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A NEW PARADIGM FOR A QUANTUM BIG BOUNCE IN THE WHEELER-DEWITT FRAMEWORK

Based on:

«Is Bianchi I a bouncing cosmology in the Wheeler-DeWitt picture?»
Eleonora Giovannetti and Giovanni Montani
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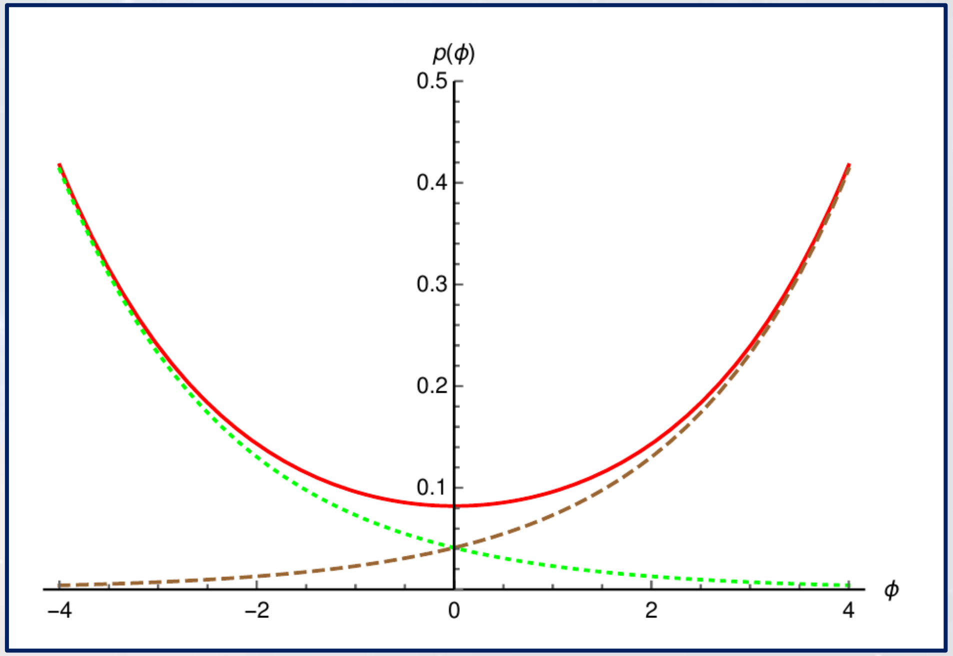
IS BIANCHI I A BOUNCING COSMOLOGY IN THE WHEELER-DEWITT FRAMEWORK?

BIG BOUNCE → best attempt to solve the initial singularity of the Big Bang in the context of Loop Quantum Cosmology thanks to the discrete spectrum of geometrical operators like area and volume



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BIG BOUNCE → best attempt to solve the initial singularity of the Big Bang in the context of Loop Quantum Cosmology thanks to the discrete spectrum of geometrical operators like area and volume

1. The Big Bounce has been characterized mainly at a semi-classical level but in the Planckian era the quantum effects are not negligible.

2. Anisotropic degrees of freedom can arise approaching the singularity. Then, a FLRW model could turn into a Bianchi one and hence the hypothesis of localized wave-packets would be violated.

IS BIANCHI I A BOUNCING COSMOLOGY IN THE WHEELER-DEWITT FRAMEWORK?

PROBLEM	PROPER DEFINITION OF A QUANTUM BIG BOUNCE
CONTEXT	WHEELER-DE WITT FORMULATION OF THE BIANCHI I MODEL IN THE MISNER VARIABLES
PROPOSAL	SCATTERING AMPLITUDE IN ANALOGY WITH THE RELATIVISTIC QUANTUM MECHANICS

THE BIANCHI I MODEL IN THE WHEELER-DEWITT FRAMEWORK

Hamiltonian of the **Bianchi I** model in the **Misner variables**

$$H = Ce^{-3\alpha}\left[-p_\alpha^2 + p_+^2 + p_-^2\right] = 0 \longrightarrow \boxed{\dot{\alpha} = -2NCe^{-3\alpha}p_\alpha}$$

collapsing and **expanding** singular solutions

Wheeler-DeWitt equation ⟶ analogy with a massless **Klein-Gordon**

$$\boxed{\hat{H}\psi = \Box\psi(\alpha,\beta_\pm) = [\partial_\alpha^2 - \partial_{\beta_+}^2 - \partial_{\beta_-}^2]\psi(\alpha,\beta_\pm) = 0}$$

α emerges as **time** (different signature)

Separation of frequencies

$$\psi_{\omega_k}^{\pm}(\alpha,\beta_\pm) = e^{\mp i\omega_k\alpha}e^{i(k_+\beta_+ + k_-\beta_-)}\,, \quad \omega_k \equiv \sqrt{k_+^2 + k_-^2}$$

Hence, by applying $\hat{p}_\alpha = -i\partial_\alpha$ to $\psi_{\omega_k}^{\pm}$ we get that

> the positive frequency solutions $\psi_{\omega_k}^{+}$ describe an expanding Universe,
> whereas the negative frequency ones $\psi_{\omega_k}^{-}$ describe a collapsing Universe.

THE BIANCHI I MODEL IN THE WHEELER-DEWITT FRAMEWORK

BIANCHI I → Two-dimensional relativistic particle → Non-linear dispersion relation

$$(\partial_\alpha^2 - \partial_{\beta_+}^2 - \partial_{\beta_-}^2)\psi(\alpha, \beta_\pm) = 0$$

$$\omega_k = \sqrt{k_+^2 + k_-^2}$$

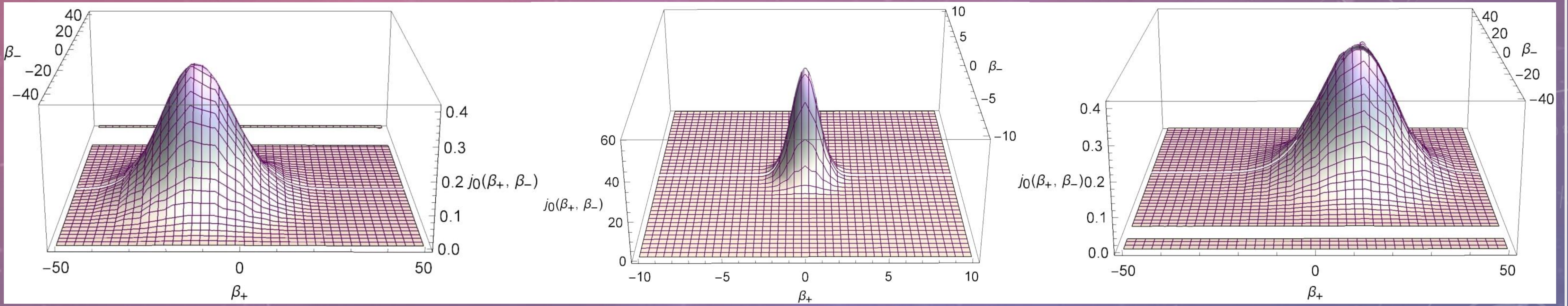
FLRW → One-dimensional relativistic particle → Linear dispersion relation

$$(\partial_\alpha^2 - \partial_\phi^2)\psi(\alpha, \phi) = 0$$

$$\omega_k = k_\phi$$

In the case of the **Bianchi I** model the non-zero **second derivative of the dispersion relation** enters in the variance of the Gaussian wave packet through a linear term in α that produces the **spreading phenomenon.**

3D-profiles of the Bianchi I wave packet for ***α=-10,0,10*** respectively.



THE RELATIVISTIC SCATTERING FORMALISM

Klein-Gordon equation

$$(\Box + m^2)\varphi(x) = 0$$

→ free relativistic particles of zero spin

$$f_{\mathbf{p}}^{(\pm)}(x) = \frac{e^{\mp ip\cdot x}}{\sqrt{(2\pi)^2 2\omega_{\mathbf{p}}}}$$

that form an orthonormal set

$$\int d^2x\, f_{\mathbf{p}'}^{(\pm)*}(x) i\overleftrightarrow{\partial_0} f_{\mathbf{p}}^{(\pm)}(x) = \pm\delta^2(\mathbf{p}-\mathbf{p}')\,,$$
$$\int d^2x\, f_{\mathbf{p}'}^{(\pm)*}(x) i\overleftrightarrow{\partial_0} f_{\mathbf{p}}^{(\mp)}(x) = 0\,.$$

Interaction potential

$$(\Box + m^2 + V(x))\phi(x) = 0$$

→ general solution in terms of the Feynman propagator (by iteration)

$$\phi(x) = \varphi(x) - \int d^3y\, \Delta_F(x-y)V(y)\phi(y)$$

“Relativistic quantum mechanics” by James D. Bjorken and Sidney D. Drell

Transition amplitude in the wave function formalism

Particles scattering	*Pair annihilation*
$S_{\mathbf{p}'_+,\mathbf{p}_+} = \delta^2(\mathbf{p}'_+ - \mathbf{p}_+) - i\int d^3y\, f^{(+)*}_{\mathbf{p}'_+}(y)V(y)\phi(y)$	$S_{\mathbf{p}_-,\mathbf{p}_+} = -i\int d^3y\, f^{(-)*}_{\mathbf{p}'_-}(y)V(y)\phi(y)$
Antiparticles scattering	*Pair production*
$S_{\mathbf{p}'_-,\mathbf{p}_-} = \delta^2(\mathbf{p}'_- - \mathbf{p}_-) - i\int d^3y\, f^{(-)*}_{\mathbf{p}'_-}(y)V(y)\phi(y)$	$S_{\mathbf{p}_+,\mathbf{p}_-} = -i\int d^3y\, f^{(+)*}_{\mathbf{p}'_+}(y)V(y)\phi(y)$

The transition probability is the square modulus of *S*.

COMPUTATION OF THE SCATTERING AMPLITUDE

Wheeler-DeWitt equation for the **Bianchi I** model with an **ekpyrotic-like matter term**

quantum interacting potential
responible for transition

$$\hat{H}\psi = \left[\partial_\alpha^2 - \partial_{\beta_+}^2 - \partial_{\beta_-}^2 + \lambda e^{-3\varepsilon\alpha}\right]\psi(\alpha, \beta_\pm) = 0$$

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$$\psi(\alpha, \beta_\pm) = \varphi(\alpha)e^{ik_+\beta_+}e^{ik_-\beta_-}$$

$$\partial_\alpha^2\varphi(\alpha) + (\omega_k^2 + \lambda e^{-3\varepsilon\alpha})\varphi(\alpha) = 0$$

Fourier expansion

$$\varphi(\alpha) = \frac{1}{\sqrt{2}}J_{-\frac{2i\omega_k}{3\varepsilon}}(2\sqrt{\lambda e^{-3\varepsilon\alpha}}/3\varepsilon)\Gamma(1 - \frac{2i\omega_k}{3\varepsilon}) + \frac{1}{\sqrt{2}}J_{\frac{2i\omega_k}{3\varepsilon}}(2\sqrt{\lambda e^{-3\varepsilon\alpha}}/3\varepsilon)\Gamma(1 + \frac{2i\omega_k}{3\varepsilon})$$

Bessel functions

COMPUTATION OF THE SCATTERING AMPLITUDE

In-going wave packet

$$\psi(\alpha,\beta_\pm) = \iint_{-\infty}^{+\infty} dk_+ dk_-\, A(k_+,k_-)\varphi(\alpha) e^{ik_+\beta_+} e^{ik_-\beta_-}$$

the collapsing solution
that emerges from the interaction
with the time-dependent potential *V*(*α*)

Out-going wave packet

$$\chi(\alpha,\beta_\pm) = \iint_{-\infty}^{+\infty} dk'_+ dk'_-\, A'(k'_+,k'_-) e^{-i\omega_{k'}\alpha} e^{ik'_+\beta_+} e^{ik'_-\beta_-}$$

the free expanding solution

Scattering amplitude

$$S_{Bounce} = -i \iiint_{-\infty}^{+\infty} d\alpha\, d\beta_+ d\beta_-\, \chi^*(\alpha,\beta_\pm) V(\alpha) \psi(\alpha,\beta_\pm)$$

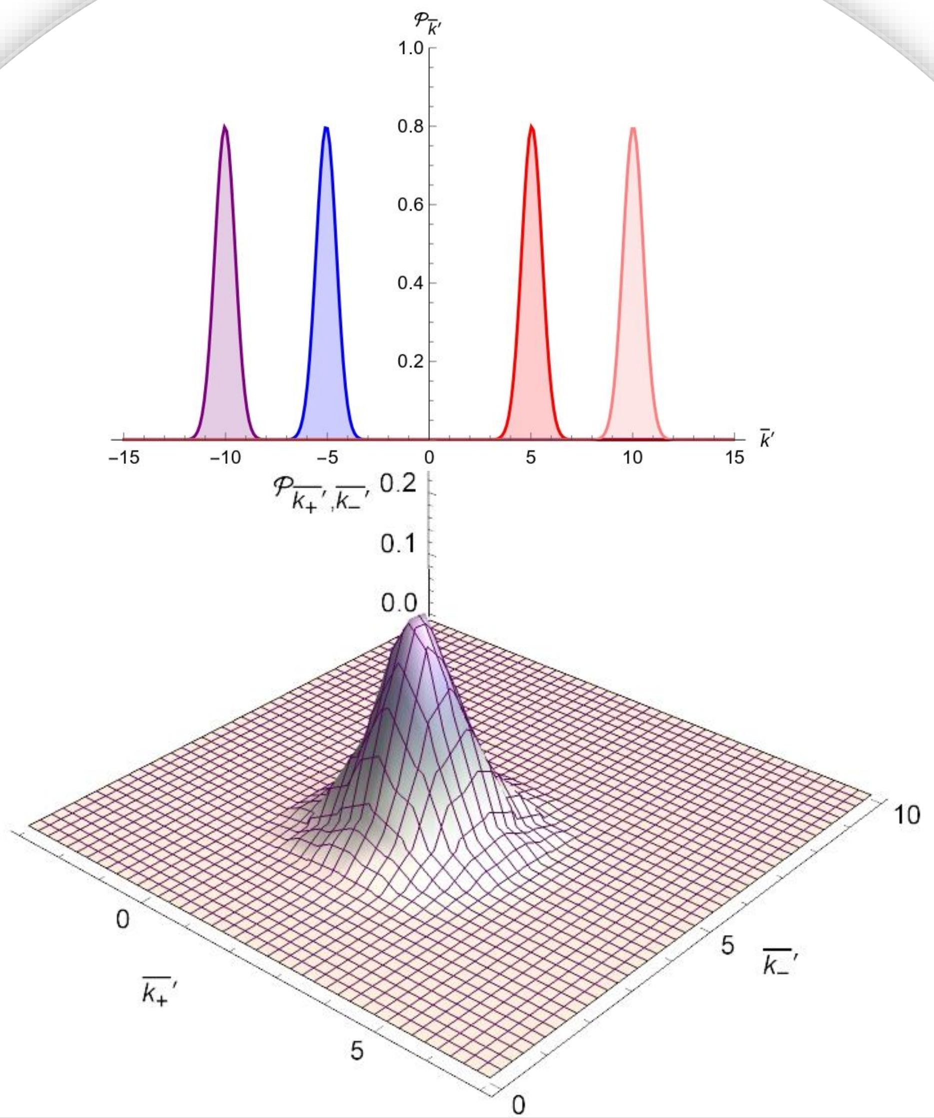
QUANTUM BIG BOUNCE AS A PROBABILISTIC PROCESS

Transition probability of the Quantum Big Bounce

$$\mathcal{P} = |S_{Bounce}(\bar{k}'_+, \bar{k}'_-, \bar{k}_+, \bar{k}_-)|^2$$

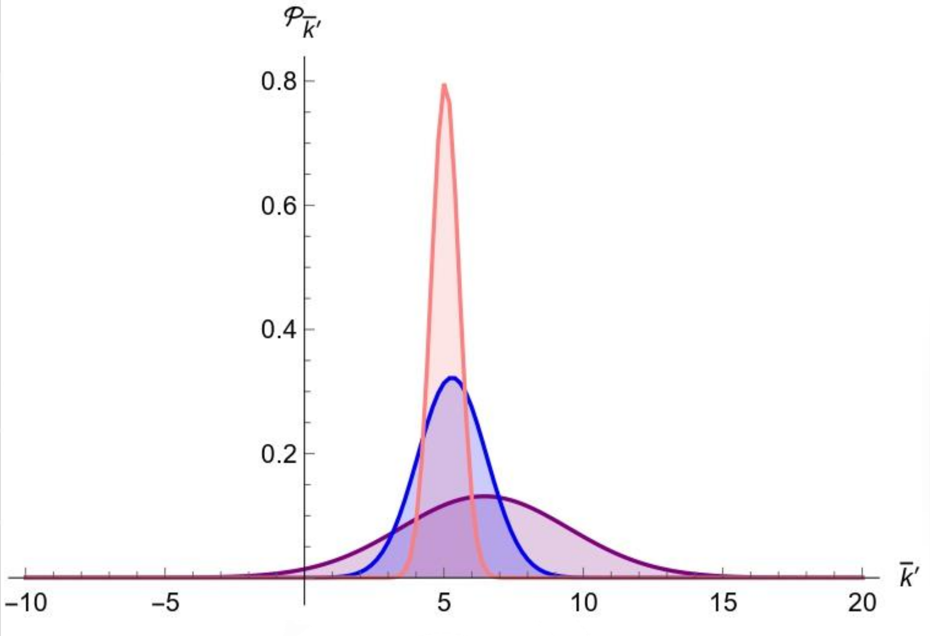
The plots highlight the **Gaussian shape** of the **probability density**. The peak occurs for $\bar{k}'_+ \sim \bar{k}_+$ and $\bar{k}'_- \sim \bar{k}_-$.

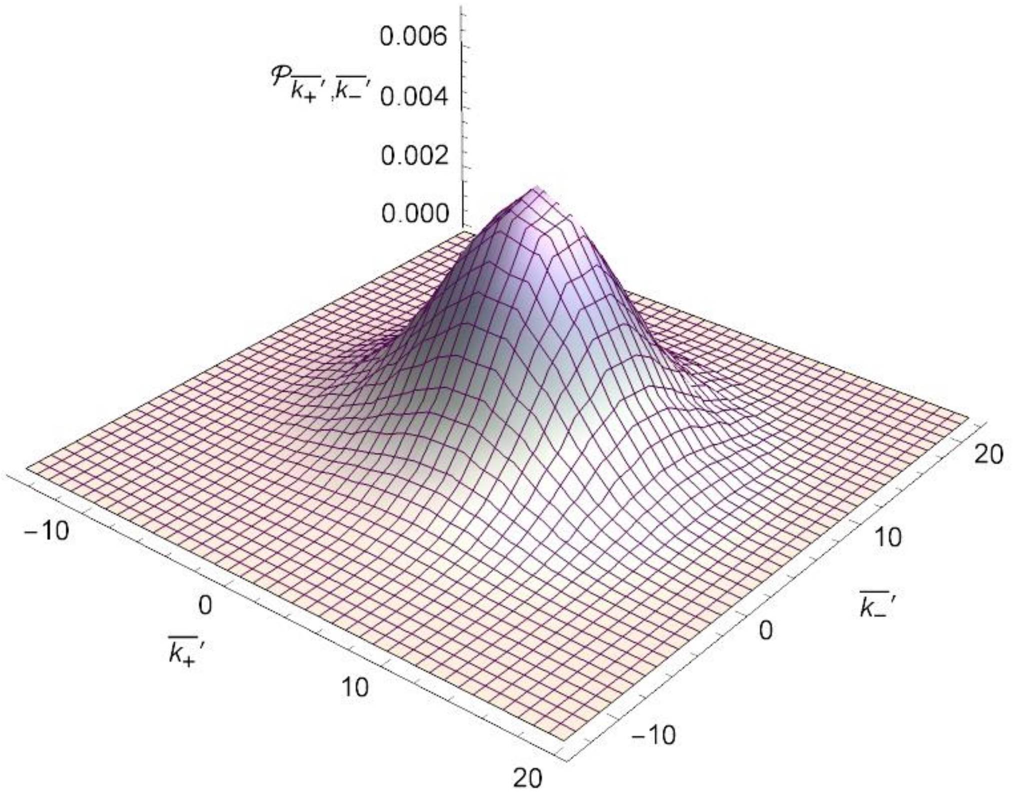
In the 2D-plots we have considered the same Gaussian distribution for the two anisotropic momenta by imposing $\bar{k}' = \bar{k}'_+ = \bar{k}'_-$ and $\bar{k} = \bar{k}_+ = \bar{k}_-$.



QUANTUM BIG BOUNCE AS A PROBABILISTIC PROCESS

The bigger the variance of the wave packet is, the more appreciable the shift of the peak $\bar{k}'_+, \bar{k}'_-$ from $\bar{k}_+, \bar{k}_-$ is.





Hence, for highly-localized wave packets this probability density reproduces the symmetrical reconnection of the semiclassical Big Bounce.

CONCLUSIONS AND FURTHER DEVELOPMENTS

- Thanks to the analogy with the Klein-Gordon formalism, we have recovered a “time after quantization” (the Misner variable α) and identified the positive and negative frequency solution with the collapsing and expanding Bianchi I Universe respectively.

- We have shown that the presence of a quantum time-dependent potential (here the ekpyrotic-like matter term) creates a mixed state near the singularity and that the transition from a collapsing to an expanding (singular) Universe has a non-zero and well-defined probability.

- In the case of highly-localized wave packets (i.e. semiclassical states) the probability shape resembles the symmetrical reconnection of the collapsing and expanding branches of the semiclassical Big Bounce, thus reproducing a Quantum Big Bounce.

THANK YOU
FOR YOUR KIND ATTENTION!

Link to the paper: https://arxiv.org/pdf/2203.01062.pdf