Quantum gravitational aspects of black holes

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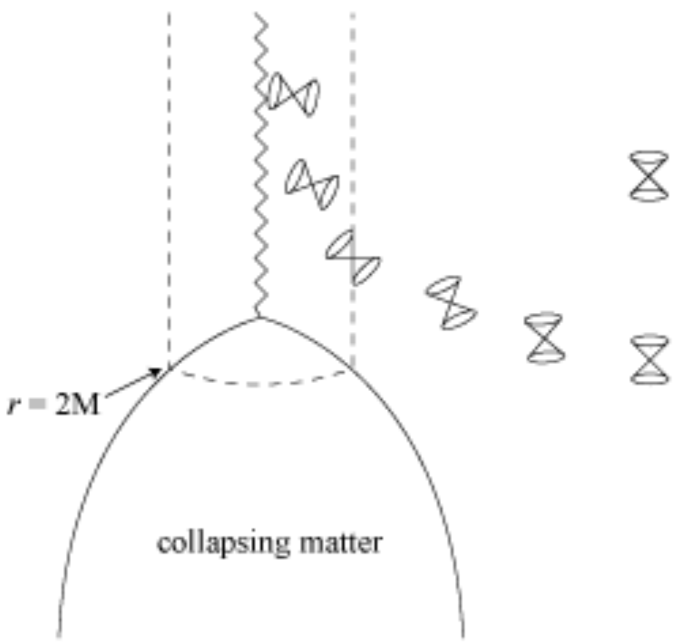
Black hole thermodynamics



[Bekenstein 72; Bardeen, Carter, Hawking 73; Hawking 74]

Black holes in their stationary phase behaves as thermodynamical systems:

0th law: the surface gravity κ is constant on the horizon.

1st law: $\delta M = \frac{\kappa_H}{8\pi G}\delta A + \Phi_H\, \delta Q + \Omega_H\, \delta J$

2nd law: $\delta A \geq 0$

3rd law: the surface gravity value $\kappa = 0$ (extremal BH) cannot be reached by any physical process.

$$S \leftrightarrow \frac{A}{8\pi G\hbar\alpha}\,, \quad T \leftrightarrow \hbar\alpha\kappa_H$$

But, in classical GR: $T = 0$

- Hawking radiation: Thermal emission of particles from a BH at $T = \frac{\hbar\,\kappa_H}{2\pi k_B}$

➥ Bekenstein-Hawking entropy formula:

$$S_{BH} = \frac{Ak_B}{4G\hbar}$$

Semiclassical result

The entropy puzzle

Statistical physics: entropy of any system is given by S = ln N

N = number of states of the system for the given macroscopic parameters

$N = e^S \sim 10^{10^{77}}$ for a solar mass black hole

Where do all these DOF live? Natural guess: On the horizon

I. Classically forbidden: no-hair th. (the horizon cannot hold any information in its vicinity)

Unique geometry of the horizon ➥ $S = \ln 1 = 0$

II. Classically over-enhanced: plenty of soft hair = residual diffeos at boundary (gauge vs physical symmetry)

Infinite-dim symmetry algebra ➥ $S = \infty$

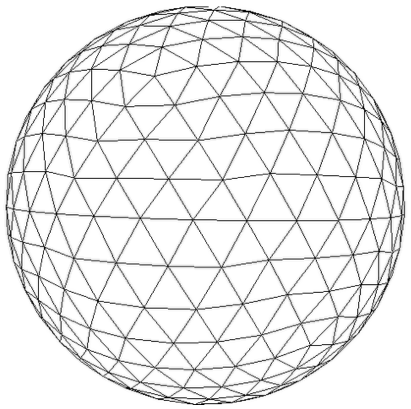
☞ Call for a quantum treatment of the gravitational DOF

Weak holographic principle:

The entropy in the 1st law is the log of the number of states of the black hole that can affect the *exterior* [Bekenstein; Jacobson; Perez; Rovelli; Sorkin; Smolin]

➥ The horizon carries some kind of information with a density approximately 1 bit per unit area

"It from Bits"



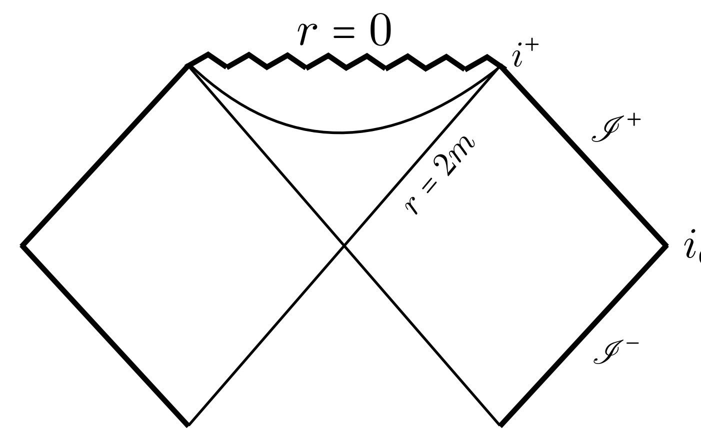
What these bits of information represent depends on the deep structure of space-time

✧ The finiteness of the BH entropy hints at discreteness of space-time at the Planck scale

Black hole singularity

[Schwarzschild 1916] first black hole solution:

$$ds^2 = -\underbrace{\left(1 - \frac{2Gm}{r}\right)}_{\xrightarrow[r \to 0]{} \infty} dt^2 + \left(1 - \frac{2Gm}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$



— A mathematical artifact due to the spherical symmetry —

[Penrose PRL 1965] singularity theorem:

Some sort of geodesic incompleteness occurs inside *any* black hole
whenever matter satisfies reasonable energy conditions ($R_{\mu\nu}k^\mu k^\nu \geq 0$).
Deviations from spherical symmetry are not able to prevent the formation of singularities.

— Singularities are generic predictions of general relativity! —

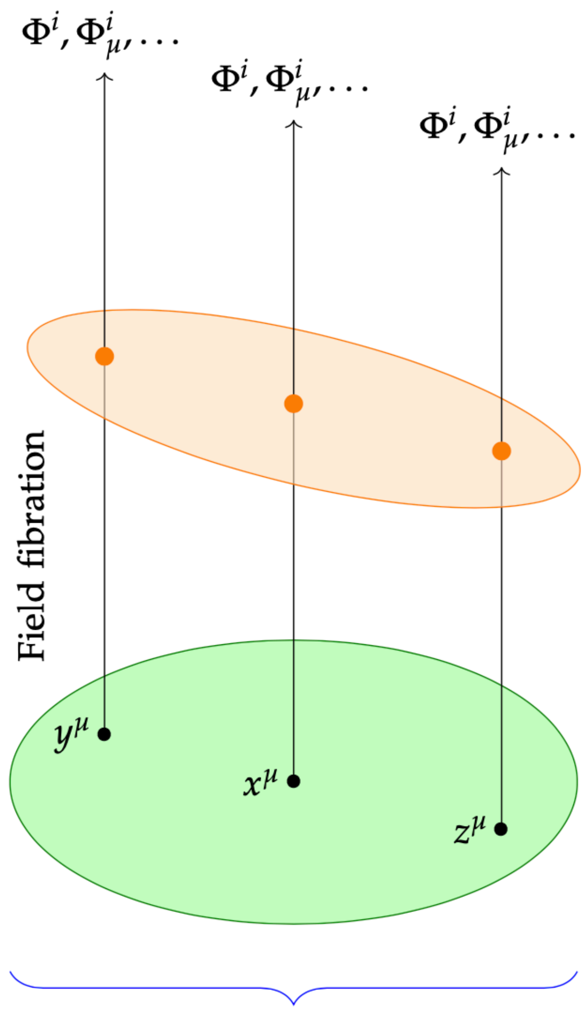
Part I

BH entropy

- Covariant phase space
- Canonical GR: metric variables & connections variables
- Isolated horizons: classical
- Quantum geometry: Discrete area spectrum
- Isolated horizons: quantum
- Entropy counting

- Laurent Freidel, Marc Geiller, DP, **Edge modes of gravity. Part I-II**, JHEP 11 (2020) 026, [hep-th/2006.12527]; JHEP 11 (2020) 027, [hep-th/2007.03563];
- Laurent Freidel, Roberto Oliveri, DP, Simone Speziale, **Extended corner symmetry, charge bracket and Einstein's equations**, *JHEP* 09 (2021) 083, [hep-th/2104.12881];
- Fernando Barbero and DP, **Black Hole Entropy in Loop Quantum Gravity**, "Handbook of Quantum Gravity", Cosimo Bambi, Leonardo Modesto, Ilya Shapiro (editors), Springer (2023), [gr-qc/2212.13469];

Covariant phase space



Field space J $\longrightarrow$ $T^*_P J \;=\; \mathrm{Span}\{\delta\Phi^i_I\}$

Section $= \{\Phi^i(x^\mu), \partial_\alpha \Phi^i(x^\mu), \ldots\}$

Spacetime M $\longrightarrow$ $T^*_P M \;=\; \mathrm{Span}\{dx^\mu\}$

Jet bundle $= \{(x^\mu, \Phi^i_{(\mu)})\}$ $\longrightarrow$

Horizontal derivative = exterior derivative d
Vertical derivative = variational operator δ

Figure credit: [Compere, Fiorucci hep-th/1801.07064]

Target spacetime = Lorentzian manifold M with set of coordinates $\{x^\mu\}$

Tangent space $T_P M$ with coordinate basis $\{\partial_\mu\}$

Cotangent space $T^*_P M$ with natural basis $\{\mathrm{d}x^\mu\}$

1. Inner product

$$\iota : \; T_P M \to \text{Linear functions on } T^*_P M;$$
$$\xi \mapsto [\iota_\xi : T^*_P M \to \mathbb{R} : w \mapsto \iota_\xi w := \xi^\mu \partial_\mu w]$$

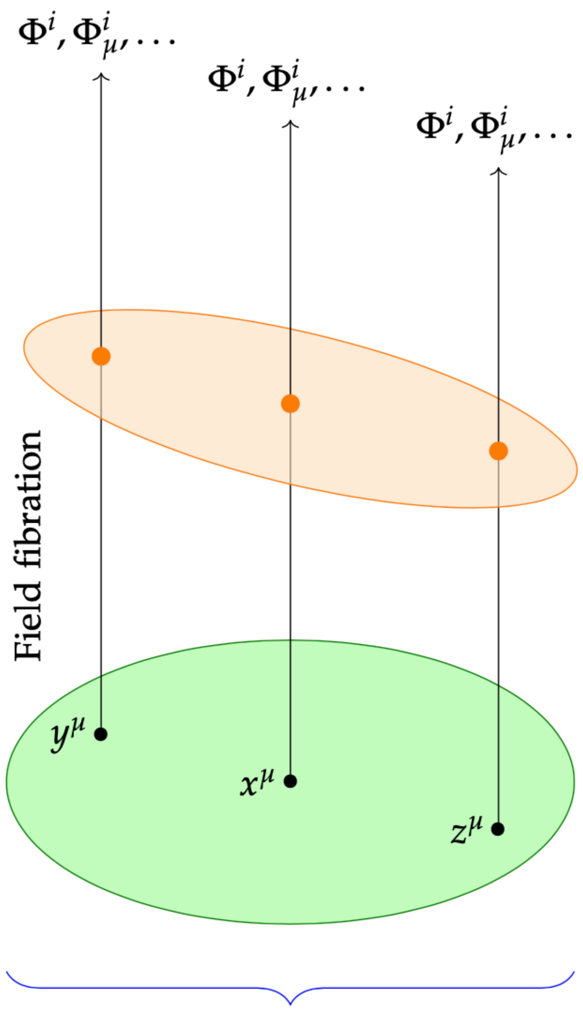
$$\iota_\xi : \Omega^k(M) \to \Omega^{k-1}(M)$$

2. Exterior derivative $\quad \mathrm{d} = \mathrm{d}x^\mu \partial_\mu$

$$\mathrm{d} : \Omega^k(M) \to \Omega^{k+1}(M)$$

Lie derivative: $\quad \mathcal{L}_\xi = \mathrm{d}\iota_\xi + \iota_\xi \mathrm{d}$

Covariant phase space



Field space J $\longrightarrow$ $\boxed{T^*_P J \; = \; \text{Span}\{\delta\Phi^i_I\}}$

Section = $\{\Phi^i(x^\mu), \partial_\alpha \Phi^i(x^\mu), \ldots\}$

Spacetime M $\longrightarrow$ $\boxed{T^\star_P M \; = \; \text{Span}\{dx^\mu\}}$

Jet bundle = $\{(x^\mu, \Phi^i_{(\mu)})\}$ $\longrightarrow$

Horizontal derivative = exterior derivative d
Vertical derivative = variational operator δ

Collection of fields $\quad \Phi = (\Phi^i_I)_{i \in I}$

Field space (or jet space) = set $\{\Phi^i, \Phi^i_\mu, \Phi^i_{\mu\nu}, \ldots\}$

symmetrized derivatives of fields

Cotangent space = $\{\delta\Phi^i, \delta\Phi^i_\mu, \delta\Phi^i_{\mu\nu}, \ldots\}$

1. Variational operator

$$\delta = \delta\Phi^i \frac{\partial}{\partial\Phi^i} + \delta\Phi^i_\mu \frac{\partial}{\partial\Phi^i_\mu} + \delta\Phi^i_{\mu\nu} \frac{\partial}{\partial\Phi^i_{\mu\nu}} + \ldots$$

$\delta =$ exterior derivative on the field space $(\delta^2 = 0)$

$\delta\Phi^i, \delta\Phi^i_\mu, \delta\Phi^i_{\mu\nu}, \ldots \quad$ 1-forms in field space

2. Inner product

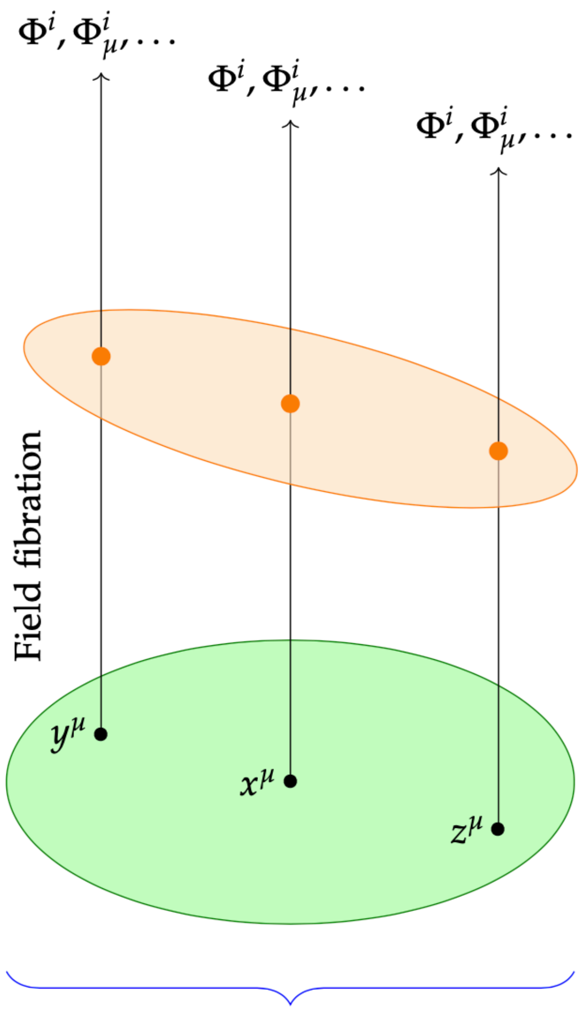
$I_\xi := I_{\mathcal{L}_\xi}$ field space contraction on field space forms

Figure credit: [Compere, Fiorucci hep-th/1801.07064]

Vector field on field space $\quad X_\xi := \int \mathrm{d}^d x \, \mathcal{L}_\xi \Phi^i(x) \frac{\delta}{\delta\Phi^i} : \quad I_\xi \delta\Phi^i(x) = \delta_\xi \Phi^i(x) = X_\xi \lrcorner \delta\Phi^i(x) = \mathcal{L}_\xi \Phi^i(x)$

Field space Lie derivative: $\quad \delta_\xi = \delta I_\xi + I_\xi \delta$

Covariant phase space



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Jet bundle = manifold with local coordinates $(x^\mu, \Phi^i_{(\mu)})$

Fields = All fibers above the target manifold

Section of the fiber = $\{\Phi^i(x^\mu), \partial_\mu \Phi^i(x^\mu), \partial_\mu \partial_\nu \Phi^i(x^\mu), \ldots\}$

Map $\phi : U^M \to U^J \,, \quad x^\mu \to \Phi^i_{(\mu)}(x^\mu)$

Bi-graded Cartan calculus:

$$[\mathrm{d}, \delta] = 0, \quad [\iota_\xi, I_\chi] = 0$$

Variational bicomplex:

Bigraded space of forms on field-space

$\Omega^{p,q}(M, J)$ = Set of (p, q)-forms,

with $p =$ number of d, $q =$ number of δ

- Lagrangian top-form $L \in \Omega^{d,0}(M,J)$

equations of motion $E_L \in \Omega^{d,1}(M,J)$,

symplectic potential $\theta_L \in \Omega^{d-1,1}(M,J)$

$$\Omega^{d,0}(M,J) \to \Omega^{d,1}(M,J) \oplus \Omega^{d-1,1}(M,J)$$
$$L \to (E_L, \theta_L)$$

$\longrightarrow$ Anderson homotopy operator [Anderson 1992]

Unique $\theta = \theta^L$ s.t. : $\quad \delta L = \mathrm{d}\theta - E \quad$ (✲)

[Lee, Wald 1990]; [Freidel, Geiller, DP 2020]

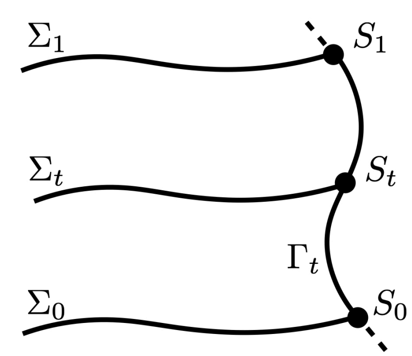
For first order Lagrangians: $E = \delta\Phi^i \left(\frac{\partial L}{\partial \Phi^i} - \partial_\mu \left(\frac{\partial L}{\partial_\mu \Phi^i} \right) \right) \epsilon$, $\qquad \theta := \delta\Phi^i \left(\frac{\partial L}{\partial \partial_\mu \Phi^i} \right) \epsilon_\mu$

Volume form $\qquad \epsilon_\mu = \iota_{\partial_\mu} \epsilon$

- Symplectic form $\quad \Omega := \int_\Sigma \delta\theta \quad$ where $\Sigma =$ codimension-1 hypersurface with boundary $S := \partial\Sigma$

(✲) $\to \mathrm{d}\omega = \delta E$ ➥

- ✅ symplectic current conserved on-shell
- ❎ symplectic current independent of Σ



On-shell

$$\partial_t \Omega_t \hat{=} \delta \mathcal{F}_\xi \quad \leftrightarrow \quad \Omega_t - \Omega_0 \hat{=} \Omega_{\Gamma_t} \qquad \text{where} \qquad \mathcal{F}_\xi := \int_S \iota_\xi \theta$$

symplectic flux associated with $\xi = \partial_t$

Noether theorems

1. For each gauge symmetry there exists a constraint which vanishes on-shell: $I_\xi E = \mathrm{d}C_\xi$ with $C_\xi = \xi^\mu G_\mu{}^\nu \epsilon_\nu$

2. Noether current conserved on-shell: $j_\xi := I_\xi \theta - \iota_\xi L = C_\xi + \mathrm{d}q_\xi$

➥ Noether charges for local gauge symmetries = corner charges on-shell

$$Q_\xi = \int_S q_\xi , \qquad \mathrm{d}q_\xi \hat{=} I_\xi \theta - \iota_\xi L$$

Uniquely determined for a given Lagrangian and a choice of field coordinates

- Fundamental canonical relation: $-I_\xi \Omega = \delta \left(\int_\Sigma C_\xi \right) + \delta Q_\xi - \mathcal{F}_\xi$

➥ When $\mathcal{F}_\xi = 0$ the Noether charge is the Hamiltonian charge

$Q_\xi =$ Canonical generator of symmetry: $\Omega(\delta_\xi, \delta_\chi) := I_\chi I_\xi \Omega \hat{=} \delta_\xi Q_\chi = \{Q_\xi, Q_\chi\} = -Q_{[\xi,\chi]}$

- Canonical boundary condition $\mathcal{B}^L :\quad \theta \overset{\Gamma}{=} 0 \quad \leftrightarrow \quad \mathcal{F}_\xi = 0 \,, \quad \forall\, \xi \parallel \Gamma$

Changing the Noetherian split

Let us consider L and $L' = L + \mathrm{d}\ell \quad \to \quad \theta' - \theta = \underbrace{\delta\ell - \mathrm{d}\vartheta}_{\text{Boundary EOM}} \quad \to \quad \Omega' = \Omega - \int_S \delta\vartheta$

[Freidel, Geiller, DP 2020]

$\ell =$ Boundary Lagrangian $\qquad \vartheta =$ Corner symplectic potential

- Shifted charge and flux: $\quad Q'_\xi - Q_\xi = \int_S (\iota_\xi \ell - I_\xi \vartheta), \qquad \mathcal{F}'_\xi - \mathcal{F}_\xi = \int_S (\delta \iota_\xi \ell - \delta_\xi \vartheta)$

Choice of Lagrangian related to different representations of the corner symmetry algebra

Local codim-2 form on Γ

In general, for a given L, stationarity of the action requires the boundary condition: $\theta \overset{\Gamma}{=} \mathrm{d}C - \delta\ell$

[Iyer, Wald 1994]; [Harlow, Wu 2019]

➥ A general boundary condition is canonical when $C = \vartheta \quad \to \quad \mathcal{B}^{L'}: \quad \theta' \overset{\Gamma}{=} 0$ ✓

Choice of Lagrangian related to a choice of canonical boundary conditions

[Freidel, Oliveri, DP, Speziale 2021]

Canonical GR

Metric variables

GR Lagrangian: $$L_{\mathrm{GR}}[\tilde{g}, n] := \frac{1}{\kappa}\epsilon\Big(\tilde{R} - (\tilde{K}^2 - \tilde{K}^{\mu\nu}\tilde{K}_{\mu\nu})\Big)$$

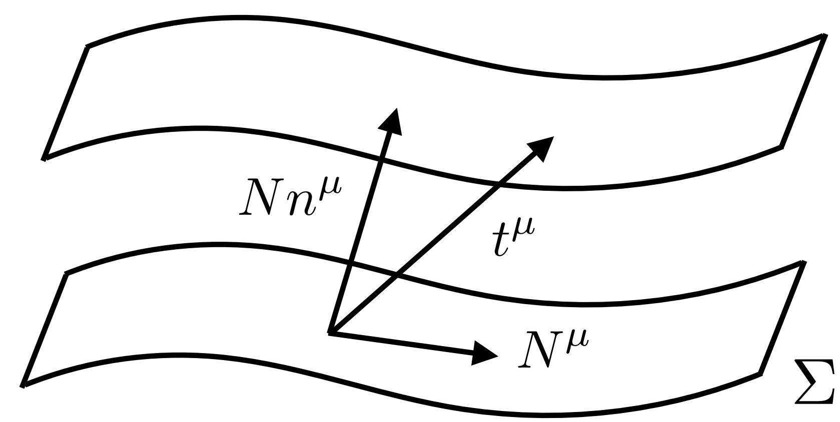
Ricci scalar of Σ   extrinsic curvature of Σ

$$\kappa = 16\pi G$$

$$\epsilon := \sqrt{|g|}\,\mathrm{d}^4x$$

Upon foliation of spacetime M in terms of space-like three dimensional surfaces Σ_t , the phase space of GR is parametrized by the canonical coordinates

$$\Theta_{\mathrm{GR}} = \frac{1}{\kappa}\int_\Sigma \underbrace{\tilde{\epsilon}(\tilde{K}\tilde{g}^{\mu\nu} - \tilde{K}^{\mu\nu})}_{:=\tilde{P}^{\mu\nu}}\delta\tilde{g}_{\mu\nu}$$



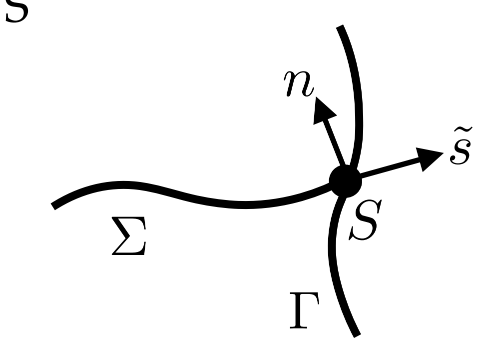
Spatial diffeomorphism constraint = Conservation eq. : $\tilde{\nabla}_\mu\tilde{P}^{\mu\nu} = 0$

- Hamiltonian charges: Diffeomorphisms within Σ and tangent to the boundary sphere S

$$-I_\xi\Omega_{\mathrm{GR}} = \delta\mathcal{H}_{\mathrm{GR}}[\xi]\,, \qquad \mathcal{H}_{\mathrm{GR}}[\xi] = \mathcal{H}^\Sigma_{\mathrm{GR}}[\xi] + \mathcal{H}^S_{\mathrm{GR}}[\xi]\,,$$

$$\mathcal{H}^\Sigma_{\mathrm{GR}}[\xi] := -\int_\Sigma \xi_\mu\tilde{\nabla}_\nu\tilde{P}^{\mu\nu} \hat{=} 0$$ smeared vector constraint of canonical gravity

$$\mathcal{H}^S_{\mathrm{GR}}[\xi] := \int_S \sqrt{q}\,\tilde{s}_\mu\xi_\nu\tilde{P}^{\mu\nu}$$ Brown–York charge



$$\xi^\mu\tilde{s}_\mu \overset{S}{=} 0$$

$\Theta_{\rm GR}$ = "Fundamental" potential capturing the bulk canonical DOF which are common to any formulation of GR:

Any other formulation F can be understood as $$\boxed{\Theta_{\rm F} = \Theta_{\rm GR} + \Theta_{\rm F/GR} - \delta(\cdots)}$$

$$\delta L_{\rm F/GR} + {\rm d}\theta_{\rm F/GR} = \theta_{\rm F} - \theta_{\rm GR}\,, \qquad \Theta_{\rm F/GR} = \int_S \theta_{\rm F/GR}$$

$\Theta_{\rm F/GR}$ = corner symplectic potential $\rightarrow$ Different set of corner charges and non-trivial representation for different components of $\mathfrak{g}_S$

$\Theta_{\rm GR} \rightarrow$ $\mathfrak{g}^S_{\rm GR} = {\rm diff}(S)$ Corner algebra non-trivially represented in all formulations of gravity

Canonical GR

Connection variables

- Einstein–Cartan–Holst Lagrangian: $L_{\mathrm{ECH}}[e,\omega] = \frac{1}{\kappa} E_{IJ}[e] \wedge F^{IJ}[\omega]$

$$F^{IJ} = \mathrm{d}\omega^{IJ} + \omega^I{}_K \wedge \omega^{KJ}$$

$$E_{IJ}[e] := (* + \frac{1}{\gamma})(e_I \wedge e_J)$$

Lorentz connection $\omega^{IJ} = -\omega^{JI}$

Tetrad coframe field $e^I = \mathbb{R}^4$-valued 1-forms

$I = 0, i$ and $i = 1, 2, 3$ internal Lorentz indices

$\gamma =$ Immirzi parameter

duality operation: $(*M)_{IJ} = \frac{1}{2}\epsilon_{IJ}{}^{KL} M_{KL}$

- Time gauge: $e^0 = n$ Triad $e^i, i = su(2)$ indices

set of three 1-forms defining a frame at each point in Σ : $\tilde{g}_{ab} = e^i_a e^j_b \delta_{ij}$

densitized triad $E^i := \epsilon^i{}_{jk} e^j \wedge e^k$

Ashtekar-Barbero connection $A^i := \Gamma^i + \gamma K^i$

where $K^i := \omega^{0i}$ = extrinsic curvature of Σ , and $\Gamma^i = -\frac{1}{2}\epsilon^{ijk}\omega_{jk}$ = spin connection: $\mathrm{d}_\Gamma e^i = 0$

solution of Cartan's structure equations

$$\kappa\,\Theta_{\rm ECH} = \frac{1}{\gamma}\int_\Sigma E_i \wedge \delta A^i \quad ➡ \quad \{E^a_j(x), A^i_b(y)\} = \kappa\gamma\delta^a_b\delta^i_j\delta(x,y)$$

Hamiltonian

$$H = N_a V^a(E^a_j, A^j_a) + NS(E^a_j, A^j_a) + N^i G_i(E^a_j, A^j_a)$$

System of 7 first class constraints
for the 18 phase space variables

☞ 2 physical DOF

$G(\alpha) = \alpha_i \mathrm{d}_A E^i = 0\,,$ internal rotations with parameter $\alpha_i \in su(2)$

$V(\xi) = (\iota_\xi F^i(A)) \wedge E_i = 0$ spatial diffeomorphisms generated by $\xi \in T(\Sigma)$

GR = background independent *SU(2)* gauge theory (partly analogous to *SU(2)* Yang-Mills theory)

$$\kappa\,\Theta_{\rm ECH} = \int_\Sigma E_i \wedge \delta K^i + \frac{1}{\gamma}\int_{\partial\Sigma} e_i \wedge \delta e^i$$

Bulk symplectic structure

$$\{E^a_i(x), K^j_b(y)\} = \kappa\delta^a_b\delta^j_i\delta^3(x,y)$$

Palatini variable: Imply the Poisson brackets of the ADM phase space

Corner symplectic structure

$$\{e^i_a(x), e^j_b(y)\} = \frac{\kappa\gamma}{2}\delta^{ij}\epsilon_{ab}\delta^2(x,y)$$

Important implications for entropy DOF interpretation

$$\text{Using}\quad P^I := -\epsilon^I{}_{JKL} n^L K^J \wedge e^K \quad \to \quad E_I \wedge \delta K^I = P_I \wedge \delta e^I - \frac{1}{2}\delta(e^I \wedge P_I)$$

$$L_{\text{ECH}} = L_{\text{GR}} + \mathrm{d}L_{\text{ECH/GR}}\,, \qquad L_{\text{ECH/GR}} = -\frac{1}{\kappa}\Big(*\,(e\wedge e)_{IJ} \wedge \mathrm{d}_\omega n^I n^J + \frac{1}{\gamma} e_I \wedge \mathrm{d}_\omega e^I \Big)$$

$$\Theta_{\text{ECH}} = \Theta_{\text{GR}} + \Theta_{\text{ECH/GR}} - \frac{1}{\kappa}\delta\left(\int_\Sigma \tilde{\epsilon}\tilde{K}\right)\,, \quad \Theta_{\text{GR}} = \int_\Sigma P_i \wedge \delta e^i\,, \quad \Theta_{\text{ECH/GR}} = \frac{1}{\kappa\gamma}\int_S e_i \wedge \delta e^i$$

- Spatial diffeo charges: $\quad -I_\xi \Omega_{\text{ECH}} = \delta \mathcal{H}_{\text{ECH}}[\xi]\,, \qquad \mathcal{H}_{\text{ECH}}[\xi] = \mathcal{H}^\Sigma_{\text{ECH}}[\xi] + \mathcal{H}^S_{\text{ECH}}[\xi]$

$$\mathcal{H}^\Sigma_{\text{ECH}}[\xi] = -\int_\Sigma \underbrace{\xi_i \mathrm{d}_\Gamma P^i}_{=V(\xi)} \mathrel{\hat{=}} 0\,, \qquad \mathcal{H}^S_{\text{ECH}}[\xi] = \frac{1}{2}\int_S \iota_\xi \omega_{IJ} E^{IJ} = \int_S \xi_i\, P^i + \frac{2}{\gamma}\int_S \xi_i\, \mathrm{d}e^i \qquad \text{with } \xi_i = \iota_\xi e_i$$

- Internal SU(2) charges: $\quad -I_\alpha \Omega_{\text{ECH}} = \delta \mathcal{H}_{\text{ECH}}[\alpha]\,, \qquad \mathcal{H}_{\text{ECH}}[\alpha] = \mathcal{H}^\Sigma_{\text{ECH}}[\alpha] + \mathcal{H}^S_{\text{ECH}}[\alpha]$

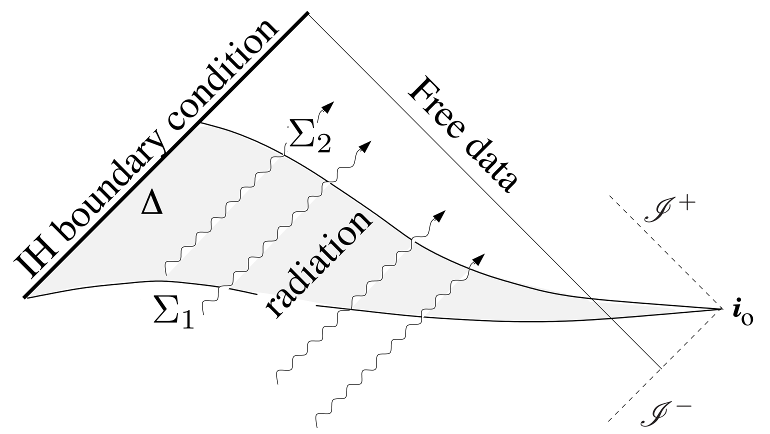
$$\mathcal{H}^\Sigma_{\text{ECH}}[\alpha] = -\int_\Sigma \underbrace{\alpha^i \mathrm{d}_A E_i}_{=G(\alpha)} \mathrel{\hat{=}} 0\,, \qquad \mathcal{H}^S_{\text{ECH}}[\alpha] = \int_S \alpha^i E_i \qquad \text{with } \delta_\alpha E^i = [E,\alpha]^i$$

$$\Theta_{\text{ECH}} \quad \to \quad \mathfrak{g}^S_{\text{ECH}} = \text{diff}(S) \oplus su(2)^S$$

Quasi local definition of BH

Isolated Horizons

IH boundary conditions



1) Null hypersurface Δ of $(M, g_{\mu\nu}, \nabla_\mu)$ with topology $\Delta = S^2 \times \mathbb{R}$

2) Einstein's equations and the stronger dominant energy condition hold at Δ

3) Δ is equipped with an equivalence class $[\ell]$ of null normals (related by a positive constant rescaling) such that the expansion of any $\ell \in [\ell]$ has to vanish within Δ

4) Intrinsic connection is conserved along Δ : $[\mathcal{L}_\ell, D_\mu]v^\nu = 0, \ \forall \ v^\mu \in T(\Delta)$

$\ell^\mu =$ normal future pointing null vector field

$q_{\mu\nu} =$ degenerate intrinsic metric of Δ

$D_\mu =$ unique intrinsic connection compatible with $q_{\mu\nu}$

$\kappa_{(\ell)} = \omega_\mu \ell^\mu$, where $D_\mu \ell^\nu = \omega_\mu \ell^\nu$

3) $\rightarrow$ I. 2-sphere cross-section A_S is constant in time

II. $\mathcal{L}_\ell q_{\mu\nu} = 0$

4) $\rightarrow$ Surface gravity κ_ℓ constant along Δ for each $\ell \in [\ell]$:

$$\ell' = c\ell \ \rightarrow \ \kappa_{(\ell')} = c\kappa_{(\ell)}, \quad c > 0$$

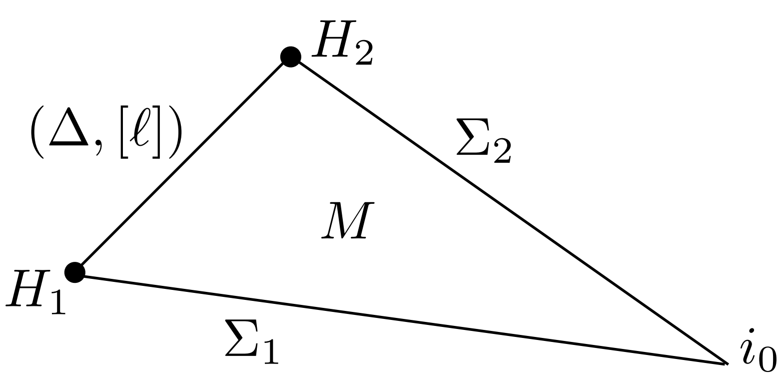
$0th$ law of BH mechanics

IH b.c. ➡ Certain geometric structures intrinsic to Δ are time independent: $(q_{\mu\nu}, D) :=$ Horizon geometry

Introducing the null-tetrad $(\ell, n, m, \bar{m}):\quad n\cdot\ell=-1\,,\ m\cdot\bar{m}=1\,,$ area 2-form on Δ $\;{}^2\epsilon := im\wedge\bar{m}$

Static (non-rotating) IH: $\quad \mathrm{Im}(\Psi_2)=0\,,\quad \Psi_2 = C_{abcd}\ell^a m^b \bar{m}^c n^d$

➥ $F^i(A) \overset{H}{=} -\dfrac{\pi(1-\gamma^2)}{A_H} E^i$ (✻) and $\epsilon^i{}_{jk} K^j \wedge K^k \overset{H}{=} \dfrac{2\pi}{A_H} E^i \quad \rightarrow \quad \xi_i P^i \overset{H}{=} 0\,,\ \forall \xi \in T(H)$



- IH phase space:

$$p=(e^i, A^i)\in\Gamma\,,\quad \delta=(\delta e^i,\delta A^i)\in T^*_p(\Gamma)$$

for the pull back of fields on the horizon δ = linear combinations of *SU(2)* gauge transformations and diffeos preserving the horizon geometry *(q, D)*

$$\alpha:\Delta\rightarrow su(2)\,,\ \xi:\Delta\rightarrow \mathrm{T}(H)$$

$$\delta_\alpha e^i = [e,\alpha]^i\,,\quad \delta_\alpha A^i = \mathrm{d}_A\alpha^i\,,$$

$$\delta_\xi e^i = \mathcal{L}_\xi e^i = \iota_\xi \mathrm{d}e^i + \mathrm{d}\iota_\xi e^i\,,$$

$$\delta_\xi A^i = \mathcal{L}_\xi A^i = \iota_\xi F^i + \mathrm{d}_A \iota_\xi A^i$$

The symplectic structure:
$$\kappa\,\Omega_{\rm GR} = \int_\Sigma \delta E_i\wedge\delta K^i$$
$$= \frac{1}{\gamma}\int_\Sigma \delta E_i\wedge\delta A^i + \frac{1}{\gamma}\int_H \delta e_i\wedge\delta e^i$$
$$\overset{(✻)}{=} \frac{1}{\gamma}\int_\Sigma \delta E_i\wedge\delta A^i - \frac{A_H}{\pi\gamma(1-\gamma^2)}\int_H \delta A^i\wedge\delta A_i$$

Corner term given by an *SU(2)* Chern-Simons symplectic structure

- 1st law of BH mechanics for spherical IH:

Require the time evolution along vector fields t^μ which are time translations at infinity and proportional to the null generators ℓ at the horizon to correspond to a Hamiltonian time evolution

$$-\kappa I_t \Omega_{\rm GR}(\delta, \delta) = \delta H_t\,, \ \forall\ \delta \in T^*_p(\Gamma) \quad ➥ \quad H_t = E^t_{\rm ADM} - E^t_{H} \quad \text{such that} \quad \delta E^t_{H} = \frac{\kappa^t_{H}}{\kappa}\delta A_{H}$$

where κ^t_H can only depend on the horizon area

The transformation δ_t on Γ defined by the spacetime evolution field t^μ is Hamiltonian *if and only if* the first law holds:

Infinite family of first laws, one associated with each permissible t^μ

'Canonical' choice : t^μ agrees with the static Killing field on the horizon which is *unit* at infinity

$$➥ \quad \kappa^{t_0}_{H} = \frac{1}{2R_{H}} \quad \rightarrow \quad E^{t_0}_{H} = M_{H}\,, \quad \delta M_{H} = \frac{\kappa_{H}}{\kappa}\delta A_{H}$$

analog of the ADM mass in the rest frame at infinity

- Symmetry group for spherical IH: $G_H = \mathrm{Diff}(H) \ltimes SU(2)^H$

Trivially represented in the GR (Palatini) formulation, but not in the ECH formulation

The Dirac program

The non perturbative quantization of GR

The phase space of gravity in the bulk can be parametrized by connection-flux variables:

$$\{E^a_j(x), A^i_b(y)\} = \kappa\gamma\delta^a_b\delta^i_j\delta(x,y)$$

- (i) Find a representation of the phase space variables of the theory as operators in an auxiliary or kinematical Hilbert space H_{kin} satisfying the standard commutation relations, i.e., $\{\ ,\ \} \to -i/\hbar[\ ,\ ]$.

- (ii) Promote the constraints to (self-adjoint) operators in H_{kin}. In the case of gravity we must quantize the seven constraints $G_i(A, E)$, $V_a(A, E)$, and $S(A, E)$.

- (iii) Characterize the space of solutions of the constraints and define the corresponding inner product that defines a notion of physical probability. This defines the so-called physical Hilbert space H_{phys}.

- (iv) Find a (complete) set of gauge invariant observables, i.e., operators commuting with the constraints. They represent the questions that can be addressed in the generally covariant quantum theory.

- Kinematical structure (connection polarization) :

The kinematical Hilbert space consists of a suitable set of functionals of the connection which are square integrable with respect to a suitable (gauge invariant and diffeomorphism invariant) measure

Holonomy along a path γ: $h_\gamma[A] = P\exp - \int_\gamma A$ $\quad\rightarrow\quad$ Generalized connection

Algebra of kinematical observables = Algebra of Cylindrical functionals of of generalized connections $\mathrm{Cyl} = \cup_\Gamma \mathrm{Cyl}_\Gamma$

union of Cyl_Γ for all graphs in Σ

$$f : SU(2)^{N^\Gamma} \rightarrow \mathbb{C}\,, \qquad \Psi_{\Gamma,f}[A] = f(h_{\gamma_1}[A], \ldots, h_{\gamma_{N^\Gamma}}[A])$$

Measure in the space of generalized connections in order to give a definition of the kinematical inner product:

Positive normalized state on Cyl $\qquad \mu_{\mathrm{AL}}(\Psi_{\Gamma,f}) = \int \prod_{i=1}^{N^\Gamma} dh_i\, f(h_{\gamma_1}, \cdots, h_{\gamma_{N^\Gamma}})$

SU(2) Haar measure: $\int_{SU(2)} dh = 1\,, \quad dh = d(\alpha h) = d(h\alpha) = dh^{-1}\,,\ \forall\, \alpha \in SU(2)$ $\quad\rightarrow\quad$ $\mu_{\mathrm{AL}}(1) = 1$ and positive

Inner product on Cyl

$$\langle \Psi_{\Gamma_1,f}, \Psi_{\Gamma_2,f}\rangle := \mu_{\mathrm{AL}}(\overline{\Psi_{\Gamma_1,f}}\Psi_{\Gamma_2,f}) = \int \prod_{i=1}^{N^{\tilde\Gamma}} dh_i\, \overline{\tilde f(h_{\gamma_1}, \cdots, h_{\gamma_{N^{\tilde\Gamma}}})}\tilde g(h_{\gamma_1}, \cdots, h_{\gamma_{N^{\tilde\Gamma}}})$$

Peter-Weyl th. : given $f \in L^2[SU(2)]$, $\quad f(g) = \sum_j f_j^{mn} \Pi^j_{mn}(g)$

$\Pi^j_{mn}(g) =$ *SU(2)* unitary irreducible representation matrices of spin j, $\; -j \leq m, n \leq j$

orthogonality relation for unitary representations of SU(2) :

$$\int_{SU(2)} dg \, \Pi^j_{mn}(g) \Pi^{j'}_{m'n'}(g) = \frac{1}{2j+1} \delta_{jj'} \delta_{mm'} \delta_{nn'}$$

Given an arbitrary cylindrical function $\Psi_{\Gamma,f}[A] \in \mathrm{Cyl}$:

$$\begin{aligned} \psi_{\Gamma,f}[A] &= f(h_{\gamma_1}[A] \cdots h_{\gamma_{N^\Gamma}}[A]) \\ &= \sum_{j_1 \cdots j_{N^\Gamma}} f^{m_1 \cdots m_{N^\Gamma}, n_1 \cdots n_{N^\Gamma}}_{j_1 \cdots j_{N^\Gamma}} \Pi^{j_1}_{m_1 n_1}(h_{\gamma_1}[A]) \cdots \Pi^{j_{N^\Gamma}}_{m_{N^\Gamma} n_{N^\Gamma}}(h_{\gamma_{N^\Gamma}}[A]) \end{aligned}$$

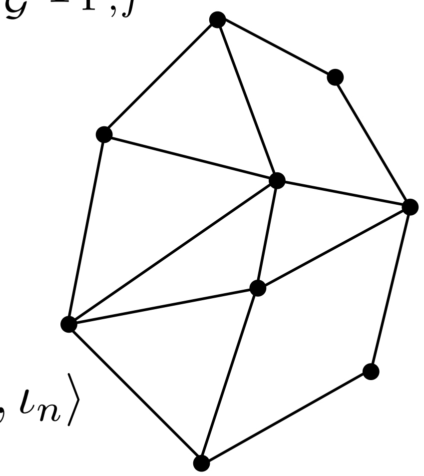
➥ The product of components of irreducible representations of SU(2) associated with the edges of a graph, for all values of *j*, *m* and for any graph is a complete orthonormal basis of H_{kin}

- Under finite gauge transformations: $h'_\gamma[A] = g(x(0))h_\gamma[A]g^{-1}(x(1)) \quad \rightarrow \quad \mathcal{U}_{\mathcal{G}}[g]\Pi^j_{mn}[h_\gamma] = \Pi^j_{mn}[g_s h_\gamma g_t^{-1}]$

Implementing gauge invariance: $P_{\mathcal{G}}\Psi_{\Gamma,f} = \int D[g]\mathcal{U}_{\mathcal{G}}[g]\Psi_{\Gamma,f} = \prod_{n\subset\Gamma} P^n_{\mathcal{G}}\Psi_{\Gamma,f}$

Intertwiners = Orthogonal set of invariant vectors (product of Clebsch–Gordan coefficients)

$$P^n : V_{j_1\ldots j_i} \to \mathrm{Inv}\,[V_{j_1\ldots j_i}]\,, \qquad P^n = \sum_\alpha \iota^\alpha \iota^{\alpha *}$$



$|\Gamma, j_\gamma, \iota_n\rangle$

➡ $H^{\mathcal{G}}_{\text{kin}}$ of spin network states = Products of *SU(2)* representation matrices contracted with intertwiners

They form a complete basis of the Hilbert space of solutions of the quantum Gauss law

- Under the action of diffeomorphisms $\phi \in \mathrm{Diff}(\Sigma)$: $h_\gamma[\phi^* A] = h_{\phi^{-1}(\gamma)}[A] \quad \rightarrow \quad \mathcal{U}_{\mathcal{D}}[\phi]\Psi_{\Gamma,f}[A] = \Psi_{\phi^{-1}(\Gamma),f}[A]$

Implementing diffeo invariance: No self-adjoint infinitesimal generator, instead $\mathcal{U}_{\mathcal{D}}[\phi]\Psi = \Psi$ for $\Psi \in \mathrm{Cyl}^\star$

$$([\Psi_{\Gamma,f}]| = \sum_{\phi\in\mathrm{Diff}(\Sigma)} \langle\Psi_{\Gamma,f}|\mathcal{U}_{\mathcal{D}}[\phi] = \sum_{\phi\in\mathrm{Diff}(\Sigma)} \langle\Psi_{\phi(\Gamma),f}| \;\rightarrow\; \langle[\Psi_{\Gamma,f}]|[\Psi_{\Gamma',g}]\rangle_{\text{diff}} = ([\Psi_{\Gamma,f}]|\Psi_{\Gamma',g}\rangle = ([\Psi_{\Gamma,f}]|\mathcal{U}_{\mathcal{D}}[\phi]\Psi_{\Gamma',g}\rangle$$

Finite diffeo invariant action

➡ $H^{\mathcal{D}}_{\text{kin}}$ of abstract spin network states = equivalence classes of spin networks under smooth deformations

They represent a quantum state of the geometry of space in a fully combinatorial manner

- Fluxes: The densitized triad naturally induces a 2-form with values in the Lie algebra of SU(2).

In the quantum theory, E_i^a becomes an operator valued distribution.

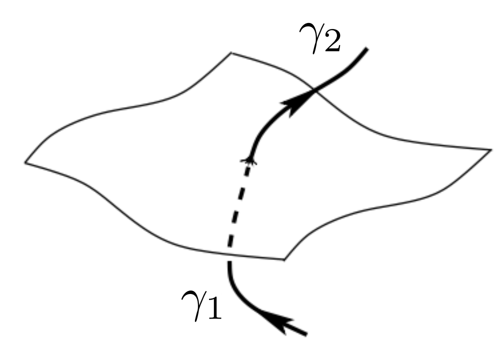
Integrals of the triad field on 2D surfaces with suitable test functions become well defined self adjoint operators in H_{kin}:

$$\hat{E}[S,\alpha] = \int_S \alpha^i \hat{E}_i^a n_a d\sigma_1 d\sigma_2 = -i\hbar\kappa\gamma \int_S \alpha^i \frac{\delta}{\delta A_a^i} n_a d\sigma_1 d\sigma_2 \qquad \text{where} \qquad n_a = \frac{\partial x^b}{\partial \sigma_1}\frac{\partial x^c}{\partial \sigma_2}\epsilon_{abc}$$

using: $$\frac{\delta}{\delta A_a^i(x)} h_\gamma[A] = \frac{\delta}{\delta A_a^i(x)}\left(P\exp\int ds\,\dot{x}^d(s)A_d^k\tau_k\right) = \int ds\,\dot{x}^a(s)\delta^{(3)}(x(s)-x)h_{\gamma_1}[A]\tau_i h_{\gamma_2}[A]$$

➥ $$\hat{E}[S,\alpha]h_\gamma[A] = -i\hbar\kappa\gamma\alpha^i h_{\gamma_1}[A]\tau_i h_{\gamma_2}[A]$$

SU(2) algebra generators



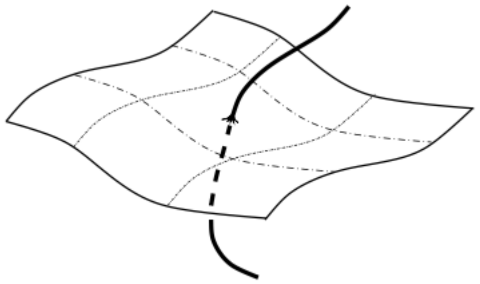
The operators $\hat{E}[S,\alpha]$ for all surfaces S and all smearing functions α contain all the information of the quantum Riemannian geometry of Σ

In terms of the operators $\hat{E}[S,\alpha]$ we can construct any geometric operator

- Area operator: $A_S = \int_S \sqrt{h}\, d\sigma_1 d\sigma_2 = \int_S \sqrt{E^a_i E^b_j \delta^{ij} n_a n_b}\, d\sigma_1 d\sigma_2\,, \quad h_{ab} = q_{ab} - n^{-2} n_a n_b$

$$A_S = \lim_{N\to\infty} A_S^N\,, \quad A_S^N = \sum_{I=1}^{N} \sqrt{E_i(S_I) E^i(S_I)}\,, \qquad \hat{E}_i(S_I)\hat{E}^i(S_I)\Pi^j_{mn}(h_\gamma[A]) = (8\pi\ell_P^2\gamma)^2 j(j+1)\Pi^j_{mn}(h_\gamma[A])$$

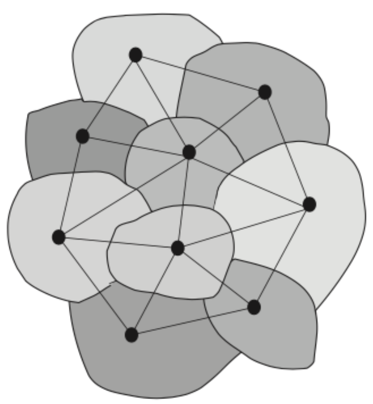
2-cells decomposition of S



The spectrum depends on the value of the Immirzi parameter

$$\rightarrow \quad \boxed{\hat{A}_S|\Gamma, j_p\rangle = 8\pi\ell_P^2\gamma \sum_{p\in\Gamma\cap S} \sqrt{j_p(j_p+1)}|\Gamma, j_p\rangle}$$

Spin network states are the eigenstates
of the quantum area operator



description of quantized geometries

Spectral analysis of geometrical operators ➡ Planck scale discreteness

"Atoms" of quantum space = Polymer-like excitations of the gravitational field

Quantum isolated horizon

- Bulk theory: LQG Hilbert space associated to a fixed graph $\Gamma \subset M$ with end points p's on H

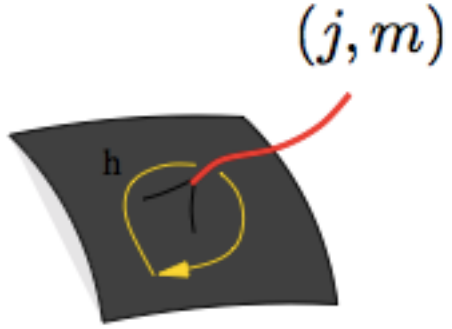
$$\hat{A}_H|\{j_p, m_p\}_1^n; \dots\rangle = 8\pi\gamma\ell_p^2 \sum_{p=1}^{n} \sqrt{j_p(j_p+1)}|\{j_p, m_p\}_1^n; \dots\rangle$$

spin network states

Boundary condition:

$$-\frac{A_H}{\pi(1-\gamma^2)}\epsilon^{ab}\hat{F}^i_{ab} = 16\pi G\gamma \sum_{p\in\gamma\cap H} \delta(x, x_p)\hat{J}^i(p)$$

Densitized triad quantum operator



- Boundary theory: *SU(2)* Chern-Simons with punctures

$$S_{\rm CS} + S_{\rm int} = \frac{k}{4\pi}\int_\Delta \mathrm{Tr}\left[A\wedge \mathrm{d}A + \frac{2}{3}A\wedge A\wedge A\right] + \sum_{p=1}^{n}\lambda_p \int_{c_p} \mathrm{Tr}\left[\tau_3(\Lambda_p^{-1}\mathrm{d}\Lambda_p + \Lambda_p^{-1}A\Lambda_p)\right]$$

Poisson brackets:

$$\{A^i_a(x), A^j_b(y)\} = \delta_{ij}\epsilon_{ab}\frac{2\pi}{k}\delta^2(x-y), \quad a,b=1,2;\ x^0=y^0$$

$$\{S^i, \Lambda\} = -\tau^i\Lambda\,, \quad \{S^i, S^j\} = i\epsilon^{ij}{}_k S^k$$

$\Lambda \in SU(2)$ particle d.o.f.

$S^i \in su(2)$ momentum conjugate to Λ

EOM:

$$\epsilon^{ab}F_{ab}(A(x)) = -\frac{2\pi}{k}\sum_p S^i_p\delta(x, x_p)$$

$$S_p\cdot S_p - \lambda_p^2 = 0 \;\rightarrow\; \lambda_p = \sqrt{s_p(s_p+1)}$$

Quantum IH DOF described by a Chern-Simons theory on a punctured 2-sphere H

$$k = \frac{A_H}{4\pi\ell_p^2\gamma(1-\gamma^2)}\,, \quad S^i = J^i \quad \rightarrow \quad H^n_{\text{IH}} = \bigoplus_{\{j\}_n} H_\Sigma^{\{j\}_n} \otimes H_H^{\text{CS}}(j_1,\ldots,j_n)$$

$\star$ Area constraint: $$A_H - \delta \leq 8\pi\gamma\ell_P^2 \sum_{i=1}^{n}\sqrt{j_i(j_i+1)} \leq A_H + \delta$$

Combinatorial quantization: an ordering of the *distinguishable* punctures needs to be introduced. A subset of diffeomorphism charges at the horizon needs to be activated in the quantum theory

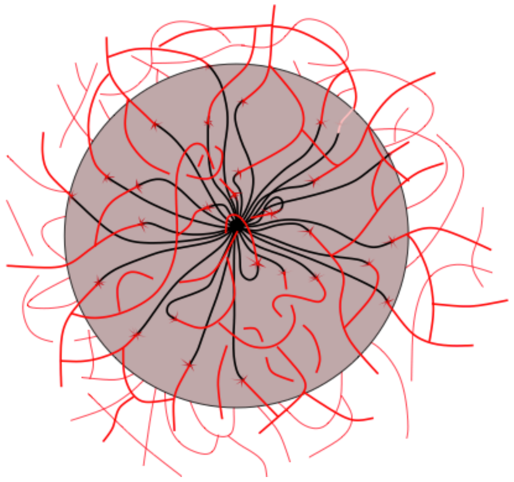
$\star$ Global constraint (due to b.c.): $H_H^{\text{CS}}(j_1\cdots j_n) \subset \text{Inv}(j_1 \otimes \cdots \otimes j_n)$

- Weak holographic principle: $S = -Tr(\rho_H \ln \rho_H) = \ln(\mathcal{N}_{\text{IH}})$

with $\mathcal{N}_{\text{IH}} =$ Dim of the IH corner Hilbert space

Due to area constraint: $\dim[H_H^{\text{CS}}(j_1\cdots j_n)] = \dim[\text{Inv}(j_1 \otimes \cdots \otimes j_n)]$

➥ We can model the IH by a single *SU(2)* intertwiner



For $A_H \propto k \to \infty$: $$S = \ln \sum_{n;\, j_1,\dots,j_n} \dim[H_H^{\mathrm{CS}}(j_1 \cdots j_n)] = \frac{A_H}{4\ell_P^2}\frac{\gamma_0}{\gamma} - \frac{3}{2}\ln\frac{A_H}{\ell_P^2}$$

- Advanced combinatorial methods; [Agullo, Barbero, Borja, Diaz-Polo, Villasenor 2009]
- CFT and quantum group representation th. methods; [Engle, Noui, Perez, DP 2010]

➡ Bekenstein-Hawking formula for $\gamma = \gamma_0$, with $\gamma_0 = 0.274067\dots$

$$\boxed{\text{BH entropy DOF} \;=\; \text{Polymer-like excitations of the gravitational field}}$$

- Entropy DOF and Immirzi parameter:

$$G_H = \mathrm{Diff}(H) \ltimes SU(2)^H$$

- GR (Palatini) formulation:

All IH corner charges vanish classically, only at the quantum level a finite set of these local charges are activated

$\to$ All the DOF accounting for the BH entropy have a purely quantum origin (counterintuitive from statistical mechanics of ordinary systems)

- ECH formulation:

  Non-vanishing *SU(2)* and translational charges: $\mathcal{H}^H_{\text{ECH}}[\alpha] = \int_H \epsilon_{ijk}\alpha^i e^j \wedge e^k\,, \quad \mathcal{H}^H_{\text{ECH}}[\xi] = \frac{2}{\gamma}\int_H \xi_i \mathrm{d}e^i$

Local holography program:

- Decompose the bulk of spacetime into a collection of subregions and attach a symmetry algebra to the corner of each subregion;

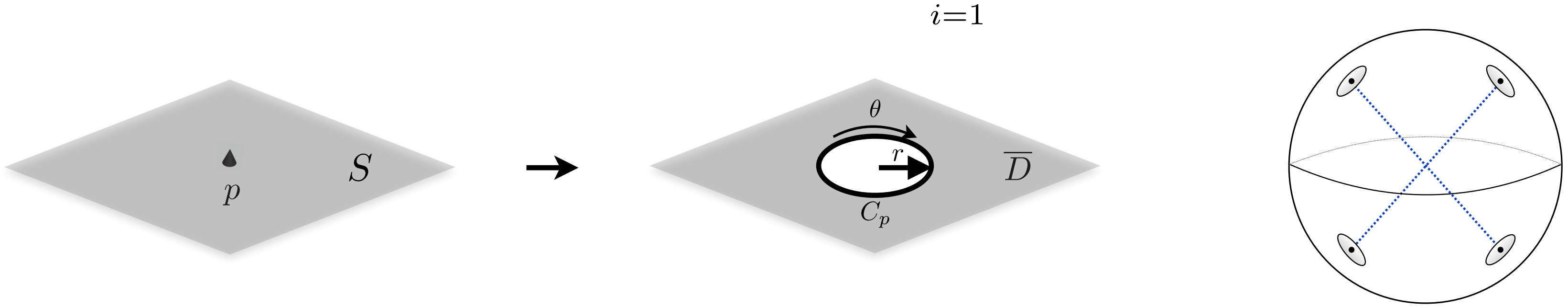
- The corner Hilbert space forms an irreducible representation of the local corner symmetry algebra, and choices of states in this corner Hilbert space then encode quantum geometries;

Corner symmetry charges = Coarse-grained information of geometrical DOF inside each region it encloses

Bulk constraints = Conservation laws for the local corner charges

Fock space representation of the corner algebra: Area discreteness from the continuum and semi-classical theory

Discrete measure density on S $\quad \mu_{\boldsymbol{\rho}}(x)\psi = \sqrt{q}(x)\psi\,, \quad \mu_{\boldsymbol{\rho}}(x) = \sum_{i=1}^N \mu_i \delta^{(2)}(x-x_i)\,, \quad \mu_i = \gamma \ell_P^2 \sqrt{j_i(j_i-1)}, \qquad j_i \in \mathbb{N}$



New observables:

[Freidel, Livine, DP, 2019]

- String vibration modes of the punctures generating a $U(1)^3$ Kac-Moody algebra.
- New momentum observable defining the boundary charge induced by the bulk invariance under 3D diffeomorphism.

In summary

- The thermodynamical BH entropy can be accounted for by considering the quantum microstates of the horizon which are distinguishable from the exterior of the hole.

- The entropy counting yields the Bekenstein-Hawking area law at leading order. Extensions in the literature: Inclusion of distortion for static IH, progress towards the inclusion of rotation, generalization to higher dimensional horizons and supersymmetry, addition of gauge fields, extension to different topologies.

In the end, the correctedness of the value of the Immirzi parameter predicted by the standard LQG BH entropy calculation can be addressed in a conclusive manner only through observational tests sensitive to the area gap (promising steps in this direction within a cosmological setting [Ashtekar, Gupt 2017]).

Alternatively, one can hope to have at least another independent theoretical model descending as close as possible from the full LQG framework, where the same numerical value is predicted by demanding a given outcome or value for an observable of physical relevance.

One of the main open issues

➢ Continuum/classical limit: The LQG Hilbert space is of a new (background-independent) kind, operators are regulated in a non-standard (background-independent) way. Does the theory that has been constructed so far indeed have General Relativity as its classical limit?

If space–time is fundamentally discrete, how does the continuum space–time we experience at low energies and macroscopic scales emerge from its fundamentally discrete building blocks, and end up being described by general relativity?

☞ Coherent states constructed out of Quantum Reduced Loop Gravity spin network states, which allow to implement symmetry reduction within the full theory: Expectation value of geometrical operators easier to compute, while retaining the information on the full graph structure. They allow to extract an effective dynamics that reproduce GR plus quantum corrections. [see second half of the course]

☞ New perspective based on the hypothesis that space–time is a sort of condensate of microscopic building blocks. Space, time and geometry would be emergent concepts, valid at macroscopic scales, whose emergence is the result of a collective dynamical process (a phase transition) of the fundamental DOF: Construction of quantum gravity condensates using the Group Field Theory formalism (second quantization of LQG) whose wave function is peaked on a few global observables. [see Oriti course]

Part II

BH singularity

- Quantum Reduced Loop Gravity program
- Semiclassical coherent states: spherical symmetry
- Effective Hamiltonian
- BH interior effective dynamics
- BH cosmology?

- Emanuele Alesci, Sina Bahrami and DP, **Quantum evolution of black hole initial data sets: Foundations**, Phys. Rev. D98 (2018) 4, 046014, [gr-qc/1807.07602];
- Emanuele Alesci, Sina Bahrami and DP, **Quantum gravity predictions for black hole interior geometry**, Phys. Lett. B797 (2019) 134828, [gr-qc/1904.12412];
- Emanuele Alesci, Sina Bahrami and DP, **Asymptotically de Sitter universe inside a Schwarzschild black hole**, Phys. Rev. D102 (2020) 6, 066010, [gr-qc/2007.06664];

The information paradox

Hawking radiation: QFT on a curved (BH) background ➡ BH radiate as thermal bodies

i.e. the spectrum does not depend on the structure of the body that collapsed to form the BH

The emitted quanta are in a mixed (thermal) state with
excitations which stay inside the hole:
Correlations between d.o.f. accessible outside the horizon and
d.o.f. unaccessible behind the horizon

♨ But what happens when the hole evaporates completely??

There is nothing left to be entangled with anymore!

➡ An initial pure state has evolved into a mixed state

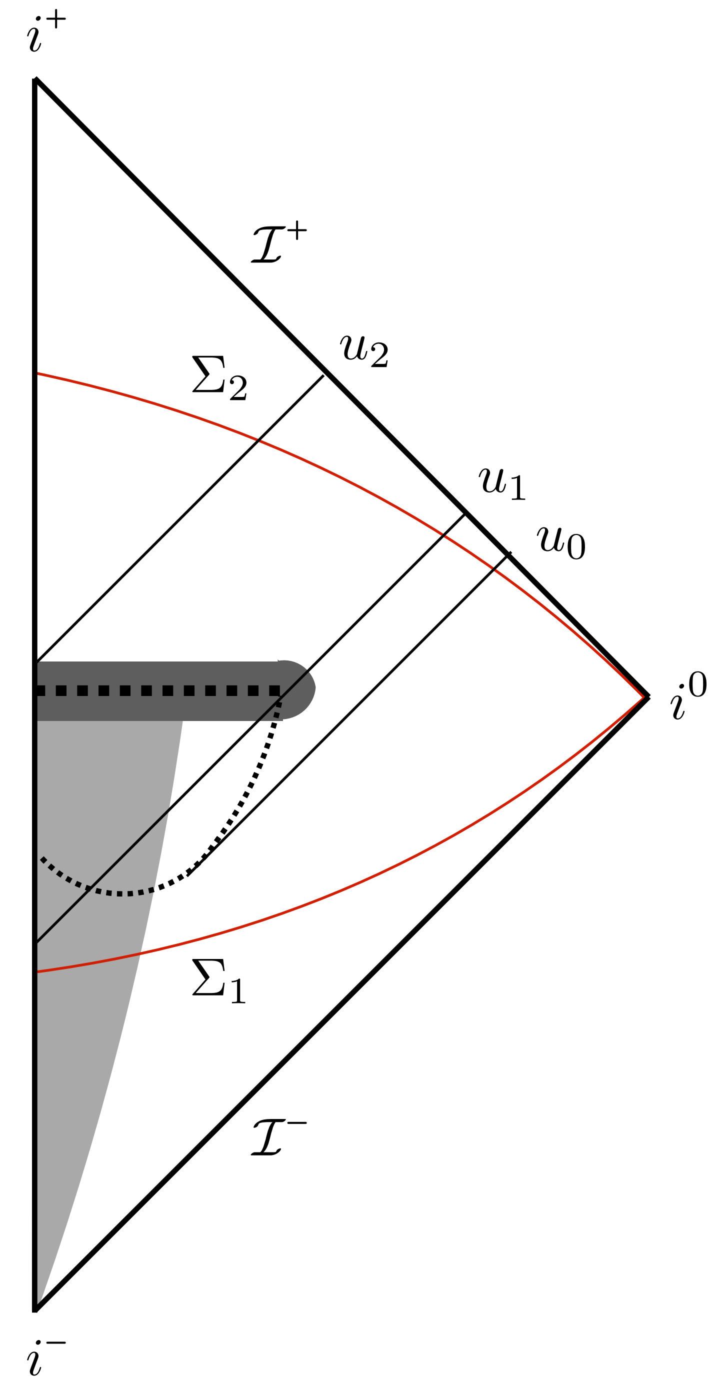
Contradiction: GR + QM lead to a non-unitary evolution of a BH!!

50 years old paradox!

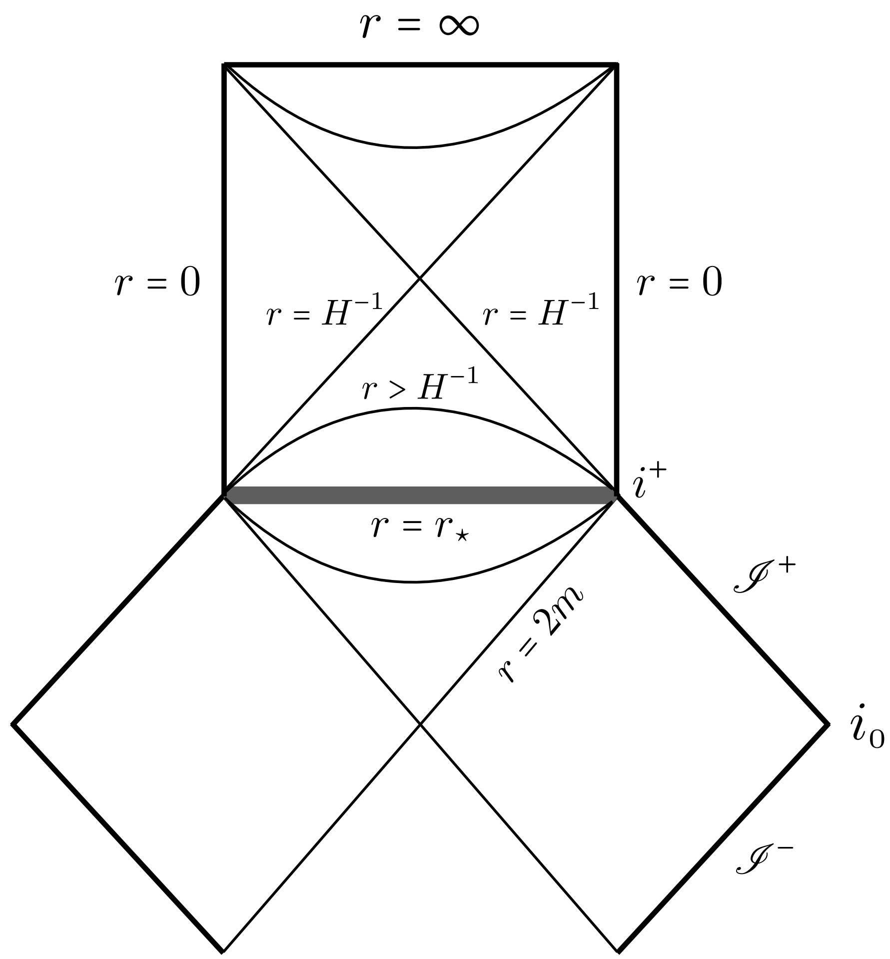
Despite their robustness, singularity theorems are reliable only in the regime where spacetime geometry is classical:

Quantum gravitational effects are expected to smooth out spacetime singularities

Conservative solutions to restore unitary evolution rely on elimination of singularities [Hossenfelder, Smolin, PRD 2010]

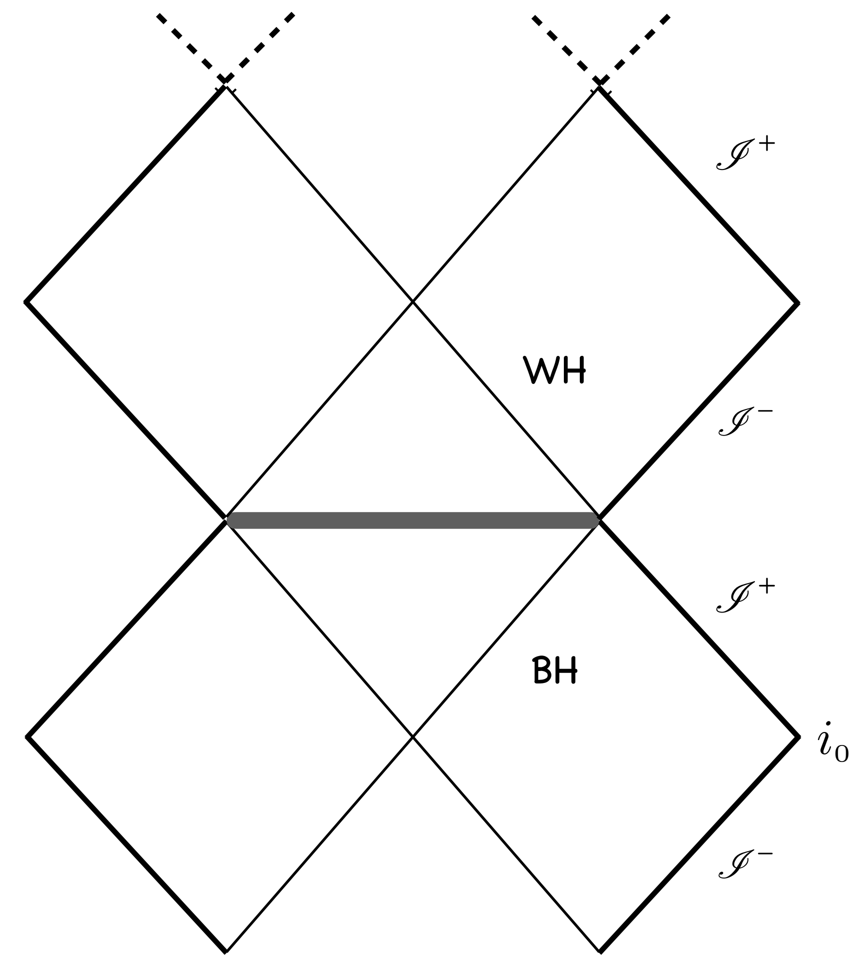


The global space-time causal structure according to the [Ashtekar, Bojowald, CQG 2006] paradigm. The black hole evaporation takes place according to semiclassical expectations until the horizon approaches Planck's area. The classical would-be-singularity is represented by the shaded region where quantum geometry fluctuations are large and no space-time picture is available. The space-time becomes classical to the future of this region: it emerges into a classical (essentially) flat background as required by energy-momentum conservation. Observers at the instant Σ_2 are in causal contact with the would-be-singularity which (in classical terms) appears to them as a naked singularity.



Closed de Sitter Universe inside

[Frolov, Markov, Mukhanov, PLB 1988];



Black hole / White hole bounce

[Hajicek, Kiefer, IJMPD 2001];
[Barcelo, Carballo-Rubio Garay, IJMPD 2014];
[Haggard, Rovelli, PRD 2015];

Examples of a wider category: "regular black holes"

[Ansoldi, 2008]; [Carballo-Rubio, Di Filippo, Liberati, Visser, PRD 2020]

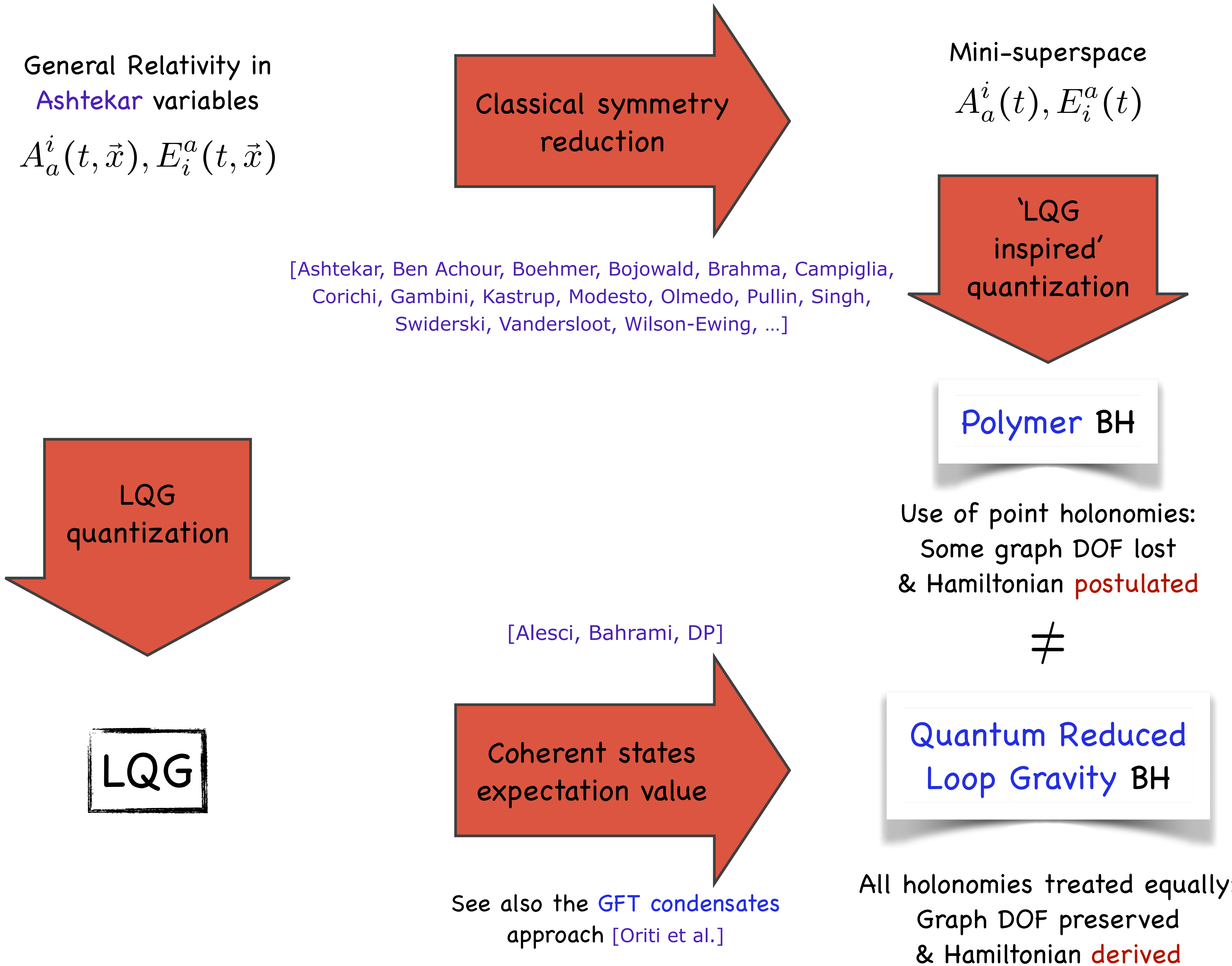
Problems related to the presence of an inner horizon

[Poisson, Israel, PRL 1989]; [Brown, Mann, Modesto, PRD 2011]; [Carballo-Rubio, Di Filippo, Liberati, Pacilio, Visser, JHEP 2018]

At the end of the day, only a full quantum gravity calculation can discriminate between different scenarios.

Loop Quantum Gravity (LQG) provides a non-perturbative framework to investigate BH singularity resolution.

However, the issue of symmetry presents itself again (in reverse):
What replaces the singularity can depend on an important choice:
Reduction or Quantization first?
The two in general do NOT commute!


General Relativity in
Ashtekar variables
$A^i_a(t,\vec{x}), E^a_i(t,\vec{x})$
Classical symmetry
reduction
Mini-superspace
$A^i_a(t), E^a_i(t)$
'LQG
inspired'
quantization
[Ashtekar, Ben Achour, Boehmer, Bojowald, Brahma, Campiglia,
Corichi, Gambini, Kastrup, Modesto, Olmedo, Pullin, Singh,
Swiderski, Vandersloot, Wilson-Ewing, ...]
Polymer BH
Use of point holonomies:
Some graph DOF lost
& Hamiltonian postulated
LQG
quantization
[Alesci, Bahrami, DP]
$\neq$
LQG
Coherent states
expectation value
Quantum Reduced
Loop Gravity BH
See also the GFT condensates
approach [Oriti et al.]
All holonomies treated equally:
Graph DOF preserved
& Hamiltonian derived

Quantum Reduced Loop Gravity program

- Symmetry reduced models have "smart" frames: systems of coordinates adapted to the symmetries
- In these coordinate systems the imposition of the symmetries allows to further simplify the form of the metric and Einstein Equations

Symmetry reduction in two steps:

1) Partial Gauge fixing of the metric (without symmetry reduction)
   Study the second class constraint system: Reduced Phase Space

   A. Solve the second class constraints
   B. Dirac Brackets
   C. Gauge Unfixing [Mitra, Rajaraman, Anishetty, Vytheeswaran]:
      - Ordinary Poisson Brackets for the non gauge fixed variables
      - Modified Constraints to preserve the gauge fixing during the evolution

2) Implement the symmetry reduction in the reduced phase space

Quantization

A. Quantize the classically Reduced Phase Space (with or without symmetry)
B. Quantize Dirac Brackets
C. QRLG: 4 steps

1. Impose the second class constraints weakly in the Full Hilbert Space:
   Selects the reduced states i.e. the quantum reduced phase space

2. Project the constraints defined in the full theory to represent the classical gauge unfixed constraints (preserving the gauge fixing)

3. Impose the symmetry reduction on the reduced states using coherent states

4. Define the effective constraints by taking the expectation value of the quantum reduced constraints on the symmetry reduced states

☞ Find quantum symmetry reduction compatible with given metrics

- Black Holes: Orthogonal gauge fixing

Quantum Reduced Loop Gravity: Black holes

The intrinsic metric on the spacelike hypersurfaces is:

$$d\sigma^2 = \Lambda^2 dr^2 + R^2\left(d\theta^2 + \sin^2\theta\ d\varphi^2\right)$$

$\Lambda(t,r), R(t,r)$ ADM phase space configuration variables

$$\begin{aligned} E &= E^r(t,r)\sin\theta\tau_3\partial_r + \left[E^1(t,r)\tau_1 + E^2(t,r)\tau_2\right]\sin\theta\partial_\theta + \left[E^1(t,r)\tau_2 - E^2(t,r)\tau_1\right]\partial_\varphi\,, \\ A &= A_r(t,r)\tau_3 dr + \left[A_1(t,r)\tau_1 + A_2(t,r)\tau_2\right]d\theta + \sin\theta\left[A_1(t,r)\tau_2 - A_2(t,r)\tau_1\right]d\varphi + \cos\theta\tau_3 d\varphi \end{aligned}$$

with Poisson brackets

$$\begin{aligned} \left\{A_r(t,r), E^r(t,r')\right\} &= 2G\gamma\,\delta(r-r')\,, \\ \left\{A_1(t,r), E^1(t,r')\right\} &= G\gamma\,\delta(r-r')\,, \\ \left\{A_2(t,r), E^2(t,r')\right\} &= G\gamma\,\delta(r-r') \end{aligned}$$

Orthogonal partial gauge fixing conditions:

$$\begin{aligned} E^r_I &= 0\,, \quad I = 1,2\,, \\ E^A_3 &= 0\,, \quad A = \theta,\phi \end{aligned}$$

Gauss constraint: $\kappa\tilde{G}_3[\alpha_3] = \int_\Sigma \mathrm{d}^3x\ \alpha_3\, {}^R G_3 = \int_\Sigma \mathrm{d}^3x\ \alpha_3\ \left[\partial_r E^r_3 + \epsilon_{3I}{}^{J} A^I_B E^B_J\right]$

Radial diffeomorphism constraint: $\kappa\tilde{H}_r[N^r] = \int_\Sigma \mathrm{d}^3x\, N^r\, ({}^R H_r + {}^e H_r)$

Hamiltonian constraint:

$$-\kappa\gamma^2\tilde{H}_E[N] = 2\int_\Sigma \mathrm{d}^3x\ \frac{N}{\sqrt{\det(E)}}\ ({}^R H_E + {}^R H_E^{ext})$$

$$-\kappa\gamma^2\tilde{H}_L[N] = (1+\gamma^2)\int_\Sigma \mathrm{d}^3x\, N\sqrt{\det(E)}\ {}^R R$$

Ricci scalar: $R = -\epsilon_{ijk} R^k_{ab} e^a_i e^b_j\,,\quad R^k_{ab} = 2\partial_{[a}\Gamma^k_{b]} + \epsilon^k{}_{lm}\Gamma^l_a\Gamma^m_b$

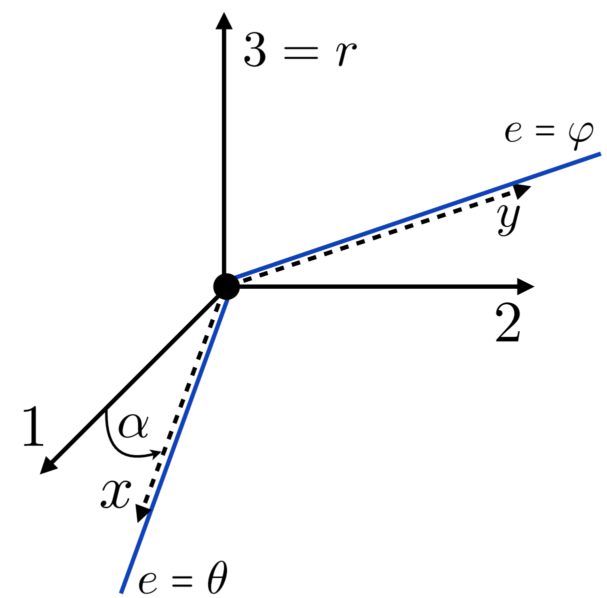
expression in terms of the fluxes and their derivatives only

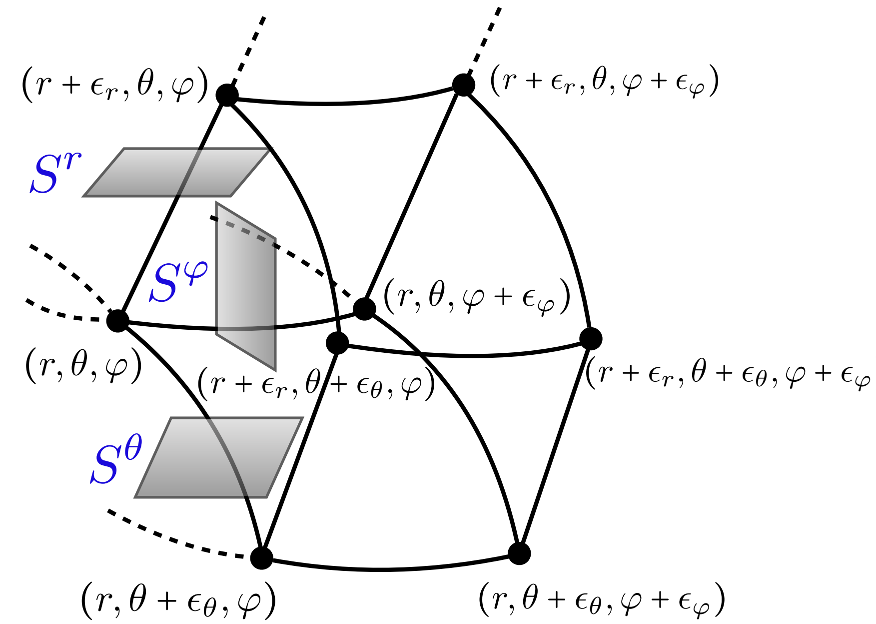
Symmetric subspace of the reduced phase space:

$$H^E_{sph}[N] = \frac{2}{\kappa}\int_\Sigma d^3x \frac{N(x)\sin\theta}{\sqrt{((E^1)^2+(E^2)^2)E^r}}\Big[2E^r A_r\left(E^1A_1+E^2A_2\right) + 2E^r(E^1A_2' - E^2A_1') + \Big((E^1)^2+(E^2)^2\Big)\Big((A_1^2+A_2^2)-1\Big)\Big]$$

Step 1:

$PH_{kin} = H^R$ $\qquad H^R = \oplus_\Gamma H^R_\Gamma$ $\qquad \Gamma =$ cubulation





Assign to each link in a given tangent direction the following basis elements

$$
\begin{aligned}
{}^x D^{j_x}_{\bar m_x \bar n_x}(g_\theta) &= \langle \bar m_x, \vec u_x | D^{j_x}(g_\theta) | \bar n_x, \vec u_x \rangle \,, \\
{}^y D^{j_y}_{\bar m_y \bar n_y}(g_\varphi) &= \langle \bar m_y, \vec u_y | D^{j_y}(g_\varphi) | \bar n_y, \vec u_y \rangle \,, \\
D^{j_z}_{\bar m_z \bar n_z}(g_r) &= \langle \bar m_z, j_z | D^{j_z}(g_r) | j_z, \bar n_z \rangle \,,
\end{aligned}
$$

where $\bar m, \bar n = \pm j$

$|\bar n_I, \vec u_I\rangle =$ SU(2) coherent state having maximum or minimum magnetic number along $\vec u_I$

2 orthogonal unit vectors in the arbitrary internal directions $I \in \{x, y\}$

$S^a =$ Orthogonal faces of the cube dual to a 6-valent node of the reduced graph (regularization of the reduced fluxes)

On H^R the gauge fixing conditions are weakly satisfied:

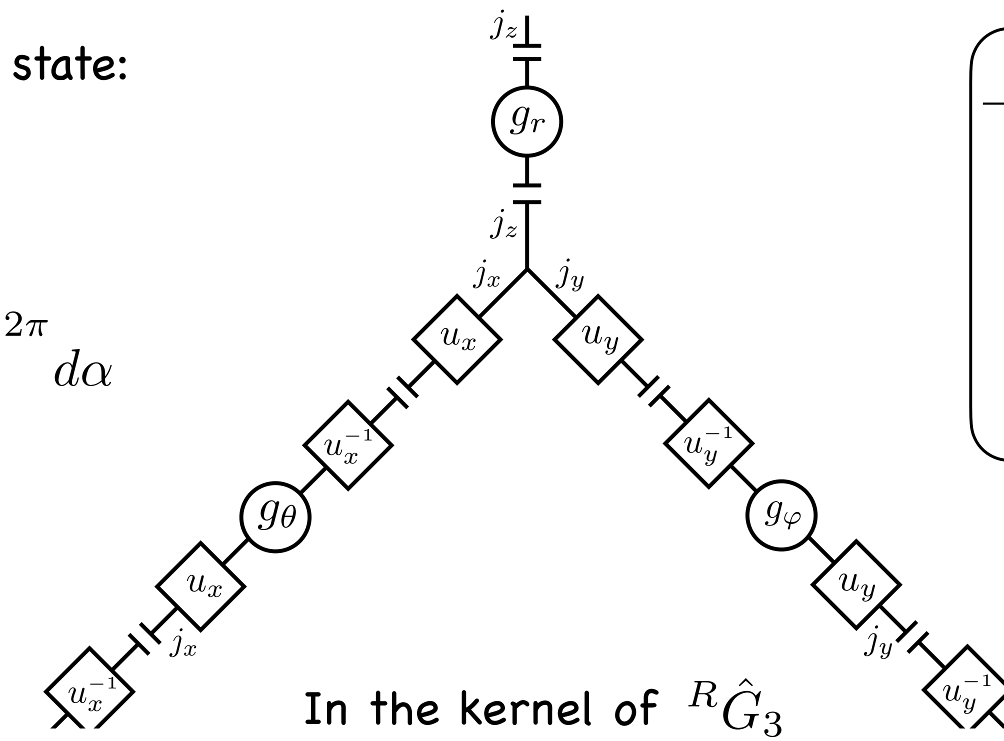
$$
\langle \hat E_I(S^r) \rangle = 0 \,, \quad I = 1, 2 \,,
$$
$$
\langle \hat E_3(S^A) \rangle = 0 \,, \quad A = \theta, \phi
$$

Basis elements: $\quad {}^{I}D^{j_I}_{\bar m_I \bar n_I}(g) = D^{j_I-1}_{\bar m_I m}(u_I)\,D^{j_I}_{mn}(g)\,D^{j_I}_{n\bar n_I}(u_I)$

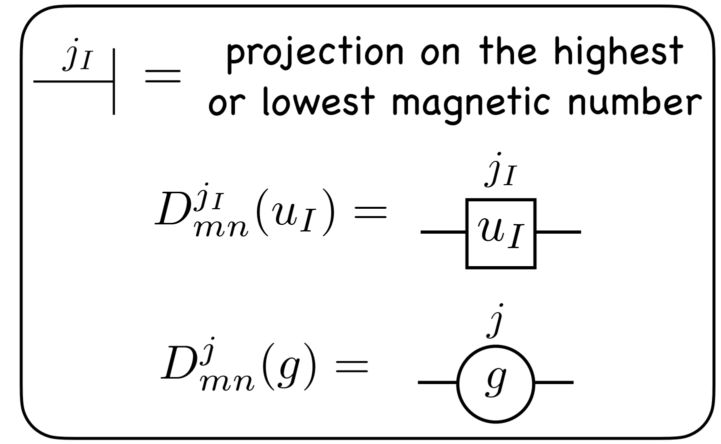
group element that rotates the 3-axis into $\vec u_I$

- Reduced 3-valent vertex state:

$$|v_3^R(j)\rangle = \int_0^{2\pi} d\alpha$$



In the kernel of ${}^{R}\hat G_3$



- Reduced flux operators:

$$\begin{aligned} {}^{R}\hat E_i(S^r) &= \hat P^z \hat E_i(S^r)\hat P^z\,, \\ {}^{R}\hat E_i(S^\theta) &= \hat P^x \hat E_i(S^\theta)\hat P^x\,, \\ {}^{R}\hat E_i(S^\varphi) &= \hat P^y \hat E_i(S^\varphi)\hat P^y \end{aligned}$$

where

$$\begin{aligned} \hat P^z &= \sum_{\bar m_z = \pm j_z} |j_z, \bar m_z\rangle\langle \bar m_z, j_z|\,, \\ \hat P^I &= \sum_{\bar m_I = \pm j_I} |\vec u_I, \bar m_I\rangle\langle \bar m_I, \vec u_I| \end{aligned}$$

✦ This procedure allows one to work with the complete structure of the full theory, consisting of quantum states of polymeric nature labelled by graphs and SU(2) representations

✦ At the same time, the reduced flux operators are diagonal on the reduced quantum states!

Reduced volume operator: $\quad {}^{R}\hat V(v)|v_3^R(j)\rangle = (\kappa\gamma)^{\frac{3}{2}}\sqrt{|j_x j_y j_z|}\,|v_3^R(j)\rangle$

• Step 2:

Extended Hamiltonian constraint (preserving the gauge fixing)

Let us focus on the reduced Euclidean Hamiltonian: $-\kappa\gamma^2 \tilde{H}_E[N] = 2\int_\Sigma \mathrm{d}^3x \, \frac{N}{\sqrt{\det(E)}} \left({}^R H_E + {}^R H_E^{ext}\right)$

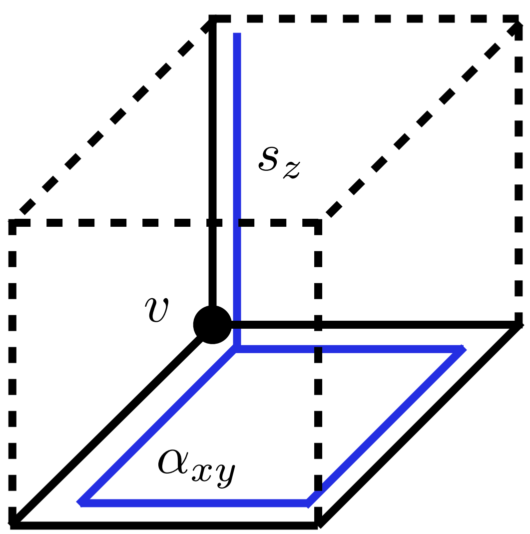
Using Thiemann's techniques ➥

$$ {}^R\hat{H}^E_\square[N] = -\frac{2}{\kappa^2\gamma} N(v)\epsilon^{ijk}\mathrm{tr}\left[\left({}^R\hat{g}_{\alpha_{ij}} - {}^R\hat{g}^{-1}_{\alpha_{ij}}\right){}^R\hat{g}^{-1}_{s_k}\left[{}^R\hat{g}_{s_k}, {}^R\hat{V}(v)\right]\right] $$

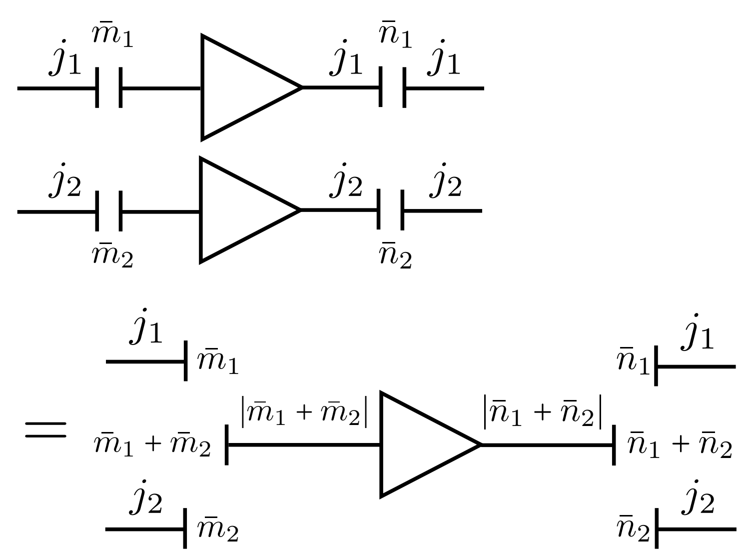
Orthogonal now!

$i,j,k = z,x,y \qquad s_k = \ell_z, \ell_x, \ell_y \qquad \alpha_{ij} = \ell_i \circ \ell_j \circ \ell_i^{-1} \circ \ell_j^{-1}$

loop in the plane (ij)

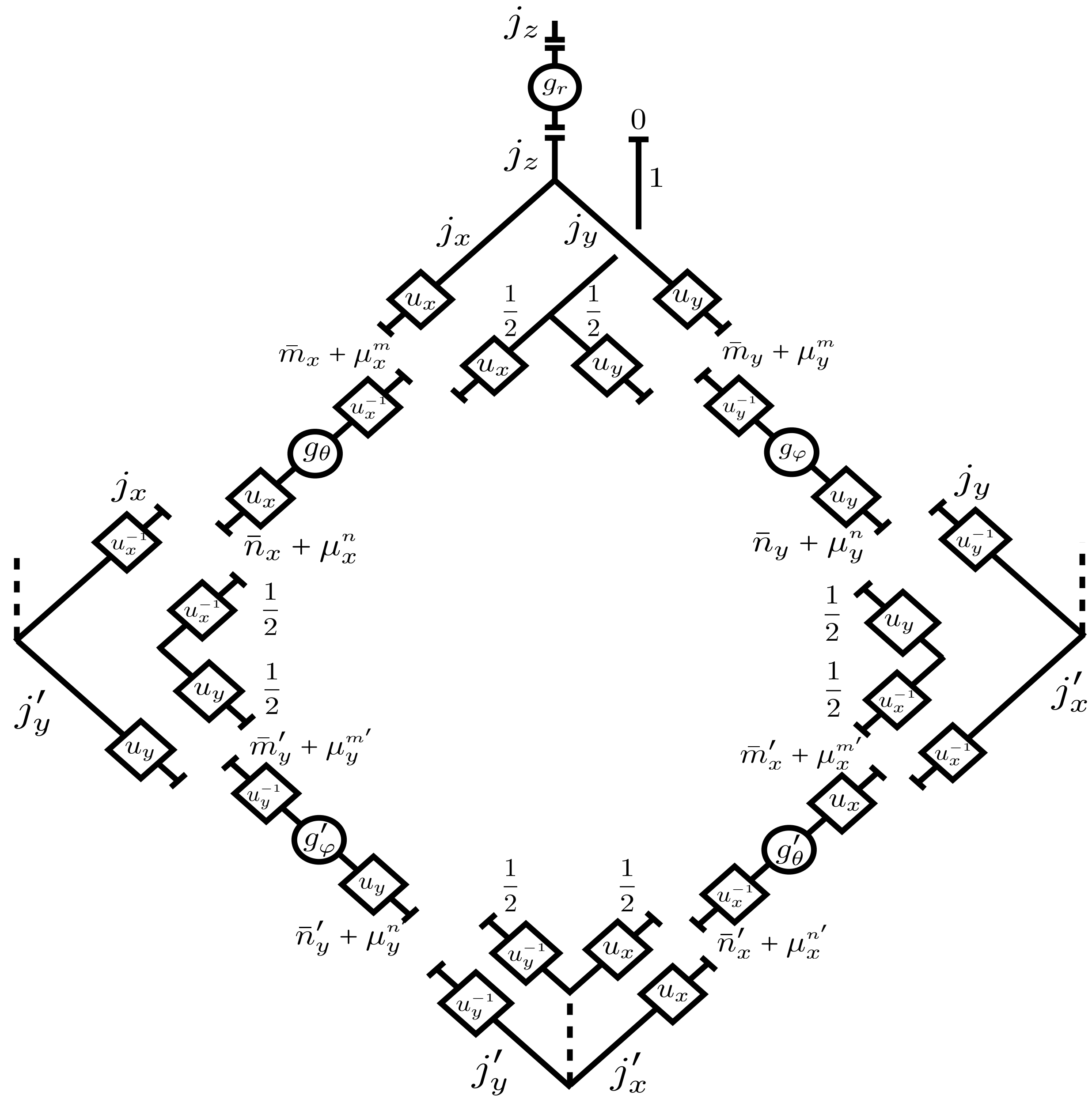


By means of the ``reduced" recoupling rule:



its action can be computed in a straightforward way

$$ {}^{R}\hat{H}^{E}_{\Box_{xy}}[N]|v^{R}_{\Box}\rangle = -8\pi\sqrt{\frac{\gamma}{\kappa}}N(v) \sum_{\mu,\mu^m_x,\mu^n_x,\mu^m_y,\mu^n_y,\mu^{m'}_x,\mu^{n'}_x,\mu^{m'}_y,\mu^{n'}_y=\pm 1/2} s(\mu)\sqrt{j_x j_y (j_z+\mu)} $$



In ${}^{R}\hat{H}^{ext}_{\Box}[N]$, ${}^{R}\hat{H}^{L}_{\Box}[N]$ only the reduced fluxes and their derivatives appear:

$$ \partial_a {}^{R}\hat{E}_i(S^b(v)) \equiv {}^{R}\hat{E}_i(S^b(v+\epsilon_a)) - {}^{R}\hat{E}_i(S^b(v))\,, $$

$$ \partial^2_a {}^{R}\hat{E}_i(S^b(v)) \equiv {}^{R}\hat{E}_i(S^b(v+2\epsilon_a)) - 2\,{}^{R}\hat{E}_i(S^b(v+\epsilon_a)) + {}^{R}\hat{E}_i(S^b(v)) $$

- Step 3:

Coherent semiclassical states

[Hall, Thiemann, Winkler, Sahlmann, Bahr]

$$\psi_G^\lambda(g_\ell) = \sum_{j_\ell}(2j_\ell+1)e^{-\frac{\lambda}{2}j_\ell(j_\ell+1)}\chi_{j_\ell}(g_\ell^{-1}G)$$

$\lambda =$ positive real number controlling the fluctuations of the state

$\chi_{j_\ell} =$ SU(2) character in the irreducible representation j_ℓ

$$G = g\exp\left(i\frac{\lambda}{\kappa\gamma}E_i(S^\ell)\tau^i\right)$$

Codes the extrinsic curvature

Codes the intrinsic geometry

SL(2,C) group element encoding the classical geometry around which we want to peak

e.g. $$\psi_G^\lambda(g_r) = \sum_{j_z=0}^{\infty}\sum_{\bar{m}_z}(2j_z+1)e^{-\frac{\lambda}{2}j_z(j_z+1)}e^{\lambda\bar{m}_z\frac{\delta_r^2 E^r\sin\theta}{\kappa\gamma}}D^{j_z}_{\bar{n}_z\bar{m}_z}(e^{\epsilon_r A_r\tau_3})\,D^{j_z}_{\bar{m}_z\bar{n}_z}(g_r^{-1})$$

for $j_x, j_y, j_z \gg 1$ the coherent states become Gaussian weights for the fluxes peaked around the semiclassical values $\widetilde{j_\ell} = \delta_\ell^2 j_\ell^0$ with

$$j_x^0 = \frac{\sin\theta\Lambda R}{\kappa\gamma} = \frac{\sin\theta}{\kappa\gamma}\left(E_1\cos\alpha + E_2\sin\alpha\right)\,,$$

$$j_y^0 = \frac{\Lambda R}{\kappa\gamma} = \frac{1}{\kappa\gamma}\left(E_1\cos\alpha + E_2\sin\alpha\right)\,,$$

$$j_z^0 = \frac{\sin\theta R^2}{\kappa\gamma} = \frac{\sin\theta E^r}{\kappa\gamma}$$

$$\delta_x^2 = \epsilon_r\epsilon_\varphi\,,$$
$$\delta_y^2 = \epsilon_r\epsilon_\theta\,,$$
$$\delta_z^2 = \epsilon_\theta\epsilon_\varphi$$

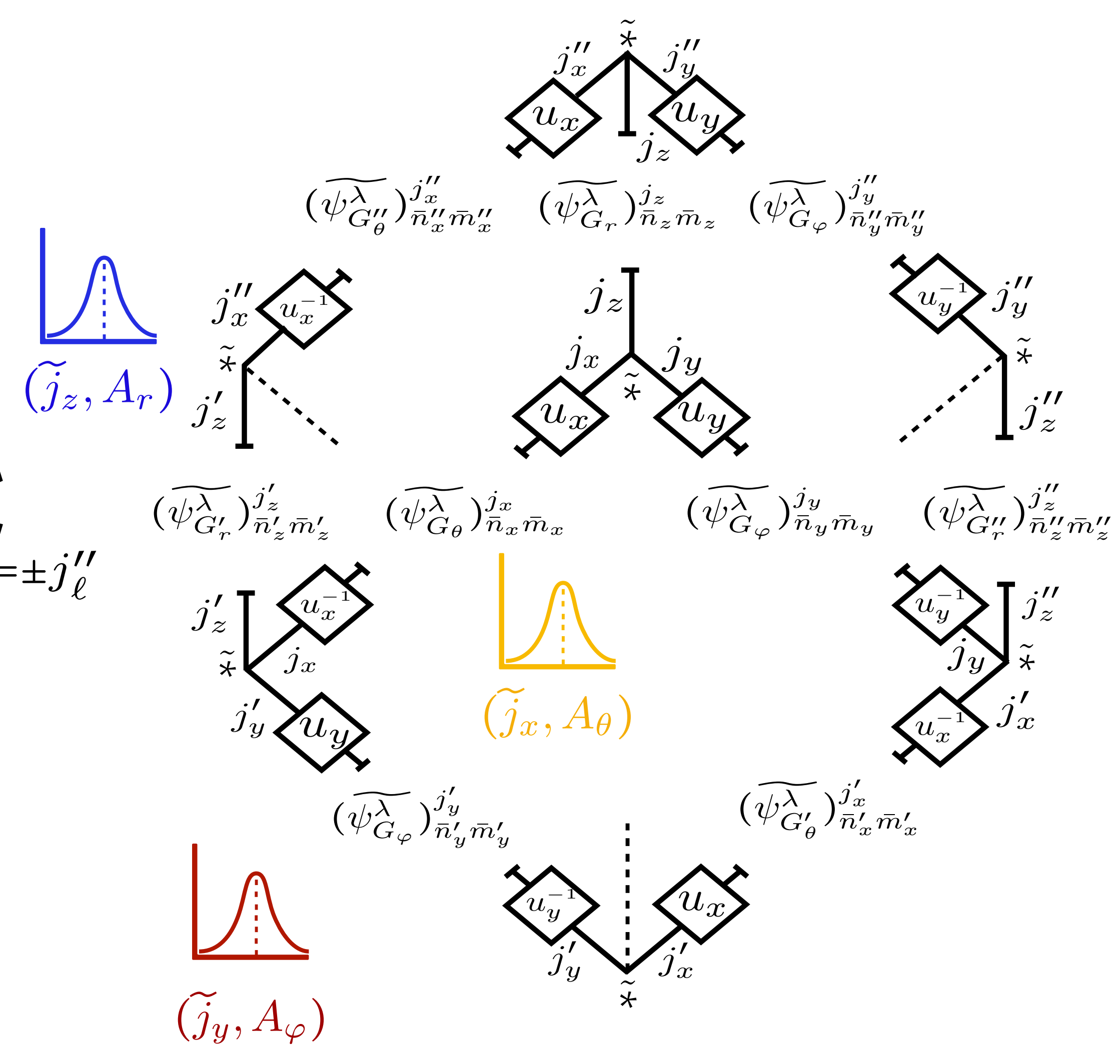
If we open up the character, we can write the quantum reduced coherent states in the compact notation:

$$\psi_G^{\lambda}(g_\ell) = \sum_{j_\ell=0}^{\infty} \sum_{\bar{m}_\ell,\bar{n}_\ell=\pm j_\ell} (2j_\ell+1)(\psi_G^{\lambda})^{j_\ell}_{\bar{n}_\ell \bar{m}_\ell}\, {}^{\ell}D^{j_\ell}_{\bar{m}_\ell \bar{n}_\ell}(g_\ell^{-1})$$

`Gaussian' wave-function

By contracting with reduced intertwiners and introducing proper normalizations, we obtain the normalized quantum reduced coherent state

$$|\widetilde{\psi^{\lambda}_{\Box}}\rangle = \prod_{\ell=x,y,z} \sum_{j_\ell,j'_\ell,j''_\ell=0}^{\infty} \sum_{\bar{m}_\ell,\bar{n}_\ell=\pm j_\ell} \sum_{\bar{m}'_\ell,\bar{n}'_\ell=\pm j'_\ell} \sum_{\bar{m}''_\ell,\bar{n}''_\ell=\pm j''_\ell}$$



$$|\widetilde{v^{R}_{\Box}}\rangle$$

$$
\begin{aligned}
-2\kappa\gamma^2\langle\widetilde{\psi^\lambda_\square}|{}^R\hat{H}^E_\square + {}^R\hat{H}^{ext}_\square + {}^R\hat{H}^L_\square|\widetilde{\psi^\lambda_\square}\rangle = \; & 4\sqrt{E^r}\Bigg[\epsilon_\theta\Bigg(\cos\Bigg[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\sin\theta\epsilon_\varphi\Bigg]\sin\Bigg[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\sin\theta\epsilon_\varphi\Bigg] \\
& \times\frac{\Big(\sin\big[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\big]A_1(r+\epsilon_r)+\cos\big[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\big]A_2(r+\epsilon_r)\Big)}{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}} \\
& -\sin\Bigg[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\sin\theta\epsilon_\varphi\Bigg]\cos\Bigg[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\sin\theta\epsilon_\varphi\Bigg] \\
& \times\frac{\Big(\sin\big[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\big]A_1(r)+\cos\big[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\big]A_2(r)\Big)}{\sqrt{A_1^2(r)+A_2^2(r)}}\Bigg) \\
& +\epsilon_\varphi\sin\theta\Bigg(\cos\Bigg[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\epsilon_\theta\Bigg]\sin\Bigg[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\epsilon_\theta\Bigg] \\
& \times\frac{\Big(\sin\big[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\big]A_1(r+\epsilon_r)+\cos\big[\frac{A_r(r)+A_r(r+\epsilon_r)}{2}\epsilon_r\big]A_2(r+\epsilon_r)\Big)}{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}} \\
& -\sin\Bigg[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\epsilon_\theta\Bigg]\cos\Bigg[\frac{\sqrt{A_1^2(r+\epsilon_r)+A_2^2(r+\epsilon_r)}}{2}\epsilon_\theta\Bigg] \\
& \times\frac{\Big(\sin\big[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\big]A_1(r)+\cos\big[\frac{A_r(r+\epsilon_r)-A_r(r)}{2}\epsilon_r\big]A_2(r)\Big)}{\sqrt{A_1^2(r)+A_2^2(r)}}\Bigg)\Bigg] \\
& +2\epsilon_r\frac{E^1}{\sqrt{E^r}}\sin\Big[\sqrt{A_1^2(r)+A_2^2(r)}\epsilon_\theta\Big]\sin\Bigg[\frac{\sqrt{A_1^2(r)+A_2^2(r)}}{2}\left(\sin\theta+\sin\left(\theta+\epsilon_\theta\right)\right)\epsilon_\varphi\Bigg] \\
& -2\gamma^2\frac{\epsilon_r\epsilon_\varphi}{\epsilon_\theta}\frac{E^1(r)}{\sqrt{E^r(r)}}\left(\sin\left(\theta+2\epsilon_\theta\right)-2\sin\left(\theta+\epsilon_\theta\right)+\sin\theta\right) \\
& -(1+\gamma^2)\frac{\epsilon_\theta\epsilon_\varphi}{\epsilon_r}\frac{\sin\theta}{2\sqrt{E^r(r)}(E^1(r))^2} \\
& \times\Big[E^1(r)\Big(\left(E^r(r+\epsilon_r)-E^r(r)\right)^2+4E^r(r)\left(E^r(r+2\epsilon_r)-2E^r(r+\epsilon_r)+E^r(r)\right)\Big) \\
& -4E^r(r)\left(E^r(r+\epsilon_r)-E^r(r)\right)\left(E^1(r+\epsilon_r)-E^1(r)\right)\Big]
\end{aligned}
$$

- Step 4:

Valid for any foliation!

Semiclassical limit: $$\lim_{\epsilon_r,\epsilon_\theta,\epsilon_\varphi\to 0}\sum_\square\langle\widetilde{\psi^\lambda_\square}|\left({}^R\hat{H}^E_\square[N]+{}^R\hat{H}^{ext}_\square[N]+{}^R\hat{H}^L_\square[N]\right)|\widetilde{\psi^\lambda_\square}\rangle\approx H_{sph}[N]+o(\epsilon^4)$$

In summary

- Applying the QRLG framework, we implemented a quantization program that is aimed at identifying a symmetric sector at the quantum level, thus reverting the process of symmetry reduction and quantization that is frequently adopted in other existing treatments of quantum black holes.

- The main result of this paper is the construction of an effective Hamiltonian that can now be used to evolve black hole initial data sets while incorporating quantum corrections. The classical data entering the coherent states—that, if sharply peaked, are the best candidates to describe classical geometries—can now be seen as the initial data set to be evolved with the effective Hamiltonian.

- The importance of this construction lies in the fact that it is not tied to a particular choice of foliation, allowing one to treat on equal footing various sets of coordinate systems such as horizon penetrating coordinates or coordinates restricted to the interior or exterior of the event horizon of a black hole.

Interior effective dynamics

We are interested in the effective description for the interior geometry of a spherically symmetric black hole, namely we restrict our search for quantum geometries to metrics in the minisuperspace of the form

$$ds^2 = -N(\tau)^2 d\tau^2 + \Lambda(\tau)^2 dx^2 + R(\tau)^2 d\Omega^2$$

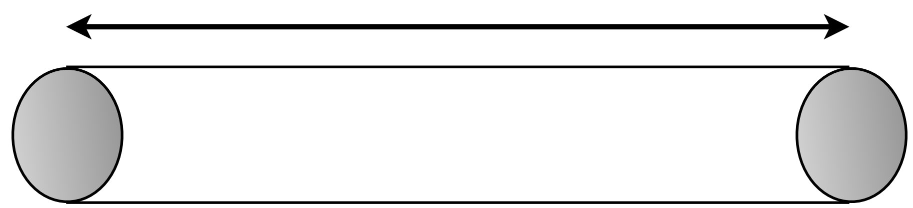
Homogeneous Cauchy slices with topology $\mathbb{R} \times S^2$

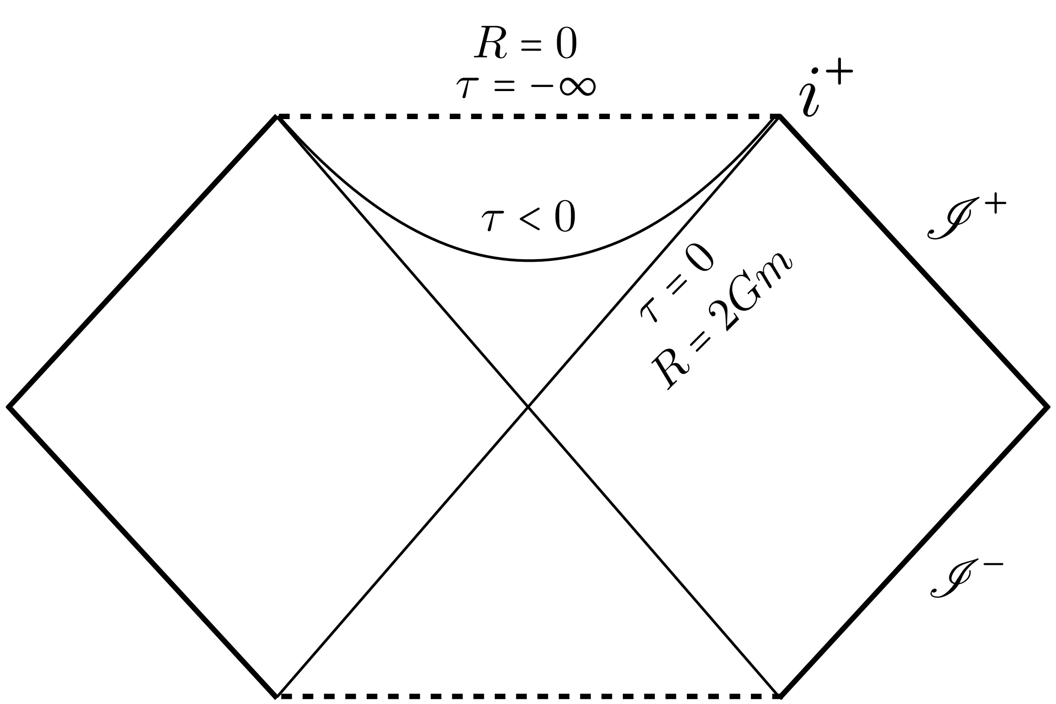
$$N_c = -\frac{R^2}{2G^2 m P_\Lambda}\,, \qquad -\infty < \tau < 0$$

$\tau = 0 \rightarrow$ BH horizon

$\tau = -\infty \rightarrow$ classical singularity

$x \in [0, L_0]$ infrared cut-off





- ADM phase space:

$$E^x = R^2\,, \quad E^1 = R\Lambda\,,$$

$$A_x = -\frac{\gamma G}{R}\left(P_R - \frac{\Lambda}{R} P_\Lambda\right)\,, \quad A_1 = -\frac{\gamma G}{R} P_\Lambda$$

$$\rightarrow \qquad \{R, P_R\} = \{\Lambda, P_\Lambda\} = 1/L_0$$

classical trajectories:

$$R_c(\tau) = 2Gm\, e^{\tau/2Gm}\,, \quad \Lambda_c(\tau) = \sqrt{e^{-\tau/2Gm} - 1}$$

★ Effective Hamiltonian:

$$H_{\rm eff} = -\frac{L_0}{4\gamma^2 G\epsilon_x\epsilon^2}\left[\epsilon R\sin\left(\frac{\gamma G\epsilon_x[P_R R - P_\Lambda\Lambda]}{R^2}\right)\left\{2\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)+\pi H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\}\right.$$
$$\left. + \epsilon_x\Lambda\left\{8\gamma^2\cos(\epsilon)\sin\left(\frac{\epsilon}{2}\right)^2 + \pi\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right\}\right]$$

Classical limit: $\hbar\to 0\,,\quad \epsilon,\epsilon_x\to 0\,,\quad H_{\rm eff}\to H_c = L_0\left(\frac{G\Lambda P_\Lambda^2}{2R^2}-\frac{GP_RP_\Lambda}{R}-\frac{\Lambda}{2G}\right)$

Struve function of order 0:

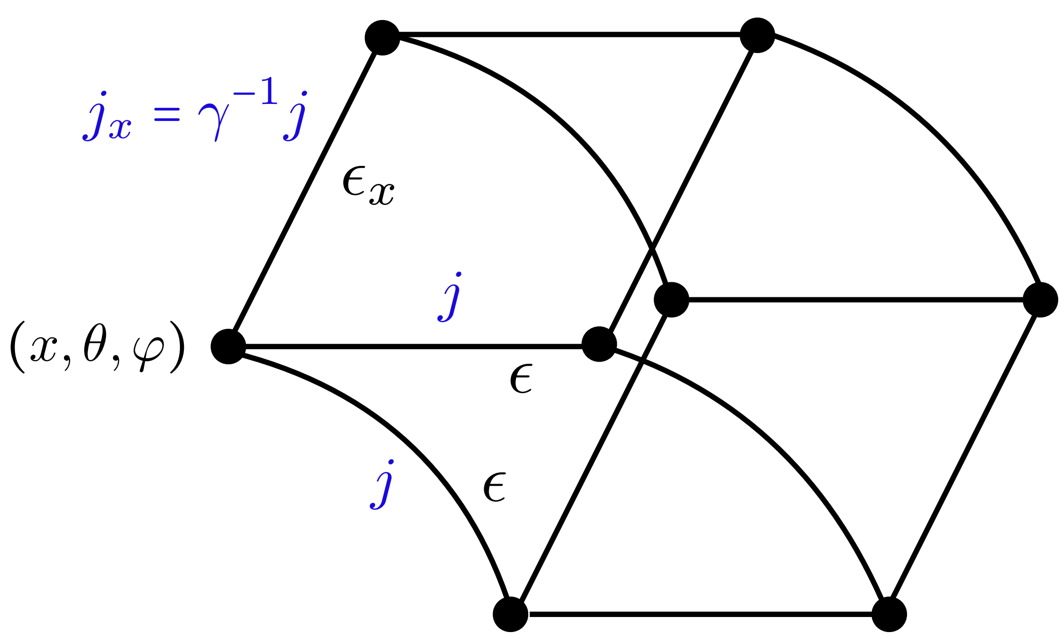
$$H_0[z] = \sum_{k=0}^{\infty}\frac{(-1)^k}{\left(\Gamma\left[k+\frac{3}{2}\right]\right)^2}\left(\frac{z}{2}\right)^{2k+1}$$

main departure from the minisuperspace quantization models: encodes the DOF associated with the 2-sphere graph structure which are frozen in all the treatments relying on the use of point holonomies.

➥ No inner horizon in the effective interior geometry

- Quantum parameters:

$$\epsilon_\theta = \epsilon_\varphi := \epsilon = \frac{2\pi}{\mathcal{N}}\,,\quad \epsilon_x := \frac{L_0}{\mathcal{N}_x}\,,\quad \mathcal{N},\mathcal{N}_x \gg 1$$



$$A(R) = 4\pi R^2 \simeq 4\pi\gamma\ell_P^2 j_x\mathcal{N}^2$$

$$V(\Sigma) = 8\pi L_0 R^2\Lambda \simeq 4(8\pi\gamma\ell_P^2)^{3/2} j\sqrt{j_x}\mathcal{N}_x\mathcal{N}^2$$

➥
$$\epsilon = \frac{\alpha}{R}\,,\qquad \alpha := 2\pi\sqrt{\gamma j_x}\,\ell_P\,,$$
$$\epsilon_x = \frac{\beta}{\Lambda}\,,\qquad \beta := \frac{4\sqrt{8\pi\gamma}\,j\,\ell_P}{\sqrt{j_x}}$$

- Effective Hamilton evolution eq.s:

Initial data on $\Sigma \rightarrow$ event horizon (the classical solution satisfies the effective EOM with vanishingly small error)

$$R_c(\tau) = 2Gm\, e^{\tau/2Gm}, \quad P_{R_c}(\tau) = \frac{1}{2G}\left[2 - e^{-\tau/2Gm}\right],$$
$$\Lambda_c(\tau) = \sqrt{e^{-\tau/2Gm} - 1}, \quad P_{\Lambda_c}(\tau) = -2m\, e^{\tau/4Gm}\sqrt{1 - e^{\tau/2Gm}}$$

$$\dot{R} = \{R, H_{\text{eff}}[N]\} = \frac{1}{L_0}\frac{\partial H_{\text{eff}}[N]}{\partial P_R},$$
$$\dot{P}_R = \{P_R, H_{\text{eff}}[N]\} = -\frac{1}{L_0}\frac{\partial H_{\text{eff}}[N]}{\partial R},$$
$$\dot{\Lambda} = \{\Lambda, H_{\text{eff}}[N]\} = \frac{1}{L_0}\frac{\partial H_{\text{eff}}[N]}{\partial P_\Lambda},$$
$$\dot{P}_\Lambda = \{P_\Lambda, H_{\text{eff}}[N]\} = -\frac{1}{L_0}\frac{\partial H_{\text{eff}}[N]}{\partial \Lambda} \quad \leftrightarrow \quad H_{\text{eff}}[N] = 0$$

$$N = -\frac{\gamma\epsilon R}{Gm\left[\sin\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right) + \frac{\pi}{2}H_0\left(\frac{\gamma G\epsilon P_\Lambda}{R}\right)\right]}$$

The effective dynamics depends on 2 free parameters: j, γ

The solutions to the evolution equations above present three different regimes labelled uniquely by:

$$\boxed{\eta := \frac{\alpha}{\beta} = \frac{\sqrt{2\pi}}{8\gamma}}$$

$\eta < 1, \eta = 1, \eta > 1$

All curvature invariants have a mass-independent bound ☞ Singularity resolution

In all cases, the classical black hole singularity is replaced by a homogeneous expanding Universe.

However, various aspects of the post-bounce effective geometry depend on the choice of γ

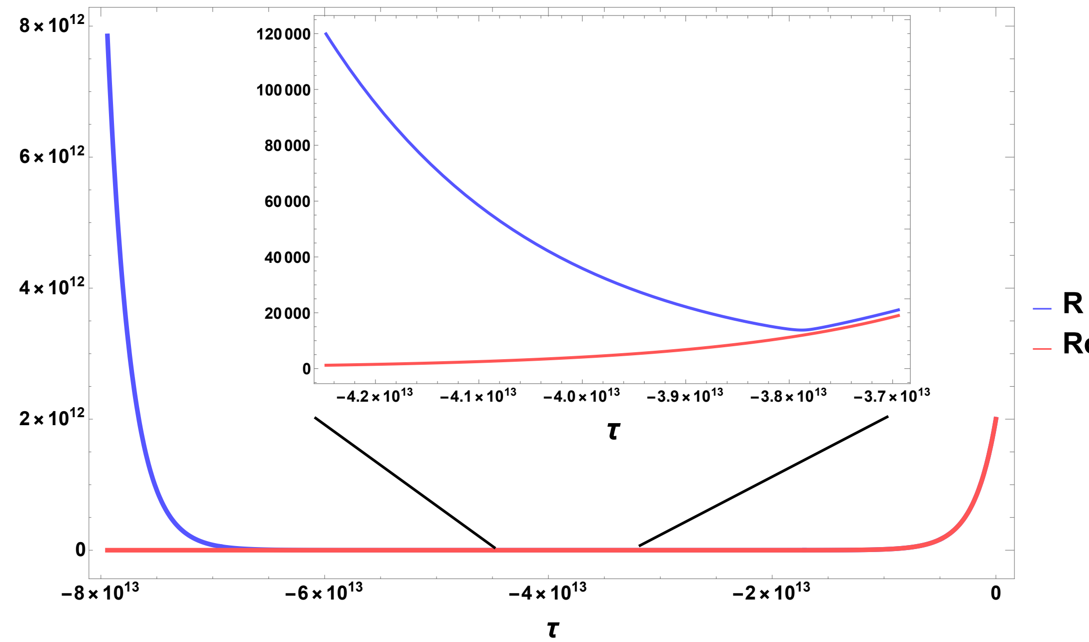
Kretschmann scalar

$$\mathcal{K}_c := R_{abcd}R^{abcd} = \frac{3\, e^{-\frac{3\tau}{Gm}}}{4(Gm)^4}$$

QG regime: $\mathcal{K}_c \sim 1/\ell_p^4\,,$ for $\tau_\star = \frac{Gm}{3}\log\left[\frac{3\ell_p^4}{(4G^4m^4)}\right]$

- The effective metric function $R(\tau)$ has a qualitatively similar behavior in all three regimes:

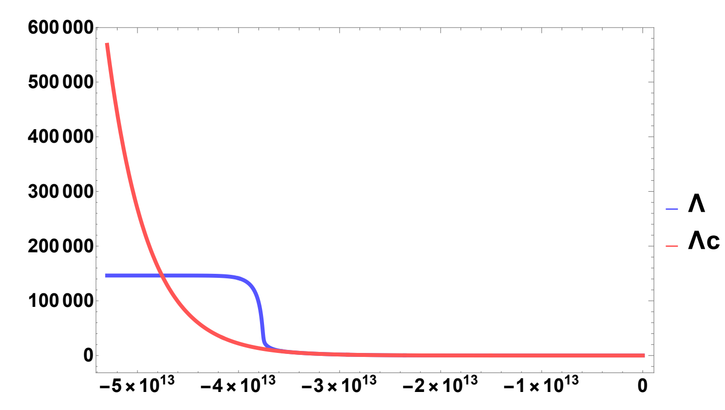
It follows the classical trajectory Rc till the quantum region $\tau \sim \tau_\star$ is reached, at which point a bounce occurs and it starts increasing exponentially



Numerical Solution: $m = 10^{12}m_p\,,\quad \tau_\star \approx -3.7\times10^{13}$

- The effective metric function $\Lambda(\tau)$ has a has a different asymptotic behavior in each sector:

  - It vanishes exponentially for $\eta < 1$;
  - It approaches a constant value for $\eta = 1$;
  - It grows exponentially for $\eta > 1$

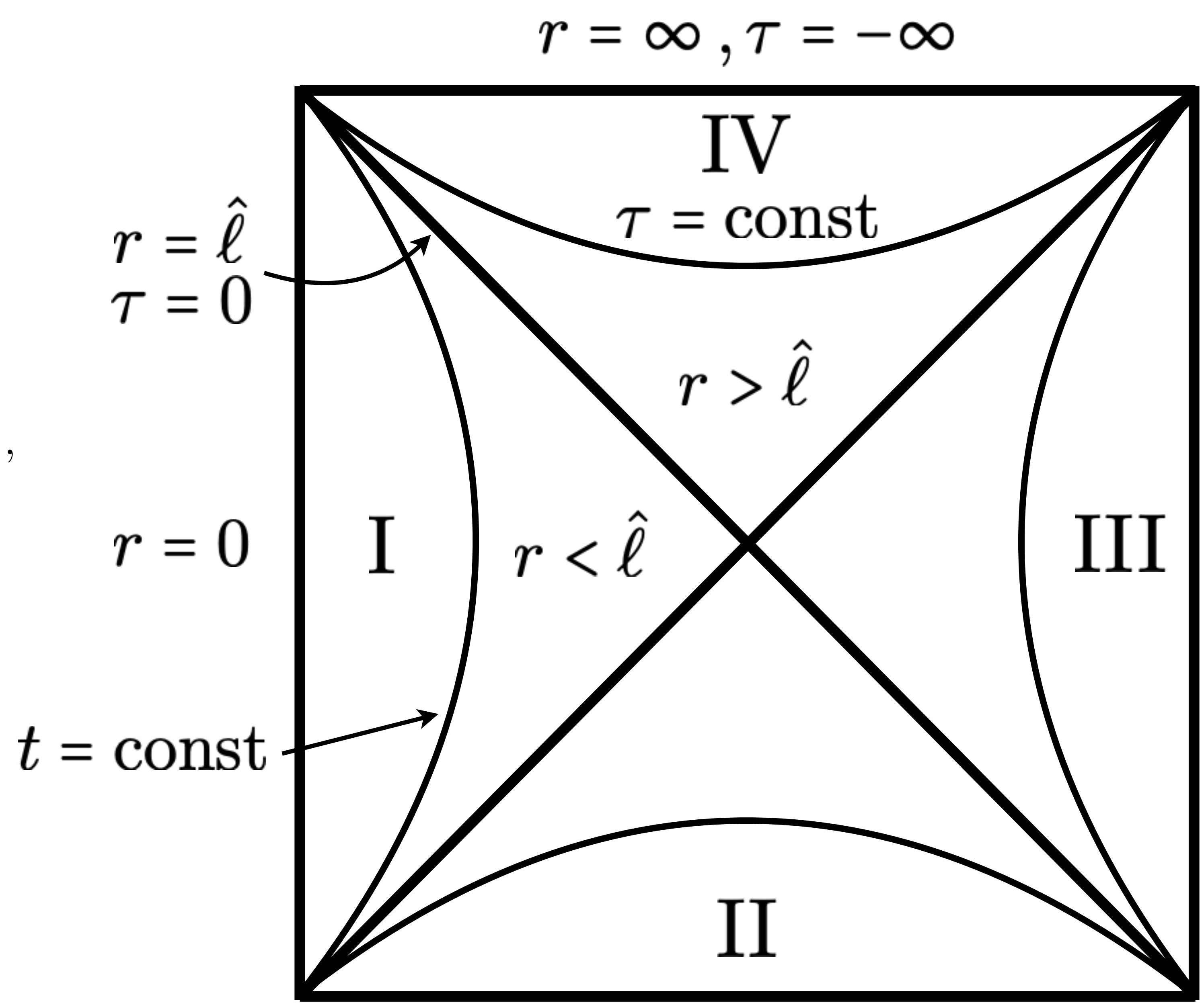


$$t = x\,, \quad \int \frac{dr}{\sqrt{\frac{r^2}{\hat\ell^2} - 1}} = \int N(\tau)\, d\tau \quad \overset{N = \text{const}}{\longrightarrow} \quad r = \hat\ell \cosh\left(\frac{N\tau}{\hat\ell}\right), \quad \ell := \frac{\hat\ell}{N}$$

$$ds^2 = -N^2 d\tau^2 + \sinh^2\left(\frac{\tau}{\ell}\right) dx^2 + N^2\ell^2 \cosh^2\left(\frac{\tau}{\ell}\right) d\Omega^2, \qquad \lambda = \frac{3}{N^2\ell^2}$$

$$ds^2 = -\Big(1 - \frac{r^2}{\hat\ell^2}\Big) dt^2 + \frac{dr^2}{1 - \frac{r^2}{\hat\ell^2}} + r^2 d\Omega^2\,,$$

$$\lambda = \frac{3}{\hat\ell^2}$$



Is there a numerical value of the Immirzi parameter such that a de Sitter Universe is recovered in the post-bounce asymptotic region?

Desiderata: $$ds^2 = -N_0^2 d\tau^2 + \sinh^2\left(\frac{\tau}{\ell}\right) dx^2 + N_0^2 \ell^2 \cosh^2\left(\frac{\tau}{\ell}\right) d\Omega^2\,, \qquad \lambda = \frac{3}{N_0^2 \ell^2}$$

★ The strategy

Start with the following asymptotic expansion:

$$z := \exp\left(-\tau/\ell\right)$$

with ℓ some length scale

$$\lim_{z\to\infty} N(z) = N_0 + \frac{N_1}{z} + \frac{N_2}{z^2} + \mathcal{O}(z^{-3})$$

$$\lim_{z\to\infty} \Lambda(z) = \frac{1}{2}\left(z - \frac{1}{z}\right) + \mathcal{O}(z^{-2}),$$

$$\lim_{z\to\infty} R(z) = \frac{N_0 \ell}{2}\left(z + \frac{1}{z}\right) + \mathcal{O}(z^{-2})$$

$$\lim_{z\to\infty} P_\Lambda(z) = \mu_0 z^2 + \mu_1 z + \mu_2 + \mathcal{O}(z^{-1}),$$

$$\lim_{z\to\infty} P_R(z) = \nu_0 z^2 + \nu_1 z + \nu_2 + \mathcal{O}(z^{-1})$$

$\mu_i\,,\nu_i$

to-be-determined constants

Total of 4 equations, which must be solved in vicinity of $z = \infty$ for up to three orders in z.

They impose a number of algebraic relations to be satisfied by the parameters of the theory

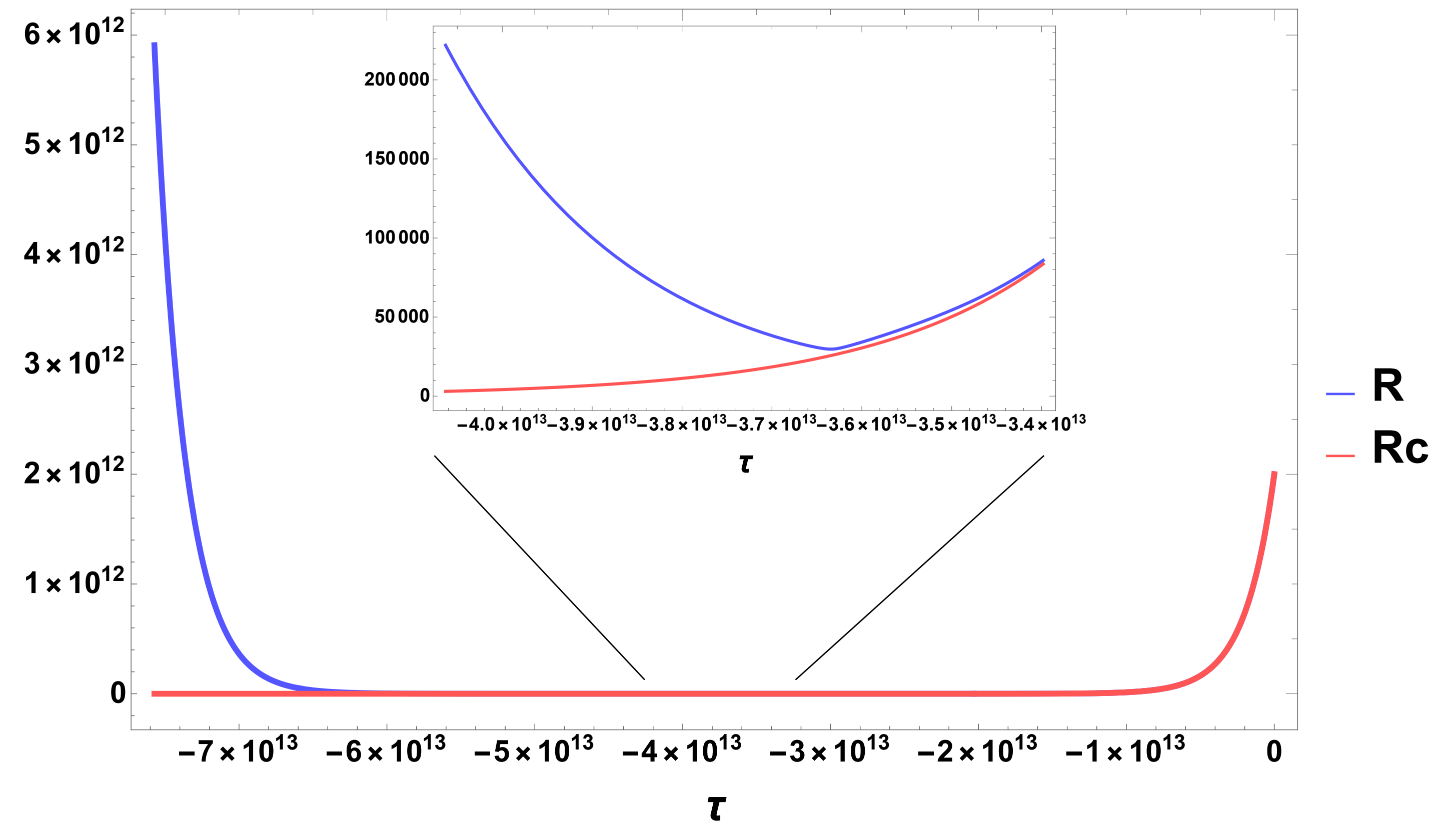
➥ $\ell \approx \dfrac{2Gm}{0.974}\,,\quad \gamma \approx 0.274$

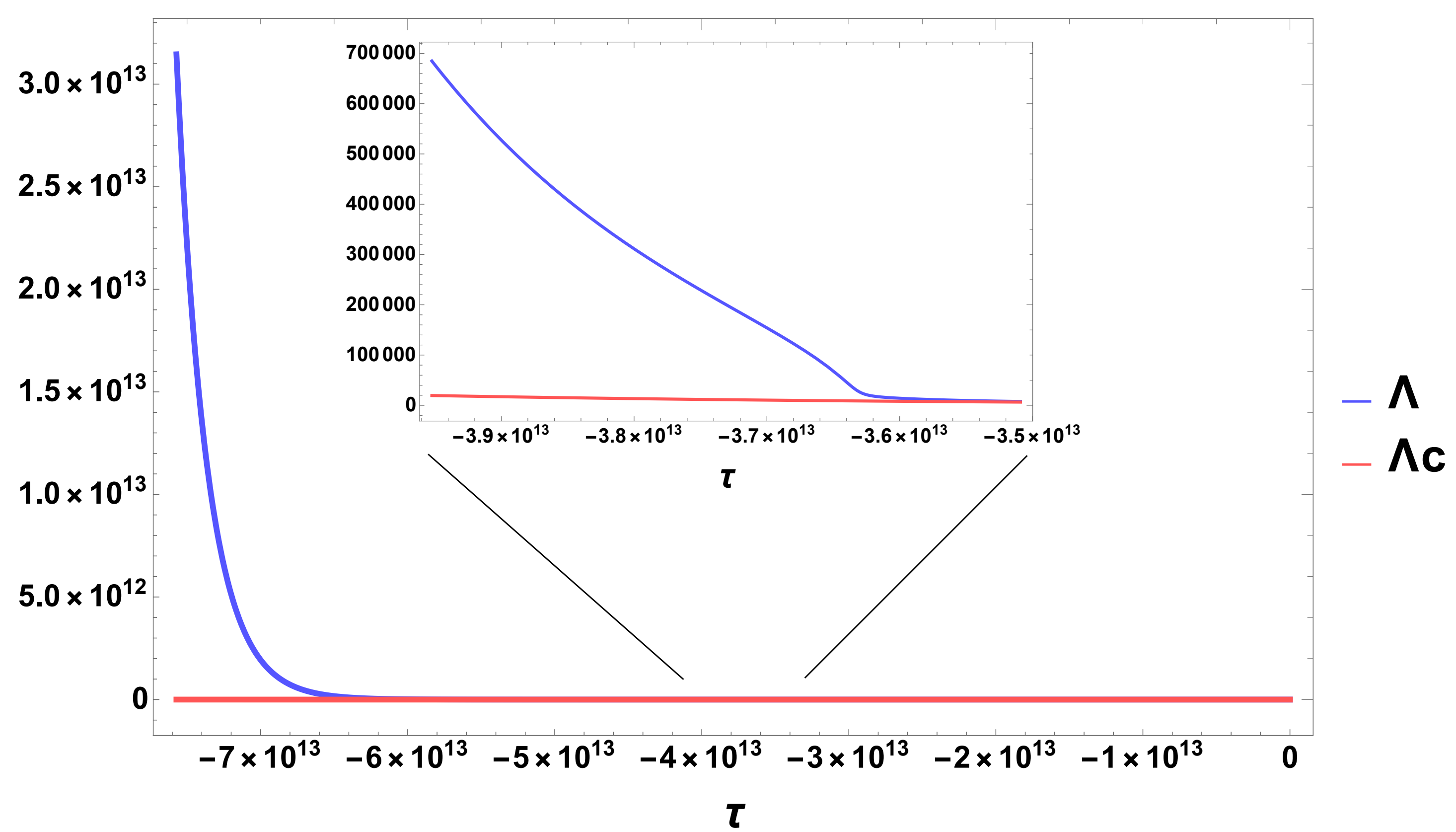
Same numerical value as from the SU(2) black hole entropy calculation!

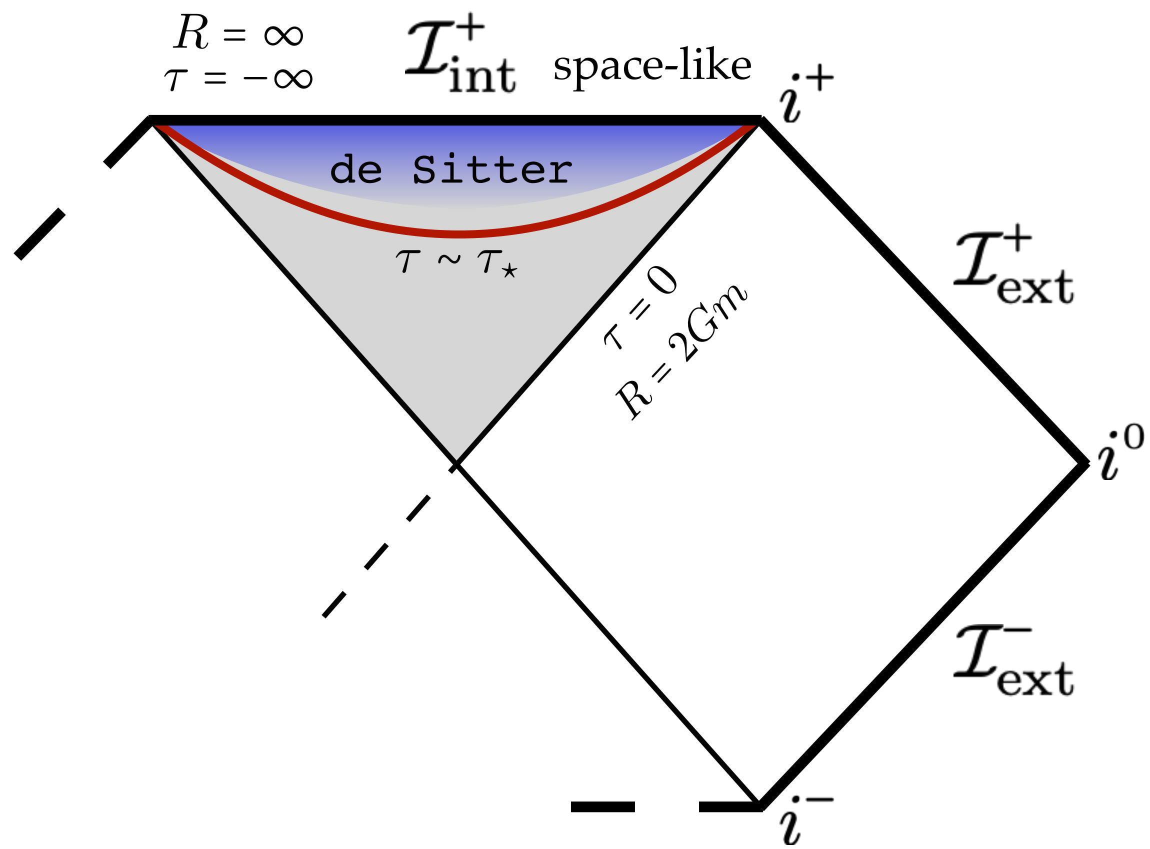
$$\boxed{\lambda = \frac{3}{N_0^2 \ell^2} \approx \frac{0.06}{\ell_P^2 j}}$$

Emergent CC purely of quantum gravitational origin

QG regime: $\mathcal{K}_c \sim 1/\ell_p^4\,,$ at $\tau = \tau_\star$ Numerical Solution: $m = 10^{12} m_P\,,$ $\tau_\star \approx -3.7 \times 10^{13}$







Conformal factor $\omega := 1/z \quad \rightarrow \quad \tilde{g}_{ab} := \omega^2 g_{ab}$

Conformal boundary $\omega = 0 \quad \Rightarrow$ Space-like hypersurface $\mathcal{I}^+_{\mathrm{int}}$ (with topology $\mathbb{R} \times \mathbb{S}^2$)

$$T^{\mathrm{eff}}_{ab} := \frac{1}{8\pi G}\left[G_{ab} + \lambda g_{ab}\right] \xrightarrow[\omega \to 0]{} 0 \quad \text{at least as fast as } \omega$$

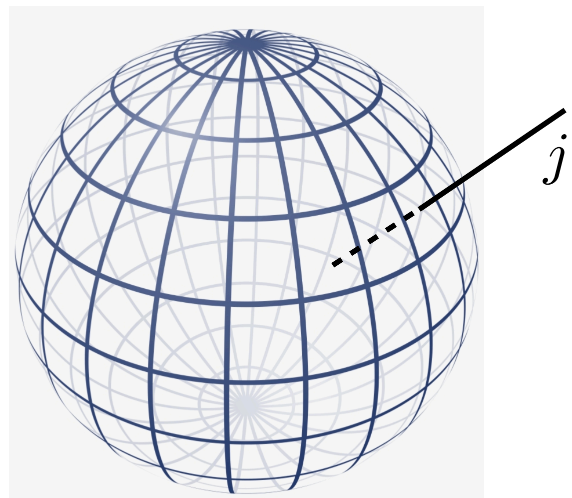
➡ The interior spacetime is asymptotically Schwarzschild-de Sitter [Ashtekar, Bonga, Kesavan, 2015]

(same Penrose diagram as "regular phantom black holes" [Bronnikov, Fabris, 2006], [Bronnikov, Melnikov, Dehnen, 2007] but with no need for exotic matter)

Are we living inside a huge black hole?

Quantum gravitational effects give birth to an expanding Universe that is both homogeneous and locally isotropic asymptotically, away from the cosmic bounce:

$$\lambda \approx \frac{0.06}{\ell_P^2 j}$$



On the initial Cauchy surface near the event horizon

$$(2m)^2 \simeq \ell_p^2 j \mathcal{N}^2$$

A prescription for evolving j is not accessible within our effective dynamics approach

We expect j to be renormalized by the microscopic dynamics, particularly in the deep Planckian regime:

λ should remain regular in the limit $\hbar \to 0$ $\Rightarrow$ $\ell_p^2 \bar{j} \sim (Gm)^2$

However, coarse graining operation derived from application of LQG techniques in the cosmological sector suggest a renormalization flow of the spin [Bodendorfer, 2017]; [Ben Achour, Livine, 2019]:

$$\boxed{\ell_p^2 \bar{j} = (Gm)^2 \frac{0.06}{\sigma}}$$

In a wild stretch of imagination, it is tempting to entertain the idea that our own Universe is couched within a Schwarzschild black hole:

Effects of a matter dominated phase encoded in a $\Omega_\lambda < 1$

$$(*) \qquad \bar{\lambda} = \frac{3H_0^2\Omega_\lambda}{c^2} \simeq \frac{\sigma c^4}{0.06(Gm)^2}$$

BH ADM mass

Baryonic + cold dark matter of the observable Universe

$$m = m_B + m_C$$

$$(**) \qquad Gm = G(\Omega_B + \Omega_C)\rho_{\rm crit}V_{\rm obs} = \frac{(\Omega_B + \Omega_C)c^3}{2H_0}\left[\int_0^\infty \frac{dz}{\sqrt{\Omega_\Lambda + (\Omega_B + \Omega_C)(1+z)^3}}\right]^3$$

determined by the comoving particle horizon $\chi \simeq \zeta H_0^{-1}$

$$(*) + (**) \qquad ➥ \qquad \boxed{\left[\int_0^\infty \frac{dz}{\sqrt{\Omega_\Lambda + (\Omega_B + \Omega_C)(1+z)^3}}\right]^3 \sqrt{\Omega_\Lambda}(\Omega_B + \Omega_C) \simeq \sqrt{\frac{4\sigma}{3 \times 0.06}}}$$

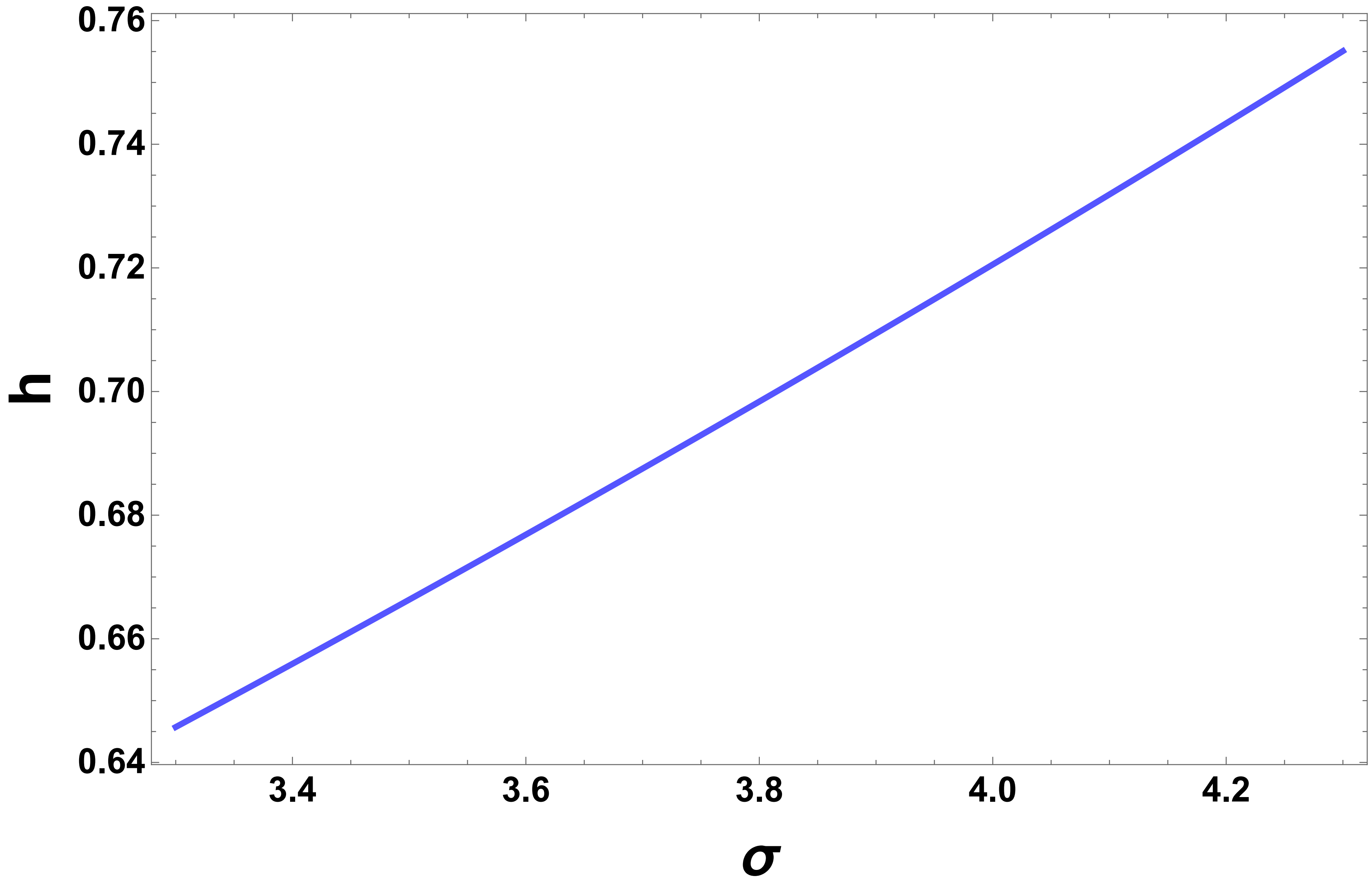
New equation among the density parameters of ΛCDM model and a quantum geometry renormalization parameter:

The Hubble constant becomes a function of the Planck scale physics parameter σ

Planck collaborations by combinig TT, TE, EE+lowE+lensing+BAO data [N. Aghanim et al. (Planck), A&A 2020]

$$\Omega_B h^2 = 0.02242 \pm 0.00014\,, \quad \Omega_C h^2 = 0.11933 \pm 0.00091\,, \quad N_{\rm eff} = 2.99 \pm 0.34$$

where $h \equiv H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$



Summary & Outlook

- [x] By performing the symmetry reduction at the quantum level all relevant DOF are encoded in the effective dynamics and the Hamiltonian can be derived for the first time:

  -> Crucial modifications w.r.t. minisuperspace quantization models: Baby Universe inside

- [x] Geometric considerations to fix the most relevant dynamics ambiguities:

  -> Asymptotically de Sitter effective metric for the same Immirzi parameter value as in SU(2) BH entropy calculation.

- [x] Emerging cosmological constant due to quantum gravity effects.

- [ ] Construct a graph changing Hamiltonian: Study renormalization of CC in terms of $\mathcal{N}$

  -> Precise prediction for the Hubble constant?

- [ ] Inclusion of matter: Does the gravitational collapse encode the history of the Universe??

- [ ] Horizon penetrating foliation (exterior and interior dynamics together for the first time):

  - Algebra of effective constraints.
  - Study the shear operator in the near horizon region and compare its modes to the QNM of the (luminosity of the) outgoing radiation of a GW flux.