



UNIVERSITÀ DI PISA

Primordial black holes from stochastic tunnelling

24 January 2023, Pisa



Chiara Animali

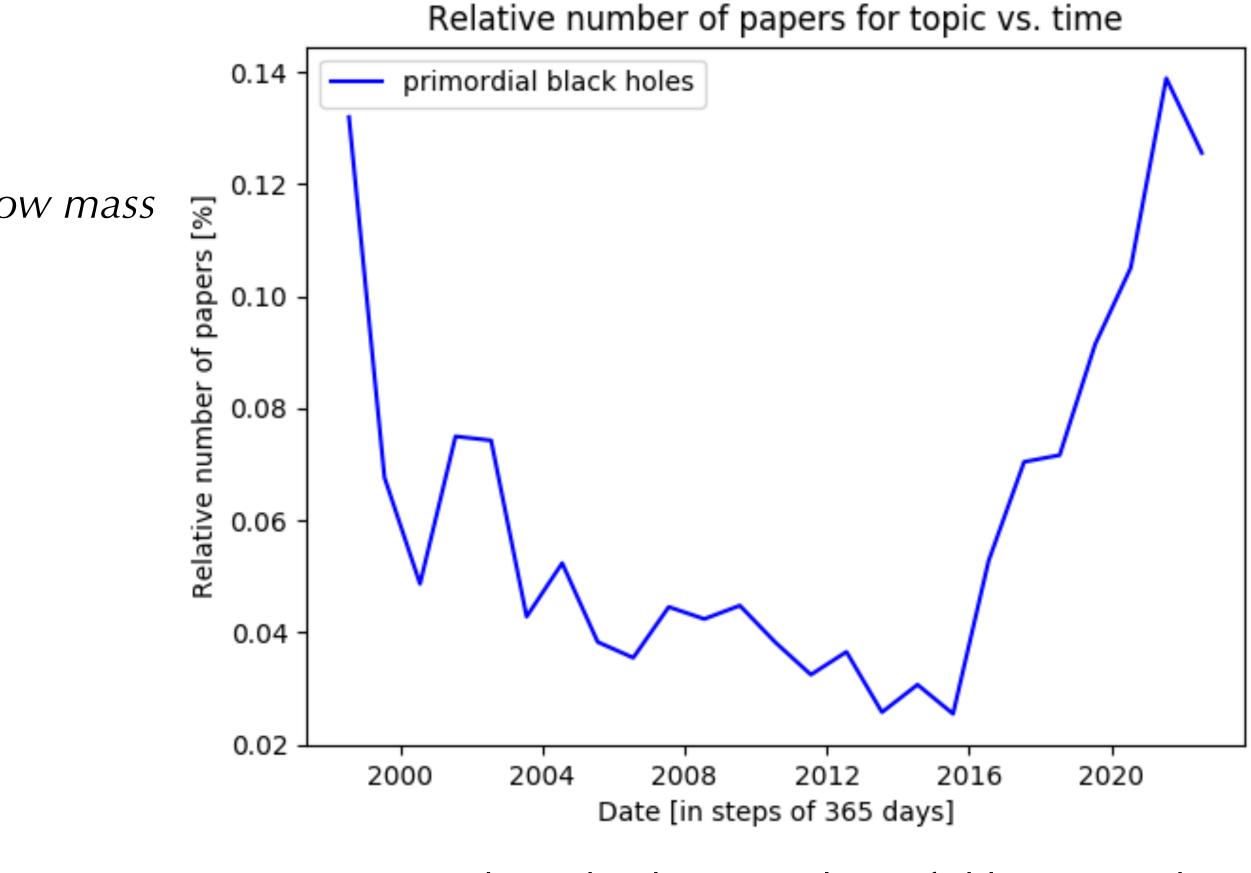
DarkCosmoGrav Conference

Black holes which could have formed in the early Universe through a non-stellar way

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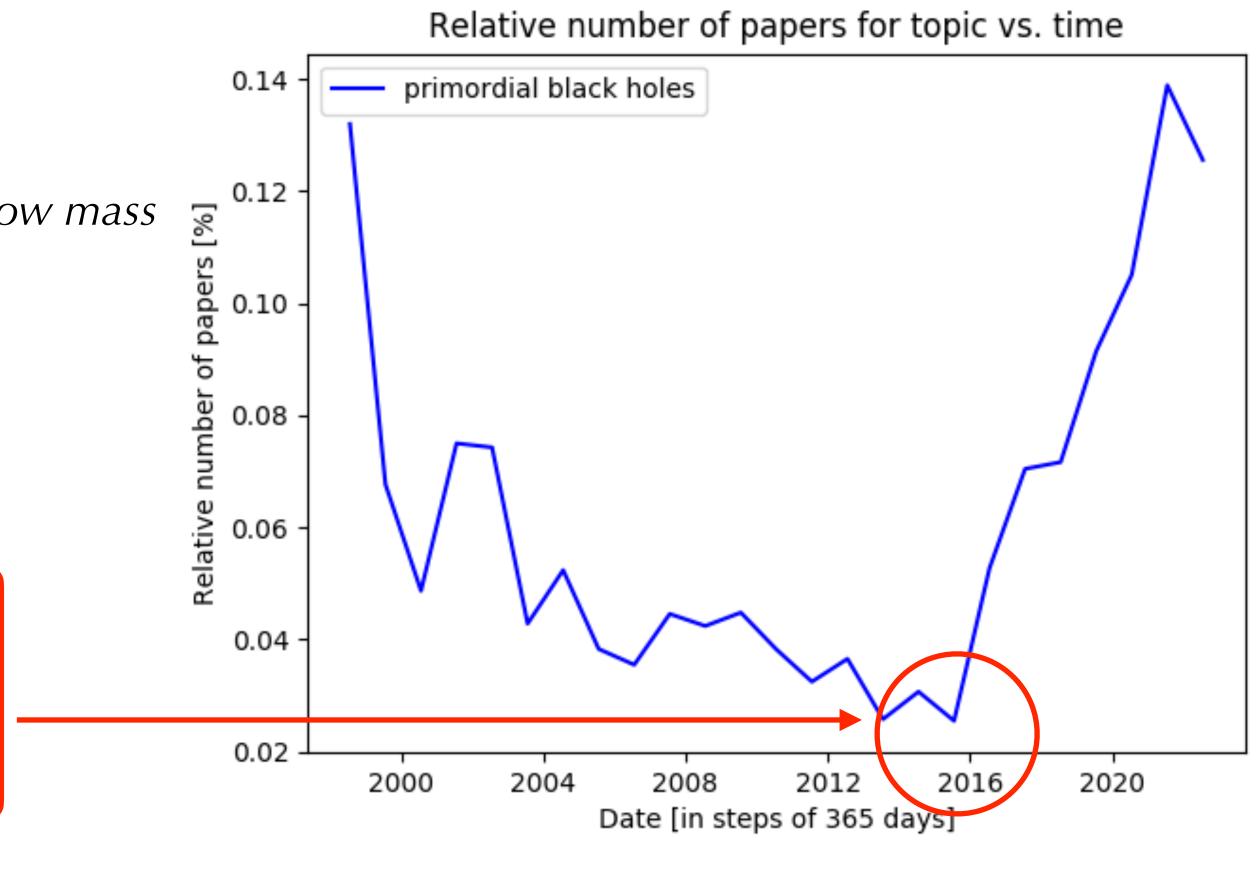
[Plot realised via www.benty-fields.com/trending]



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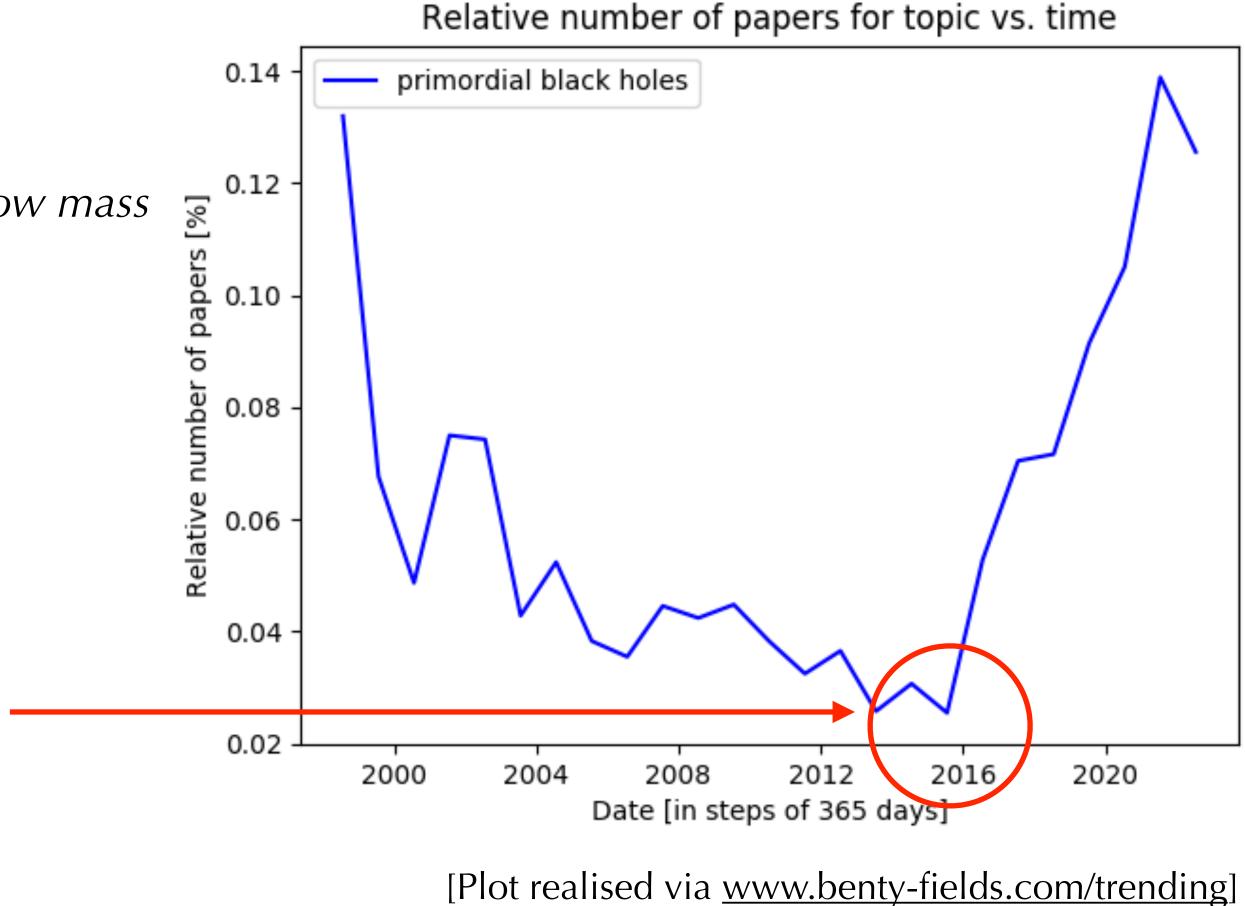


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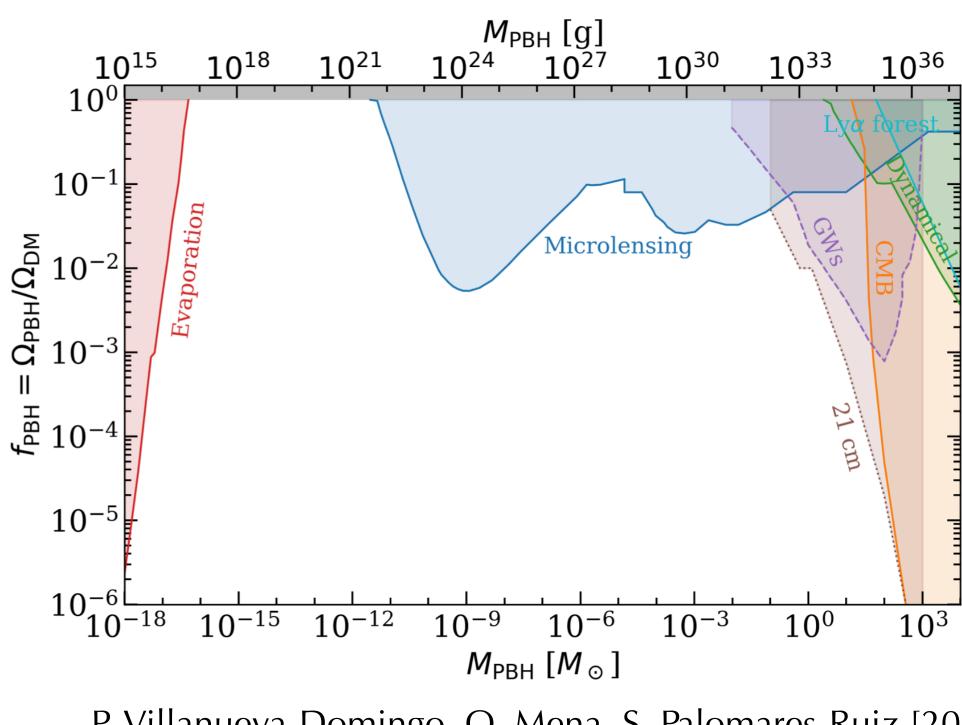
S. Bird, I. Cholis, J.B. Muñoz, Y. Ali-Haïmoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, A. G. Riess [2016]: Did LIGO detect dark matter?





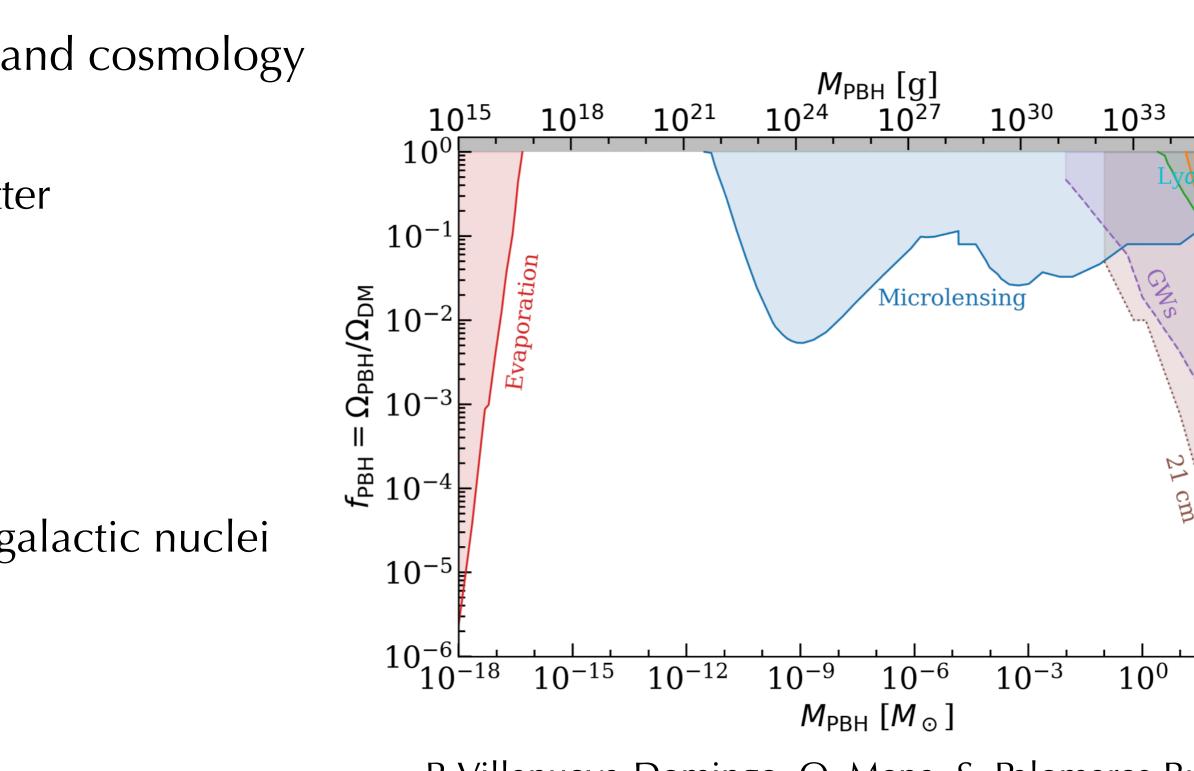
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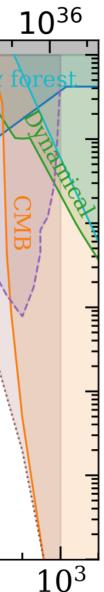


P. Villanueva-Domingo, O. Mena, S. Palomares-Ruiz [2021] A brief review on primordial black holes as dark matter

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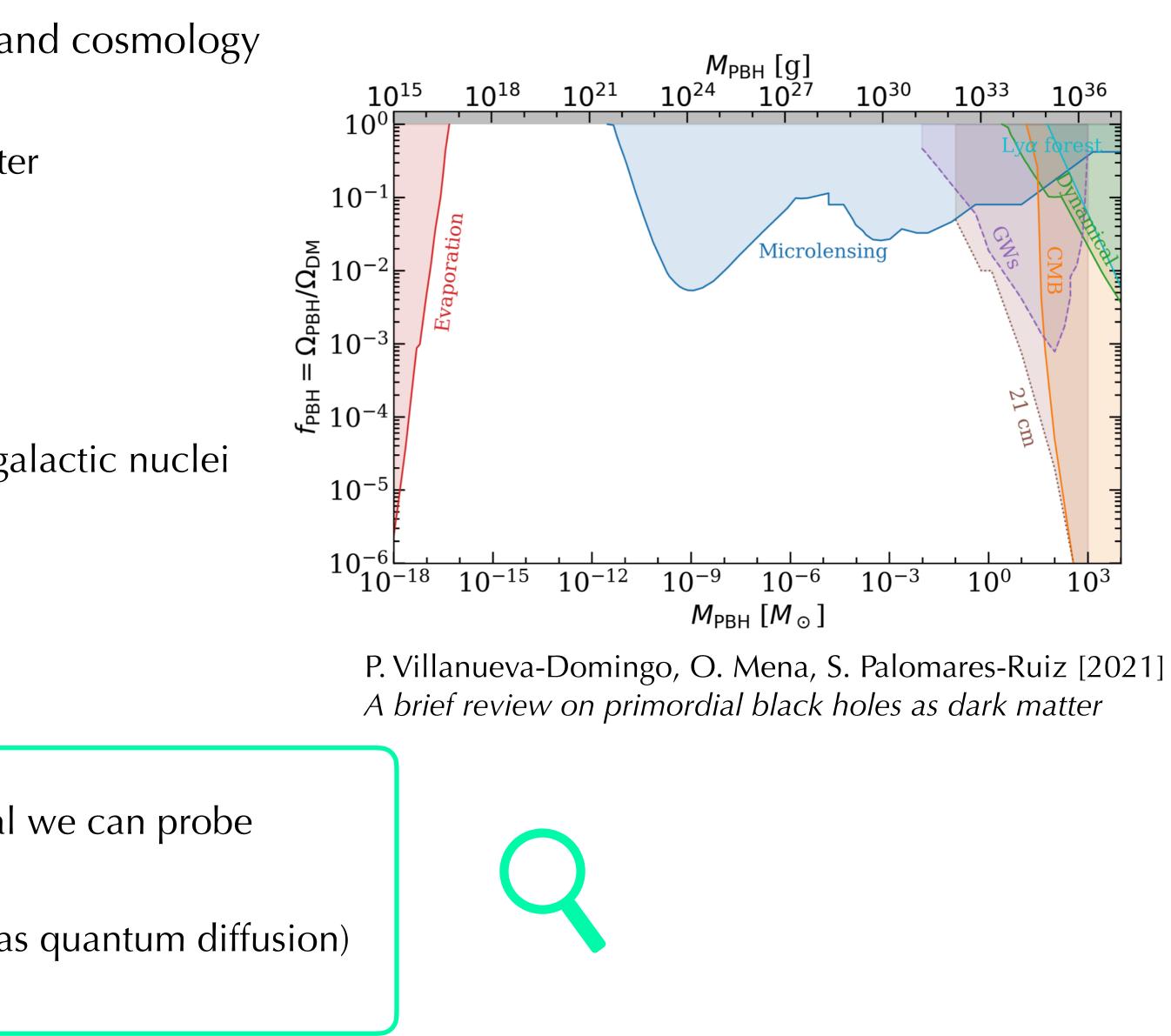


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- They could extend the region of the inflationary potential we can probe
- They provide a place to look for quantum effects (such as quantum diffusion)





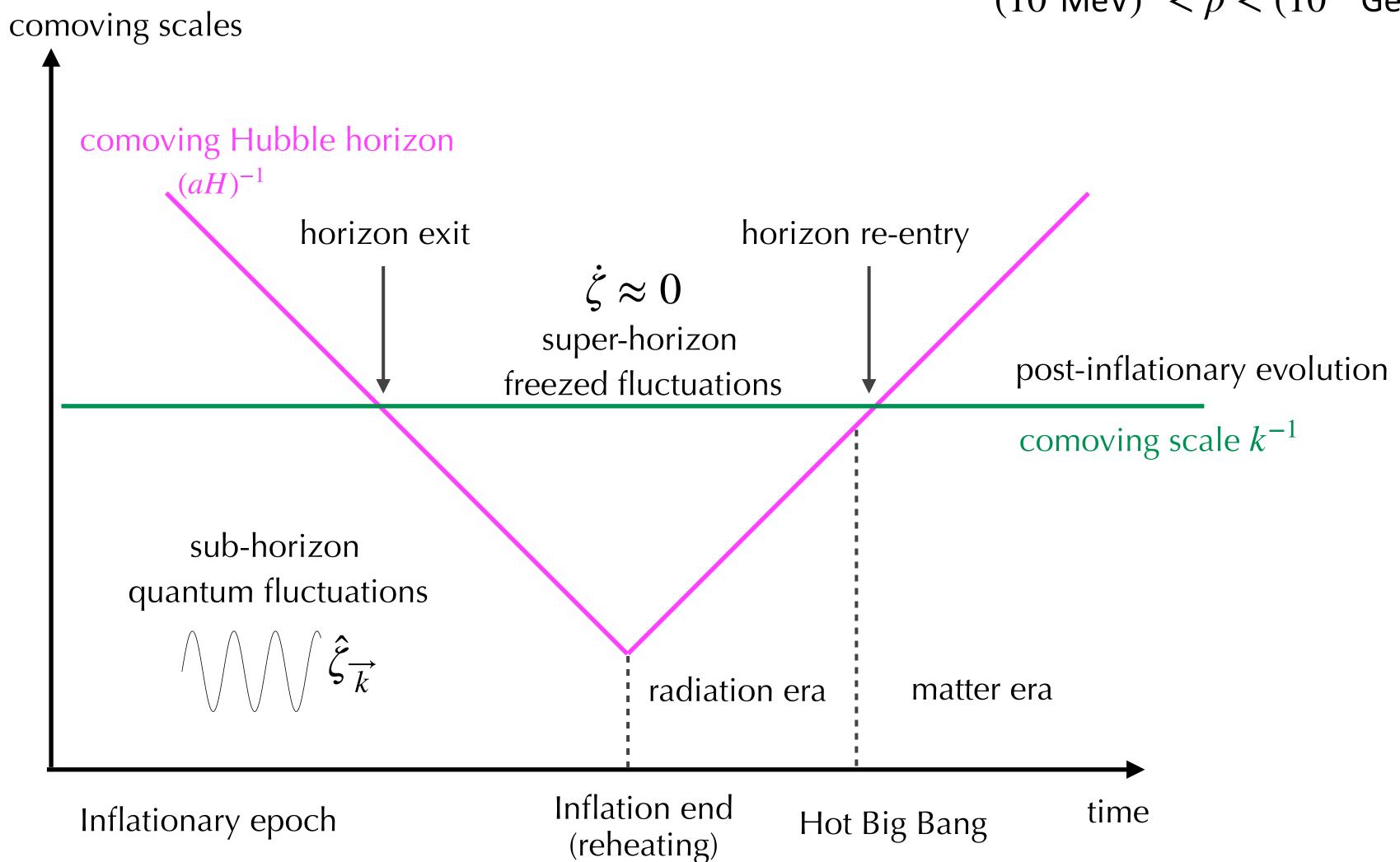
High energy phase of accelerated expansion of spacetime

$$ds^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^{2} + a^{2}(t)d\vec{x}^{2} \qquad \dot{a}, \ddot{a} > 0$$

 $(10~{\rm MeV})^4 < \rho < (10^{16}\,{\rm GeV})^4$



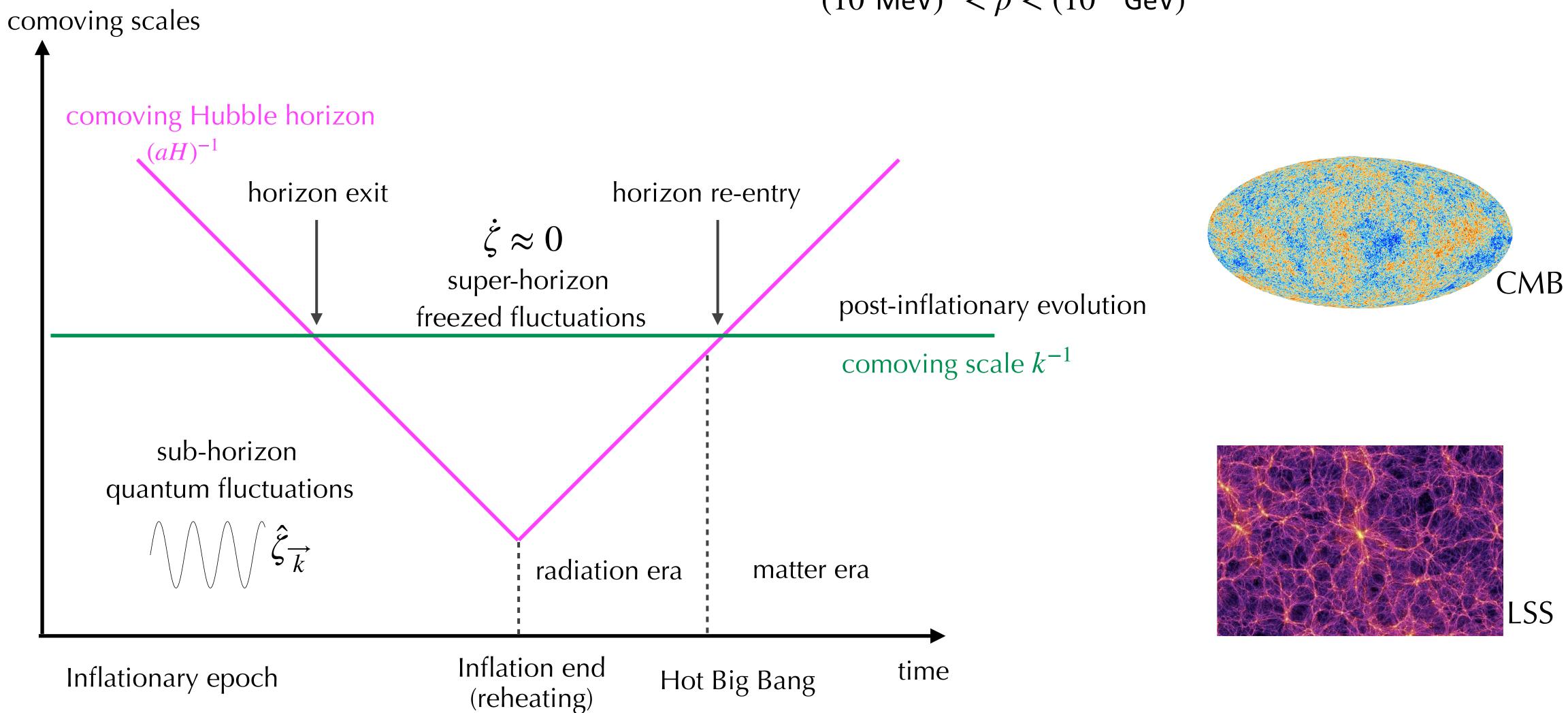
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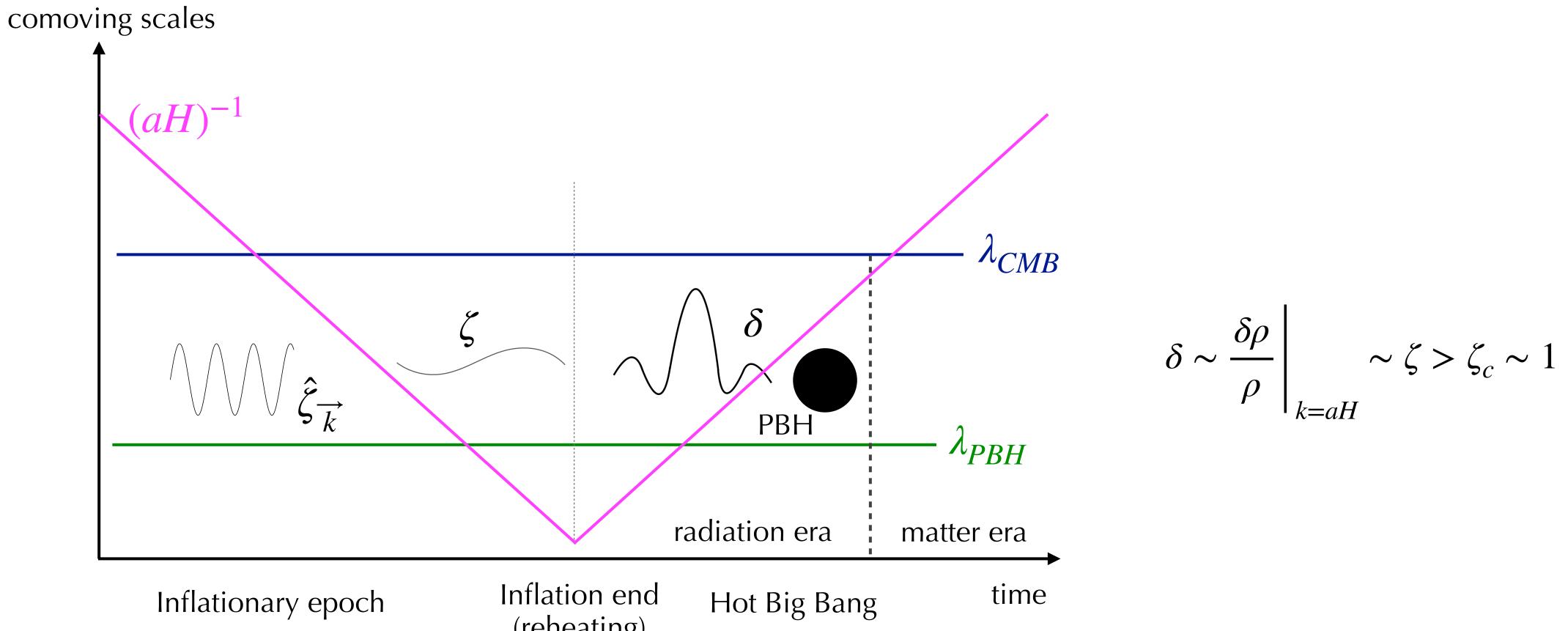
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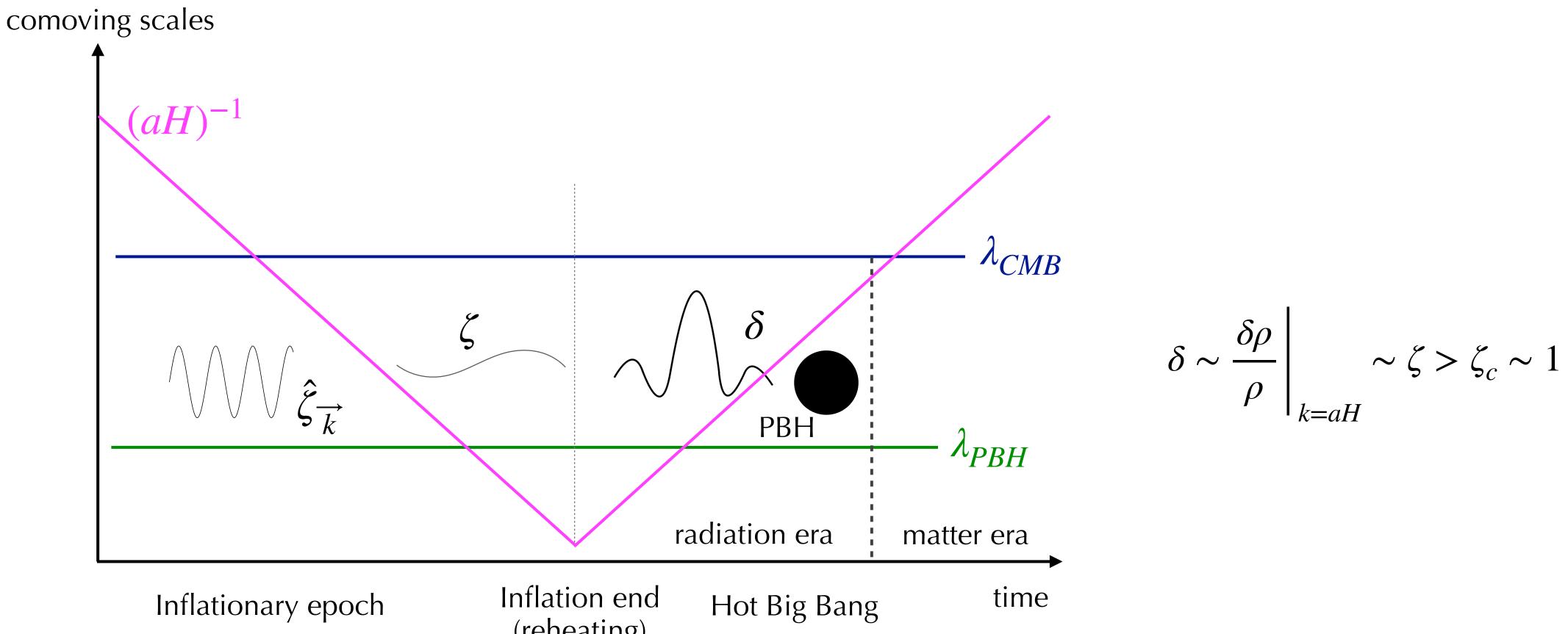


BHs may be originated from peaks of the density perturbations generated in the early universe



(reheating)

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Can inflation give rise to such enhanced perturbations?



Simplest realisation: slow-roll inflation

scalar field ϕ (inflaton) slowly rolling towards the minimum of its potential

$$S_{\phi} = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right)$$

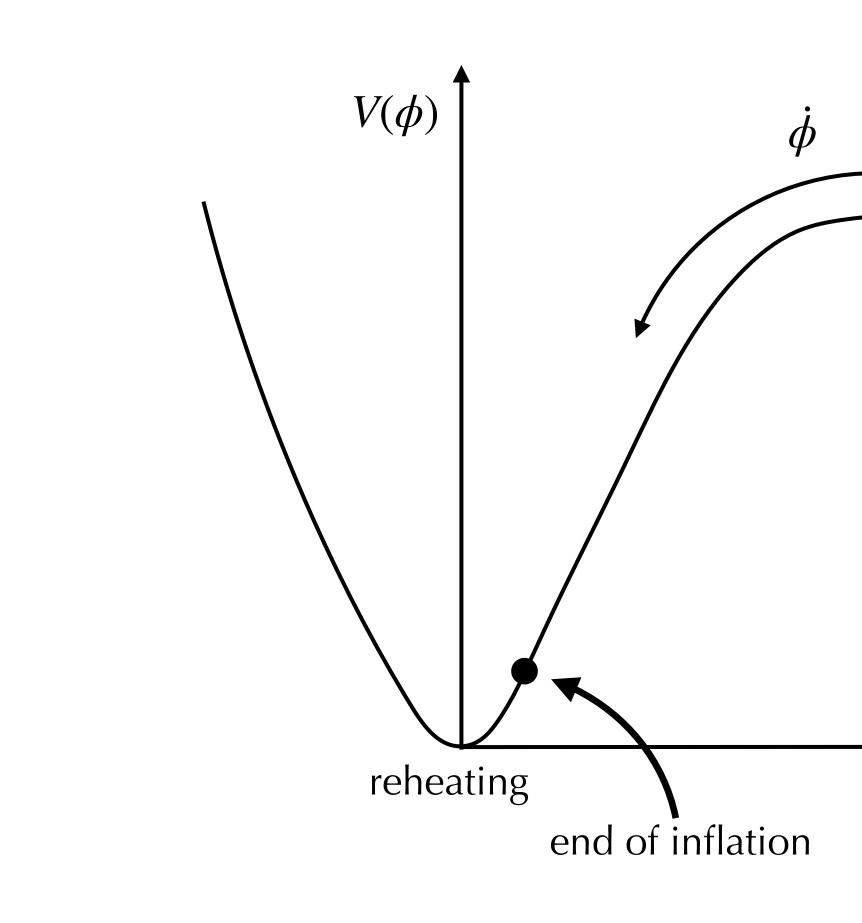
$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16 \pi G} \left(\frac{V_{,\phi}}{V}\right)^2$$
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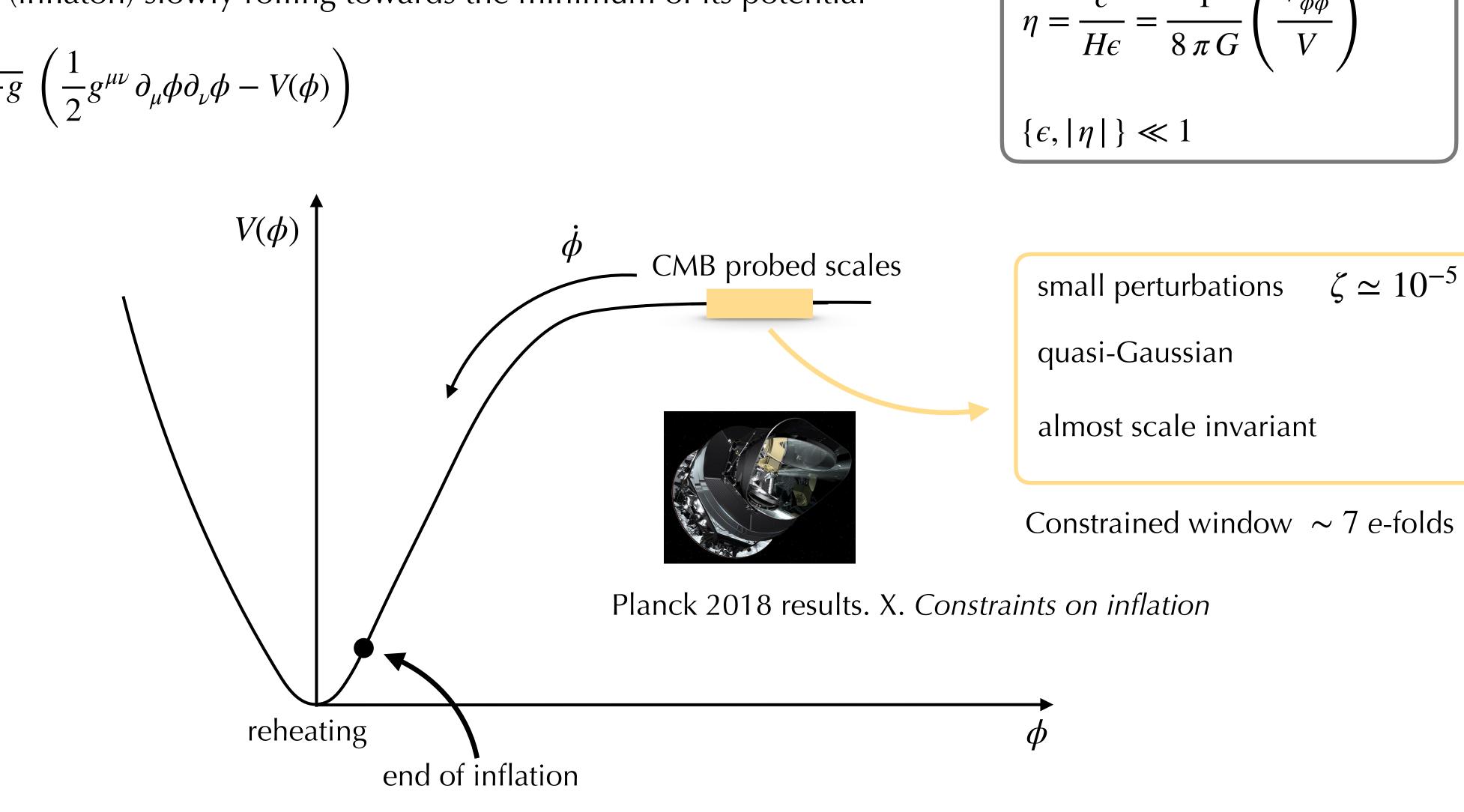
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 $V_{,\phi}$

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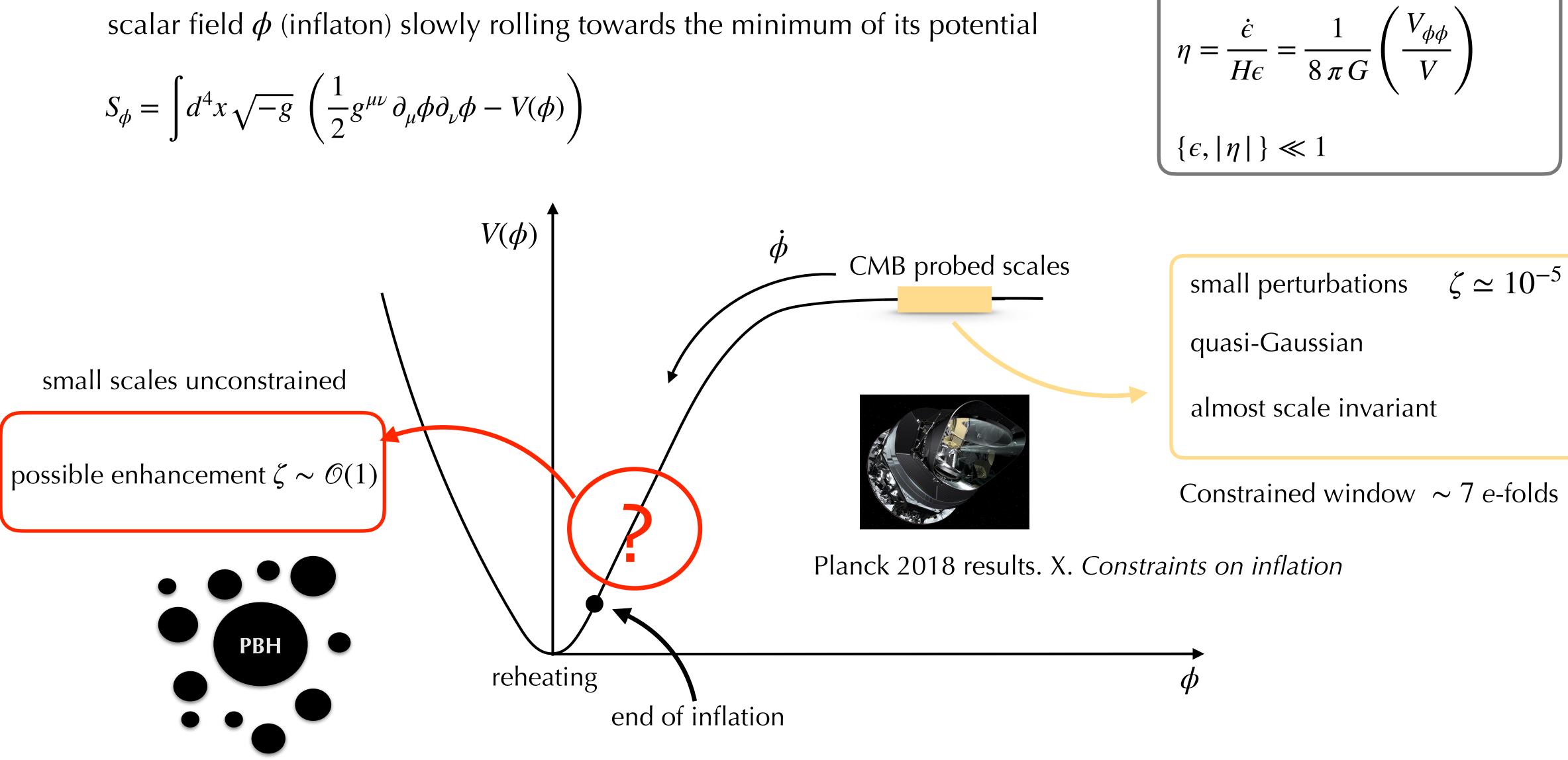




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- Large fluctuations are needed to form PBHs They could backreact on the expansion dynamics
 - Backreaction can be incorporated in an effective (stochastic) theory

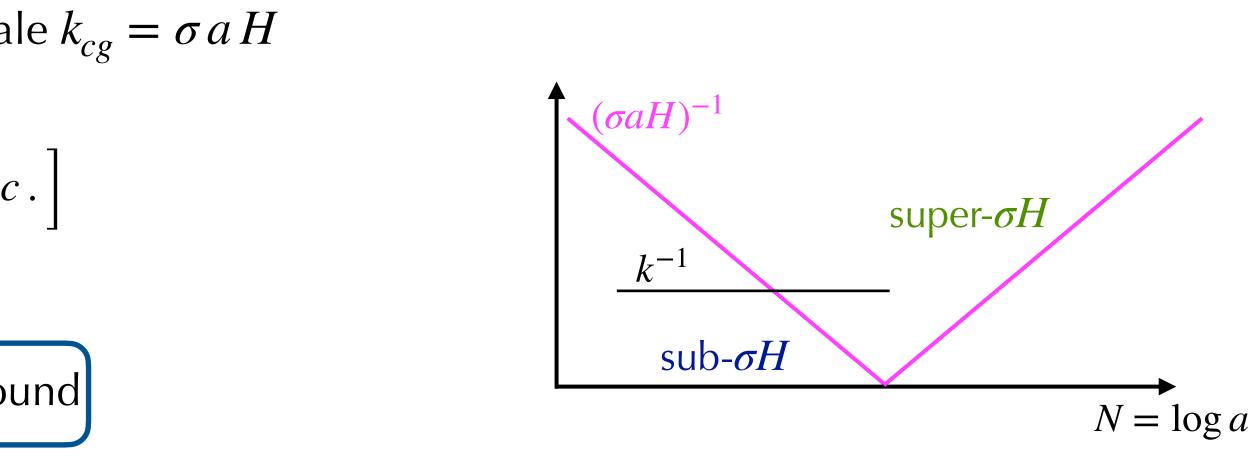


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- Stochastic inflation Splitting fields into UV and IR part: coarse-graining scale $k_{cg} = \sigma a H$

$$\phi(x) = \phi_{cg} + \int \frac{dk}{(2\pi)^{3/2}} \theta\left(\frac{k}{\sigma aH}\right) \left[\phi_k(N) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + h \cdot dx\right]$$

Quantum subhorizon fluctuations source the background

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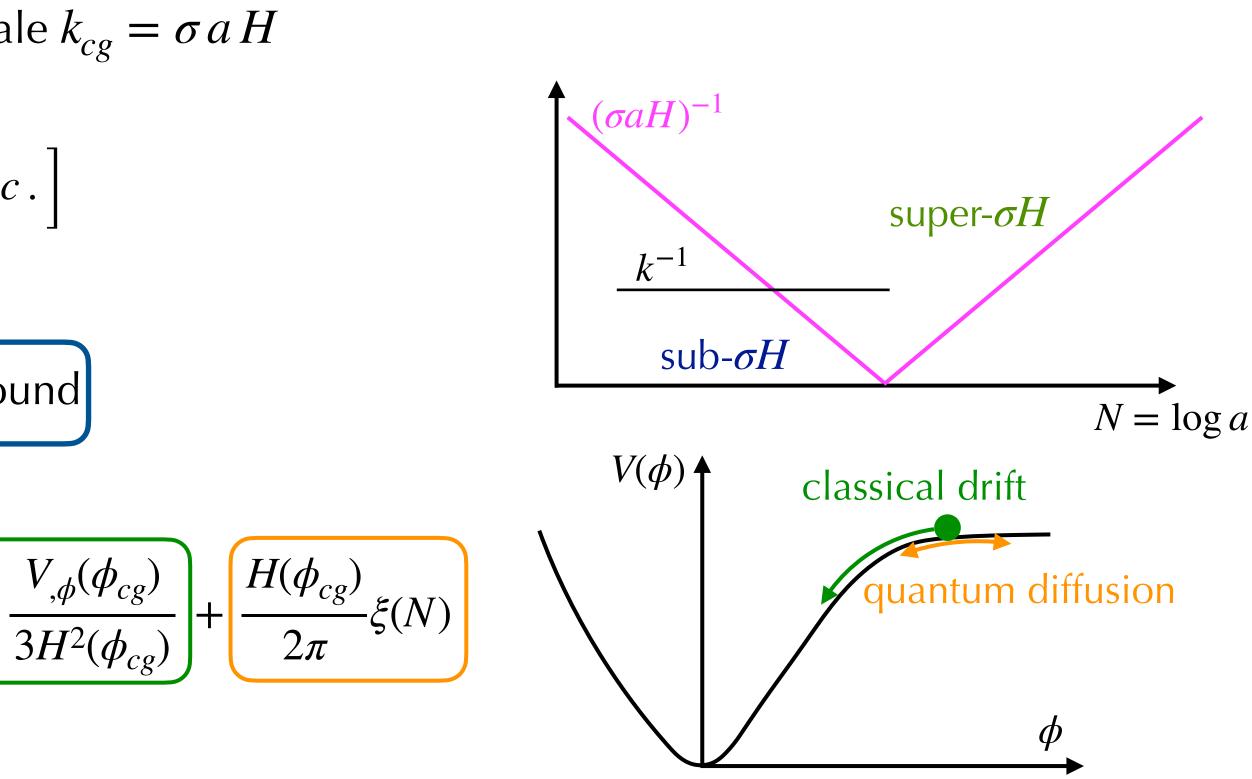
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Dynamics at leading order in slow roll:

$$\frac{d}{dN}\phi_{cg} = \left[-\right]$$

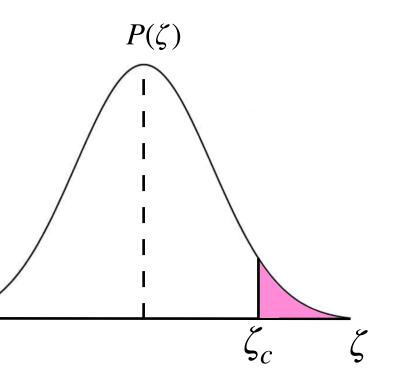
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Abundance of PBHs $\beta(M) = \int_{\zeta_c}^{\infty} P(\zeta) d\zeta$

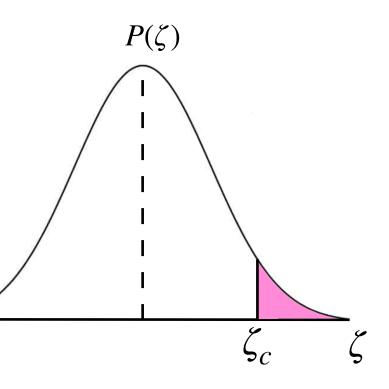






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How to reconstruct the statistics of ζ in presence of quantum diffusion?

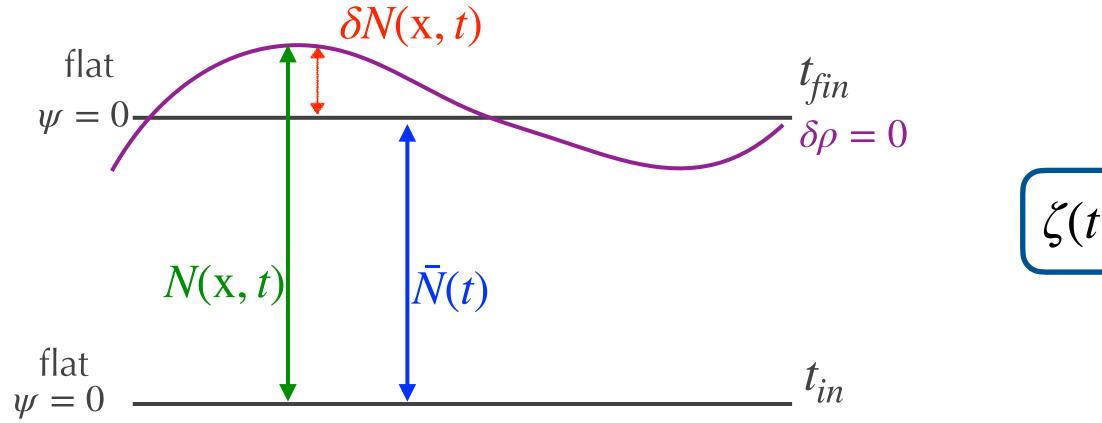


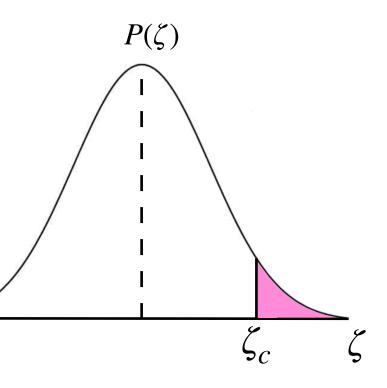




How to reconstruct the statistics of ζ in presence of quantum diffusion?

 δN formalism





$$(t, \mathbf{x}) = N(t, \mathbf{x}) - \overline{N}(t) \equiv \delta N$$

Lifshitz, Khalatnikov [1960] Starobinsky [1983] Wands, Malik, Lyth, Liddle [2000]

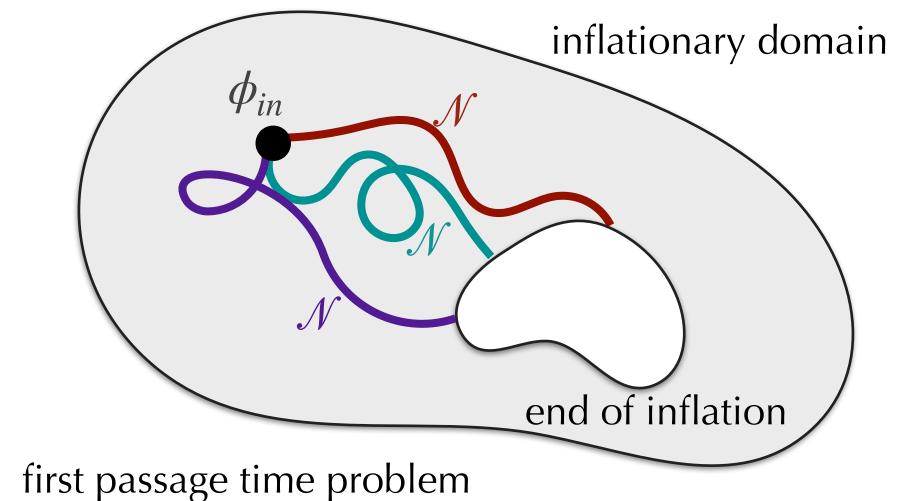




• Stochastic- δN formalism

[Enqvist, Nurmi, Podolsky, Rigopoulos [2008] Vennin, Starobinsky [2015]

Number of *e*-folds is a stochastic variable \mathcal{N}





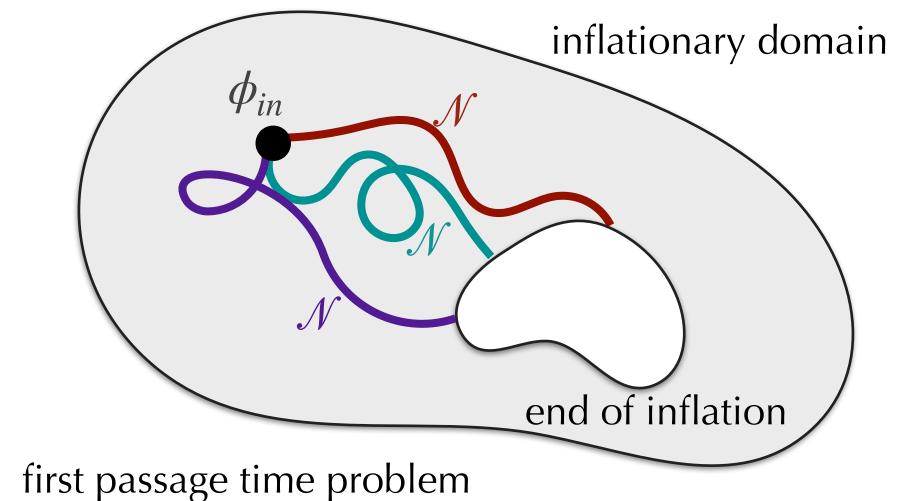
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Statistics of ζ from the statistics of ${\mathscr N}$

$$\zeta_{cg}(\mathbf{x}) = \mathcal{N}(\mathbf{x}) - \langle \mathcal{N} \rangle$$





Stochastic- δN formalism

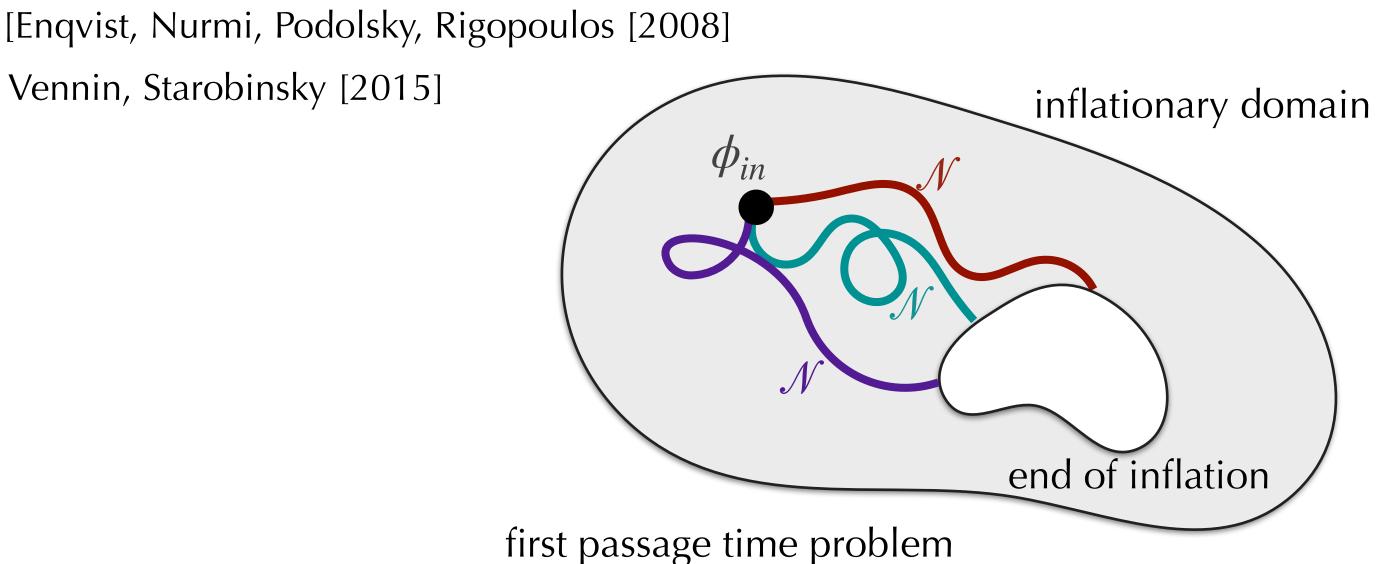
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Distribution function for the duration of inflation (first passage time)

$$\frac{\partial}{\partial \mathcal{N}} P(\mathcal{N}, \phi) = \mathscr{L}_{FP}^{\dagger}(\phi) \cdot P(\mathcal{N}, \phi) \qquad \qquad \frac{1}{M_{Pl}^2} \mathscr{L}_{FP}^{\dagger}(\phi) = -\frac{v'(\phi)}{v(\phi)} \frac{\partial}{\partial \phi} + v(\phi) \frac{\partial^2}{\partial \phi^2} \qquad \qquad v = \frac{V}{24\pi^2 M_{Pl}^4}$$





Stochastic- δN formalism: exponential tails



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Full PDF of the first passage time

Characteristic function (includes all moments)

$$\chi(t,\phi) \equiv \langle e^{it\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{it\mathcal{N}} P(\mathcal{N},\phi) \, d\mathcal{N} \qquad \longrightarrow \qquad \mathcal{L}_{FP}^{\dagger} \cdot \chi(t,\phi) = -it\chi(t,\phi) \qquad \longrightarrow \qquad P(\mathcal{N},\phi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\phi) \, d\mathcal{N} = -it\chi(t,\phi)$$

Pattison, Vennin, Assadullahi, Wands [2017]

Obeys differential equation Full PDF given by inverse Fourier transform







Stochastic- δN formalism: exponential tails

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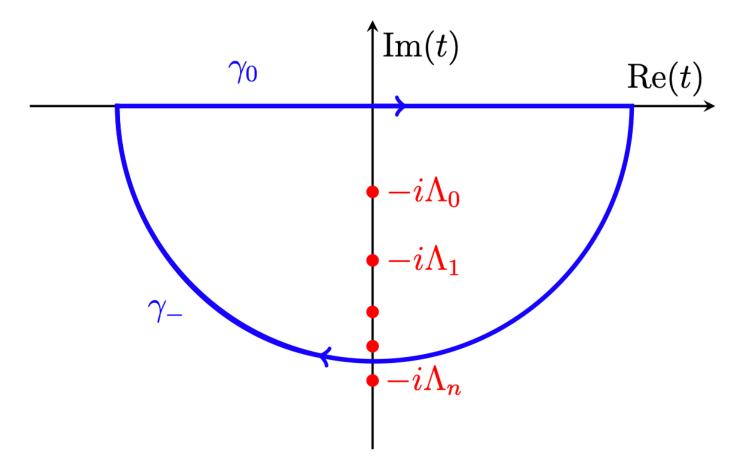
Useful trick: pole expansion

[Ezquiaga, Garcia-Bellido, Vennin (2020)]

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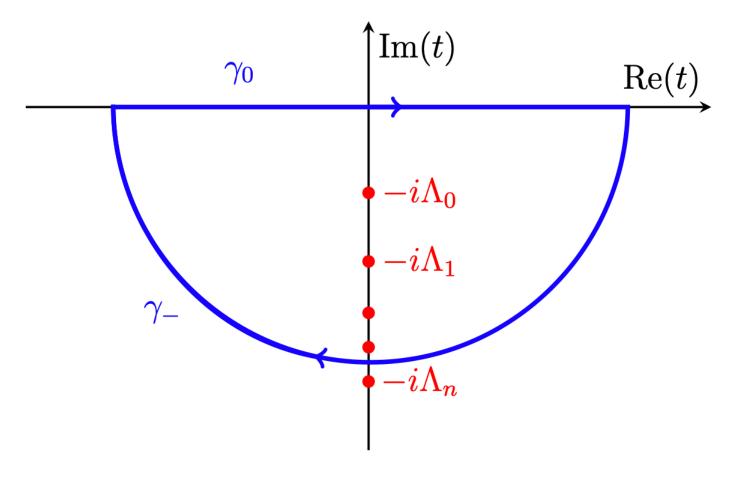
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Tail of the PDF for ζ has an exponential fall-off behaviour

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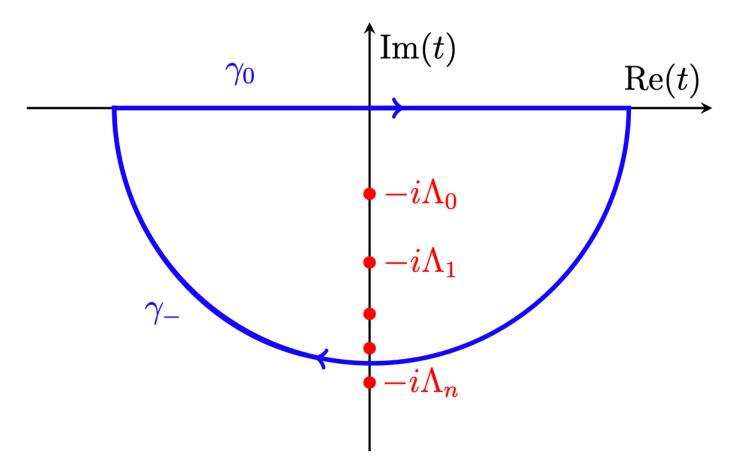
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This type of non-Gaussianities cannot be captured by perturbative parametrisations (such as the fNL expansion)

Pattison, Vennin, Assadullahi, Wands [2017]

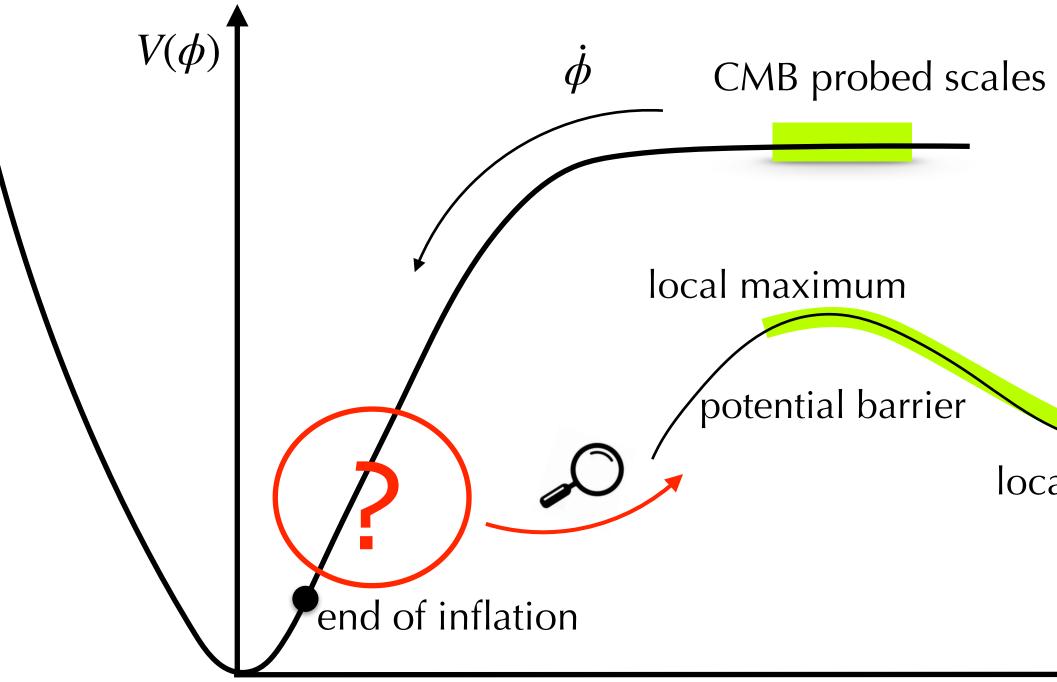










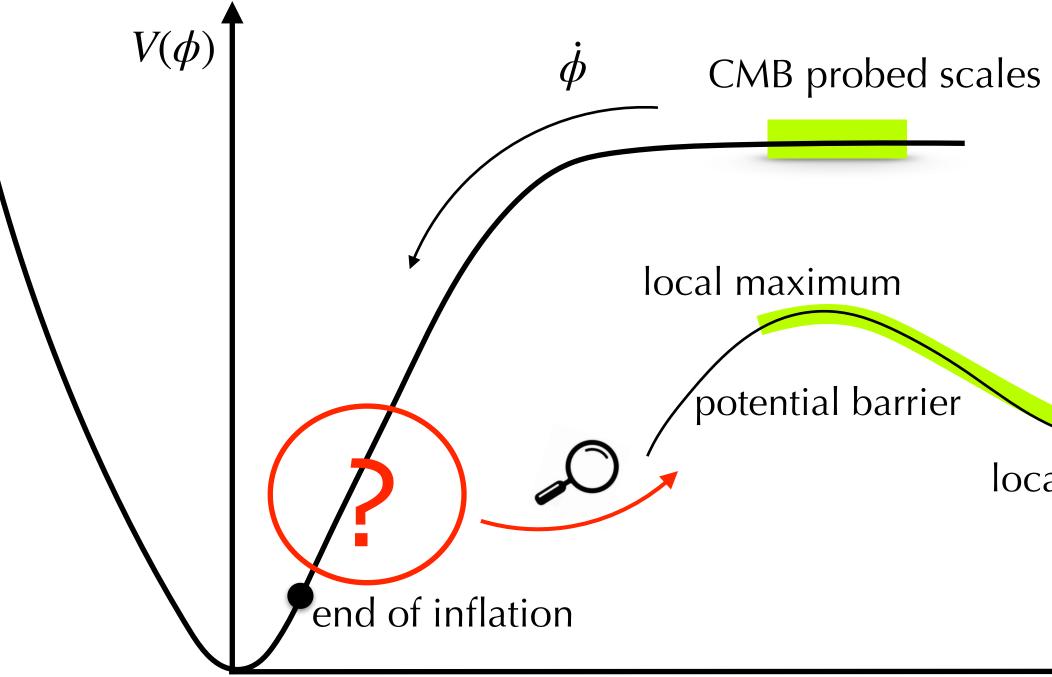


<u>C.A.</u>, V. Vennin [2022] "Primordial black holes from stochastic tunneling"

local minimum



False vacuum state



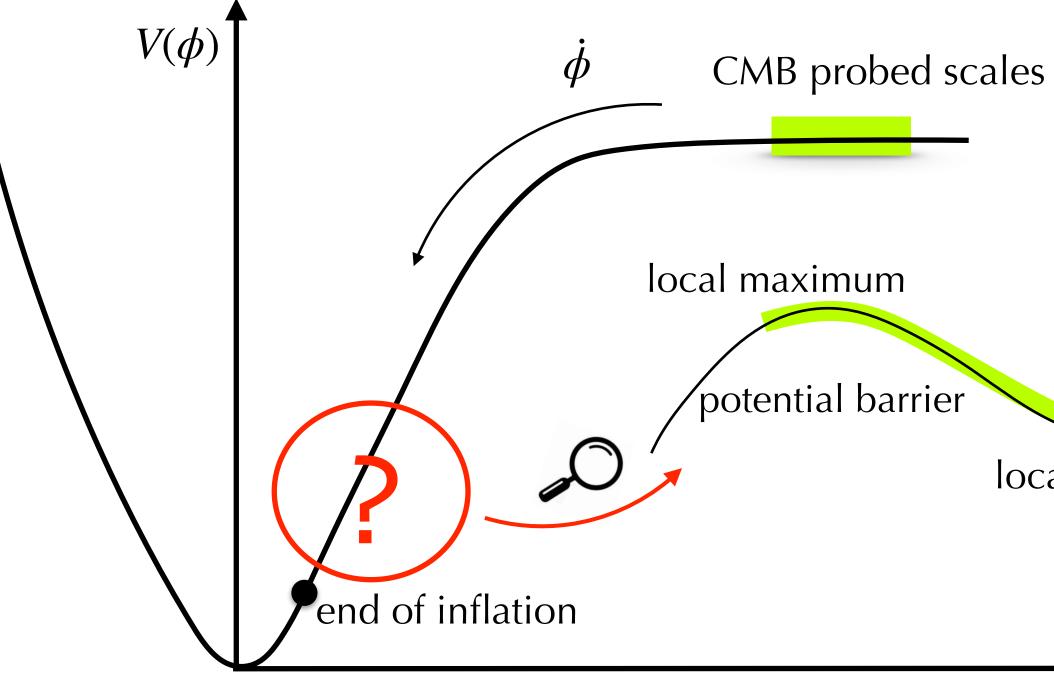
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- Local minima naturally appear in various contexts: high energy constructions (supersymmetry, supergravity)

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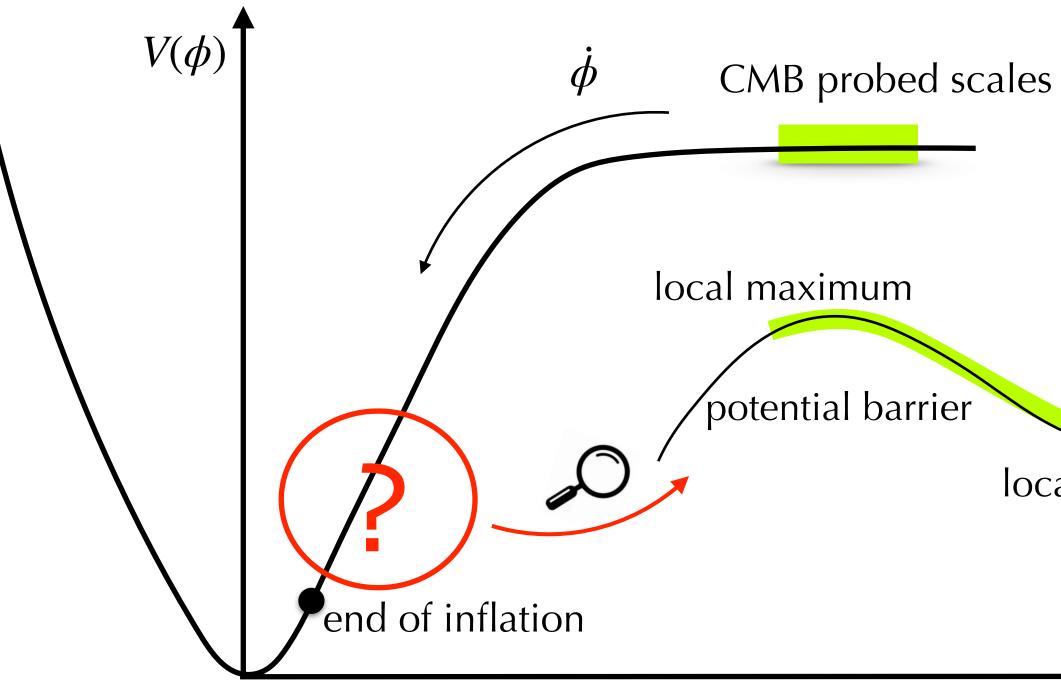
- breaking of flat-inflection point condition through radiative corrections

local minimum



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 - etc.
- 1) Large classical velocity How to escape?



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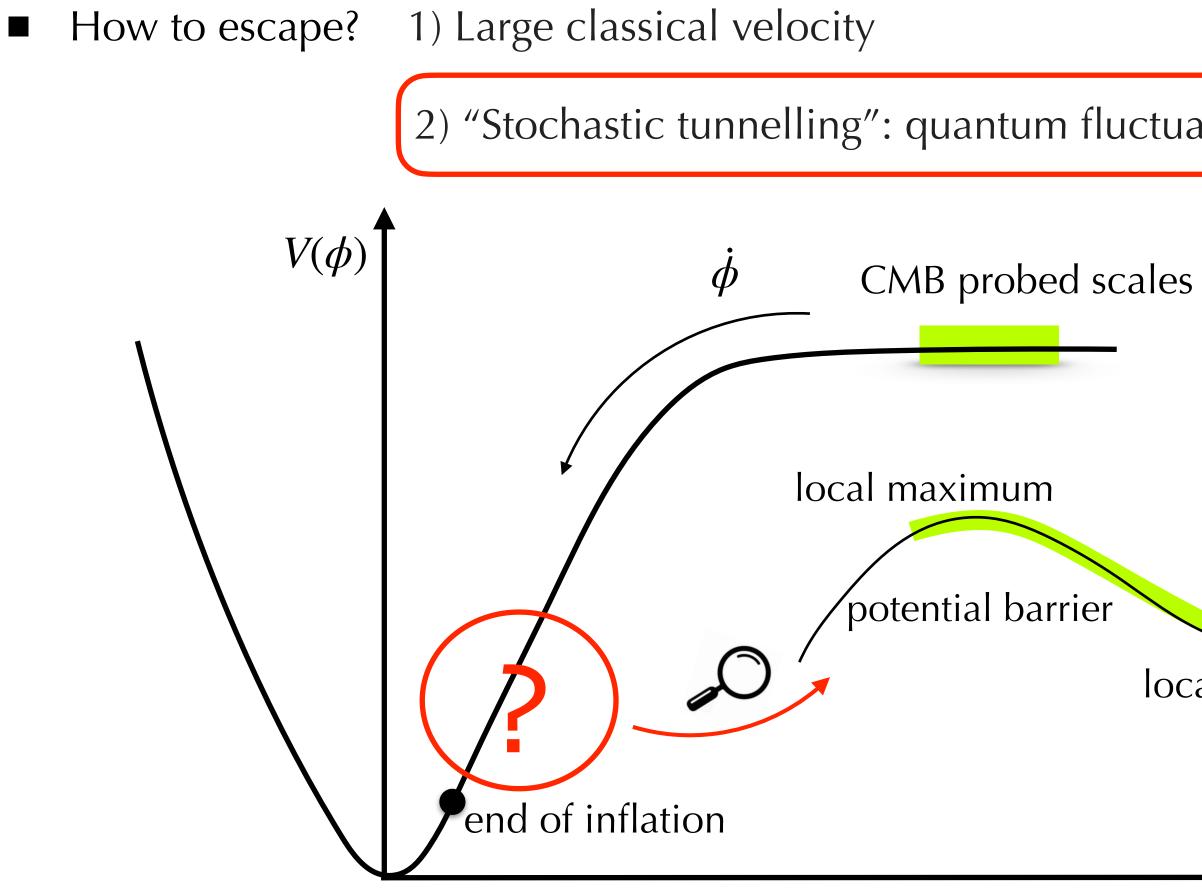
2) "Stochastic tunnelling": quantum fluctuations jiggle the inflaton and push it outwards

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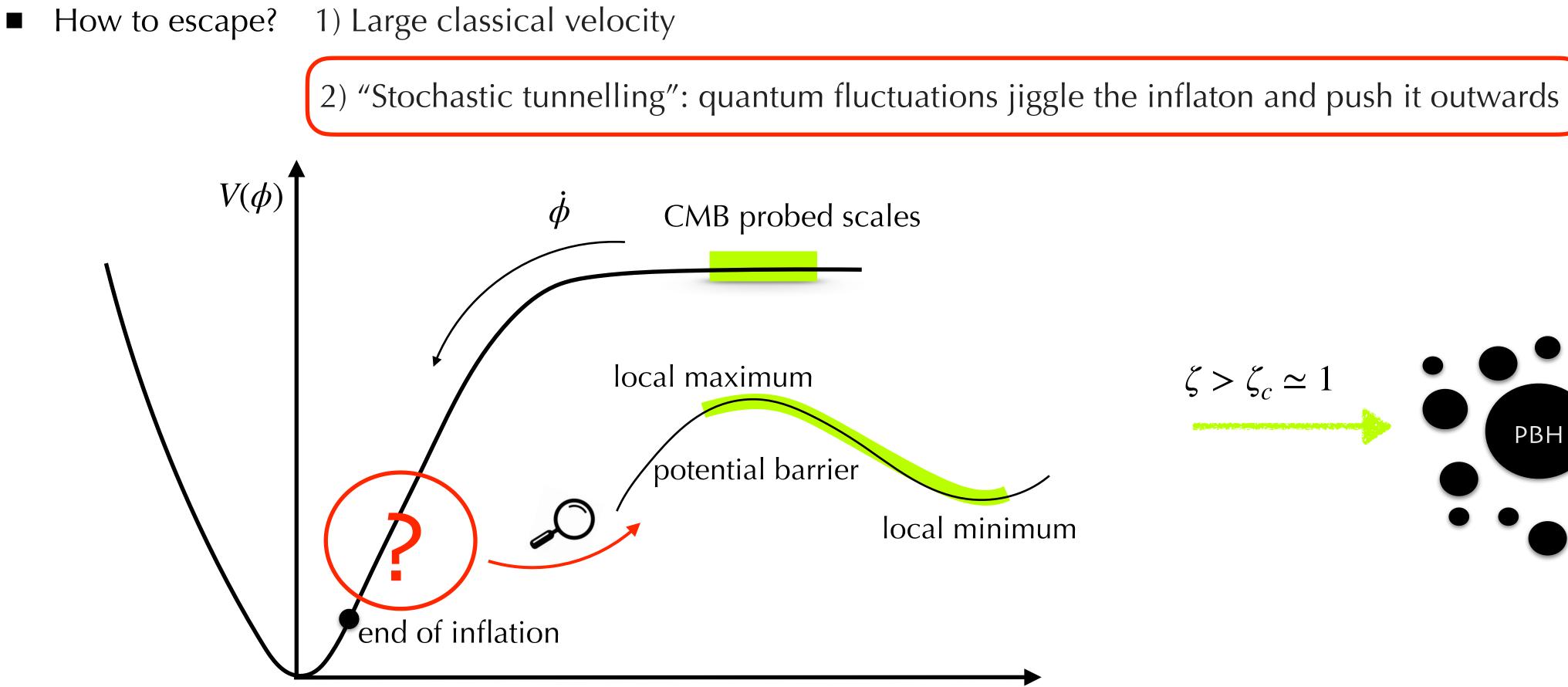
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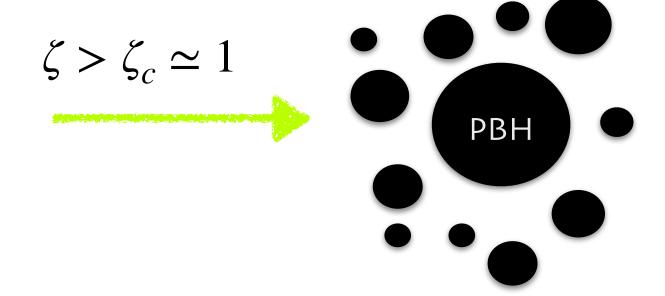
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- Local minima naturally appear in various contexts: high energy constructions (supersymmetry, supergravity)

 - etc.



<u>C.A.</u>, V. Vennin [2022] "Primordial black holes from stochastic tunneling"

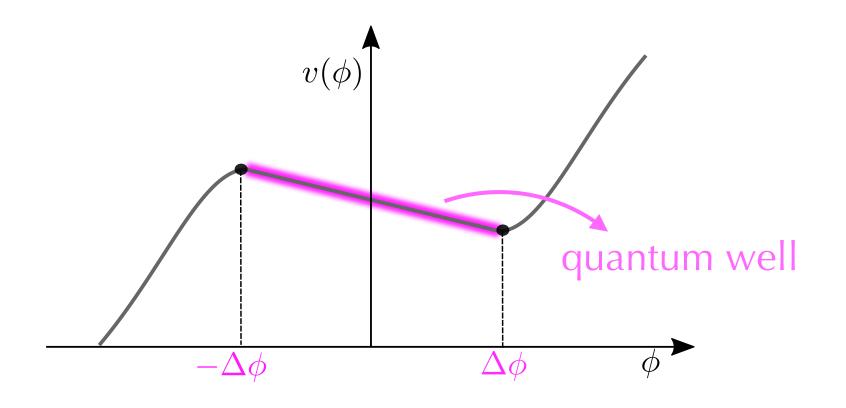
- breaking of flat-inflection point condition through radiative corrections





Linear model

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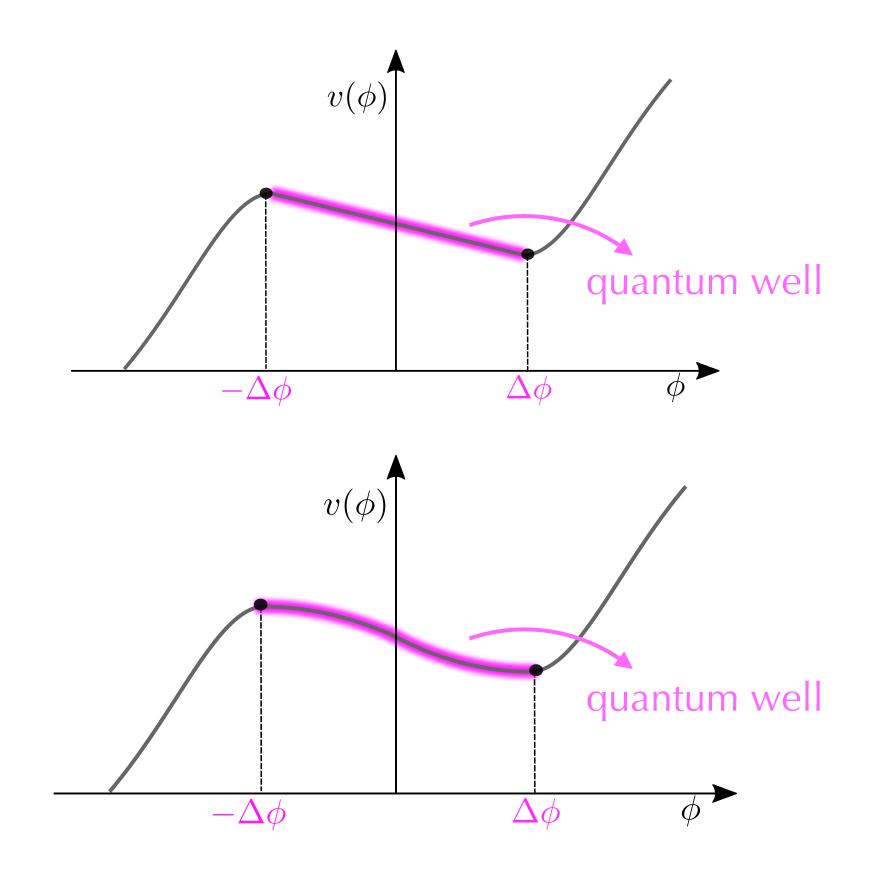


Linear model

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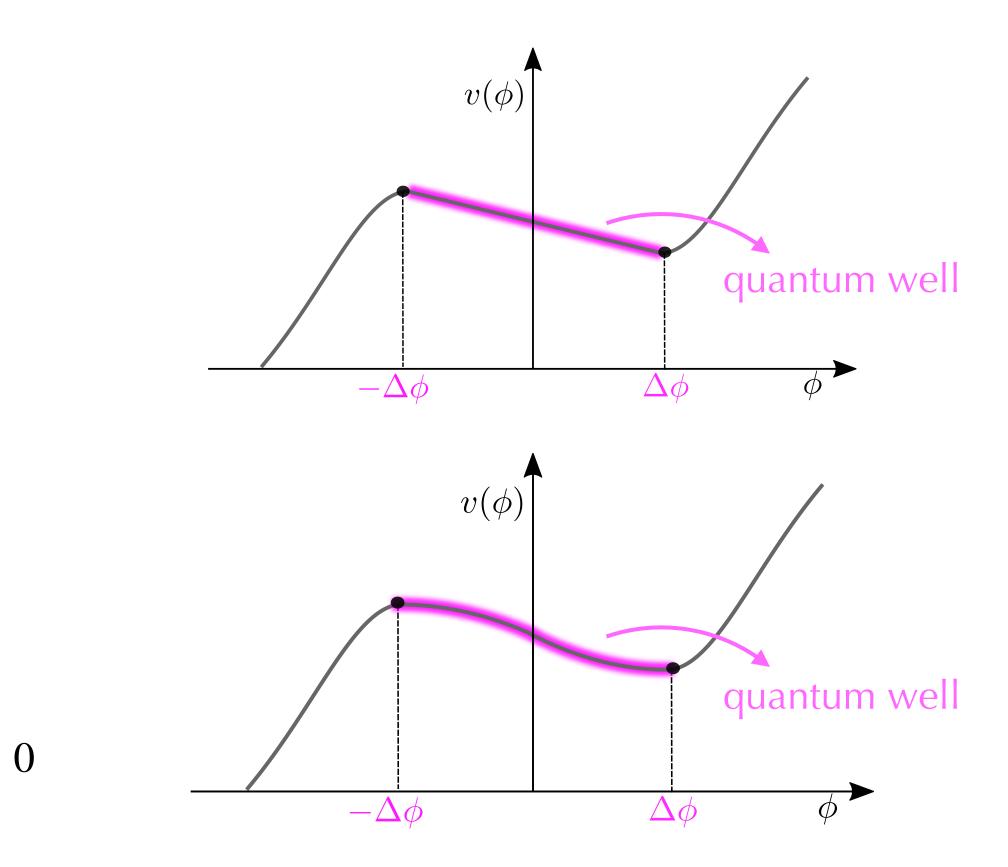
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Quantum diffusion in highlighted regions, potential gradient elsewhere

Slow roll preserved:
$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{v'}{v}\right)^2 \ll 1, |\eta| = \left|M_{Pl}^2 \frac{v''}{v}\right| \ll$$

 $\langle \mathcal{N} \rangle$ smaller than ~ 50: $\Delta v = v(-\Delta \phi) - v(\Delta \phi) \ll v_0$



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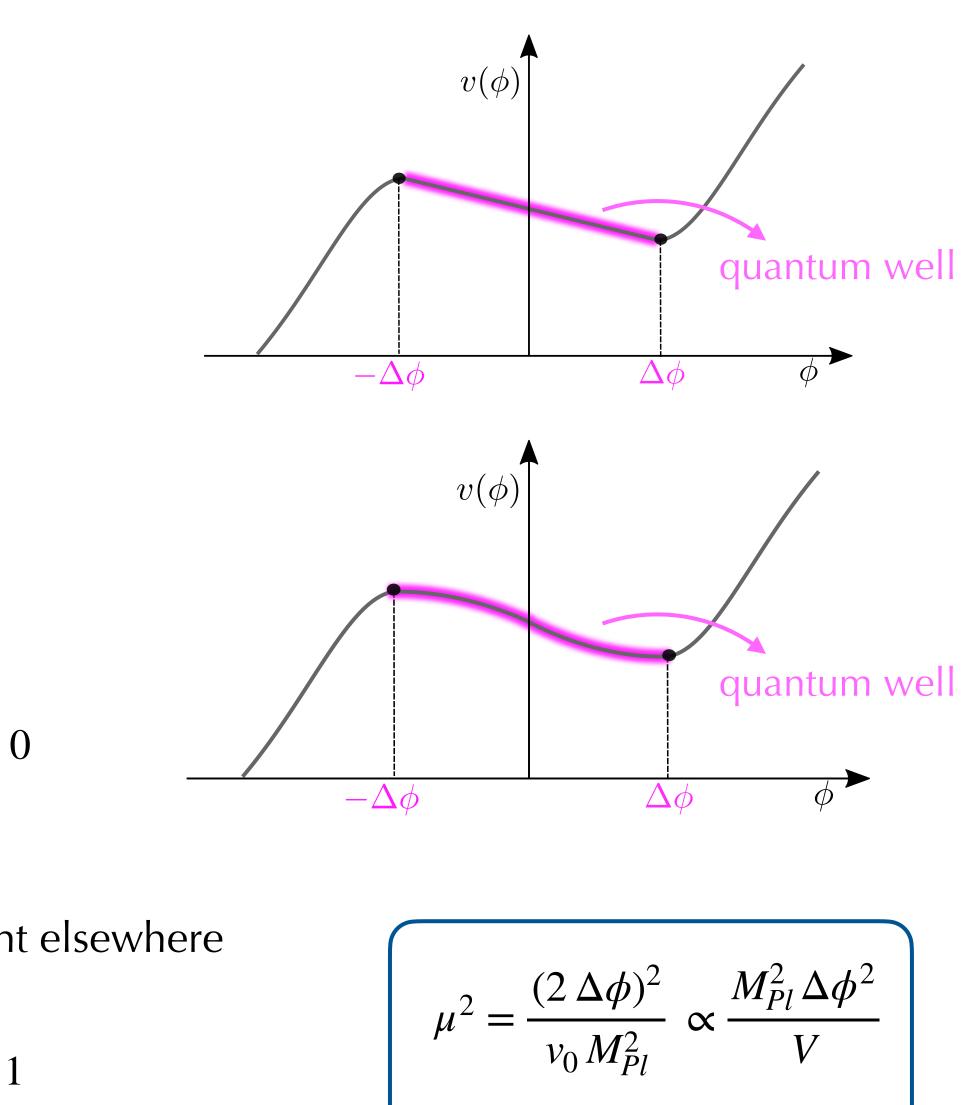
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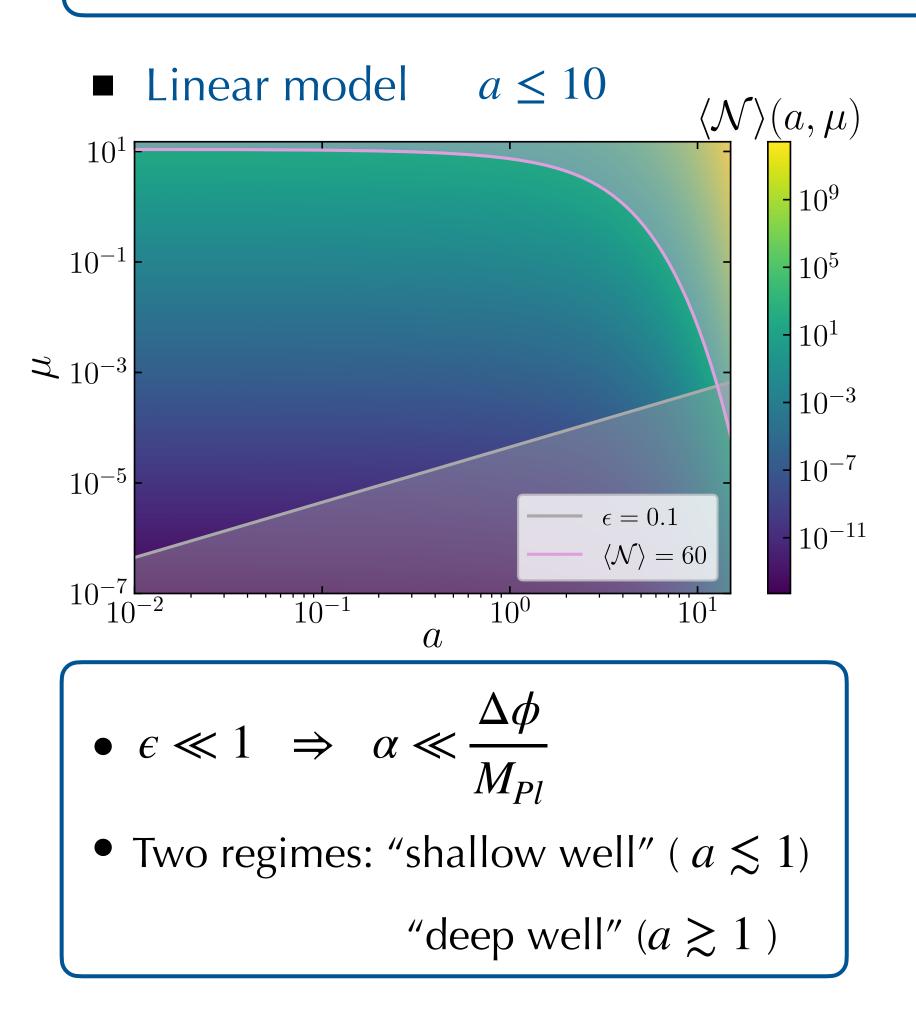
$$a = \frac{\alpha}{v_0} \propto \frac{M_{Pl}^4 \Delta V}{V^2}$$

- $\langle \mathcal{N} \rangle$ features quadratic dependence on μ and exponential dependence on a
- μ constrained from below by slow-roll conditions

a not much larger than 1

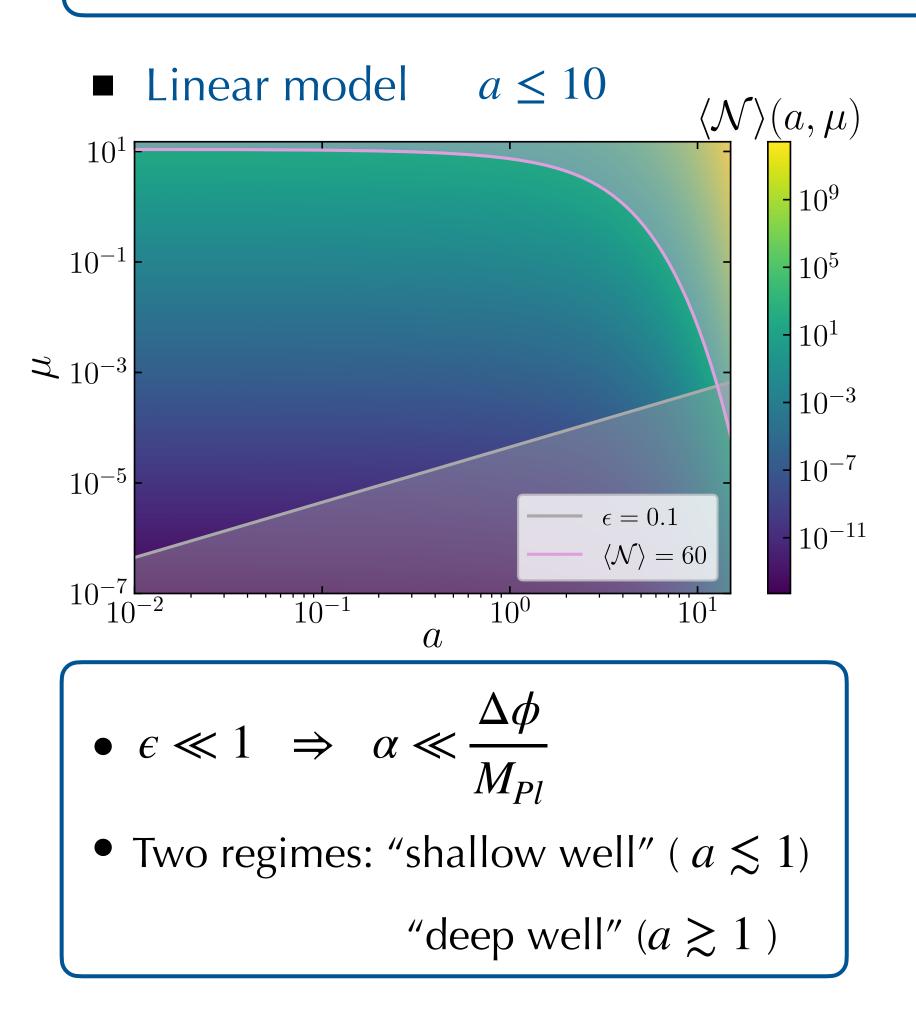
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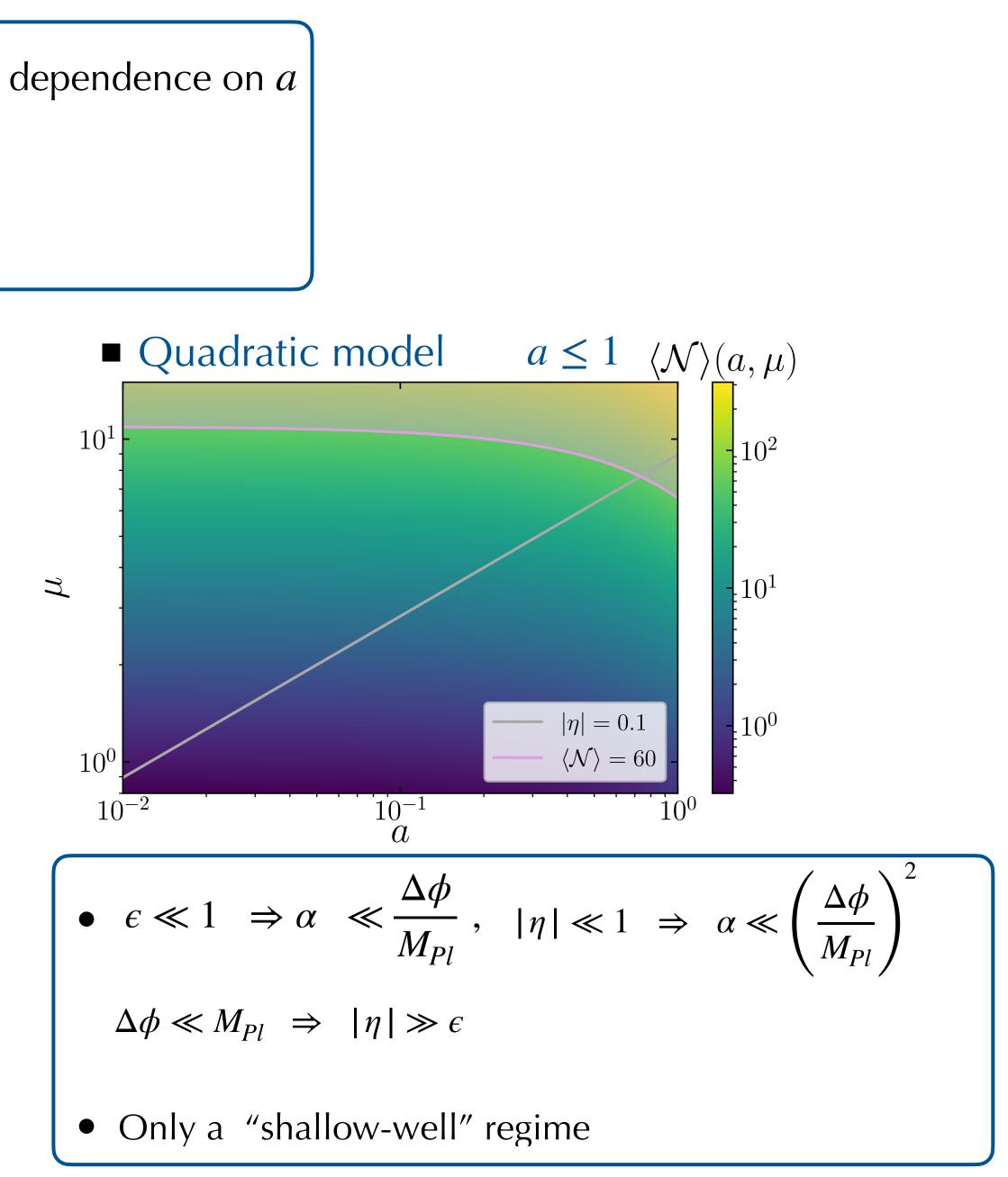
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shallow-well limit

$$\Lambda_n^{shallow} = \frac{1}{\mu^2} \left[\pi^2 \left(n + \frac{1}{2} \right)^2 - 2a + \mathcal{O}(a^2) \right]$$
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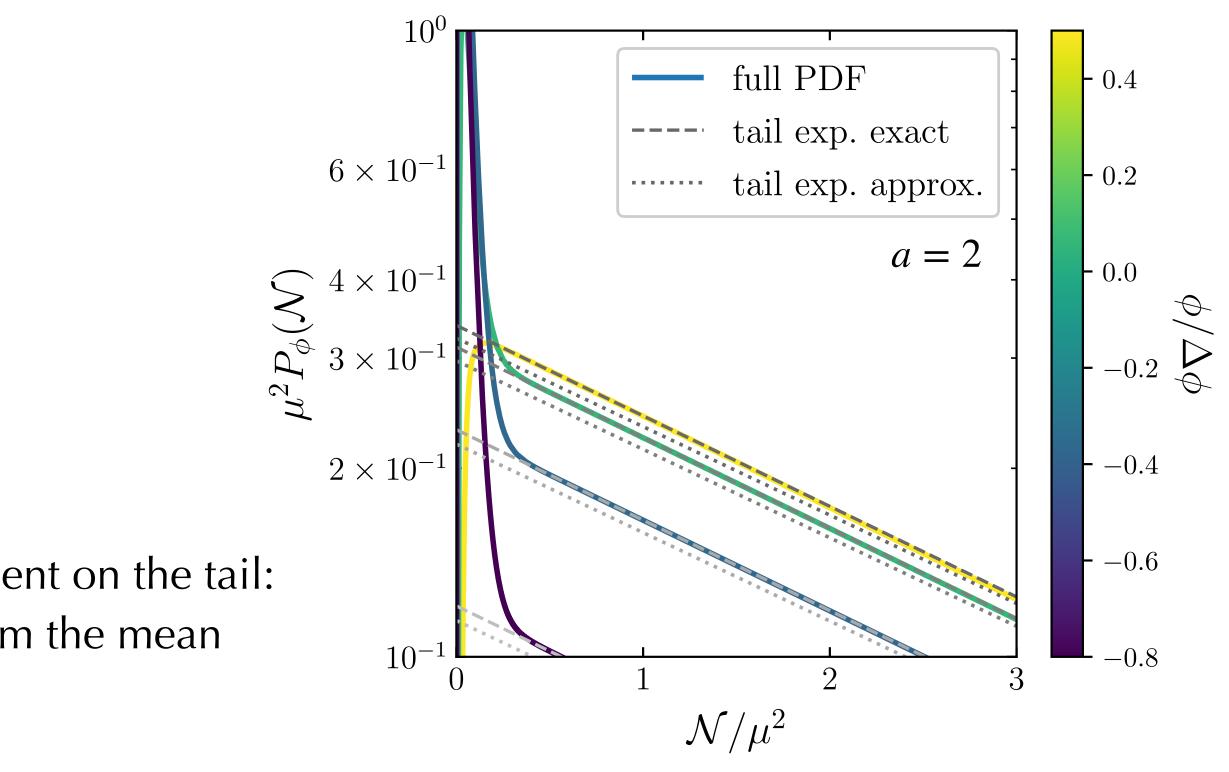
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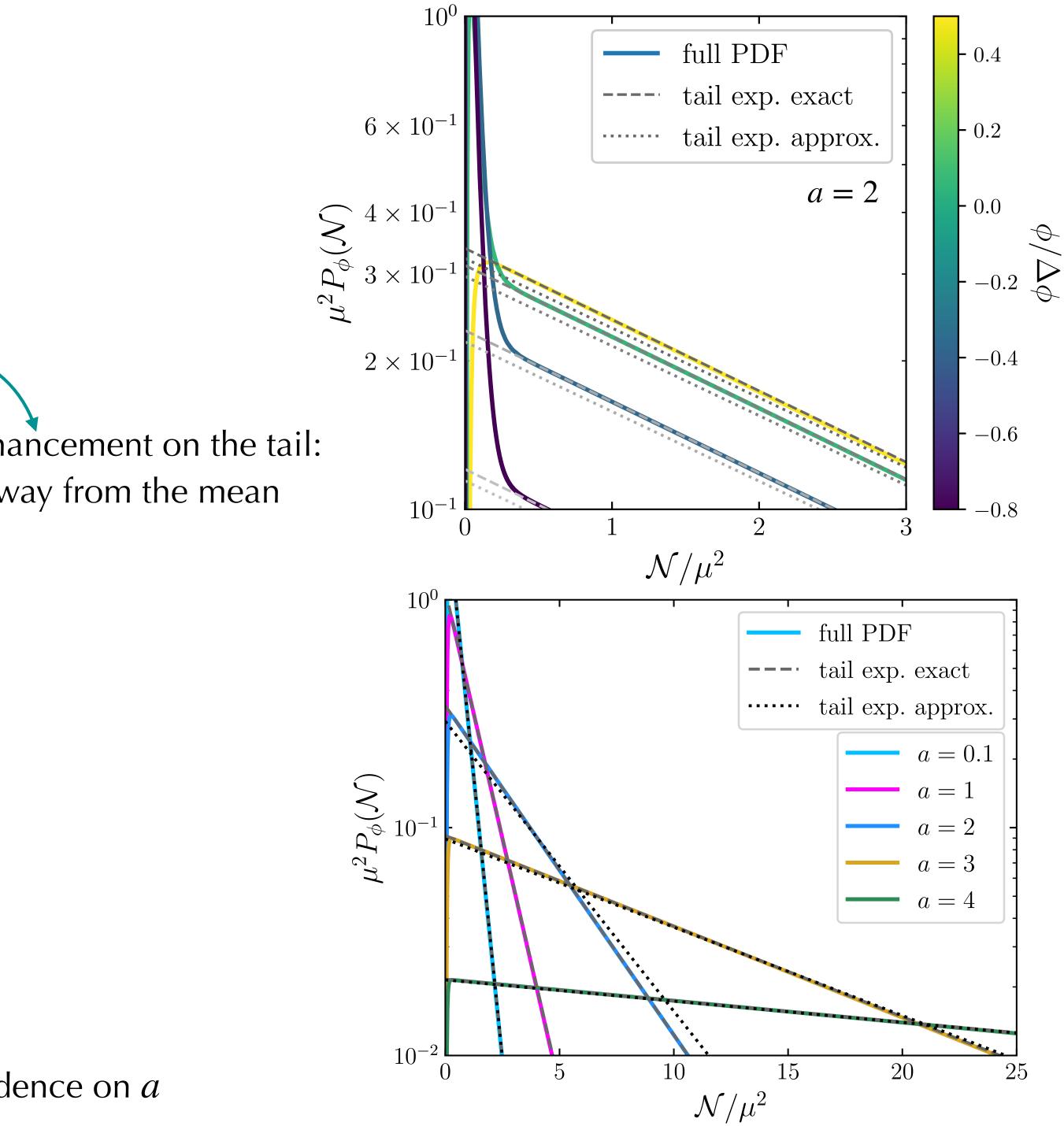
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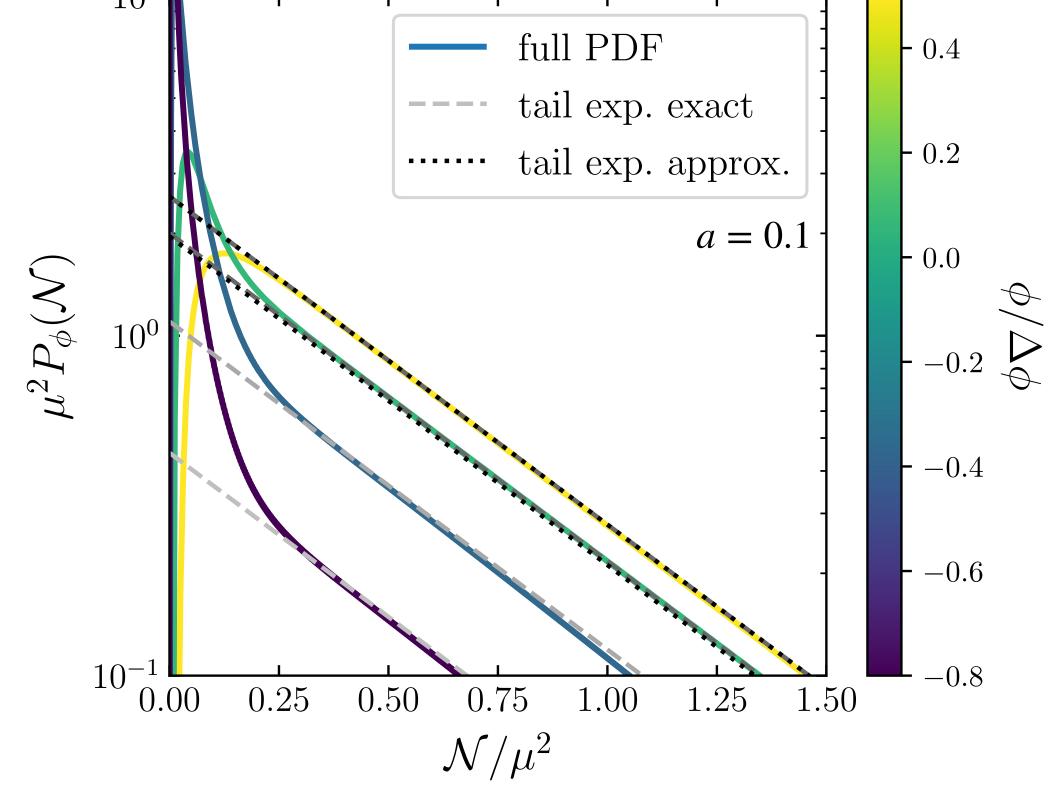
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$$10^{1}$$

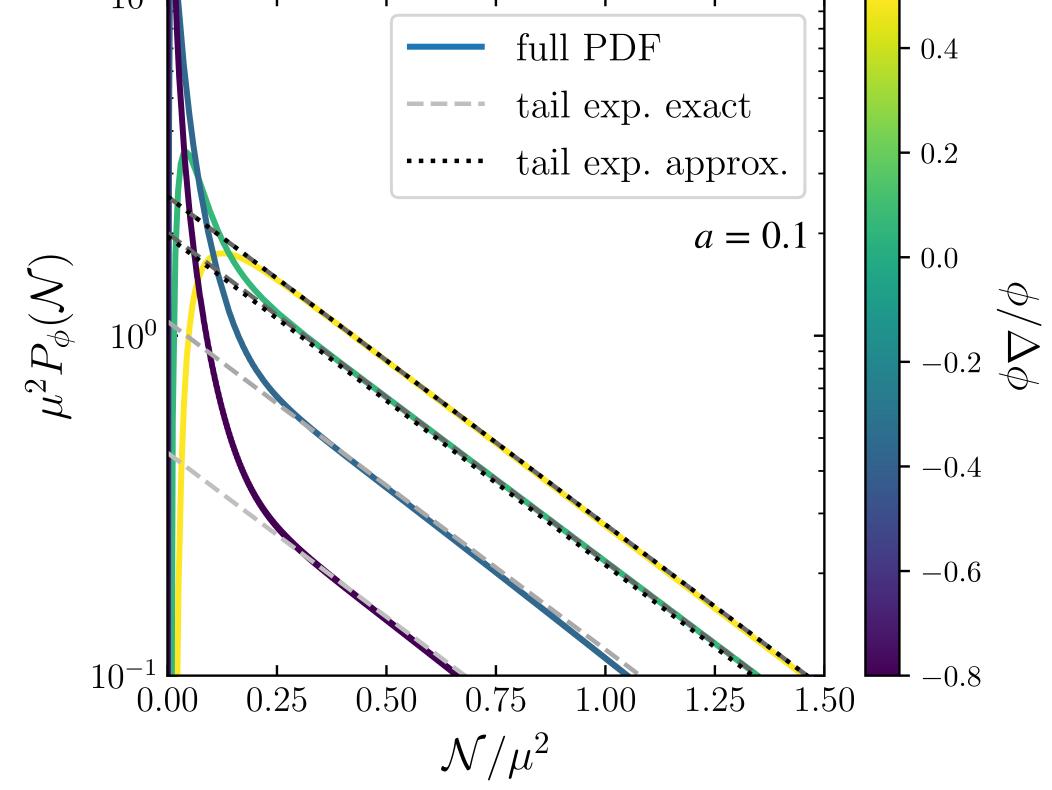


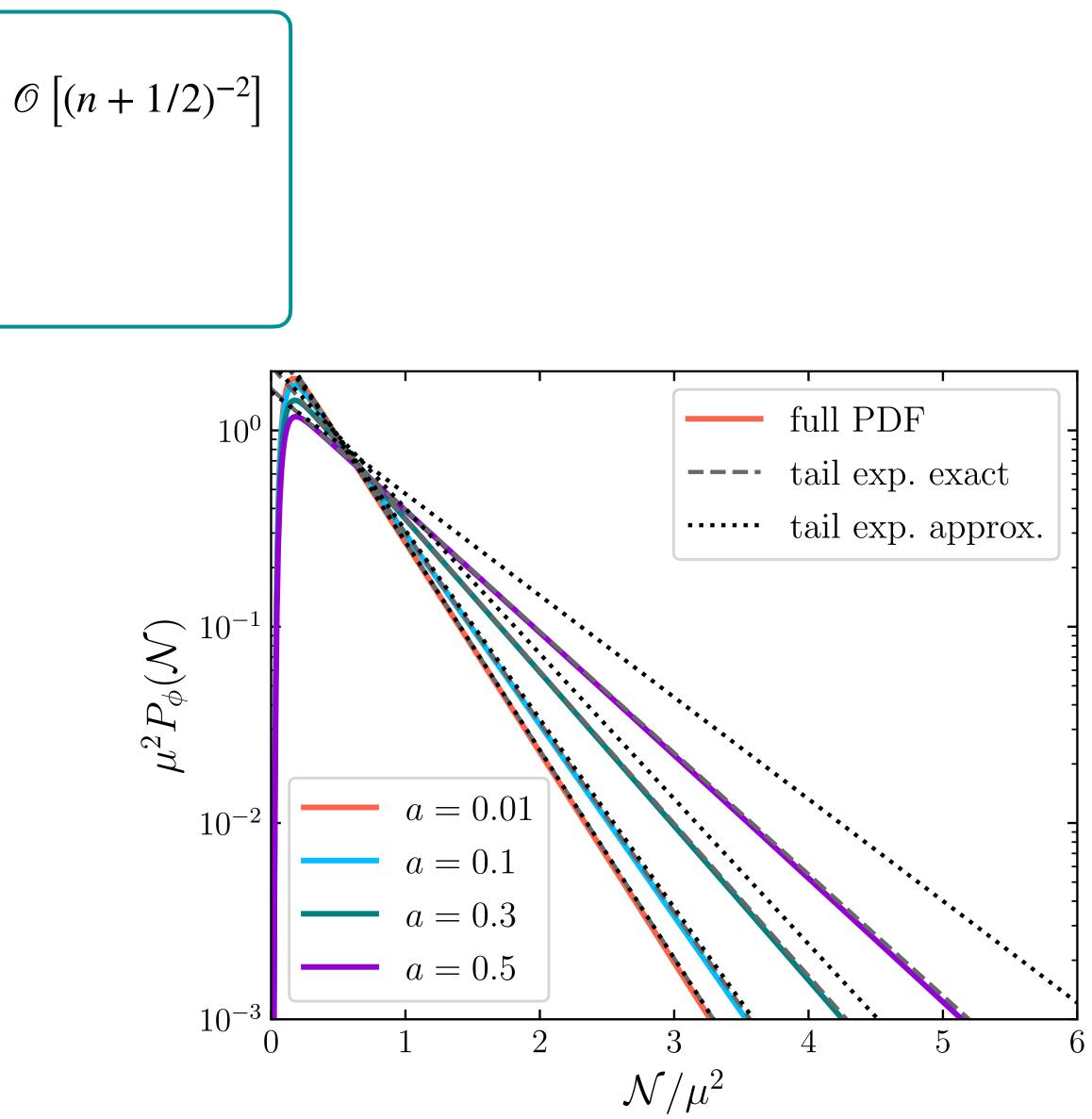
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$$10^{1}$$







Typical abundance: Press-Schechter estimate

$$\beta \sim \int_{\zeta_c}^{\infty} P(\zeta) \ d\zeta \quad \longrightarrow \quad \beta \sim \int_{\langle \mathcal{N} \rangle + \zeta_c}^{\infty} P(\mathcal{N}, \phi = \Delta \phi) \ d\mathcal{N}$$





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$$\beta = \sum_{n} \frac{a_{n}(\Delta \phi)}{\Lambda_{n}} e^{-\Lambda_{n} \left[\zeta_{c} + \langle \mathcal{N} \rangle (\Delta \phi)\right]}$$
$$\langle \mathcal{N} \rangle(\phi) = \sum_{n} \frac{a_{n}(\phi)}{\Lambda_{n}}$$





Typical abundance: Press-Schechter estimate

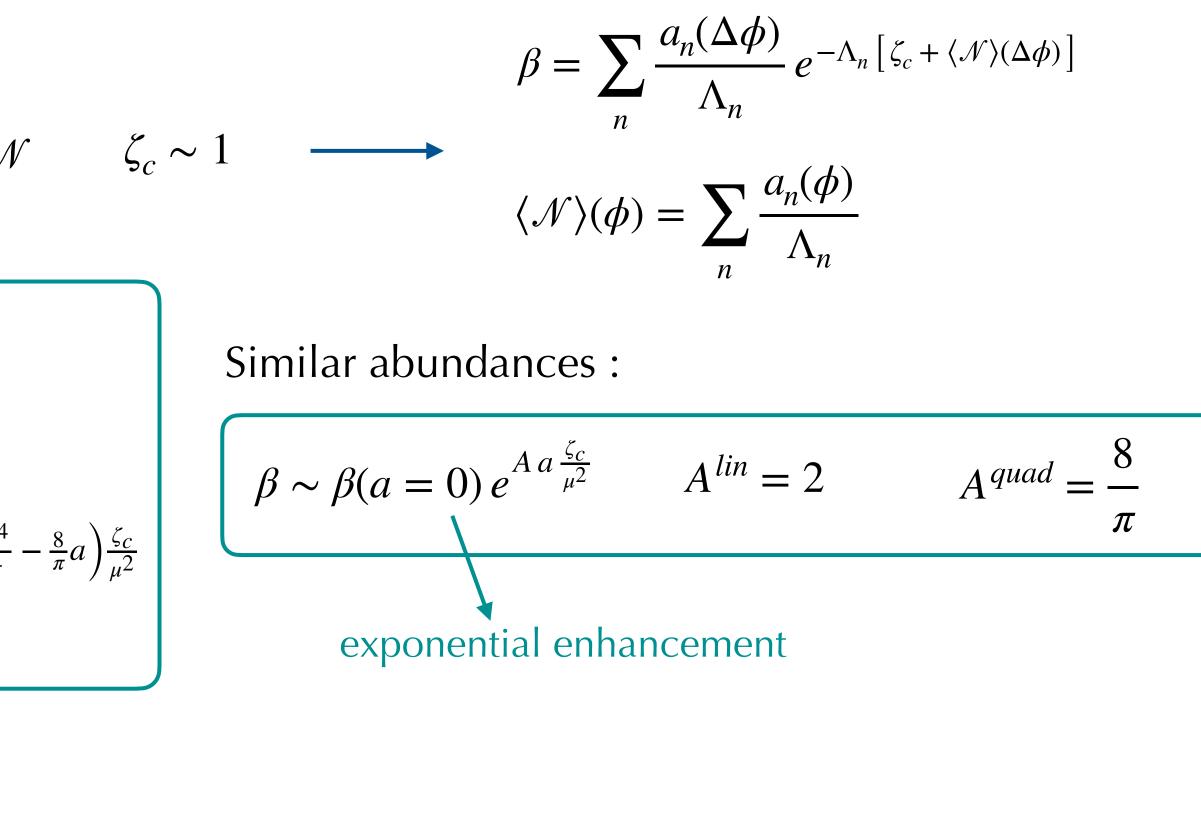
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What the slow-roll assumption implies?

quadratic model: $\mu \gg \sqrt{a} \longrightarrow$ exponential factor negligible \longrightarrow flat-well limit applies where slow roll satisfied linear model: $\mu \gg a \sqrt{v_0}$ \longrightarrow exponential factor large even at small *a* values



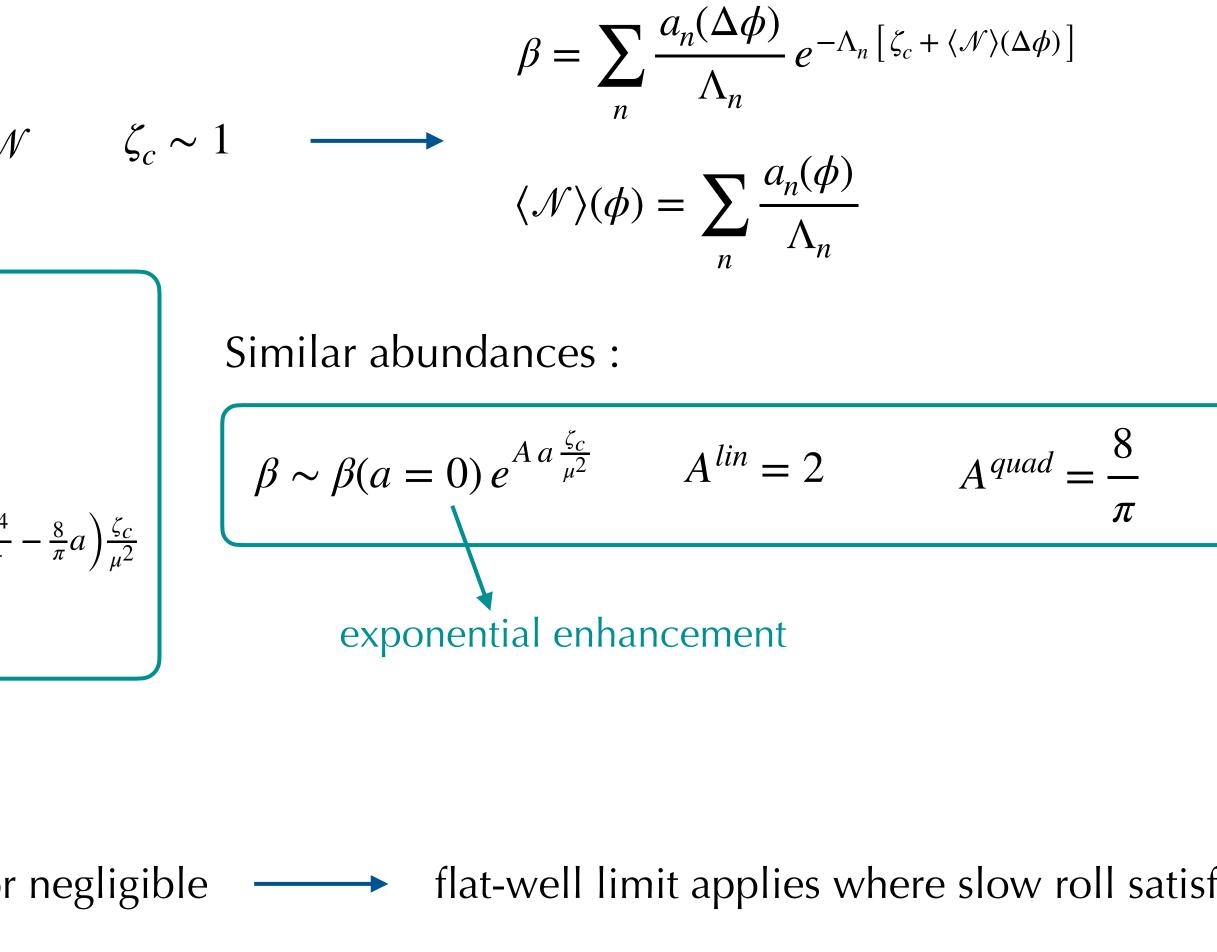
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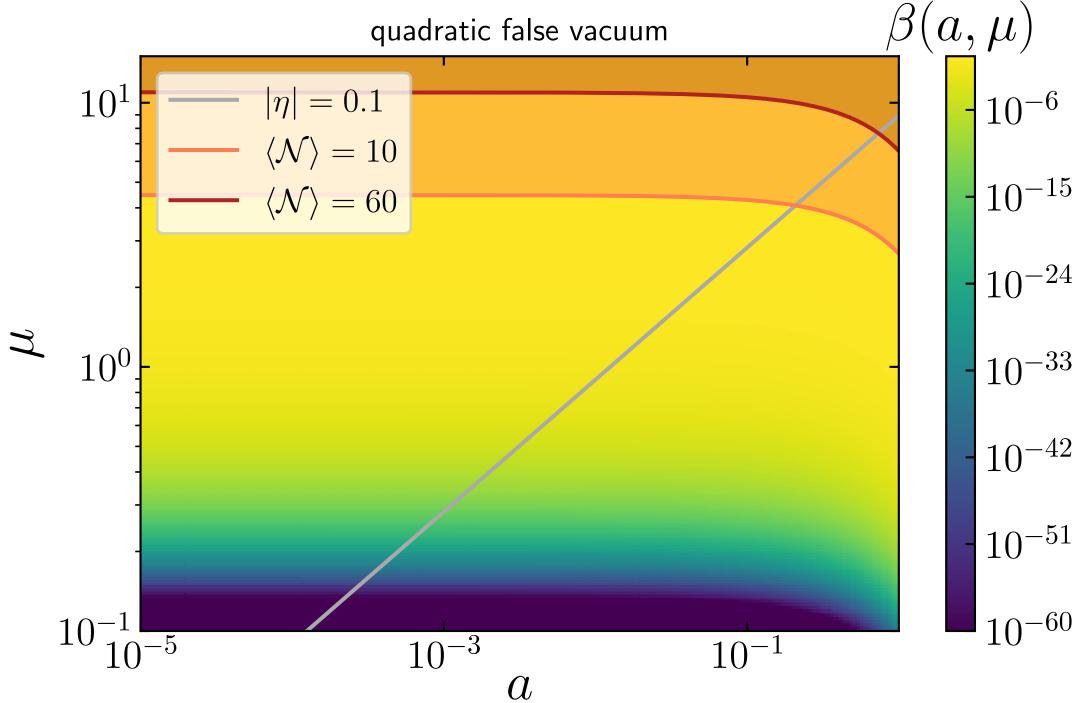
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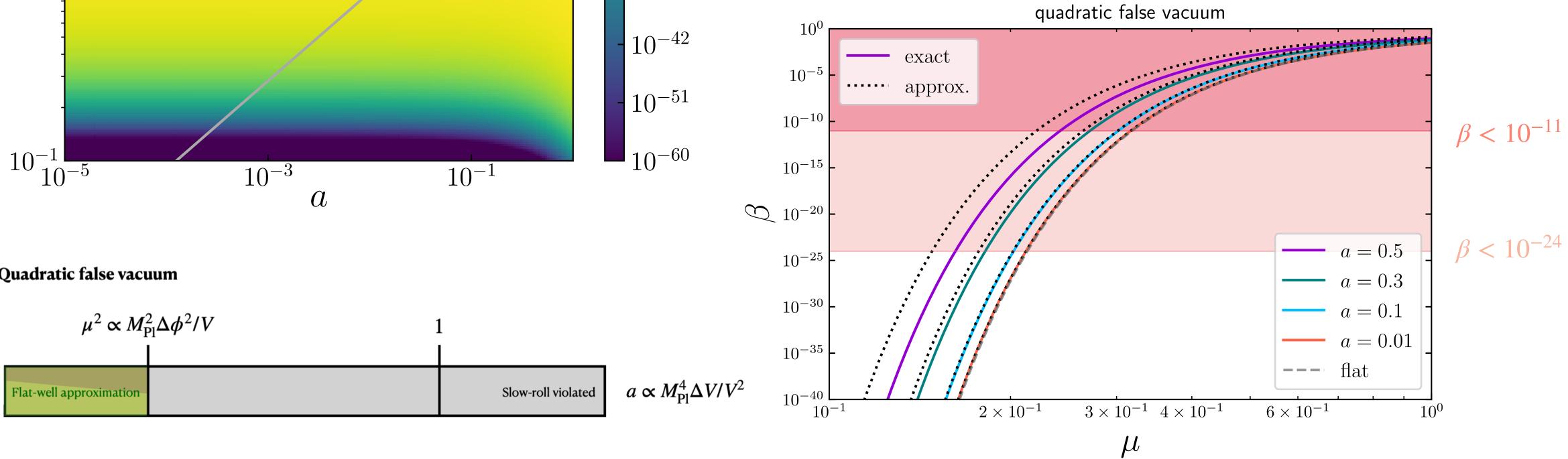
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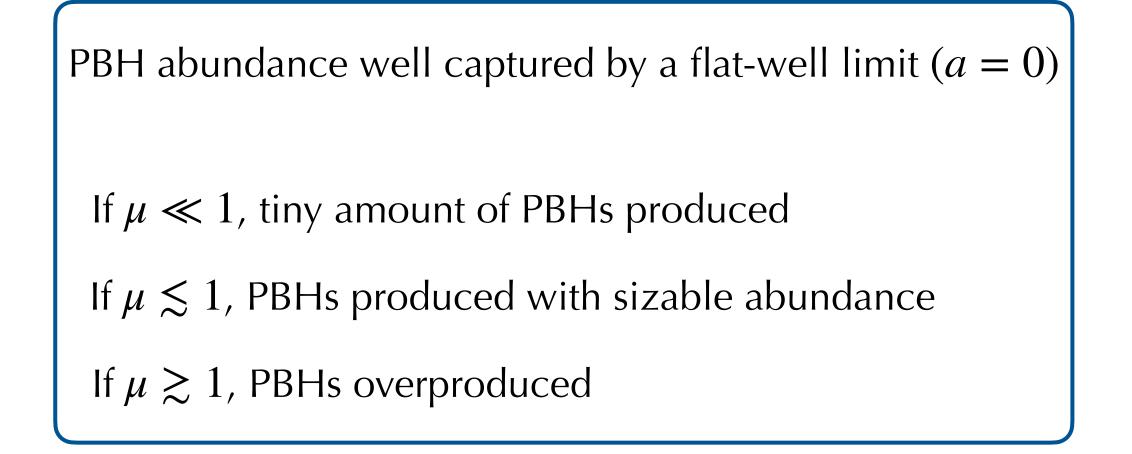


Quadratic model



Quadratic false vacuum

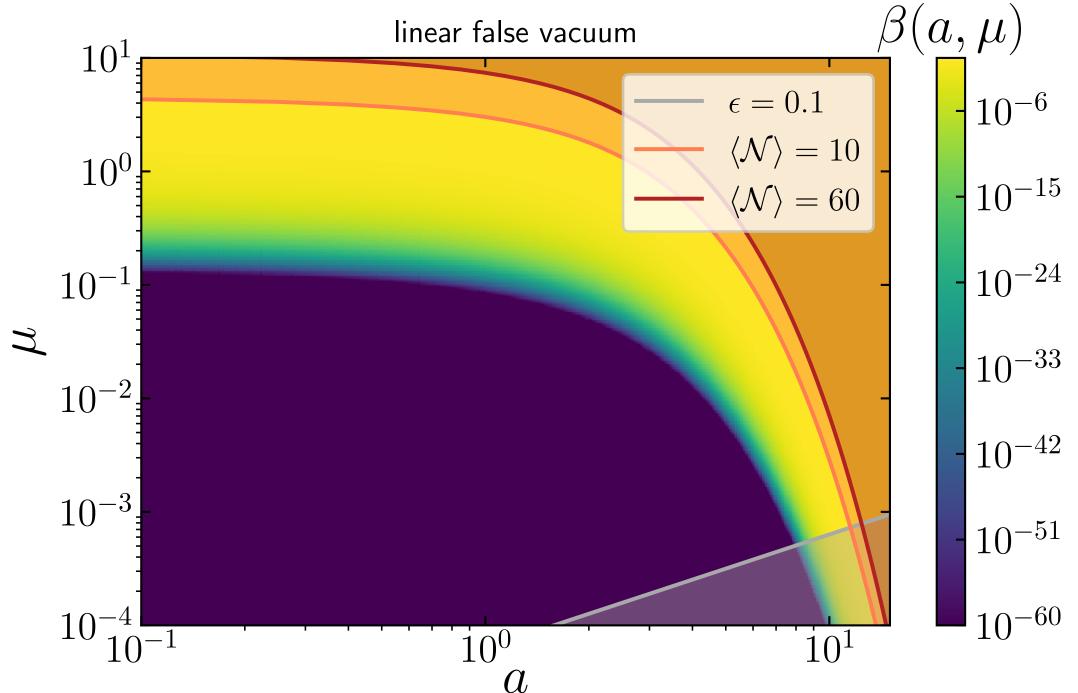


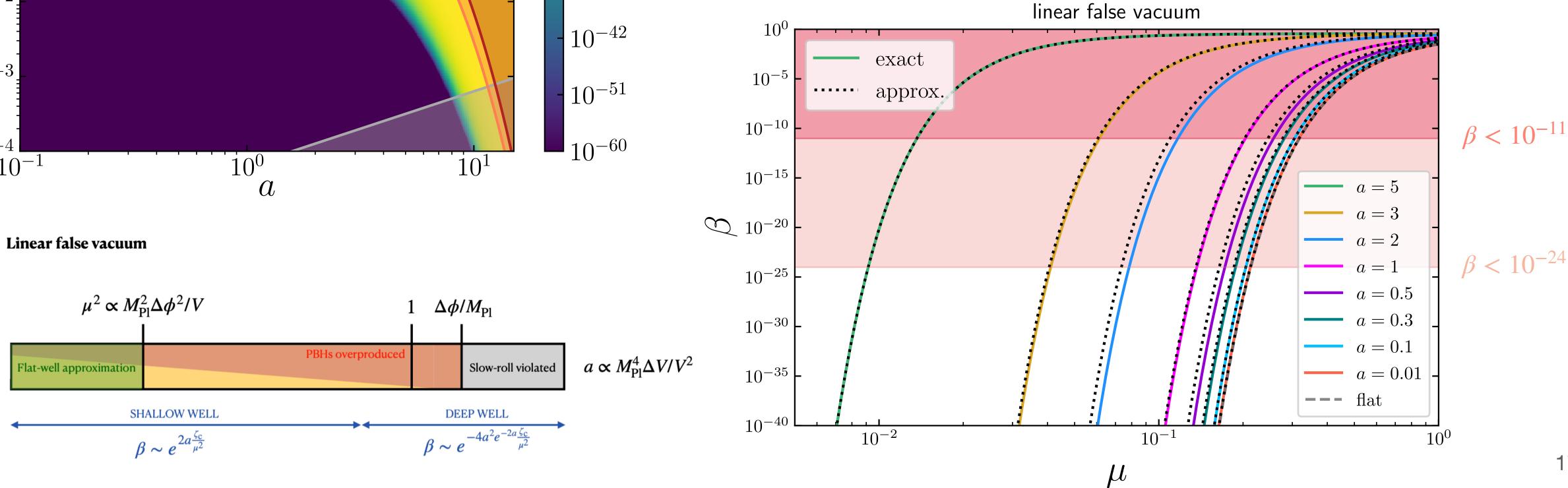




False vacuum: implications for Primordial Black Holes

Linear model





Additional regimes:

If $\mu^2 \ll a \ll 1$ (μ small): large deviations from flat-well, still shallow-well domain; non-trivial imprint of the false-vacuum profile

If $a \sim \mathcal{O}(1)$: large PBH production



17



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- Generalisation beyond slow roll
- Impact of non-gaussian tails on different properties of PBHs, and in different scenarios, even at CMB and LSS scales







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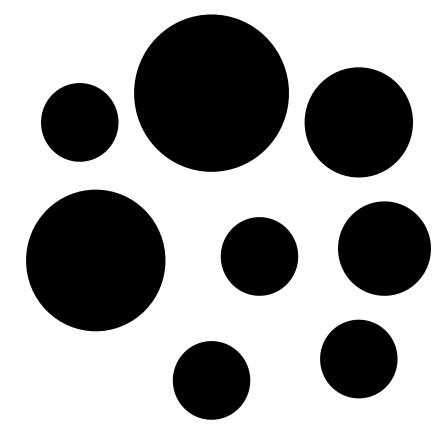




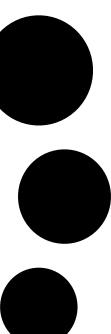


Many thanks for the attention!

chiara.animali@phys.ens.fr







False vacuum: preserving slow roll Slow roll requires: $|\ddot{\phi}| \ll 3H|\dot{\phi}|, |V_{,\phi}|$

$$\ddot{\phi} + 3 H(\phi, \dot{\phi}) \dot{\phi} + V_{,\phi} = 0 \qquad H^2(\phi, \dot{\phi}) = \frac{1}{3M_{Pl}^2} \left(V(\phi) + \frac{\dot{\phi}^2}{2} \right)$$

Linearised Klein-Gordon equation

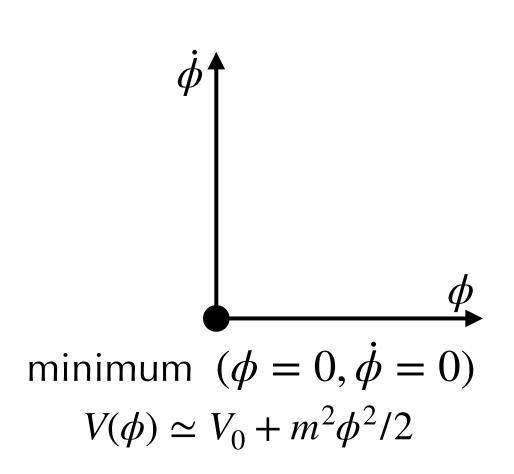
$$\begin{split} \dot{\phi} &= A \exp\left[-\frac{3}{2}\left(1 + \sqrt{1 - \frac{4m^2}{9H_0^2}}\right)H_0t\right] + B \exp\left[-\frac{3}{2}\left(-1 - \sqrt{1 - \frac{4m^2}{9H_0^2}}\right)H_0t\right] \end{split}$$

 $m \gg 3H_0/2$: damped oscillations, friction term $3H\dot{\phi}$ subdominant: far from slow-roll regime

$$m \ll 3H_0/2 \qquad \phi \simeq A \exp\left(-3H_0t\right) + B \exp\left(-\frac{1}{3}\frac{m^2}{H_0^2}H_0t\right) \simeq B \exp\left(-\frac{m^2t}{3H_0}\right)$$

$$3H\dot{\phi} \simeq -m^2\phi = -V_{,\phi}(\phi) \qquad \qquad \ddot{\phi} \simeq \frac{m^4}{9H_0^2}\phi = \frac{m^2}{9H_0^2}V_{,\phi} \ll V_{,\phi}(\phi)$$

What happens if $|V_{,\phi}| = 0$?



 (ϕ)

slow-roll regime: acceleration term subdominant (m^2/H_0^2 - suppressed)

Primordial black holes: observational constraints

Depends on the mass at which PBHs form

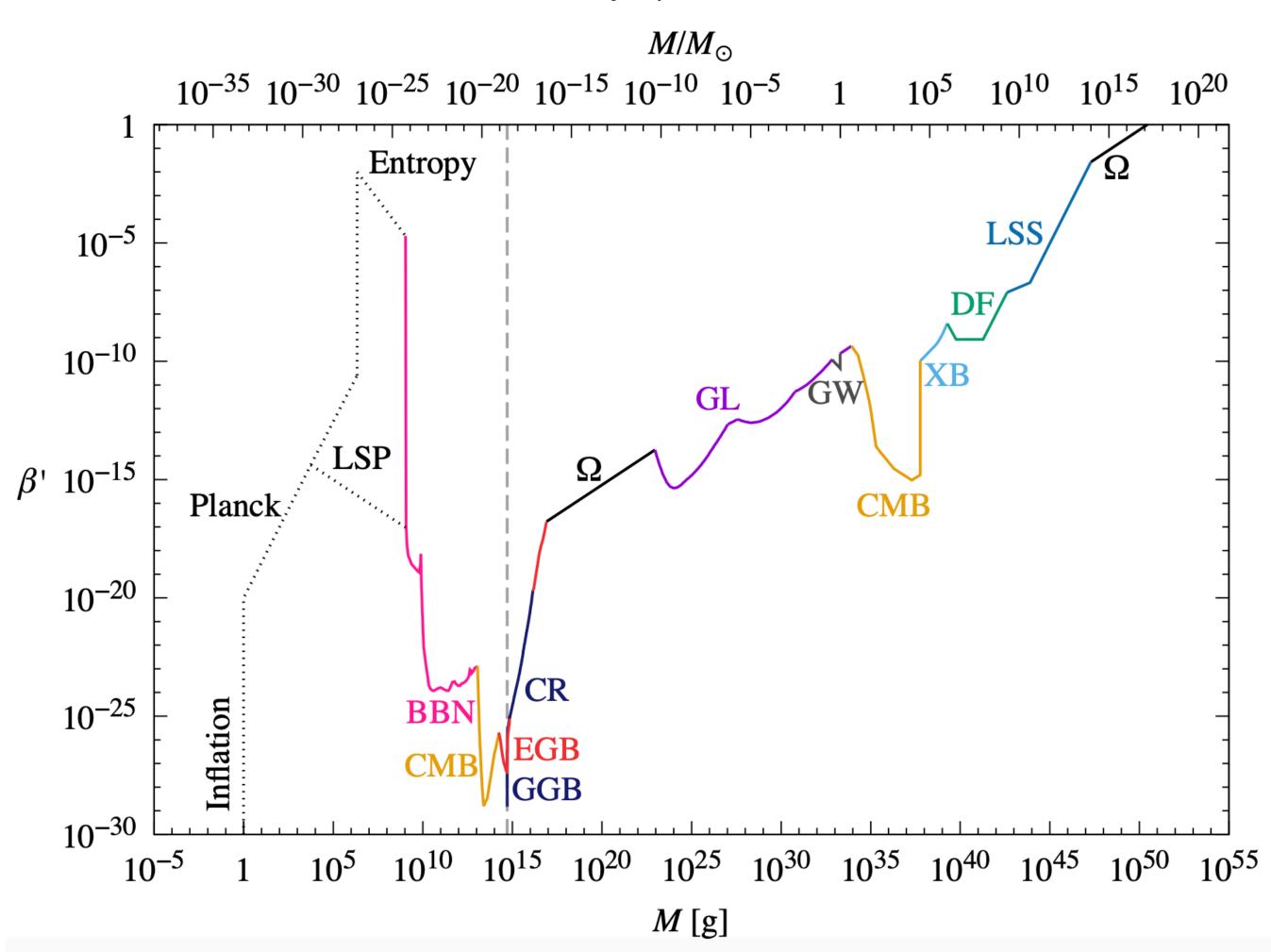
 $10^{9}g < M_{PBH} < 10^{16}g \longrightarrow \text{from } \beta < 10^{-24} \text{ to } \beta < 10^{-17}$ $10^{16}g < M_{PBH} < 10^{50}g \longrightarrow \text{from } \beta < 10^{-11} \text{ to } \beta < 10^{-5}$

 $M_{PBH} < 10^9 g$

Not yet evaporated: no direct observational constraints

B. Carr, K. Kohri, Y. Sendouda, J. Yokoyama [2021] *Constraints on Primordial Black Holes* 17 PBH Hawking evaporation on Big Bang Nucleosynthesis and on the extragalactic photon background

Gravitational and astrophysical effects



Primordial black holes: observational constraints

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