# Does thermodynamics rely on Lorentz invariance?

Francesco Del Porro

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ArXiv:23xx.xxxx

In collaboration with:
S. Liberati, M. Herrero-Valea and M. Schneider

# Outline

- Lorentz Violating gravity: why and how
- Black Holes in LV gravity
- Hawking radiation in LV
- Conclusion

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# Lorentz Violating gravity

- Violating Local Lorentz Invariance is a way to build a power counting renormalizable theory of gravity

Maybe fully renormalizable:
J. Bellorin, C. Borquez, B.Droguet, ArXiv:2207.08938

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$$\vec{x} \to b\vec{x}, \quad \tau \to b^z \tau$$

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$z = 3$
arXiv:0901.3775, P.Horava , 2009

# Lorentz Violating gravity

- Local Lorentz Invariance is broken by having a notion of absolute time: $\mathscr{M} \simeq \mathbb{R} \times \Sigma$

$$FDiff \subsetneq Diff$$

- One can introduce a Stueckelberg vector field, the Aether, that parametrizes the time direction:

$$U_\mu = \frac{\partial_\mu \tau}{\sqrt{g^{\alpha\beta}\partial_\alpha \tau \partial_\beta \tau}} \qquad S[g,\tau] = -\frac{1}{16\pi G}\int_{\mathscr{M}} \sqrt{-g}\,\left(R + c_\theta \theta^2 + c_\omega \omega_{\mu\nu}\omega^{\mu\nu} + c_\alpha a_\mu a^\mu\right)$$

# Matter Fields

- The theory allows the presence of higher (spatial) derivative operators:

$$S_m[\phi] = \frac{1}{2}\int_{\mathscr{M}} \sqrt{-g}\ \phi\left[\nabla_\mu \nabla^\mu - \sum_{j=2}^{n} \frac{\alpha_{2j}}{\Lambda^{2j-2}}(\Delta)^j\right]\phi \qquad \Delta = \nabla_\mu \gamma^{\mu\nu} \nabla_\nu$$

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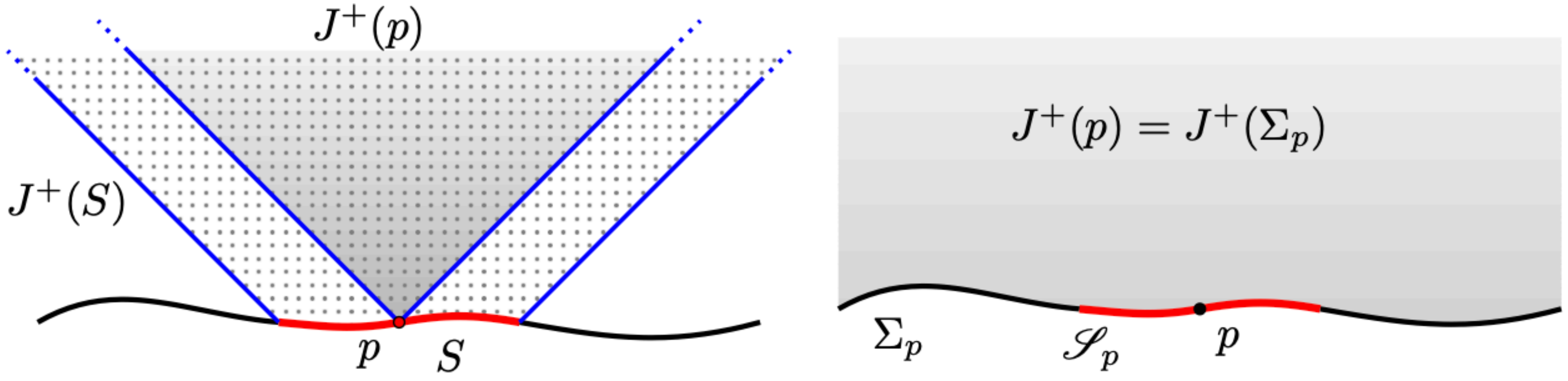
Superluminal particles:

$$\omega^2 = q^2 + \alpha_4 \frac{q^4}{\Lambda^2} + \ldots + \alpha_{2n}\frac{q^{2n}}{\Lambda^{2(n-1)}}$$

Different notion of causality:




J.Bhattacharyya, M.Colombo, T.P. Sotiriou, ArXiv: 1509.01558

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# Horizons

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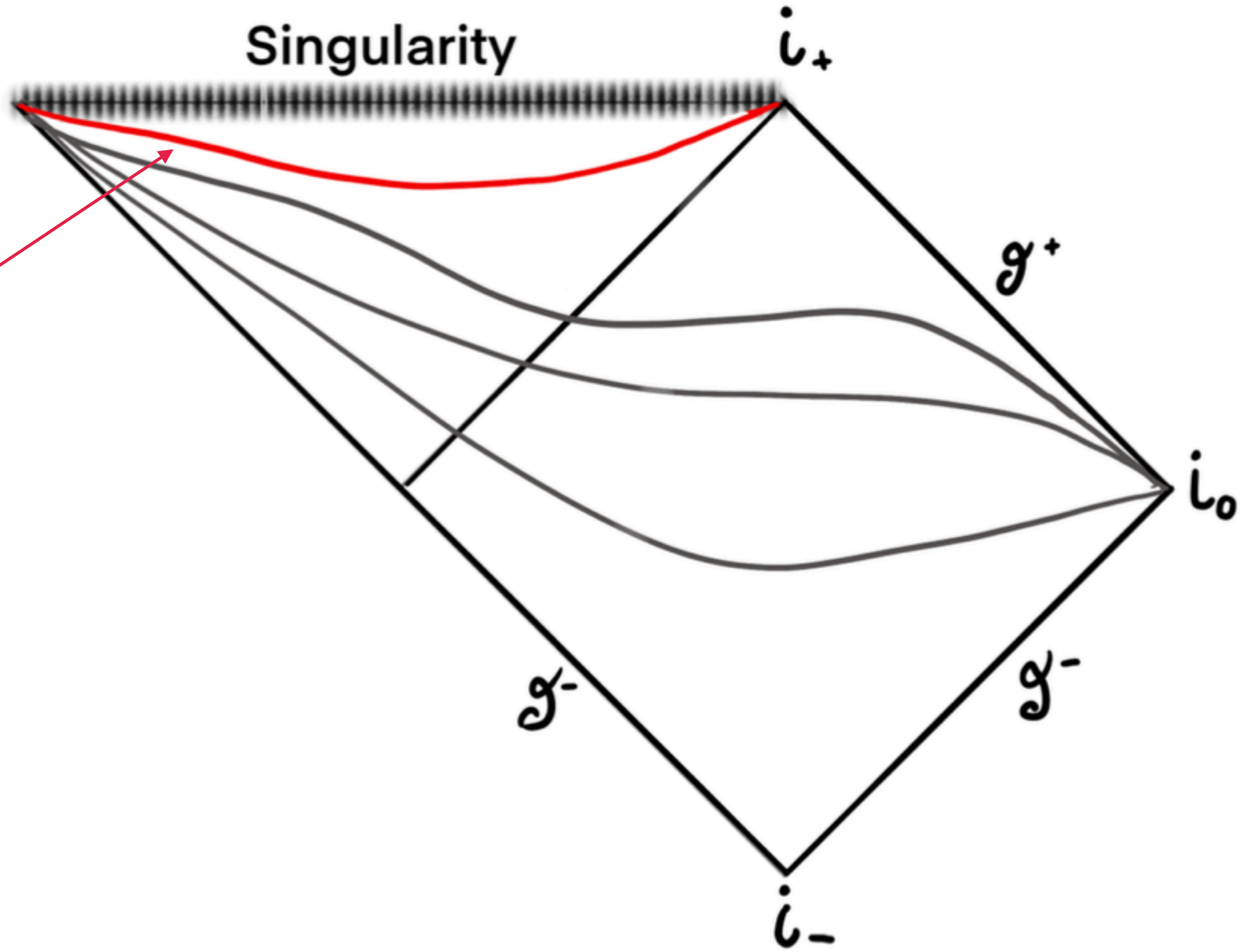
If $U_\mu$ becomes orthogonal to a compact surface, we have a <u>Universal Horizon</u>




S. Liberati, M. Herrero-Valea, R. Garcia-Santos, ArXiv:2101.00028

# Universal Horizons

- Also stationarity assumes a different taste. Let $\chi = \partial_t$ be a Killing Vector:

$$\mathscr{M}^+ = \{(\chi \cdot U) > 0\} \neq \emptyset$$

- Then we can give a local definition for the Universal Horizon:

$$UH = \{(\chi \cdot U) = 0\,, \quad (\chi \cdot a) \neq 0\}$$

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<u>Surface gravity:</u> $\kappa = \frac{1}{2}(\chi \cdot a)\Big|_{UH}$

# UH in Schwarzschild

- We will consider a Schwarzschild solution of the theory:

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 dS_2$$

$$U_\mu dx^\mu = \left(1 - \frac{M}{r}\right)dt + \frac{M}{r - 2M}dr$$

$$\chi^\mu \partial_\mu = \frac{\partial}{\partial t}$$

- The Universal Horizon is located at:

$$UH = \{1 - \frac{M}{r} = 0\} \implies \{r = M\}$$

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# Does the UH radiate?

- The derivation of Hawking Radiation is based on the fact that Killing Horizons are causal boundaries. We may expect similar properties from the UH...

- Let us take a massless scalar field on a BH geometry: $$\left[\nabla_\mu \nabla^\mu - \sum_{j=2}^{n} \frac{\alpha_{2j}}{\Lambda^{2j-2}} (\Delta)^j \right] \phi = 0$$

- In the $(t, r)$ plane we have the dispersion relation:

$$\begin{cases} iU^\mu \partial_\mu \phi = \omega \phi \\ iS^\mu \partial_\mu \phi = q \phi \end{cases} \implies \omega^2 = q^2 + \sum_{j=2}^{n} \frac{\alpha_{2j}}{\Lambda^{2j-2}} q^{2j} + F(\partial q, \partial \omega, \Lambda)$$

# Near-Horizon Solutions

- Since $[U^\mu \partial_\mu, S^\nu \partial_\nu] \neq 0$, neither $\omega$ nor $q$ are conserved quantities. The Killing energy $\Omega$, it is:

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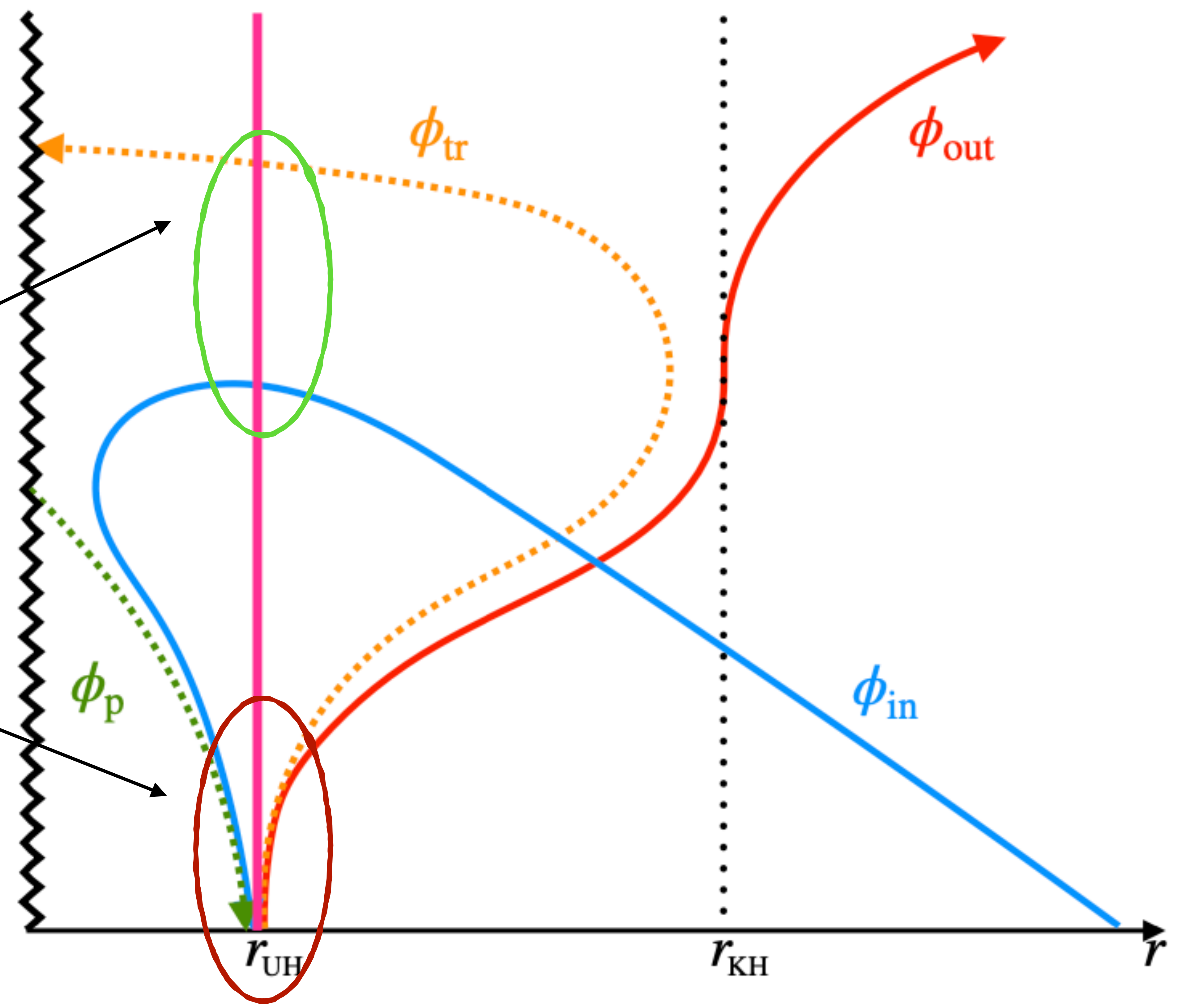
$(U \cdot \chi) \simeq 0$

- $|q|$ is regular at the UH: soft modes

- $|q| \to +\infty$ is regular at the UH: hard modes




FDP, S.Liberati, M.Herrero-Valea, M.Schneider
Phys.Rev.D 106 (2022)

# Near-Horizon Solutions

- We are interested in studying the hard mode which eventually reaches the asymptotic region, for which:

$$|q| \sim \frac{1}{(U \cdot \chi)^{\frac{1}{n-1}}} \qquad |\omega| \sim \frac{1}{(U \cdot \chi)^{\frac{n}{n-1}}}$$

- That mode is adiabatic, $|\dot{\omega}| \ll \omega^2$, so the dispersion relation reduces to:

$$\omega^2 = q^2 + \sum_{j=2}^{n} \frac{\alpha_{2j}}{\Lambda^{2j-2}} q^{2j} \rightarrow \omega^2 \simeq \frac{q^{2n}}{\Lambda^{2n-2}}$$

# Near-Horizon Solutions

- Coupling to the conservation equation we get

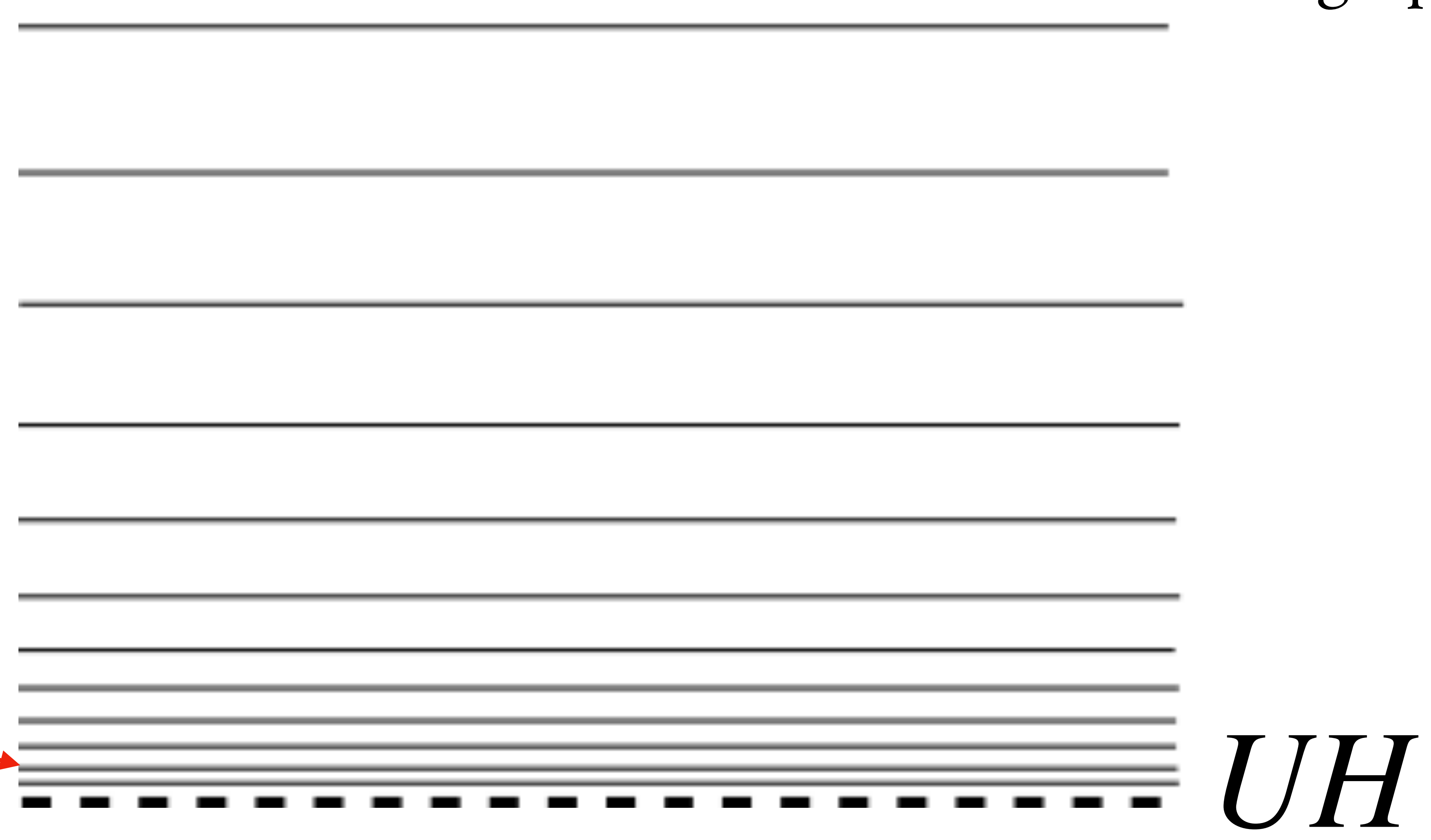
$$\omega = -\frac{\Lambda(S\cdot\chi)^{\frac{n}{n-1}}}{(U\cdot\chi)^{\frac{n}{n-1}}} + \frac{n}{n-1}\frac{\Omega}{(U\cdot\chi)} + O(1) \qquad q = \frac{\Lambda(S\cdot\chi)^{\frac{1}{n-1}}}{(U\cdot\chi)^{\frac{1}{n-1}}} - \frac{\Omega}{(S\cdot\chi)(n-1)} + O(U\cdot\chi)$$

- This gives the form of the mode, which is non-analytical at the Horizon:

T. Jacobson, ArXiv:gr-qc/0308048

$$\phi_\Omega = A\exp\left[\frac{i}{2\kappa}\Omega\log(U\cdot\chi)\right] = Ae^{-i\Omega\tau}$$

Phase contours



# UH temperature

- A similar mode can be found on the inside of the UH. Following a standard treatment, one can combine the two, building a solution analytical at the Horizon:

$$\phi_\Omega^{out} = Ae^{\frac{i}{2\kappa}\Omega \log(U\cdot\chi)}$$

$$\phi_\Omega^{in} = Ae^{-\frac{i}{2\kappa}\Omega \log[-(U\cdot\chi)]}$$

$$\Phi_\Omega^{\pm}(x) = c^{\pm}[\phi_\Omega^{out}(x) + e^{\mp\frac{\pi\Omega}{2\kappa}}(\phi_\Omega^{in}(x))^{\dagger}]$$

- The particle content of the mode is given by:

$$\mathcal{N}_\Omega = \langle \Phi_\Omega^- \Phi_\Omega^- \rangle = \frac{1}{e^{\frac{\pi\Omega}{\kappa}} - 1} \implies T_{UH} = \frac{\kappa}{\pi}$$

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LV Black Holes radiates!!

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- Thermodynamical properties of BH seem to be robust enough to survive losing Local Lorentz Invariance
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- Further developments: what arrives at infinity? Does the KH play any role?
- Linked projects: Are the 4 laws of thermodynamics fulfilled? Unruh effect? Rotating BH?

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**Thank you!**

DarkCosmoGrav