Dissipative processes at the acoustic horizon

Silvia Trabucco
with M.L. Chiofalo, D. Grasso, M. Mannarelli

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email: silvia.trabucco@gssi.it

Outline

IDEA OF ANALOGUE GRAVITY

> Physical systems (*analogue*) with emergent metric
> $g_{\mu\nu}$ *analogue metric* ruling low-energy excitations

Analogue systems can be classical (hydrodynamics, surface waves, ...) or quantum (BEC, Helium, ...)

In which sense *analogy*?

- ★ phenomenological, formal

Which physics?

- ► geometrical aspects of GR, BH phenomenology, aspects of FT in curved background

Dynamics of the background is not determined by Einstein equation.
At small scales, we cannot neglect microphysics.

Sound waves in fluids

Units: $\hbar = c = k_B = 1$

Continuity	Newton law (Euler)
$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$	$\rho \left[\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = \mathbf{f}$

Unruh studied an inviscid ($\mathbf{f} = -\nabla P$), irrotational ($\mathbf{v} = \nabla \phi$), barotropic ($\rho(P)$) fluid:

Linear expansion around a background configuration (ρ_0, P_0, ϕ_0): $0 < \epsilon \ll 1$

$$\begin{aligned} \rho =& \rho_0 + \epsilon \rho_1 + \mathcal{O}(\epsilon^2) \\ P =& P_0 + \epsilon P_1 + \mathcal{O}(\epsilon^2) \\ \phi =& \phi_0 + \epsilon \phi_1 + \mathcal{O}(\epsilon^2) \end{aligned}$$

Ex: continuity equation

$$\begin{aligned} &\partial_t \rho_0 + \nabla \cdot (\rho_0 \nabla \phi_0) = 0 \\ &\partial_t \rho_1 + \nabla \cdot (\rho_1 \nabla \phi_0 + \rho_0 \nabla \rho_1) = 0 \end{aligned}$$

Unruh, PRL 46(21), 1981

Combining together the linearized continuity and Euler equations, we have

$$\partial_t \left(\frac{\rho_0}{c_s^2} (\partial_t \phi_1 + \nabla \phi_0 \cdot \nabla \phi_1) \right) - \nabla \cdot \left(\rho_0 \nabla \phi_1 - \frac{\rho_0}{c_s^2} \nabla \phi_0 (\partial_t \phi_1 + \nabla \phi_0 \cdot \nabla \phi_1) \right) = 0$$

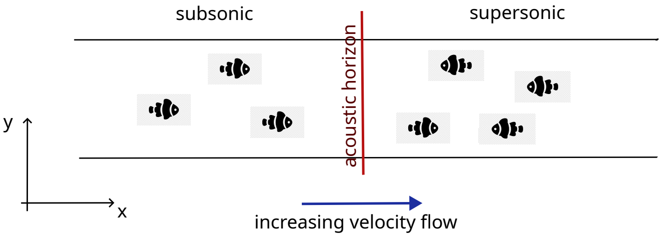
with $c_s^2 = \left. \frac{\partial P}{\partial \rho} \right|_{S=cost}$ adiabatic sound speed.
We interpret the acoustic perturbation as a massless scalar field propagating over a non trivial background

$$\Box \phi_1 = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi_1) = 0 \qquad \text{sound waves}$$

once we introduce a suited emergent metric $g_{\mu\nu}$,

$$ds^2 = \frac{\rho_0}{c_s^2} [(c_s^2 - v^2) dt^2 - v_i dt dx_i - \delta_{ij} dx_i dx_j] \quad \text{acoustic metric}$$

Acoustic horizon: $g_{tt} = 0 \ \Rightarrow v^2 = c_s^2$



Phonons at the acoustic horizon have a thermal spectrum at temperature

$$T = \frac{1}{2\pi} \left| \frac{d(c_s - v_x)}{dx} \right|_H$$

OBSERVATION OF HAWKING RADIATION

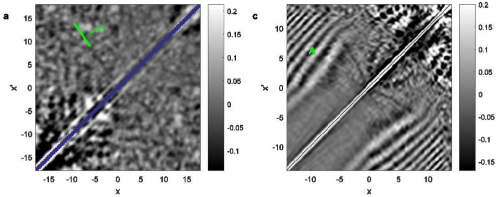
^{87}Rb Bose-Einstein condensate confined in 1d geometry

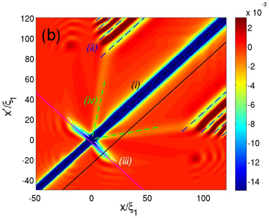
Steinhauer, *Nature Physics* 12, 959–965 (2016)

Analysis: density-density correlation function

$$G^{(2)}(x,x') \propto \frac{\langle n(x)n(x')\rangle}{\sqrt{n_x n_{x'}}}$$

Balbinot, Fabbri, Fagnocchi, Recati, Carusotto, PRA 78, 021603(R) (2008)





Covariant formulation

Inviscid, barotropic, irrotational fluid on flat spacetime $\eta_{\mu\nu} = \mathrm{diag}(+,-,-,-)$ at zero temperature.

Implement a scale separation $\theta(x) = \bar{\varphi}(x) + \phi(x)$, then the saddle point approximation

$$\mathcal{S}[\theta] = \mathcal{S}[\bar{\varphi}] + \frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \dots$$

$$g^{\mu\nu} = \Omega\left(\eta^{\mu\nu} + (c_s^{-2} - 1)v^\mu v^\nu\right)$$

$$g_{\mu\nu} = \Omega^{-1}\left(\eta_{\mu\nu} + (c_s^2 - 1)v_\mu v_\nu\right)$$

Set $\Omega = 1 \;\Rightarrow\; g = -c_s^2$

Mannarelli, Manuel, PRD 77, 103014 (2008)

KINETIC THEORY FOR FLUCTUATIONS

Phonons distribution function $f(x,p)$ $\Rightarrow$ Liouville equation $L[f] = df/d\tau$

$$L[f] = p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha} = C[f]$$

At equilibrium, we make the ansatz that the distribution function f is

$$f_{\mathrm{B.E.}}(\beta_\mu p^\mu) = \frac{1}{\exp(\beta_\mu p^\mu) - 1}$$

$$L[f_{\mathrm{B.E.}}] = 0 \quad \Rightarrow \quad \boxed{\nabla_\rho \beta_\lambda + \nabla_\lambda \beta_\rho = 0 \qquad \text{Killing's equation}}$$

For $\beta^\mu = \beta(1, \mathbf{0})$, $\beta = 1/T$.

Lindquist, Annals of Physics, vol.37, n.3 (1966)

We neglect phonon-phonon collision

$$\nabla_\alpha n^\alpha_{\text{ph}} = \int C[f] d\mathcal{P} = 0$$

$$\partial_\alpha n^\alpha_{\text{ph}} = -\frac{1}{\sqrt{-g}} \partial_\alpha \sqrt{-g}\, n^\alpha_{\text{ph}} = -\frac{1}{c_s} (\partial_\alpha c_s) n^\alpha_{\text{ph}} \qquad \text{geometrical effect}$$

Energy momentum tensor

$$T^{\alpha\beta}_{\text{ph}} = \int p^\alpha p^\beta f(x,p) d\mathcal{P}$$

Entropy density

$$s^\alpha_{\text{ph}} = -\int p^\alpha \left[f \ln f - (1+f) \ln (1+f) \right] d\mathcal{P}$$

$d\mathcal{P}$ covariant momentum measure.

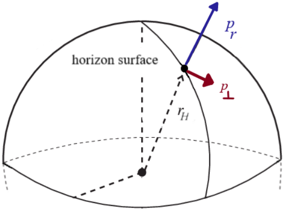
Spherical acoustic hole

Spherically symmetric stationary flow $v = v(r)\,\hat{r}\,,\, v(r) < 0$
$v_\mu = \gamma(1, -v(r), 0, 0) \quad \gamma = (1 - v^2)^{-1/2}$

$$ds^2 = dt^2(c_s^2 - v^2)\gamma^2 + 2\gamma^2(1 - c_s^2)v dr dt - [(1 - c_s^2)\gamma^2 v^2 + 1]dr^2 - r^2 d\Omega^2$$

Modeling Hawking emission at the acoustic horizon $c_s - v(r_H) = 0$.
Dimensional reduction

$$f_{\text{ph}} = f(p_r)\frac{2\pi}{L_c^2}\delta(p_\perp^2) \qquad \text{effective 1+1 phonon gas}$$



Close to acoustic horizon,

$$v + c_s = Cr\,, \qquad C = (v + c_s)'_H$$

and $E_\pm = K_\pm p_r$.

Mannarelli, Grasso, T., Chiofalo, PRD 103, 076001 (2021)

Phonon's emission at the horizon happens at the expenses of the bulk fluid, hence we can write an entropy balance equation $dS_H = dS_{\rm ph}$.

$$dS_{\rm ph} = d_g 4\pi r_H^2 dr_H \tilde{s}_{\rm ph} \,, \qquad \tilde{s}_{\rm ph} = \frac{\pi T}{6L_c^2 K_+}$$

For

$$S_H = \alpha \frac{A}{L_c^2} \,, \qquad \alpha = const.$$

$$dS_H = 8\pi\alpha \frac{r_H dr_H}{L_c^2}$$

$$T = \frac{1}{2\pi} \left(\frac{c_s - |v|}{1 - c_s |v|} \right)' \Big|_H \qquad \textbf{Hawking temperature}$$

We find $\alpha = 1/4$, as for BH.

THE KSS BOUND

It has been conjectured that for any physical system in 3+1 dimension

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

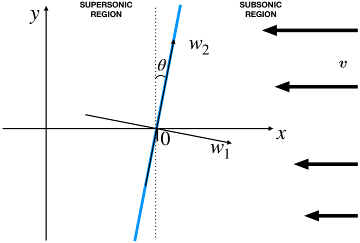
η shear viscosity, s entropy density.

The result is derived using string theory methods, for strongly interacting QFT dual to BH in AdS.

Kovtun, Son, Starinets, PRL 94, 111601 (2005)

DISSIPATION

$\mathbf{v} = v\hat{x}$, $v = c_s - Cx + ky$



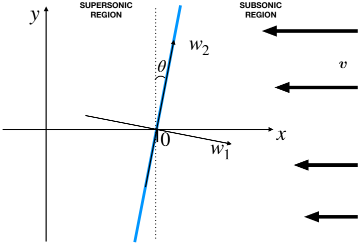
Viscous stress tensor $\sigma'_{ij} = \eta(\partial_i v_j + \partial_j v_i) + \zeta_{\text{eff}}\delta_{ix}\delta_{jx}\nabla \cdot \mathbf{v}$. At leading order in k/C

$$T^{\text{ph}}_{ij} = \sigma'_{ij} \qquad \text{yields} \qquad \frac{\zeta_{\text{eff}}}{s} = \frac{1}{4\pi} = \frac{\eta}{s}$$

Chiofalo, Grasso, Mannarelli, T, e-print:2202.13790[gr-qc]

Let us consider now a time dependent tilt $\dot{k} \neq 0$, for $\dot{k}/k \ll T$

$$S_H = \frac{A_H}{4L_c^2}\,, \qquad \dot{S}_H = \frac{\dot{A}_H}{4L_c^2}\,.$$



The emission still occurs along the w_1 direction, that depends on time

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(+\frac{\dot{k}}{2\pi T k}\right)$$

Conclusions & Future perspective

- At the acoustic horizon there are dissipative phenomena taking place
- Non zero *geometrical* viscosities are localized at the acoustic horizon
- Dissipation tends to reduce the area of the acoustic horizon
- We found a system that it seems to saturate the bound

Next:

- Is $1/4\pi$ a *lower* bound for η/s ?
- Test the bound with viable BEC platforms

Thank you for the attention!