

Can we spoil the robust
predictions?

THEORY OF COSMOLOGICAL PERTURBATIONS

PHYSICS REPORTS (Review Section of Physics Letters) 215, Nos. 5 & 6 (1992) 203–333. North-Holland

V.F. MUKHANOV^{a,b}, H.A. FELDMAN^c and R.H. BRANDENBERGER^a

<i>Part III. Extensions</i>	296
16. Introduction	296
17. Microwave background anisotropies	297
18. Gravitational waves	301
18.1. Quantization	302
18.2. Observables	305
18.3. Spectrum of gravitational waves in de Sitter space	306
18.4. Spectrum of gravitational waves in the inflationary universe	307
18.5. Spectrum of gravitational waves in double inflation models	310
19. Entropy perturbations	313
19.1. General remarks	313
19.2. A model for entropy perturbations	314
19.3. Evolution of the homogeneous field	314
19.4. Perturbations	316
19.5. Mountain and valley spectra	319
19.6. Suppression of long-wavelength perturbations	320
19.7. Modulation of the spectrum in double inflation models	321

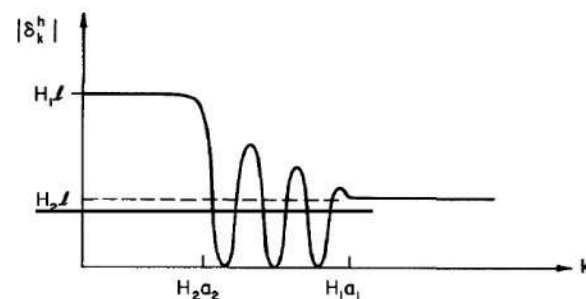
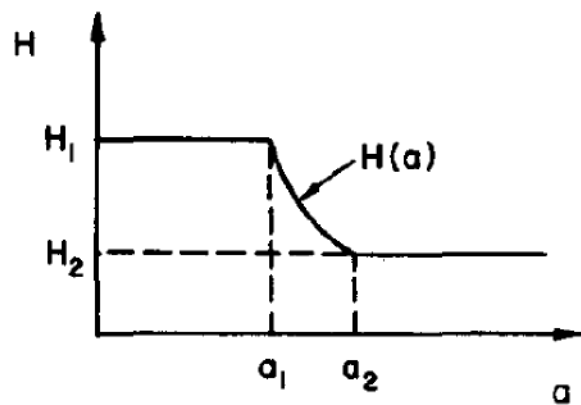


Fig. 18.7. Power spectrum $\delta_k = h$ of gravitational radiation in the double inflation model of (18.40) with the ratio of Hubble constants $H_1/H_2 = 10$.

Note that the effect discussed in this section can arise for both adiabatic and entropy perturbations. It is possible to obtain a suppression of the long-wavelength part of cosmological perturbation

$$S = \int \left[\frac{1}{2} \chi_{;\mu} \chi^{;\mu} - V(\chi) + \frac{1}{2} \varphi_{;\mu} \varphi^{;\mu} - \frac{1}{2} m_0^2 \varphi^2 - V_1(\chi, \varphi, \dots) \right] \sqrt{-g} d^4x ,$$

nontrivial spectra with mountains and valleys can also be obtained

It is also possible to generate non-Gaussian fluctuations

However, this procedure is extremely unappealing since it implies a complete loss of predictability.

MFB, Physics Report, 1992

Inflation is **THE theory** only when it is understood as the stage of unbroken accelerated expansion due to the same ingredient which is responsible for quantum fluctuations.

Otherwise it is rubbish without any predictions!!!

In this case it is unbeatable as predictive theory because it allows us to calculate the effect of amplification of quantum fluctuations in completely controllable weak coupling regimes

while most alternatives cannot even compete with "rubbish inflation" in a sense of controllable reproduction of outcome for quantum fluctuations

COSMOLOGY - Theology = $\exp(Ht)$

during at least $70 H^{-1}$, but less than $10^6 H^{-1} \rightarrow$

no any problems with predictions, which could
falsify the theory in Popper's sense

What is relevant for predictions?

– ε energy density

– p pressure

$$1 + w \equiv \frac{\varepsilon + p}{\varepsilon} \ll 1$$

during last 70 e-folds ($a = a_f \cdot e^{-N}$)

a) $1 + w \ll 1$ for $N \gg 1$

b) $1 + w \approx O(1)$ for $N \simeq O(1)$

c) $1 + w$ is a smooth function of N

Inflation is not a unique theory, but rather a class of models based on similar principles.

WRONG!

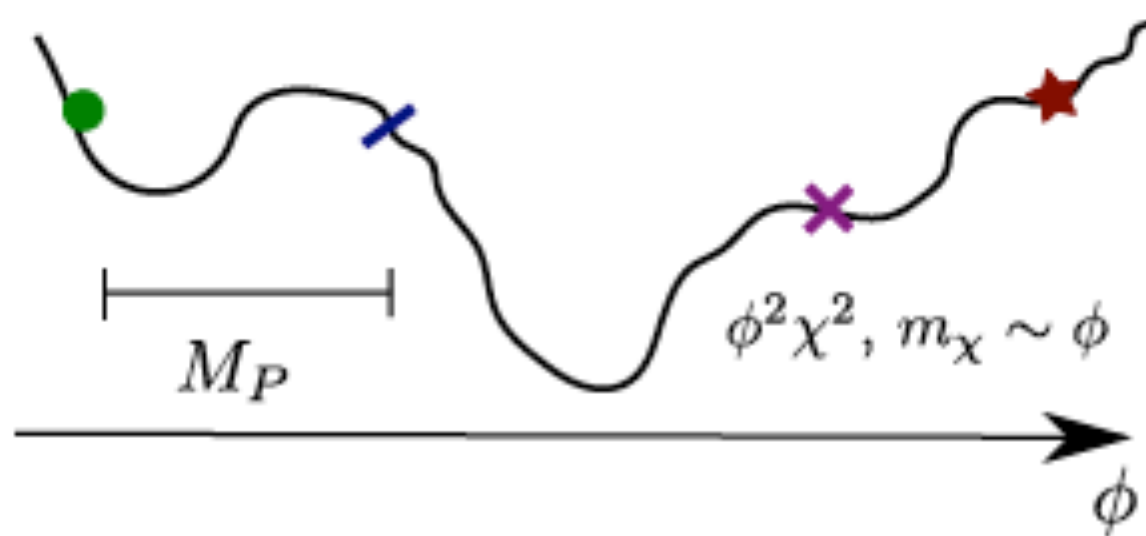
The only purpose of inflationary models relevant for observation is a mapping

$$V(\varphi) \quad \text{to} \quad p \approx -\varepsilon$$

and this mapping happened to be not crucial for robust predictions but important only for excluding definite potentials $V(\varphi)$, which anyway we will never be able to verify in any other independent experiments

$$V(\tau, \theta) = \frac{12W_0^2\xi}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)^2} + \frac{D_1 + 12e^{-2a_2\tau} \xi A_2^2}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)^2} + \frac{D_2 + \frac{16(a_2 A_2)^2}{3\alpha\lambda_2} \sqrt{\tau} e^{-2a_2\tau}}{(2\mathcal{V}_m + \xi)} \quad (25)$$

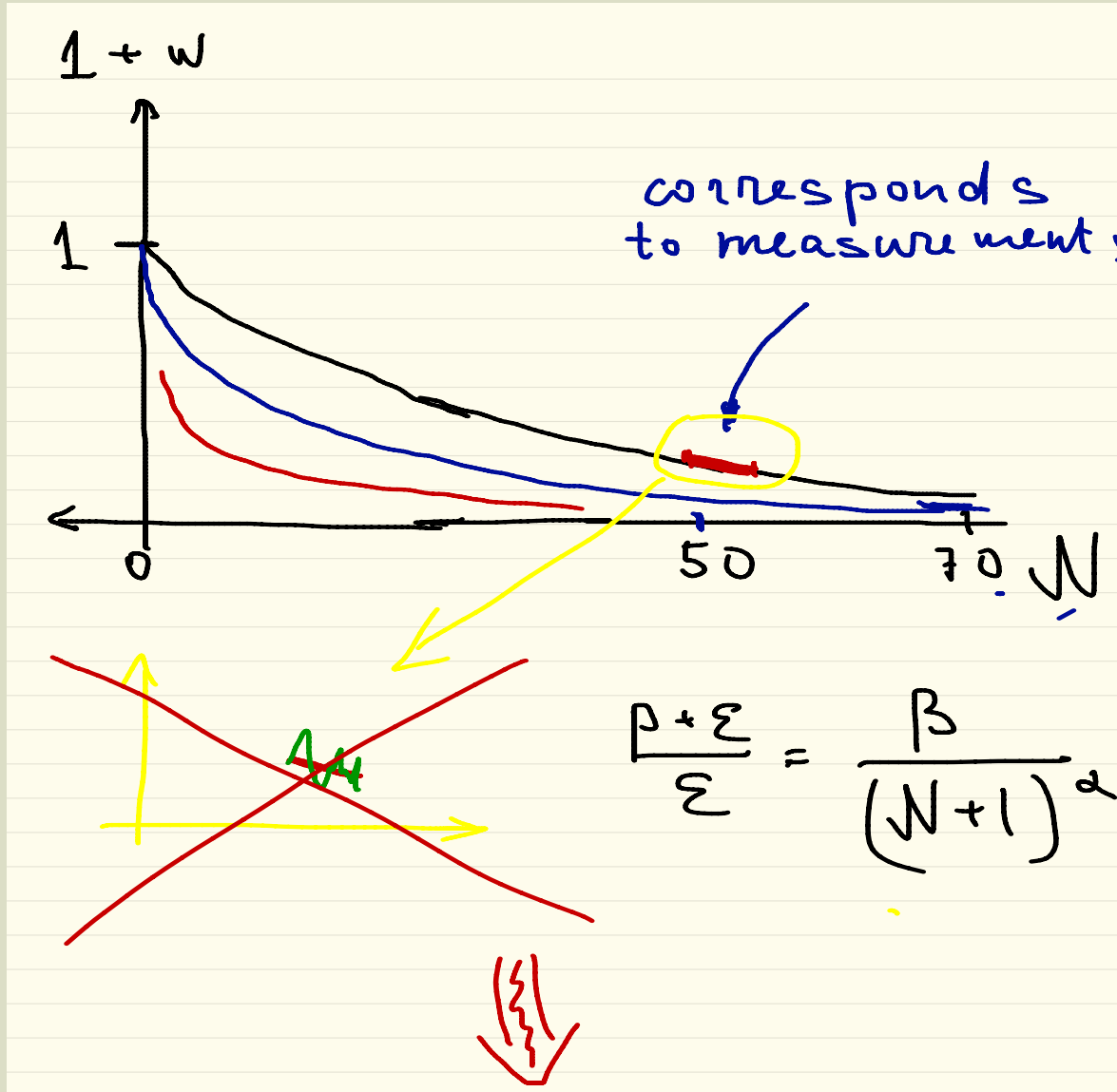
$$+ \frac{D_3 + 32e^{-2a_2\tau} a_2 A_2^2 \tau (1 + a_2\tau)}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)} + \frac{D_4 + 8W_0 A_2 e^{-a_2\tau} \cos(a_2\theta)}{(4\mathcal{V}_m - \xi)(2\mathcal{V}_m + \xi)} \left(\frac{3\xi}{(2\mathcal{V}_m + \xi)} + 4a_2\tau \right) + \frac{\beta}{\mathcal{V}_m^2}.$$



a) $1+w \ll 1$ for $N \gg 1$

b) $1+w \approx O(1)$ for $N \approx O(1)$

c) $1+w$ is a smooth function of N



PREDICTIONS

("smoking guns"-nonconfirming any of them would falsify THE theory)

- flat universe
- adiabatic perturbations
- small non-gaussianity ($f_{NL} \sim O(1)$)
- red-tilted spectrum

$$\Phi^2 \propto \lambda^{1-n_s}$$

$$1 - n_s = 3(1 + w) - \frac{d \ln(1 + w)}{dN}$$

of the faint ripples that we detect in the cosmic microwave background (CMB). First, the ripples should be nearly scale-invariant, meaning that they have nearly the same intensity at

The theory always predicts **red-tilted** spectrum

From GR content \Rightarrow one postulate \equiv
 stage of accelerated expansion \Rightarrow
 explanation of hom, isotn.
 + 2 nontrivial predictions
 - $\Omega_{total} = 1 \pm 10^{-5}$
 - spectrum of perturb.
 spectrum is never HZ for
 generic inflation. It is tilted

$$\Phi_{\lambda}^2 = \frac{\epsilon^{10^{-12}}}{\epsilon_{pl}} \frac{1}{1 + \frac{p}{\epsilon}} \Big|_{\lambda^{-1} = H_0}$$

$$n_s - 1 = -3 \left(1 + \frac{p}{\epsilon} \right)_{\lambda^{-1} = H_0} - \frac{1}{H} \left(\ln \left(1 + \frac{p}{\epsilon} \right) \right)_{\lambda^{-1} = H_0}$$

$$0.9 \lesssim n_s < 0.96 !$$
 not too much grav. waves!
 gaussian perturbations

Cambridge, 2000

[1]. Contrary to an erroneous belief inflation does not predict the scale-invariant, Harrison-Zel'dovich spectrum. The spectral index should be in the range of $0.92 < n_s < 0.97$. The physical

V. Mukhanov, CMB, Quantum Fluctuations

and the Predictive Power of Inflation,

arXiv:astro-ph/0303077 (2003)

Red-tilted log spectrum (MC, H, 1981-1982) →

$$n_s = 1 - \frac{A}{\ln(B\lambda_{gal} / \lambda_{CMB})},$$

where $A > 1,5$ and $B \simeq 1-100$ depending on $50 < N < 55$ →

$$n_s < 0.97$$

irrespective of any particular model!

L.P. 9/6/2003:

We are writing a proposal to get money to do our small angular scale CMB experiment. If I say that simple models of inflation require $n_s = 0.95 \pm 0.03$ (95% cl) is it correct?

I'm especially interested in the error. **Specifically, if $n_s = 0.99$ would you throw in the towel on inflation?**

V.M. 9/8/2003

The "robust" estimate for spectral index for inflation is $0.92 < n_s < 0.97$.

The upper bound is more robust than lower. The physical reason for the deviation of spectrum from the flat one is the necessity to finish inflation....

If you find $n_s = 0.99 \pm 0.01$ (3 sigma) I would throw in the towel on inflation.

The unavoidable uncertainty in B is **bad news** for "model builders"!
It leads to theoretical uncertainty in prediction of n_s of order 0.005
for any model of inflaton and hence further increasing of experimental
accuracy in n_s will not help us much in model selection

Further predictions ("non-smoking guns"):

- Primordial gravitational waves
- Nongaussianities due to nonlinearity of

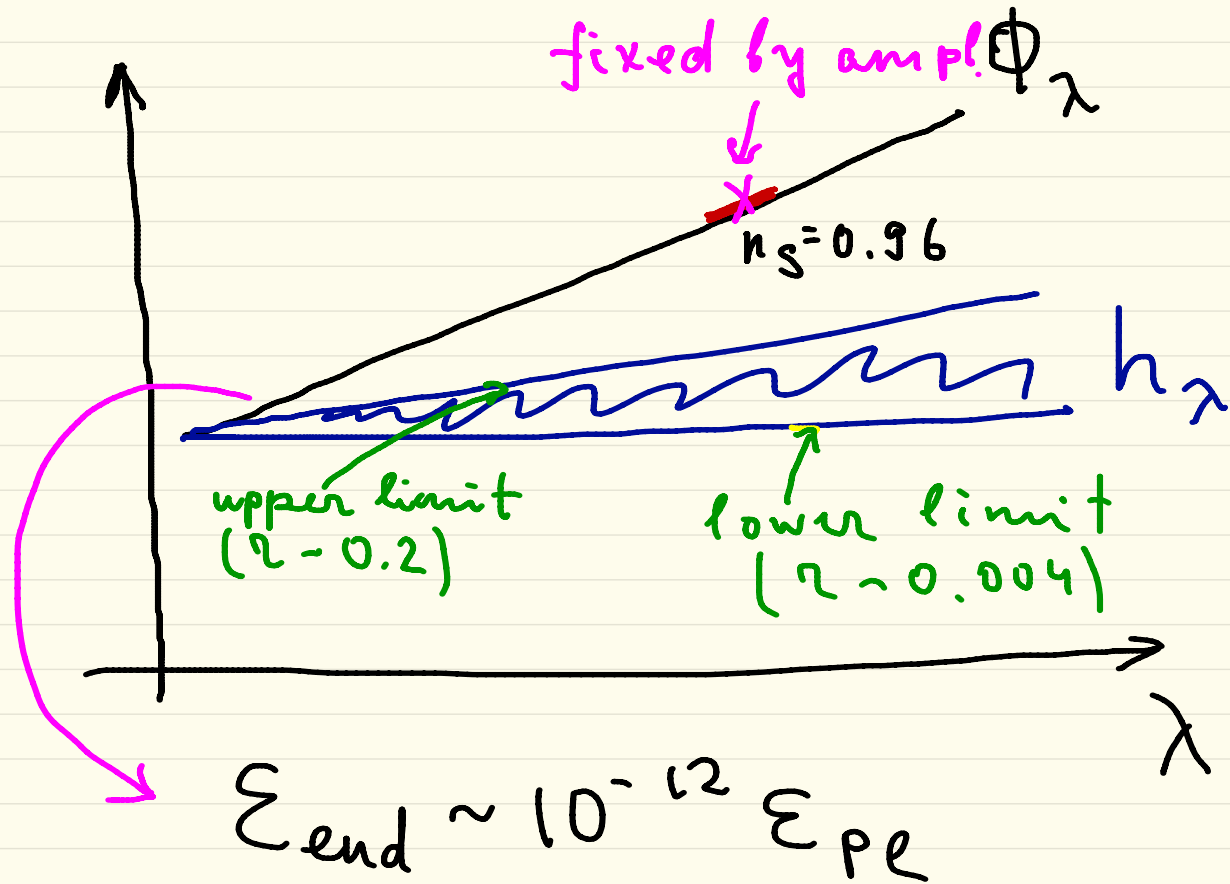
Einstein equation (3,4,...points correlaton functions)

There must be primordial gravitational waves

$$r \equiv \frac{T}{S} = 24 \cdot (1 + p / \varepsilon) = \frac{\beta}{N^\alpha}$$

No a priori low bound on their amplitude!

but if n_s is measured



However, keeping mind theoretical and experimental uncertainty, n_s within 2-sigma can be equal to 0.95.

In this case the lower bound on r becomes 0.0006

(unrealistic from the point of view of future measurements)

- Thus, detection of the primordial gravitational field will provide us an extra confirmation that quantum fluctuations were amplified on the stage of accelerated expansion.
- Failing to detect them at the level 0.04 would not have any implications and in no way can be considered as a prove of alternative for amplification of quantum fluctuation

Non-gaussianities

$$\Phi = \Phi_g + f_{NL} \Phi_g^2$$

$f_{NL} \simeq 0.04$ from inflation and $f_{NL} \simeq 2 - 4$ from subsequent evolution of perturbations

- What are the perspectives of measuring f ?

Not extremely promising