The Axion Flavour connection

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Outline

- Strong CP problem;

- Axion solution and benchmark axion models;
  - Focus on the *origin* and *quality* issues

- The Axion-Flavour connection
  - Cook our recipe: a simple (simplistic?) realisation

Strong CP Problem

- The QCD Lagrangian contains a CP-violating term:

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f - \frac{1}{4}(G_{\mu\nu}, G^{\mu\nu}) + \theta \frac{1}{32\pi^2}(G_{\mu\nu}, \tilde{G}^{\mu\nu})$$

- However, experimental bounds indicate that, in strong interactions,

$$\theta \lesssim 10^{-10}$$

- How can we explain this? Anthropic reasoning? No…

$\rightarrow$ mechanism that constrains $\theta \lesssim 10^{-10}$ or drives it to 0.

Axion Solution

- In 1977, Peccei and Quinn devised a mechanism to dispose of CP violation in strong interactions. Introducing a new anomalous, SB, global U(1) invariance at the Lagrangian level and imposing that at least one quark flavour acquires its mass by coupling to a scalar with non-vanishing VEV, they managed to preserve CP in strong interactions.

- We will now focus on the benchmark axion models, explaining the *origin* and *quality* issues of the PQ symmetry.

Steal a glance on the axion mass

Keep in mind!

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} \Longrightarrow m_a \approx 5.7 \left(\frac{10^{12}\mathrm{GeV}}{f_a} \right) \mu\mathrm{eV}$$

Benchmark Axion Models

- Different ways of implementing the PQ mechanism can be organised in three large classes:

1. The Peccei-Quinn-Weinberg-Wilczek (PQWW) model;
2. The Dine-Fishler-Srednicki-Zhitnitsky (DFSZ) model;
3. The Kim-Shifman-Vainshtein-Zakharov (KSVZ) model.

PQWW Model

- In the PQWW model, the SM is extended by adding a new complex scalar, namely a second Higgs doublet. The Lagrangian contains:

$$\mathcal{L} \supset -y_u \bar{q}_L H_u u_R - y_d \bar{q}_L H_d d_R - V(H_u, H_d) + \text{h.c.}$$

$$V = \frac{\lambda_u}{4}\left(|H_u|^2 - \frac{v_u^2}{2}\right)^2 + \frac{\lambda_d}{4}\left(|H_d|^2 - \frac{v_d^2}{2}\right)^2 + \lambda_{ud}\left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right) + \ldots$$

- One could naively say that, since there are two Higgs fields, there are two independent symmetries, which can be redefined to obtain the hypercharge and an orthogonal *accidental* Peccei-Quinn symmetry. Is it true? NO!

PQWW Model

- The general two Higgs doublet model potential, in fact, can be written as

$$V(H_u, H_d) = m_u^2 H_u^\dagger H_u + m_d^2 H_d^\dagger H_d - \left[m_{ud}^2 H_u^\dagger H_d + \text{h.c.}\right] + \frac{1}{2}\lambda_1 \left(H_u^\dagger H_u\right)^2 +$$
$$+ \frac{1}{2}\lambda_2 \left(H_d^\dagger H_d\right)^2 + \lambda_3 \left(H_u^\dagger H_u\right)\left(H_d^\dagger H_d\right) + \lambda_4 \left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right) +$$
$$+ \left[\frac{1}{2}\lambda_5 \left(H_u^\dagger H_d\right)^2 + \lambda_6 \left(H_u^\dagger H_u\right)\left(H_u^\dagger H_d\right) + \lambda_7 \left(H_d^\dagger H_d\right)\left(H_u^\dagger H_d\right) + \text{h.c.}\right],$$

- Therefore, if we want a PQ symmetry, we must *impose* that some terms are absent. *No accidental PQ symmetry*

ACHTUNG!

- In general, all the aforementioned axion models **DO NOT** feature an *accidental* PQ symmetry.
- “But, Clemente, why did we focus on the fact that the PQ symmetry was not *accidental*?”
  For two reasons:
  - Global symmetries are widely believed not to be fundamental in QFT;
  - Being anomalous, $U(1)_{PQ}$ is not a symmetry of the quantum world.

Origin and quality of the PQ symmetry

- Therefore, we should account for the *origin* of the PQ symmetry. It would be desirable that it came accidentally, as a result of imposing “sacred” principles: Lorentz and gauge invariance

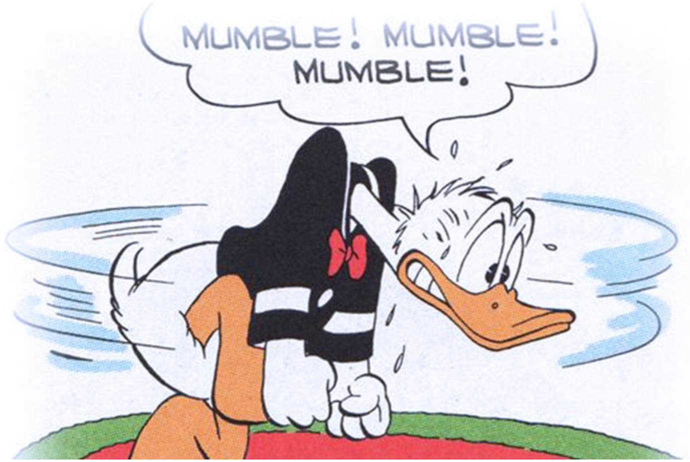
- Moreover, experimental bounds constrain $\theta \lesssim 10^{-10}$, so our symmetry must be highly protected. This is commonly referred to as the PQ *quality* issue.

Origin and quality of the PQ symmetry

- Various constructions that enforce a *high quality, accidental* PQ symmetry have been proposed, but they all rely on imposing the dimension of the first PQ breaking operator *by hand*

Not satisfying...

- Thus, we need a mechanism that enforces a *high quality, accidental* PQ symmetry without imposing any condition by hand.



The Flavour Puzzle

- On a completely different note, the fermion mass hierarchy problem represents one among the most puzzling features of the Standard Model. In two lines, there is a 5 order of magnitude difference between the Yukawa couplings of the top quark and of the up quark.

The Axion Flavour connection: a top-down approach

- We will assume that the SM flavour pattern is generated by a SB flavour symmetry, which will be identified requiring that
  - It must automatically enforce an *accidental* global anomalous PQ symmetry;
  - It must protect $U(1)_{PQ}$ up to a sufficiently large operator dimension.
- Then, we will analyse whether it reproduces, upon SSB, the observed pattern of quark mass hierarchies.

The Axion Flavour Connection: Rectangular Symmetries

- Let us consider the following examples of flavour symmetries:
  - $SU(N)_L \times SU(N)_R$: Y in the bifundamental, *det(Y)* is PQ-violating;
  - $SU(M)_L \times SU(N)_R\ M \neq N$: Y in the bifundamental, we cannot write *det(Y)*

- Therefore, rectangular symmetries are more effective, as for the *quality* issue.

The Axion Flavour Connection: Model Building

- In order to construct a sensible model, our *modus operandi* will be oriented by the following pillars and requirements:
  - *Simplicity*: simplest consistent gauge group and smallest number of fermions;
  - *Phenomenology*:
    - *Masses*: top mass at tree level from renormalizable coupling to Higgs. Up and charm from effective operators
    - *Mixings*: field content sufficiently rich to generate all the masses and mixings of the quarks
  - *Gauge anomalies*: gauge symmetries must be anomaly free
  - *PQ origin and quality*: accidental and highly protected PQ symmetry.

Cook our recipe: a simple realisation

- The “simplest” model complying with all these requirements is a 2HDM containing 6 scalars (*X, Y, Z, K,* H_u, H_d) and a 7x7 fermion mass matrix, transforming under the (G_{SM} x) G_F x $U(1)_F$ gauge group, where
  $G_F = SU(3)xSU(2)$
- Set of fermions:

$$q_L \sim (\mathbf{3},\mathbf{1}),\ u_R \sim (\mathbf{1},\mathbf{2}),\ t_R \sim (\mathbf{1},\mathbf{1}),\ Q_L \sim (\mathbf{1},\mathbf{1}),$$
$$U_R \sim (\mathbf{3},\mathbf{1}), U_L \sim (\mathbf{1},\mathbf{2}), T_L \sim (\mathbf{1},\mathbf{1}), Q_R \sim (\mathbf{1},\mathbf{1}).$$

Cook our recipe: a simple realisation

- The terms we should allow for in the Lagrangian should be chosen according to the aforementioned pillars. Among them:

  $H_u \bar{q}_L^\alpha U_{R,\alpha}$, $\Lambda_u \bar{U}_L^i u_{R,i}$ → necessary to ensure nonzero determinant;

  $H_u \bar{Q}_L t_R$ → necessary to ensure nonzero determinant and gives rise to a mass term of order *O(v)* at tree level;

  $\bar{q}_L^\alpha Z_\alpha Q_R$ → Only three up type quarks present below the EW scale.

Cook our recipe: a simple realisation

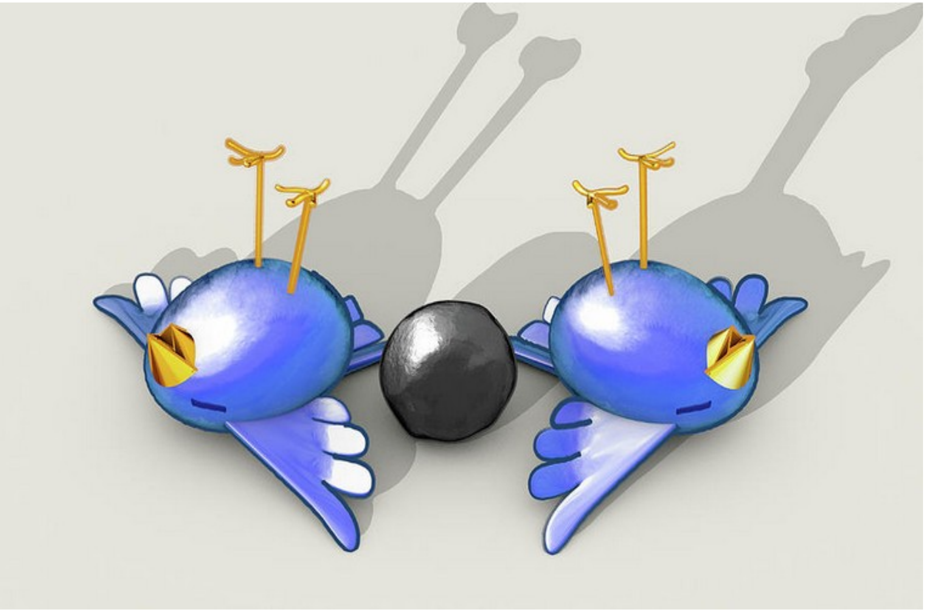
- In the models analysed, we were able to retrieve compatible quark mass hierarchies from non-hierarchical (or mildly hierarchical) input parameters without imposing any number *by hand.*

	model	experimental
m_b(GeV)	1.5	1.5
m_c(GeV)	0.5	0.4
m_s(MeV)	20	30
m_d(MeV)	0.5	1.5
m_u(MeV)	0.3	0.7

Numerical values of the quark masses and their experimental values evolved at the scale of 10^8 GeV. m_t = 102.5GeV

The Axion Flavour connection: conclusion and prospects

- As we have shown, the axion-flavour connection is a sensible ansatz to tackle the SM flavour puzzle and the strong CP problem in one fell swoop.
- Moreover, the QCD-axion could also be a good CDM candidate



The Axion Flavour connection: conclusions and prospects

- I have a dream! Flavour + Strong CP problem + CDM
- Research strategies
  - Implement powerful minimisation routines (any suggestion accepted!!!) and try to obtain CKM + Hierarchies simultaneously
  - Extend the gauge symmetry (trade off with simplicity though)
  - Extend to the lepton sector
  - Dream! (and pray...)

Thanks for your attention!

BACKUP SLIDES

Hierarchical structure of the VEVs

- The minimisation of the scalar potential of the theory under examination yields hierarchical entries in the VEVs of the scalar fields, namely:

$$|x_1|,\ |y_{1,2}|,\ |z_3|,\ |k_2|\ \sim\ O(\Lambda),\ \ |x_2|, |z_{1,2}|\ \sim O(10^{-5}\Lambda),\ |k_{1,3}|\ \sim O(10^{-6}\Lambda)$$

$$\Lambda\ =\ 10^{11}\,\text{GeV}$$

- This intuitively explains how we can get hierarchies starting from non-hierarchical (or mildly hierarchical) input parameters

PQ quality: the role of higher-dim operators

- In general, a PQ-breaking operator can be written as:

$$\eta \frac{\Phi^D}{M_P^{D-4}} + \text{h.c.} \approx \left(\frac{f_a}{M_P}\right)^{D-4} f_a^4 \cos\left(\frac{a}{f_a} + \xi_\eta\right)$$

- The total potential (QCD + PQ-break) would then be

$$V \supset -m_\pi^2 f_\pi^2 \cos\left(\frac{a}{f_a}\right) + \left(\frac{f_a}{M_P}\right)^{D-4} f_a^4 \cos\left(\frac{a}{f_a} + \xi_\eta\right)$$

- To comply with $\theta \lesssim 10^{-10}$, then

$$\left(\frac{f_a}{M_P}\right)^{D-4} f_a^4 \lesssim 10^{-10} m_\pi^2 f_\pi^2 \to D \gtrsim 8\,(13) \Leftrightarrow f_a \lesssim 10^8\,(10^{13})$$

You cannot have your cake and eat it too: a no-go theorem

- The Yukawa Lagrangian can be written as

$$\mathcal{L}_Y = \sum_{i,j} \lambda_{ij}\, \bar{Q}_{m_i} Y_{m_i n_j} q_{n_j}$$

for which the mass matrix is

$$\mathcal{M} = \left(\begin{array}{c|c|c|c|c} \lambda_{m_1 n_1} Y_{m_1 n_1} & \lambda_{m_1 n_2} Y_{m_1 n_2} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \lambda_{m_2 n_3} Y_{m_2 n_3} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \lambda_{m_i n_j} Y_{m_i n_j} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right)$$

You cannot have your cake and eat it too: a no-go theorem

- In order to have massive quarks, we need to require

$$\mathrm{Det}\left(\mathcal{M}\right) = \varepsilon_{\alpha_1\alpha_2...\alpha_N}\mathcal{M}_{1\alpha_1}\mathcal{M}_{2\alpha_2}...\mathcal{M}_{N\alpha_N} \neq 0$$

- Such an operator has a charge proportional to the anomaly

$$\mathcal{A}_{PQ} \propto \sum_i m_i \mathcal{X}_{Q_{m_i}} - \sum_j n_j \mathcal{X}_{q_{n_j}} \neq 0$$

and thus breaks PQ at dimension N. Thus, if we want massive quarks and a PQ colour anomaly, we cannot help but break PQ at dimension N.

Axion Dark Matter

- The axion is an excellent candidate for CDM. It is mainly produced through the *misaligment mechanism.* Here we give some equations

$$\ddot{a} + 3H\dot{a} - \frac{1}{R^2(t)}\nabla^2 a + V'(a,T) = 0$$

$$\rho_a = \frac{1}{2}\dot{a}^2 + \frac{1}{2}m^2(T)a^2$$

$$\dot{\rho}_a = \dot{a}\ddot{a} + m\dot{m}a^2 + m^2 a\dot{a}$$

$$\langle \dot{\rho}_a \rangle = \left\langle m^2 a^2 \right\rangle \left(-3H + \frac{\dot{m}}{m} \right)$$

$$\frac{\langle \rho_a \rangle R^3}{m} = const \Rightarrow \langle n \rangle = \frac{\langle \rho_a \rangle}{m} \propto \frac{1}{R^3}$$

Axion Dark Matter

- For reference, the resulting dark matter abundance is

$$\Omega_a h^2 \approx 0.12 \left(\frac{28\mu\text{eV}}{m_a}\right)^{7/6} = 0.12 \left(\frac{f_a}{2 \times 10^{11}\text{GeV}}\right)^{7/6}$$

- Thermal production is negligible, as it gives

$$\Omega_a \approx 10^{-8} \left(\frac{10^{12}\text{GeV}}{f_a}\right)$$