String Cosmology

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Motivation

- Superstring theory is a candidate for a unification of all four forces of Nature at the quantum level.
- Superstring theory may **remove the usual UV divergences** of point particle-based quantum field theories.
- Superstring theory has **new degrees of freedom** and **new symmetries** compared to point particle theories.
- Differences between string theory and point particle theories become large at large energies.
- Question: What does string theory predict for the evolution of the early universe?.

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Addressing Some Criticisms

see e.g.., C. Rovelli, arXiv:1108.0868

1. **String Theory Predicted Supersymmetry at the LHC Scale**
   - Generically string theory predicts supersymmetry breaking at the string scale, not the LHC scale.
   - However, this also means that string theory requires a separate solution of the hierarchy problem.
2. **Landscape Problem: String Theory is not Predictive**
   - Without guidance from string theory, there is a vast swampland of effective field theories (EFT): any dimension of space-time is o.k., any number of fields, any field range, any shape of a scalar field potential.
   - String theory is constraining: only a small set of islands in the vast swampland of EFTs is viable.

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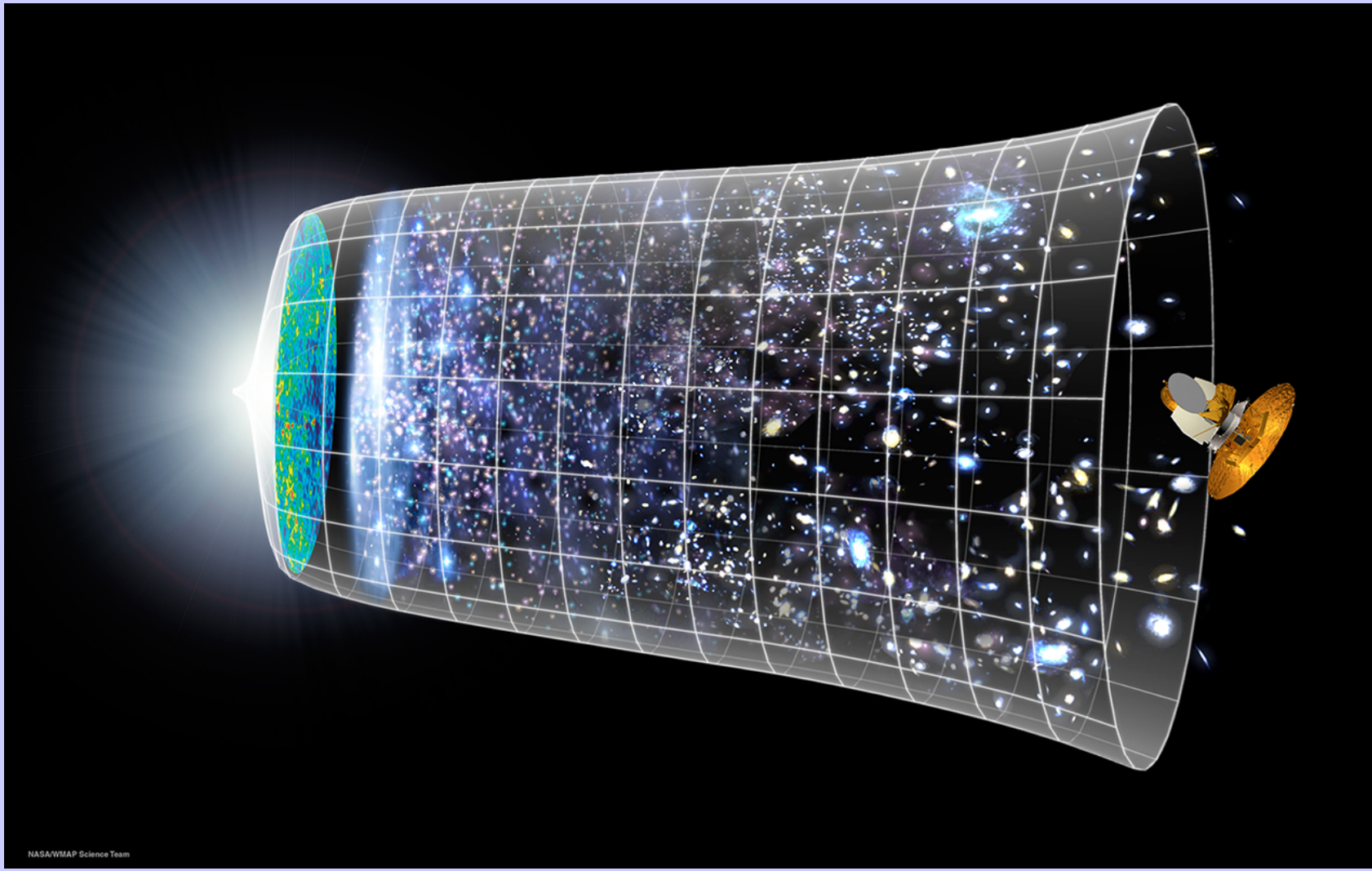
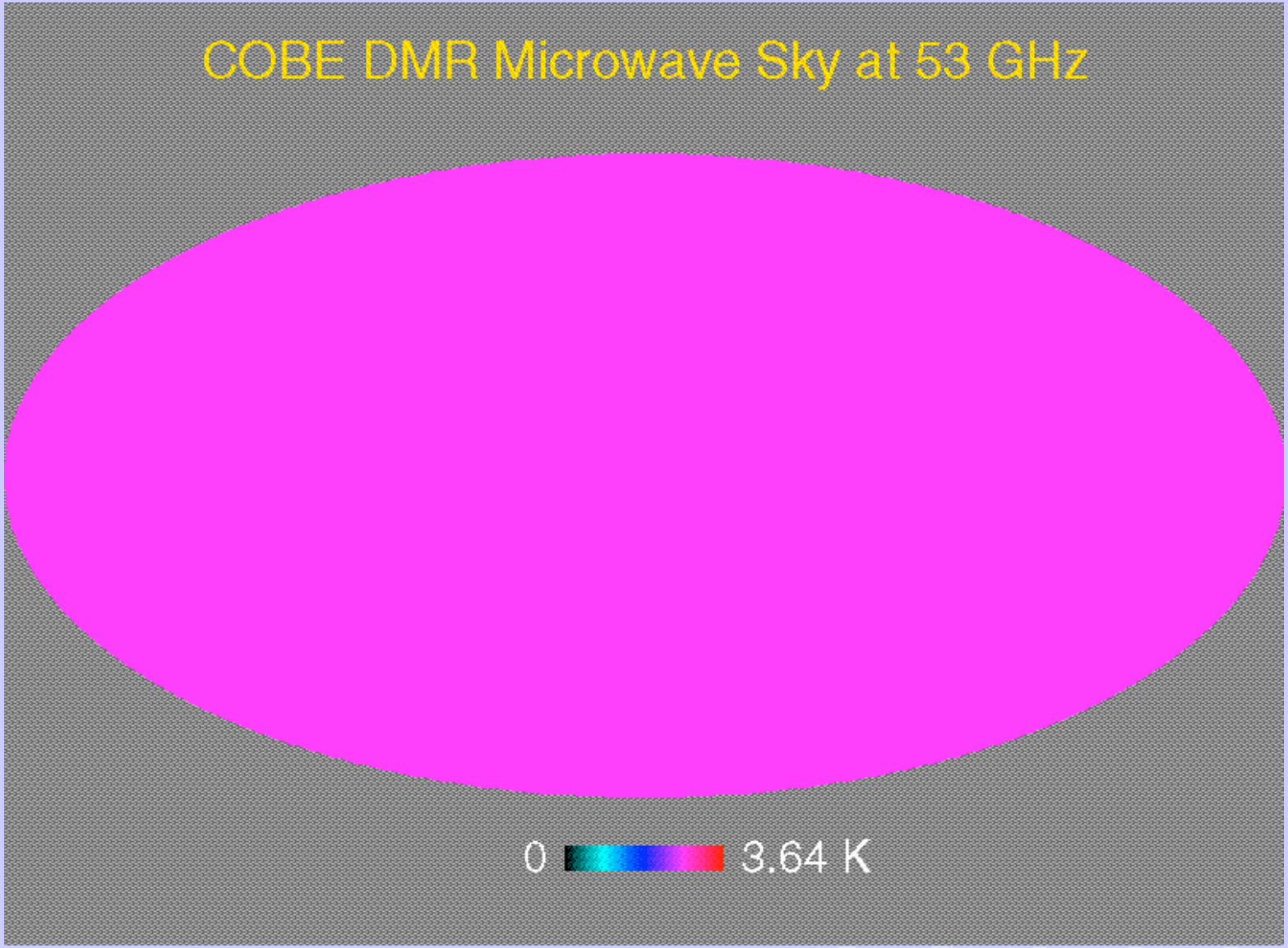
Outline

Plan

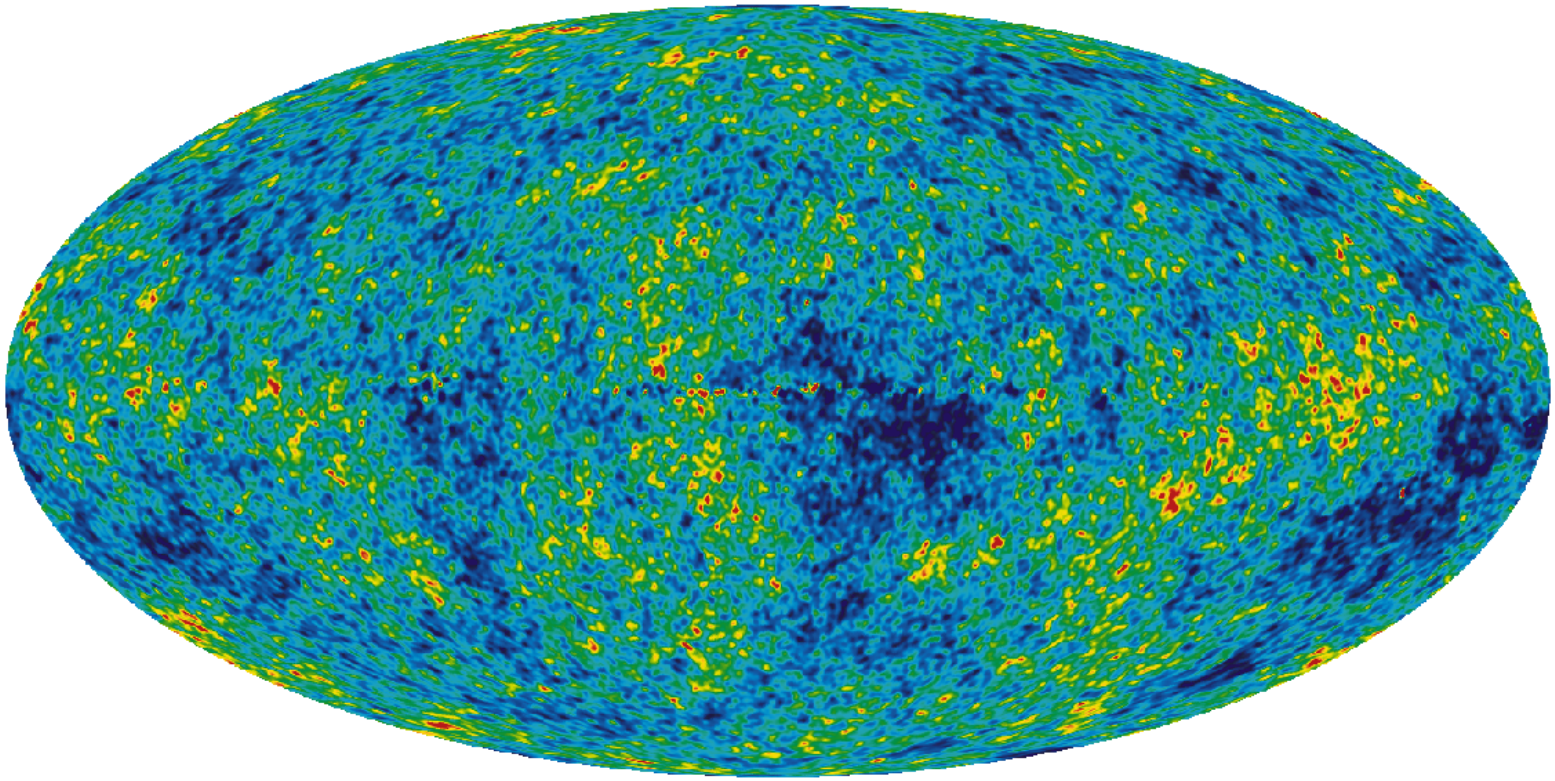
Time Evolution of the Universe



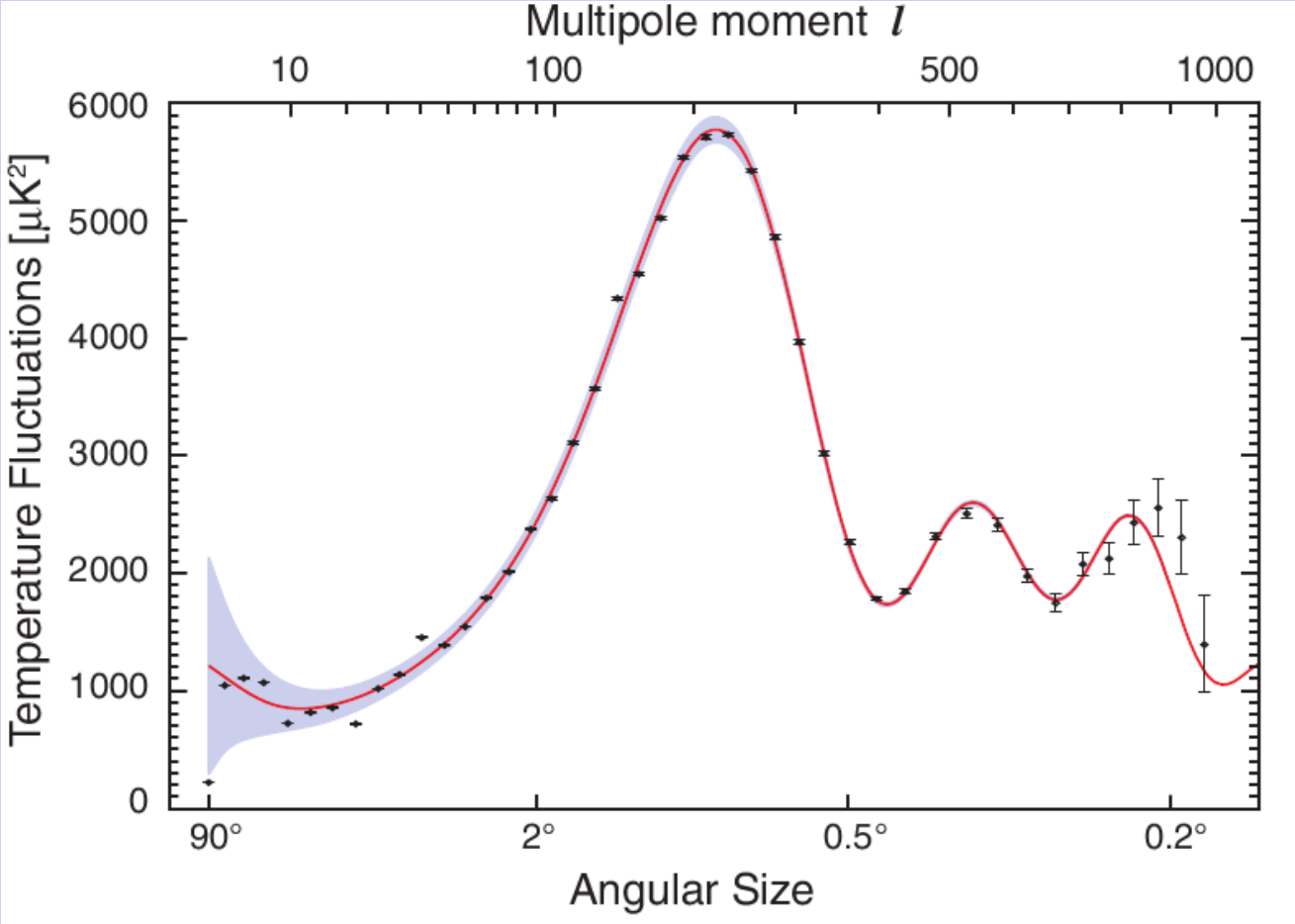
Isotropy of the Universe - the CMB Sky



CMB Anisotropies



Angular Power Spectrum of CMB Anisotropies

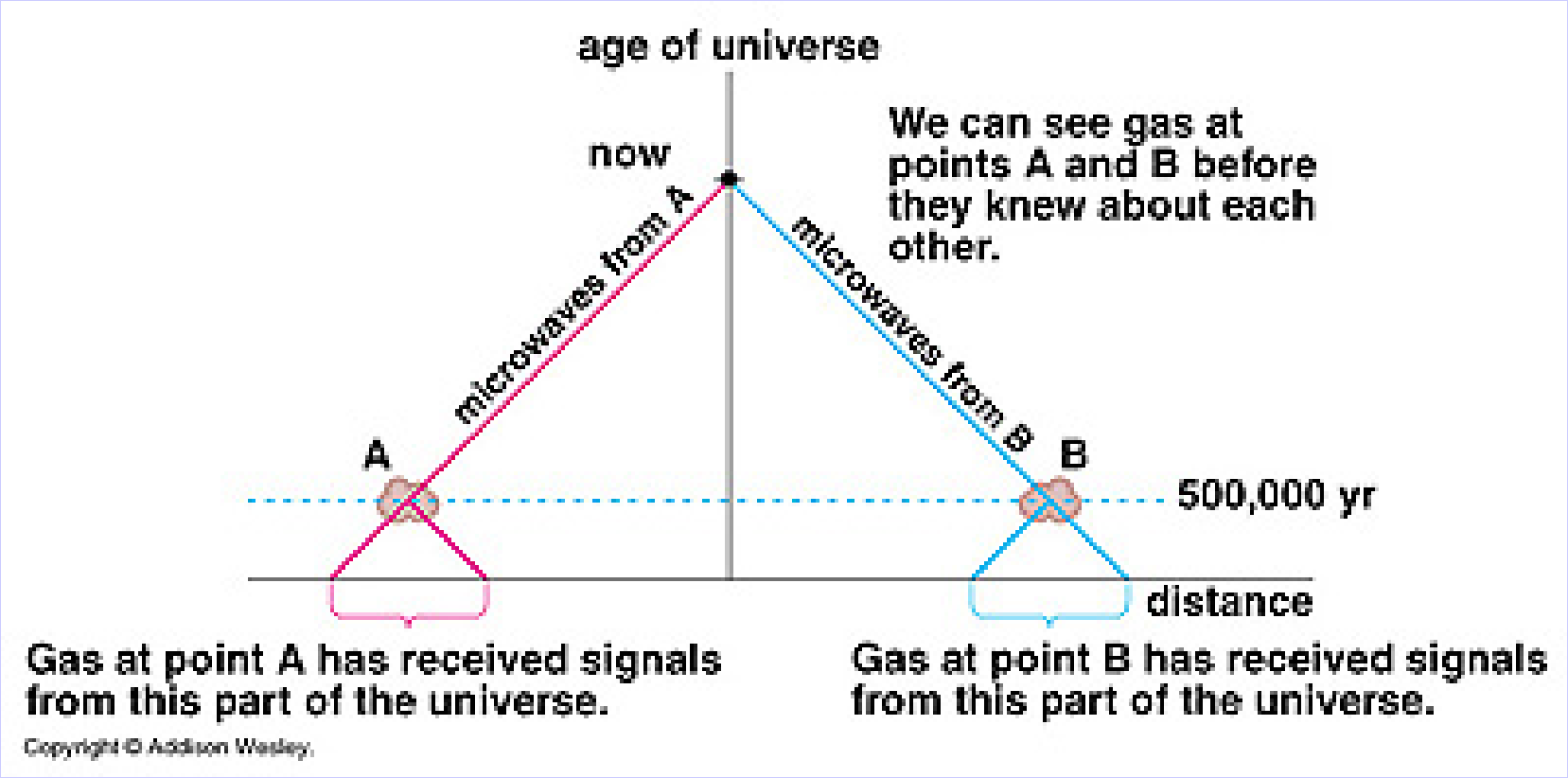


Credit: NASA/WMAP Science Team

Challenges

- Horizon Problem: explain the (near) isotropy of the CMB.
- Flatness Problem: explain the (near) spatial flatness of the universe.
- Formation of Structure Problem: Explain the origin of structure using causal physics.

Standard Big Bang Cosmology cannot address these Challenges



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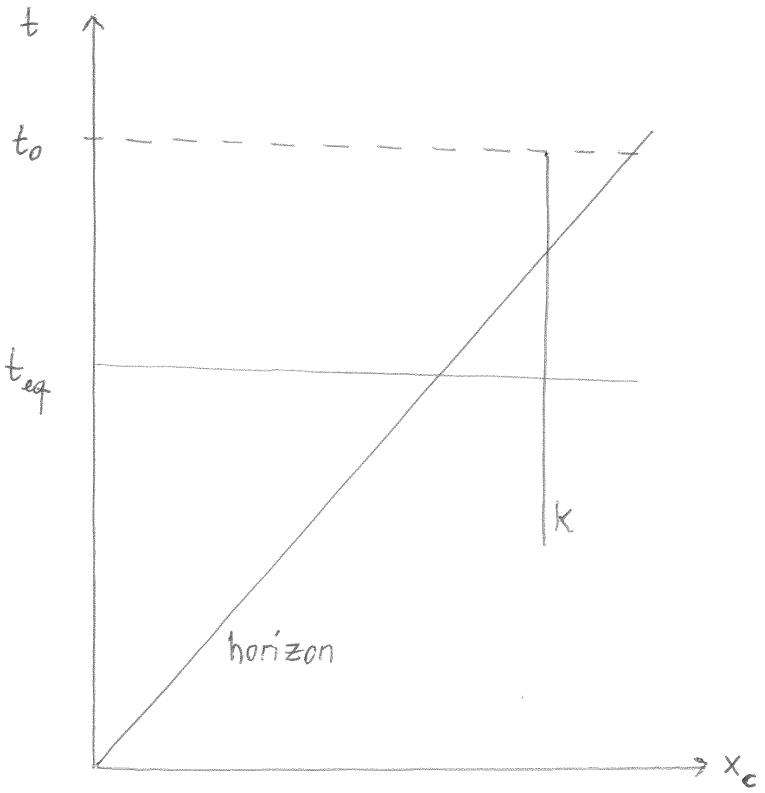

Early Universe to the Rescue

- Quantum gravity $\rightarrow$ modification of the time line of cosmology at very early times.
- $\rightarrow$ solution of the challenges?
- $\rightarrow$ predictions for observations?

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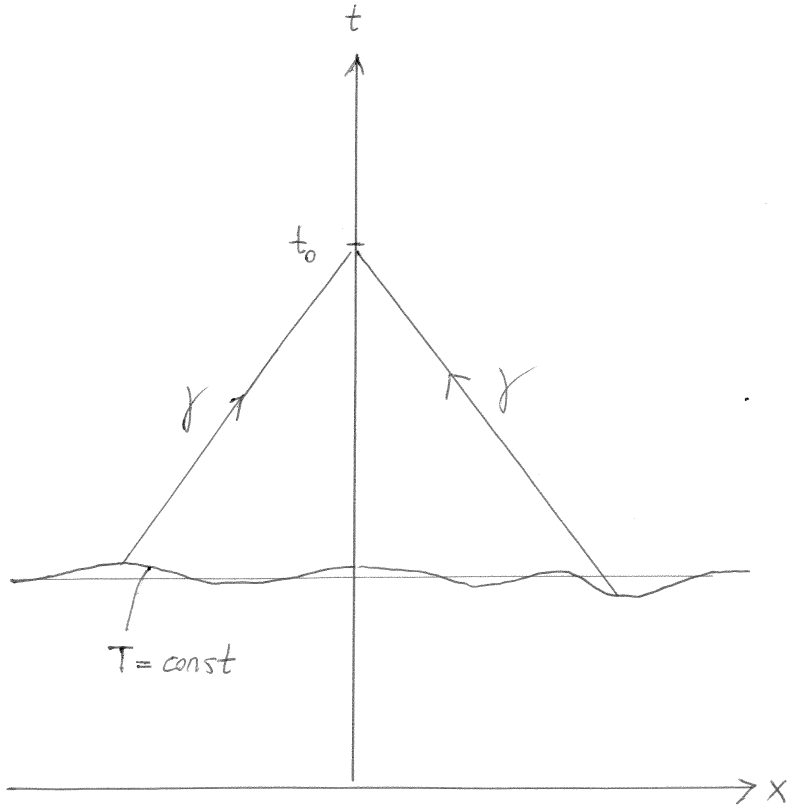
Density Fluctuations lead to CMB Anisotropies



Early Work

1970 p

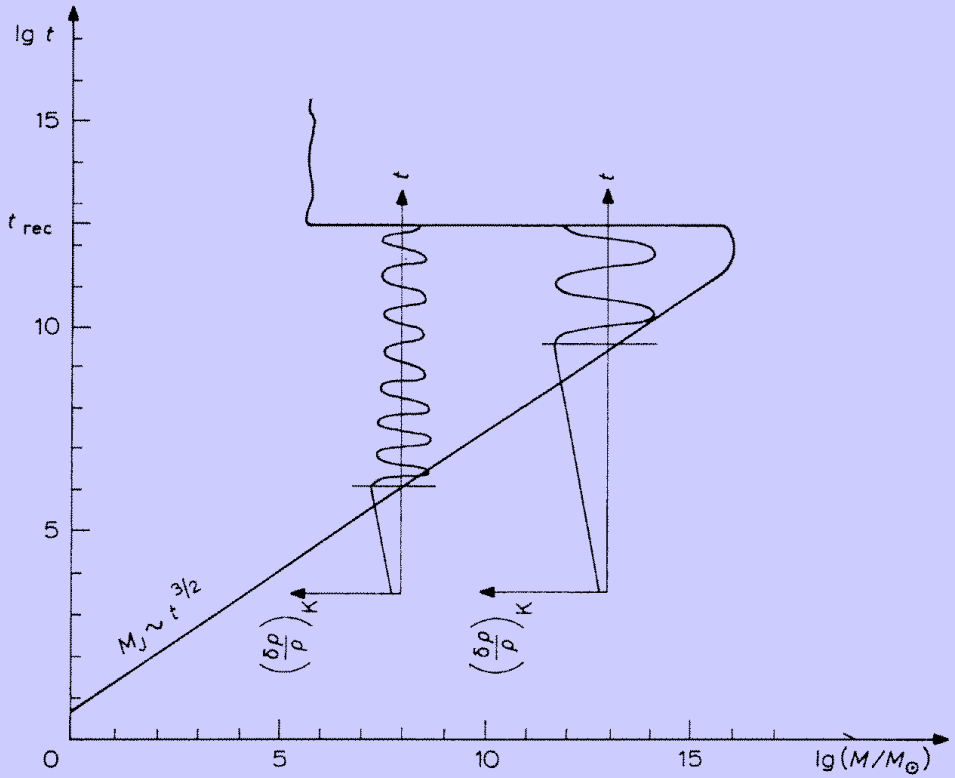


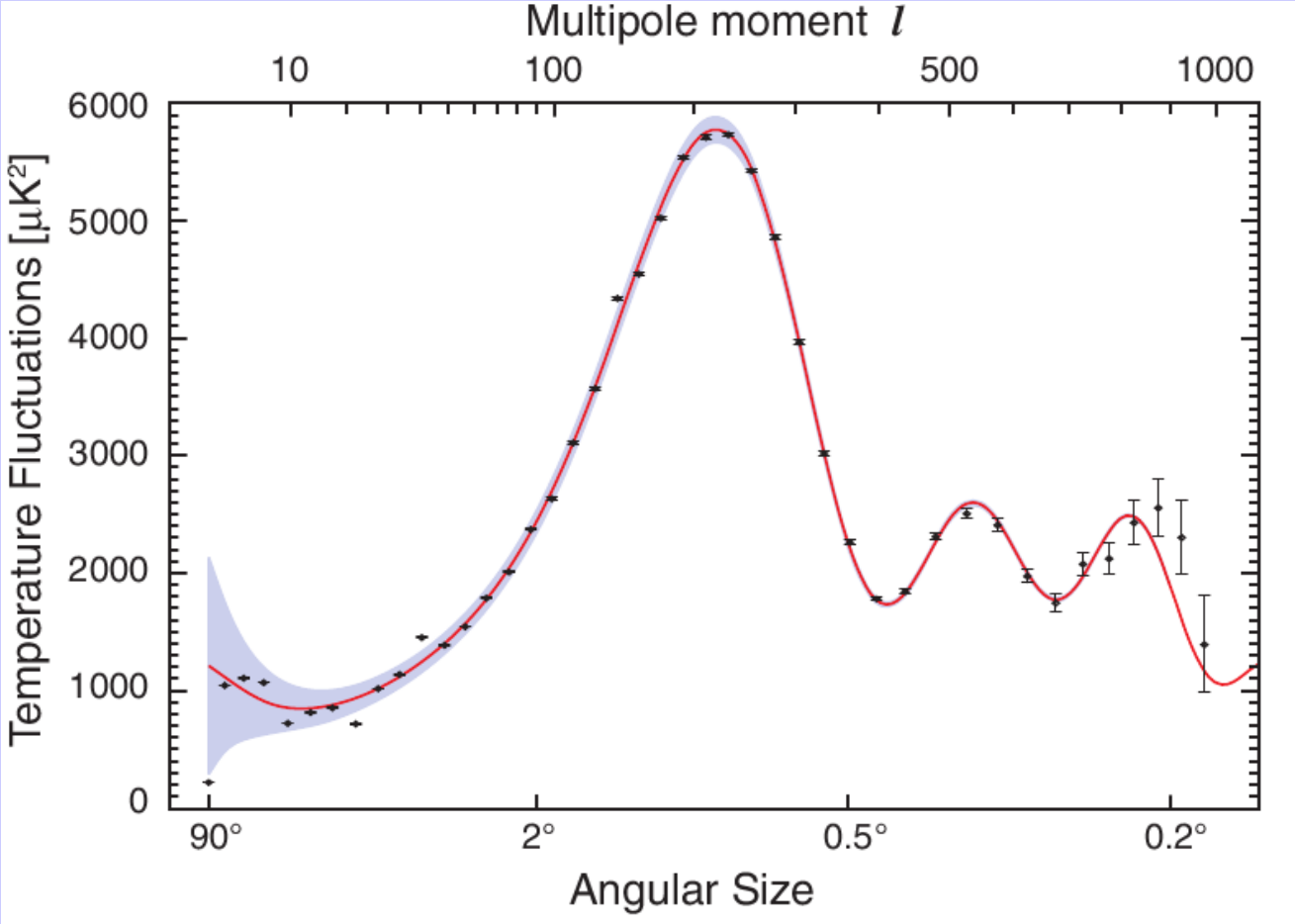
Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Challenge

Is there a causal mechanism for producing the fluctuations which are super-horizon (and hence acausal) from the point of view of Standard Big Bang Cosmology?

Hubble radius vs. Horizon

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 .$$

- Horizon: Forward light cone of a point on the initial condition Cauchy surface (at time t_i)
- Horizon at time t: $l_f(t) = a(t) \int_{t_i}^{t} a(t')^{-1} dt'$.
- Horizon carries information about **causality**.
- Hubble radius: Inverse expansion rate at time t.
- $l_H(t) = [a^{-1} da/dt]^{-1}$.
- Hubble radius relevant to the generation and evolution of cosmological fluctuations.

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Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the "horizon problem" of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on scales which are super-Hubble at t_{eq}.

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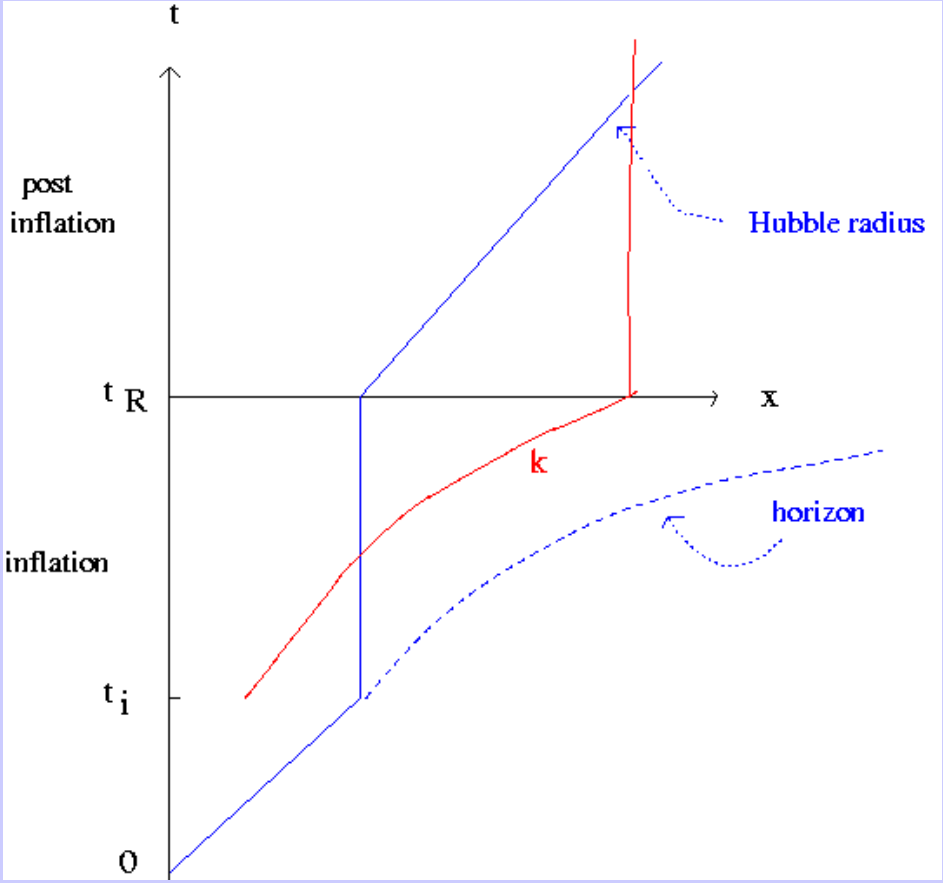
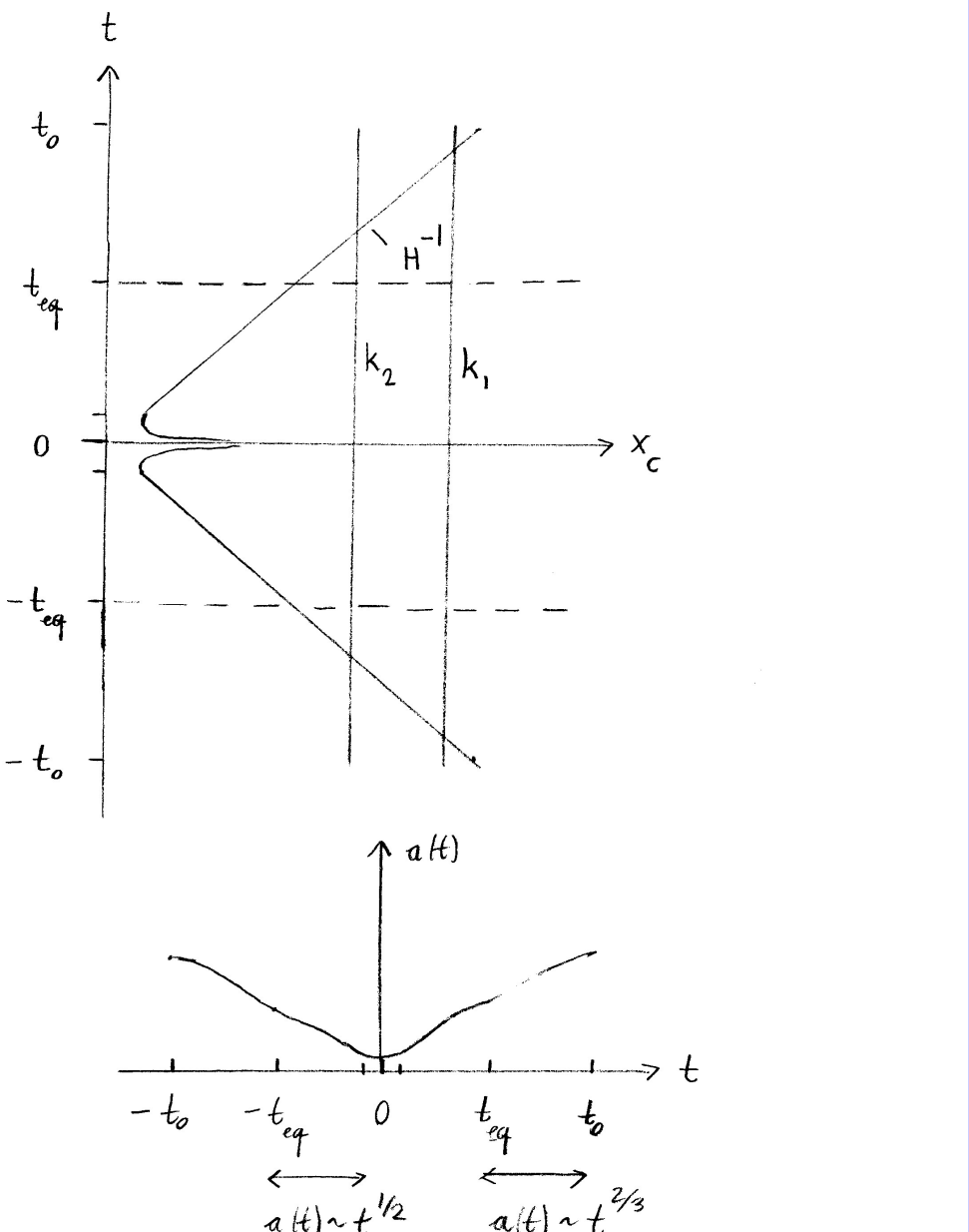
Inflation as a Solution

V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33, 532-535 (1981)**



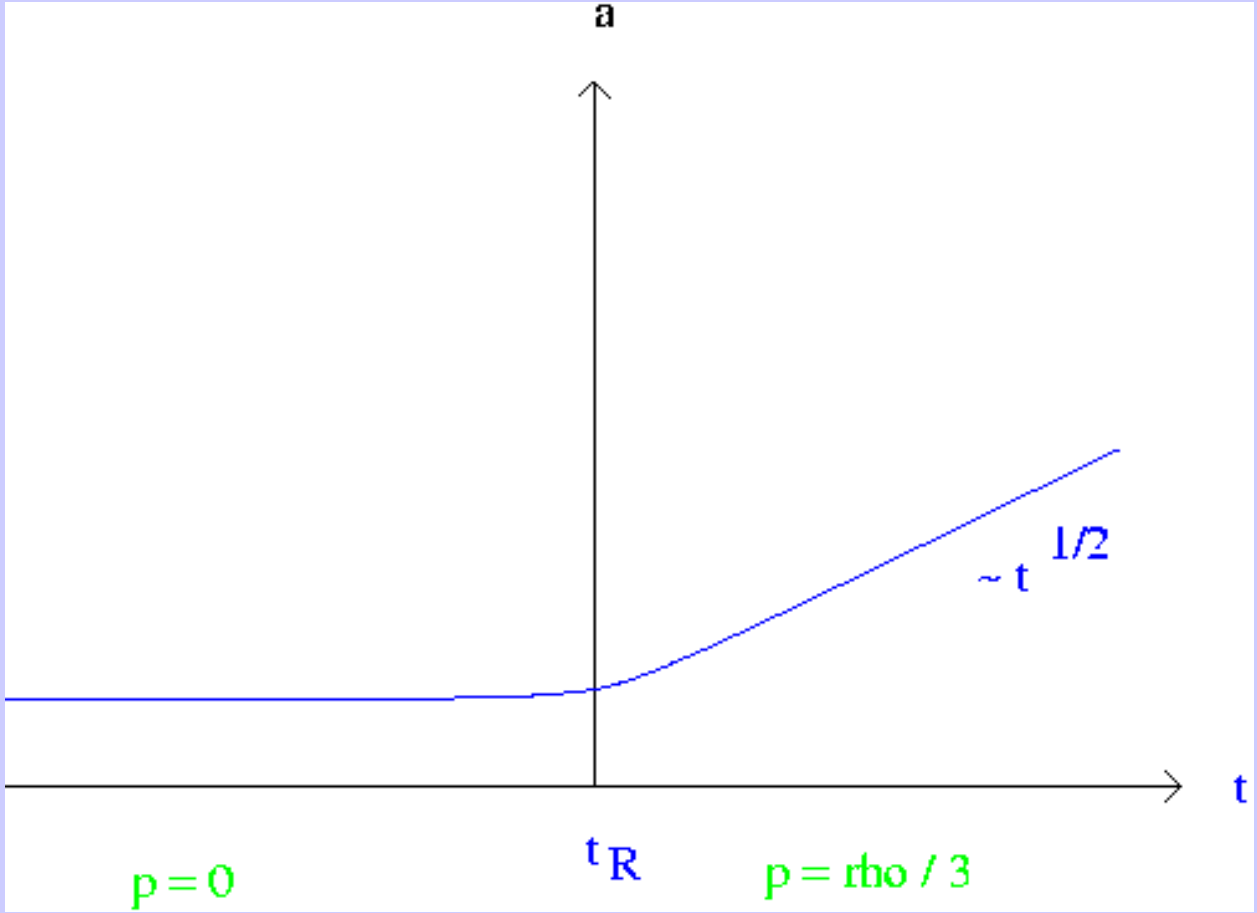
Bouncing Cosmology as a Solution

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



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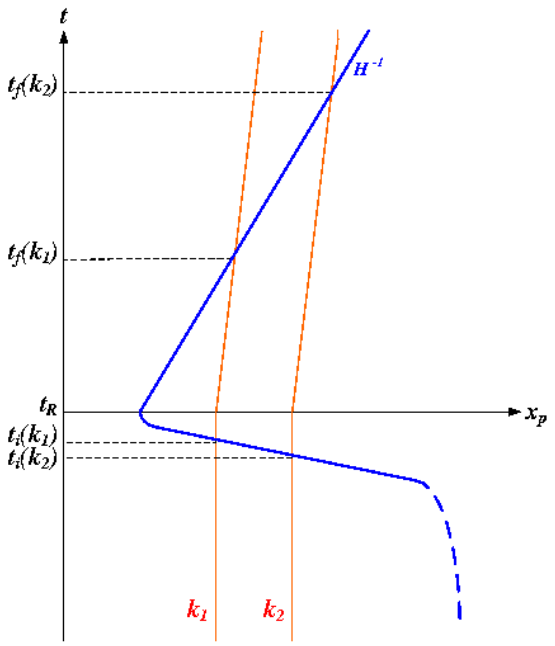
A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Comments on Inflation

- Of these scenarios, only inflation has been predictive.
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Key Question

Which of these scenarios (if any) naturally arises from superstring theory?

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Some Notation, Background

Background space-time metric:

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}^2]$$

Hubble scale:

$$l_H(t) = \frac{a}{\dot{a}}$$

Perfect fluid matter: energy density ρ, pressure p, equation of state parameter w

$$w \equiv \frac{p}{\rho}$$

Note: $w = 1/3$ for radiation, $w = 0$ for cold matter, $w = -1$ for cosmological constant.

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Cosmological Perturbations

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
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V. Mukhanov, H. Feldman and R.B., *Phys. Rep* **215**, 203 (1992

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Expand the action for matter and gravity to second order about the cosmological background:

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Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

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- **Amplitude** of the power spectrum of curvature fluctuations (scalars) $\mathcal{P}_\zeta(k)$.
- **Slope** of this spectrum $\mathcal{P}_\zeta(k) \sim k^{n_s - 1}$
- **Amplitude** of the power spectrum of gravitational waves (tensors) $\mathcal{P}_h(k)$
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Basic Observables

- **Amplitude** of the power spectrum of curvature fluctuations (scalars) $\mathcal{P}_\zeta(k)$.
- **Slope** of this spectrum $\mathcal{P}_\zeta(k) \sim k^{n_s - 1}$
- **Amplitude** of the power spectrum of gravitational waves (tensors) $\mathcal{P}_h(k)$
- **Slope** of this spectrum: $\mathcal{P}_h(k) \sim k^{n_t}$.
- Scale-invariance: $n_s = 1$ and $n_t = 0$.
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Key Assumption

Whatever the action which describes the background, the action for long wavelength (IR) cosmological fluctuations is **given by that of GR**.

Plan

Effective field theory (EFT) is

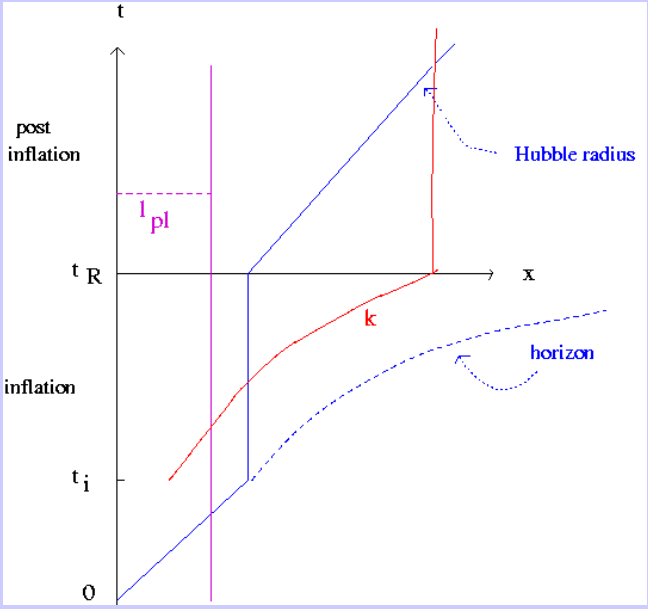
- Einstein gravity describing space and time.
- Matter obeying the usual energy conditions.

Bouncing and Emergent Scenarios Require Going Beyond EFT

- Bouncing cosmology: $H = 0$ at the bounce point cannot be achieved using EFT.
- Emergent cosmology: $H = 0$ in the emergent phase cannot be achieved using EFT.

Trans-Planckian Issue for Inflation

J. Martin and R.B., *Phys. Rev. D63, 123501 (2002)*



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- $\rightarrow$ breakdown of effective field theory; new physics MUST be taken into account when computing observables from inflation.

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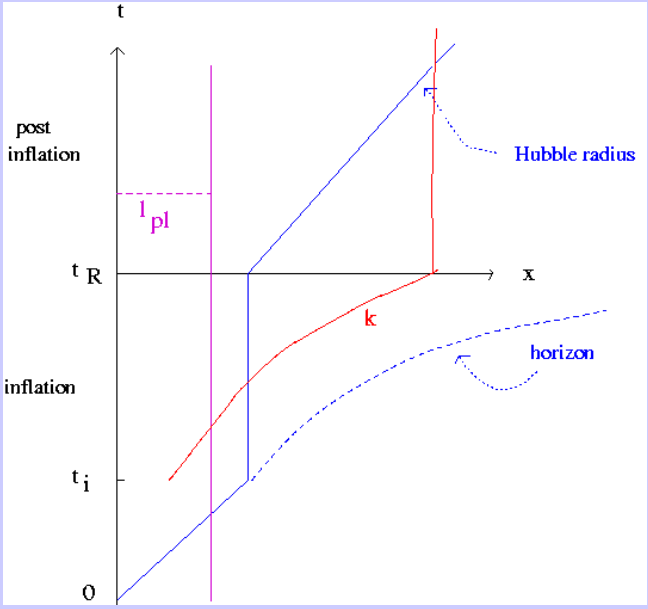

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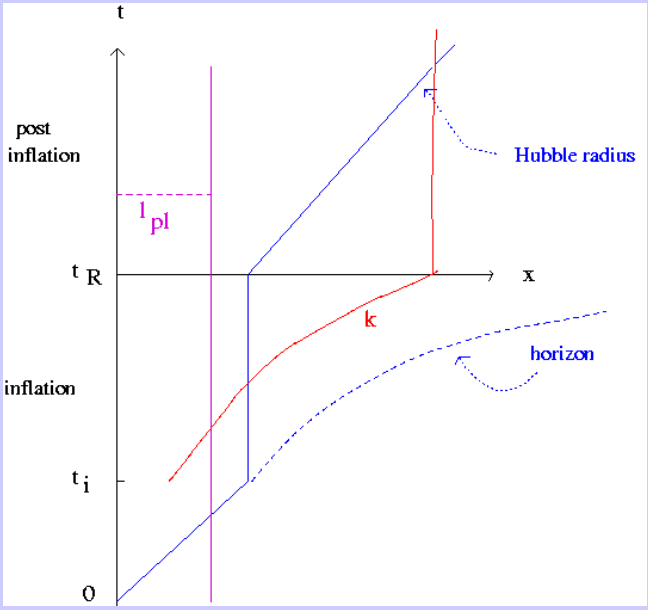

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Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

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TCC Time Scale

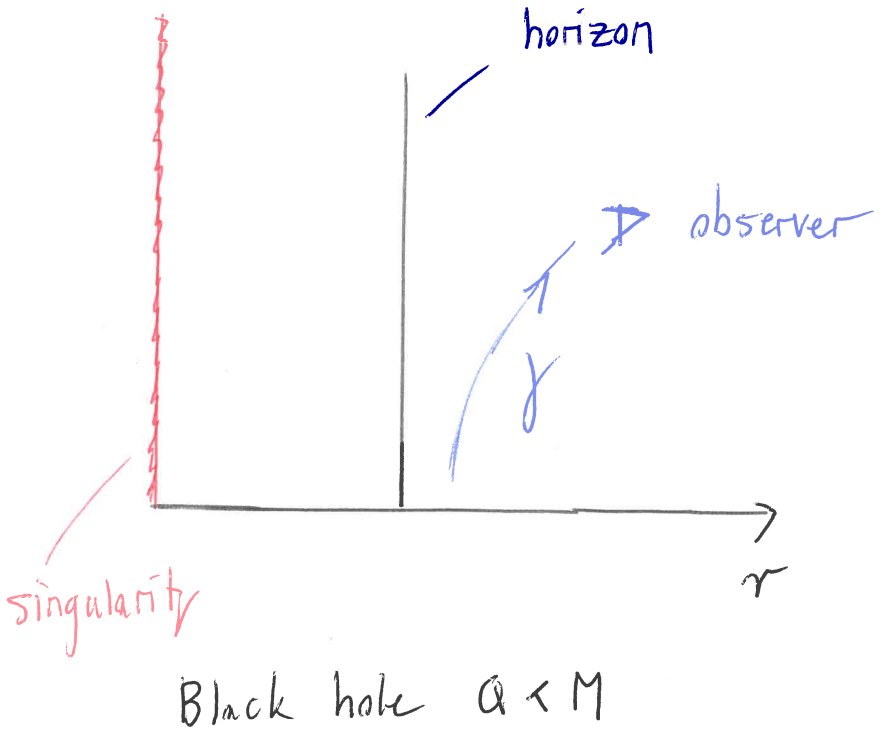
Applied to a phase of exponential expansion:

$$\delta t < \frac{m_{pl}}{H} \ln\left(\frac{m_{pl}}{H}\right)$$

Justification

R.B. arXiv:1911.06056

Analogy with Penrose's Cosmic Censorship Hypothesis:



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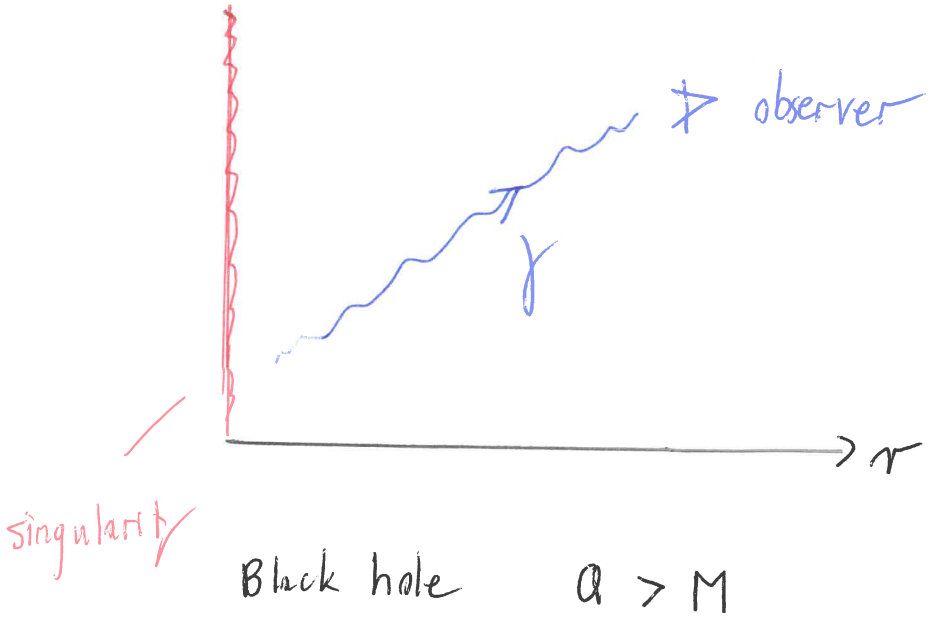

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- Effective field theory of General Relatiivty allows for solutions with timelike singularities: super-extremal black holes.
- $\rightarrow$ Cauchy problem not well defined for observer external to black holes.
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Cosmological Version of the Censorship Conjecture

Translation

- Position space $\rightarrow$ momentum space.
- Singularity $\rightarrow$ trans-Planckian modes.
- Black Hole horizon $\rightarrow$ Hubble horizon.

Observer measuring super-Hubble horizon modes must be shielded from trans-Planckian modes.

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- Recall: Fluctuations only oscillate on sub-Hubble scales.
- Recall: Fluctuations freeze out, become **squeezed states** and **classicalize** on super-Hubble scales.
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- Recall: non-unitarity of **effective field theory** in an expanding universe (N. Weiss, Phys. Rev. D32, 3228 (1985); J. Cotler and A. Strominger, arXiv:2201.11658).
- $\mathcal{H}$ is the product Hilbert space of a harmonic oscillator Hilbert space for all **comoving** wave numbers k
- UV cutoff: time dependent k_{max} : $k_{max}(t)a(t)^{-1} = m_{pl}$
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- EFT: expand **fields** in comoving Fourier space.
- Quantize each Fourier mode like a harmonic oscillator $\rightarrow$ ground state energy.
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Application of the Second Law of Thermodynamics

S. Brahma, O. Alaryani and RB, arXiv:2005.09688

- Consider entanglement entropy density $s_E(t)$ between sub- and super-Hubble modes.
- Consider an **phase of inflationary expansion**.
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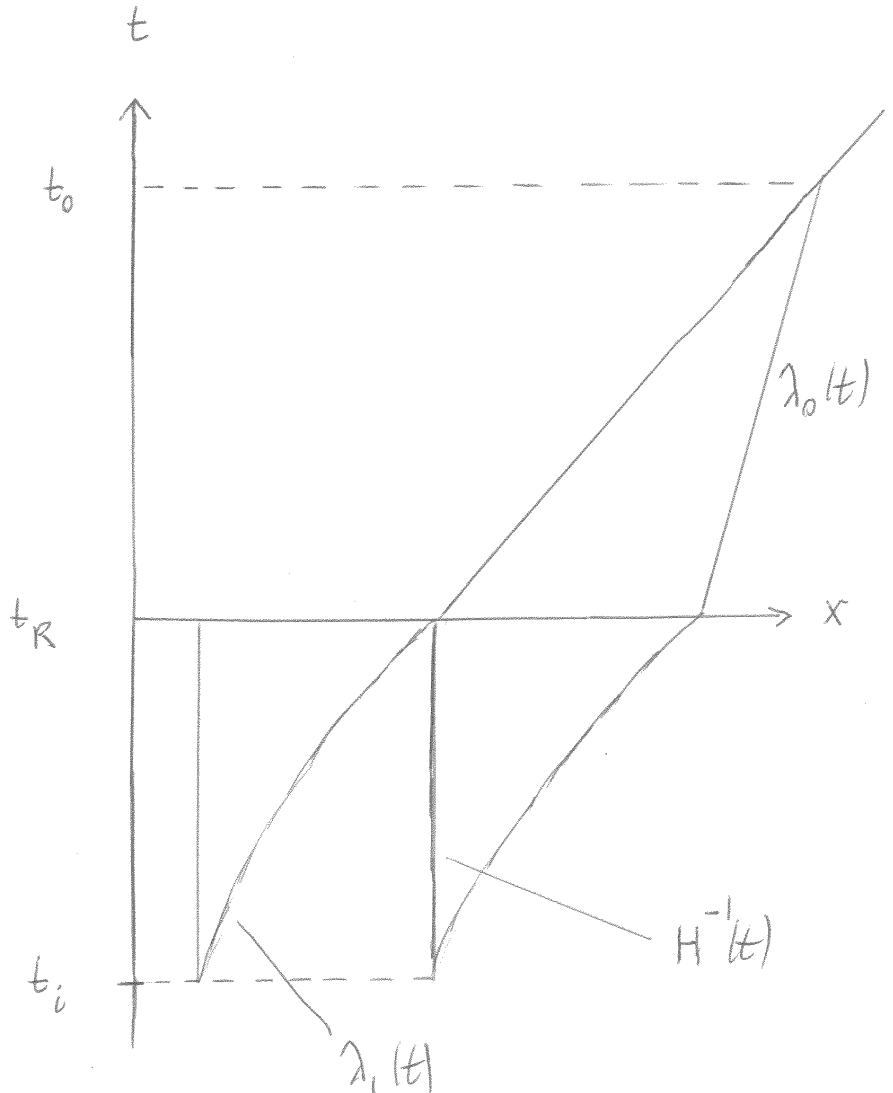

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Implications

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \mathrm{GeV}$$

$\rightarrow$ upper bound on the **primordial tensor to scalar ratio** r:

$$r < 10^{-30}$$

Note: Secondary tensors will be larger than the primary ones.

Implications for Dark Energy

- **Dark Energy cannot be a bare cosmological constant.**
- Quintessence models of Dark Energy are constrained (L. Heisenberg et al. arXiv:2003.13283]

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- Black holes decay by emitting Hawking radiation.
- Beyond the scrambling time the EFT analysis of BH evaporation is questionable (P.-M. Ho and H. Kawai, arXiv:2207.07122).
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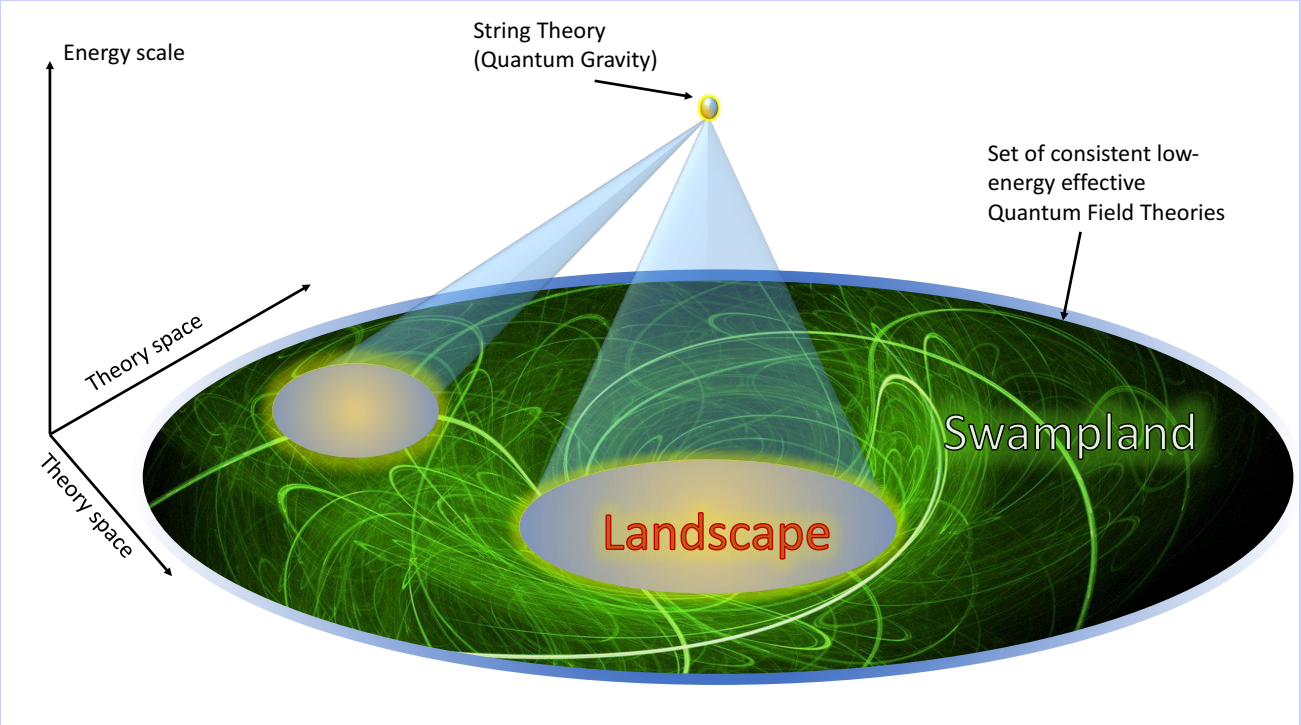
Swampland vs. Landscape



Swampland Constraints

C. Vafa, hep-th/0509212; E. Palti, arXiv:1903.06239

Swampland program: conditions which EFTs with scalar fields φ need to satisfy if they derive from string theory.

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- De Sitter Conjecture (G. Obied et al, arXiv:1806.08362): $V(\varphi)$ obeys the constraints

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Constraints on the Ground State of an EFT consistent with Supergravity

- Maldacena-Nunez no-go theorem: no de Sitter compactifications of supergravity. (J. Maldacena and C. Nunez, hep-th/0007018)
- No-go theorem: no de Sitter solutions of string theory (at the EFT level) with time-independent internal manifolds (K. Dasgupta et al, arXiv:1808.07498).

Implications

- Inflation from string theory requires going beyond an EFT analysis.
- Dark Energy is not a cosmological constant.
- Constraints on Quintessence models of dark energy (L. Heisenberg et al, arXiv:1808.02877).

Plan

Challenge

- Point particle cosmology: having a non-perturbative formulation changes everything.
- We lack a non-perturbative formulation of superstring theory.
- Different approaches to superstring cosmology: each picks an incomplete set of particular features which distinguish point particle theories from string theory.

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Inflation from Nonperturbative Approaches to String Theory

Four-dimensional de Sitter space as a Glauber-Sudarshan state in string theory (S. Brahma, K. Dasgupta and R. Tatar, arXiv:2007.00786; H. Bernardo et al, arXiv:2009.04504).

- 4-d de Sitter space is a coherent state over a supersymmetric solitonic background in full string theory, not a vacuum state.
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- Lifetime bounded by the TCC time scale.

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de Sitter as a coherent state of gravitons (G. Dvali, C. Gonez and S. Zell, arXiv:1701.08776).

- Starting point: Minkowski space-time.
- Build a coherent state of gravitons with de Sitter symmetry on top on $\mathcal{M}_4$.
- Deviations between the quantum and classical mean field evolution build up over time.
- $\rightarrow$ finite lifetime of the state quantum break time:

$$t_q \sim H^{-1}(\frac{m_{pl}}{H})^2$$

- Note: **quantum break time**: analogous to the Page time for black holes.
- Note: **quantum break time**: back-reaction of long wavelength cosmological perturbations becomes important (L. Abramo, RB and V. Mukhanov, 1997).

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Inflation from Nonperturbative Approaches to String Theory

de Sitter as a coherent state of gravitons (G. Dvali, C. Gonez and S. Zell, arXiv:1701.08776).

- Starting point: Minkowski space-time.
- Build a coherent state of gravitons with de Sitter symmetry on top on $\mathcal{M}_4$.
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Pre-Big-Bang Cosmology

M. Gasperini and G. Veneziano, Astropart. Phys. 1, 317 (1993)

Consider action of massless sector of bosonic string theory: dilaton gravity.

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right]$$

Look for homogeneous and isotropic solutions in $d = 3$ spatial dimensions.

New symmetry: **Scale factor duality**.

$$a(t) \to a(t)^{-1} \quad \bar{\Phi} \to \bar{\Phi}$$

with shifted dilaton

$$\bar{\Phi} \equiv \Phi - d \ln(a)$$

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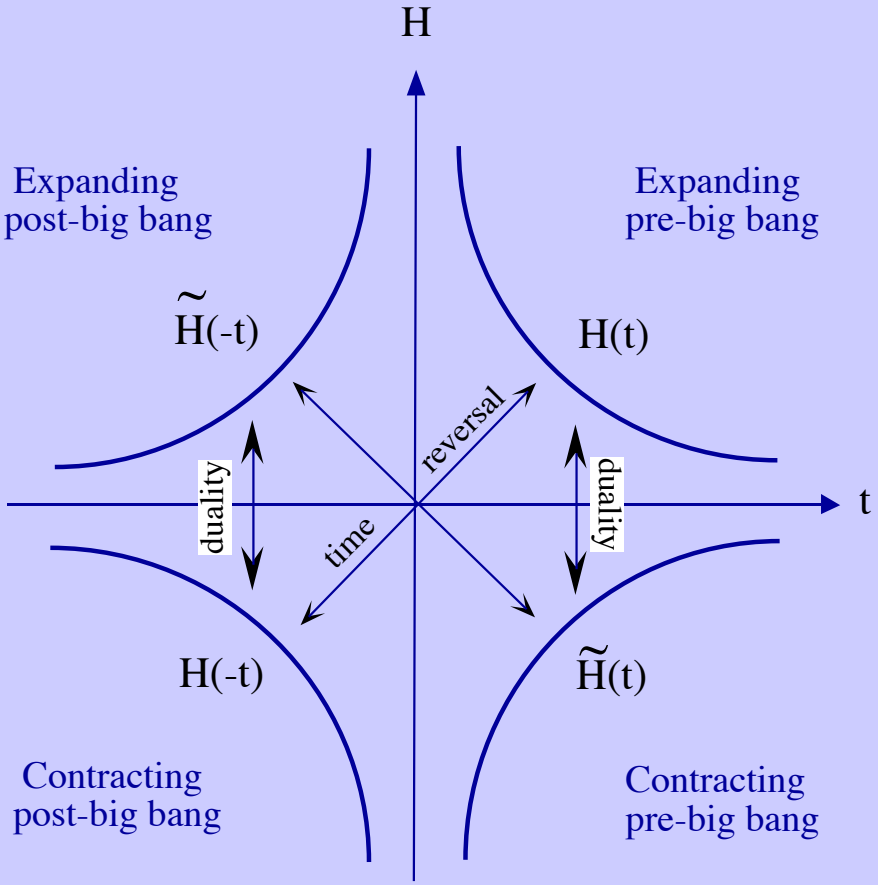
Duality relates a **contracting dilaton-driven phase** to the standard **expanding phase** with fixed dilaton.



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- Effective equation of state of the contracting branch: $w = 1$.
- Problem: **no non-singular transition between the contracting and expanding branch**.
- Problem: marginally unstable to anisotropy generation.
- Problem: initial vacuum fluctuations do not evolve into scale-invariant ones.
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J. Khouri, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D64, 123522 (2001)

- Ekpyrotic contraction: super-slow contraction with $w \gg 1$.
- Ekpyrotic contracting trajectory: global attractor in initial condition space.
- Generates isotropy and spatial flatness.
- The Bardeen potential Φ for cosmological fluctuations attains a scale-invariant spectrum.
- Ekpyrotic constraction achieved from a **scalar field** with a **negative steep exponential potential**.
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Obtaining an Ekpyrotic Universe

$$V(\varphi) = -V_0 \exp(-\sqrt{2/p}\varphi/m_{pl}) \quad p \ll 1$$

$$a(t) \sim (-t)^p$$

$$w \simeq \frac{4}{3p}$$

$$\varphi(t) = \sqrt{2p} m_{pl} \log\left(-\sqrt{\frac{V_0}{m_{pl}^2 p(1-3p)}}\, t\right)$$

Ekpyrosis: Small Field and Large Slope

Consider $\delta t = H^{-1}$:

$$\varphi \sim p^{1/2} \log(p^{-1/2}) m_{pl} \ll m_{pl} \, .$$

Relative slope of the potential:

$$|\frac{V'}{V}| m_{pl} \sim p^{-1/2} \gg 1 \, .$$

Relative curvature of the potential:

$$\frac{V''}{V} m_{pl}^2 = \frac{2}{p} \gg 1 \, .$$

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Safe Ekpyrosis

- Ekpyrotic scenario consistent with the swampland criteria.
- Ekpyrotic scenario consistent with the TCC.

S-Brane and Ekpyrosis

RB and Z. Wang, arXiv:2001.00638, arXiv:2004.06437

Challenges for Ekpyrotic Cosmology:

- How do we get the bounce?
- How do we obtain a scale-invariant spectrum of curvature fluctuations?
- Can we obtain a spectrum of gravitational waves relevant to current observations?

Adding an S-Brane to the EFT action can solve all three problems, and leads to two consistency relations for cosmological observables.

Consistency relation for the tilts

Consistency relation for the tilts:

$$n_t = (1 - n_s)\,.$$

Recall:

$$P_h(k) \sim k^{n_t}$$

$$P_\Phi(k) \sim k^{n_s - 1}$$

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Consistency relation for r

Comparing the results for the GW spectrum and the spectrum of cosmological perturbations yields

$$r \simeq 144 (k_C \tau_B)^{4q} 2^{-2\mu} \Gamma(\mu)^{-2} q^2 \, .$$

Since the value of q is given by the scalar tilt $q = (1 - n_s)/2$ we get

$$r \sim 36 (k_C \tau_B)^{4q} (1 - n_s)^2 \, .$$

Action

Idea: At the string scale a new tower of string states becomes comparable in mass to the usual low energy degrees of freedom;
$\rightarrow$ they must be included in the low energy effective action.

Included as an S-Brane.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right] - \int d^4x\kappa\delta(\tau - \tau_B)\sqrt{\gamma}\,,$$

$$\kappa \equiv N\eta_s^3\,,$$

Note: The S-brane has $\rho = 0$ and $p < 0 \rightarrow$ can mediate the transition between contraction and expansion.

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Background Evolution

Matching conditions

- Continuity of the induced metric.
- Jump in the extrinsic curvature given by the tension of the S-brane.

$$\delta H \equiv \lim_{\epsilon \to 0} H(t_B + \epsilon) - H(t_B - \epsilon) = 4\pi G \kappa \, .$$

$$\delta H = \frac{2}{\sqrt{3}} \eta_s^2 m_{pl}^{-1} \, ,$$

$$N = \frac{4}{\sqrt{3}} \frac{m_{pl}}{\eta_s} \, .$$

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Gravitational Waves Passing Through the S-Brane

$$\tilde{h}'' + \left[k^2 - \frac{a''}{a} - \kappa a m_{pl}^{-2} \delta(\tau - \tau_B)\right] \tilde{h} = 0 \,.$$

Mathematical Aside

- Consider the equation

$$X''_k(\tau) + \left[k^2 + m\delta(\tau - \tau_B)\right] X_k(\tau) \; = \; 0 \, .$$

- Solutions: plane waves for $\tau < \tau_B$ and for $\tau > \tau_B$.
- Positive frequency solutions f_k and negative frequency ones f_k^*.
- **Bogoliubov mode mixing across the transition surface**.
- Pure positive frequency before τ_B can be written for $\tau > \tau_B$ as

$$X_k \; = \; \alpha_k f_k + \beta_k f_k^* \, ,$$

- where α_k and β_k are the Bogoliubov mode matching coefficients.

Mathematical Aside II

By integrating over time τ against a test function (a smooth function which decays exponentially at $\tau \to \pm\infty$) $f(\tau)$ it can be easily shown that

$$\beta_k = \frac{m}{k} .$$

This is the factor which transforms a vacuum spectrum into a scale-invariant one.

Gravitational Waves Passing Through the S-Brane

$$\tilde{h}'' + \left[k^2 - \frac{a''}{a} - \kappa a m_{pl}^{-2} \delta(\tau - \tau_B)\right] \tilde{h} = 0 \,.$$

- Spectrum before passage through the S-Brane is a vacuum spectrum with a small blue tilt.
- $\rightarrow$ Spectrum after passage through the S-brane is scale-invariant with a slight blue tilt!.
- Power spectrum of gravitational waves;

$$\mathcal{P}_h(k) \simeq \frac{1}{2\pi^2} \kappa^2 m_{pl}^{-6} (k\tau_B)^{2q} \,.$$

Curvature Fluctuations Passing Through the S-Brane I

Solution for Φ on super-Hubble scales in the contracting phase:

$$\Phi_-(k,\tau) = A_-(k)\frac{\mathcal{H}}{a^2} + B_-(k)\,,$$

Solution for Φ on super-Hubble scales in the expanding phase:

$$\Phi_+(k,\tau) = A_+(k)\frac{\mathcal{H}}{a^2} + B_+(k)\,.$$

Matching Conditions

- Continuity of the induced metric fluctuations.
- Extrinsic curvature jump given by the fluctuations of the tension of the S-brane.

Matching conditions for a zero shear S-brane (R. Durrer and F. Vernizzi, Phys. Rev. D **66**, 083503 (2002)):

$$
\begin{aligned}
A_+ &= \frac{\mathcal{H}_-}{\mathcal{H}_+} A_- + \frac{a_B^2}{\mathcal{H}_+}(B_- - B_+) \\
B_+ &= \left(\frac{\mathcal{H}_+(\mathcal{H}_-'/\mathcal{H}_- - \mathcal{H}_-) - \mathcal{H}_+' + \mathcal{H}_+^2}{2\mathcal{H}_+^2 - \mathcal{H}_+'}\right)\frac{\mathcal{H}_-}{a_B^2} A_- \\
&\quad + \left(1 + \frac{\mathcal{H}_-\mathcal{H}_+ - \mathcal{H}_+^2}{2\mathcal{H}_+^2 - \mathcal{H}_+'}\right) B_- \,,
\end{aligned}
$$

Curvature Fluctuations Passing Through the S-Brane II

Result of the matching:

$$B_+(k) \simeq -\frac{\mathcal{H}_+}{a_B^2}\frac{1}{3q}A_-(k)\,.$$

Using vacuum initial conditions to determine $A_-(k)$:

$$A_-(k) \simeq 2^\mu \Gamma(\mu) m_{pl}^{-1} k^{-3/2} (k\tau_B)^{-q}\,,$$

Power Spectrum of Cosmological Perturbations:

$$\mathcal{P}_\Phi(k) \simeq \frac{1}{2\pi^2}(k\tau_B)^{-2q}\left(\frac{\mathcal{H}_+}{a_B^2 m_{pl}}\right)^2 \frac{1}{9q^2} 2^{2\mu}\Gamma(\mu)^2\,.$$

Scale-invariant spectrum with a slight red tilt.

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Plan

Obtaining an Emergent Cosmology: String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
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- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
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- Symmetry of vertex operators
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Adiabatic Considerations

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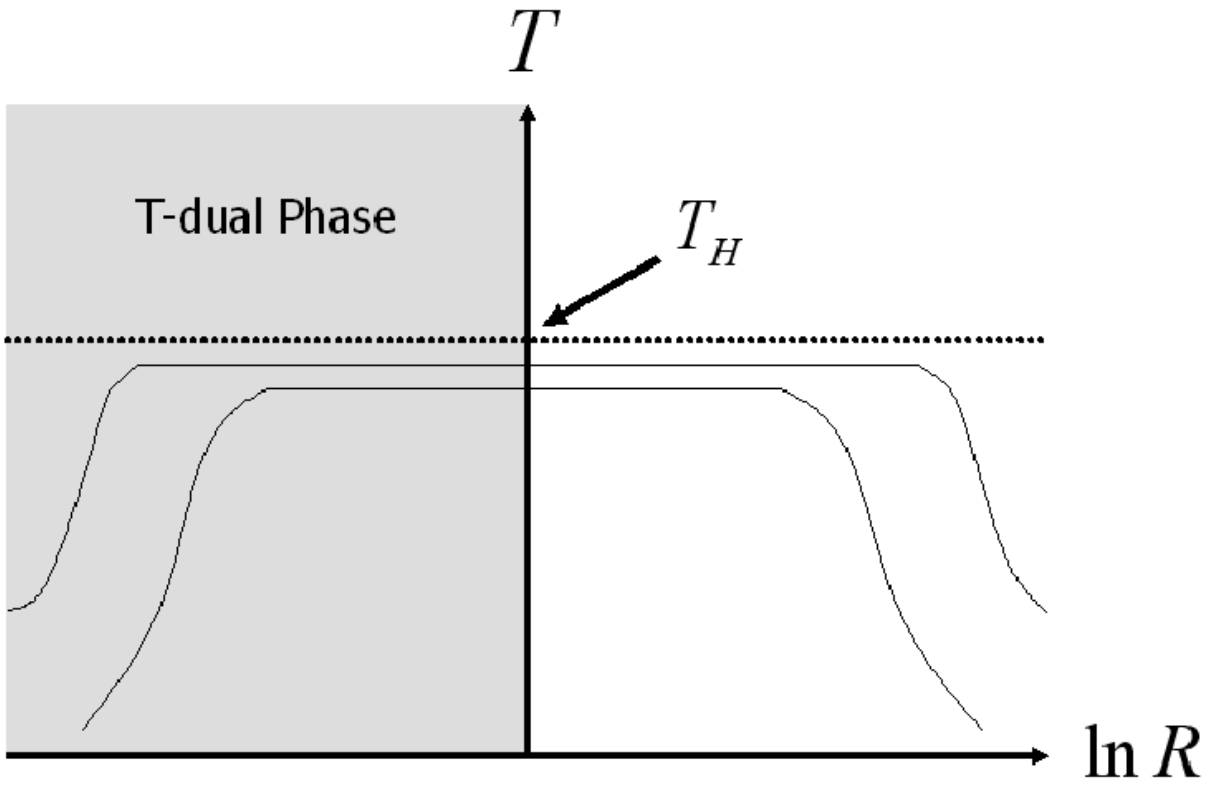
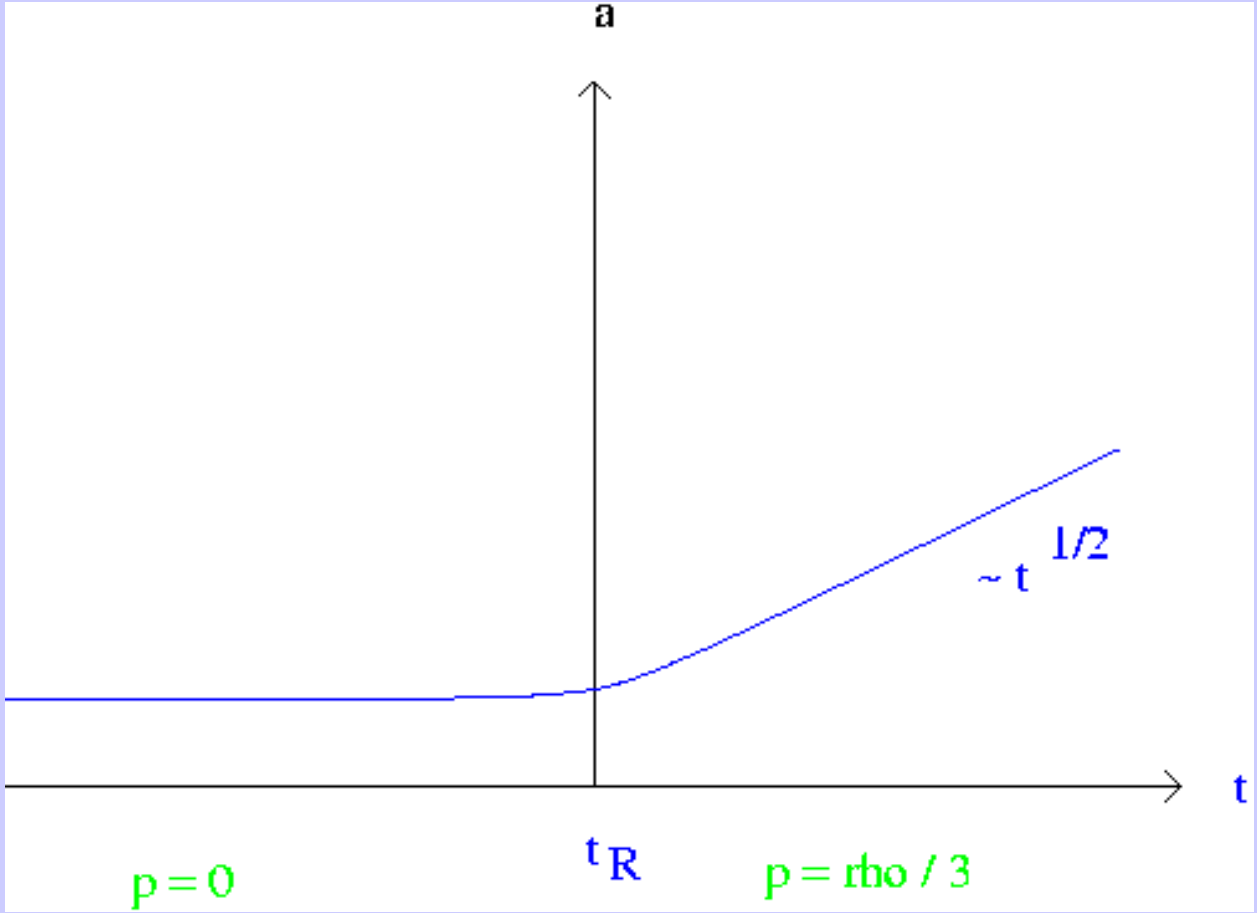
Temperature-size relation in string gas cosmology



Background for string gas cosmology



Emergence of 3-d Space

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

- Initial conditions: **Background space**: T^9 $SO(9)$ symmetry.
- All dimensions wrapped by winding strings (net winding number 0).
- A spatial dimension can only expand if winding modes about that dimension can annihilate.
- Dimension counting $\rightarrow$: windings can only disappear in ≤ 3 dimensions.
- Symmetry breaking: $SO(9) \rightarrow SO(3) \times SO(6)$.
- $\rightarrow$ dynamical emergence of 3-d large space.
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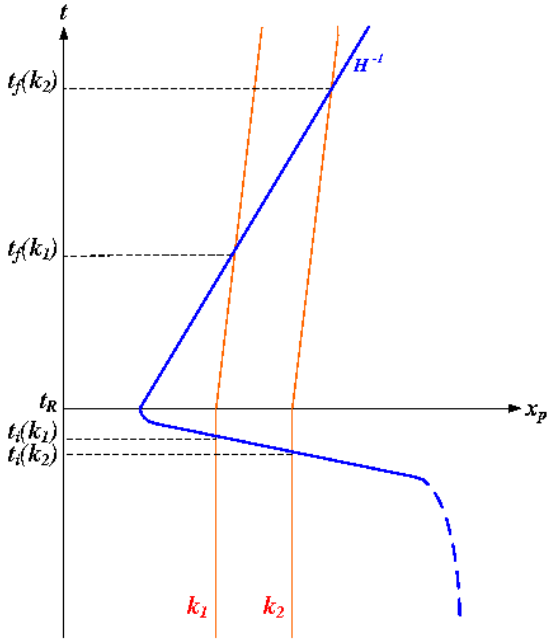
- A spatial dimension can only expand if winding modes about that dimension can annihilate.
- Dimension counting $\rightarrow$: windings can only disappear in ≤ 3 dimensions.
- Symmetry breaking: $SO(9) \rightarrow SO(3) \times SO(6)$.
- $\rightarrow$ dynamical emergence of 3-d large space.
- Remaining 6-d dimensions remain at the string scale.

Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Note: the matter correlation functions are given by partial derivatives of the **finite temperature string gas partition function** with respect to T (density fluctuations) or R (pressure perturbations).

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Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\left((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

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Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \, .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T\,(1 - T/T_H)} \, .$$

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Power spectrum of cosmological fluctuations

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Key features:

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R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

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Two consistency relations

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Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

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Dilaton stabilization in SGC

- The only remaining modulus is the dilaton.
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum.
- $\rightarrow$ diltaton is stabilized.
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008].
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Achilles Heel

No dynamical description of the Hagedorn phase.

Plan

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ Hermitean matrices.
- Note: no space!
- Note: no singularities!
- Note: BFSS matrix model is a proposed non-perturbative definition of M-theory: 10 dimensional superstring theory emerges in the $N \to \infty$ limit.

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BFSS Model (bosonic sector)

T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997)

$$L = \frac{1}{2g^2}\left[\mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right)\right]$$

- $X_i, i = 1, ...9$ are $N \times N$ Hermitean matrices.
- D_t: gauge covariant derivative (contains a matrix A_0)

't Hooft limit: $N \to \infty$ with $\lambda \equiv g^2 N = g_s l_s^{-3} N$ fixed.

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- $S_{BFSS} = S_{IKKT} + \mathcal{O}(1/T)$
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IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S_{IKKT} = -\frac{1}{g^2}\mathrm{Tr}\Big(\frac{1}{4}[A^a, A^b][A_a, A_b] + \frac{i}{2}\bar{\psi}_\alpha(\mathcal{C}\Gamma^a)_{\alpha\beta}[A_a, \psi_\beta]\Big)\,,$$

Partition function:

$$Z = \int dA d\psi e^{iS}$$

Emergent Time from Matrix Theory

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

- Eigenvalues of A_0 become emergent time.
- Work in the basis in which A_0 is diagonal.
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- $\sum_i \langle |A_i|^2_{ab} \rangle$ decays when $|a - b| > n_c$
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Emergent Space from Matrix Theory

S. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540

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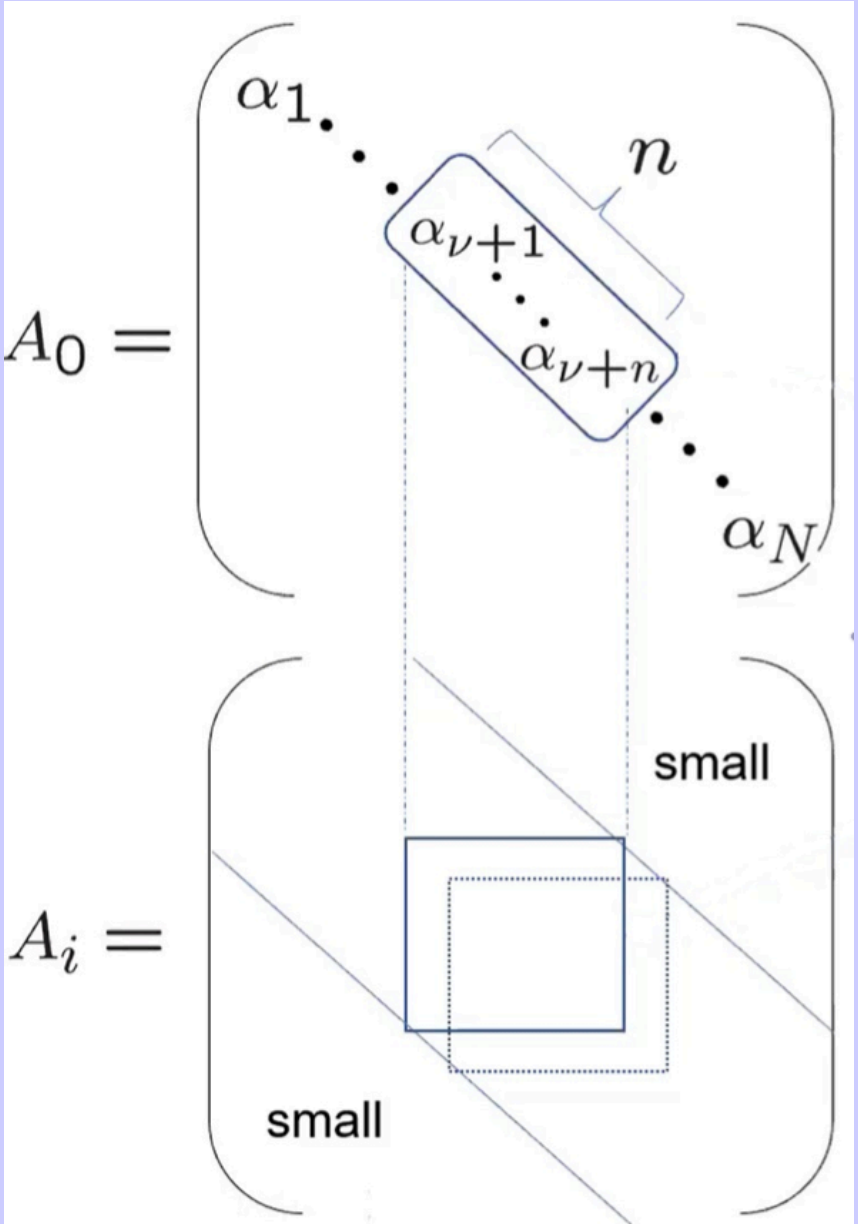
$A_0 =$
α_1
n
$\alpha_{\nu+1}$
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α_N
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Spontaneous Symmetry Breaking in Matrix Theory

J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

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- **Extent of space** in direction i

$$x_i(t)^2 \equiv \left\langle \frac{1}{n} \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- In a thermal state there is spontaneous symmetry breaking: $SO(9) \to SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined. [J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

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Emergent Metric from Matrix Theory

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: pick n (comoving spatial coordinate) and consider the block matrix $\tilde{A}_i(t)$.
- Physical distance between $n_i = 0$ and n_i (emergent space):

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- $l_{phys,i}(n_i) \sim n_i$ (for $n_i < n_c$)
- Emergent infinite and continuous space in $N \to \infty$ limit.
- Emergent metric (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

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- Physical distance between $n_i = 0$ and n_i (emergent space):

$$l^2_{phys,i}(n_{,i}\, t) \equiv \left\langle \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- $l_{phys,i}(n_i) \sim n_i$ (for $n_i < n_c$)
- Emergent infinite and continuous space in $N \to \infty$ limit.
- Emergent metric (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

$$g_{ii}(n_i)^{1/2} = \frac{d}{dn_i} l_{phys,i}(n_i)$$

Emergent Metric from Matrix Theory

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

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No Flatness Problem in Matrix Theory Cosmology

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Emergent metric:

$$g_{ii}(n_i)^{1/2} = \frac{d}{dn_i} l_{phys,i}(n_i)$$

Result:

$$g_{ii}(n_i, t) = \mathcal{A}(t)\delta_{ii} \;\; i = 1, 2, 3$$

$SO(3)$ symmetry $\rightarrow$

$$g_{ij}(\mathbf{n}, t) = \mathcal{A}(t)\delta_{ij} \;\; i = 1, 2, 3$$

$\rightarrow$ spatially flat.

Note: Local Lorentz invariance emerges in $N \rightarrow \infty$ limit.

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Late Time Dynamics

$$\mathcal{A}(t) \sim t^{1/2}$$

Note: no sign of a cosmological constant.

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

- We assume that the spontaneous symmetry breaking $SO(9) \to SO(3) \times SO(6)$ observed in the IKKT model also holds in the BFSS model.
- Using the Gaussian approximation method we have shown the existence of a symmetry breaking phase transition in the IKKT model (S. Brahma, RB and S. Laliberte, arXiv:2209.01255).
- **Thermal correlation functions** in the three large spatial dimensions calculated in the high temperature state of the BFSS model (following the formalism developed in String Gas Cosmology).
- $\to$ curvature fluctuations and gravitational waves.

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Matrix Theory Cosmology: Thermal Fluctuations

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

- Start with the BFSS partition function .
- Note: $\frac{1}{T}$ correction terms in the BFSS action are crucial!
- Calculate matter correlation functions in the emergent phase.
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$.
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations.

Note: the matter correlation functions are given by partial derivatives of the **finite temperature partition function** with respect to T (density fluctuations) or R (pressure perturbations).

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Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\left((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right) .$$

Inserting into the perturbed Einstein equations yields

$$\langle |\Phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle ,$$

$$\langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle .$$

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Computation of Fluctuations I

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

$$P(k) = k^3(\delta\Phi(k))^2 = 16\pi^2 G^2 k^2 T^2 C_V(R)$$

$$C_V(R) = \frac{\partial}{\partial T} E(R)$$

$$E = -\frac{\partial}{\partial \beta} \ln Z(\beta)$$

Computation of Fluctuations II

N. Kawahara, J. Nishumura and S. Takeuchi, arXiv:0710.2188

$$E^2 = N^2 < \mathcal{E} >_{BFSS}, \quad \mathcal{E} = -\frac{3}{4N\beta}\int_0^\beta dt \mathrm{Tr}([X_i, X_j]^2)$$

- Insert Matsubara expansion of the matrices: leading term in the BFSS action in the high T limit is the IKKT action.
- Express expectation values in terms of IKKT expectation values

To next to leading order in $1/T$:

$$E^2 = \frac{3}{4}N^2\chi_2 T - \frac{3}{4}N^4\alpha\chi_1 T^{-1/2}$$

$$\chi_1 = < R^2 >_{BFSS} T^{-1/2}$$

Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Thermal fluctuations in the emergent phase $\rightarrow$

- Scale-invariant spectrum of curvature fluctuations
- With a Poisson contribution for UV scales.
- Scale-invariant spectrum of gravitational waves.

$\rightarrow$ BFSS matrix model yields emergent infinite space, emergent infinite time, emergent spatially flat metric and an emergent early universe phase with thermal fluctuations leading to scale-invariant curvature fluctuations and gravitational waves.

Note: Horizon problem automatically solved.

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Open Problems

- Include the effects of the fermionic sector.
- Understand **phase transition** to the expanding phase of Big Bang Cosmology.
- Understand the **emergence of GR** in the IR.
- Spectral indices?
- What about Dark Energy?
- Emergent low energy effective field theory for localized excitations.

Plan

Conclusions

- **Several early universe scenarios** exist which are consistent with current observations.
- A consistent analysis of early universe cosmology requires going **beyond effective field theory.**
- Superstring theory is a promising approach to study the very early universe - unified theory at the quantum level.
- Non-perturbative formulation of superstring theory is missing.
- Several approaches to superstring theory exist, each picks particular aspects of string theory which distinguish string theory from point particle theories.
- String Gas Cosmology: **emergent universe** scenario based on the study of new degrees of freedom and new symmetries of string theory.

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Relationship between IKKT Model and Type IIB String Theory

Consider action of the Type IIB string theory in Schild gauge

$$S_{\text{Schild}} = \int d^2\sigma\alpha\left[\sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\} - \frac{i}{2}\bar{\psi}\Gamma^\mu\{X^\mu, \psi\}\right) + \beta\sqrt{g}\right].$$

$$\text{Partition function}: Z = \int \mathcal{D}\sqrt{g}\mathcal{D}X\mathcal{D}\psi e^{-S_{\text{Schild}}}.$$

$$\begin{aligned} \text{Correspondence}: \{,\} &\rightarrow -i[,] \\ \int d^2\sigma\sqrt{g} &\rightarrow \text{Tr} \end{aligned}$$

Obtain grand canonical partition function of IKKT model.

Some Details

Starting point: finite temperature partition function:

$$Z(\beta) = \int \mathcal{D}A\mathcal{D}X_i e^{-S(\beta)}$$

Internal energy

$$E = -\frac{d}{d\beta}\ln Z(\beta)$$

$$E = -\frac{3}{4}\lambda^{-1}\frac{N}{\beta}\int_0^\beta dt \mathrm{Tr}[X_, X_j]^2$$

Matsubara expansion:

$$X_i = \sum_n X_i^n e^{i(2\pi n\ \beta)t}$$

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Some Details

Matsubara expansion of the action:

$$S_{BFSS} = S_0 + S_{kin} + S_{int}$$

At high temperature: S_{kin} and S_{int} suppressed compared to S_0.

To next to leading order:

$$\begin{aligned} E &\simeq \lambda^{-1}\frac{3N^2}{4}\chi_2 T \\ &\quad -\lambda^{-1}\frac{3N^2}{4}\mathcal{O}(1)\chi_2\chi_1 T^{-1/2} \end{aligned}$$

where $\chi_1 \simeq R^2\lambda^{4/3}T^{-1/2}$.

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Some Details

- Derivative w.r.t. $T \to$ density fluctuations: both terms contribute.
- Derivative w.r.t. $R \to$ pressure fluctuations: only second term contributes.

Power spectrum $P(k)$ of density fluctuations: ($k = R^{-1}$)

- First term dominates in the UV: Poisson spectrum.
- Second term dominated in the IR: Scale-invariant spectrum.

$$P(k) = 16\pi^2 G^2 \lambda^{4/3} N^2 \mathcal{O}(1) \sim (l_s m_{pl})^{-4}$$

using the scaling $G^2 N^2 \lambda^{4/3} \sim (l_s m_{pl})^{-4}$.

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Plan

Geodesic Completeness

R.B., R. Costa, G. Franzmann and A. Weltman, arXiv:1710.02412 [hep-th]

Recall: For each dimension of the underlying topological space there are two position operators [R.B. and C. Vafa]:

- x: dual to the momentum modes
- $\tilde{x}$: dual to the winding modes

We measure **physical length** in terms of the **light** degrees of freedom.

$$
\begin{aligned}
l(R) &= R \quad \text{for} \quad R \gg 1\,, \\
l(R) &= \frac{1}{R} \quad \text{for} \quad R \ll 1\,.
\end{aligned}
$$

Doubled Space Approach

$$dS^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j - a^{-2}(t)\delta_{ij}d\tilde{x}^i d\tilde{x}^j$$

Point particle geodesic:

$$\frac{d}{dS}\left(\frac{dx^i}{dS}a^2\right) = 0$$
$$\frac{d}{dS}\left(\frac{d\tilde{x}^i}{dS}a^{-2}\right) = 0$$

Initial conditions: related by duality.

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Initial conditions: related by duality.

Proper Time along Geodesic

Assume $a(t)$ as in Standard Big Bang Cosmology.

Proper distance into the future from some time t_0 to some time $t_2 \gg t_0$:

$$\Delta S = \int_{t_0}^{t_2} a(t)\gamma(t)^{-1} dt + T_2 \,,$$

Proper distance into the past from some time t_0 to some time $t_1 \ll t_0$:

$$\Delta S = \int_{t_1}^{t_0} a(t)^{-1}\tilde{\gamma}^{-1}(t) dt + T_1 \,,$$

Interpretation

- Expansion of the scale factor in the dual spatial directions as time decreases $\equiv$ expansion in the regular directions as time increases.
- Dynamics of the dual spatial dimensions as t decreases **is measured** as expansion when the dual time $t_d = \frac{1}{t}$ decreases.

Proposal:

$$\begin{aligned} t_p(t) &= t \quad \text{for} \quad t \gg 1 \,, \\ t_p(t) &= \frac{1}{t} \quad \text{for} \quad t \ll 1 \,. \end{aligned}$$

Conclusion: Point particle geodesics can be extended in both time directions to infinite proper time.

Nonsingular String Cosmology

R.B., R. Costa, G. Franzmann and A. Weltman, arXiv:1805.06321 [hep-th]

Consider Dilaton gravity

$$\begin{aligned} \left(\dot{\phi} - dH\right)^2 - dH^2 &= e^{\phi}\rho \\ \dot{H} - H\left(\dot{\phi} - dH\right) &= \frac{1}{2}e^{\phi}p \\ 2\left(\ddot{\phi} - d\dot{H}\right) - \left(\dot{\phi} - dH\right)^2 - dH^2 &= 0 \end{aligned}$$

coupled to string gas matter.

$$w(a) = \frac{2}{\pi d}\arctan\left(\beta \ln\left(\frac{a}{a_0}\right)\right),$$

Limiting Solutions

Large radius limit:

$$\rho\,(a\,\text{large}) \to \rho_0\,(a/a_0)^{-(d+1)}\,,$$

Small radius limit:

$$\rho\,(a\,\text{small}) \to \rho_0\,(a/a_0)^{-d+1}$$

Ansatz:

$$\begin{aligned} a(t) &\sim \left(\frac{t}{t_0}\right)^{\alpha} \\ \bar{\phi}(t) &\sim \beta\,\ln(t/t_0)\,, \end{aligned}$$

Where

$$\bar{\phi} \equiv \phi - d\ln(a)$$

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Where

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Limiting Solutions

Hagedorn phase, $w = 0$:

$$(\alpha, \beta) = (0, 2)\,.$$

Note: Static in string frame.

Large a phase, $w = 1/d$:

$$(\alpha, \beta) = \left(\frac{2}{D}, \frac{2}{D}(D-1)\right)\,.$$

Note: constant dilaton.

Small a phase, $w = -1/d$:

$$(\alpha, \beta) = \left(-\frac{2}{D}, \frac{2}{D}(D-1)\right)\,.$$

Interpretation

- Bouncing cosmology in the string frame $\rightarrow$ nonsingular.
- Contracting cosmology for $t \rightarrow 0$ in the Einstein frame.
- As $t \rightarrow 0$ the energy of the string gas drifts to winding modes.
- Physical space is measured in terms of winding modes.
- In terms of winding modes the contraction as $t \rightarrow 0$ corresponds to expansion.
- $t \rightarrow 0 \;\equiv\; t_d \rightarrow \infty$
- In terms of physical variables: bouncing cosmology.
- Conclusion: nonsingular cosmology.

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Next Step: Double Field Theory as a Background for String Gas Cosmology

Idea Describe the low-energy degrees of freedom with an action in doubled space in which the T-duality symmetry is manifest.

Candidate for dynamics in the Hagedorn phase: Double Field Theory [W. Siegel, 1993, C. Hull and B. Zwiebach, 2009, L. Freidel et al., 2017]

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R},$$

$$\begin{aligned} \mathcal{R} &= \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \\ &+ 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d \\ &+ 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{2}\eta^{MN}\eta^{KL}\partial_M\mathcal{E}^A{}_K\partial_N\mathcal{E}^B{}_L\mathcal{H}_{AB}\,. \end{aligned}$$

$$
\begin{aligned}
\mathcal{H}_{MN} &= \begin{bmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{bmatrix}. \\
X^M &= (\tilde{x}_i, x^i), \\
\eta^{MN} &= \begin{bmatrix} 0 & \delta_i{}^j \\ \delta^i{}_j & 0 \end{bmatrix}.
\end{aligned}
$$

Cosmology of DFT

R.B., R. Costa, G. Franzmann and A. Weltman, in preparation

Add matter action S_m to the background action of SGC:

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R} + S_m$$

Consider generalized Friedmann metric:

$$ds^2 = dt^2 + d\tilde{t}^2 - a(t)^2 dx^2 - \frac{1}{a^2(t)} d\tilde{x}^2$$

Physical time constraint:

$$|\tilde{t}| = \frac{1}{|t|}$$

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Equations of Motion

$$
\begin{aligned}
2\bar{\phi}'' - (\bar{\phi}')^2 - (D-1)\tilde{H}^2 + 2\ddot{\bar{\phi}} - \\
(\dot{\bar{\phi}})^2 - (D-1)H^2 &= 0 \\
(D-1)\tilde{H}^2 - \bar{\phi}'' - (D-1)H^2 + \ddot{\bar{\phi}} &= \frac{1}{2}e^{\bar{\phi}}\bar{\rho} \\
\tilde{H}' - \tilde{H}\bar{\phi}' + \dot{H} - H\dot{\bar{\phi}} &= \frac{1}{2}e^{\bar{\phi}}\bar{p}
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\phi} &= \phi - (D-1)\ln a \\
' &= \frac{\partial}{\partial \tilde{t}} \\
\tilde{H} &= \frac{a'}{a}
\end{aligned}
$$