SCALE-INVARIANT INFLATION

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UNIVERSITA' DEGLI STUDI DI TRENTO

MAIN GOALS

- Consider an exactly scale-invariant $f(R, \phi)$ theory
- Show that a mass scale emerges dynamically (no need for quantum correction to break scale invariance)
- Study an inflationary scenario in this theory and show that it fits data
- terminology: here “quadratic” means “exactly scale-invariant”

OUTLINE

- Quick facts about quadratic gravity theories
- Spontaneous scale-invariance breaking
- Inflation
- Conclusions and further developments

Work in collaboration with S. Boudet, C. Cecchini, A. Ghoshal, D. Mukherjee, S. Silveravalle, L. Vanzo, S. Vicentini, S. Zerbini

Quick facts about quadratic gravity

In the literature one often refers to the action

$$S = \int d^4x \sqrt{g} \left(\frac{M^2}{2}(R - 2\Lambda) + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} \right) + S_m \,, \qquad (R^{\alpha}{}_{\mu\lambda\sigma} = \partial_\lambda \Gamma^{\alpha}_{\mu\sigma} - \ldots)$$

as to "quantum gravity".

This theory is renormalisable (Stelle '77). DOF are (flat space)

- massless spin-2 field
- ghost massive spin-2 field
- massive scalar field

At large curvature the theory becomes scale-invariant.

Invariance is typically broken by quantum corrections or by the linear term in R.

...But it can also be broken dynamically!

Simplest case of scale-invariant theory:

$$\mathscr{L}_J = \frac{\alpha}{36}\sqrt{|g|}R^2$$

$S_J = \int \mathscr{L}_J \, d^4x$ invariant under $\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(\ell x)$

Weyl - rescaling from Jordan frame to Einstein frame

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega = \frac{\sqrt{\alpha R}}{3M}$$

Note that the rescaling can be done only for $R \neq 0$

Weyl - rescaling from Jordan frame to Einstein frame:

$$\mathscr{L}_E = \sqrt{|\tilde{g}|}\left[\frac{M^2}{2}(\tilde{R} - 2\Lambda) - \frac{1}{2}\tilde{g}^{\mu\nu}\tilde{\partial}_\mu\psi\tilde{\partial}_\nu\psi\right], \quad \Lambda = \frac{9M^2}{4\alpha}$$

$$\psi = \sqrt{6}M \ln\Omega = \sqrt{\frac{3}{2}}M \ln\left(\frac{\alpha R}{9M^2}\right)$$

M is arbitrary and "redundant"

E-frame Lagrangian is still scale-invariant!

$$\bar{g}_{\mu\nu}(x) = g_{\mu\nu}(\ell x), \quad \bar{\psi}(x) = \ell\psi(\ell x), \quad \bar{M} = \ell M$$

E-frame equations of motion:

$$\tilde{G}_{\mu\nu} + \Lambda\tilde{g}_{\mu\nu} = M^2\left(\tilde{\partial}_\mu\psi\tilde{\partial}_\nu\psi - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\partial}_\mu\psi\tilde{\partial}^\mu\psi\right)$$

By comparing eom:

JF: $$RR_{\mu\nu} - \frac{1}{4}R^2 g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu)R = 0$$

EF: $$\tilde{G}_{\mu\nu} + \Lambda \tilde{g}_{\mu\nu} = M^2 \left(\tilde{\partial}_\mu \psi \tilde{\partial}_\nu \psi - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\partial}_\mu \psi \tilde{\partial}^\mu \psi \right)$$

The JF admits Ricci-flat solutions, the EF does not!

Minkowski space is a solution in JF but it is not in EF.

Same for any other Ricci-flat solution.
...careful when quantising around flat space in EF!

Emergence of a scale

SCALE-INVARIANT INFLATIONARY SCENARIO

$$\mathscr{L}_J = \frac{\alpha}{36}\sqrt{|g|}R^2, \quad ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$$

EOM: $2HH'' + H'^2 + 6HH' = 0 \quad N = \ln a, \quad H = a'/a$

One class of solutions is (in terms of e-foldings N)

$$H = (c_1 + c_2 e^{-3N})^{2/3}, \quad N = \ln a$$

The solution interpolates between a radiation-dominated Universe ($H = exp(-2N)$) and a de Sitter Universe ($H =$ const).

Note that there is no EM source!

However, the solution is no good for inflation. We need another DOF.

Scalar-tensor theory (higher-derivative "induced gravity"):

$$\mathscr{L}_{\rm inv} = \sqrt{g}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right]$$

Effective scalar potential for fixed R:

$$V_{\rm eff} = -\frac{\xi}{6}\phi^2 R + \frac{\lambda}{4}\phi^4$$

Max and min correspond to fixed points:

$$\phi = 0\,, \qquad \phi_0^2 = \frac{\xi R}{3\lambda}$$

EOM for flat RW metric. $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$:

$$H^2\phi'' + (HH' + 3H^2)\phi' - 2\xi\phi HH' - \phi(4\xi H^2 - \lambda\phi^2) = 0\,,$$

$$\alpha H^2\left(2HH'' + H'^2 + 6HH'\right) + 2\xi H^2\phi\phi' - \frac{1}{2}\phi'^2 H^2 + \frac{\phi^2}{4}(4\xi H^2 - \lambda\phi^2) = 0\,.$$

NB if we add $\beta R_{\mu\nu}R^{\mu\nu}$ then $\alpha \to \alpha + 12\beta$ in the above eq.

There are only two fixed points

$$(H,\phi) = (H,0)$$ H arbitrary, saddle point

$$(H,\phi) = (H, 2\sqrt{\xi/\lambda}H)$$ H arbitrary, stable point

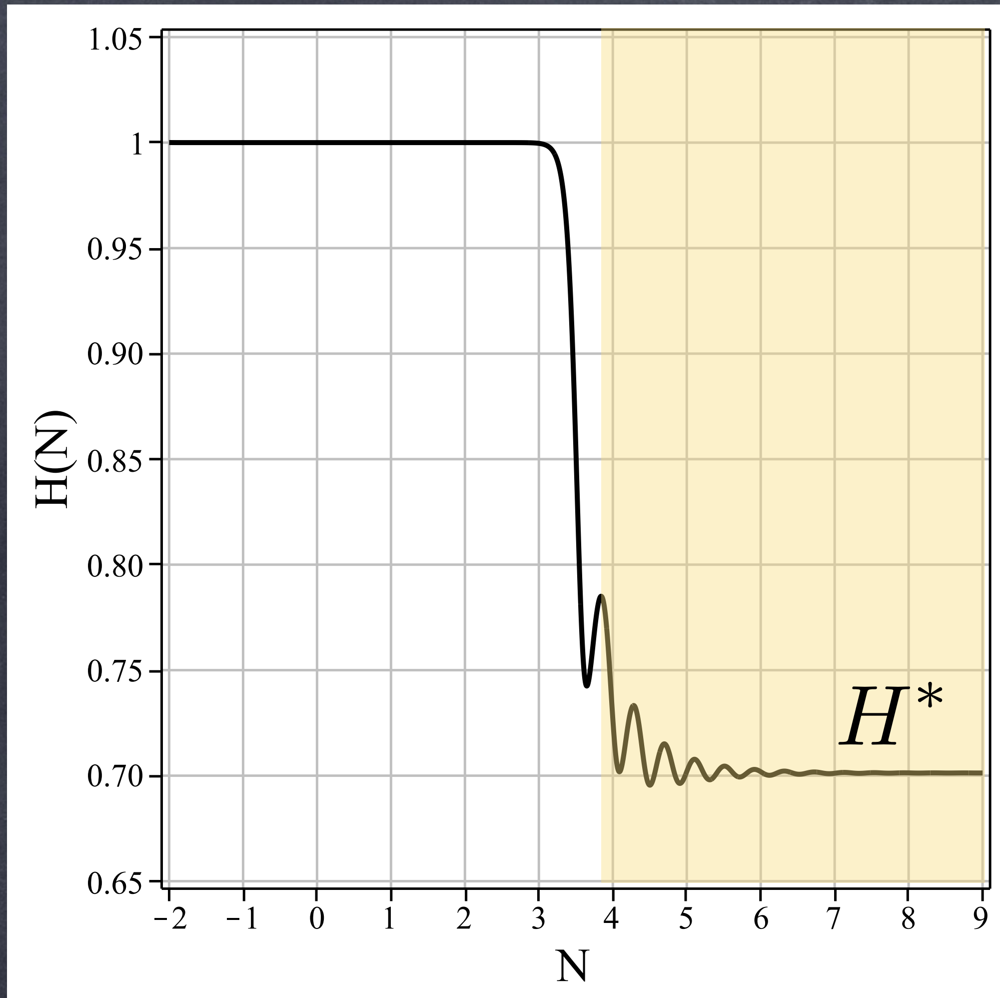
At the stable fixed point: $\Lambda_{\text{eff}} = \frac{\xi^2 R^2}{36\lambda} - \frac{\lambda\phi^4}{4} \to 0$

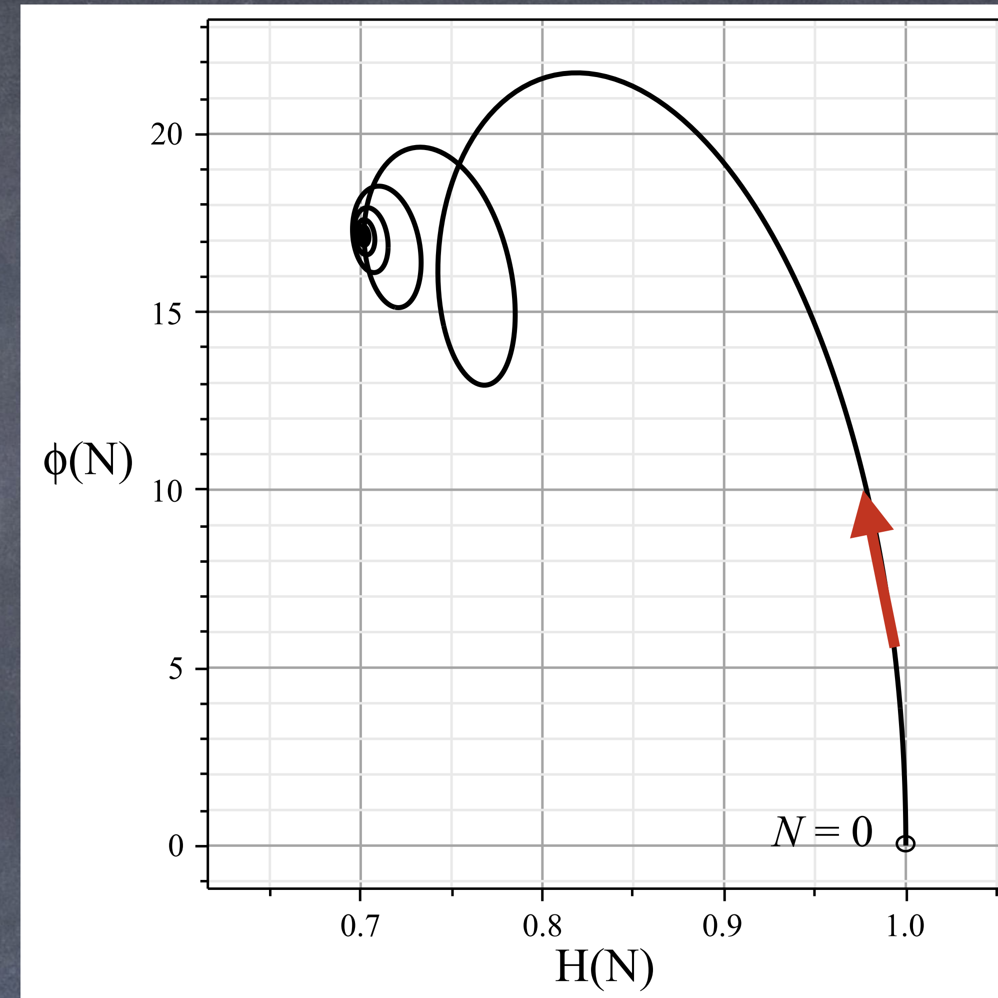
The scalar field is stabilized and a mass scale emerges:

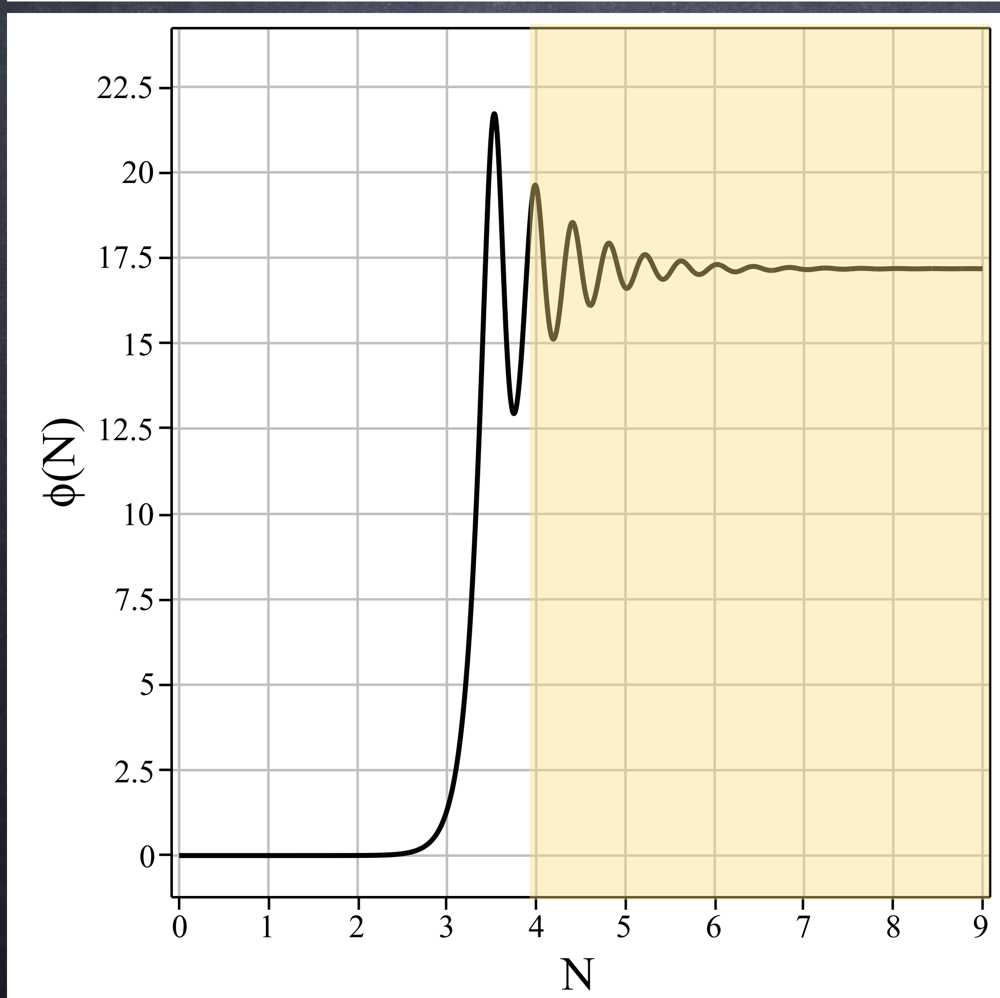
$$M = \sqrt{\xi/3}\,\phi_0 \;\Leftrightarrow\; \alpha = \xi^2/\lambda$$

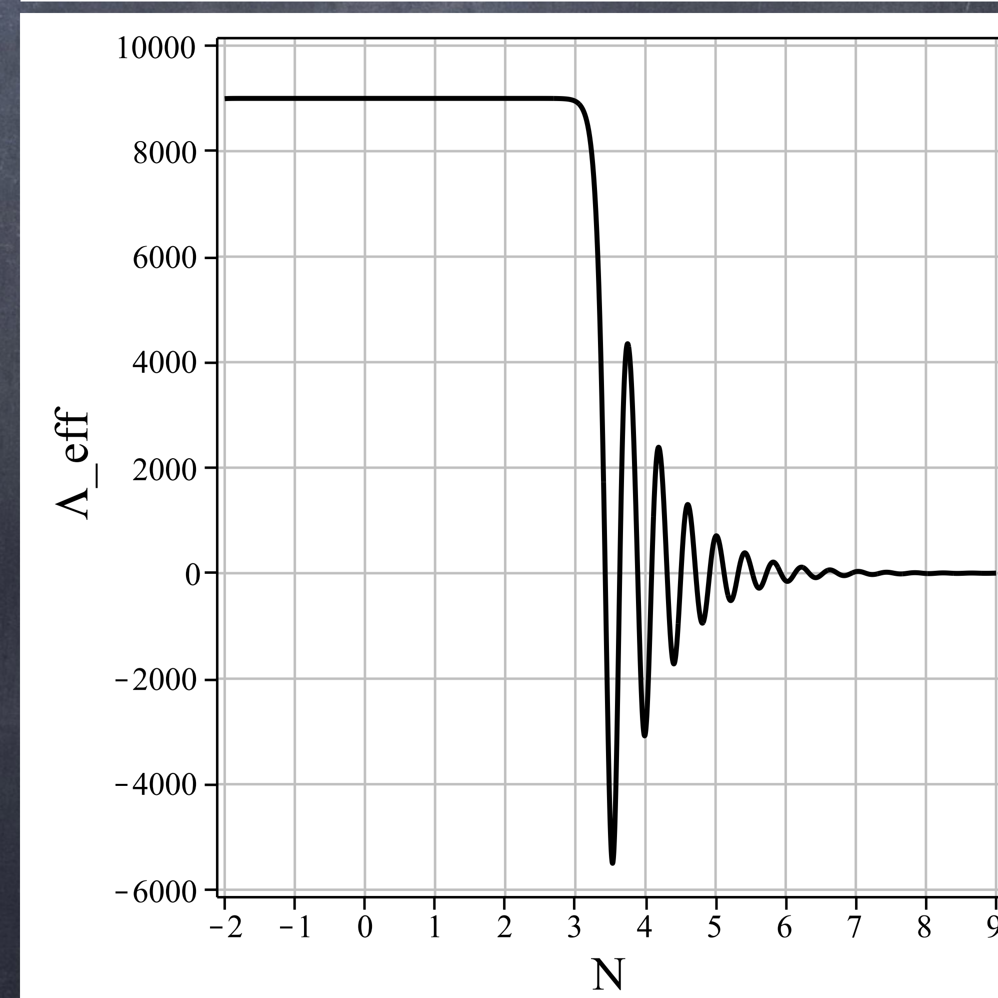
0

$$\mathcal{L}_{\text{inv}} = \sqrt{|\det g|}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right]$$


1.05
1
0.95
0.90
0.85
0.80
0.75
0.70
0.65
H(N)
H*
−2 −1 0 1 2 3 4 5 6 7 8 9
N


20
15
10
5
0
ϕ(N)
N = 0
0.7 0.8 0.9 1.0
H(N)


22.5
20
17.5
15
12.5
10
7.5
5
2.5
0
ϕ(N)
0 1 2 3 4 5 6 7 8 9
N


10000
8000
6000
4000
2000
0
−2000
−4000
−6000
Λ_eff
−2 −1 0 1 2 3 4 5 6 7 8 9
N

What happens in the Einstein frame?

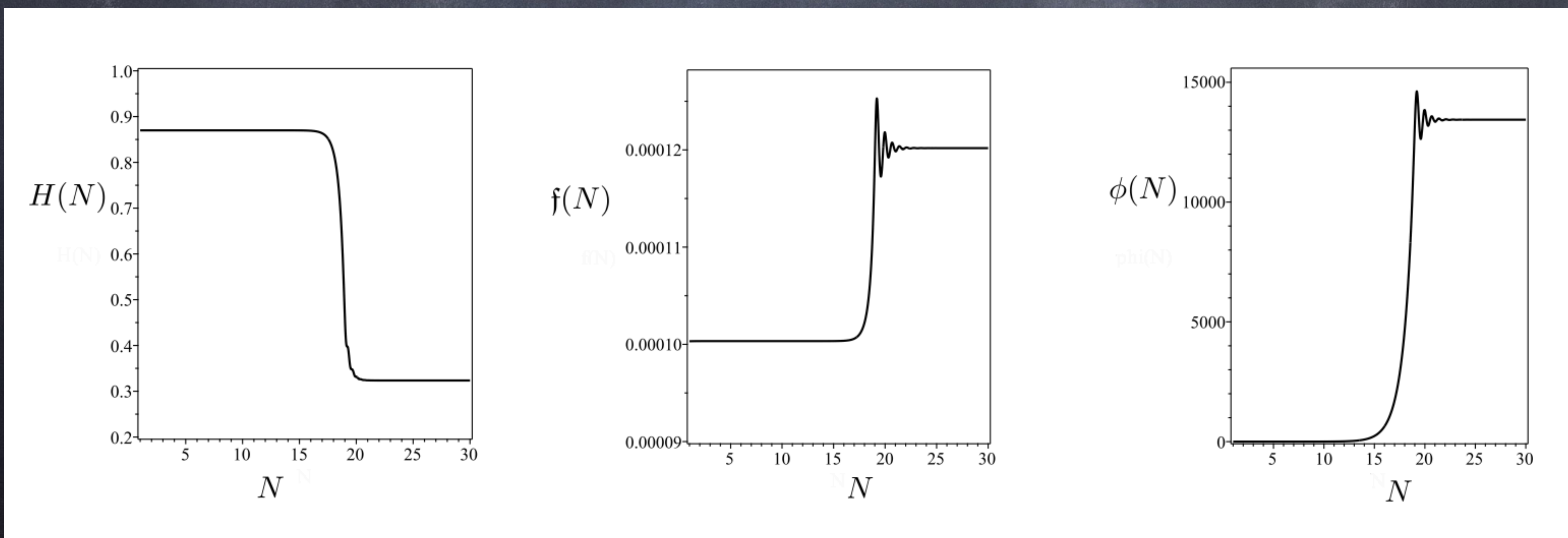
$$\mathscr{L}_{JF} = \sqrt{g}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right]$$

Fields redefinitions ->

$$\mathscr{L}_{EF} = \sqrt{g}\left[\frac{M^2}{2}R - \frac{9M^4}{4\alpha} - \frac{3M^2}{f^2}(\partial f)^2 - \frac{f^2}{2M^2}(\partial\phi)^2 + \frac{3\xi\phi^2 f^2}{2\alpha} - \frac{\Omega\phi^4 f^4}{4\alpha M^4}\right]$$

EH term | non-canonical kinetic terms | quartic potential

$$\Omega = \alpha\lambda + \xi^2$$



Inflationary dynamics

Further simplification - due to scale-invariance

Perform a new field redefinition:

$$\zeta = \sqrt{6}M \operatorname{arcsinh}\left(\frac{f\phi}{\sqrt{6}M^2}\right), \qquad \rho = \frac{M}{2}\ln\left(\frac{\phi^2}{2M^2} + \frac{3M^2}{f^2}\right)$$

New lagrangian:

$$\mathscr{L} = \sqrt{g}\left[\frac{M^2}{2}R - \frac{1}{2}(\partial\zeta)^2 - 3\cosh^2\left(\frac{\zeta}{\sqrt{6}M}\right)(\partial\rho)^2 - U(\zeta)\right]$$

One-field potential only:

$$U(\zeta) = -\frac{9\xi M^4}{\alpha}\sinh^2\left(\frac{\zeta}{\sqrt{6}M}\right) + \frac{9\Omega M^4}{\alpha}\sinh^4\left(\frac{\zeta}{\sqrt{6}M}\right) + \frac{9M^4}{4\alpha}$$

Just recall where we started off:

$$\mathcal{L}_{\rm inv} = \sqrt{|\det g|}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right]$$

We now search for an inflationary solution (1 field inflation):

Small field limit $\zeta \ll M$ leads to "hilltop inflation"? No!

$$U(\zeta) = -\frac{9\xi M^4}{\alpha}\sinh^2\left(\frac{\zeta}{\sqrt{6}M}\right) + \frac{9\Omega M^4}{\alpha}\sinh^4\left(\frac{\zeta}{\sqrt{6}M}\right) + \frac{9M^4}{4\alpha}$$

$$U(\zeta) \approx \frac{9M^4}{4\alpha}\left[1 - \frac{2\xi}{3}\frac{\zeta^2}{M^2} - \frac{1}{9}\left(\frac{\xi}{3} - \Omega\right)\frac{\zeta^4}{M^4} + \mathcal{O}\left(\frac{\zeta^6}{M^6}\right)\right]$$

should vanish (Planck)

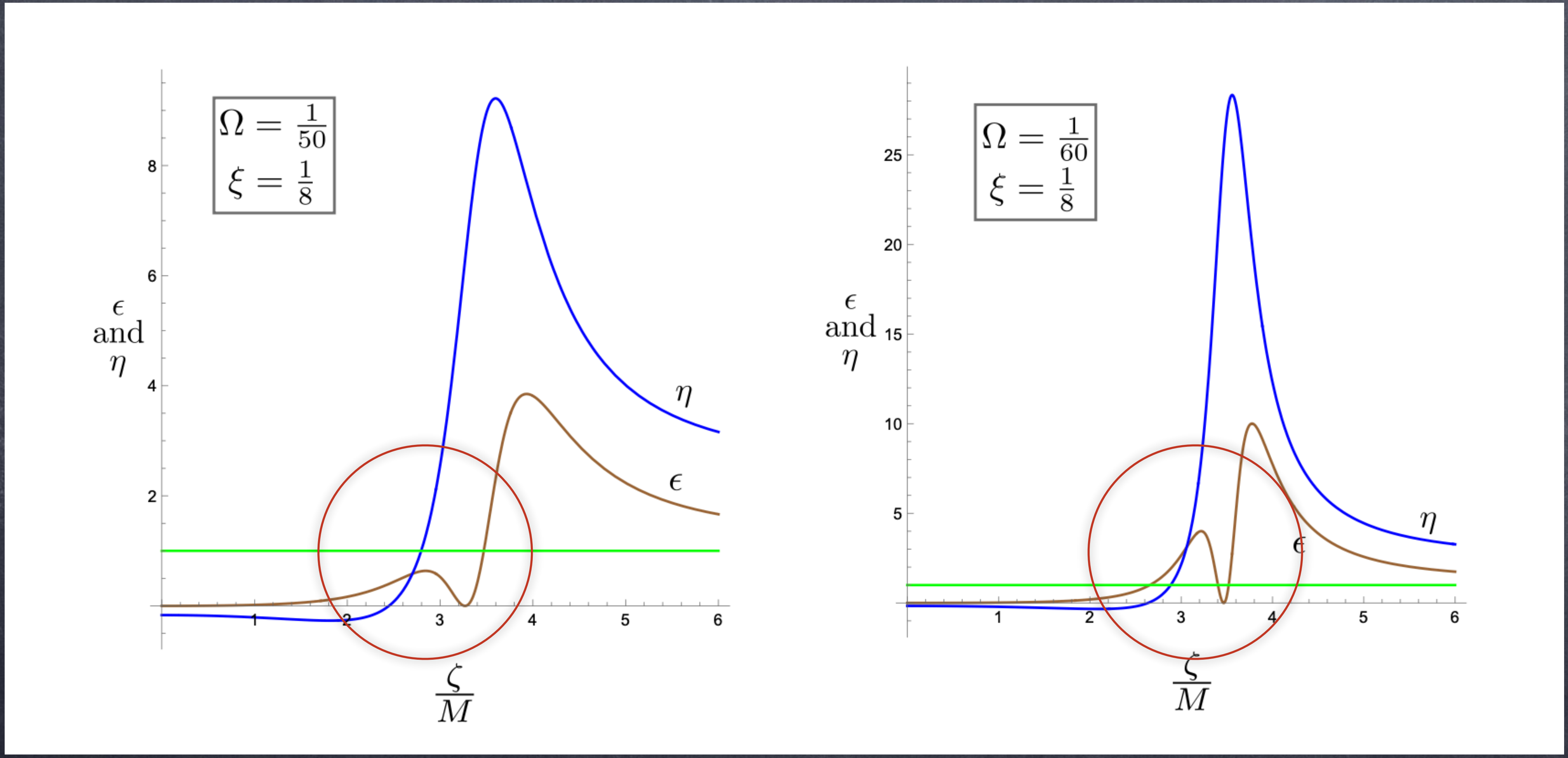
then it has the wrong sign

One can also show that $\Omega > \frac{4}{3}$, $\quad n_s \simeq 1 + \frac{8}{3}\Omega\frac{\zeta}{M}^2 > 1$

Consider $\zeta \geq M$:

Slow-roll parameters $\epsilon = \frac{M^2}{2}\left(\frac{1}{U}\frac{dU}{d\zeta}\right)^2, \qquad \eta = \frac{M^2}{U}\frac{d^2U}{d\zeta^2}$

$$\Delta N = -\frac{1}{M}\int_{\zeta_*}^{\zeta_{\rm end}} \frac{d\zeta}{\sqrt{2\epsilon}} < \infty \;\Leftrightarrow\; \Omega < \frac{2\sqrt{3}}{3}\xi^2 \simeq 1.1547\xi^2$$



Also, slow-roll parameters are well-defined for $\Omega < \frac{1}{64}$

All this implies $\xi < 0.11$

Analytic solutions if approximate (large-ish field approx)

$$U(\zeta) = -\frac{9\xi M^4}{\alpha}\sinh^2\left(\frac{\zeta}{\sqrt{6}M}\right) + \frac{9\Omega M^4}{\alpha}\sinh^4\left(\frac{\zeta}{\sqrt{6}M}\right) + \frac{9M^4}{4\alpha}$$

$$U \to \tilde{U} = \frac{9M^4}{4\alpha}\left[1 - \xi\,\exp\left(\frac{\sqrt{6}\zeta}{3M}\right) + \frac{\Omega}{4}\,\exp\left(\frac{2\sqrt{6}\zeta}{3M}\right)\right]$$

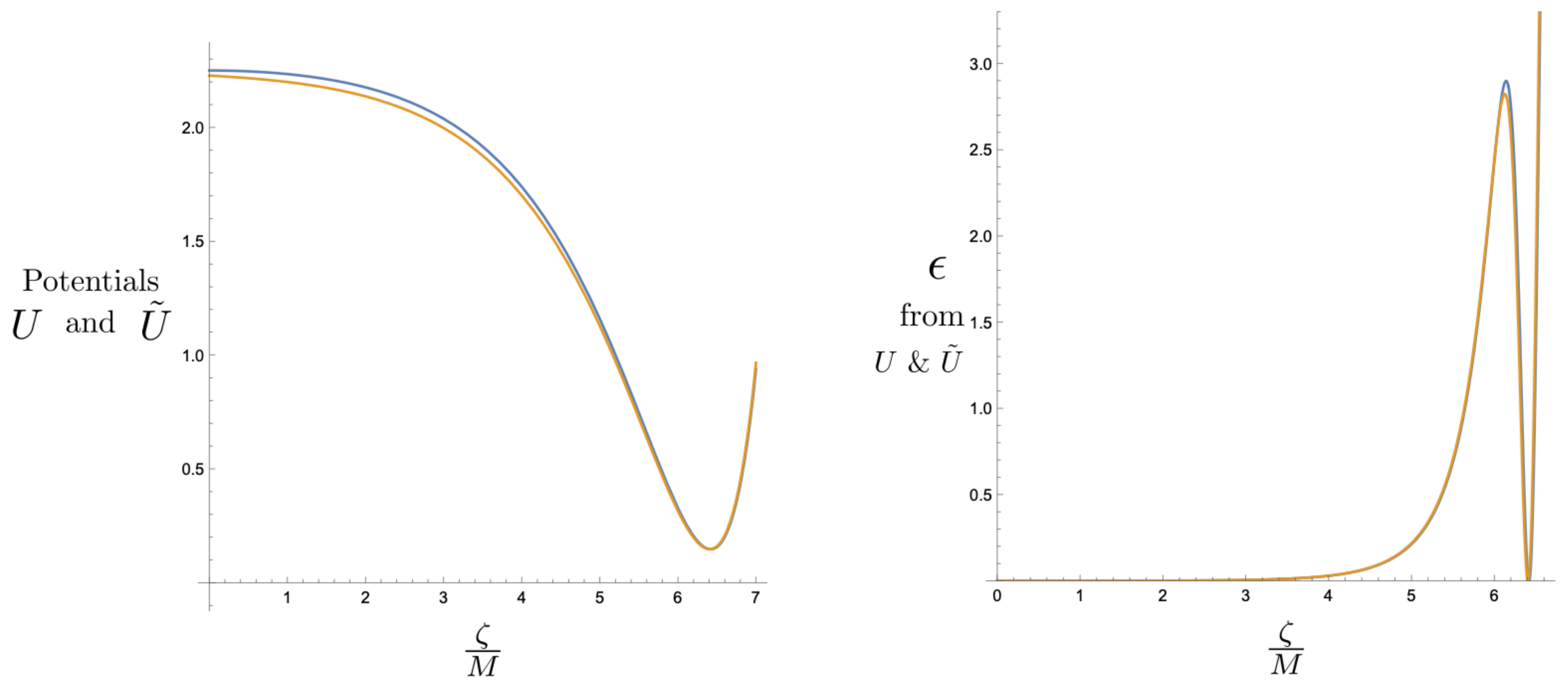


Figure 5. *Plot of U (blue curve) and $\tilde{U}$ (yellow curve) on the left and of ϵ constructed with U (blue curve) and $\tilde{U}$ (yellow curve) with $\Omega = 1.07\,\xi^2$, $\xi = 10^{-2}$, $\alpha = 1$ and $M = 1$.*

Example:

$$\Omega = 1.07\,\xi^2, \quad \xi = 10^{-2}, \quad \Delta N = 60$$

$$\zeta_i = 1.04\,M \quad \rightarrow \quad \zeta_f = 5.637\,M$$ Large field

$$n_s = 0.9679\,, \quad r = 0.003\,, \quad \xi_v^2 = 0.00024\,.$$

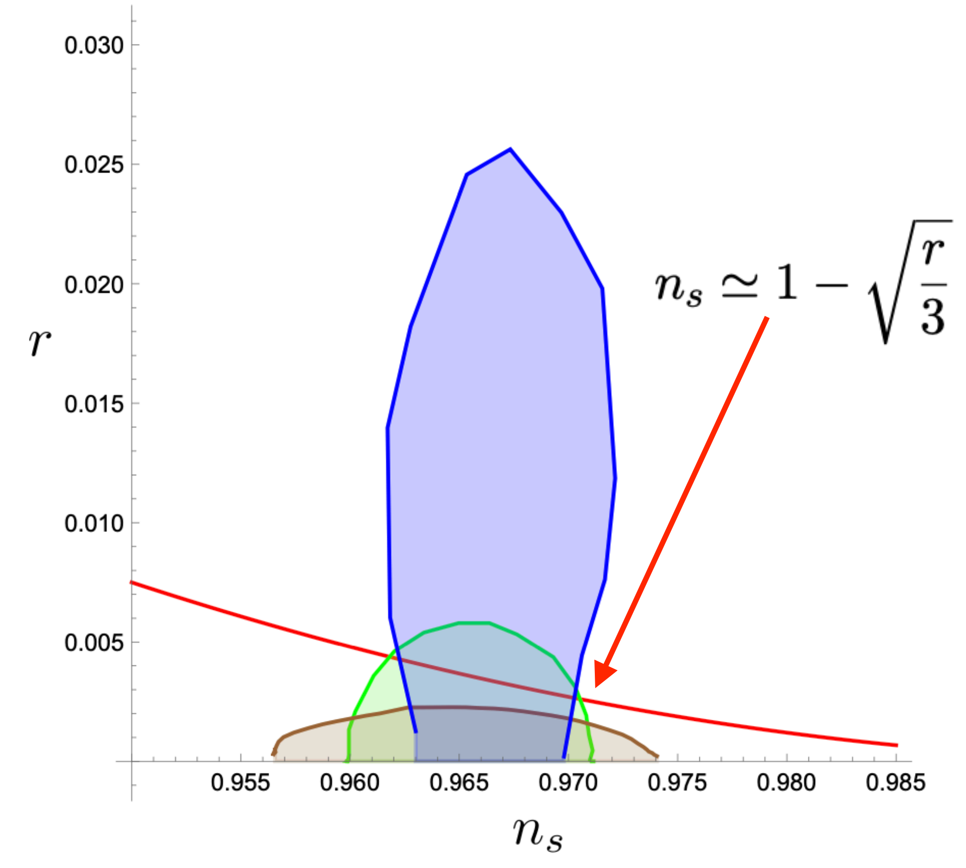


Figure 4. *Plot of r vs n_s following from (5.26). The blue region represents the current PLANCK + BICEP constraints [66–69] , the green region represents the future reach of Simon's Observatory[70] while the brown region depicts the detection range of LiteBIRD [71]*

Summary: starting from the scale-invariant theory

$$\mathscr{L}_{JF} = \sqrt{g}\left[\frac{\alpha}{36}R^2 + \frac{\xi}{6}\phi^2 R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4\right]$$

we show that a mass scale emerges dynamically on a flat RW metric.

In the process, inflation occurs.

The three-parameter space is mostly restricted by the inflationary mechanism (not by observations - the only input is the number of e-foldings).

Compared to the Starobinski model $R + R^2/m^2$: one extra DOF but no energy scale m to be determined.

Compared to Higgs inflation: ξ must be small.

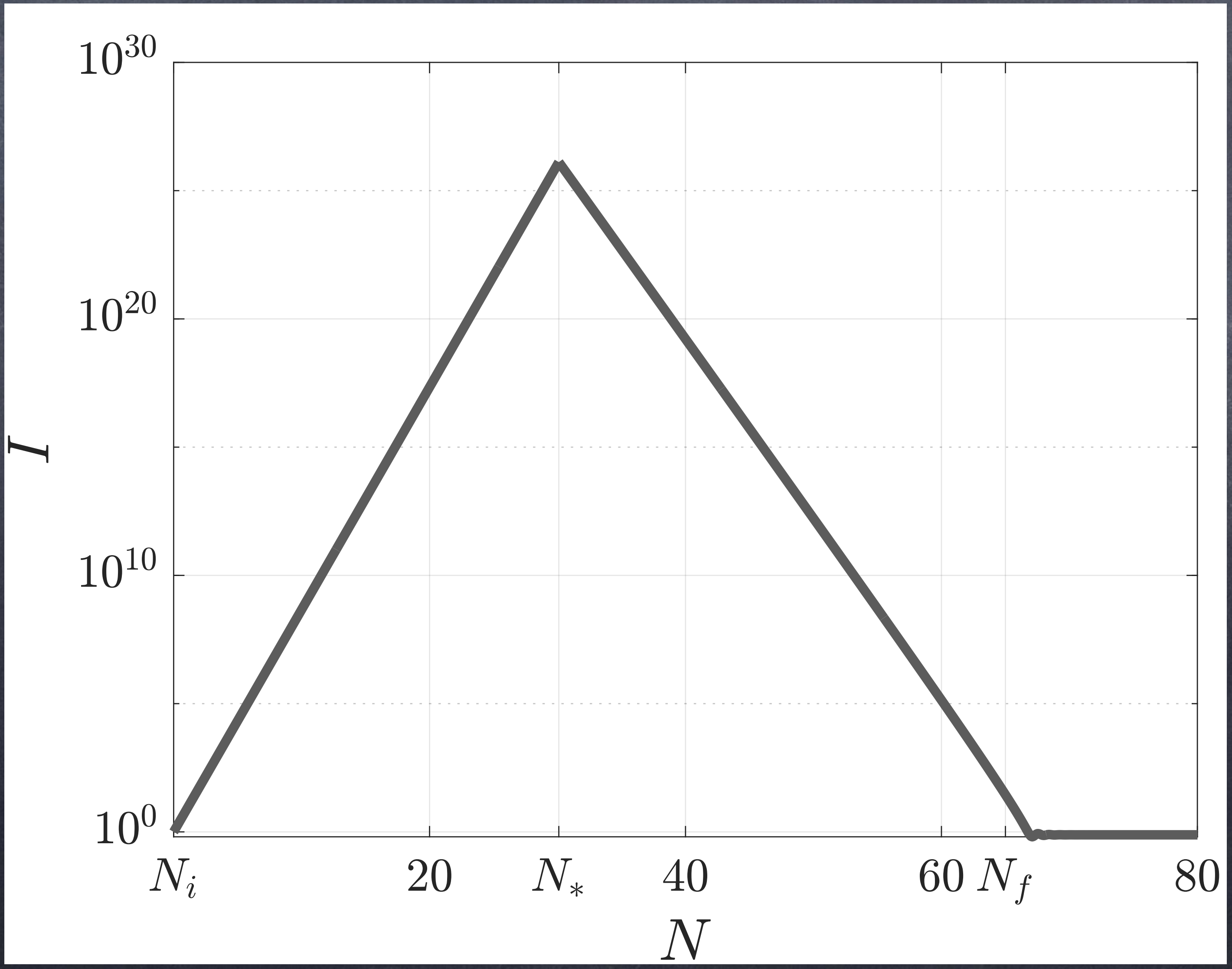
Other directions

Inflationary helical magnetic fields with a sawtooth coupling

Chiara Cecchini (INFN, Trento), Massimiliano Rinaldi (INFN, Trento and TIFPA-INFN, Trento) (Jan 18, 2023)

e-Print: 2301.07699 [astro-ph.CO]

$$S = -\frac{1}{16\pi}\int d^4x\sqrt{-g}\, I^2(\phi)\left(F_{\mu\nu}F^{\mu\nu} + \gamma F_{\mu\nu}\tilde{F}^{\mu\nu}\right) + \int d^4x\sqrt{-g}\,\mathscr{L}_\phi$$



On the stability of scale-invariant black holes

Simon Boudet (INFN, Trento and TIFPA-INFN, Trento), Massimiliano Rinaldi (INFN, Trento and TIFPA-INFN, Trento), Samuele Marco Silveravalle (INFN, Trento and TIFPA-INFN, Trento)

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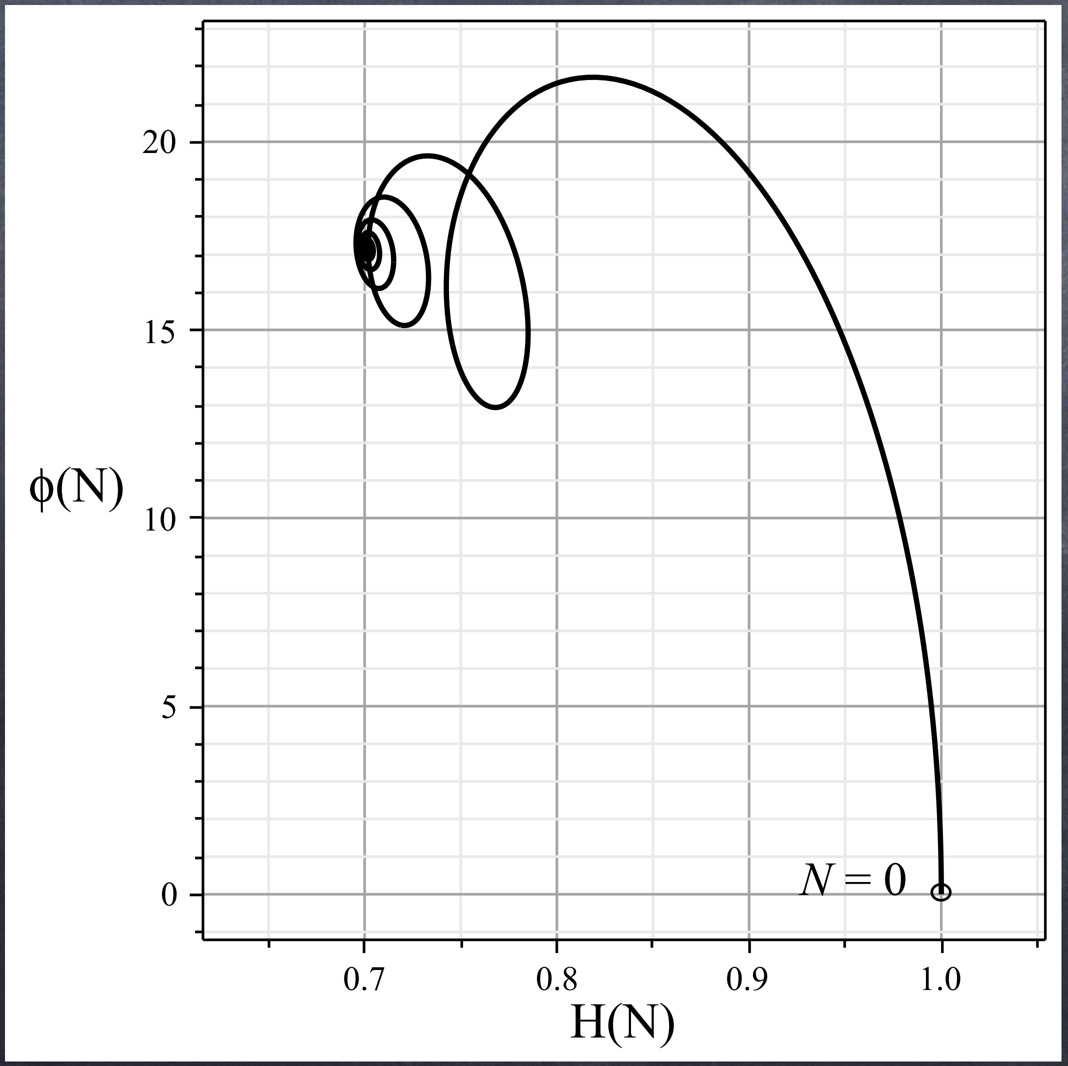
Black holes in the full $f(R, \phi)$ theory

- two dS black hole solutions
- stability/instability transition



CURRENT/FUTURE WORK

- Perturbations - non-gaussianities
- add spatial (positive) curvature: quasi-De Sitter bounce
- non-symmetric solution (hierarchy problem?)

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Marco Silveravalle (INFN, Trento and TIFPA-INFN, Trento)
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