Black Holes in String Theory

Jan de Boer, Amsterdam

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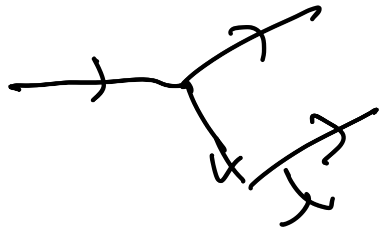
Disclaimer: there lectures are intended to be somewhat sketchy and kaleidoscopic.

Lessons from the 1970’s (Bekenstein, Hawking, Carter, Bardeen,…)

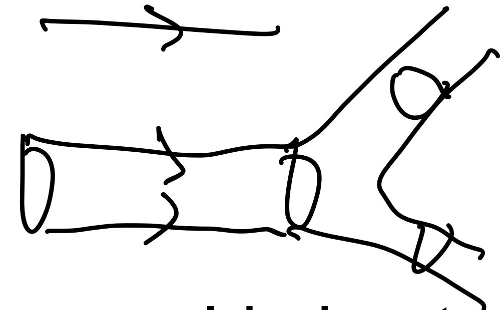
- Black holes have “no hair”
- Black holes carry an “entropy” proportional to the area of the horizon (S=A/4G)
- Black holes have a temperature
- Black holes emit Hawking radiation and can evaporate

Black holes behave like thermodynamic or hydrodynamical systems.

String Theory



Replace point particles by strings.

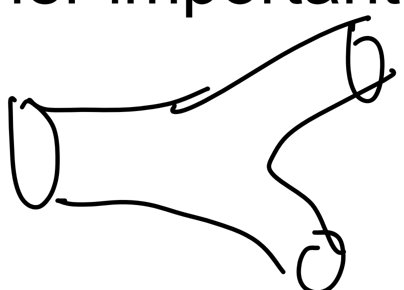


Action is given by the area of the string world-sheet multiplied by the string tension T

$$T \sim \frac{1}{\alpha'} \sim \frac{1}{l_s^2}$$

$$S = T \cdot \text{Area(world-sheet)}$$

Another important parameter is the string coupling constant



$\longleftrightarrow g_s$

In terms of these parameters, for strings propagating in d spacetime dimensions

$\cancel{d = 26}$

$d = 10$

$$\frac{1}{G_N} = \frac{1}{g_s^2 \, l_s^{d-2}}$$

$$\frac{1}{G_N} \propto \frac{1}{g_s^2 \ell_s^2}$$

One can think of a closed string as some sort of random walk in space generated by infinitely many harmonic oscillators.

$$N = \sum_{i,k} \langle k a^{\dagger}_{i,k} a_{i,k} \rangle \equiv \sum_{i,k} k n_{i,k}$$

Then

$$M \sim L \sim \sqrt{N}$$

stat. phys

$$e^S \sim e^{2\pi\sqrt{\frac{c}{6}N}}$$

+ sublead.

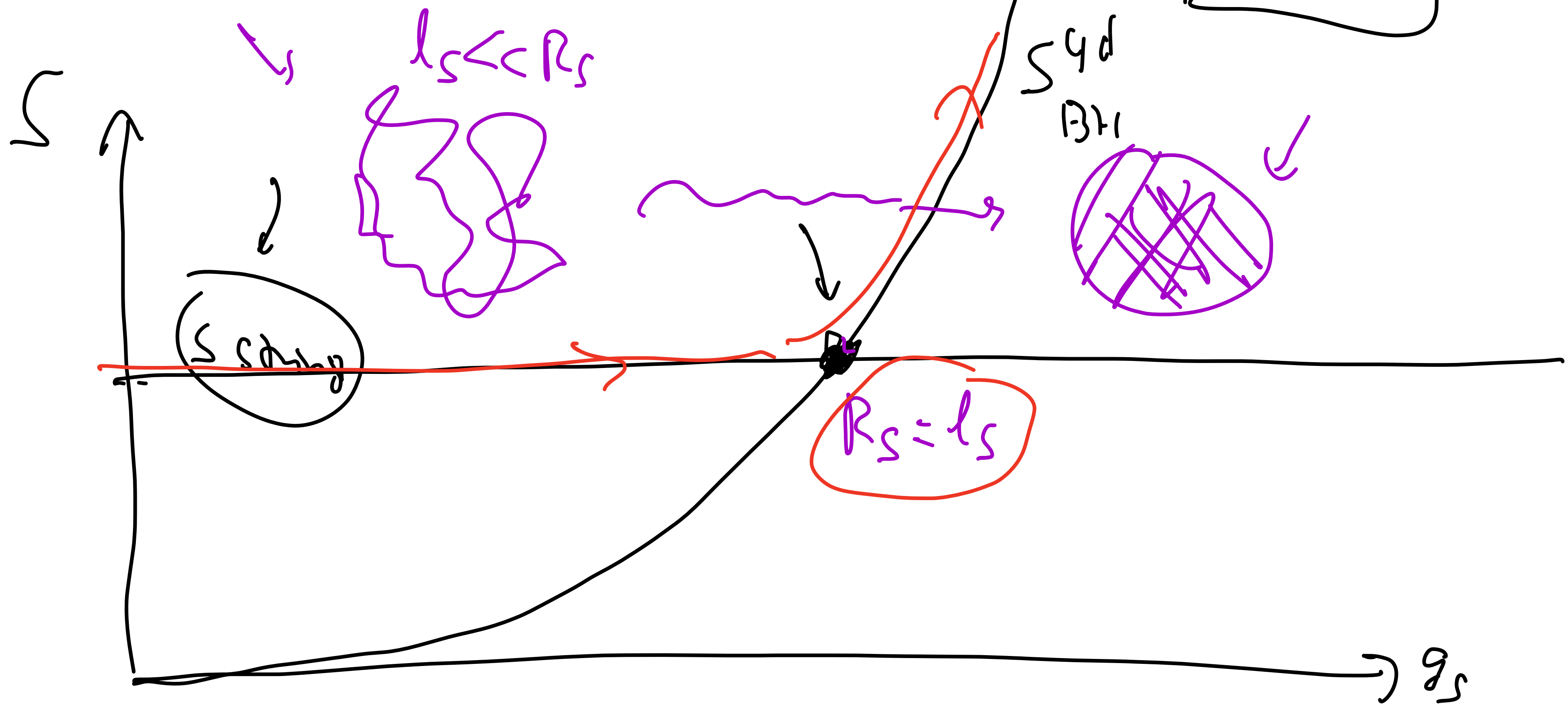
Putting back units

$$M \sim L\ell_s^{-2}, \qquad S \sim L\ell_s^{-1} \;\Rightarrow\; \boxed{S \sim M\ell_s}$$

For a 4d black hole

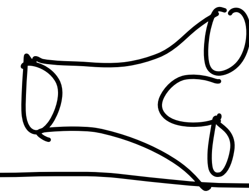
$$\sim \frac{R_S^2}{G_N}$$

$$R_S = MG_N, \quad S_{BH} = M^2 G_N = MR_S = \boxed{M^2 g_s^2 \ell_s^2}$$



Correspondence Principle (Horowitz-Polchinski)

String theory effective actions



particle $\leftrightarrow$ QFT $\longrightarrow$ "

strings $\leftrightarrow$ string field theory $\xrightarrow[\text{energies}]{\text{low}}$ effective actions

The low-energy dynamics of string theory (first quantized picture) can be described by an effective field theory (second quantized picture).

$$\frac{1}{g_s^2 \ell_s^{d-2}} \int d^d x \sqrt{-g} \left(R + \ell_s^2 R^2 + \ell_s^4 R^3 + \dots \right)$$

Heterotic string, IIA, IIB (10 dimensions) etc: all give rise to supergravity theories at low energies.

Most controlled examples are supersymmetric.

To get lower dimensional examples, need to compactify.

Kaluza-Klein reduction. $10 = 6 + 4$

$\downarrow$

circles small

$$\xi \sim \frac{h}{R}$$

p=0 D:1

p=1 black string

etc

Brane solutions

$$ds^2 = f_p^{-1/2}(-dt^2 + dx_1^2 + \ldots + dx_p^2) + f_p^{1/2}(dx_{p+1}^2 + \ldots + dx_9^2),$$

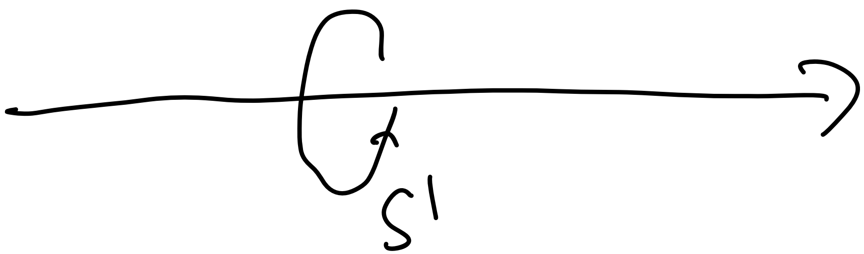
$$e^{-2(\phi-\phi_\infty)} = f_p^{(p-3)/2},$$

$$\underline{A_{0\ldots p}} = -\frac{1}{2}(f_p^{-1} - 1),$$

$$\alpha'^2 f_p = \alpha'^2 + \frac{d_p g_{YM}^2 N}{U^{7-p}}\ , \qquad d_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma(\frac{7-p}{2})\ .$$

These are extremal solutions. Can also find solutions with a finite temperature.

$$\int_{S^2} F = q_e$$



$$\int_{S^1} A = q_s$$

$\boxed{\text{(p+1)-dimensional object in d-dimensions}}$

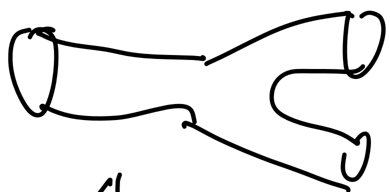
$$\int_{S^{d-p-2}} F_{d-p-2} = q_p$$

A	p
B	$p+1$
C	$d-p$
D	d
E	$d-p-1$
F	$d-p-2$
G	$d-p-3$

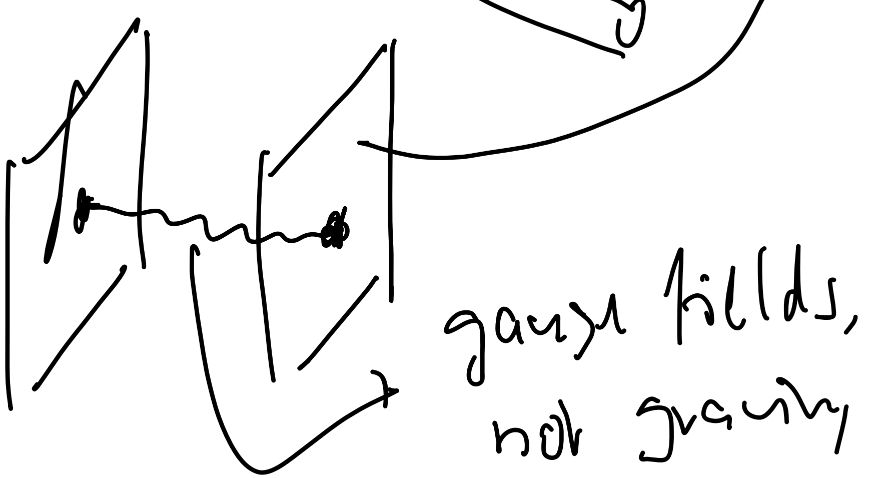
D-branes

Besides closed string, can also consider open strings. We can force the endpoint to lie on some fixed surface (Dirichlet boundary condition).

Low-energy effective action for D-branes: Dirac-Born-Infeld action



demand that endpoint lives here!



gauge fields, not gravity

$$S_{open} \sim \frac{1}{g_s l_s^{p+1}} \int d^{p+1}x \sqrt{\det\left[\delta_{\alpha\beta} + l_s^2 F_{\alpha\beta}\right]}$$

$U(1)$

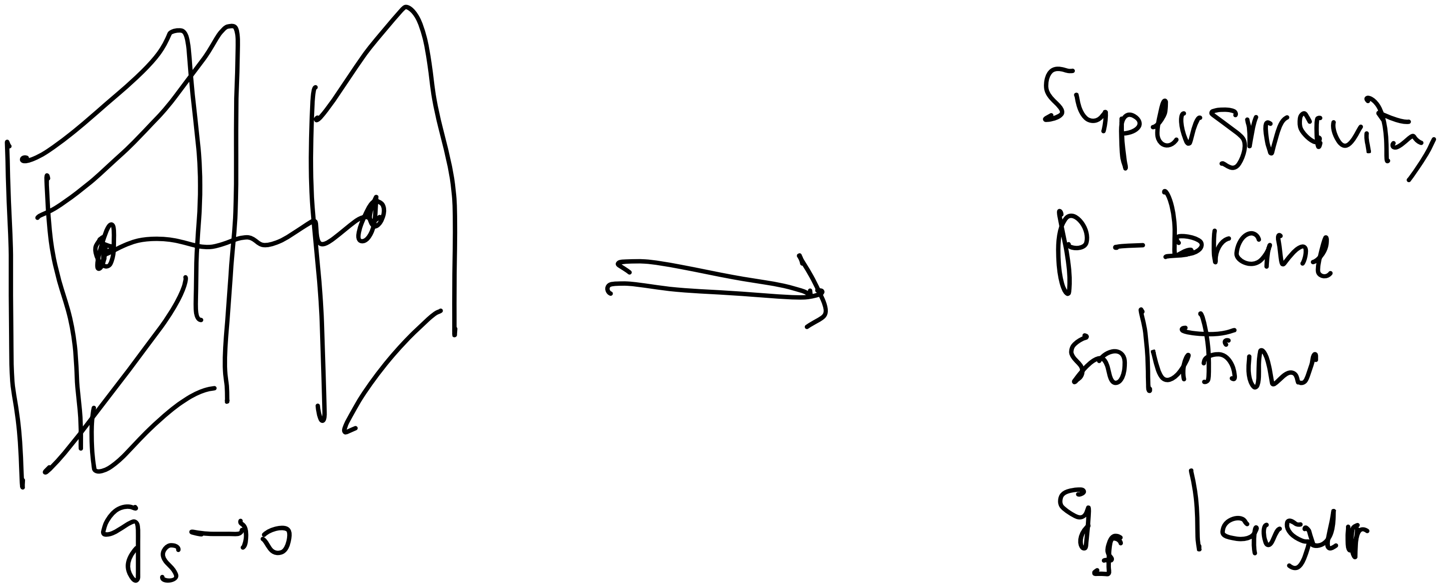
S_{Maxwell}

$S_{\text{Yang-Mills}}$

$$+ \int d^{p+1}x \, A_{\mu_0 \ldots \mu_p}$$

D-branes = p-branes

Highly excited D-branes go over into finite temperature p-branes as their Schwarzschild radius becomes string length (correspondence principle).



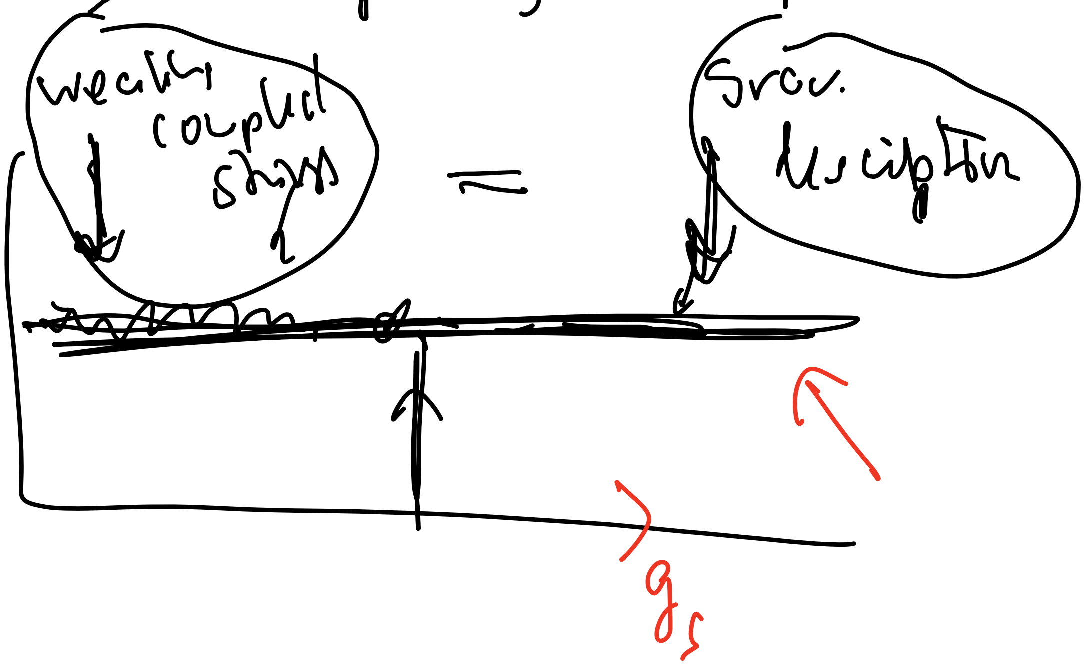
Can put the spatial part of a brane on a compact manifold (torus, sphere, more complicated). Result is a massive point-like object in the remaining dimensions.
By combining branes we can create a large number of brane bound states.

black branes $\quad \mathbb{R}^4 \times (S^1)^6$

black string

4d: black hole

many examples, "wrap branes"



$$\frac{1}{G_N} = \frac{1}{g_s^2 l_s^2} = \frac{1}{l_p^2}$$

$g_s \sim 0{,}01$

$l_s \sim 10^{-32}\,m$

$l_p \sim 10^{-35}\,m$

Supersymmetric black hole entropy (Strominger, Vafa)

Key: representation theory of supersymmetry

$$\{Q, Q^\dagger\} = H, \qquad [H, Q] = [H, Q^\dagger] = 0$$

$H = 0$

$H|\psi\rangle = 0$

$Q|\psi\rangle = Q^\dagger|\psi\rangle = 0$

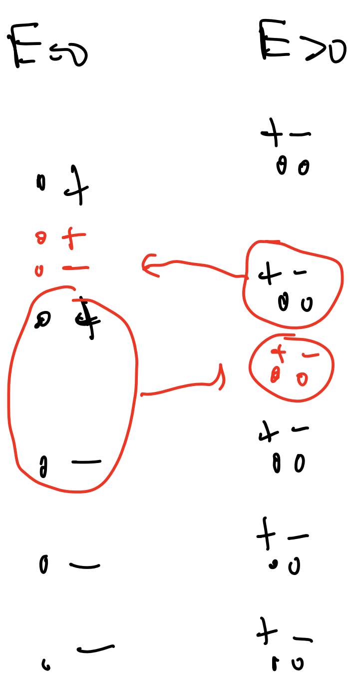
$H > 0$

$$|\chi\rangle \underset{Q}{\overset{Q^\dagger}{\rightleftarrows}} |\psi\rangle$$

$H|\chi\rangle = H|\psi\rangle = E|\chi\rangle = E|\psi\rangle$

$$\mathrm{Tr}((-1)^F) = \mathrm{Tr}((-1)^F e^{-\beta H})$$

Index does not depend on continuous parameters



H depends on g_s

$N_+ - N_-$ only has contributions at $E=0$

$\mathrm{Tr}\left((-1)^F\right) = N_+ - N_-$

$\therefore$ does not depend on g_s

$\mathrm{Tr}\left(e^{-\beta H}(-1)^F\right) = N_+ - N_-$

D5-D1-P bound state in IIB on T^5

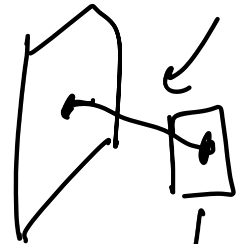
$\Downarrow$

5 dimensional black hole

extremal + supersymmetric

$$S = \frac{A}{4G_N}$$

2d Conformal Field Theory



open strings

$\Downarrow$

$$\mathrm{tr}\left((-1)^F e^{-\beta H}\right)$$

g_s

Weakly coupled D-brane computation reproduces

$$S = \frac{A}{4G_N}$$

Computations have improved over the years, one can now not just get the leading behavior but even an actual integer in favorable circumstances.

AdS/CFT (see Alberto Zaffaroni's lectures)

$U^2 = x_{p+1}^2 + \dots x_9^2$

$$ds^2 = f_p^{-1/2}(-dt^2 + dx_1^2 + \dots + dx_p^2) + f_p^{1/2}(dx_{p+1}^2 + \dots + dx_9^2),$$

$$e^{-2(\phi-\phi_\infty)} = f_p^{(p-3)/2},$$

$$A_{0\dots p} = -\frac{1}{2}(f_p^{-1} - 1),$$

D-brane p brane

$\longrightarrow g_s$

$$\alpha'^2 f_p = \alpha'^2 + \frac{d_p g_{YM}^2 N}{U^{7-p}}, \qquad d_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma\left(\frac{7-p}{2}\right).$$

Take p=3 and send U→0

$= AdS_5 \times S^5$

P-brane: $$ds^2 \sim U^2(-dt^2 + d\vec{x}^2) + \frac{1}{U^2}(dU^2 + U^2 d\Omega_5^2)$$

D-brane: Supersymmetric N=4 Yang-Mills theory

g_s p-brane d-brane

$N \to \infty$ $N \to \infty$

$AdS_5 \times S^5$

super sym. YM theory in $d=4$ ($N=4$) $SU(N)$

→ proof of AdS/CFT

Map of parameters $G_N = \ell_{ads}^3 / N, \quad \ell_{ads} = \ell_s (g_{YM}^2 N)^{1/4}$

AdS/CFT is a weak/strong coupling duality

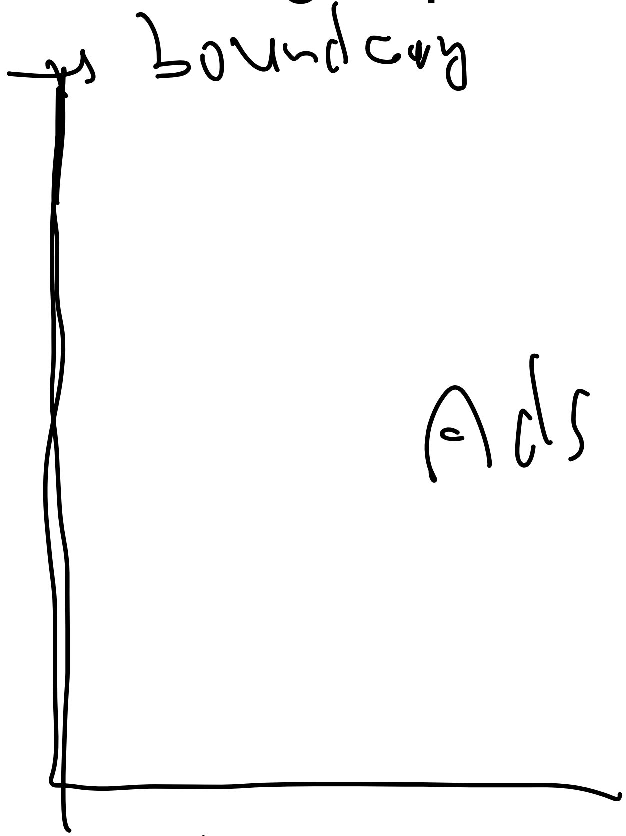
AdS provides a box with a (conformal) boundary where fluctuations die out and observables can be defined.

AdS/CFT realizes the idea of holography.

Reconstructing the interior is indirect (cf medical imaging)

The radius of AdS corresponds to an energy scale.

Asymptotic Symmetry Group (ASG) of AdS is precisely the conformal group in one dimension less.



near bdy

$$z^{\Delta} \qquad z^{4-\Delta}$$

$$\phi_{AdS} \xrightarrow{r\to\infty} e^{ar}\phi_1 + e^{br}\phi_2 + \cdots$$

fixed

fluctuations

$\begin{cases} \text{keep } \phi_1 \text{ fixed} \\ \text{change } \phi_2 \end{cases}$

diffeom. invariance

ASG = conformal group of the CFT.

AdS3 and the BTZ black hole (NB: no propagating gravitons)

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}dt^2 + \frac{r^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2\left(d\phi \,{}^{-}_{+}\, \frac{r_+ r_-}{r^2}dt\right)^2$$

$$M = r_+^2 + r_-^2, \quad J = 2r_+ r_-$$

$$S = \frac{2\pi r_+}{4G_3} = \frac{\pi}{4G_3}\left(\sqrt{M+J} + \sqrt{M-J}\right)$$

$$S_{CFT} = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} + 2\pi\sqrt{\frac{c}{6}\left(\bar{L}_0 - \frac{c}{24}\right)}$$

$L_0 \sim M + J$

$\bar{L}_0 \sim M - J$

$G_3 \sim c$

$\to$ from AdS/CFT

Dual description is a 2d CFT.

Asymptotic symmetry group is two copies of the Virasoro algebra.

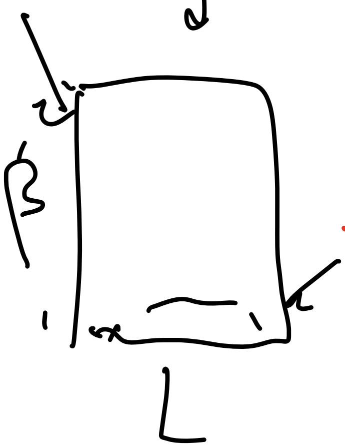
Universal formula for the high-temperature density of states (Cardy formula).

$\frac{L}{\beta}$ $\quad$ $\beta \to 0$ $\quad$ $E_c = \frac{\gamma}{L}$

$\alpha?$

$$Z_L(\beta) = Z_\beta(L) = Z_L(L^2/\beta) \simeq e^{-E_C L^2/\beta} = e^{-\gamma L/\beta}$$

$\beta \to 0$



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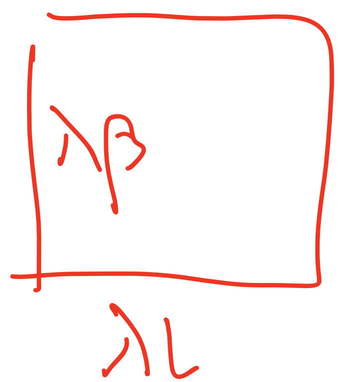
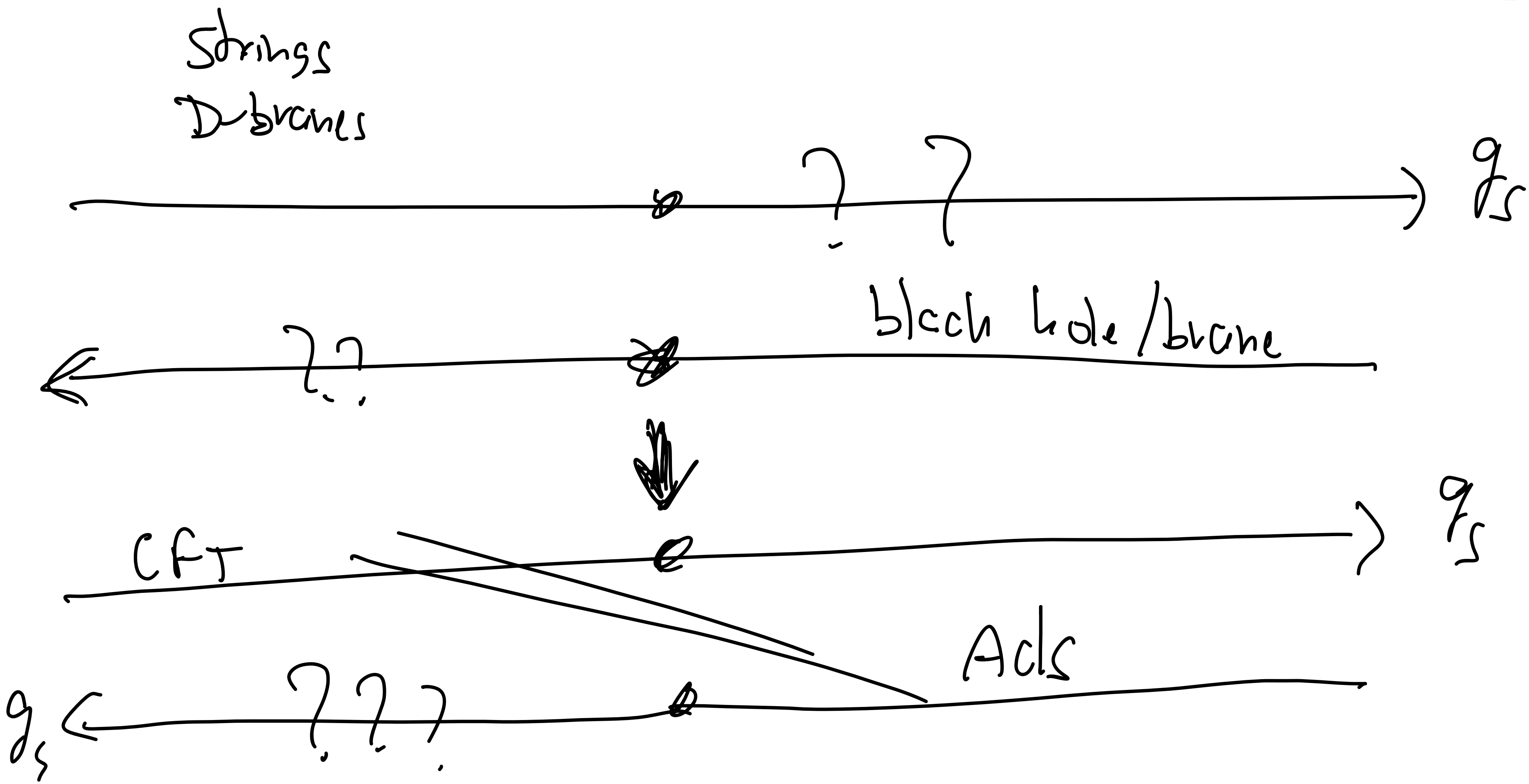


$\sum_i e^{-\beta E_i}$

Modular invariance in this form requires a well defined CFT which obeys locality and unitarity. Many versions of this argument in the literature where it is not clear whether requirements are met.



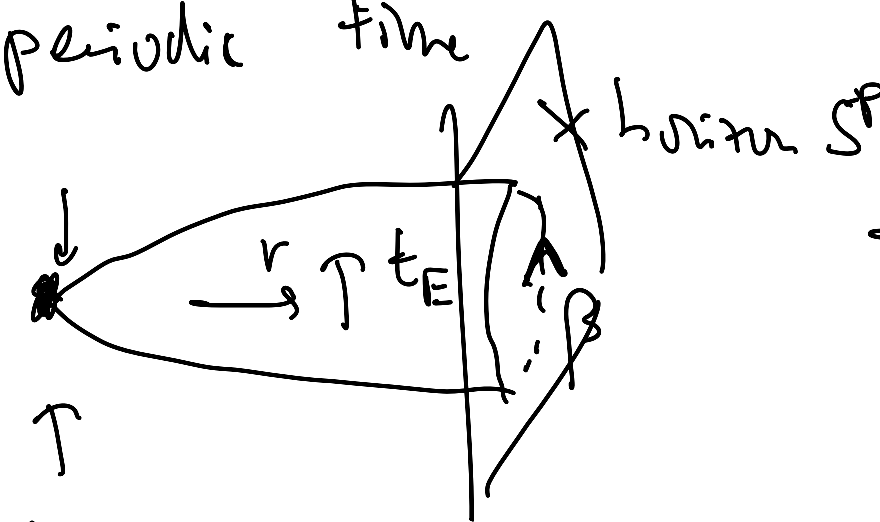
Black Holes in AdS

Black holes in AdS = thermal states on the boundary

in the CFT

Euclidean signature

periodic time



$S^p \times S^1_\beta$

CFT at finite temp.

horizon

BH $r = r_s + \varepsilon\rho$

do this exercise!

Transition at finite temperature.

AdS: Gibbons-Hawking transition.

"quark-gluon plasma"

CFT: confinement-deconfinement transition.

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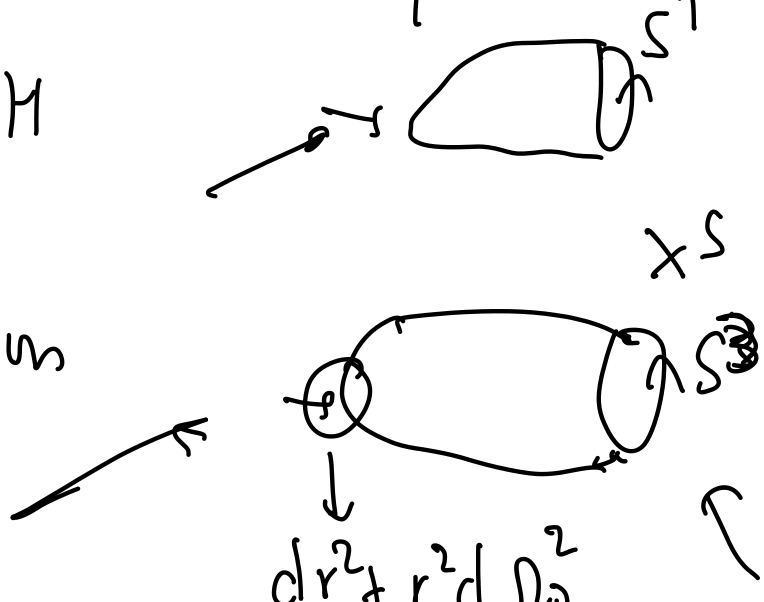
field theory on $S^3 \times S^1_\beta$ $\times S^3$

1) Euclidean BH

[high temp]

2) Other solution

[low temp]



dictionary: $Z_{CFT}[\text{sources}] = Z_{GRAV}[\text{boundary conditions}]$

$$= \sum_{\text{geometries}} \left(\qquad \right)$$

Thermalization and Thermodynamics

Boundary: finite temperature state (fluid, plasma) in the conformal field theory. Small perturbations governed by hydrodynamics.

AdS: Black holes. Small perturbations governed by Einstein equations.

Can indeed be mapped to each other!

Black hole absorption = dissipation

shear viscosity

$$\langle T^{xy} T^{xy} \rangle_R(\omega, 0) = i\omega\eta + \mathcal{O}(\omega^2)$$

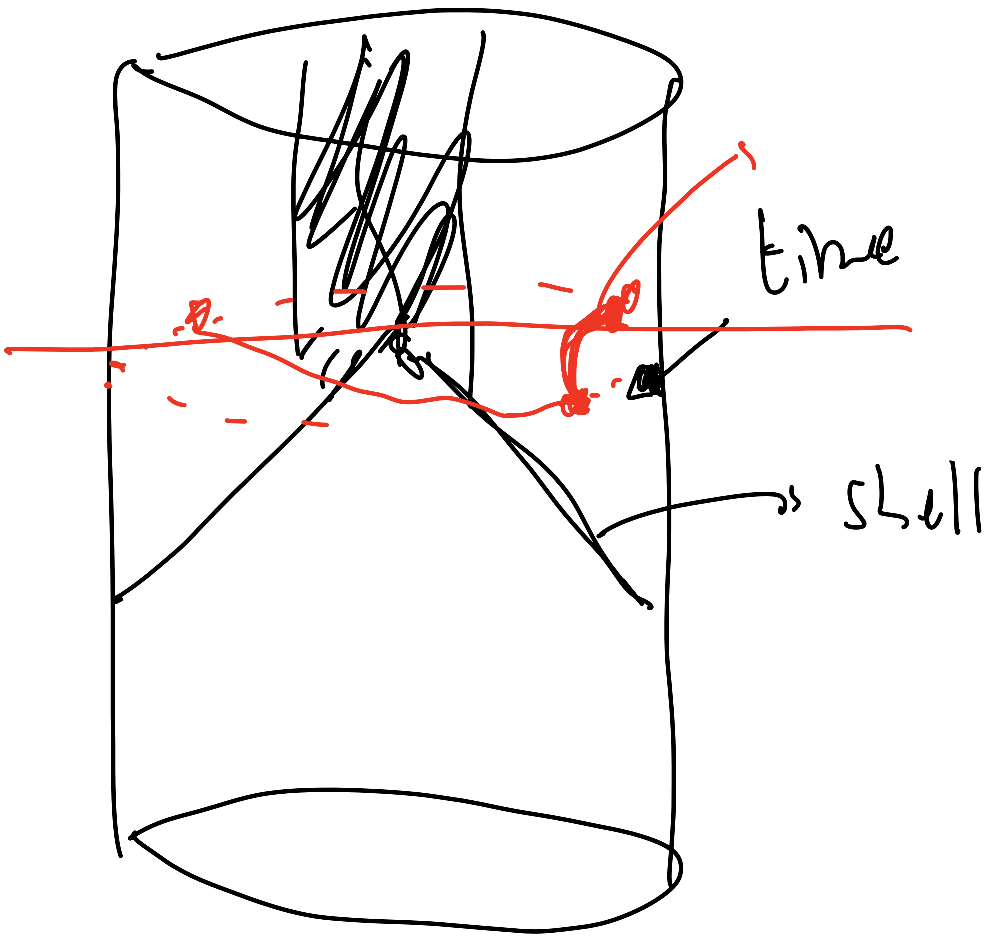
$\omega \to 0$

Universal prediction from Einstein gravity:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

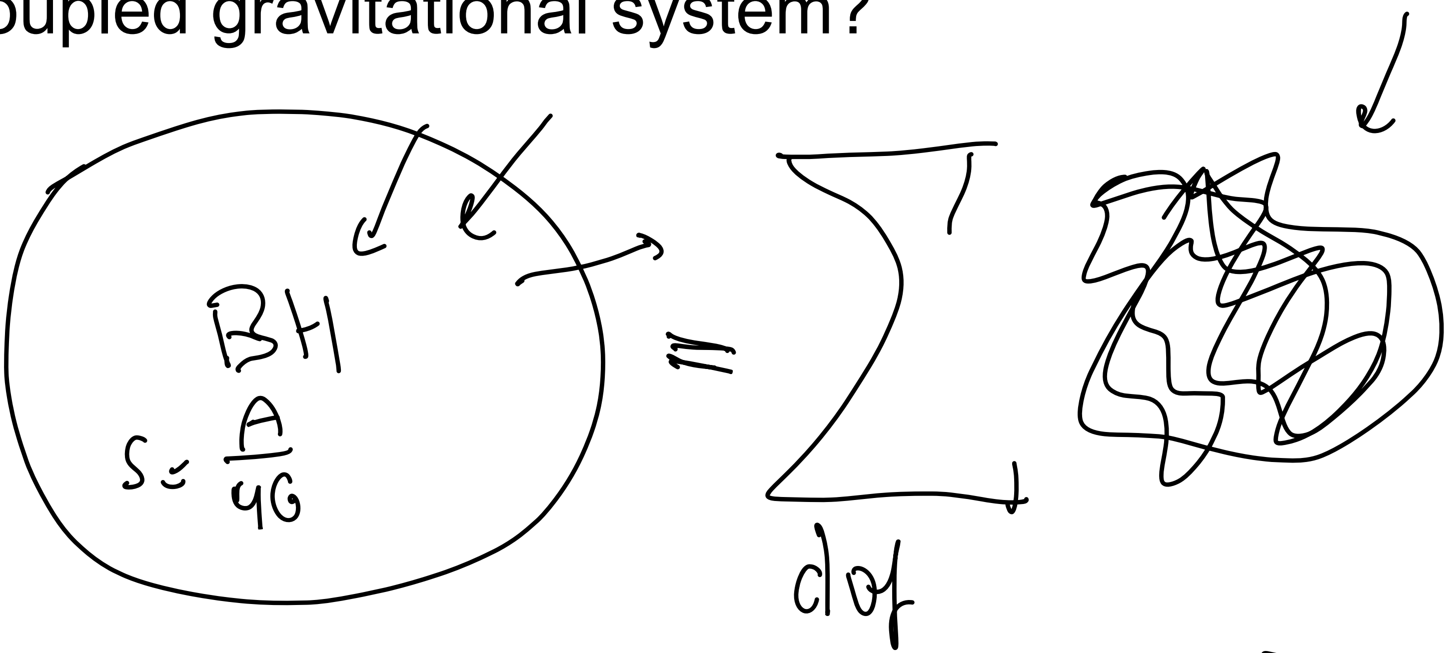
Extremely low shear viscosity, of the same order of magnitude as what is seen in the quark gluon plasma in heavy ion colliders.

Perhaps related, thermalization (=black hole creation) happens very fast.

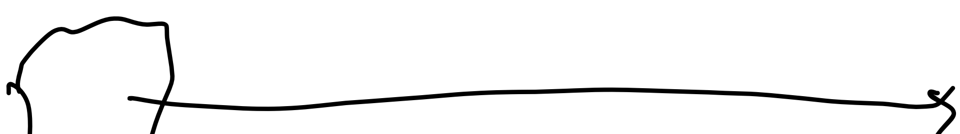


Microstates and fuzzballs

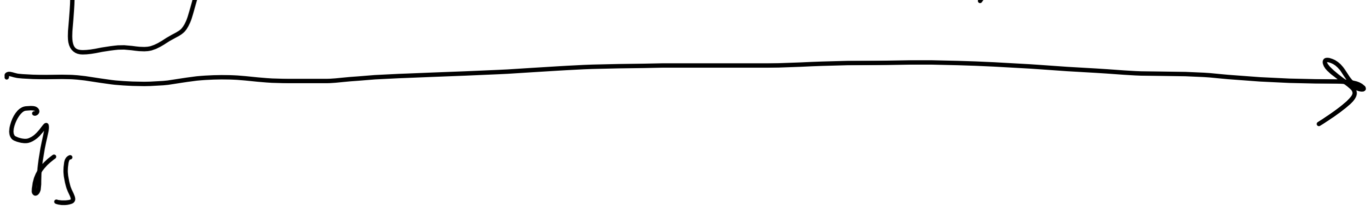
Can one not just follow the entropy, but also the individual degrees of freedom, as one increases the coupling constant from a weakly coupled conformal field theory to a weakly coupled gravitational system?



enough susy: this can be done?



grav version (smooth)



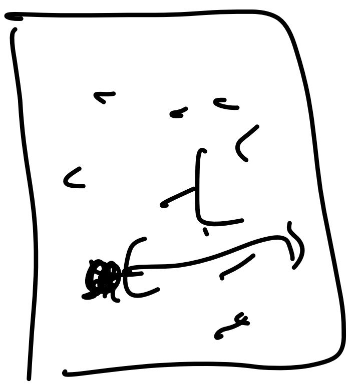
$$\sum \left(\text{smooth gravity solutions} \right) \Leftarrow \text{tiny black hole}$$

Open question: extend to large extremal black hole

Scrambling and chaos

Strongly coupled field theories scramble very fast
“black holes are the fastest scrambles of information in nature”

Scrambling time $t_S \sim \beta \log S$

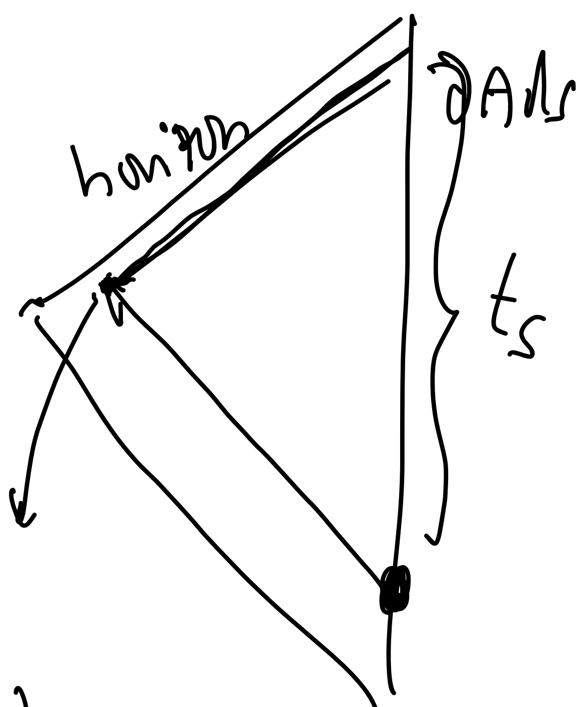


$t_S \sim L$

$S \sim L^{d-1}$ #

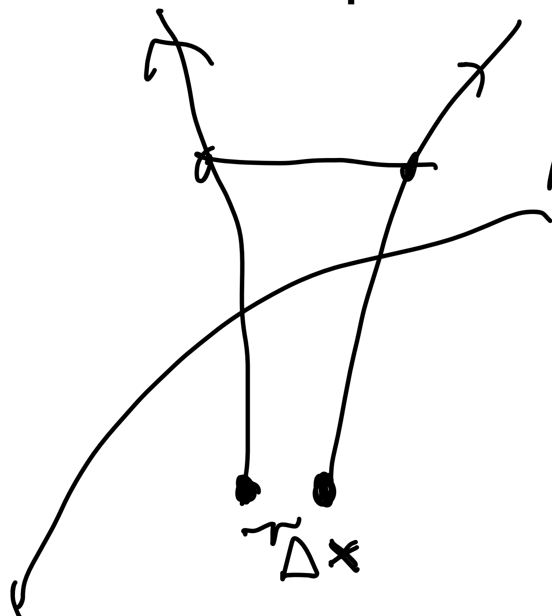
$t_S \sim S^{1/(d-1)}$

$d \to \infty$



l_p from horizon.

Classical chaos is the exponential separation of classical trajectories.



$\Delta x\, e^{\lambda t}$

Lyapunov exponent.

Quantum mechanically, one considers

$$\frac{\partial q(t)}{\partial q(0)} \sim \{p(0), q(t)\} \rightarrow [p(0), q(t)]$$

or rather the square to have a non-zero expectation value

$$C(t) = \langle [V(t), W(0)]^2 \rangle$$

Which is ofter refered to as an “OTOC” (an Out of Time Ordered Correlator)

Exponential growth $\sim e^{\lambda_L t}$

Field theory: can be shown using analyticity of correlators.

$$\lambda_L \leq \frac{2\pi}{\beta}$$

Gravity: can be shows using near-horizon Rindler geometry and corresponding red- and blueshifts.

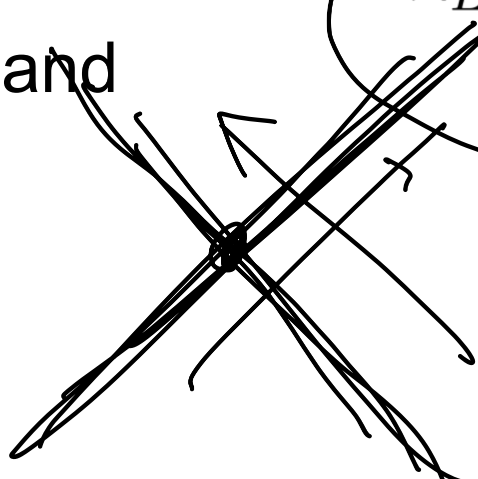
$$\lambda_L \simeq \frac{2\pi}{\beta}$$

horizon

$$ds^2 = dx^+ dx^-$$

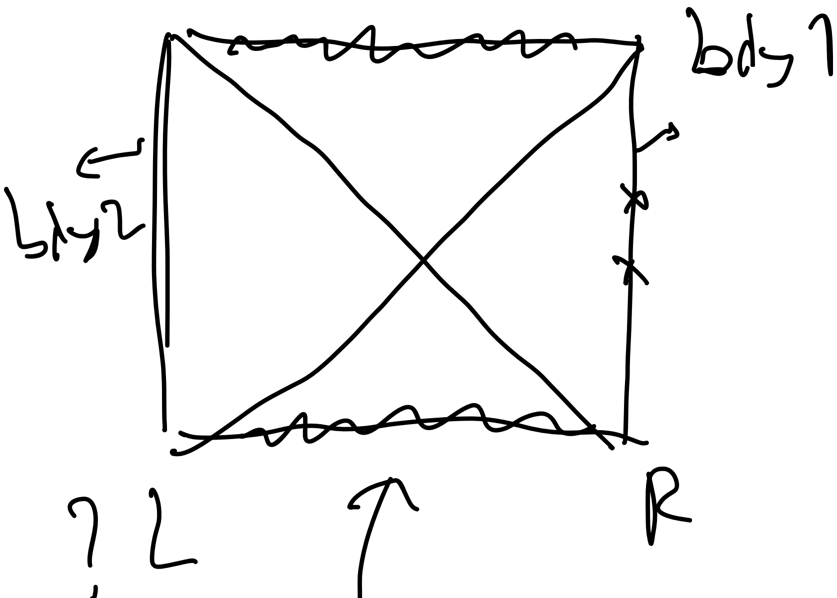
$$x^+ \to \lambda x^+$$

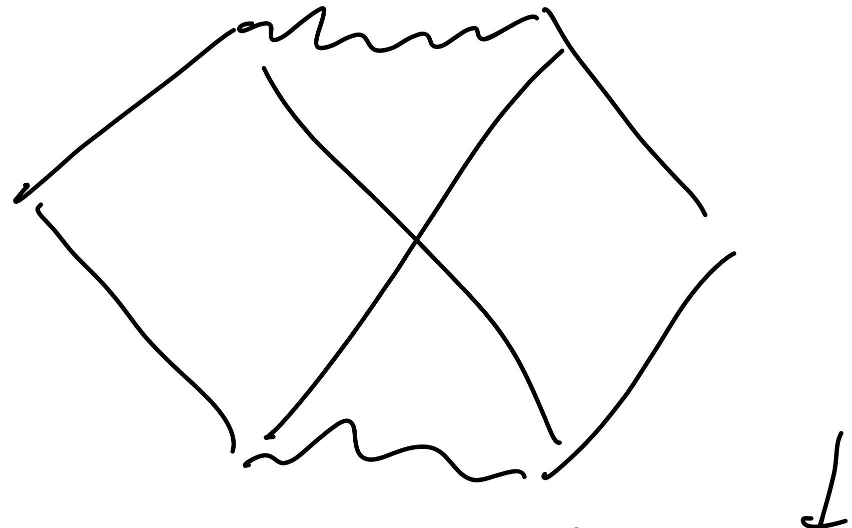
$$x^- \to \lambda^{-1} x^-$$



Eternal black holes and ER=EPR

Penrose diagram of an eternal AdS black hole



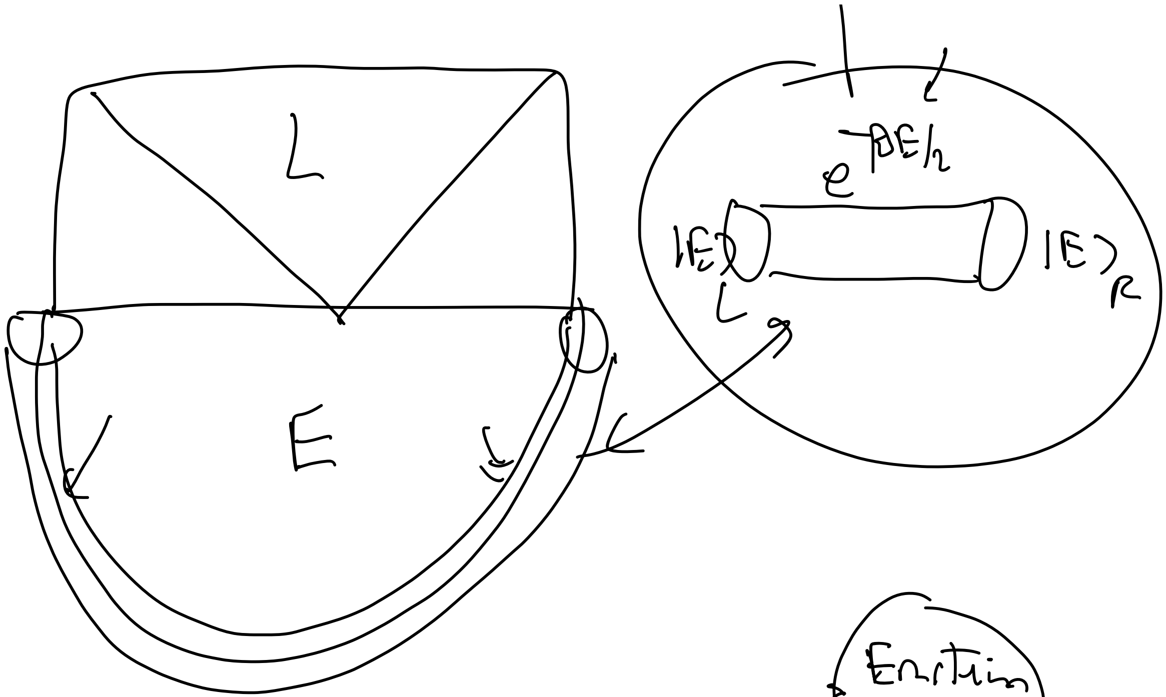


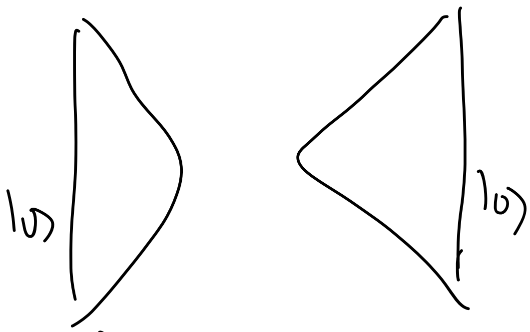
$$\mathrm{Tr}_L\left(|TFD\rangle\langle TFD|\right) = \frac{1}{Z}\sum e^{-\beta E}|E\rangle_R\langle E|_R$$

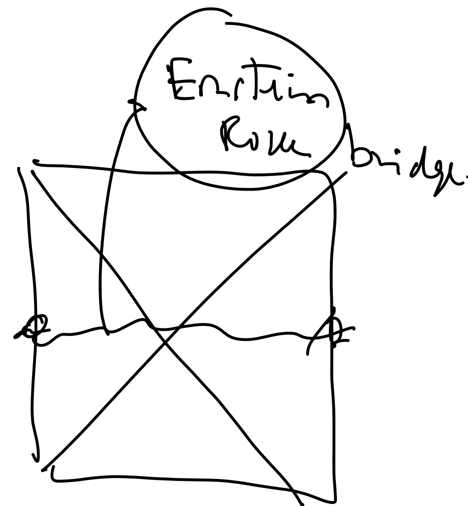
$|\psi\rangle_L|\chi\rangle_R$

$$\frac{1}{\sqrt{2}}\left(|0\rangle_L|0\rangle_R + |1\rangle_L|1\rangle_R\right)$$

$$|\mathrm{TFD}\rangle = \sum_E e^{-\beta E/2}|E\rangle_L|E\rangle_R \frac{1}{\sqrt{Z}}$$







∄ entanglem.

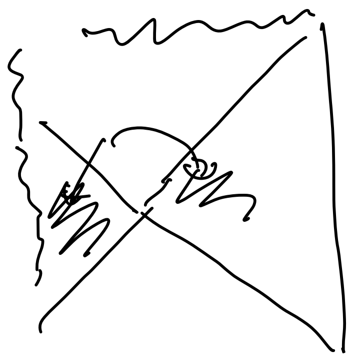
$|\chi\rangle_L \; |\psi\rangle_R$

∃ entangl.

$\frac{1}{\sqrt{Z}}\sum e^{-\beta E/2} |E\rangle_L |E\rangle_R$

EPR

One sided case



entangle

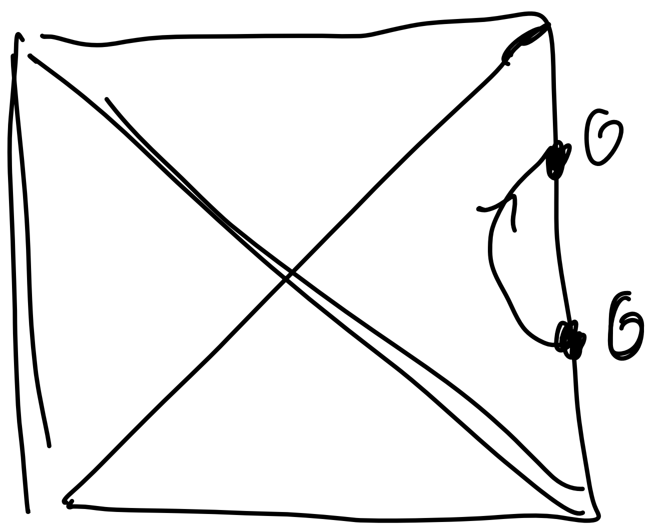
low & high-energy

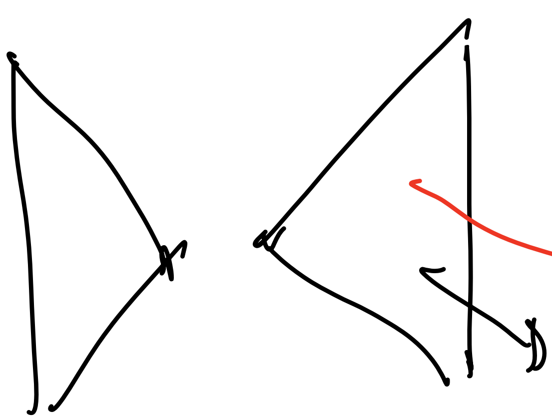
dof.

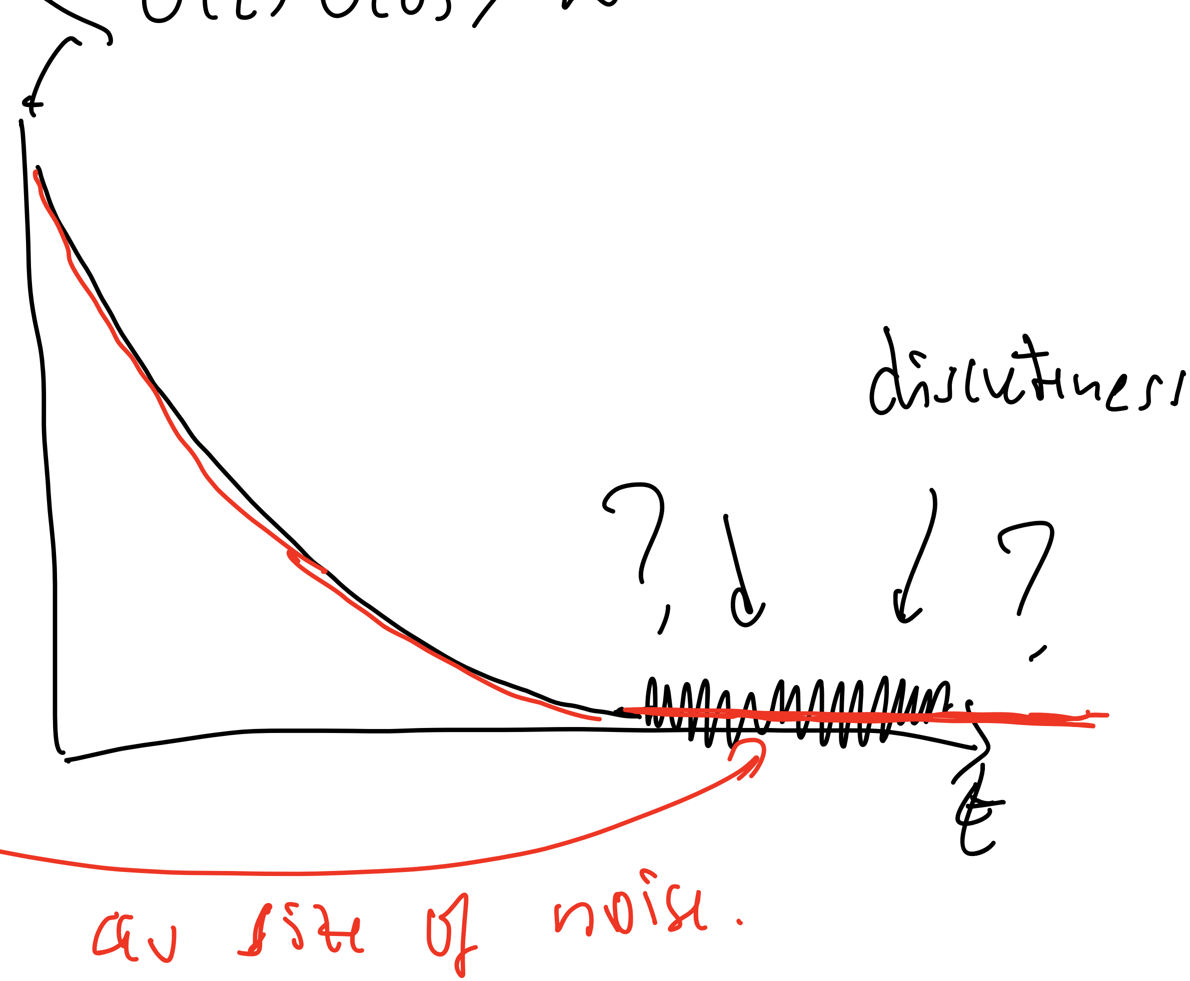
A puzzle is that this geometry does not produce the right late time two-point function in the theory.

Black hole sees a coarsed grained version of the correct answer.

$$\langle O(t)\, O(0) \rangle \sim e^{-\beta t}$$







ER=EPR refers to the general philosophy that quantum entanglement is crucial in order to connect spacetime regions.

A precise statement is currently lacking as far as I know.

One can obtain linearized and second order Einstein equations from properties of quantum entanglement but it is conceivable one needs more input to get the full non-linear Einstein equations.

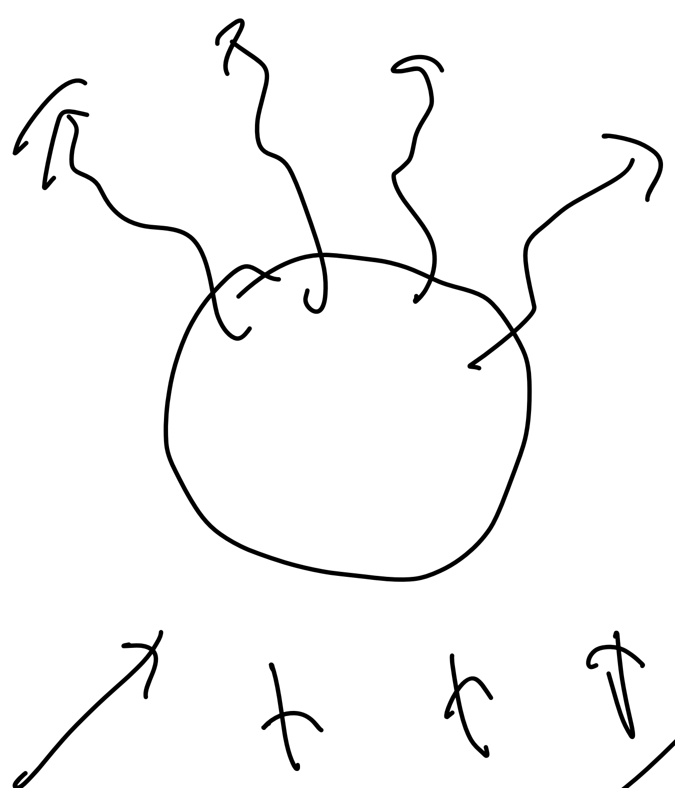
More on quantum information, the information paradox, wormholes, complexity, the factorization puzzle, the fate of the infalling obsever, firewalls, quantum error correction, SYK versus JT gravity,..

semiclassical gravity has a problem

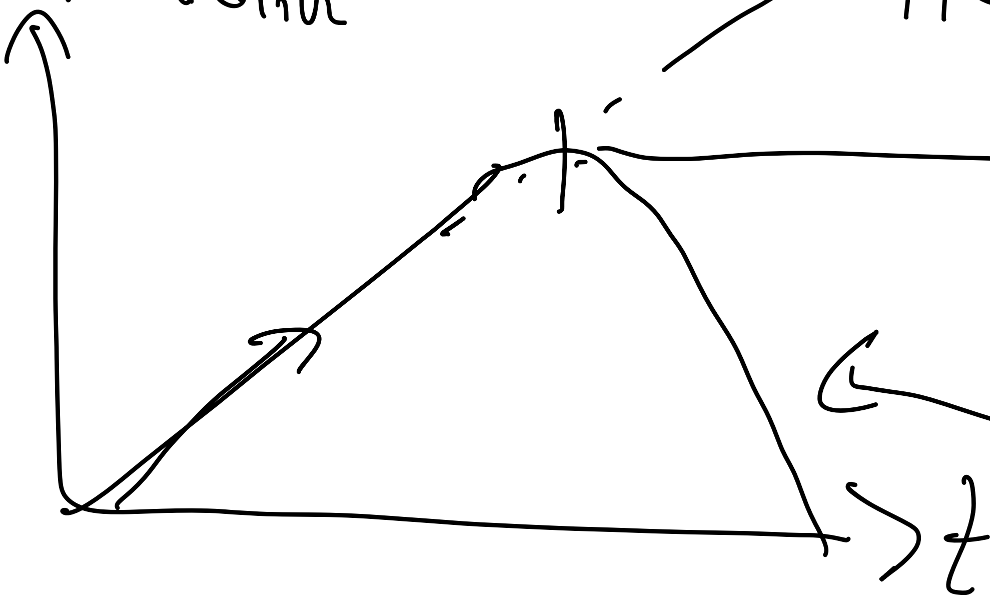
$$\rho \sim \sum_E e^{-\beta E} |E\rangle\langle E|$$

↯ unitarity &

+ higher order corrections keep locality ↯



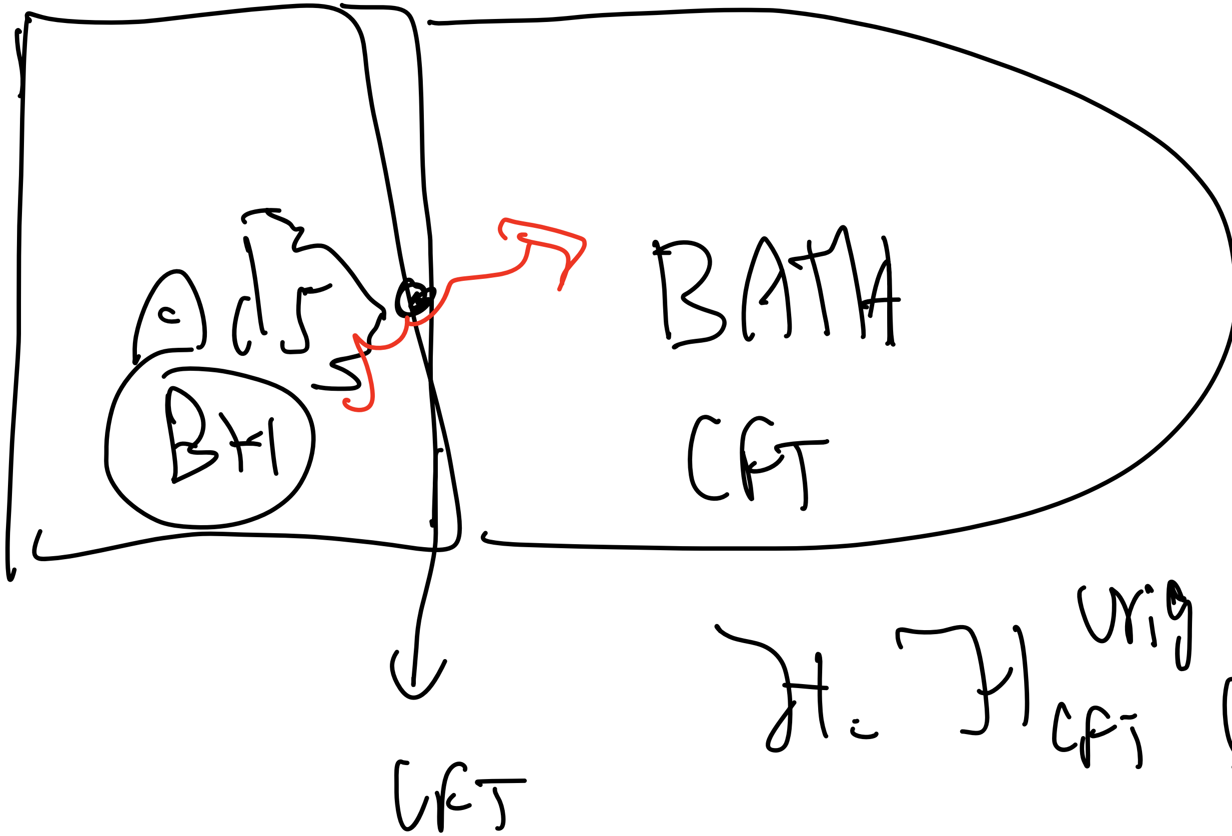
$|\psi\rangle$



$\dim \mathcal{H}_B \approx \dim \mathcal{H}_{rad}$

$$|\psi\rangle \longrightarrow \mathcal{H}^{(t)}_{BH} \otimes \mathcal{H}^{(t)}_{radiation}$$

$$S_{rad} \leq \min \dim\left(\mathcal{H}^{(t)}_{B}, \mathcal{H}^{(t)}_{rad}\right)$$



$$\mathcal{H} = \mathcal{H}^{\text{orig}}_{CFT} \otimes \mathcal{H}_{BATH}$$

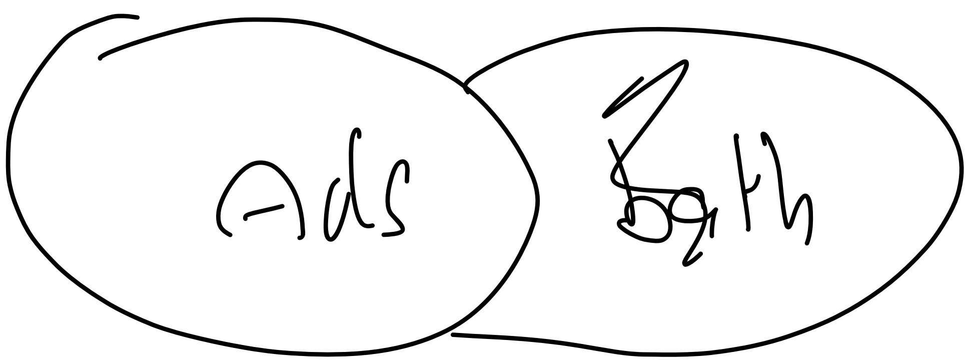
$S_{BATH}(t)$ good question.

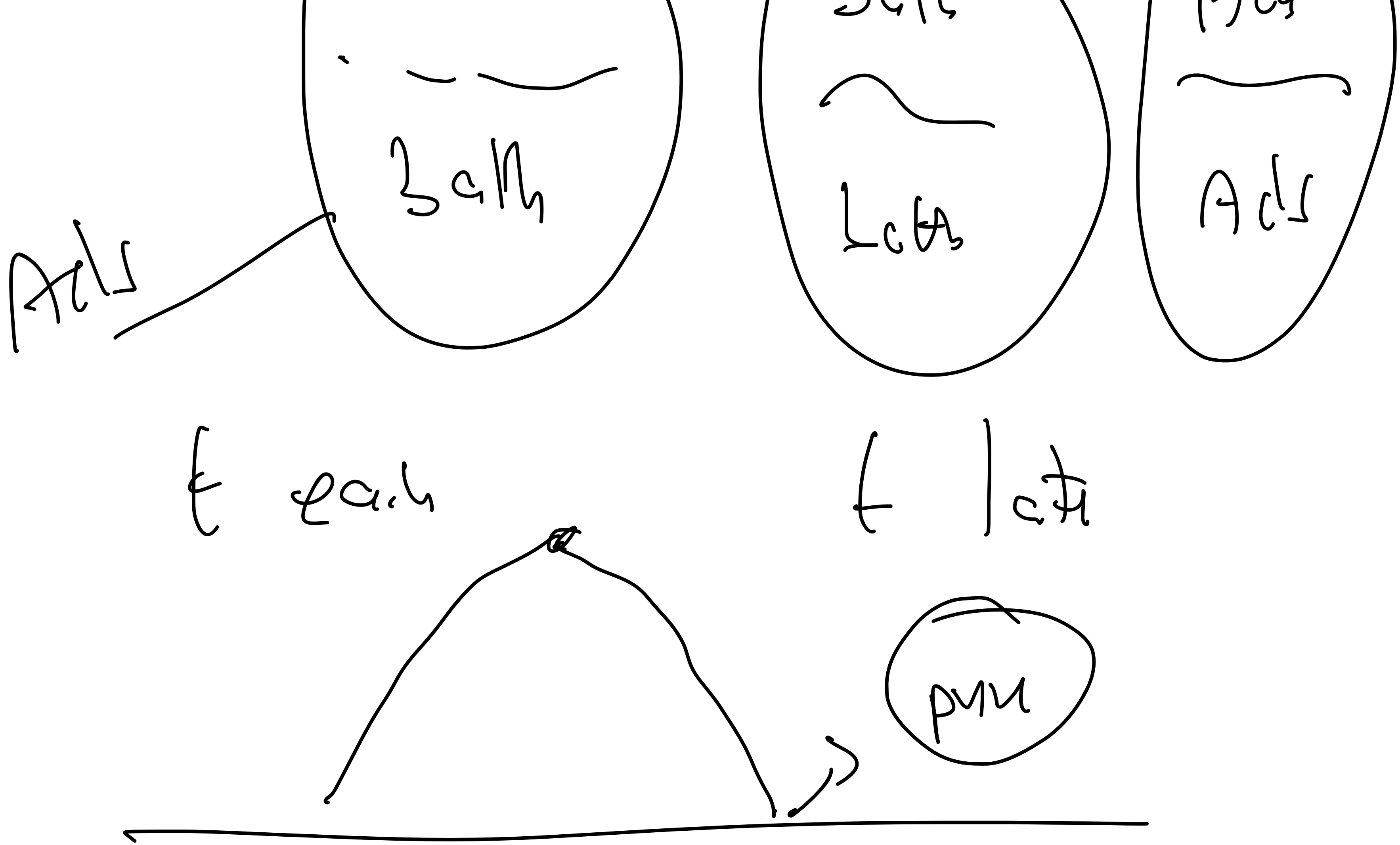
$\mathrm{tr}(\rho^2_{BATH}(t))$ // 1 pure

<1 mixed

$\exists$ Technology $\rightarrow$ replica method

$\rightarrow$ $\mathrm{Tr}(\rho^n)$ $\Longrightarrow$ grav solution w/ particular bc

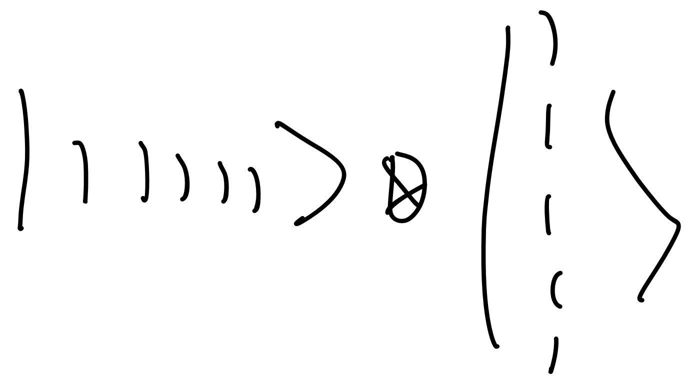



Acts
Acts

$$\begin{pmatrix} e \\ & e^{-S} \\ & & e^{-S} \\ & & & e^{-S} \\ & & & & e^{-S} \end{pmatrix} \longrightarrow \begin{pmatrix} e & e & \cdots \\ e^{-S} \\ & \ddots \\ & & e^{-S} \end{pmatrix}$$

mixed pure

e^{-S}



We have not quite seen the individual degrees of freedom which make up the black hole….