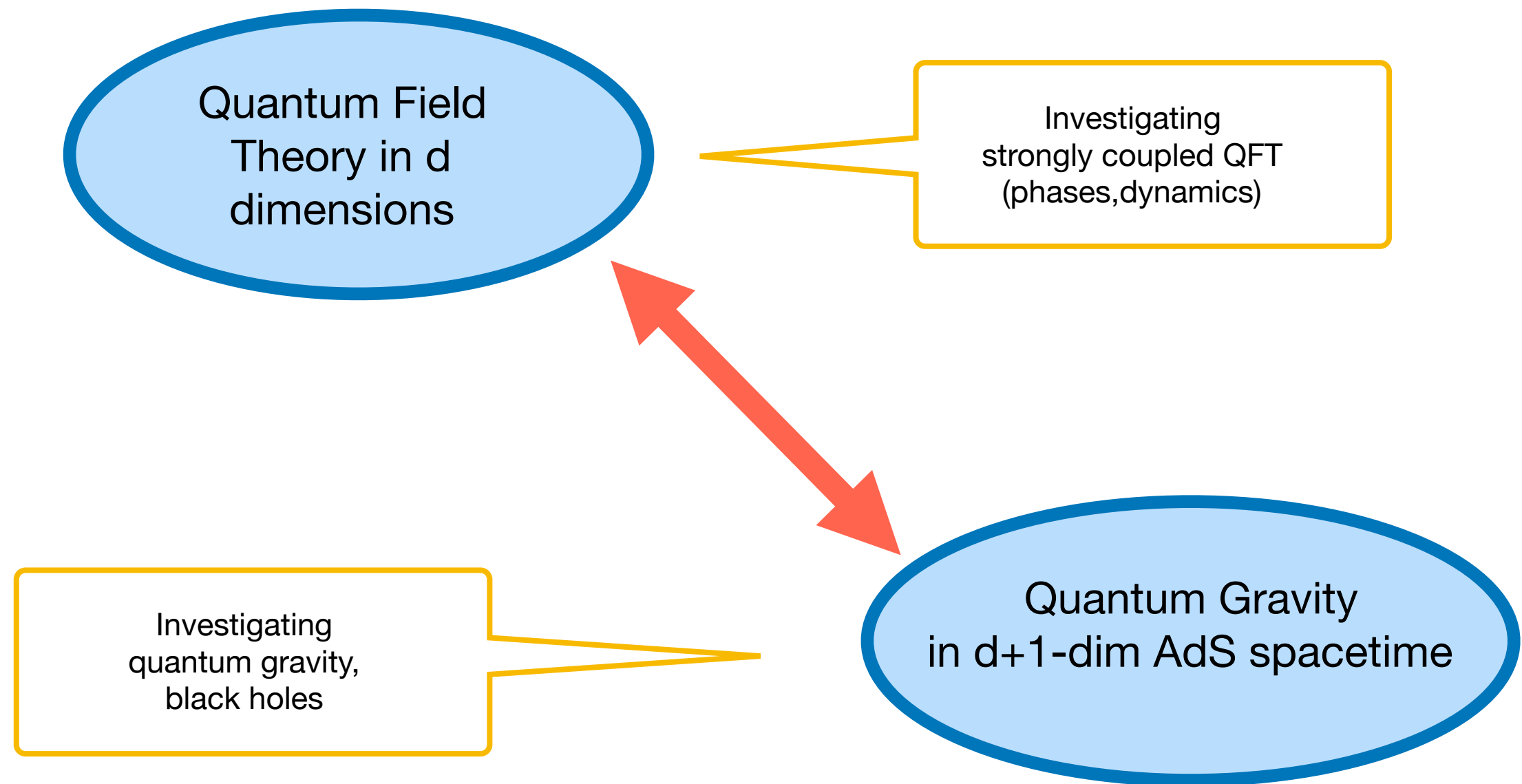
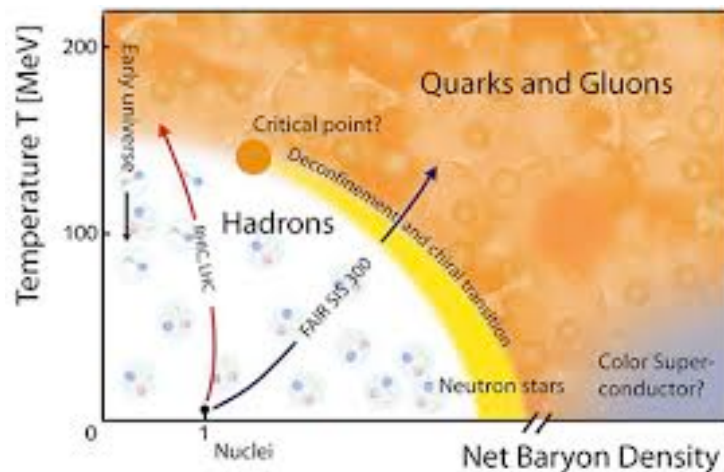


# Introduction to the AdS/CFT correspondence

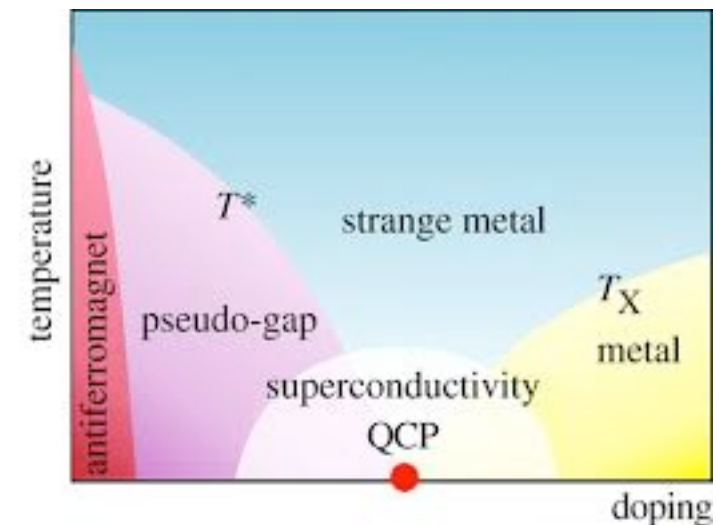


## Strongly coupled QFTs (particle physics)



QCD, confinement, quark-gluon plasma

## Strongly coupled electron systems (condensed matter)



Quantum critical point

The AdS/CFT correspondence provide a tool to investigate strongly coupled QFT (critical points/phase transition) and deformation thereof

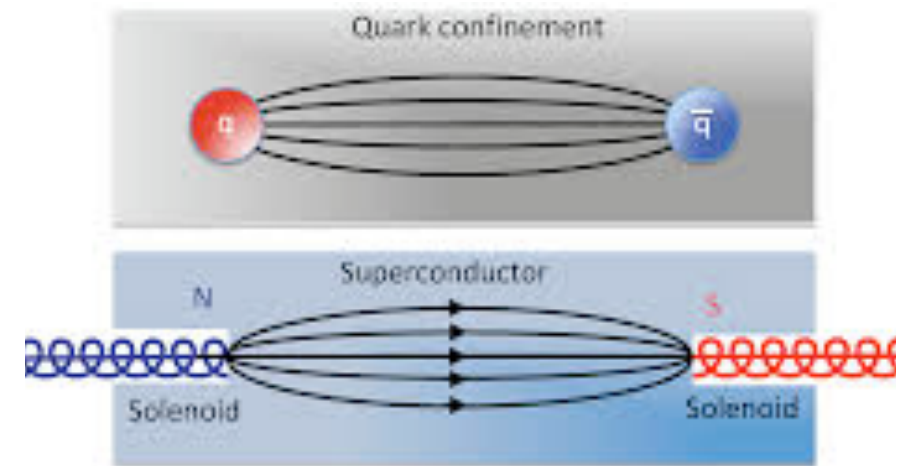
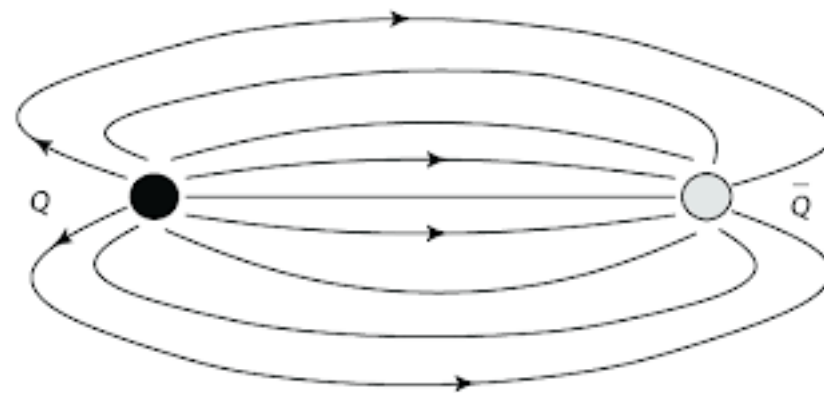
Alternative to

- Lattice quantum field theory (no real time)
- Bootstrap methods

# Gauge theories as string theories

It is an old idea from the seventies that strongly interacting gauge theories have an effective description as a theory of strings

- Confinement (QCD strings)



- Large N expansion

$$O(N^2) \left( \text{planar graphs} \right) + O(1) \left( \text{non-planar graphs} \right)$$

planar graphs:  $f(x)$

$$O(\bar{g}_s^2) \left( \text{circle} \right) + O(1) \left( \text{annulus} \right) + \dots O(\bar{g}_s^{2+2g}) \left( \text{genus } g \right)$$

# Holographic principle

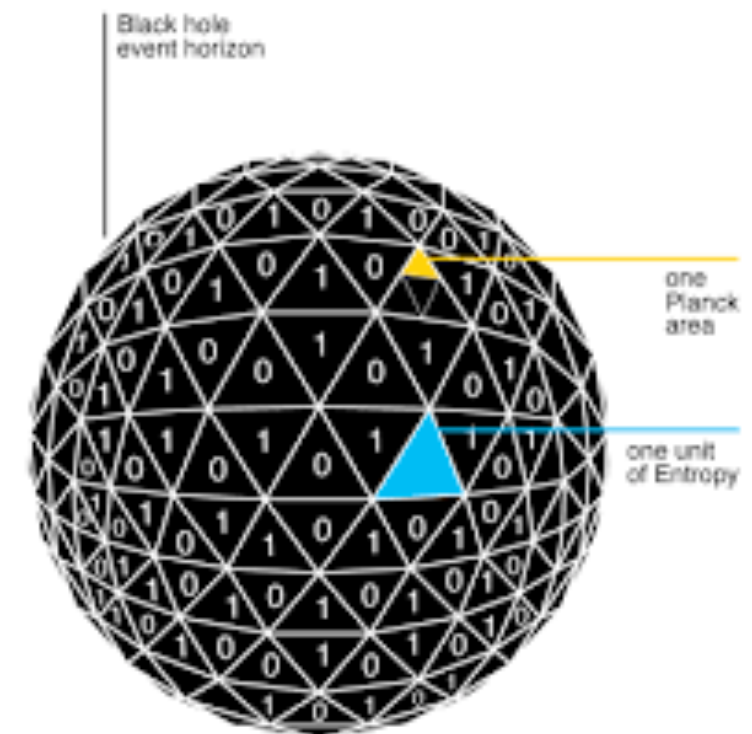
- A volume of space in quantum gravity can be described in terms of boundary degree of freedom
- Number of states in quantum gravity scales as the area of a region

beautiful Bekenstein-Hawking formula

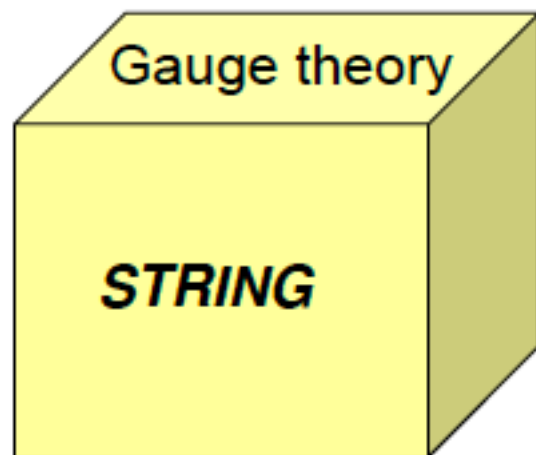
$$S = k_B \frac{Ac^3}{4G\hbar}$$

Diagram illustrating the components of the Bekenstein-Hawking formula:

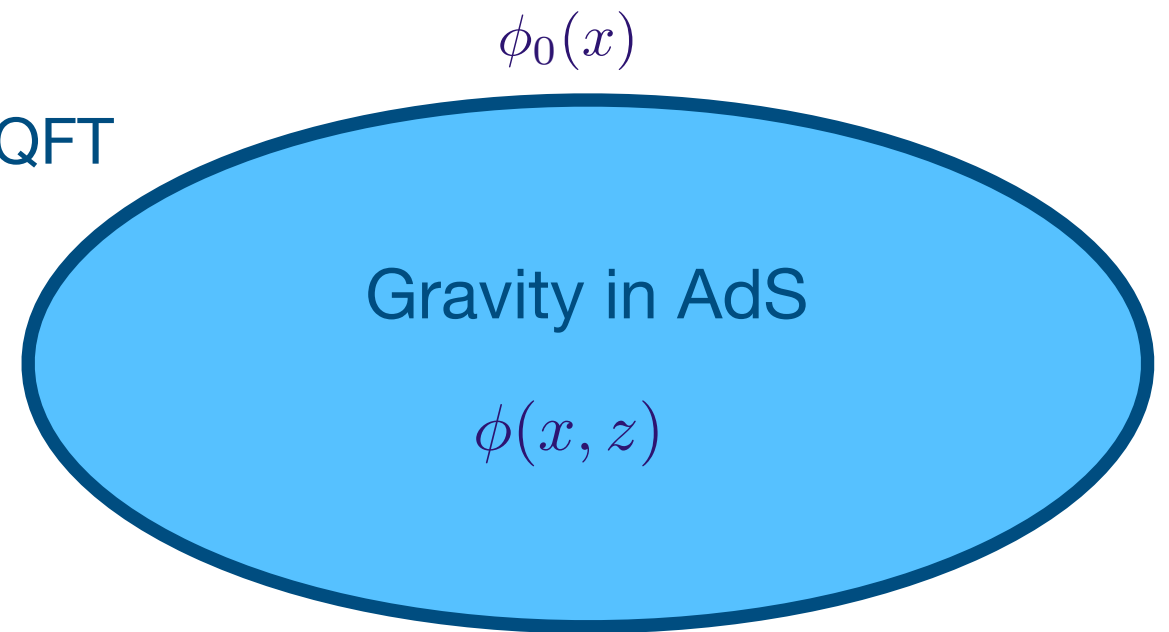
- holography** (red text) points to the  $A$  term.
- relativity** (pink text) points to the  $c^3$  term.
- statistical mechanics** (blue text) points to the  $k_B$  term.
- gravity** (blue text) points to the  $G$  term.
- quantum mechanics** (black text) points to the  $\hbar$  term.



# AdS/CFT in a nutshell



QFT



$$Z_{QFT}(\phi_0(x)) = Z_{gravity}(\phi|_{\partial} = \phi_0(x))$$

AdS/CFT

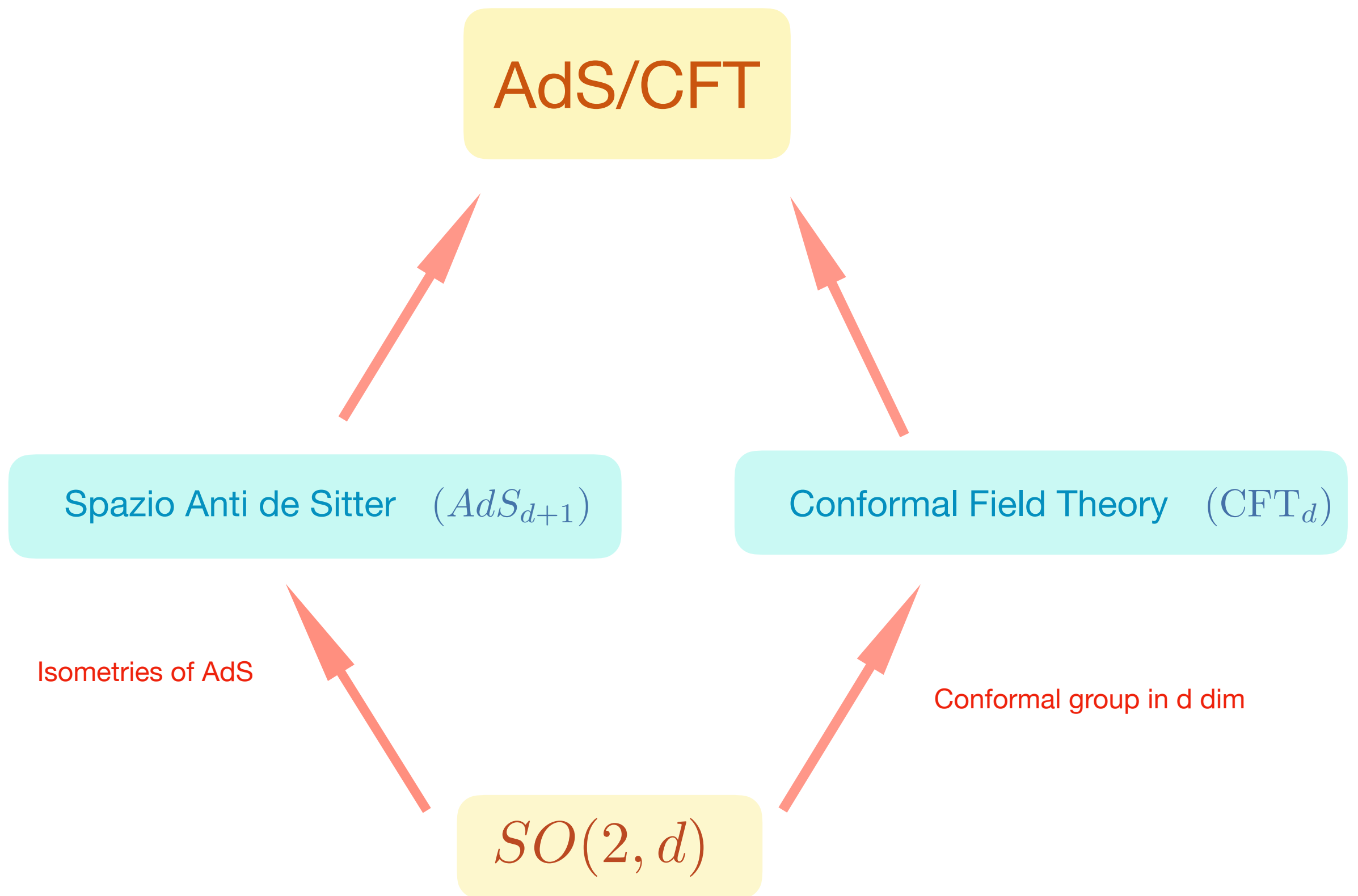
Spazio Anti de Sitter ( $AdS_{d+1}$ )

Conformal Field Theory ( $CFT_d$ )

Isometries of AdS

$SO(2, d)$

Conformal group in d dim



# CFT = conformal field theory

Conformal field theories describe systems with scale invariance

- Renormalization group fixed points
- Phase transitions

- Classical theories without dimensionfull parameters are scale invariant (massless fermions and scalars, Yang-Mills theories)

$$S = \int dx^4 \left( (\partial\phi)^2 + \frac{\lambda}{4!} \phi^4 \right) \quad \phi(x) \rightarrow \lambda^\Delta \phi(\lambda x) \quad \Delta = 1$$

- Scale invariance is typically broken by quantum effects (renormalisation introduces scales=regulators). Scale invariant QFT are either finite or endpoints of the RG flow

$$\mu \frac{d}{d\mu} g = \beta(g) \rightarrow g(\mu), \Lambda_{QCD}$$
$$T_\mu^\mu \sim \beta(g) F_{\mu\nu}^2$$

$$A) \quad \beta(g) = 0$$

$$B) \quad \beta(g): \quad \begin{array}{c} g=0 \qquad \qquad \qquad g^* \\ \xrightarrow{\hspace{1.5cm}} \\ \text{curved arrow from } g=0 \text{ to } g^* \end{array}$$

$\beta(g^*)=0$

# Conformal Invariance versus Scale Invariance

Scale invariance  $x_\mu \rightarrow \lambda x_\mu \quad (dx)^2 \rightarrow \lambda^2 (dx)^2$

Conformal invariance  $x_\mu \rightarrow x'_\mu \quad (dx)^2 \rightarrow (dx')^2 = \Omega^2(x)(dx)^2$

Infinitesimal level

$$x'_\mu = x_\mu + v_\mu(x)$$

$$\partial_\mu v_\nu + \partial_\nu v_\mu - \frac{2}{D}(\partial^\tau v_\tau)\eta_{\mu\nu} = 0.$$

Two dimensions: infinite dimensional group (Virasoro)

D space-time dimensions:

$$x_\mu \rightarrow \frac{x_\mu + c_\mu x^2}{1 + 2cx + (cx)^2} \longrightarrow$$

$\delta x_\mu =$	$a_\mu$	$P_\mu$	
	$\omega_{\mu\nu} x_\nu$	$J_{\mu\nu}$	$(\omega_{\mu\nu} = -\omega_{\nu\mu})$
	$\lambda x_\mu$	$D$	
	$(b_\mu x^2 - 2x_\mu(bx))$	$K_\mu$	

group of dimension:  $D + \frac{D(D-1)}{2} + 1 + D = \frac{(D+1)(D+2)}{2}$



# Conformal group = SO(2,d)

Algebra:

$$[J_{\mu\nu}, J_{\rho\sigma}] = i\eta_{\mu\rho}J_{\nu\sigma} \pm \text{permutation}$$

$$\eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$$

$$[J_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu)$$

$$[J_{\mu\nu}, K_\rho] = i(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu)$$

$$[J_{\mu\nu}, D] = 0$$

$$[D, P_\mu] = iP_\mu$$

$$[D, K_\mu] = -iK_\mu$$

$$[K_\mu, P_\nu] = -2iJ_{\mu\nu} - 2i\eta_{\mu\nu}D$$

Combining:

$$J_{MN} = \begin{pmatrix} J_{\mu\nu} & \frac{K_\mu - P_\mu}{2} & -\frac{K_\mu + P_\mu}{2} \\ -\frac{K_\mu - P_\mu}{2} & 0 & D \\ \frac{K_\mu + P_\mu}{2} & -D & 0 \end{pmatrix} \quad M, N = 1, \dots, D+2$$

$$[J_{MN}, J_{RS}] = i\eta_{MR}J_{NS} \pm \text{permutation}$$

$$\eta_{MN} = \text{diag}(-1, 1, \dots, 1, -1)$$

# AdS: Anti de Sitter space in $D=d+1$

Maximally symmetric space (Einstein in particular)

$$\mathcal{R}_{\mu\nu\tau\rho} = \frac{\Lambda}{(D-1)(D-2)} (g_{\mu\tau}g_{\nu\rho} - g_{\mu\rho}g_{\nu\tau}) \longrightarrow \mathcal{R}_{\mu\nu} = \frac{\Lambda}{D-2}g_{\mu\nu}$$

Pseudo-hyperboloid

$$x_0^2 + x_D^2 - x_1^2 - x_2^2 - x_3^2 - \dots x_{D-1}^2 = R^2$$

$$ds^2 = -dx_0^2 - dx_D^2 + dx_1^2 + dx_2^2 + dx_3^2 + \dots dx_{D-1}^2$$

Isometry

$$SO(2, D-1) \equiv SO(2, d)$$

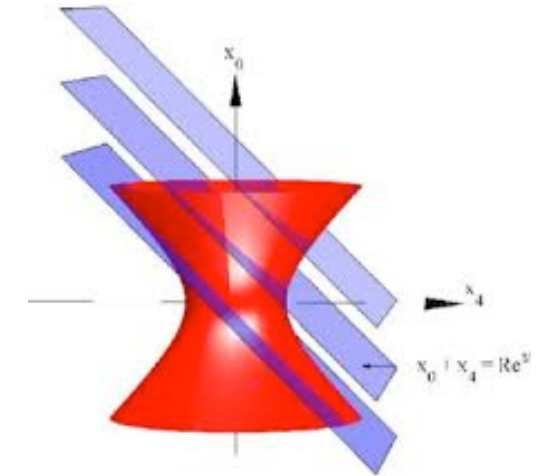
Vacuum of gravitational theory

$$S = \frac{1}{16\pi G_D} \int dx^D \sqrt{|g|} (\mathcal{R} - \Lambda)$$
$$\mathcal{R}_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{R} = -\frac{\Lambda}{2} g_{\mu\nu}$$

# Metrics for AdS5

## Global coordinates

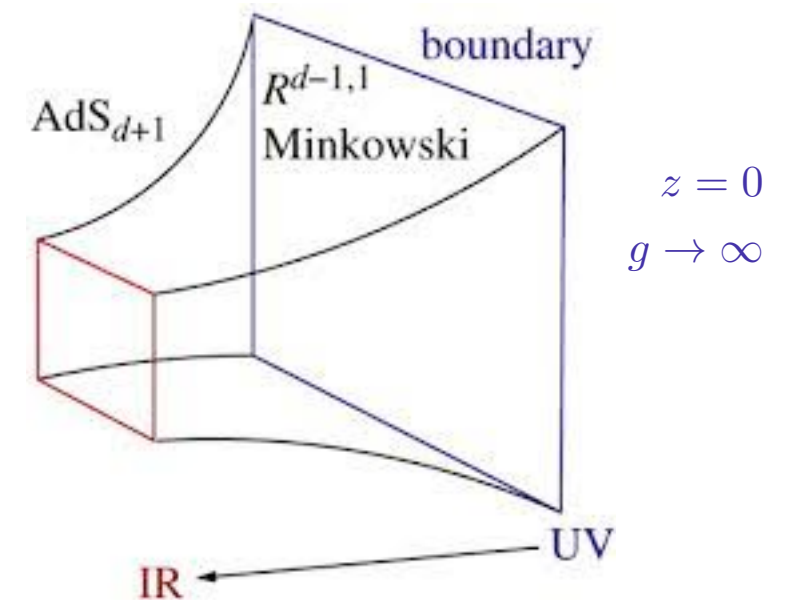
$$ds^2 = R^2 \left( -\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_3 \right)$$



## Poincare coordinates

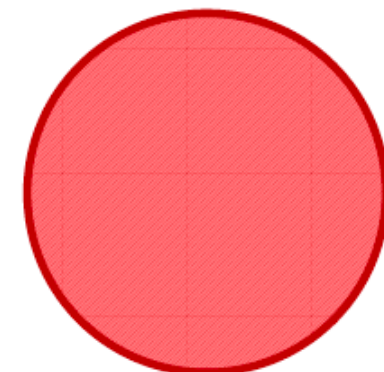
$$ds^2 = R^2 \left( \frac{dz^2 + dx_\mu dx^\mu}{z^2} \right)$$

↑  
 $z = \infty$   
horizon



## Euclidean AdS

$$\mathbb{R}^5 : \quad y_1^2 + \cdots + y_5^2 \leq R^2, \quad ds^2 = \frac{dy^2}{(R^2 - |y|^2)^2}$$



AdS/CFT

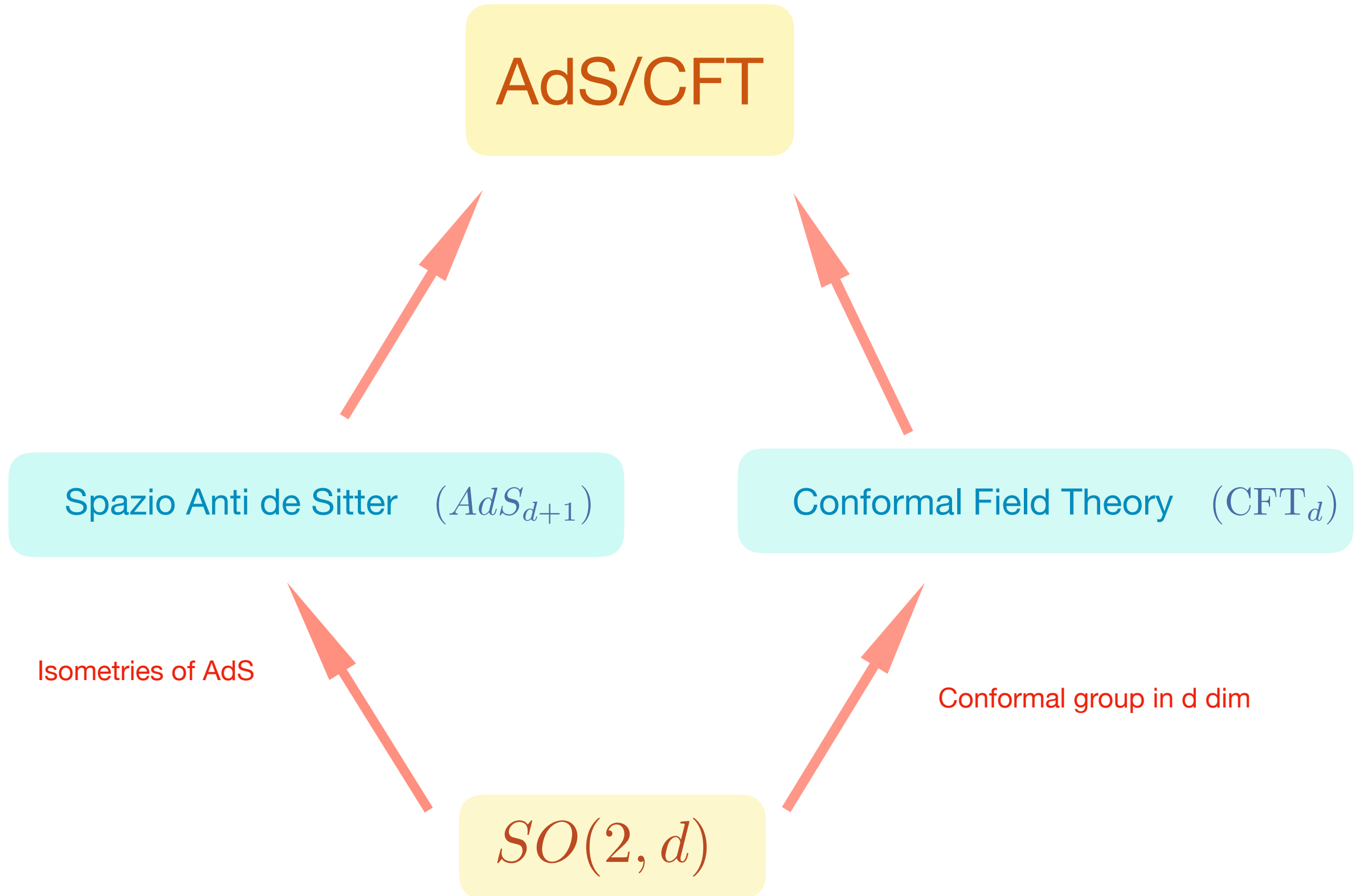
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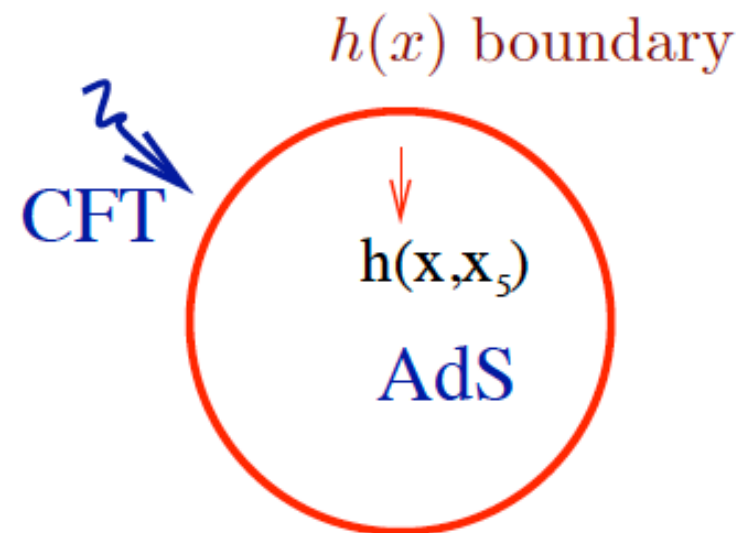
# AdS/CFT working package

A dual pair of:

- CFT in  $d$  dimensions with local operators  $O(x)$
- Quantum gravity theory in  $d+1$  dimensional AdS (string theory etc...) with fields  $h(x,z)$

Couple the operators to background sources  $O(x) \rightarrow h(x)$

$$L_{CFT} + \int d^4x h O$$



$$Z_{\text{QFT}}(h(x)) = \left\langle e^{\int O h} \right\rangle_{\text{QFT}} = Z_{\text{QG}}(h(x, z)|_{\partial} = h(x))$$

# AdS/CFT working package

This is what you do in QFT

$$L_{CFT} + \int d^4x h O$$

$$e^{W(h)} = \left\langle e^{\int h O} \right\rangle_{QFT} \longrightarrow \langle O \dots O \rangle_c = \left. \frac{\delta^n W}{\delta h^n} \right|_{h=0}$$

$$L_{CFT} + \int d^4x \sqrt{g} (g_{\mu\nu} T_{\mu\nu} + A_\mu J_\mu + \phi F_{\mu\nu}^2 + \dots)$$

Field theory path integral with background fields

$$Z_{QFT}(g_{\mu\nu}, A_\mu, \phi)$$

Gravity partition function with boundary values

$$Z_{GQ} \Big|_{b.c.=(g_{\mu\nu}, A_\mu, \phi)}$$

# Symmetries

Global symmetries in the boundary  $\longrightarrow$  Gauge symmetries in the bulk

$$J_\mu \implies A_\mu$$

conserved current

$$\partial^\mu J_\mu = 0$$

gauge field

stress-energy tensor

$$T_{\mu\nu} \implies g_{\mu\nu}$$

# Weakly coupled gravity

Although the correspondence is generic, for QFT purposes it is useful when gravity is weakly coupled and can be treated classically

$$Z_{\text{GQ}} \Big|_{b.c.=(g_{\mu\nu}, A_\mu, \phi)} = \int [d \text{ fields}] e^{iS_{\text{AdS}}(\hat{g}_{\mu\nu}, \hat{A}_\mu, \hat{\phi}, \dots)} \equiv e^{iS_{\text{AdS}}(\hat{g}_{\mu\nu}, \hat{A}_\mu, \hat{\phi}, \dots)} \Big|_{\substack{\text{classical solution} \\ \hat{g}_{\mu\nu}|_{\text{boundary}}=g_{\mu\nu}, \dots}}$$



string theory?

Quantum gravity path integral



In the semiclassical approximation, just evaluate the effective gravitational action with AdS vacuum on the solution of the eqs of motion with prescribed boundary values



# Boundary conditions

Gravity and fields blow up or vanish at the boundary  $z=0$   $ds^2 = R^2 \left( \frac{dz^2 + dx_\mu dx^\mu}{z^2} \right)$

$$S \sim \int dx^5 \sqrt{g} (g^{mn} \partial_m \phi \partial_n \phi + m^2 \phi^2) = \int dz dx \frac{1}{z^5} (z^2 (\partial_z \phi)^2 + z^2 (\partial_\mu \phi)^2 + m^2 \phi^2)$$

Asymptotically

$$\phi \sim z^\Delta$$

$$m^2 = \Delta(\Delta - 4)$$

$\phi \leftrightarrow O$  the scaling dimension of the boundary operator is related to the mass of the bulk field

The general solution

$$\phi \sim \phi_0 z^{4-\Delta} + \phi_1 z^\Delta$$

Boundary conditions:

$$\phi(z, x_\mu) \rightarrow z^{4-\Delta} \phi_0(x_\mu) .$$

# Correlation functions

Given a solution with

$$\phi(z, x_\mu) \rightarrow z^{4-\Delta} \phi_0(x_\mu) .$$

Plugging it into

$$S_{AdS} \sim \int_{\text{boundary}} \sqrt{g} \phi \partial^n \phi + \int \sqrt{g} \phi (-\square + m^2) \phi$$

One obtains

$$S_{AdS} \sim \int dx \phi^0(x) \phi^1(x) \sim \int dx dx' \frac{\phi^0(x) \phi^0(x')}{|x - x'|^{2\Delta}}$$

$$\langle O(x) O(x') \rangle = \frac{\delta^2 S_{AdS}}{\delta \phi^0(x) \delta \phi^0(x')} \Big|_{\phi^0=0} = \frac{1}{(x - x')^{2\Delta}}$$

Operator of scaling dimension  $\Delta$

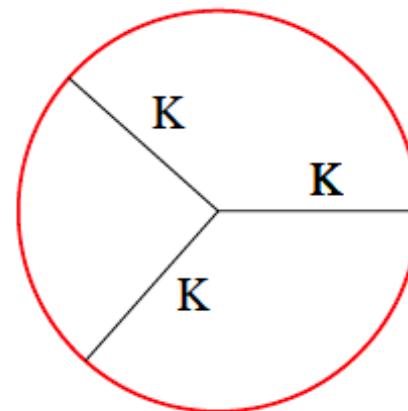
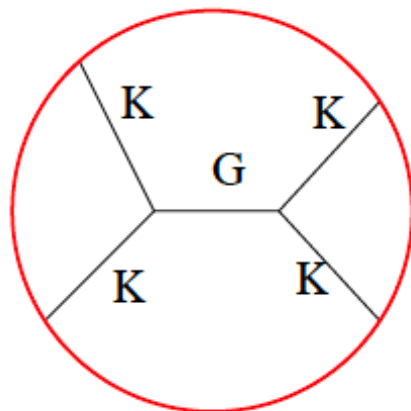
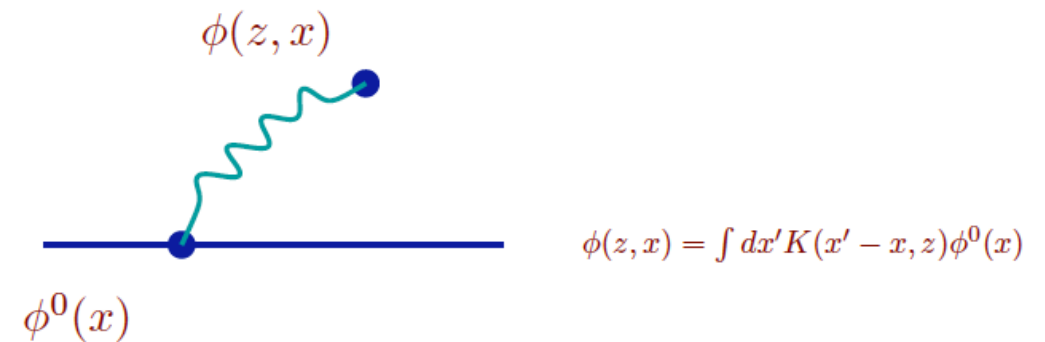
# Higher points correlation functions

$$S_{AdS} = \int dx^5 \left( \frac{1}{2} \sum_i (\partial \phi_i)^2 + \frac{m_i^2}{2} \phi_i^2 + \sum_{k=3}^n \lambda_{i_1 \dots i_k} \phi_{i_1} \dots \phi_{i_k} \right)$$

3-point functions fixed by conformal invariance

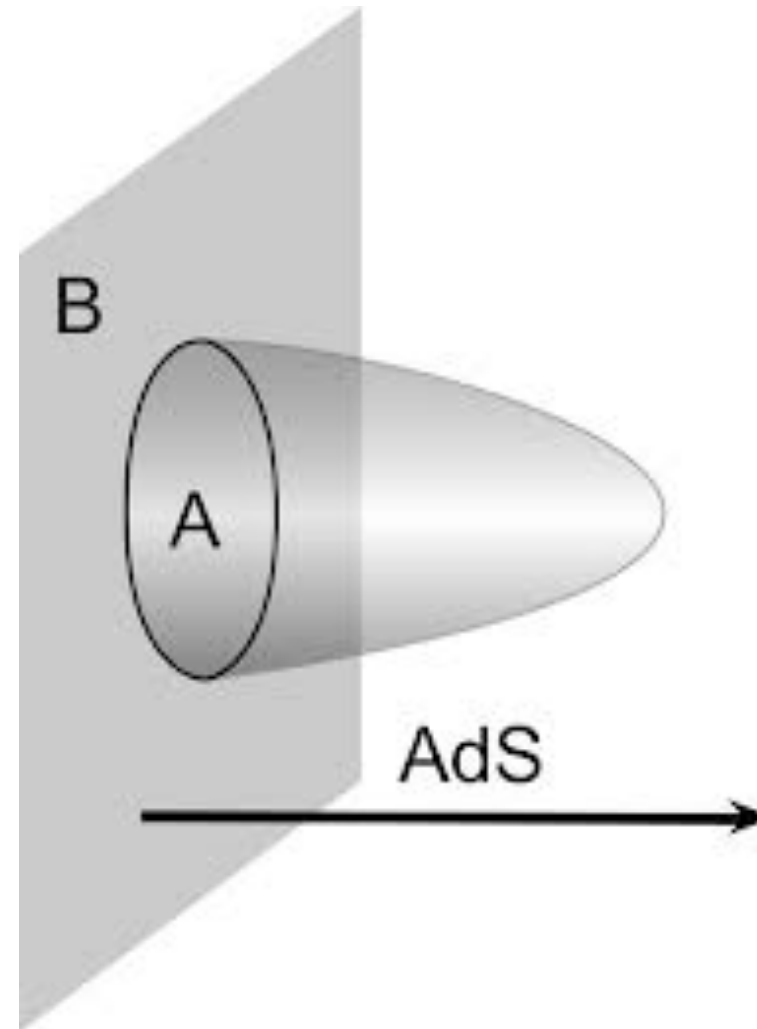
$$\langle O_i(x_i) O_j(x_j) O_k(x_k) \rangle \sim \frac{\lambda_{ijk}}{|x_i - x_j|^{\Delta_i + \Delta_j - \Delta_k} |x_j - x_k|^{\Delta_j + \Delta_k - \Delta_i} |x_k - x_i|^{\Delta_k + \Delta_i - \Delta_j}}$$

Feynman diagram technique (Witten diagrams)



# Other observables

- Wilson loops
- Entanglement entropy



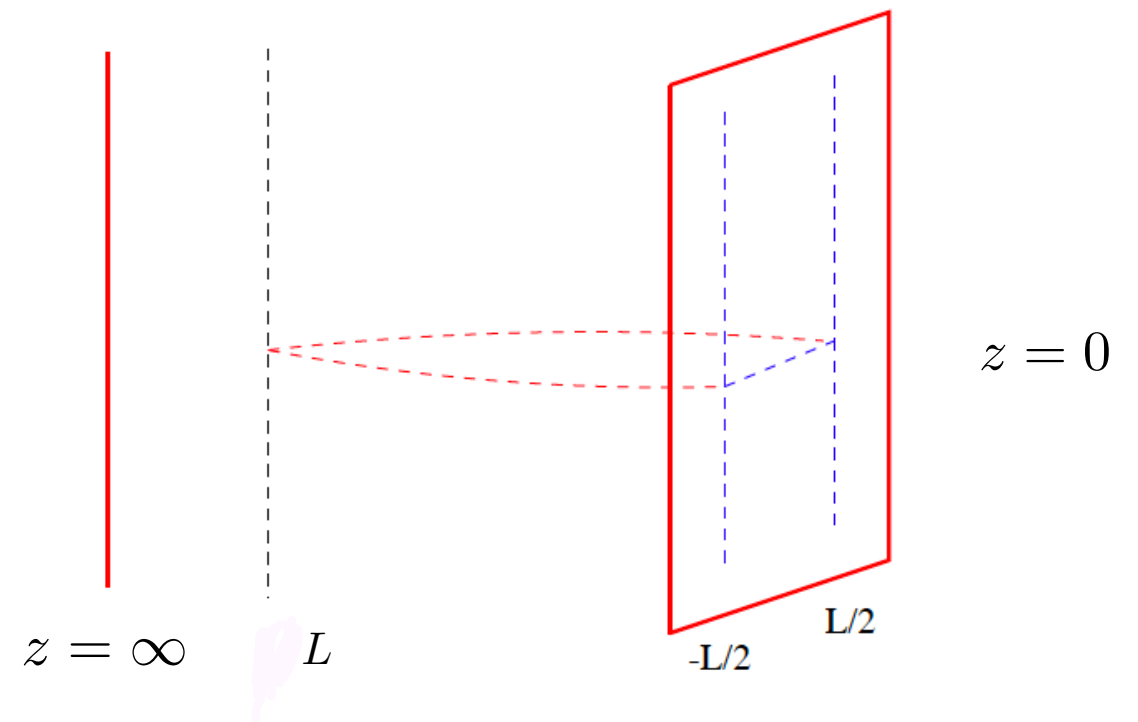
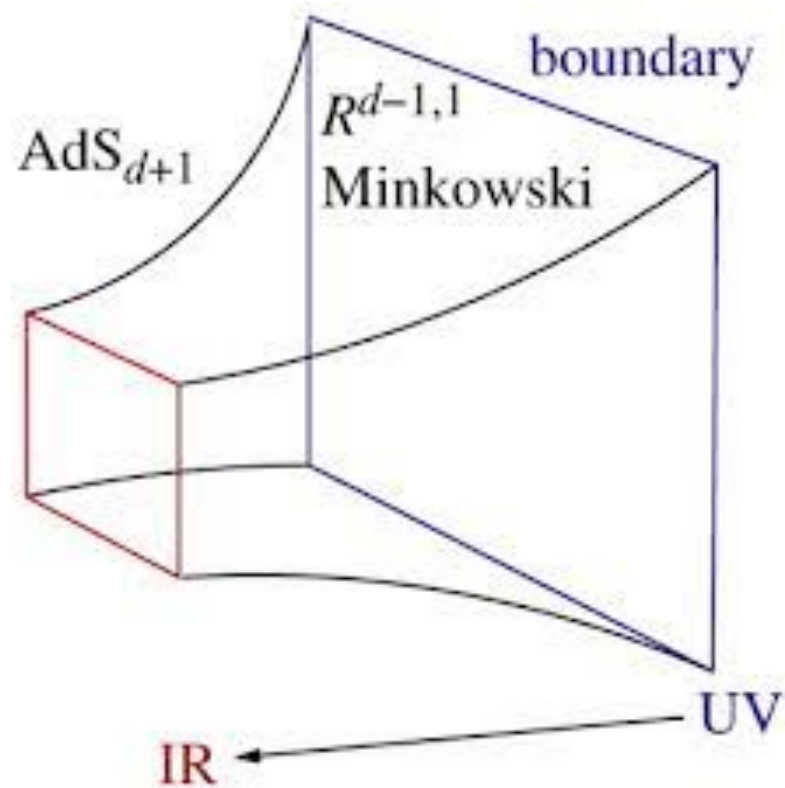
minimal area surfaces

# Scale invariance

$$ds^2 = R^2 \left( \frac{dz^2 + dx_\mu dx^\mu}{z^2} \right)$$

Dilatation:  $x_\mu \rightarrow \lambda x_\mu, \quad z \rightarrow \lambda z$

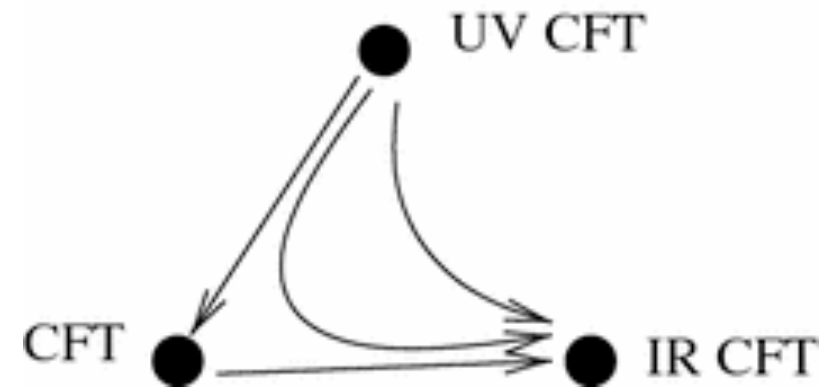
- $1/z$  energy scale
- IR physics controlled by interior of AdS



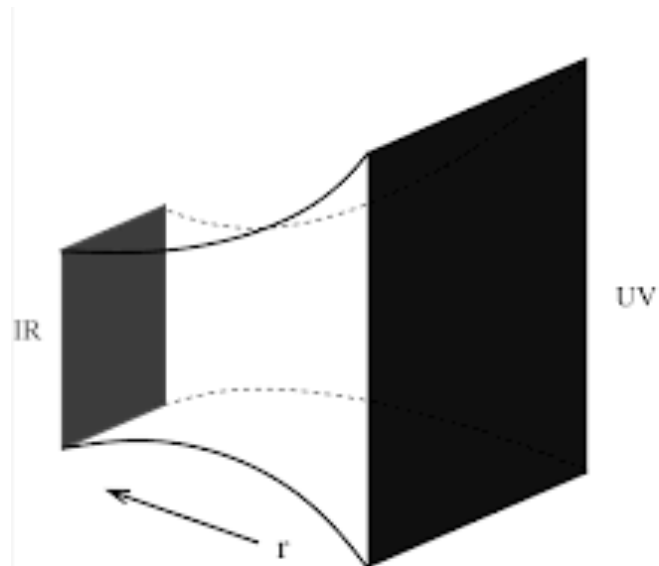
# Breaking conformal invariance

Deforming a CFT by relevant operator

$$L_{CFT} + \int d^4x h O$$



Asymptotically AdS background



$$ds^2 = e^{2A(z)} (dz^2 + dx_\mu dx^\mu)$$

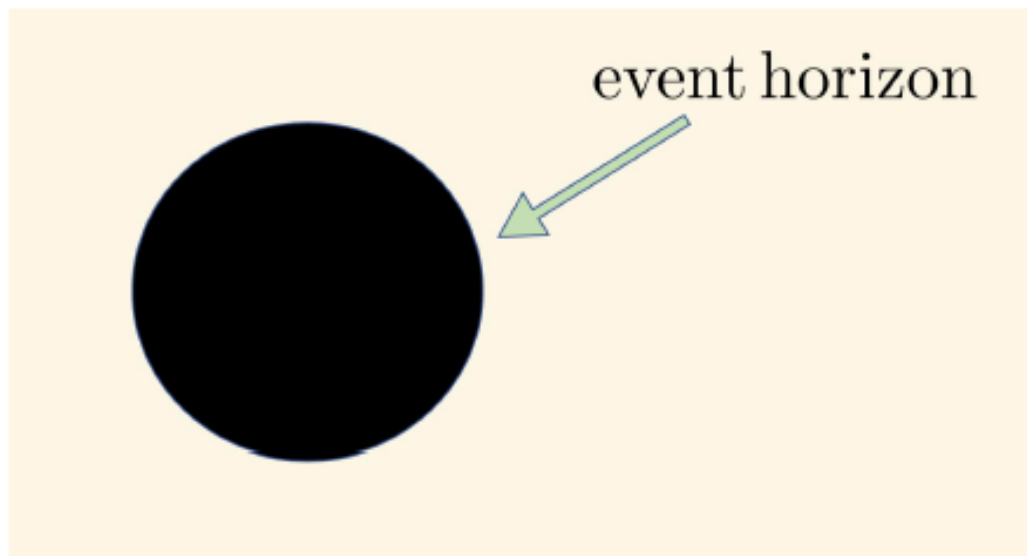
$$e^{2A(z)} \sim_{z \rightarrow 0} \frac{1}{z^2}$$

# CFT at finite temperature

It is an old result that there is striking similarity between black hole mechanics and thermodynamics [Bekenstein; Hawking 70's]

$$dM = \frac{1}{8\pi G} \kappa dA \quad \Longleftrightarrow \quad dE = T dS$$

where  $A$  is the area of the horizon.



The black hole has a temperature and an entropy

$$T = \frac{\hbar \kappa}{2\pi}$$

$$S = \frac{A}{4G\hbar}$$

# CFT at finite temperature

Put a black hole in AdS

$$ds^2 = \frac{L^2}{z^2} \left( \frac{dz^2}{1 - \frac{z^d}{z_0^d}} + \left(1 - \frac{z^d}{z_0^d}\right) dt^2 + d\vec{x}^2 \right)$$

horizon  
 $z = z_0$

boundary  
 $z = 0$

Solution of the effective action

$$S_{EH} = -\frac{1}{16\pi G_N} \int d^{d+1}x \sqrt{g} \left( R + \frac{d(d-1)}{L^2} \right)$$

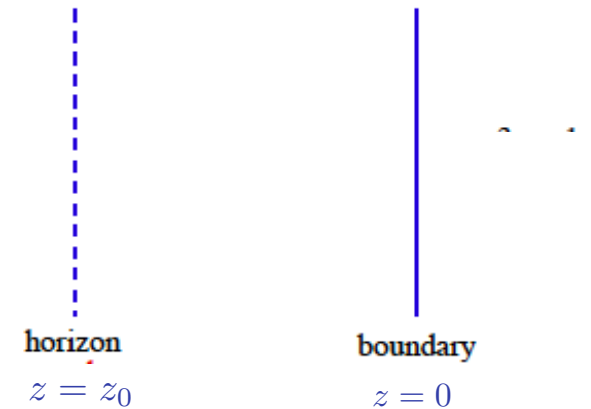


# CFT at finite temperature

Put a black hole in AdS

$$ds^2 = \frac{L^2}{z^2} \left( \frac{dz^2}{1 - \frac{z^d}{z_0^d}} + \left(1 - \frac{z^d}{z_0^d}\right) dt^2 + d\vec{x}^2 \right)$$

$$S = \frac{A}{4G_N} = \frac{L^{d-1}V}{4G_N z_0^{d-1}} \quad T = \frac{d}{4\pi z_0}$$



- Temperature: surface gravity. Go to Euclidean signature and compactly time: metric is smooth for  $t_E \rightarrow t_E + 1/T$

$$ds^2 =_{r \sim \sqrt{z_0 - z}} dr^2 + \frac{4\pi^2}{T^2} r^2 dt_E^2 + \frac{R^2}{z_0} d\vec{x}^2 = dr^2 + r^2 d\phi^2 \dots, \quad \phi \in [0, 2\pi]$$

- Entropy: area

Holography: we can compute the partition function

$$Z_{CFT} \equiv e^{-\beta F} = e^{-S_g[\underline{g}]}$$

$$-\frac{F}{V} = \frac{L^2}{16\pi G_N} \frac{1}{z_0^d} \implies S = -\partial_T F$$

# CFT at finite density

$$L_{\text{CFT}} + \int A_\mu J^\mu \quad \Longrightarrow \quad Z_{\text{CFT}} = \text{Tr} e^{-\frac{1}{T}(H - \mu J)}$$

In the bulk:

$$ds^2 = \frac{L^2}{z^2} \left( -f dt^2 + d\vec{x}^2 + \frac{dz^2}{f} \right)$$

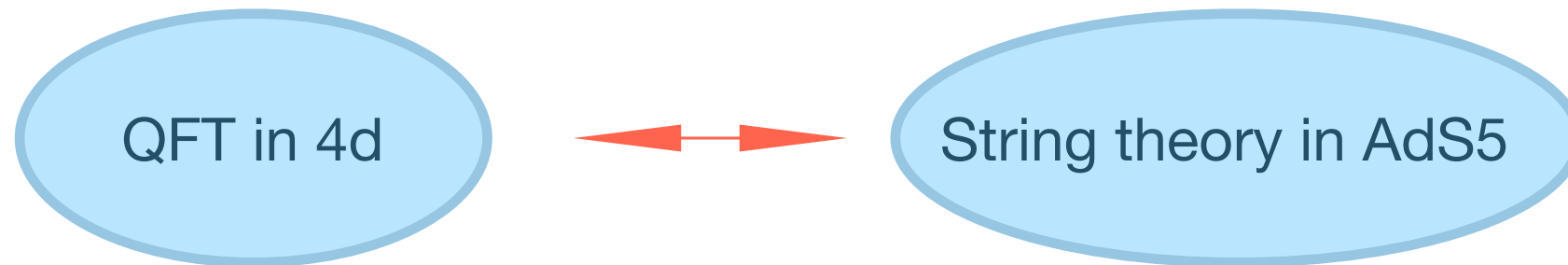
now the black hole is charged

$$f = 1 - Mz^d + Qz^{2d-2}$$

$$A = dt (\mu + \rho z^{d-2})$$

$\mu$  chemical potential       $\rho$  charge density

# Main example



$\mathcal{N} = 4$  Super Yang-Mills  
gauge group  $SU(N)$

Type IIB String theory  
on  $\text{AdS}_5 \times S^5$

*Operators  $O(x)$*

*Bulk Fields  $\phi(x)$*

$$Z_{\text{QFT}}(\phi_0(x)) = \left\langle e^{\int dx^4 O(x) \phi_0(x)} \right\rangle_{\text{QFT}} = Z_{\text{string}}(\phi|_{\partial} = \phi_0)$$

# What we need to know about String theory for these lectures

It reduces to an effective theory of gravity and massless fields at low energy

$$\begin{aligned} S_{IIB} = & \frac{1}{(2\pi)^7(\alpha')^4} \int dx^{10} \sqrt{g} e^{-2\phi} \left( R + 4(\partial\phi)^2 - \frac{1}{12} H_{\mu\nu\tau}^2 \right) \\ & - \frac{\sqrt{g}}{2} \left( (F_\mu)^2 + \frac{\tilde{F}_{\mu\nu\tau}^2}{3!} + \frac{\tilde{F}_{\mu\nu\tau\rho\sigma}^2}{5!} \right) \\ & - \frac{1}{2} \epsilon_{\mu_1 \dots \mu_{10}} C_{\mu_1 \dots \mu_4} H_{\mu_5 \dots \mu_7} F_{\mu_8 \dots \mu_{10}} + \text{fermions} \end{aligned}$$

It contains infinitely many massive fields:  $m^2 = \frac{n}{\alpha'}$

It has two coupling constants:

$\alpha' \implies$  string tension

$g_s = \langle e^{-\phi} \rangle \implies$  string coupling

# What we need to know about String theory for these lectures

The two coupling constants:

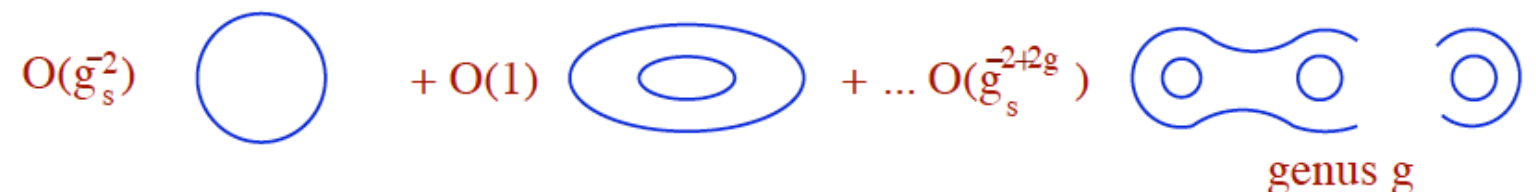
$$\alpha' \implies \text{string tension}$$

$$g_s = \langle e^{-\phi} \rangle \implies \text{string coupling}$$

correspond to a double expansion in the effective action

$$\int dx^{10} \sqrt{g} e^{-2\phi} \mathcal{R} + \dots + \sqrt{g} e^{-2\phi} (\alpha' \mathcal{R}^2 + \dots) \\ + \sum_{g=1}^{\infty} e^{(2g-2)\phi} \sqrt{g} (\dots)$$

Physical quantities have an expansion  $\sum_{g=0}^{\infty} g_s^{2g-2} f_g(\frac{\alpha'}{R^2})$

$$O(\bar{g}_s^{-2}) \quad \bigcirc \quad + O(1) \quad \bigcirc \quad + \dots O(\bar{g}_s^{-2+2g}) \quad \text{genus } g$$


# What we need to know about N=4 SYM for these lectures

$$S = \frac{1}{g_{YM}^2} \int d^4x \text{Tr} \left( -\frac{F_{\mu\nu}^2}{2} - i\bar{\psi}^a \not{D} \psi_a - (D_\mu \phi^i)^2 + C_i^{ab} \psi_a [\phi^i, \psi_b] + [\phi^i, \phi^j]^2 \right)$$

Non-abelian gauge fields  $A_\mu$

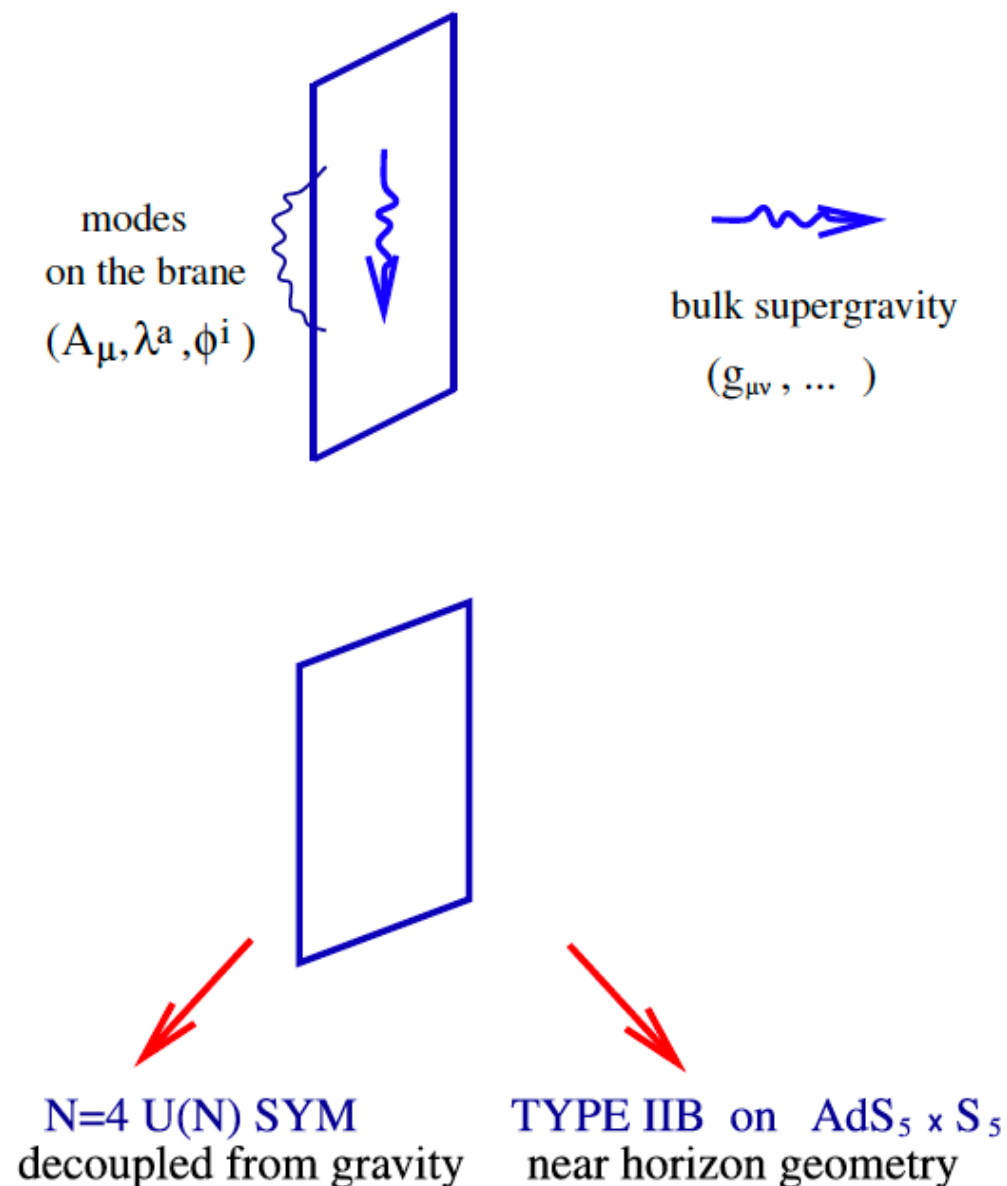
6 real scalar fields  $\phi^i$

4 well fermions  $\psi_a$

symmetry  $SO(6) = SU(4)$  rotating scalars and fermions

# Near horizon geometry

Examples of dual theories arise in string theory by taking the near horizon geometry of stacks of D-branes, extended objects supporting gauge theories on the world-volume



# Near horizon geometry

Near a brane for  $r$  small

$$ds^2 = H^{-1/2} dx_\mu dx^\mu + H^{1/2} (dr^2 + r^2 \Omega_5)$$

$$F^{(5)} = \text{flux of a charged object}$$

$$e^\phi = g_s \equiv \frac{g_{YM}^2}{4\pi}$$

$$H = 1 + \frac{g_{YM}^2 N \alpha'^2}{r^4}$$

$$ds^2 \sim \alpha' R^2 \left( \frac{dz^2}{z^2} + \frac{dx^2}{z^2} + \Omega_5 \right)$$

$$z = \alpha' / r$$

with the identification

$$g_s \equiv \frac{g_{YM}^2}{4\pi}$$

$$R^2 = \alpha' \sqrt{g_{YM}^2 N}$$



# Symmetries

$N = 4$ SYM	Type IIB on $AdS_5 \times S^5$
Conformal group $SO(4, 2)$	Isometry group of $AdS_5$ $SO(4, 2)$
Supersymmetries 8 = 4 linear +4 conformal	Type IIB on $AdS_5 \times S^5$ has $N = 8$ susy
$R$ -symmetry $SU(4)$	Isometry of $S^5$ $SO(6) = SU(4)$

# Parameters

$$4\pi g_s = \frac{x}{N}$$
$$\frac{R^2}{\alpha'} = \sqrt{x}$$

where  $x = g_{YM}^2 N$   
is the t'Hooft coupling

string theory useful when weakly coupled reducing to supergravity:

- string loop suppressed
- higher derivative suppressed

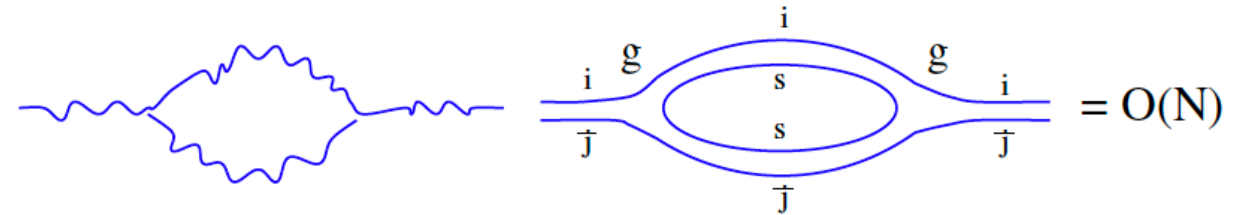
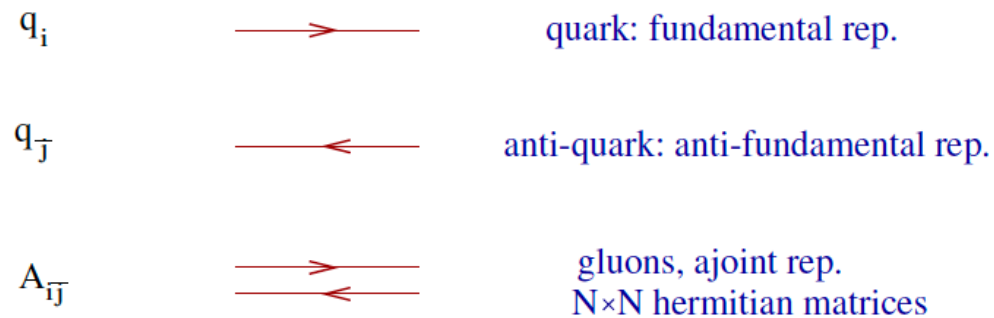
$$g_s \rightarrow 0$$
$$\alpha'/R^2 \rightarrow 0$$

both implemented in the t'Hooft limit

$$N \rightarrow \infty, x = g_{YM}^2 N \text{ fixed,}$$

# t'Hooft limit or large N expansion

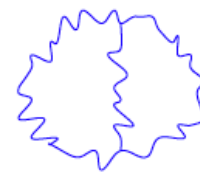
A way of repacking Feynman graphs



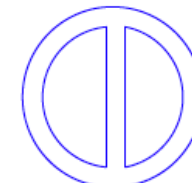
Graphs are finite in the t'Hooft limit

$$N \rightarrow \infty, x = g_{YM}^2 N \text{ fixed.}$$

and organise in terms of topology



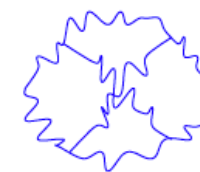
planar graph



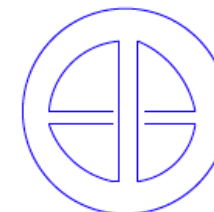
$O(N^2)$



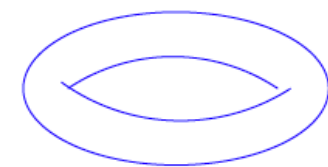
sphere



non planar  
(first non-trivial topology)



$O(1)$



torus

# Double expansion

Expansions implemented in the t'Hooft limit

$$N \rightarrow \infty, x = g_{YM}^2 N \text{ fixed,}$$

$$4\pi g_s = \frac{x}{N}$$

$$\frac{R^2}{\alpha'} = \sqrt{x}$$

$$\sum_{g=0}^{\infty} N^{2-2g} f_g(x)$$

CFT

$$\langle O O \rangle = \left( \text{planar graphs} + \dots \right) + \frac{1}{N^2} \left( \text{non-planar graphs} + \dots \right)$$

planar graphs:  $f(x)$

$$\sum_{g=0}^{\infty} g_s^{2g-2} f_g\left(\frac{\alpha'}{R^2}\right)$$

AdS

$$\langle h h \rangle =$$



world-sheet corrections  
higher derivatives terms

# Weak coupling/strong coupling duality

$$4\pi g_s = \frac{x}{N}$$
$$\frac{R^2}{\alpha'} = \sqrt{x}$$

where  $x = g_{YM}^2 N$   
is the t'Hooft coupling

string theory useful when weakly coupled reducing to supergravity:

- string loop suppressed
- higher derivative suppressed

$$g_s \rightarrow 0$$
$$\alpha'/R^2 \rightarrow 0$$

$$N \rightarrow \infty \quad x \gg 1$$

Weakly coupled string theory/ strongly coupled CFT

# Why should we believe it?

Many checks of the duality (made easy by supersymmetry):

- spectrum of operators at strong coupling/ gravity modes
- Wilson loop
- quantitative comparison with integrability results
- baryons and non/perturbative objects

# Many extensions

- many extensions to other supersymmetric AdS vacua of string theory/ M theory  
(zoo of superconformal CFT in various dimensions)
- higher spin theories ( $O(N)$  models)
- JT gravity (SYK models)

# Basic Questions

- ▶ 1. What kind of field theories are described by gravity?

gauge theories at large number of colors (or flavors)

$$L/l_P \sim \text{number of d.o.f} \sim N^2$$

- ▶ 2. What is the UV completion of the gravitational theory?

string theory (two parameters  $l_s$  and  $g_s$ )

M theory (still mysterious: 11 d supergravity, membranes)

- ▶ 3. Can we use field theory to learn about gravity in curved space?

AdS/CFT as a non-perturbative completion of some M/string backgrounds



# Decoupling of operators

Two kind of gravity modes:

supergravity fields (KK modes on S5)

CFT operators of finite dimensions  
of maximum spin 2

string modes

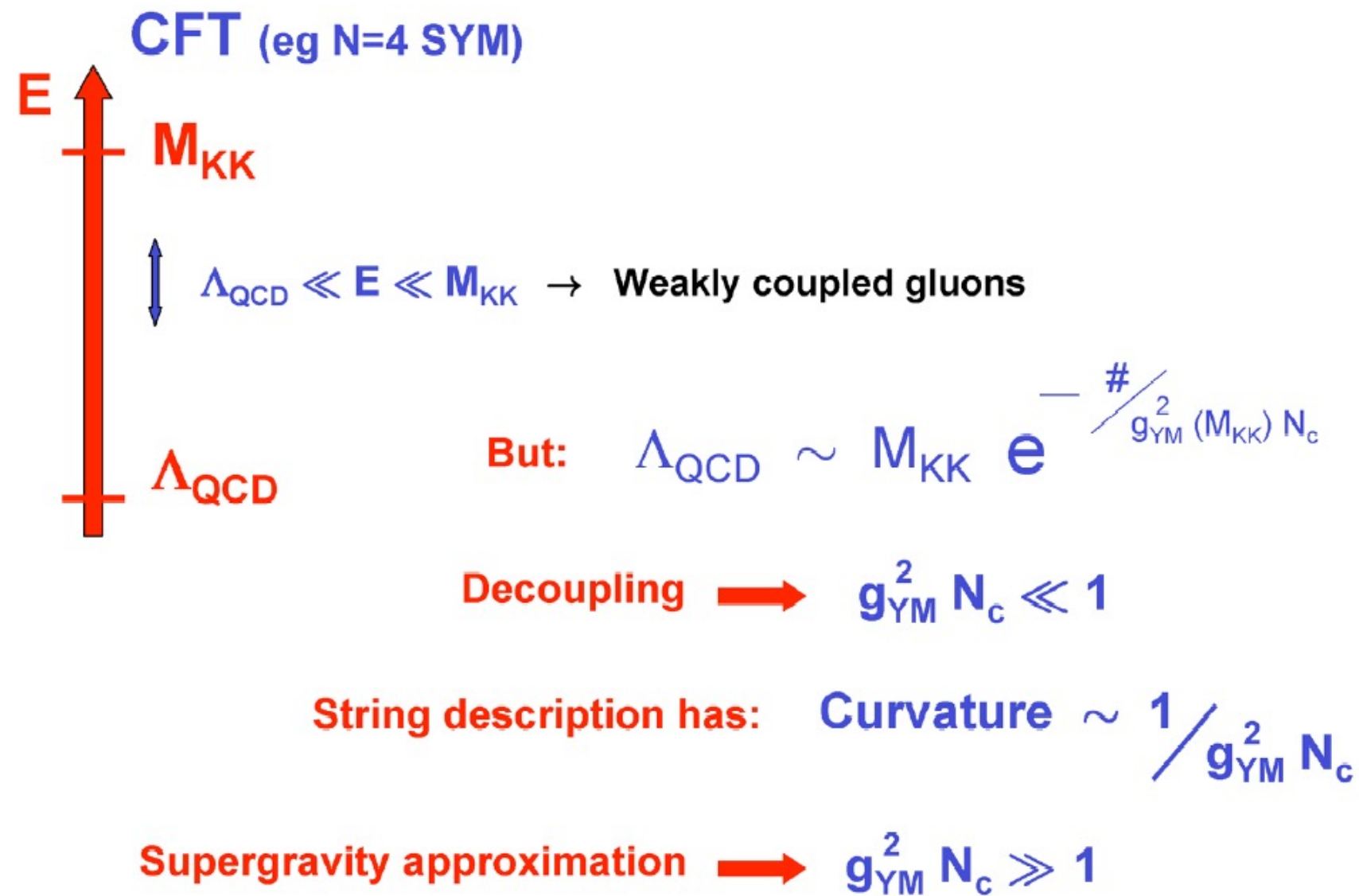
CFT operators with large anomalous  
dimensions at strong coupling

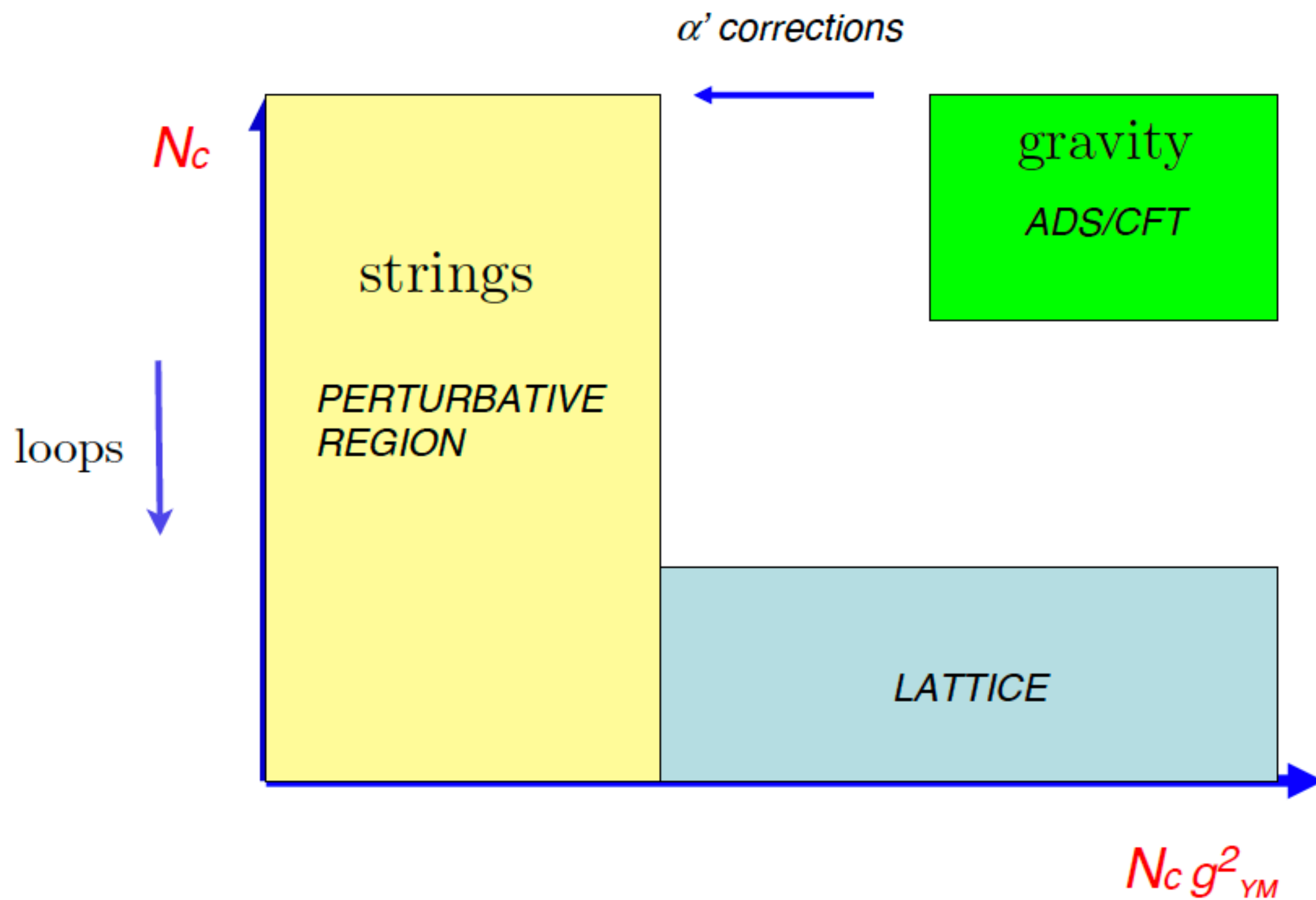
$$m^2 = \frac{\Delta(\Delta - 4)}{R^2} \sim \frac{1}{\alpha'} = \frac{\sqrt{x}}{R^2} \quad \rightarrow \quad \Delta \sim x^{1/4}$$

This is not what happens in QCD: operators and bound states of all spins

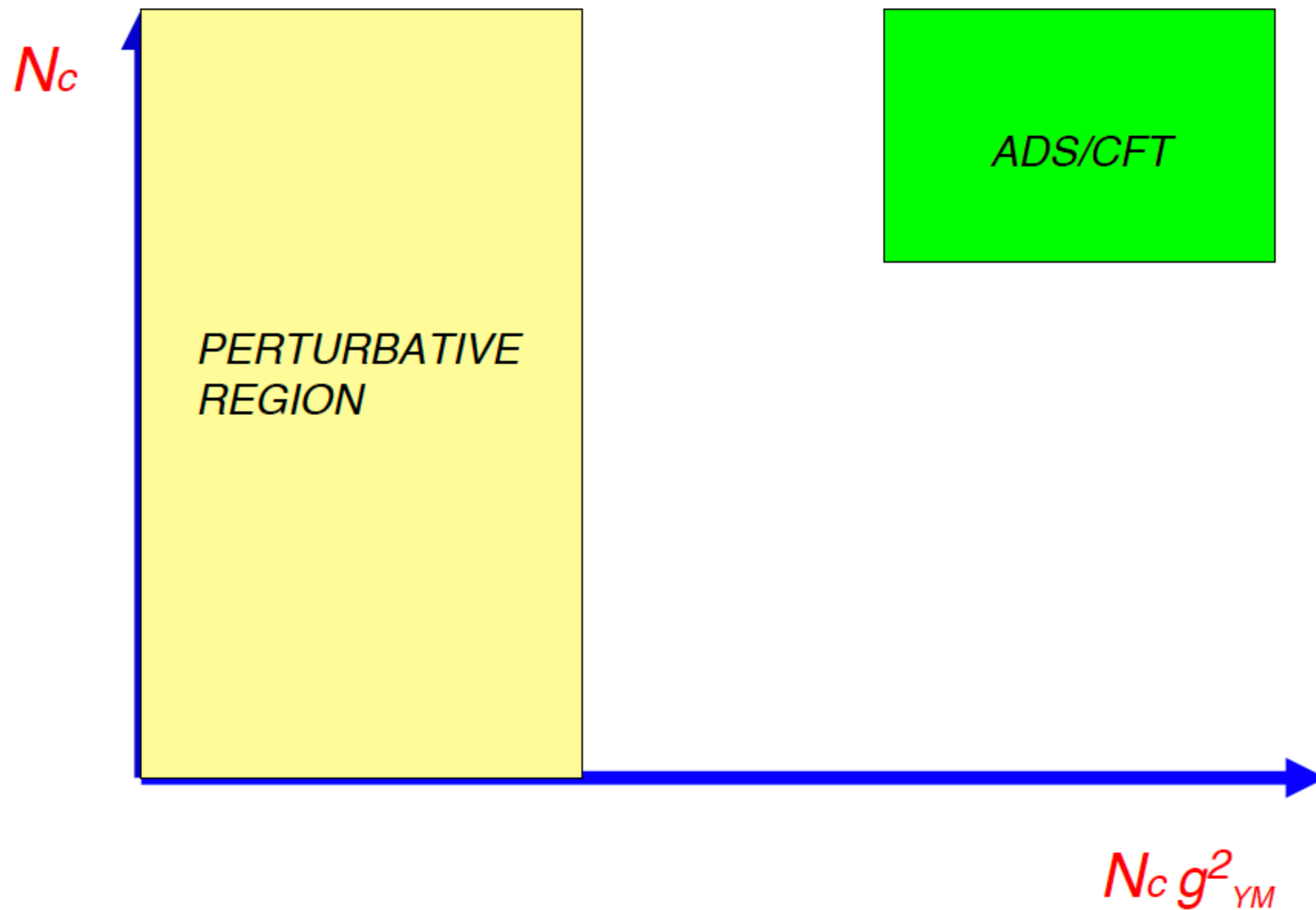
Can we study QCD (or pure YM) ?

No, which means:





*Non equilibrium – Time dependent processes*



1) Resummation of higher derivatives corrections is in principle doable in string theory: it can be done in flat space and selected backgrounds. **Only a technical problem prevent solution of QCD at large N.** Can be used as an effective model: AdS/QCD

2) Interesting application to non equilibrium physics: Hydrodynamic properties of hot dense QCD plasma (**shear viscosity, jet quenching parameter -- RHIC**).

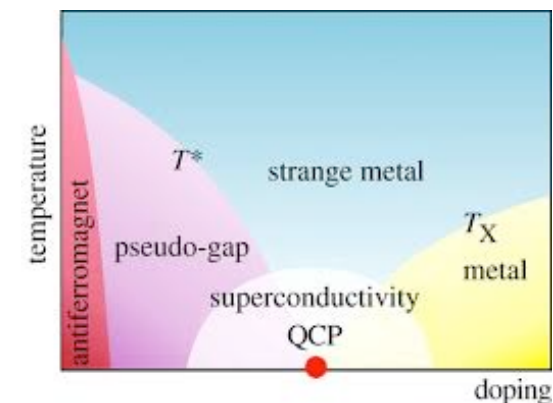
**Strongly coupled:** no perturbation theory!

**Time dependent process:** no lattice simulation!

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

Kovtun, Son & Starinets

3) application to condensed matter theory: a new Landau-Ginsburg paradigm for strongly correlated electronic systems (high-T superconductors, non-Fermi liquids)



4) interesting connection between holography and quantum information (entanglement, error-corrections codes...)



AdS/CFT today



AdS/CFT

