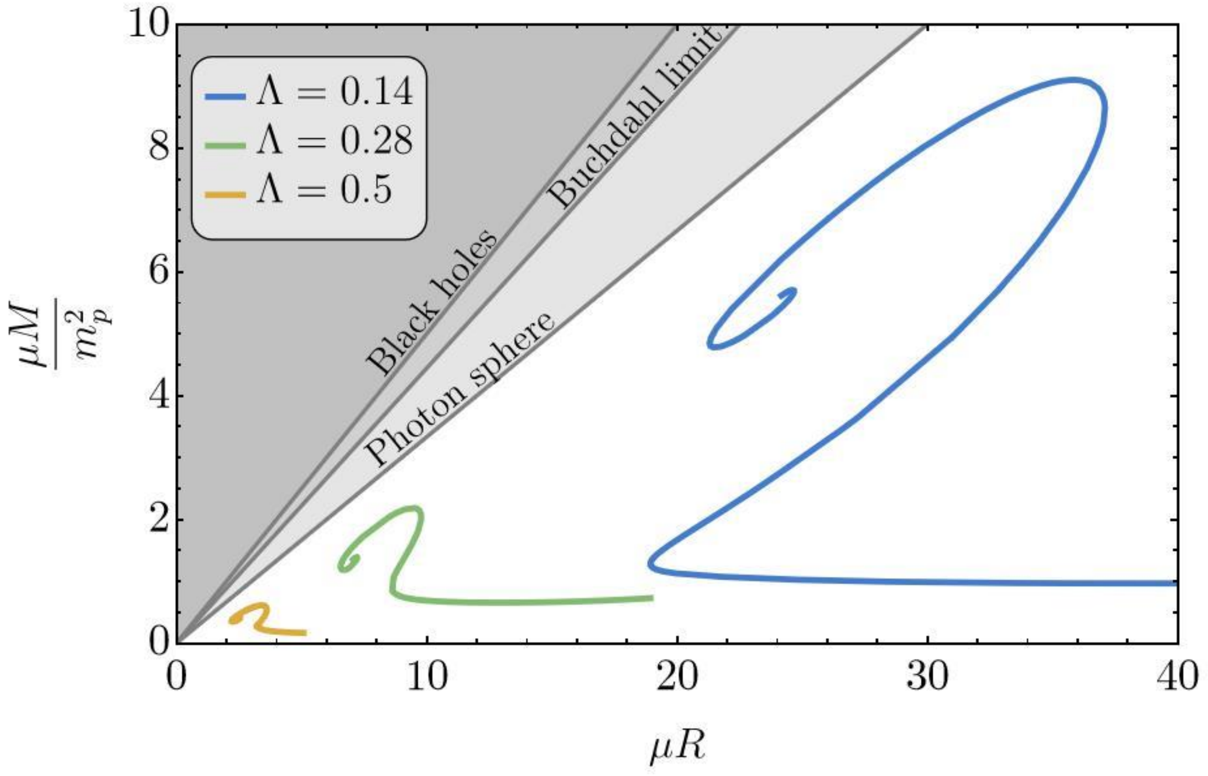
$$\Lambda = \frac{\sqrt{8\pi}\phi_0}{m_p} \qquad \eta = \frac{m_f}{\mu^{1/2}\phi_0^{1/2}}$$

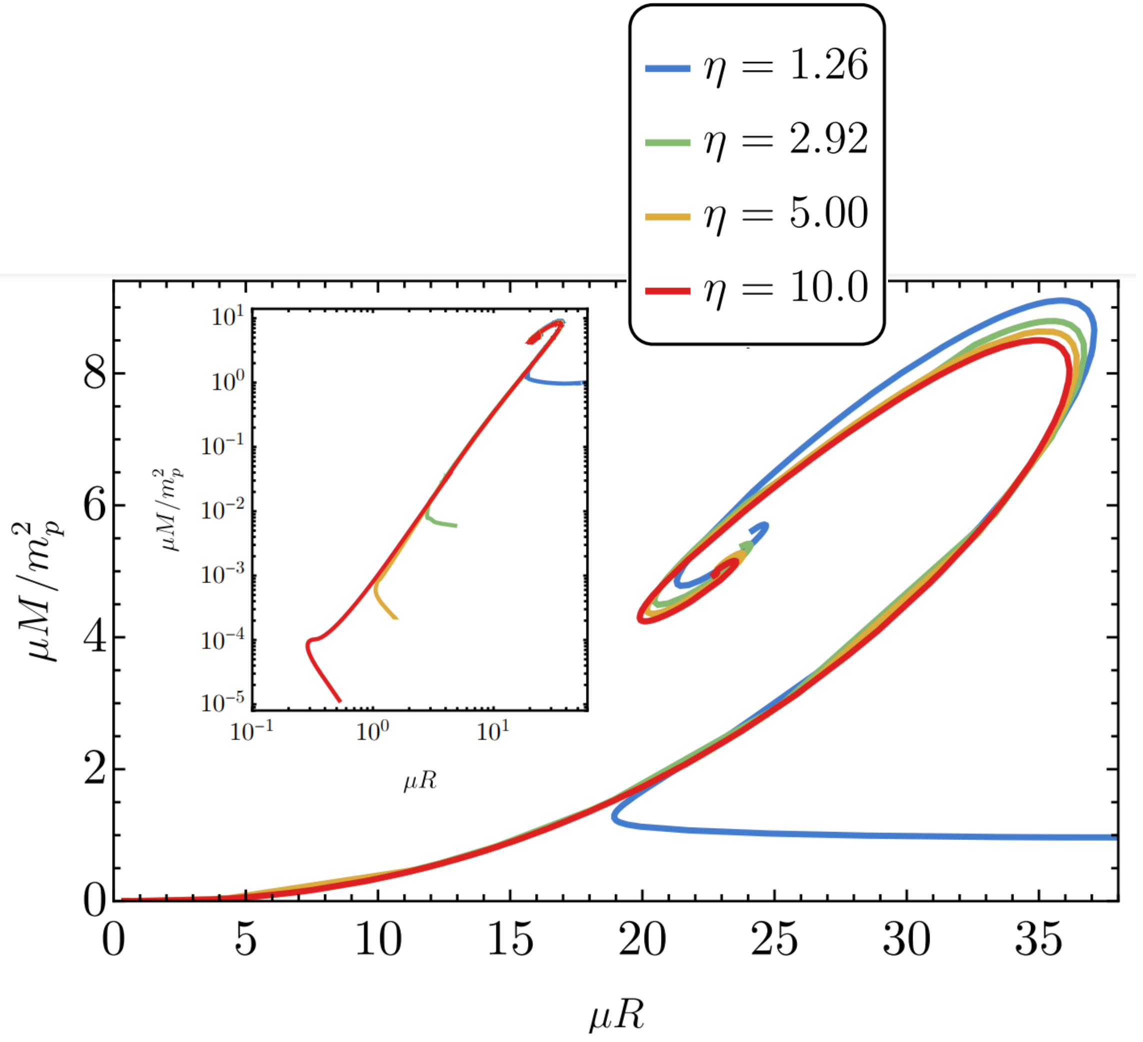
- Critical scaling

$$\frac{\mu M_c}{m_p^2} \approx \frac{0.19}{\Lambda^2} \qquad \mu R_c \approx \frac{0.71}{\Lambda^2} \qquad C_c \approx 0.27$$

Independent of η

Mass-radius diagrams





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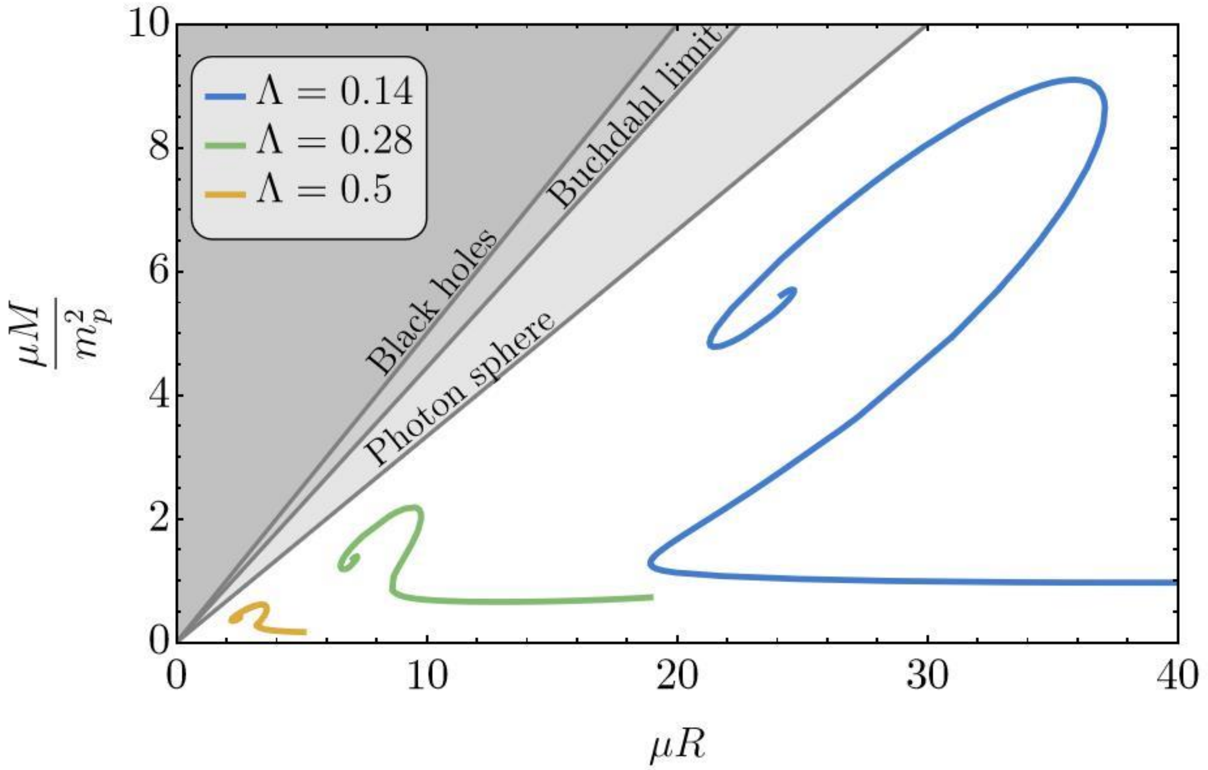
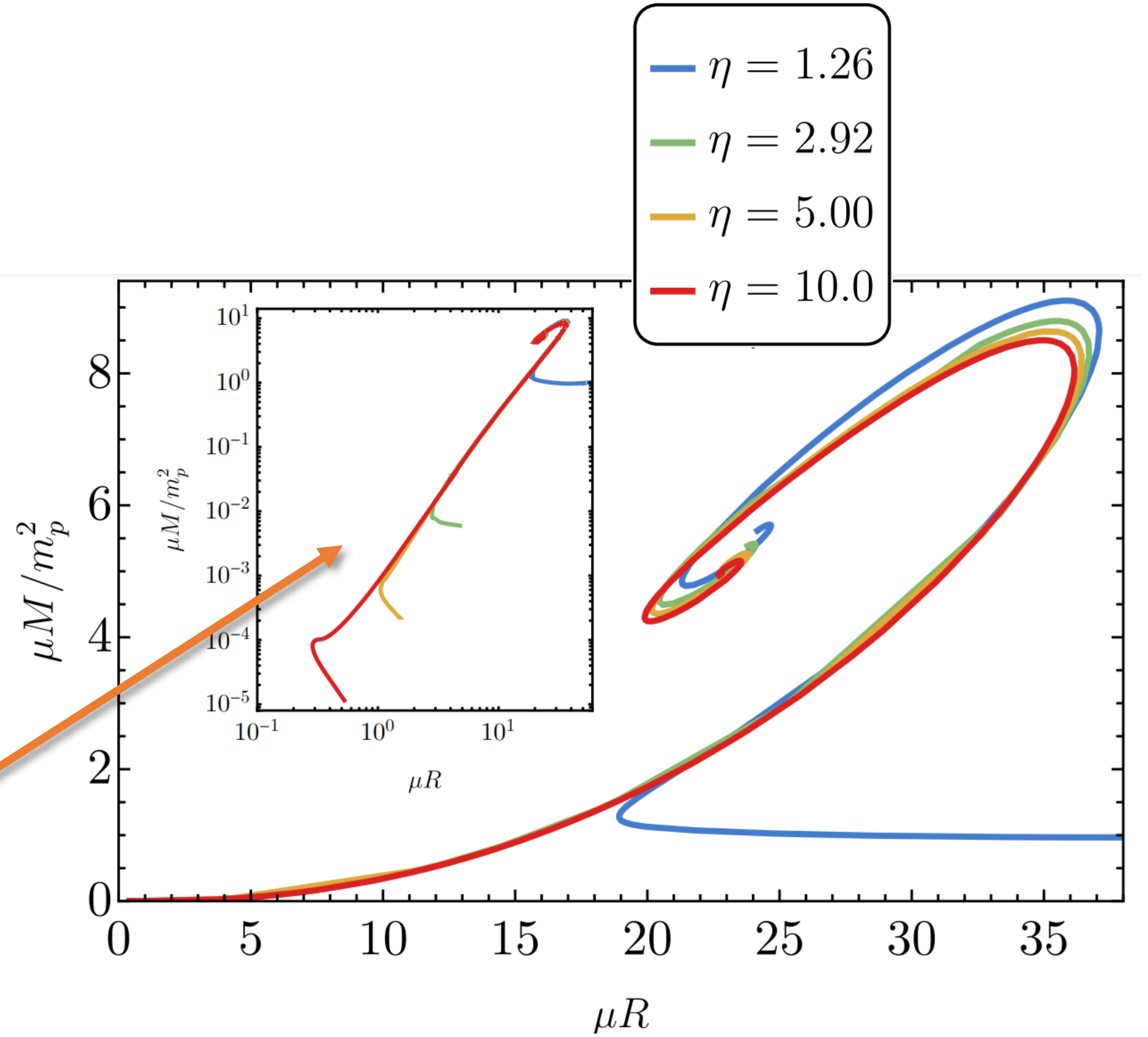
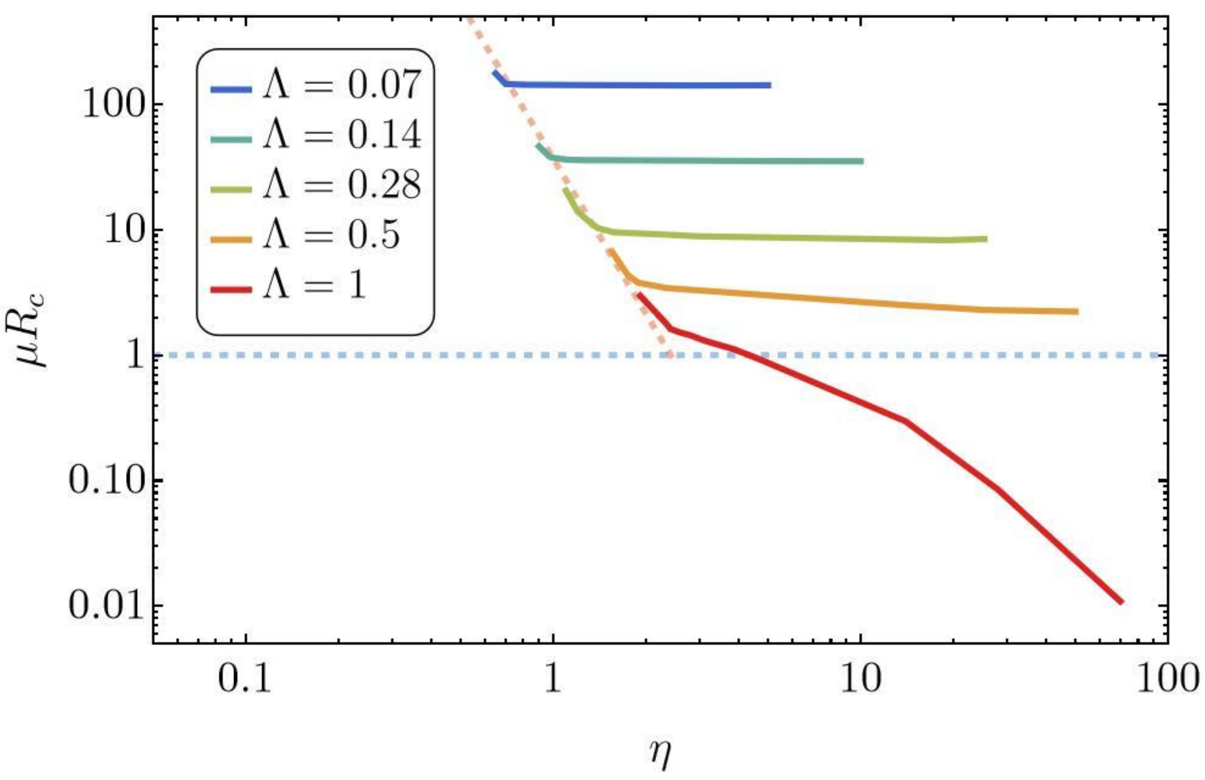
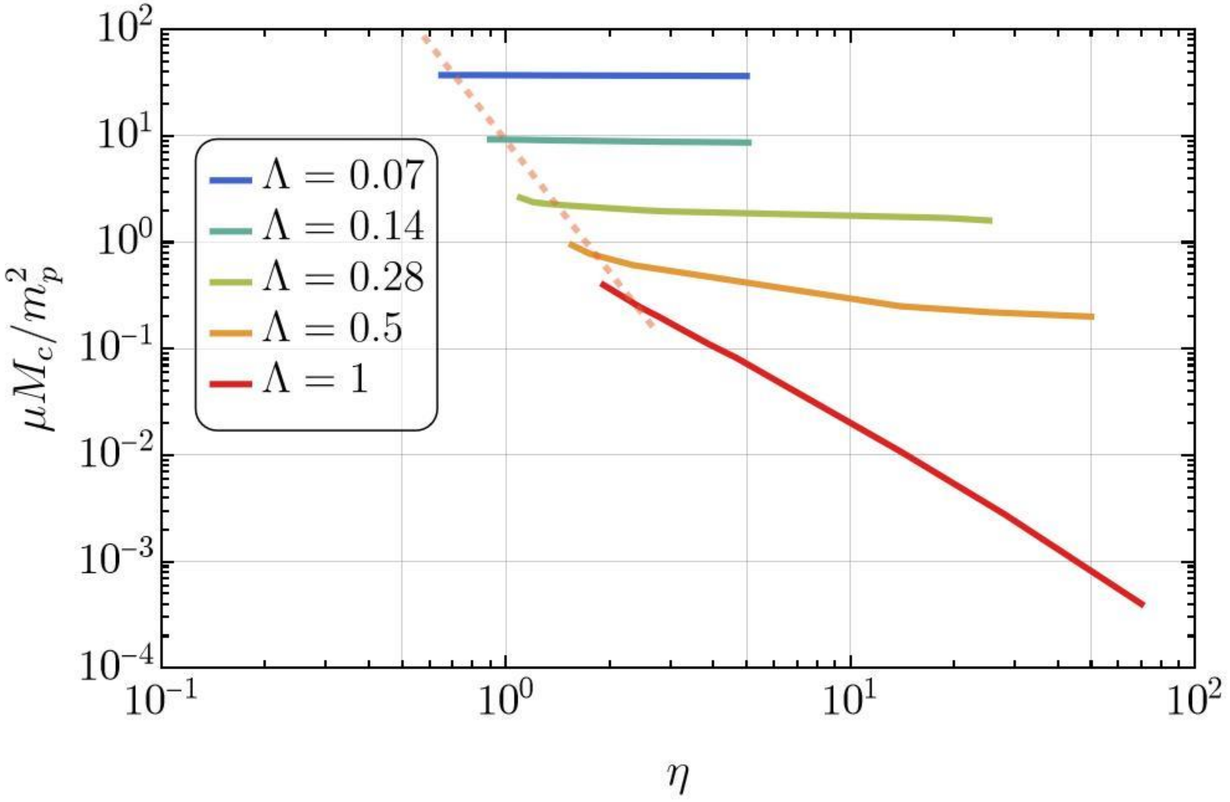




Newtonian regime not always present

Critical radius and mass





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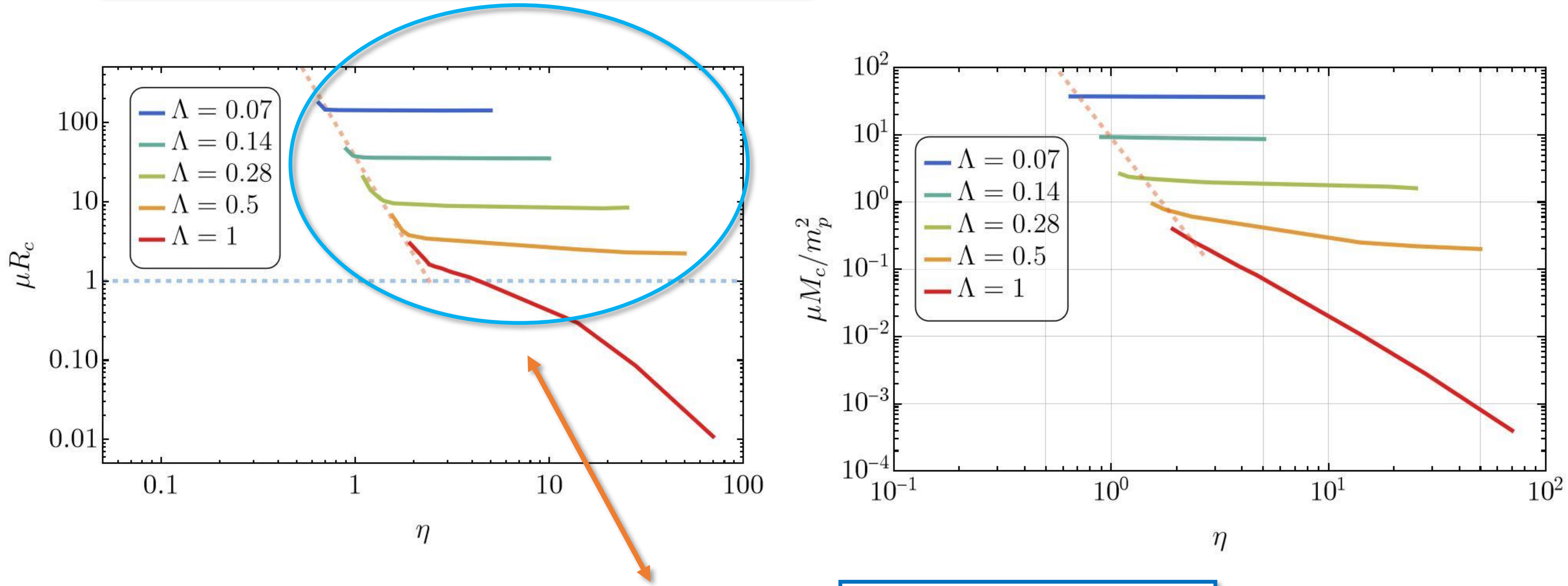

Confining regime $\eta > \eta_c \approx 2.7\,\Lambda^{1/2}$

Astrophysical implications

$$M_c \sim \frac{0.19}{8\pi}\frac{m_p^4}{q^3} \sim 1.27\,M_\odot \left(\frac{q}{5\times 10^5\,\mathrm{GeV}}\right)^{-3}$$

$$R_c \sim \frac{0.71}{8\pi}\frac{m_p^2}{q^3} \sim 6.5\,\mathrm{km} \left(\frac{q}{5\times 10^5\,\mathrm{GeV}}\right)^{-3}$$

$$q \equiv (\mu\phi_0^2)^{1/3}$$

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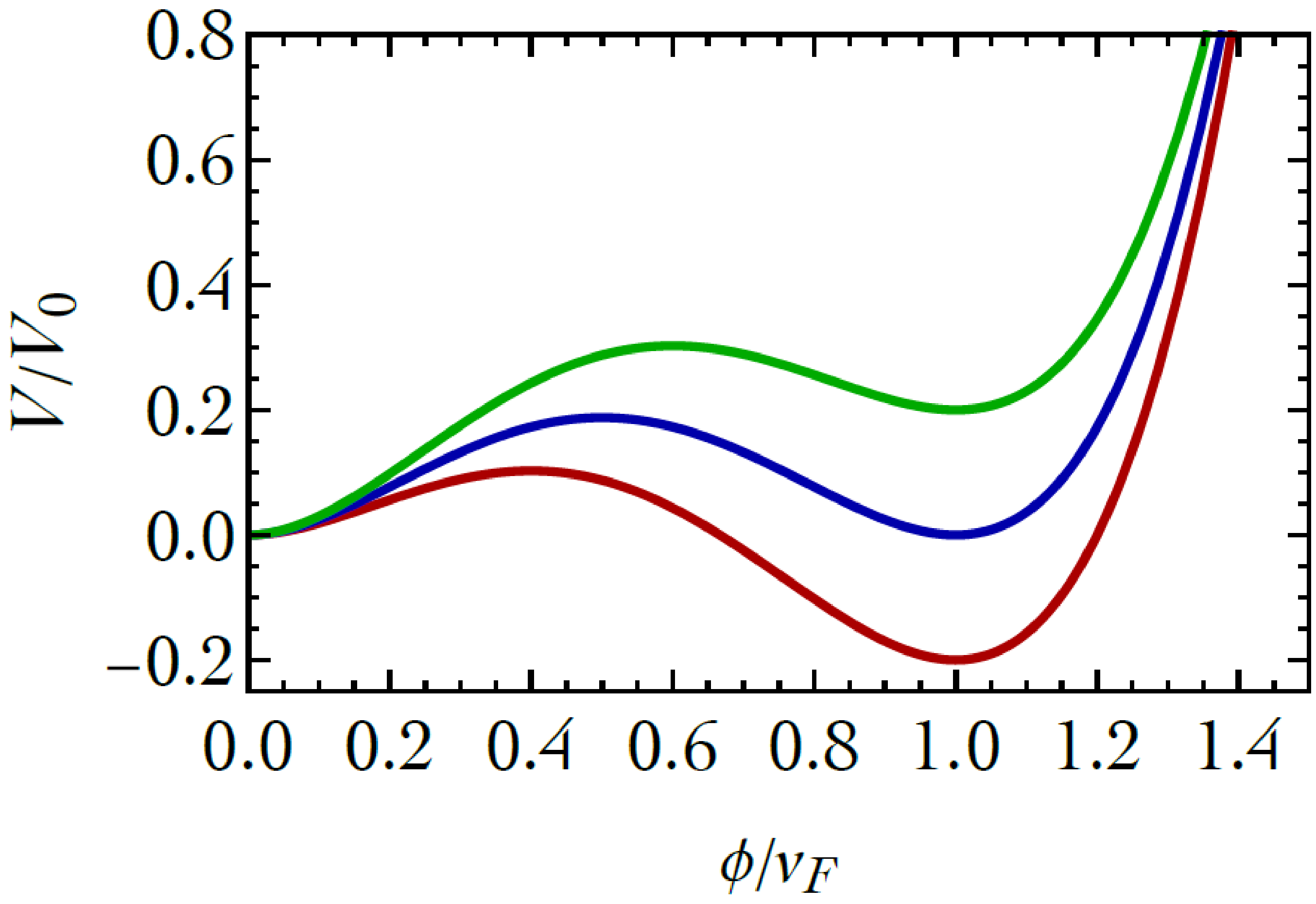
$$m_f > m_f^c \equiv 2.7\left(\frac{\sqrt{8\pi}q^3}{m_p}\right)^{1/2} \sim 0.6\,\mathrm{GeV}\left(\frac{q}{5\times 10^5\,\mathrm{GeV}}\right)^{3/2}$$

$$\mu \gtrsim 8.4\times 10^{-11}\left(\frac{q}{5\times 10^5\,\mathrm{GeV}}\right)^{3}\,\mathrm{eV}$$

<u>Solar-mass soliton stars with a degenerate gas of neutrons</u>

Gravitational solitons in the standard model?



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- Fermion soliton stars exist beyond the thin-shell approximation and are a way to evade no-go theorems
- Astrophysical compact objects can be built also with a standard degenerate gas of neutrons
- Hint at the existence of classical gravitational solitons in the standard model

Conclusions

Thank you!

Gravitational soliton in **electro-vacuum** GR?

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

- Wheeler's influential idea of **geons** [Wheeler, 1955]
- The answer is again **NO** [Heusler, 1996]

Fermionic quantities

$$W = \frac{2}{(2\pi)^3} \int_0^{k_{\rm F}(\rho)} d^3k \sqrt{k^2 + (m_f - f\phi(\rho))^2} \tag{1}$$

$$P = \frac{2}{(2\pi)^3} \int_0^{k_{\rm F}(\rho)} \frac{d^3k\, k^2}{3\sqrt{k^2 + (m_f - f\phi(\rho))^2}} \tag{2}$$

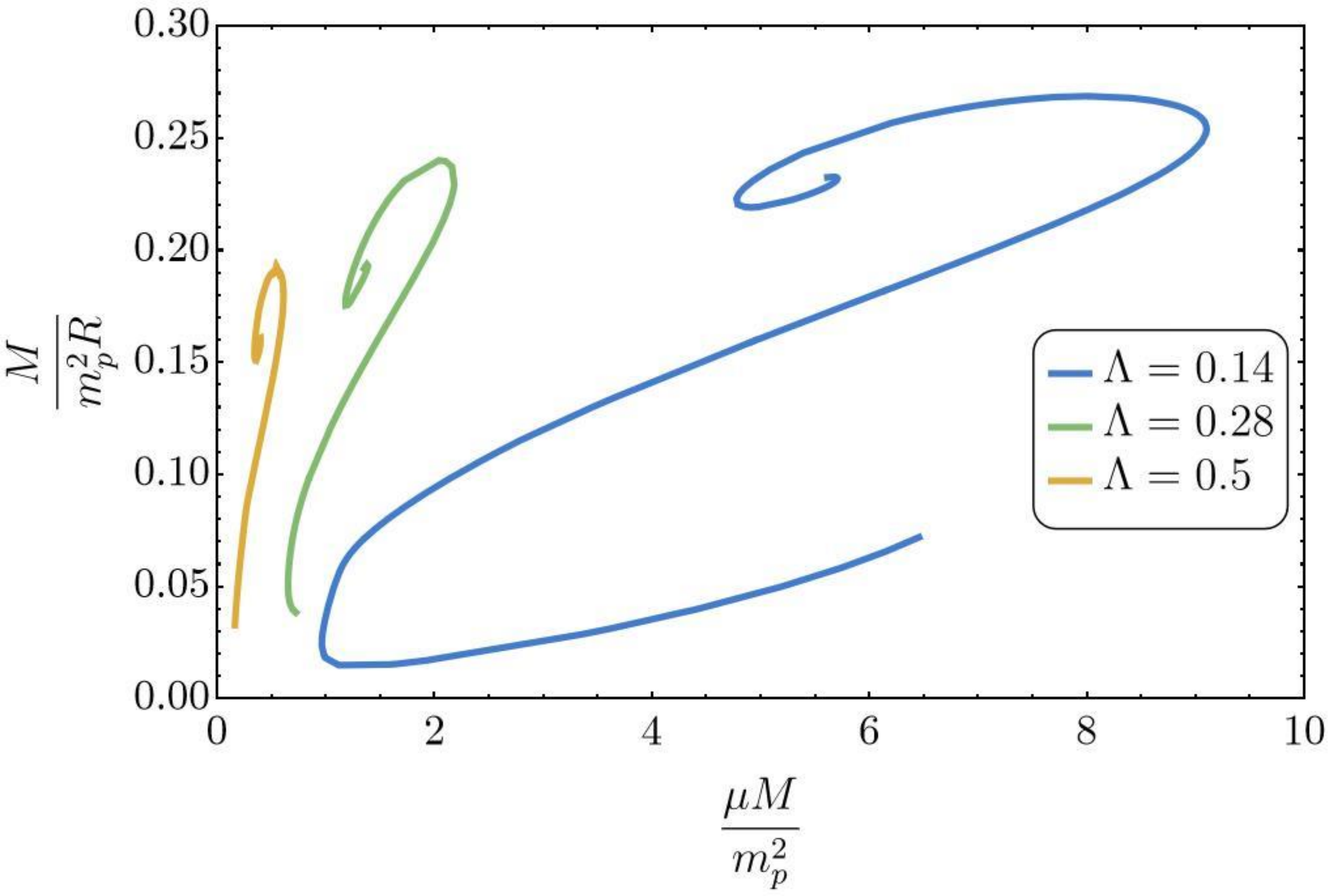
$$S = \frac{2}{(2\pi)^3} \int_0^{k_{\rm F}(\rho)} d^3k \frac{m_f - f\phi(\rho)}{\sqrt{k^2 + (m_f - f\phi(\rho))^2}} \tag{3}$$

Fermi momentum equation

- Enforce a constant number of fermions N
- Introduce the Lagrangian multiplier ω_F
- Minimize the fermion total energy at fixed N

$$k_F^2(\rho) = \omega_F^2 e^{-2u(\rho)} - m_{\text{eff}}^2$$

Compactness-mass diagram



Astrophysical implications

- $\Lambda \lesssim 0.5 \Rightarrow$

$$\phi_0 \lesssim \frac{0.5}{\sqrt{8\pi}} m_p \sim 10^{18}\,\mathrm{GeV}$$

$$\mu \gtrsim 8.4 \times 10^{-11} \left(\frac{q}{5 \times 10^5\,\mathrm{GeV}} \right)^3 \mathrm{eV}$$