DarkCosmoGrav: New Frontiers in Particle Physics, Gravity, and Cosmology, Pisa, Italy

Regular black holes, universes without singularities, and phantom-scalar field transitions

A.Yu. Kamenshchik

University of Bologna and INFN, Bologna

January 23-25, 2023

Based on:

Leonardo Chataignier, Alexander Yu. Kamenshchik, Alessandro Tronconi and Giovanni Venturi,

Regular black holes, universes without singularities, and phantom-scalar field transitions,

Physical Review D 107, 023508 (2023)

arXiv:2208.02280 [gr-qc].

Content

Introduction and Motivations

- Appearance of singularities is one of the most important phenomena in General Relativity and its generalizations and modifications.
- The singularities were first discovered in such simple geometries as those of Friedmann and Schwarzschild and later their general character was established (Penrose, Hawking).
- The investigation of the oscillatory approach to the cosmological singularity (Belinsky, Khalatnikov, Lifshitz) known also as Mixmaster universe (Misner) has opened the way to the birth of a new branch of the mathematical physics chaotic cosmology and hyperbolic Kac-Moody algebras (Damour, Henneaux, Nicolai).

- One can try to study the opportunity to cross the singularity.
- Another choice: to construct the spacetimes (static or cosmological) free of singularities.
- Simpson and Visser, 2019 have suggested to eliminate the singularity from the Schwarzschild black hole, by a simple substitution

$$ds^2 = \left(1 - \frac{2m}{\sqrt{u^2 + b^2}}\right) dt^2 - \left(1 - \frac{2m}{\sqrt{u^2 + b^2}}\right)^{-1} du^2$$
$$-(u^2 + b^2)(d\theta^2 + \sin^2\theta d\varphi^2),$$

where b is a regularising parameter. If $b > 2m$, the formula above represents a wormhole with a throat at $u = 0$; if $b < 2m$, one has a black hole with two horizons at $u = \pm\sqrt{4m^2 - b^2}$ and $b = 2m$, we see an extremal black hole with the only horizon at $u = 0$.

- In the black hole case, the hypersurface $u = 0$ is not a throat since u is a temporal coordinate there and $u = 0$ corresponds to a bounce in one of the two scalar factors, of the Kantowski-Sachs universe in the inner region of the black hole. This phenomenon was called black bounce.
- Bronnikov, 2022 has considered a static spherically symmetric geometry in the presence of the scalar field and of the magnetic field. An interesting phenomenon of the transformation between the phantom scalar field and the standard field is observed.
- A similar phenomenon arises in a simple non-singular cosmological models.

Our goals here:

- To study non-singular cosmological models.
- To find a non-singular spherically symmetric static black-hole like geometry sustained by a scalar field.

Flat Friedmann model with a scalar field

Let us consider a flat Friedmann universe filled with a massless scalar field.

$$ds^2 = dt^2 - t^{\frac{2}{3}}(dx_1^2 + dx_2^2 + dx_3^2),$$

$$\dot{\phi} = \sqrt{\frac{2}{3}}\frac{1}{t}.$$

Let us now construct the regularized metric:

$$ds^2 = dt^2 - (t^2 + b^2)^{\frac{1}{3}}(dx_1^2 + dx_2^2 + dx_3^2).$$

$$R_0^0 = \frac{2t^2 - 3b^2}{3(t^2 + b^2)^4},$$

$$R_1^1 = R_2^2 = R_3^3 = -\frac{b^2}{3(t^2 + b^2)^2}.$$

$$R = \frac{2t^2 - 6b^2}{3(t^2 + b^2)^2}.$$

The Friedmann equations give the expressions for the energy density and for the isotropic pressure of matter

$$\rho = \frac{t^2}{3(t^2 + b^2)^2},$$

$$p = \frac{t^2 - 2b^2}{3(t^2 + b^2)^2}.$$

Let us suppose that the universe is filled with a spatially homogeneous scalar field with a potential $V(\phi)$.

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

Then

$$\dot{\phi} = \pm\sqrt{\frac{2}{3}}\frac{\sqrt{t^2 - b^2}}{t^2 + b^2},$$

$$V = \frac{b^2}{3(t^2 + b^2)^2}.$$

What happens at $|t| < b$?

The kinetic energy of ϕ changes sign and the standard scalar field transitions to a phantom scalar field.

We can study the behaviour of the potential V in the vicinity of $t = b$.

$$t = b + \tau,$$

where τ is small.

$$\frac{d\phi}{d\tau} = \frac{\sqrt{\tau}}{\sqrt{3b^3}},$$

$$\phi(\tau) = \phi_0 + \frac{2\tau^{3/2}}{3\sqrt{3b^3}}.$$

$$\tau = 3b\left(\frac{\phi - \phi_0}{2}\right)^{\frac{2}{3}}.$$

In the vicinity of the critical point:

$$V(\phi) = \frac{1}{3b^2\left[\left(1 + 3\left(\frac{\phi - \phi_0}{2}\right)^{\frac{2}{3}}\right)^2 + 1\right]^2}.$$

By keeping only the leading terms:

$$V(\phi) = \frac{1}{12b^2}\left[1 - 6\left(\frac{\phi - \phi_0}{2}\right)^{\frac{2}{3}}\right].$$

The distinguishing feature of this expressions is the presence of a non-analyticity of the cusp type, which is responsible for the transition from the standard scalar field to its phantom counterpart and vice versa.

It is interesting that a similar phenomenon of the transition from the phantom and non-phantom phases of the scalar field was found in another context in Andrianov, Cannata and Kamenshchik, 2005, Smooth dynamical crossing of the phantom divide line of a scalar field in simple cosmological models, Phys. Rev. D 72, 043531.

The potential of the scalar field had also a cusp with the same type of non-analyticity $(\phi - \phi_0)^{2/3}$.

A slightly more general model

$$ds^2 = dt^2 - t^{2\alpha}(dx_1^2 + dx_2^2 + dx_3^2).$$

Such an evolution arises in a universe filled with a perfect fluid with the equation of state parameter

$$w = \frac{2 - 3\alpha}{3\alpha}.$$

This is a particular solution to the equations of motion for the flat Friedmann model with a minimally coupled scalar field with an exponential potential:

$$V(\phi) = \alpha(3\alpha - 1)\exp\left(-\sqrt{\frac{2}{\alpha}}(\phi - \phi_0)\right).$$

Singularity-free solution:

$$ds^2 = dt^2 - (t^2 + b^2)^\alpha dl^2.$$

The expressions for the potential and the time derivative of the scalar field realizing this evolution are

$$V(\phi) = \frac{\alpha(b^2 + (3\alpha - 1)t^2)}{(t^2 + b^2)^2},$$

$$\dot{\phi}^2 = \frac{2\alpha(t^2 - b^2)}{(t^2 + b^2)^2}.$$

Again phantom–non-phantom transformation occurs at $|t| = b$. The potential at this point has a cusp.

The behaviour of the potential in the vicinity of the cusp:

$$V(\phi) = \frac{\alpha}{4b^2}\left[3\alpha - \frac{2\cdot 3^{2/3}}{\alpha^{1/3}}\left(\frac{\phi-\phi_0}{2}\right)^{2/3}\right].$$

This expression has the same non-analyticity ($\sim (\phi-\phi_0)^{2/3}$) as that seen in the preceding model, and when $\alpha = \frac{1}{3}$ these expressions coincide.

Bianchi-I universes without singularities

Let us consider a Bianchi-I universe with the metric

$$ds^2 = dt^2 - (a_1^2(t)dx_1^2 + a_2^2(t)dx_2^2 + a_3^2 dx_3^2).$$

It is convenient to introduce the following variables:

$$a_1(t) = A(t)e^{\beta_1(t)},$$
$$a_2(t) = A(t)e^{\beta_2(t)},$$
$$a_3(t) = A(t)e^{\beta_3(t)},$$

where the anisotropic factors β_i satisfy the identity

$$\beta_1 + \beta_2 + \beta_3 = 0.$$

The Ricci tensor components and the Ricci scalar:

$$R_0^0 = -3\frac{\ddot{A}}{A} - \sum_{i=1}^{3} \dot{\beta}_i^2,$$

$$R_1^1 = -\left(\frac{\ddot{A}}{A} + 2\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}}{A}\dot{\beta}_1 - \ddot{\beta}_1\right),$$

$$R_2^2 = -\left(\frac{\ddot{A}}{A} + 2\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}}{A}\dot{\beta}_2 - \ddot{\beta}_2\right),$$

$$R_3^3 = -\left(\frac{\ddot{A}}{A} + 2\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}}{A}\dot{\beta}_3 - \ddot{\beta}_3\right),$$

$$R = -\left(6\frac{\ddot{A}}{A} + 6\frac{\dot{A}^2}{A^2} + \sum_{i=1}^{3} \dot{\beta}_i^2\right).$$

In an empty space or a space filled with matter with an isotropic pressure, we have

$$R_1^1 = R_2^2 = R_3^3,$$

and

$$R_2^2 + R_3^3 - 2R_1^1 = 0.$$

Then,

$$\ddot{\beta}_1 + 3\frac{\dot{A}}{A}\dot{\beta}_1 = 0$$

and

$$\dot{\beta}_1 = \frac{\beta_{10}}{A^3},$$

where β_{10} is an integration constant. Similarly,

$$\dot{\beta}_2 = \frac{\beta_{20}}{A^3},$$

$$\dot{\beta}_3 = \frac{\beta_{30}}{A^3}.$$

$$R_0^0 - \frac{1}{2}R = 3\frac{\dot{A}^2}{A^2} - \frac{1}{2}\frac{\bar{\beta}^2}{A^6},$$

where

$$\bar{\beta}^2 \equiv \sum_{i=1}^{3} \beta_{i0}^2.$$

In an empty universe

$$A(t) = A_0 t^{1/3}$$

and then one can find also the anisotropy factor coming to the Kasner solution.

The regularized geometry is

$$A(t) = (t^2 + b^2)^{\frac{1}{6}}.$$

The corresponding potential and kinetic term of a scalar field are

$$\dot{\phi}^2 = \frac{t^2(2 - 3\bar{\beta}^2) - b^2(2 + 3\bar{\beta}^2)}{3(t^2 + b^2)^2}.$$

$$V = \frac{b^2}{3(t^2 + b^2)^2}.$$

The phantom-scalar transition occurs at

$$|t| = b\sqrt{\frac{2 + 3\bar{\beta}^2}{2 - 3\bar{\beta}^2}}.$$

The presence of the scalar field imposes the restriction on the value of anisotropy:

$$\bar{\beta}^2 \leq \frac{2}{3}.$$

Spherically symmetric static regular geometries sustained by a scalar field

Bronnikov,2022 considered regularized Fisher-type solutions, where the role of matter was played by the scalar field and by the electromagnetic field. Here we wish to construct a spherically symmetric static spacetimes filled exclusively with the scalar field.

We shall look for this solution in the following form:

$$ds^2 = A(r)dt^2 - B(r)dr^2 - (r^2 + b^2)(d\theta^2 + \sin^2\theta d\varphi^2).$$

The components of the Ricci tensor for this metric:

$$R_0^0 = \frac{A''}{2AB} - \frac{A'^2}{4A^2B} - \frac{A'B'}{4AB^2} + \frac{A'r}{AB(r^2+b^2)},$$

$$R_r^r = \frac{A''}{2AB} - \frac{A'^2}{4A^2B} - \frac{A'B'}{4AB^2} - \frac{B'r}{B^2(r^2+b^2)} + \frac{2b^2}{B(r^2+b^2)^2},$$

$$R_\theta^\theta = R_\varphi^\varphi = \frac{1}{B(r^2+b^2)} - \frac{B'r}{2B^2(r^2+b^2)} + \frac{A'r}{2AB(r^2+b^2)} - \frac{1}{r^2+b^2},$$

where the "prime" means the derivative with respect to r.

Let us suppose that our spacetime is filled with the scalar field which depends only on the radial coordinate r. Its Lagrangian

$$L = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) = -\frac{1}{2B} \phi'^2 - V(\phi).$$

The components of the energy-momentum tensor are

$$T_0^0 = \frac{1}{2B} \phi'^2 + V(\phi),$$

$$T_r^r = -\frac{1}{2B} \phi'^2 + V(\phi),$$

$$T_\theta^\theta = T_\varphi^\varphi = \frac{1}{2B} \phi'^2 + V(\phi).$$

$$T_0^0 = T_\theta^\theta = T_\varphi^\varphi,$$

$$R_0^0 = R_\theta^\theta = R_\varphi^\varphi .$$

It gives us a constraint

$$\frac{A''}{2AB} - \frac{A'^2}{4A^2B} - \frac{A'B'}{4AB^2} + \frac{A'r}{2AB(r^2+b^2)} - \frac{1}{B(r^2+b^2)} + \frac{B'r}{2B^2(r^2+b^2)} + \frac{1}{r^2+b^2} = 0,$$

but we still have a lot of freedom.
One of the simplest possible choices is the "Schwarzschild-like" condition

$$AB = 1.$$

It gives the equation

$$A'' - \frac{2A}{r^2+b^2} + \frac{2}{r^2+b^2} = 0.$$

Its general solution is

$$A = 1 + c_1(r^2+b^2) + \frac{c_2}{2b^3}\left[(r^2+b^2)\mathrm{arctan}\frac{r}{b} + br\right],$$

where c_1 and c_2 are arbitrary constants. It is reasonable to require that the general solution would have a well-defined limit at $b \to 0$. To find it, we shall use the expansion of the arctan function when its argument tends to infinity:

$$\mathrm{arctan}\frac{r}{b} = \frac{\pi}{2} - \frac{b}{r} + \frac{b^3}{3r^3} - \frac{b^5}{5r^5} + \cdots.$$

The condition for the regularity at $b \to 0$ is

$$c_1 = -\frac{c_2\pi}{4b^3}.$$

At $r \to \infty$:

$$A = 1 - \frac{c_2}{3r} + \frac{c_2 b^2}{15 r^3} + \cdots .$$

It is convenient to introduce $r_0 = \frac{c_2}{3}$ when at $r \to \infty$ this expression has a "Schwarzschild-like" form

$$A = 1 - \frac{r_0}{r} + \frac{b^2 r_0}{5 r^3} + \cdots .$$

Finally,

$$A = 1 - \frac{3\pi r_0}{4b^3}(r^2 + b^2) + \frac{3r_0}{2b^3}\left[(r^2 + b^2)\mathrm{arctan}\frac{r}{b} + br\right] .$$

This geometry does not have any singularity at $r = 0$.

One can show that at

$$b \geq \frac{3\pi r_0}{4}$$

one has a wormhole. In the opposite case we have a regular black hole with a black bounce.

We can connect the obtained geometry of the spacetime with a scalar field, living in it, using the Einstein equations.

$$\phi'^2 = -\frac{b^2}{(r^2 + b^2)^2}.$$

The negative definiteness of the right-hand side of this equation indicates that the scalar field should be phantom and that we should change the sign at the kinetic term of scalar field Lagrangian. Then

$$\phi'^2 = \frac{b^2}{(r^2 + b^2)^2},$$

and

$$\phi' = \pm \frac{b}{r^2 + b^2}$$

Then

$$\phi = \arctan \frac{r}{b},$$

$$r = b \tan \phi.$$

The potential of this field is

$$V = -\frac{3\pi r_0}{4b^3} + \frac{3r_0}{2b^3}\phi + \frac{9r_0 \sin 2\phi}{4b^3} - \frac{3\pi r_0 \sin^2\phi}{2b^3} + \frac{3r_0\phi \sin^2\phi}{b^3}.$$

Here the domain of the function is

$$0 \leq \phi < \frac{\pi}{2}$$

and the potential is an analytic function without any irregularities of the cusp type. It is in agreement with the fact that the scalar field does not undergo in this solution the phantom - non-phantom transition.

The same solution was found earlier in

K.A. Bronnikov and J.C. Fabris, Regular phantom black holes, Phys. Rev. Lett. 96 (2006) 251101.

Conclusions

- Consideration of rather simple cosmological or static (black-hole type) models with a naive modification of the solutions aimed to exclude singularities can imply the appearance of interesting physical effects such as transformations between different kinds of matter.
- Phantom scalar fields can be of interest from phenomenological point of view. In recent paper A. S. de Jesus, N. Pinto-Neto, F. S. Queiroz, J. Silk and D. R. da Silva, The Hubble Rate Trouble: An Effective Field Theory of Dark Matter, arXiv:2212.13272 [hep-ph] the authors combining a phantom-like cosmology with modification of the Standard Model of particle physics (adding non-renormalizable interactions) manage to suggest a solution of the Hubble tension puzzle.