

CMB-slow: How to estimate cosmological parameters by hand

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Parameters

Metric

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(t)((1 - 2\Phi)\delta_{ik} + h_{ik}^{TT})dx^i dx^k$$

gravitational potential gravity waves

After inflation

$$|k^3 \Phi_k^2| = A k^{n_s - 1}$$

$$|k^3 h_k^2| = B k^{n_T}$$

Matter

$$1 = \Omega_{tot}^0 = \Omega_\gamma^0 + \Omega_\nu^0 + \Omega_b^0 + \Omega_{CDM}^0 + \Omega_{\Lambda,Q}^0$$

Prediction of inflation!

● Main unknown parameters determining the spectrum

$$1) h_{75} = \frac{H_0}{75 \frac{km}{s \cdot Mpc}}$$

$$2) \Omega_m^0 = \Omega_b^0 + \Omega_{CDM}^0$$

$$3) \Omega_b^0$$

$$4) \Omega_{\Lambda,Q}^0$$

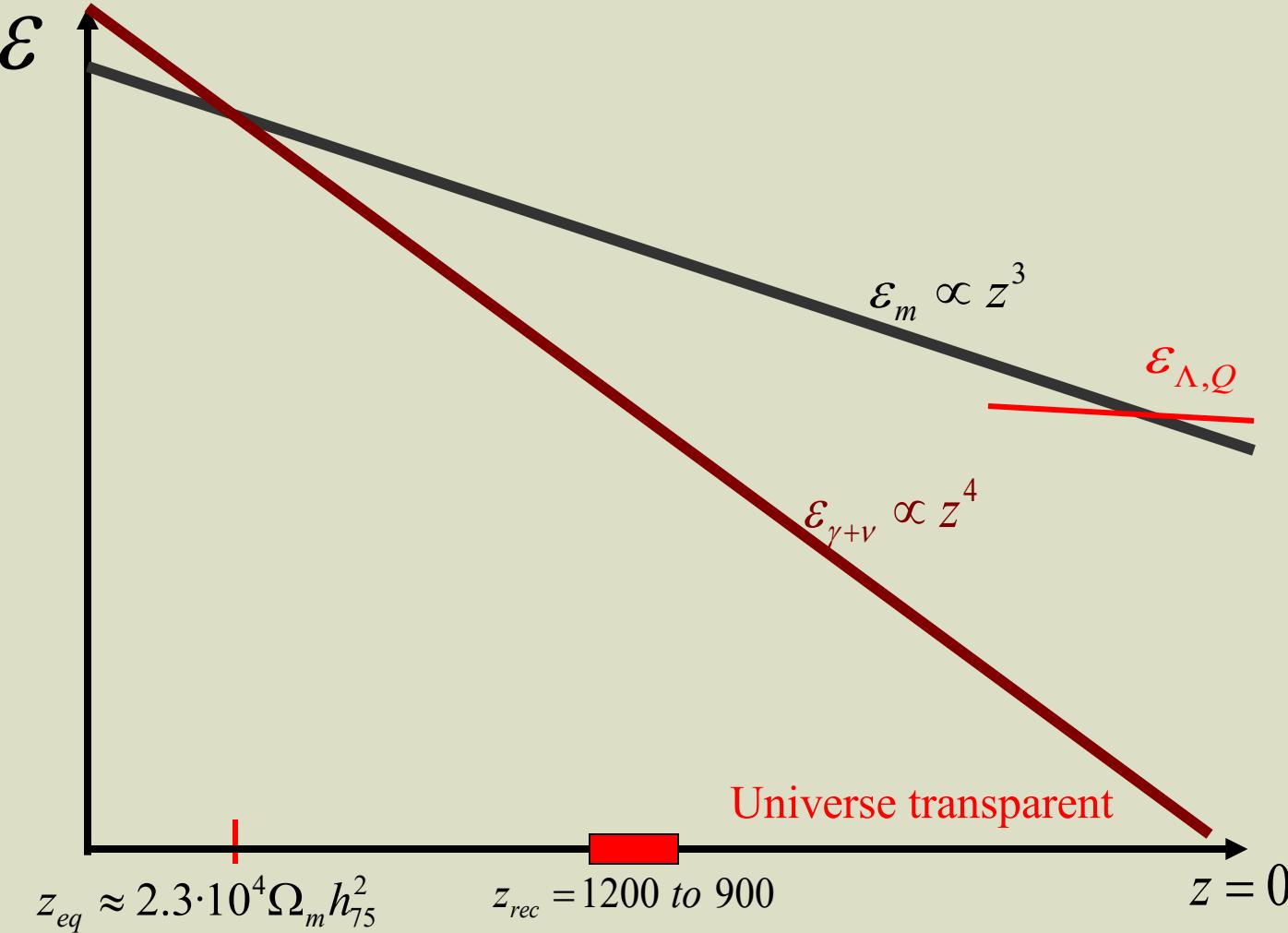
5) amplitude A

6) spectral index n_s

If $\Omega_{tot}^0 = 1$ then $\Omega_{\Lambda,Q}^0 = 1 - \Omega_m^0$

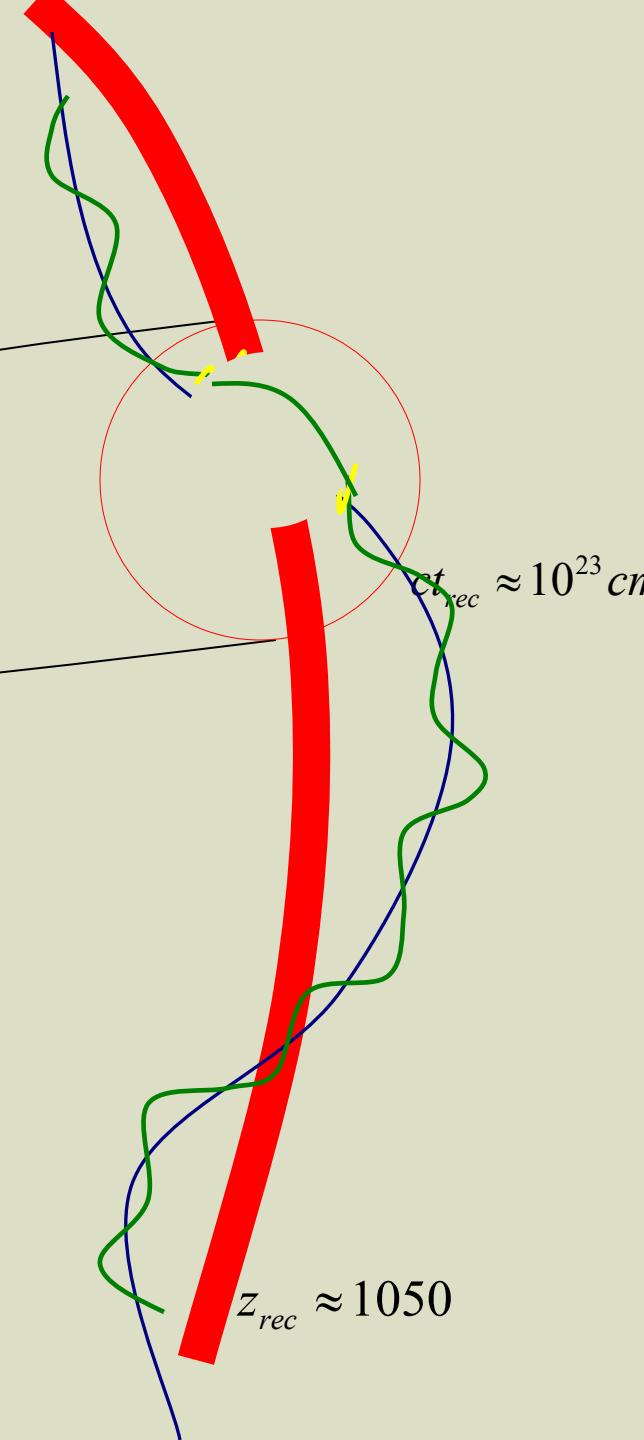
● Extra parameters influencing the spectrum

number of light neutrinos, gravity waves, reionization





$0.87^\circ \Omega_{tot}^{1/2}$



● Before recombination

- (Im)perfect fluid: coupled baryon-radiation plasma

$$T_{\alpha}^{\beta} = (\varepsilon_{\gamma} + \varepsilon_b + p_{\gamma}) u_{\alpha} u^{\beta} - p_{\gamma} \delta_{\alpha}^{\beta} + \text{viscosity terms}$$

- CDM particles which interact with plasma only gravitationally

The fractional density perturbations in radiation $\delta = \delta \varepsilon_{\gamma} / \varepsilon_{\gamma}$ satisfy the equation:

$$a \frac{\partial}{\partial t} \left(\frac{\delta'}{c_s^2} \right)' - \frac{3\zeta}{\varepsilon_{\gamma} a} \Delta \delta' - \Delta \delta = \frac{4}{3c_s^2} \Delta \Phi + \left(\frac{4\Phi'}{c_s^2} \right)' - \frac{12\zeta}{\varepsilon_{\gamma} a} \Delta \Phi'$$

↑ speed of sound
↓ viscosity
↑

Gravitational potential is generated mainly by radiation
before equality and by cold matter after that

● After recombination

$$\left(\frac{\partial}{\partial \eta} + n^i \frac{\partial}{\partial x^i} \right) \left(\frac{\delta T}{T} + \Phi \right) = 0$$



$$\left(\frac{\delta T}{T} + \Phi \right) = const$$

along photon's geodesic

$$x^i(\eta) = x_0^i + n^i(\eta - \eta_0)$$



$$\frac{\delta T}{T}(\eta_0, x_0^j, \mathbf{n}^i) = \frac{\delta T}{T}(\eta_r, x^j(\eta_r), \mathbf{n}^i) + \Phi(\eta_r, x^j(\eta_r))$$



$$\mathbf{n}^i$$

$$rec \frac{\delta T}{T}(\eta_r, x^j(\eta_r), \mathbf{n}^i) - ?$$

$$\frac{\delta T}{T}(\eta_r, x^j(\eta_r), \mathbf{n}^i) - ?$$

- Matching conditions for T_α^β

Instantaneous recombination



$$\frac{\delta T}{T}(\eta_0, \mathbf{x}_0, \mathbf{n}) = \int \left(\Phi_k + \frac{\delta_k}{4} - \frac{3\delta'_k}{4k^2} \frac{\partial}{\partial \eta_0} \right)_{\eta_r} e^{i\mathbf{k}(\mathbf{x}_0 + \mathbf{n}(\eta_r - \eta_0))} \frac{d^3 k}{(2\pi)^{3/2}}$$

- Correlation function

$$C(\theta) = \left\langle \frac{\delta T}{T}(\mathbf{n}_1) \frac{\delta T}{T}(\mathbf{n}_2) \right\rangle = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l P_l(\cos \theta)$$

$\cos \theta = \mathbf{n}_1 \cdot \mathbf{n}_2$

Recombination is non-instantaneous

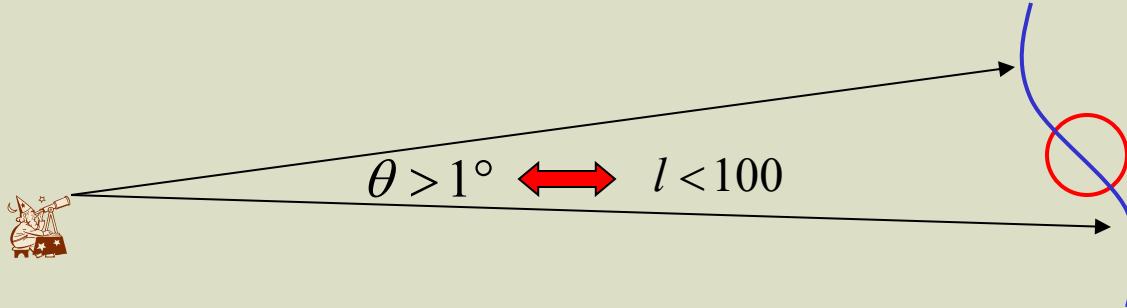
where

$$C_l = \frac{2}{\pi} \int \left| \left(\Phi_k + \frac{\delta_k}{4} \right)_{\eta_r} j_l(k\eta_0) - \frac{3\delta'_k(\eta_r)}{4k} \frac{dj_l(k\eta_0)}{d(k\eta_0)} \right|^2 e^{-2\sigma k\eta_r} k^2 dk$$

$\sigma \approx 2 \cdot 10^{-2}$

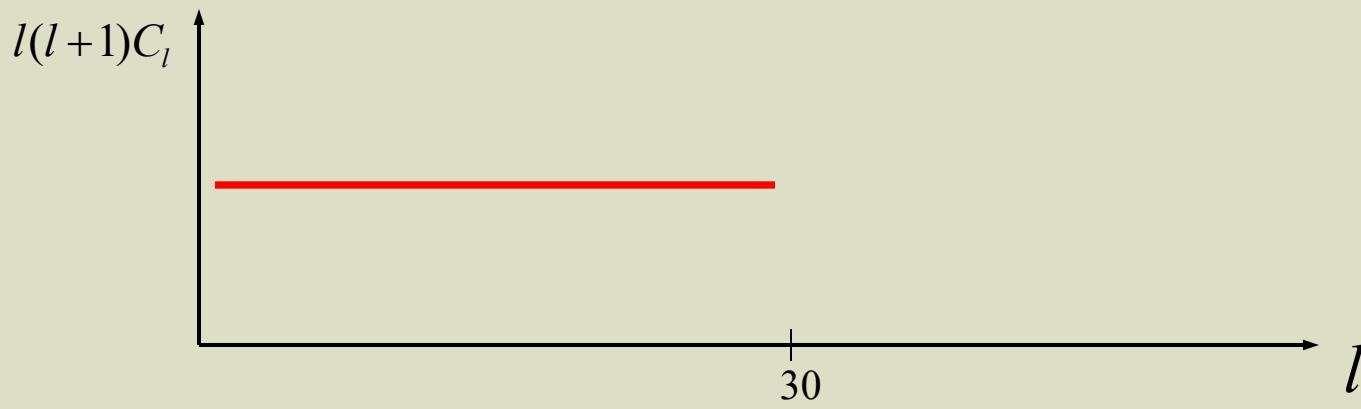
$l(l+1)C_l$ characterizes fluctuations on angular scales $\theta \approx \frac{\pi}{l}$

- Longwave inhomogeneities ($k\eta_r \ll 1$)



$$\left| k^3 \Phi_k^2 \right|_{\eta_r} = A' k^{n_S - 1}, \quad \delta(\eta_r) = \frac{\delta \varepsilon_r}{\varepsilon_r} = \frac{4 \delta \varepsilon_m}{3 \varepsilon_m} = -\frac{8}{3} \Phi, \quad \delta'(\eta_r) = 0$$

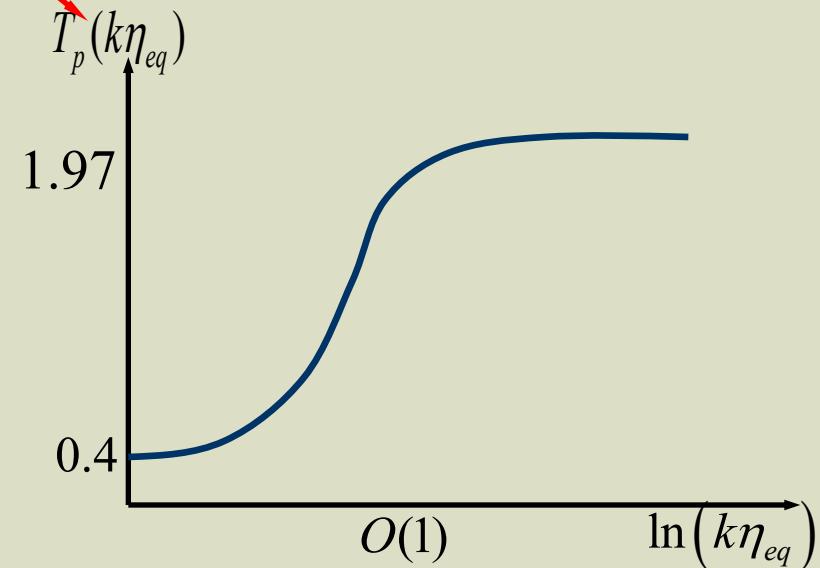
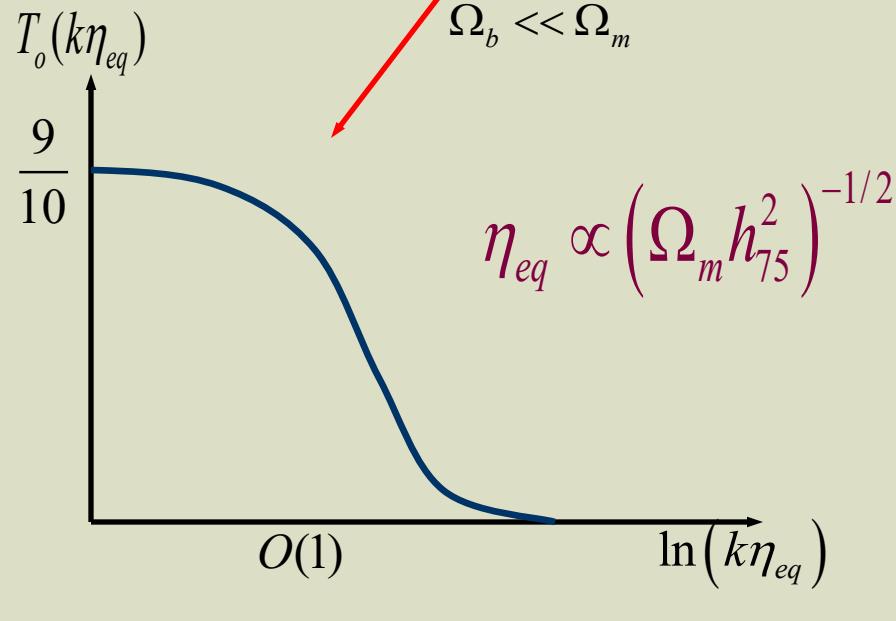
$$l(l+1)C_l \approx \frac{9A'}{100\pi} \quad \text{for } l < 30 \quad \text{if } n_S \approx 1$$



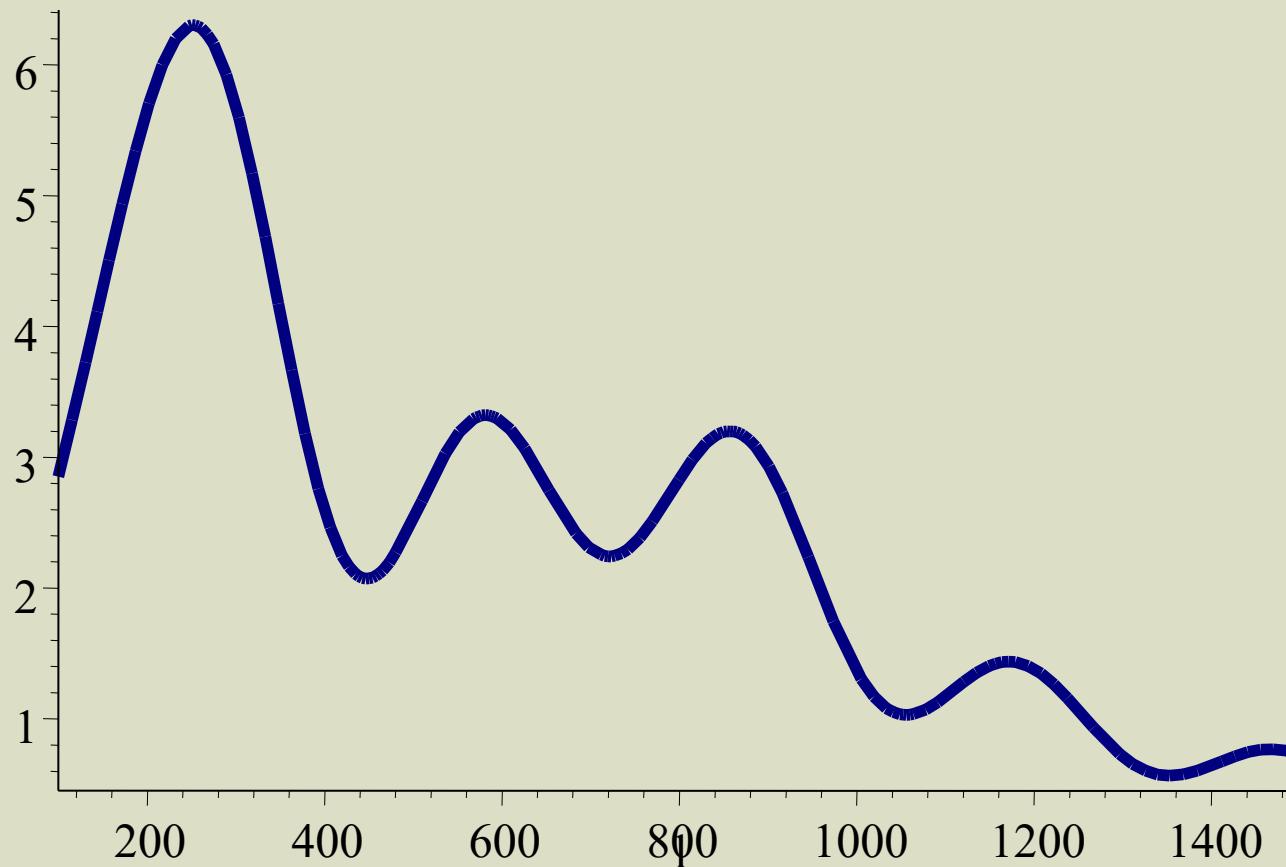
● Shortwave inhomogeneities ($k\eta_r \gg 1$)

$$C_l \approx \frac{1}{16\pi} \int_{l\eta_0^{-1}}^{\infty} \left(\frac{|\Phi + 4\delta| k^2}{(k\eta_0)\sqrt{(k\eta_0)^2 - l^2}} + \frac{9\sqrt{(k\eta_0)^2 - l^2}}{(k\eta_0)^3} \delta' \right) e^{-2\sigma k\eta_r} dk \quad \text{for } l \gg 1$$

$$\left(\Phi_k + \frac{\delta_k}{4} \right)_{\eta_r} \approx \left[T_p \left(1 - \frac{1}{3c_S^2} \right) + T_o \sqrt{c_S} \cos \left(k \int_0^{\eta_r} c_S d\eta \right) e^{-(k/k_D)^2} \right] \Phi_l^0$$



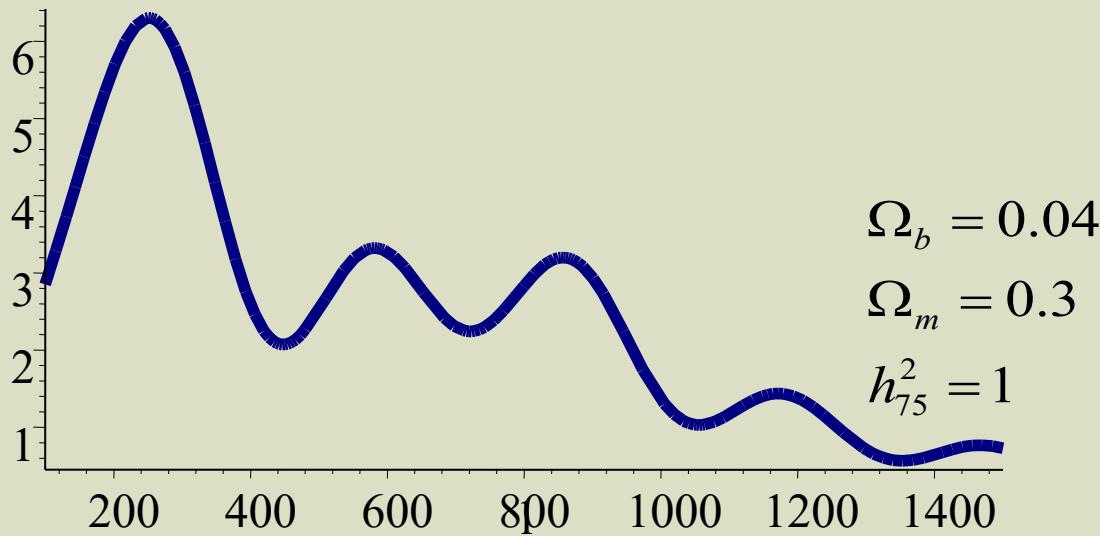
- Spectrum



- Determining the cosmological parameters

$$h_{75}, \Omega_m, \Omega_b, \Omega_{tot} = \Omega_m + \Omega_{\Lambda,Q}, A, n_s$$


 $\propto \frac{1}{\sqrt{\Omega_{tot}}} \quad (\Lambda=0)$

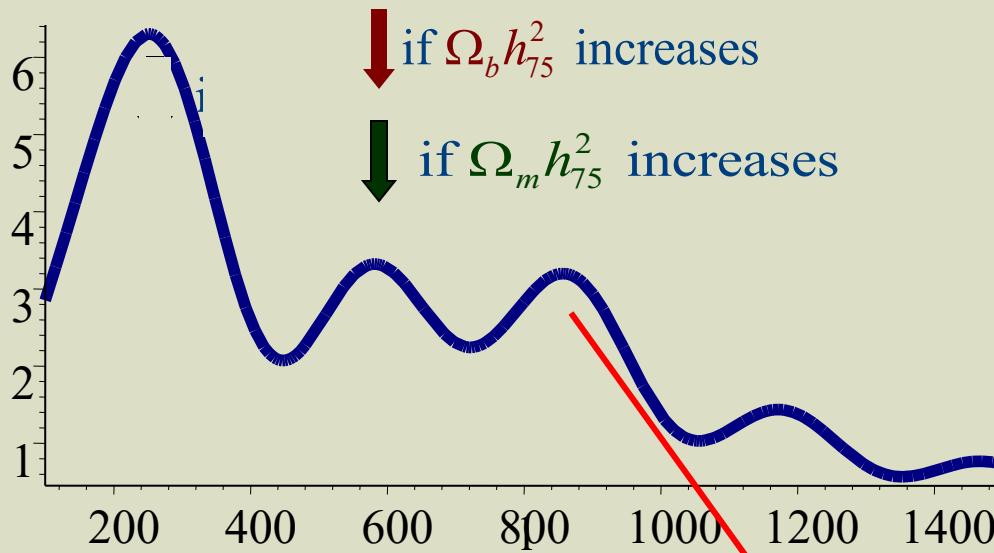


– The location of the peaks ($\Omega_{tot} = 1$)

$$l_n \approx \frac{\pi}{\rho} n \propto \frac{\left(1 + 2.2\Omega_b h_{75}^2\right)}{\left(\Omega_m h_{75}^{3.1}\right)^{0.16}}$$


 $\Delta l_1 = +40$ when $\Omega_b h_{75}^2$ increases twice
 $\Delta l_1 = -40$ when $\Omega_m h_{75}^2$ increases twice

– The heights of the peaks



The second peak exists $\rightarrow \Omega_b h_{75}^2 < 0.07$

$\Omega_m h_{75}^2 < 0.3$ if $\Omega_b h_{75}^2 > 0.03$

$\Omega_{\Lambda,Q} \neq 0!!!$ because $\Omega_{tot} = 1$

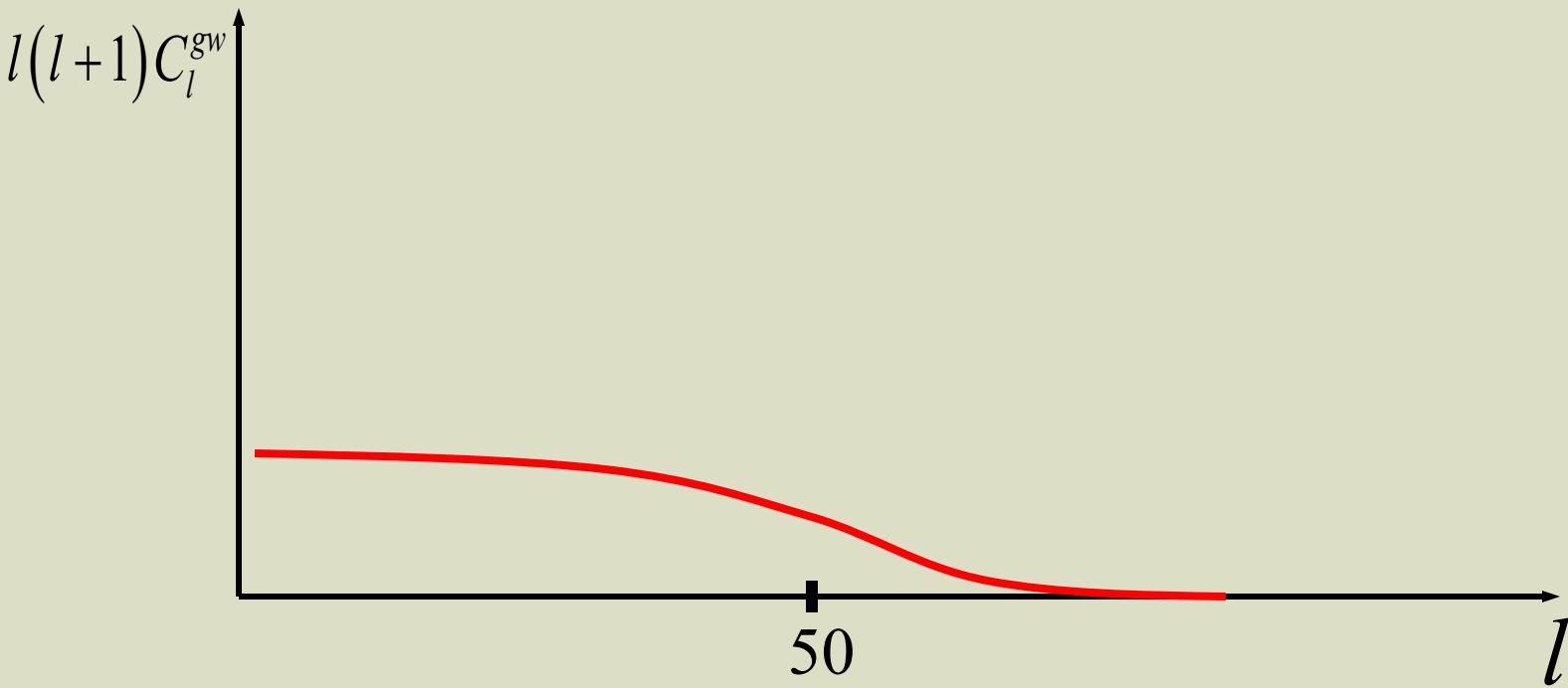
$n_s - ?$ for given $\Omega_b h_{75}^2$ and $\Omega_m h_{75}^2$

$h_{75}^2 - ?$ peaks location depends on $\Omega_m h_{75}^2$
peaks heights depends on $(\Omega_m h_{75}^{3.1})^{0.16}$

$\rightarrow 1\% \text{ in location} \leftrightarrow$

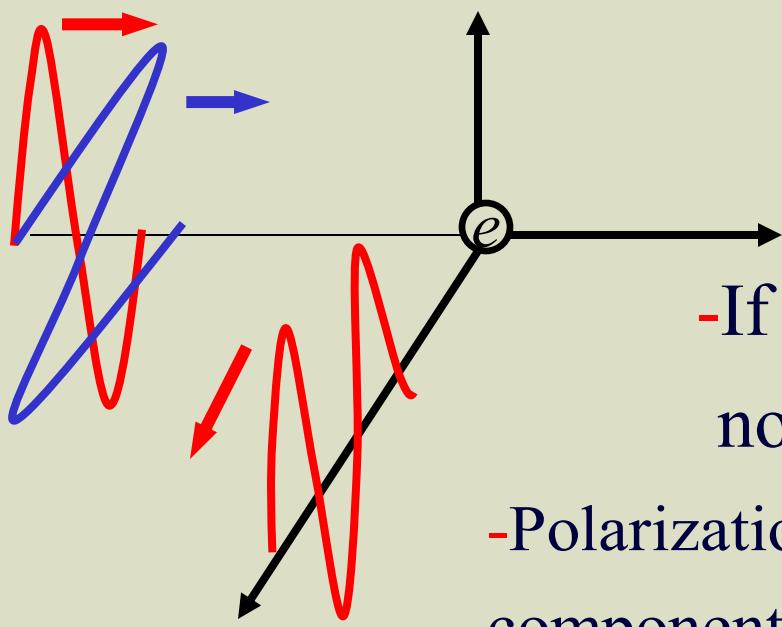
$7\% \text{ in } h_{75}$

- Gravitational waves



- CMB polarization-way to detect primordial gravitational waves

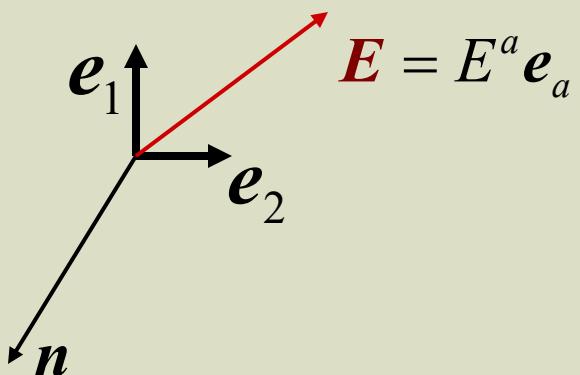
Thomson scattering \rightarrow linear polarization



100% polarized

- If incident radiation is isotropic-
no polarization is generated
- Polarization is proportional to the quadrupole component of incoming radiation
- Quadrupole anisotropy is generated by both,
scalar perturbations and gravity waves, only after
the recombination begins \rightarrow *polarization \propto duration of recombination*

• Polarization tensor



$$P_{ab}(\mathbf{n}) = \frac{1}{I} \left(\langle E_a E_b \rangle - \frac{1}{2} g_{ab} \langle E_c E^c \rangle \right)$$

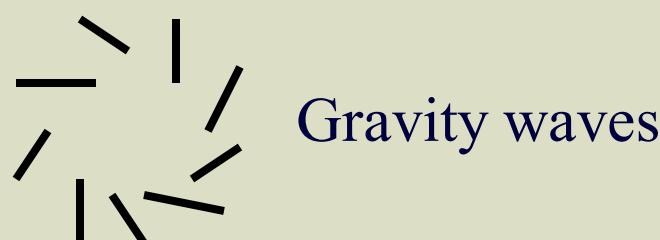
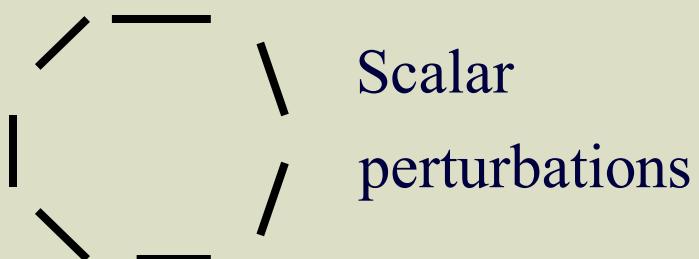
2d second rank traceless tensor


E – mode: B – mode:

$$E(\mathbf{n}) = P_{a;b}^{b;a} \quad B(\mathbf{n}) = P_a^{b;ac} \epsilon_{bc}$$

Only gravitational waves contribute to B !!!

$$P_{ab} = p_a p_b - \frac{1}{2} g_{ab} p^2 \quad p_a(\mathbf{n}) - ?$$



$\langle BB \rangle$ gets maximum at $l \approx 100$