

Sterile neutrinos in $0\nu\beta\beta$

Wouter Dekens

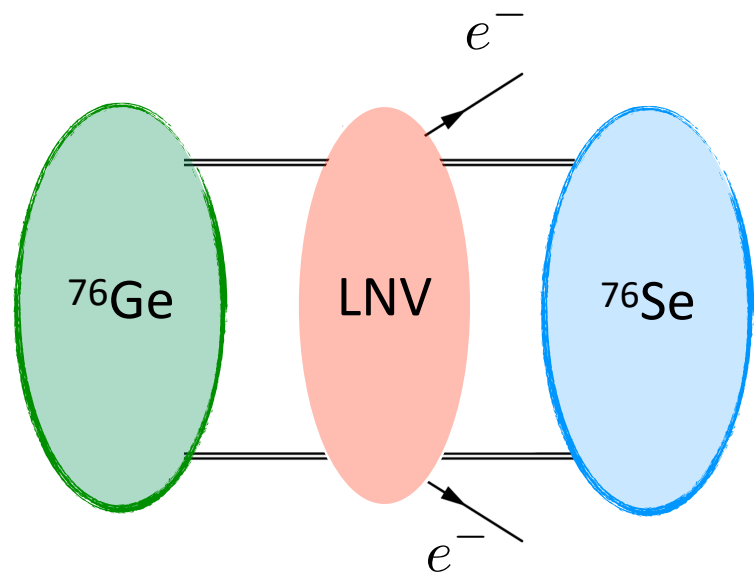
with

J. de Vries, K. Fuyuto, J. Menéndez,
E. Mereghetti, P. Soriano, V. Plakkot, G. Zhou

arXiv:2303.04168, 2002.07182



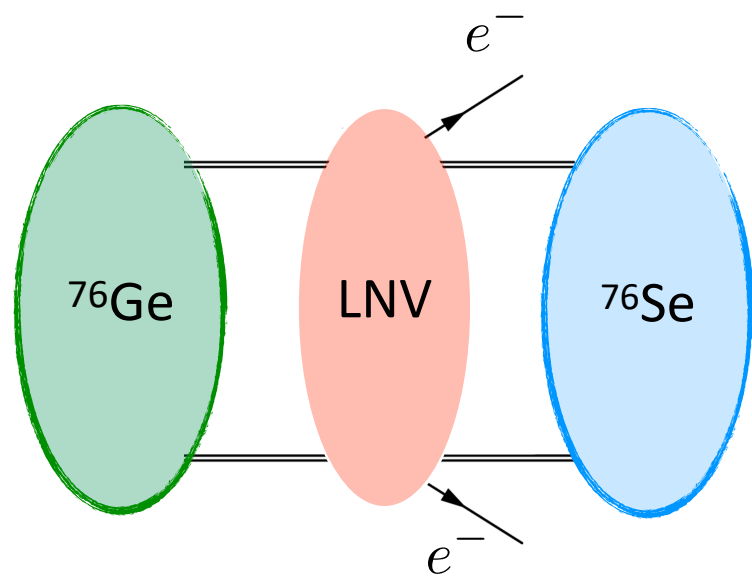
$0\nu\beta\beta$



- Violates lepton number, $\Delta L=2$
- Stringently constrained experimentally
 - To be improved by 1-2 orders

$T_{1/2}^{0\nu}(^{76}\text{Ge})$	$T_{1/2}^{0\nu}(^{130}\text{Te})$	$T_{1/2}^{0\nu}(^{136}\text{Xe})$
Gerda	Cuore	KamLAND-zen
$> 9 \cdot 10^{25} \text{ yr}$	$> 3.2 \cdot 10^{25} \text{ yr}$	$> 1.1 \cdot 10^{26} \text{ yr}$

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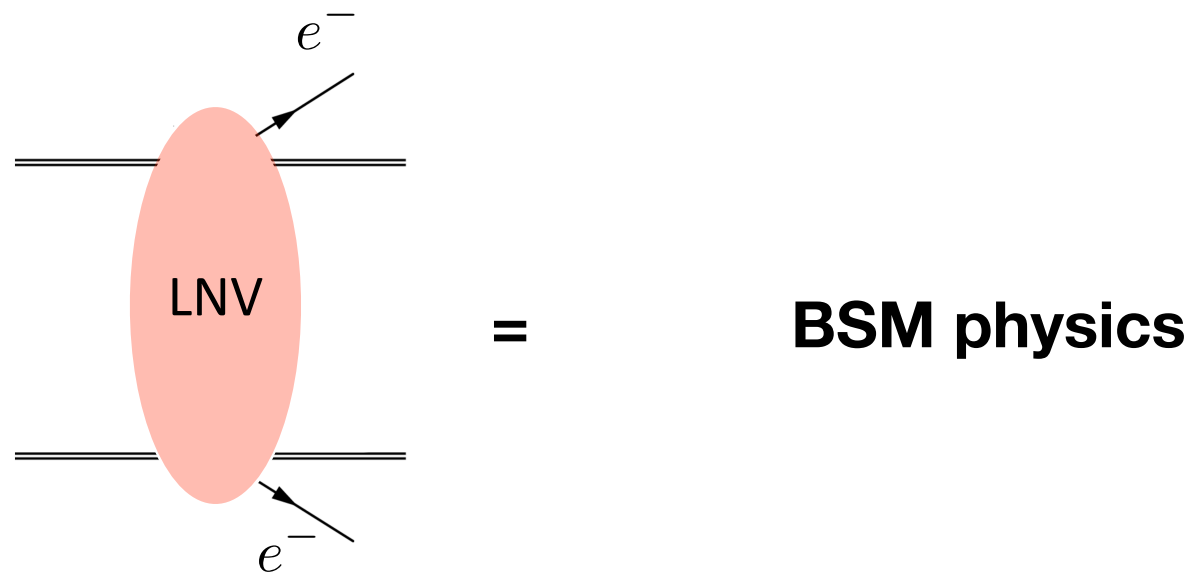


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- Would imply that:
 - Neutrino's are Majorana particles
 - Physics beyond the SM

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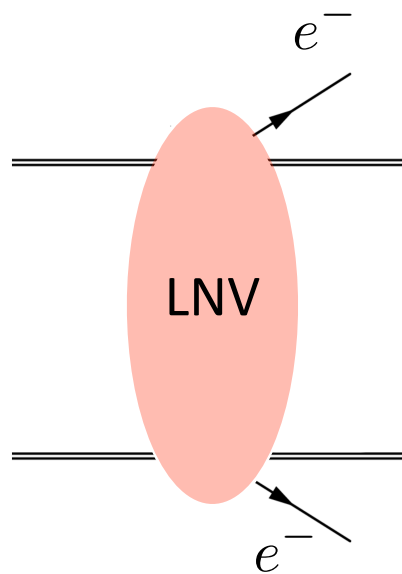


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=

This talk: Sterile Neutrinos

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Sterile neutrinos

- ν_R 's could help solve several SM deficiencies:
 - Neutrino masses
 - Leptogenesis
 - Dark matter candidate
- Appear in Left-Right/Leptoquarks/GUTs

Canetti et al. '13

Boyarski et al. '19

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- Add n singlets, ν_R , to the SM(EFT):

$$\mathcal{L}_{\nu_R} = i\bar{\nu}_R \not{\partial} \nu_R - \frac{1}{2} \bar{\nu}_R^c M_R \nu_R - \bar{L} \tilde{H} Y_D \nu_R + \mathcal{L}_{\nu_R}^{(6)} + \mathcal{L}_{\nu_R}^{(7)}$$

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Heavy BSM physics

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Majorana mass

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This talk: minimal scenario

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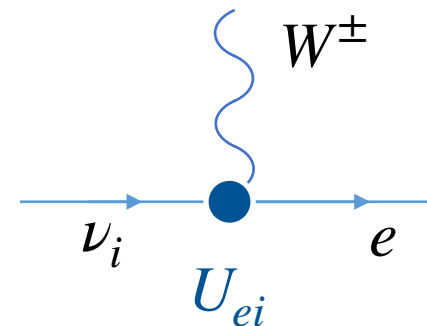
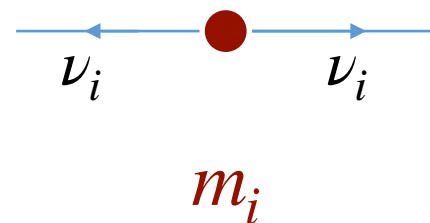
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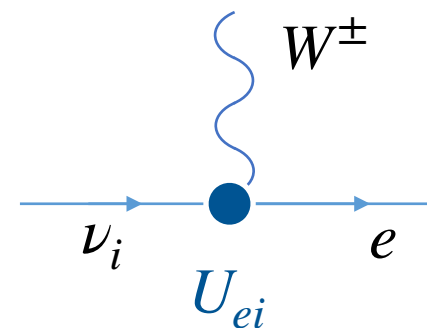
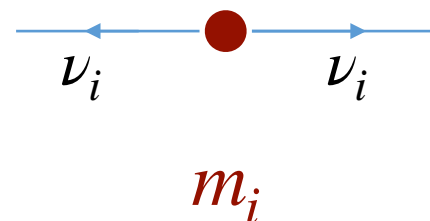
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$$\Delta L = 2 \implies 0\nu\beta\beta$$



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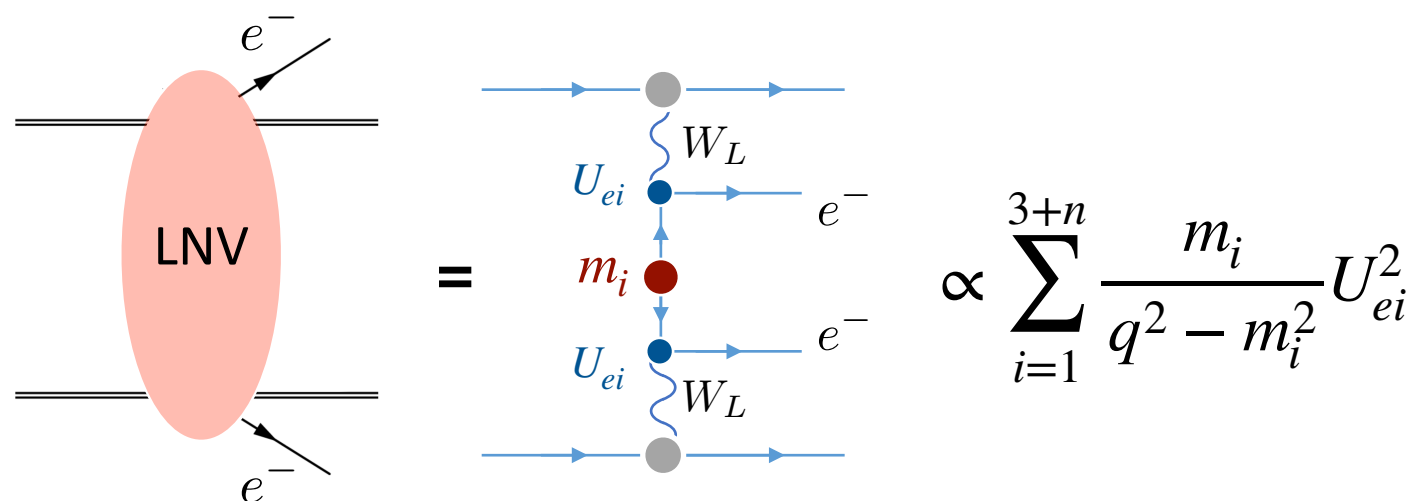
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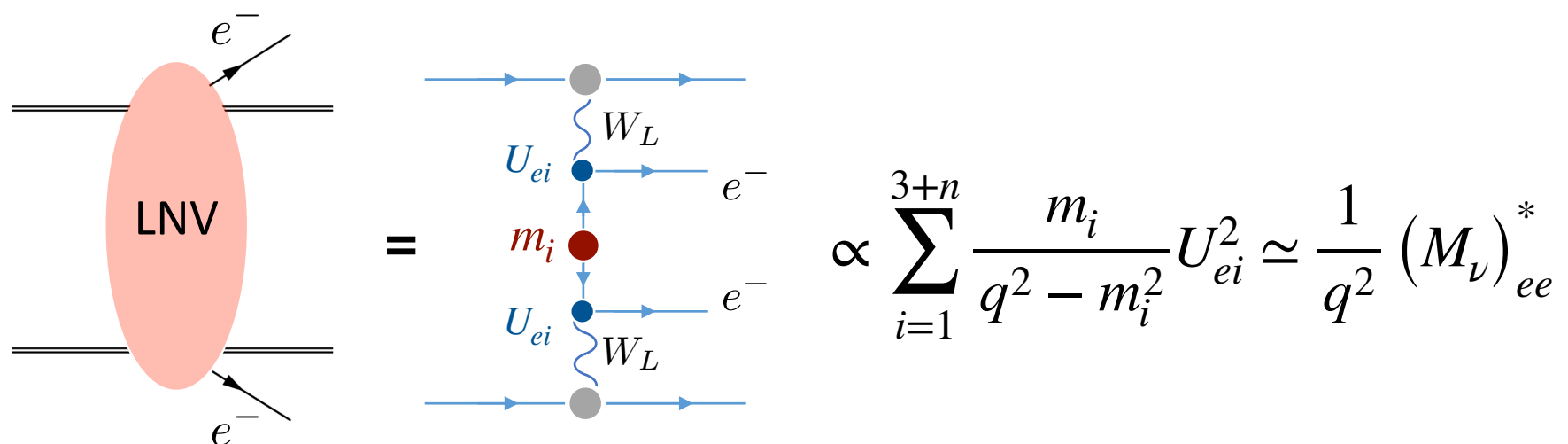
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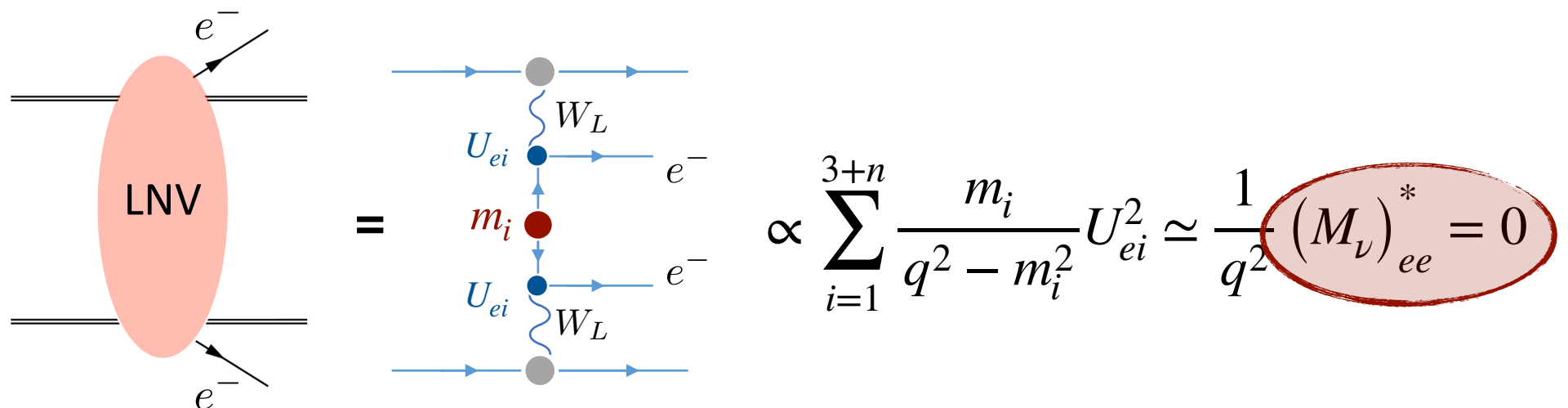
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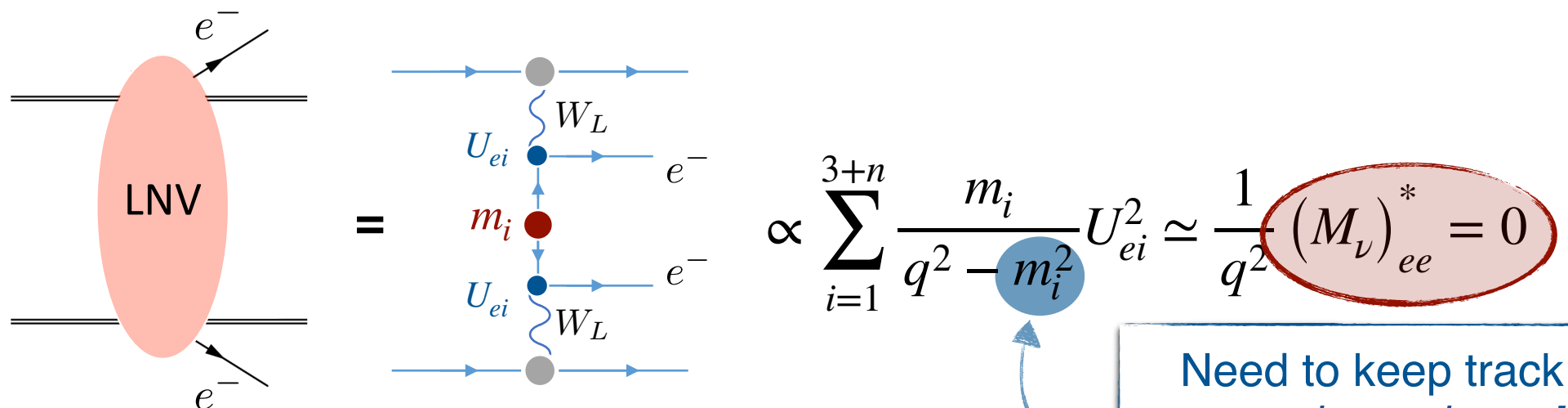
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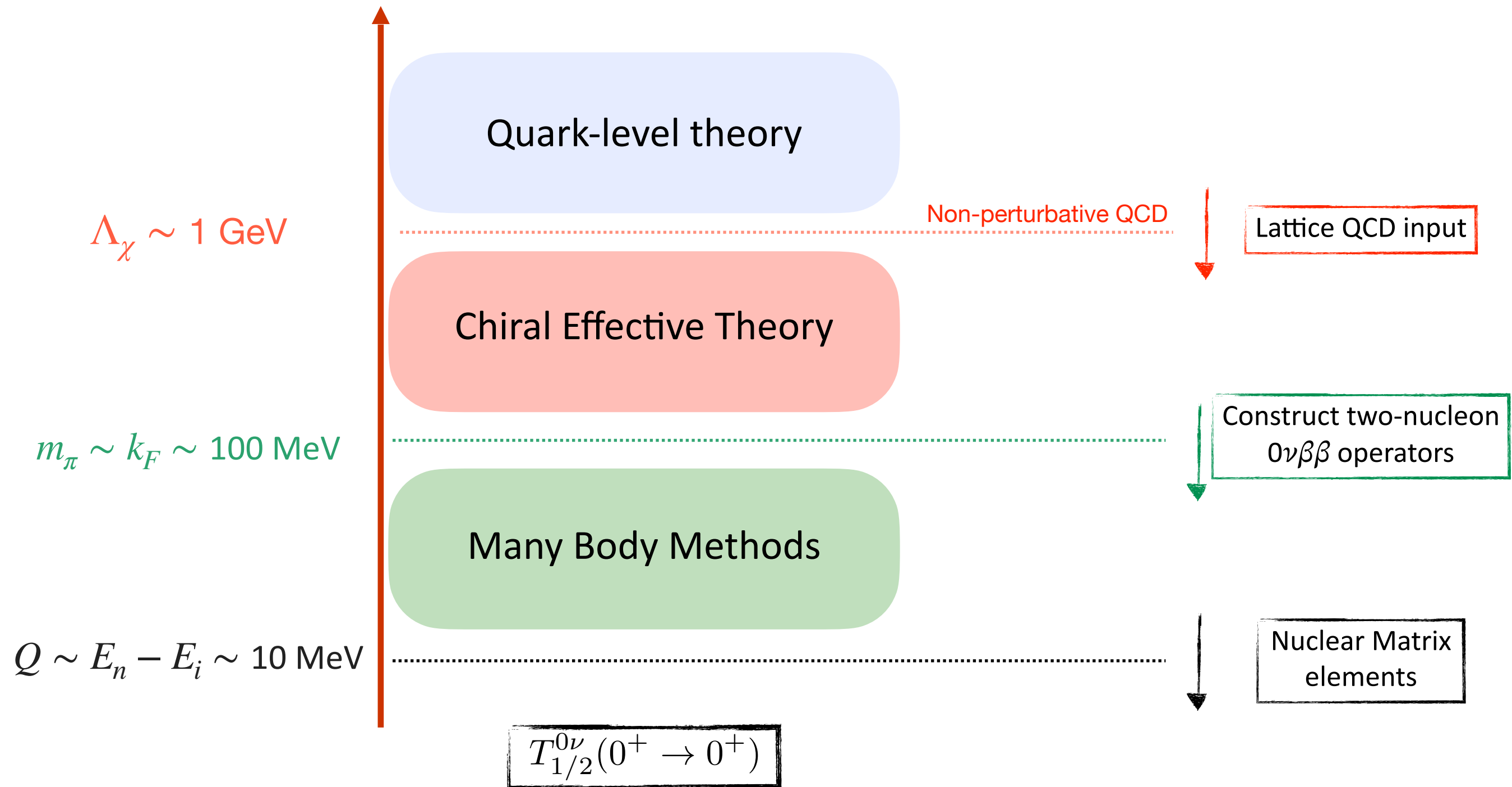
- In the mass basis:



Need to keep track of m_i dependence!

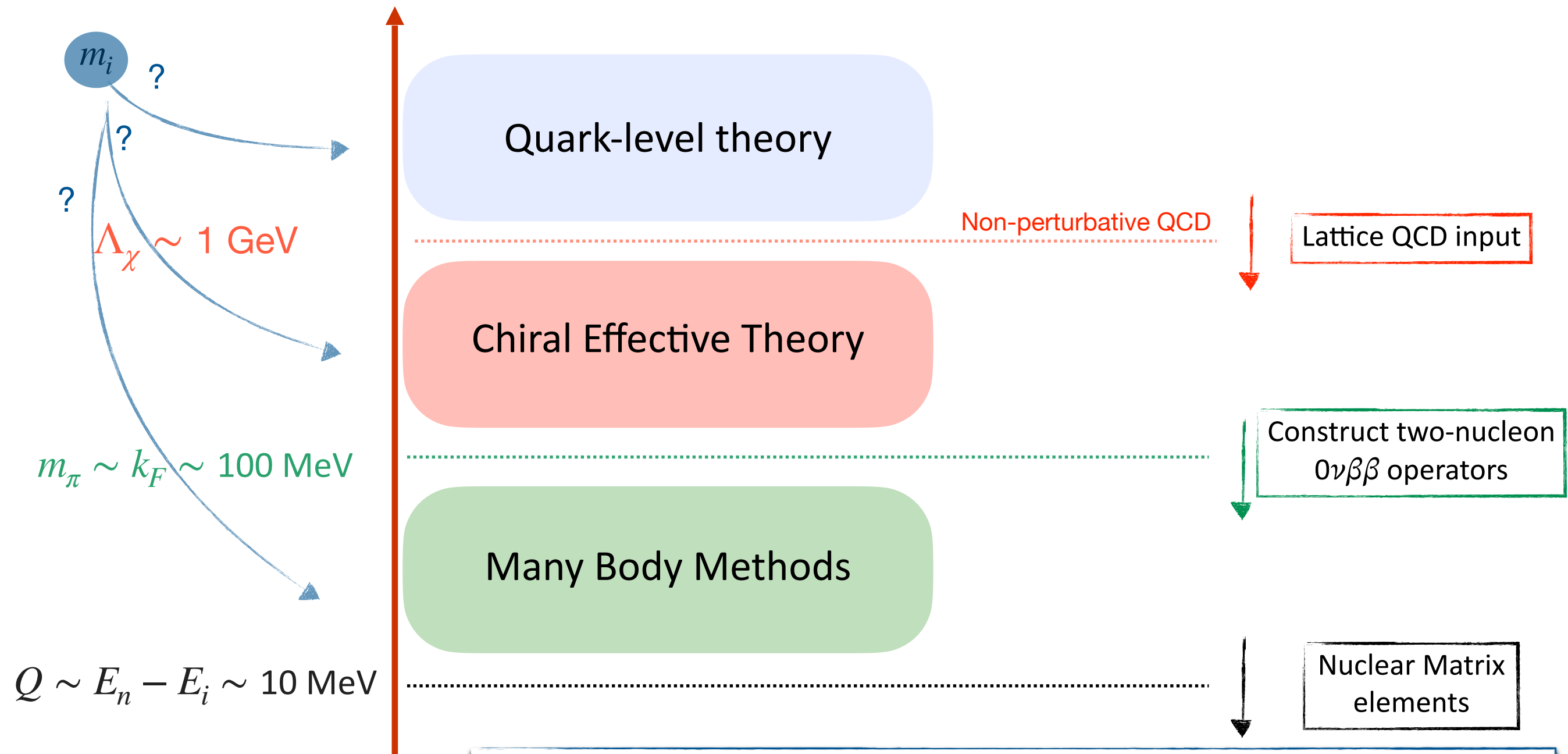
EFT approach

One scale at a time



EFT approach

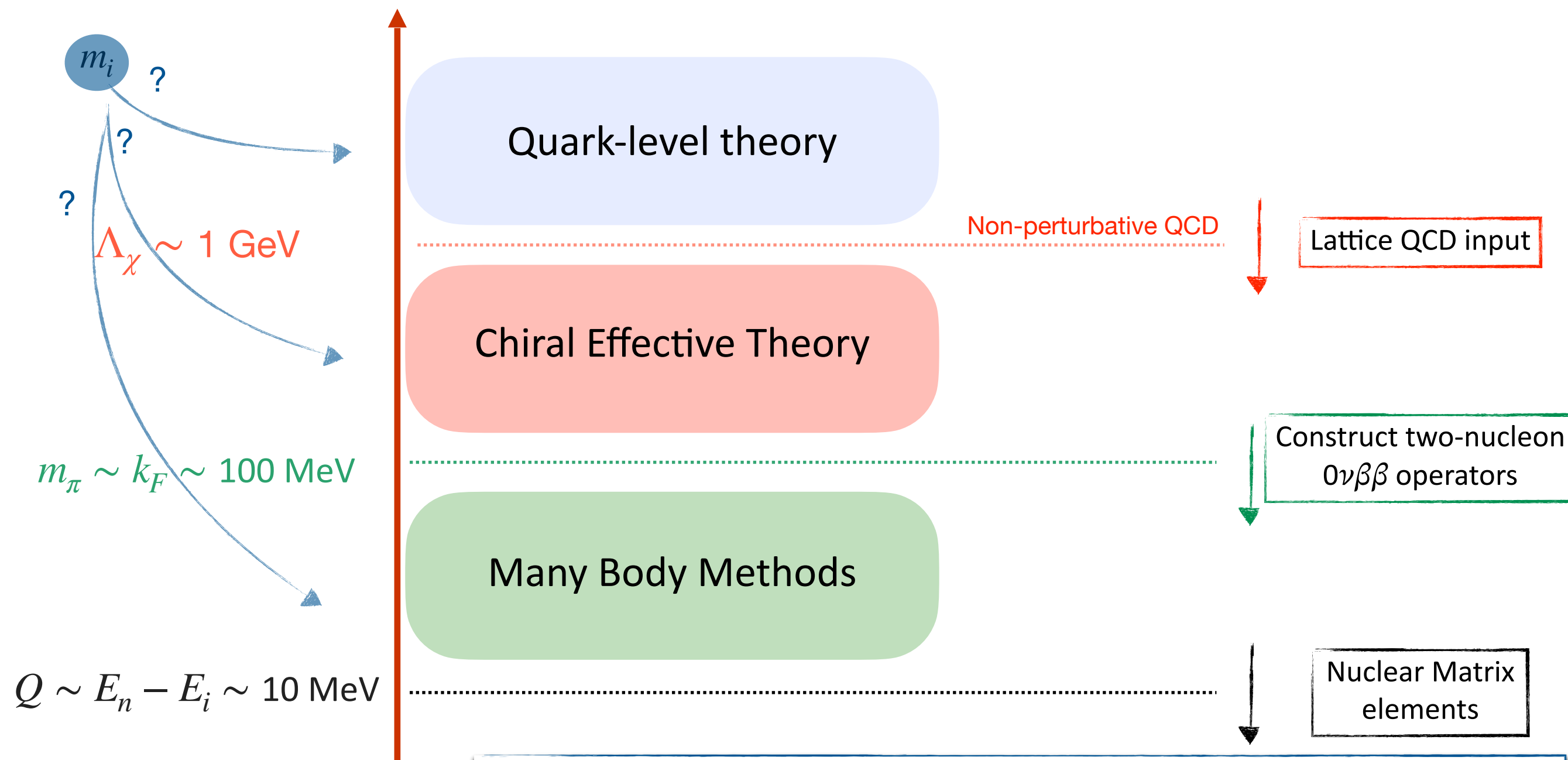
One scale at a time



- How to include ν_i depends on its mass

EFT approach

One scale at a time



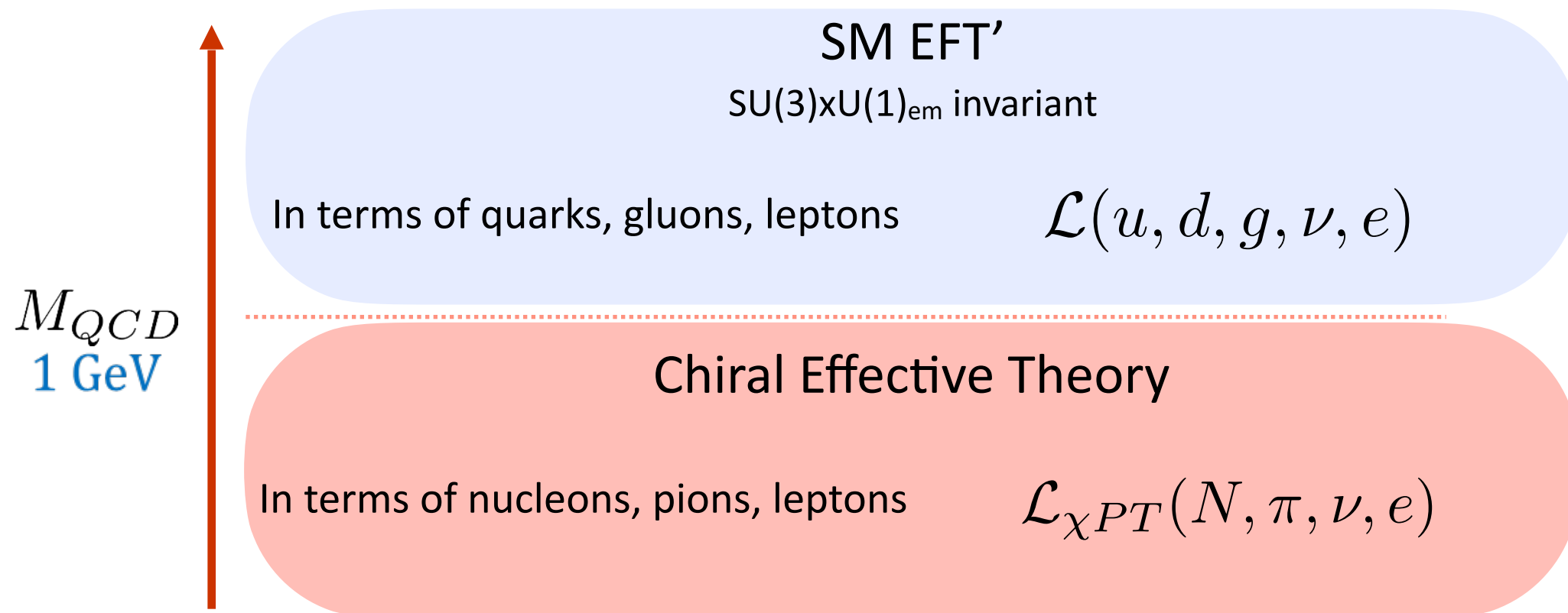
- How to include ν_i depends on its mass
- First: simpler case of $m_i \ll Q, k_F, \Lambda_\chi$ **[ignoring cancellations]**
Similar to the 'usual' mechanism

$$m_i \ll Q, k_F, \Lambda_\chi$$

Ignoring cancellations



Matching to Chiral EFT



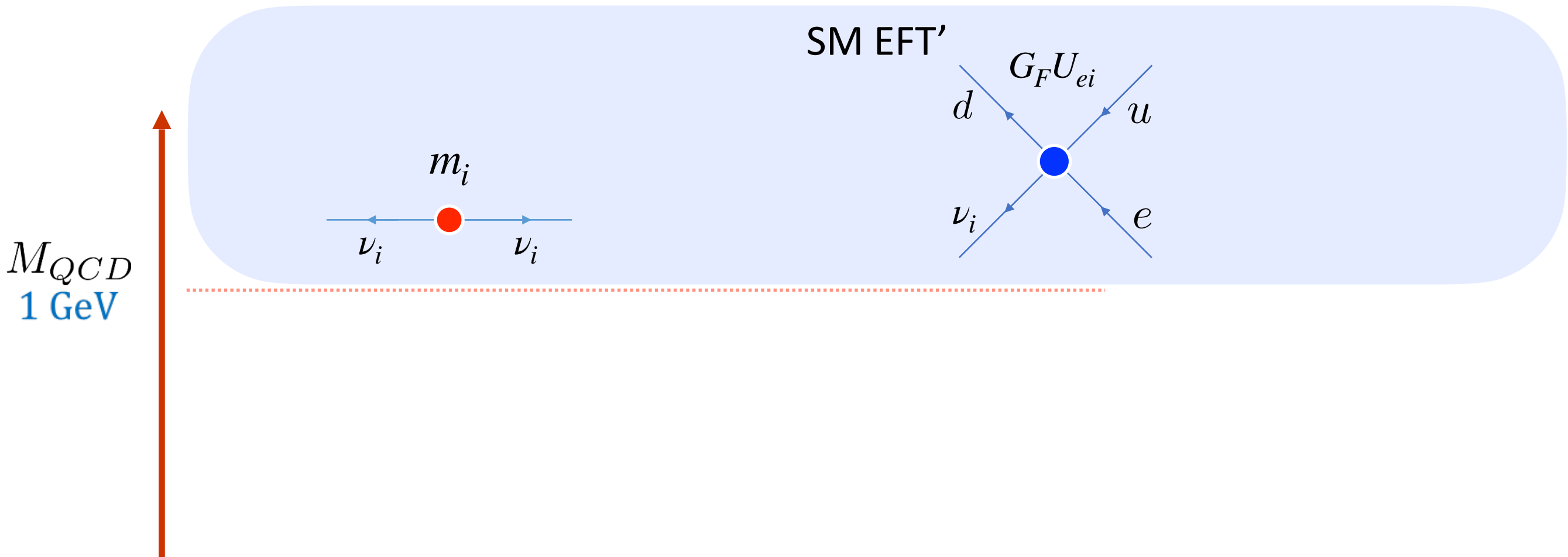
Form of operators determined by chiral symmetry

The operators come with unknown constants (LECs)

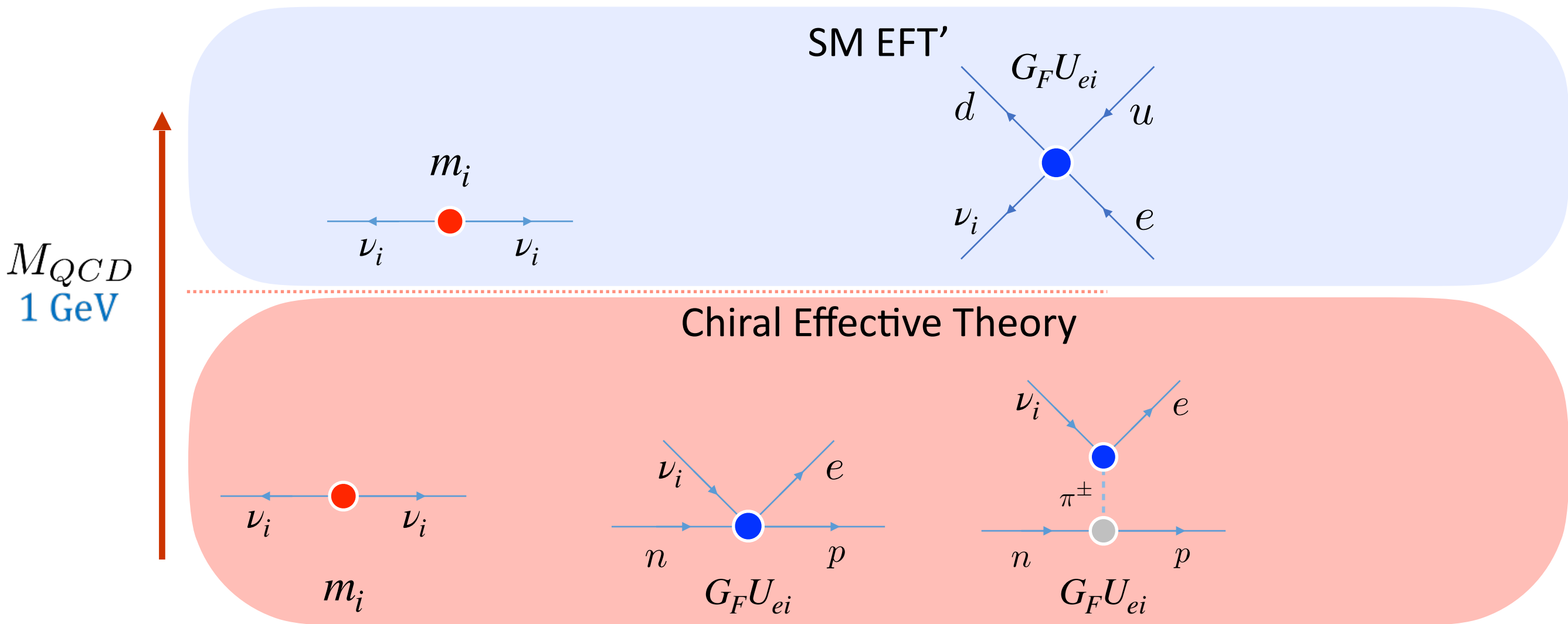
Need a power-counting scheme

- Often used: Weinberg counting / Naive dimensional analysis (NDA)

Matching to Chiral EFT

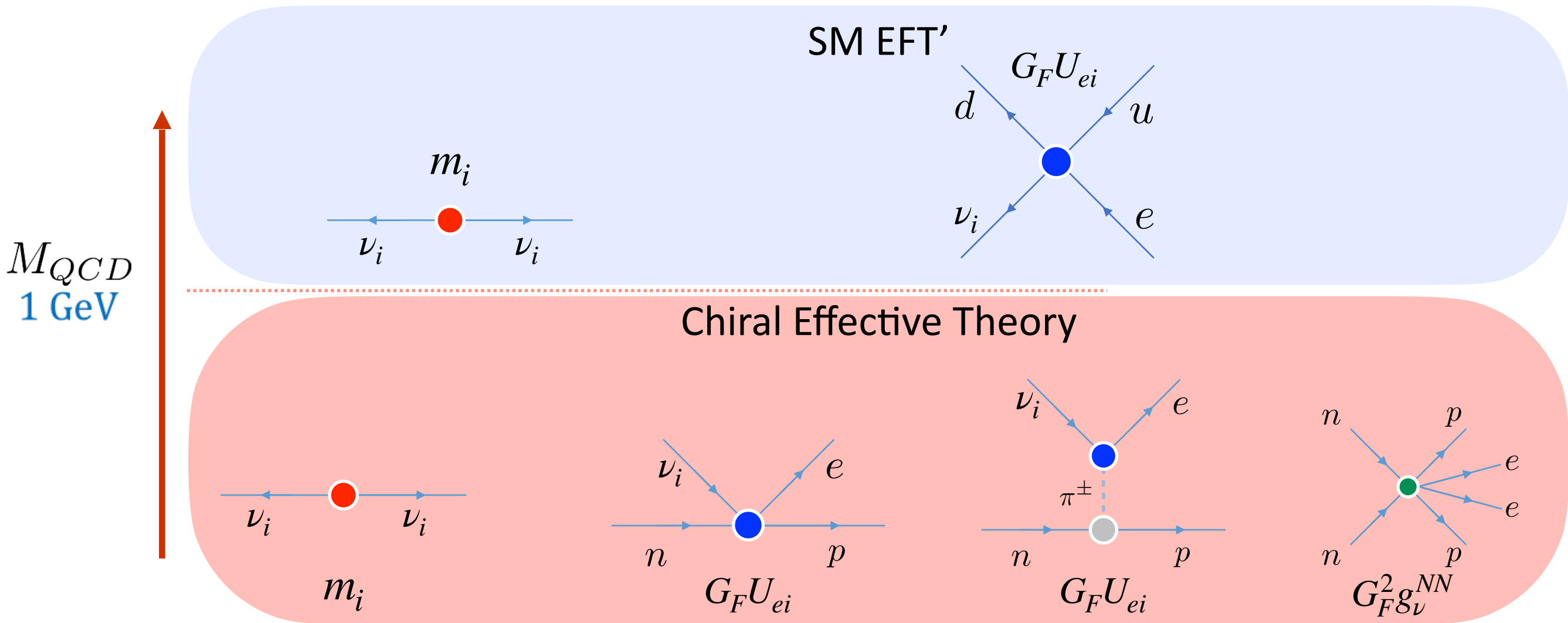


Matching to Chiral EFT



- At LO in Weinberg counting all needed low-energy constants are known

Matching to Chiral EFT



- At LO in Weinberg counting all needed low-energy constants are known

- Additional 'non-NDA' contact interaction needed for renormalization

Cirigliano et al '18,'19

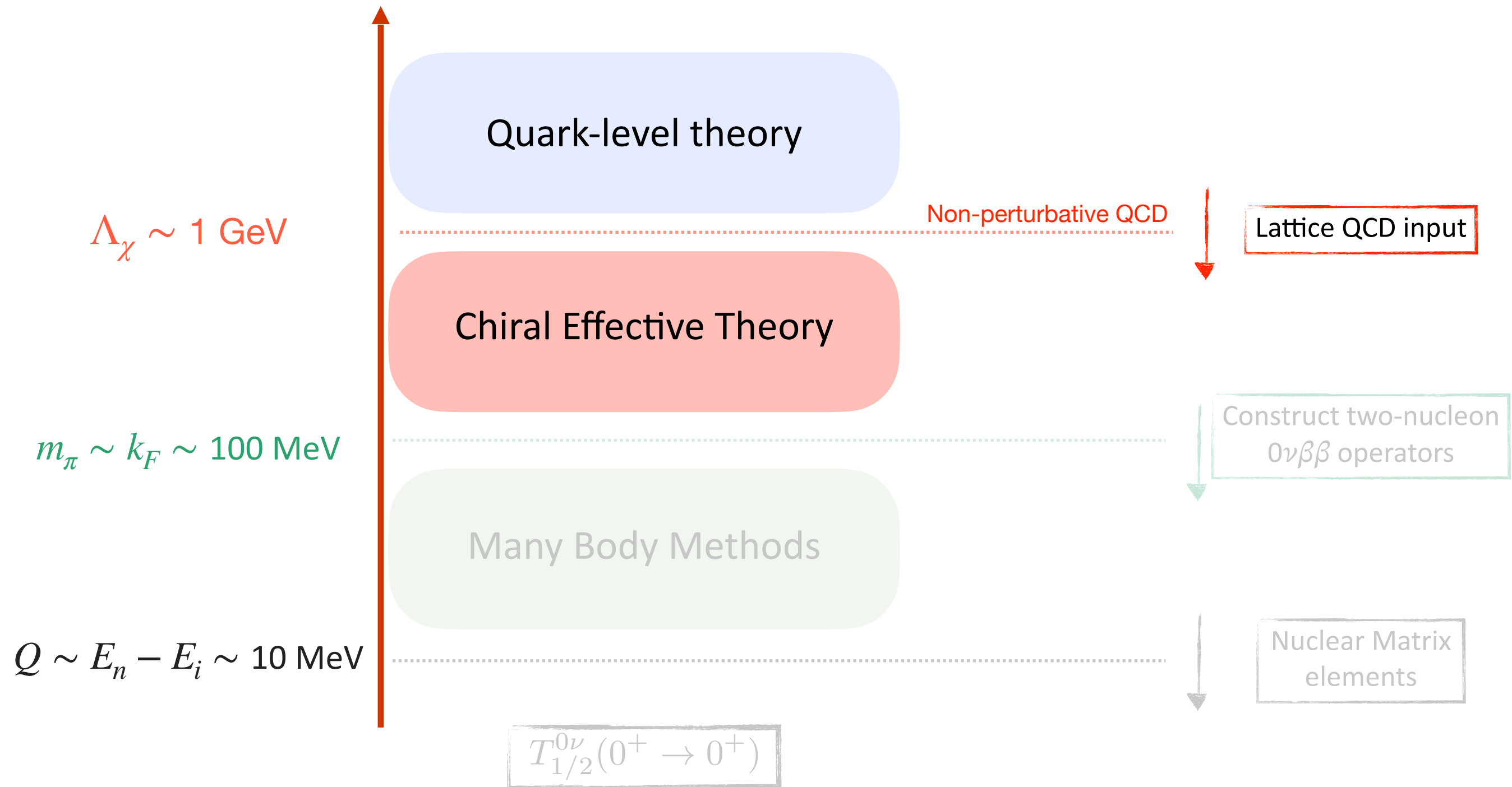
- New LEC, g_ν^{NN} .

- Currently unknown only model estimates

Cirigliano, et al, '21; Richardson et al, '21

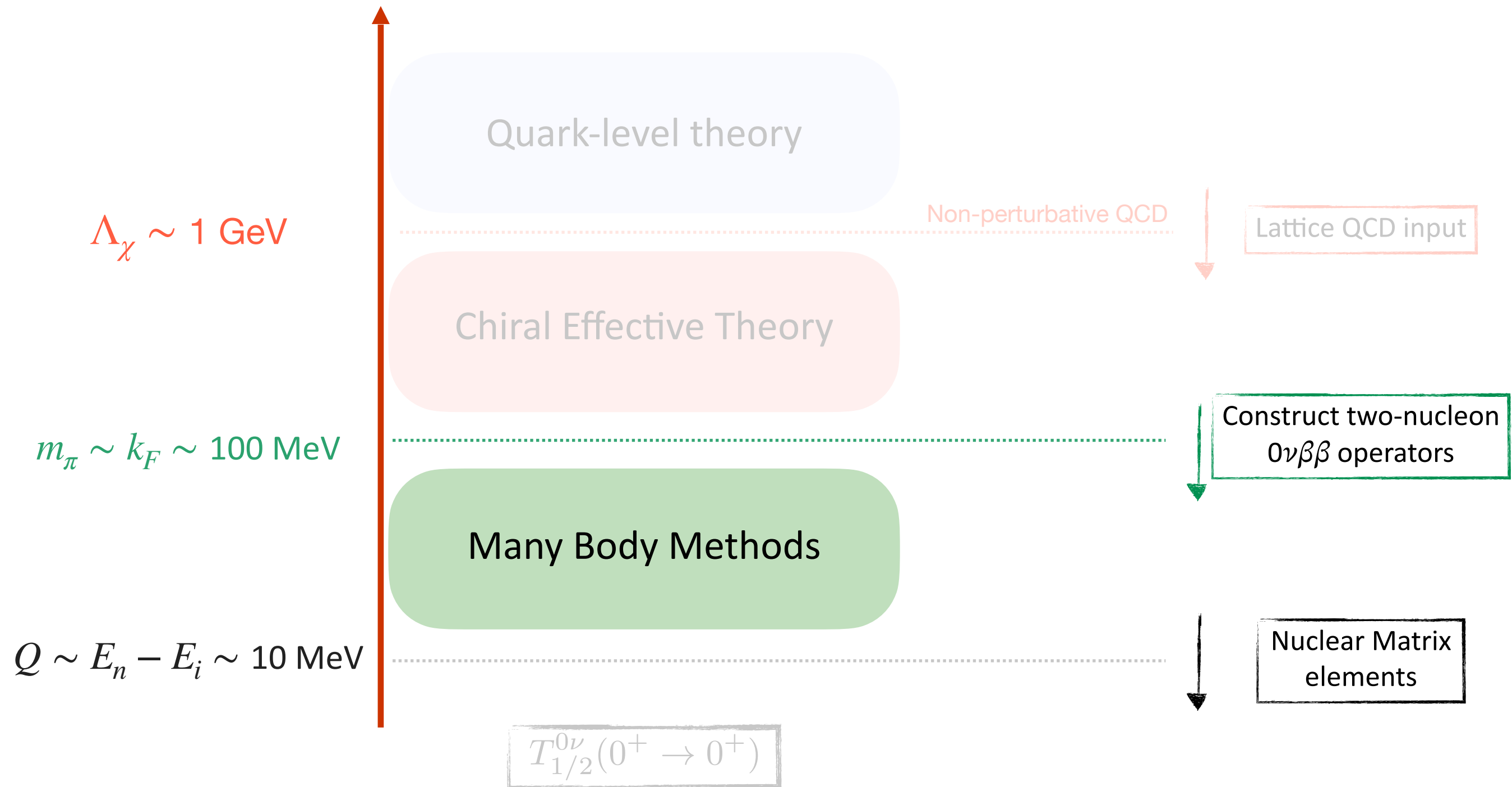
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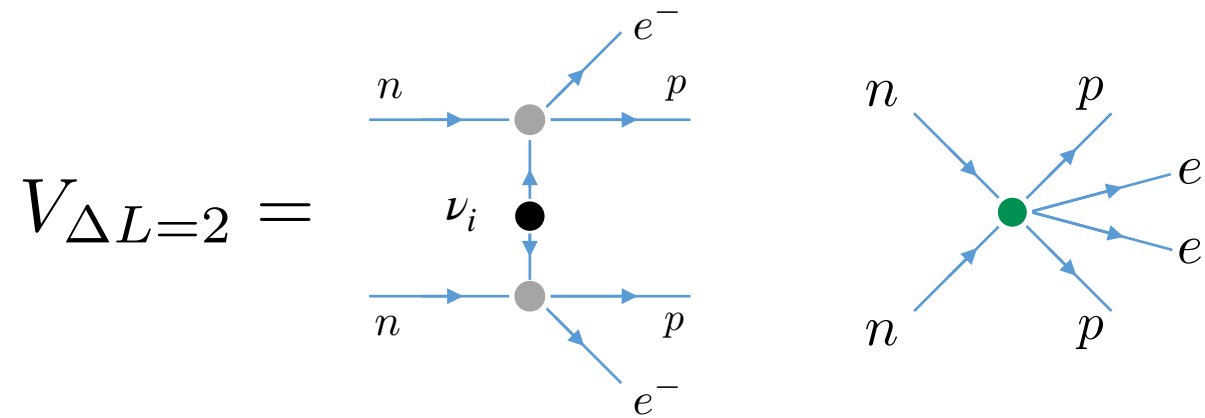
One scale at a time



Chiral EFT

Active ν 's: leading order

Leading order

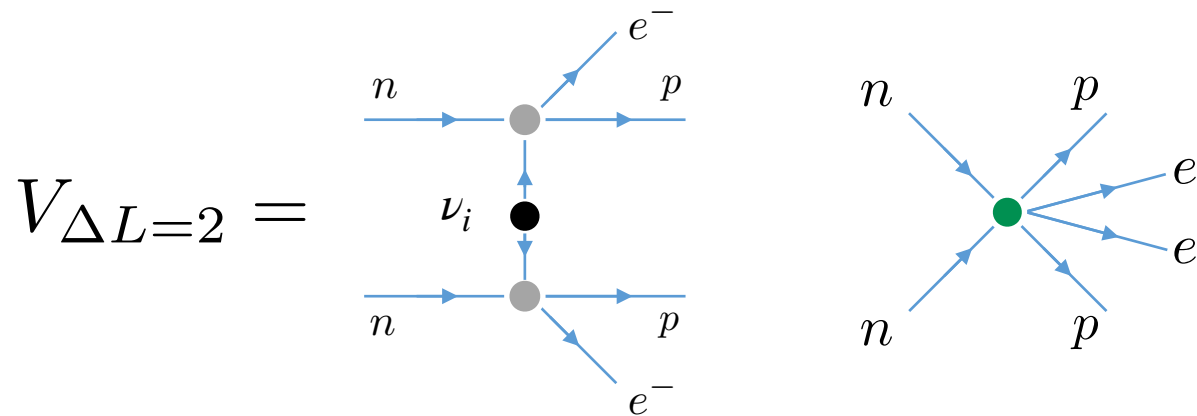


Need to evaluate $A_\nu = \langle {}^{136}\text{Ba} | V | {}^{136}\text{Xe} \rangle$

Chiral EFT

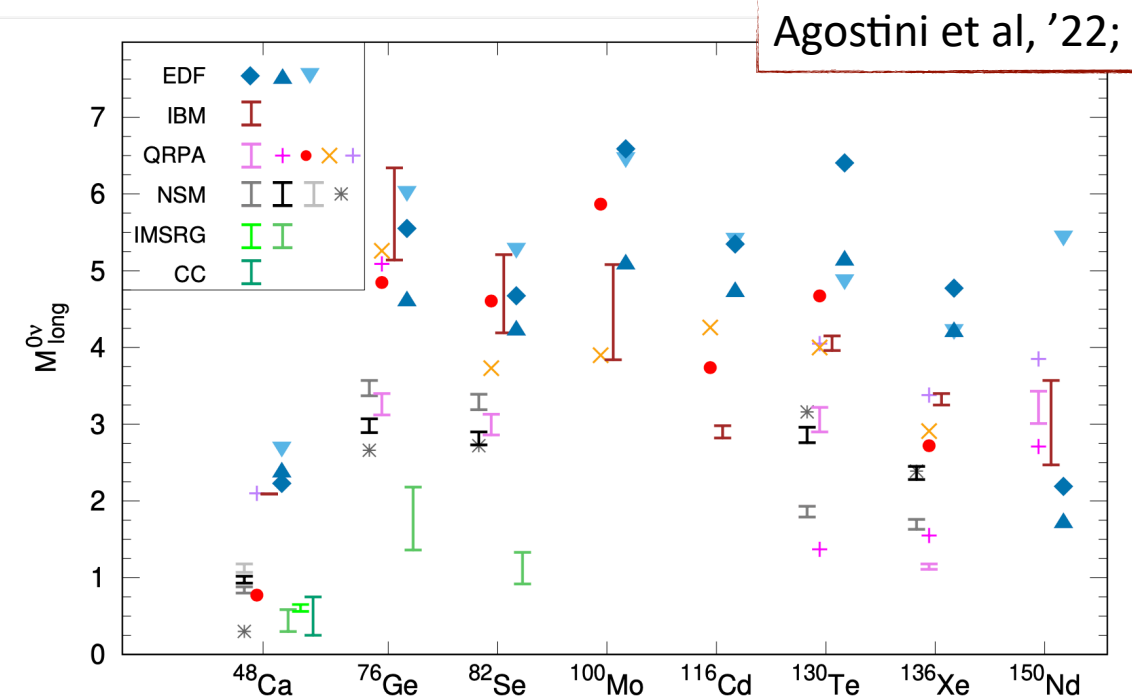
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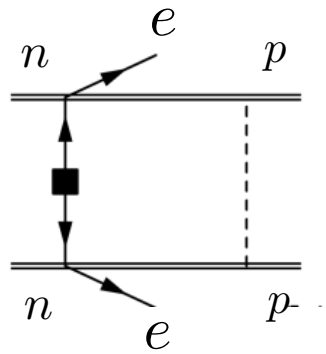
- Requires many-body methods
- Matrix elements differ factor 2-3 between methods
- *Ab initio* NMEs for $A \geq 48$ are starting to appear
- Including estimates of g_ν^{NN} effects



Chiral EFT

Active ν 's: N2LO

Loops and counterterms



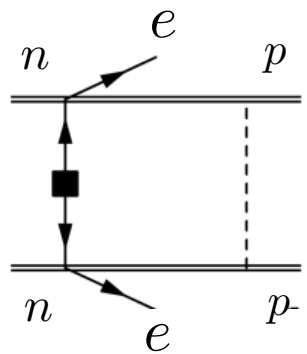
- Leads to:
 - form factors
 - correction to the potential
- Estimated to be $\lesssim \mathcal{O}(10\%)$ in light nuclei

Pastore et al '17

Chiral EFT

Active ν 's: N2LO

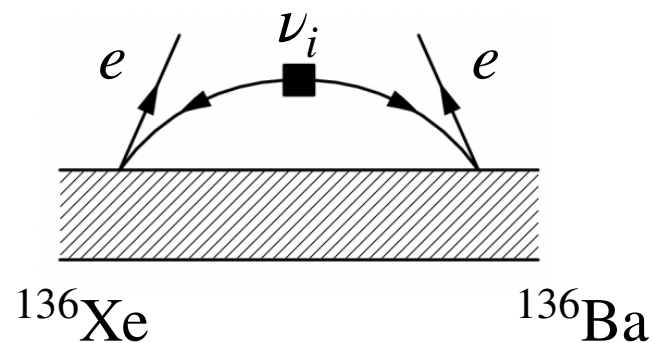
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'Ultrasoft' neutrinos



$$A_{\nu}^{\text{usoft}} \sim \sum_N \langle f | J_{\mu} | n \rangle \langle n | J^{\mu} | i \rangle \Delta E \left(\ln \frac{\mu}{2\Delta E} + 1 \right)$$

$$\Delta E = E_n - E_i + E_e$$

- Due to neutrinos with small momenta

$$q^0 \sim |\vec{q}| \sim k_F^2 / m_N \sim \Delta E$$
- See the nucleus as a whole
- Dependence on intermediate state, 'closure' correction

Minimal SM extension

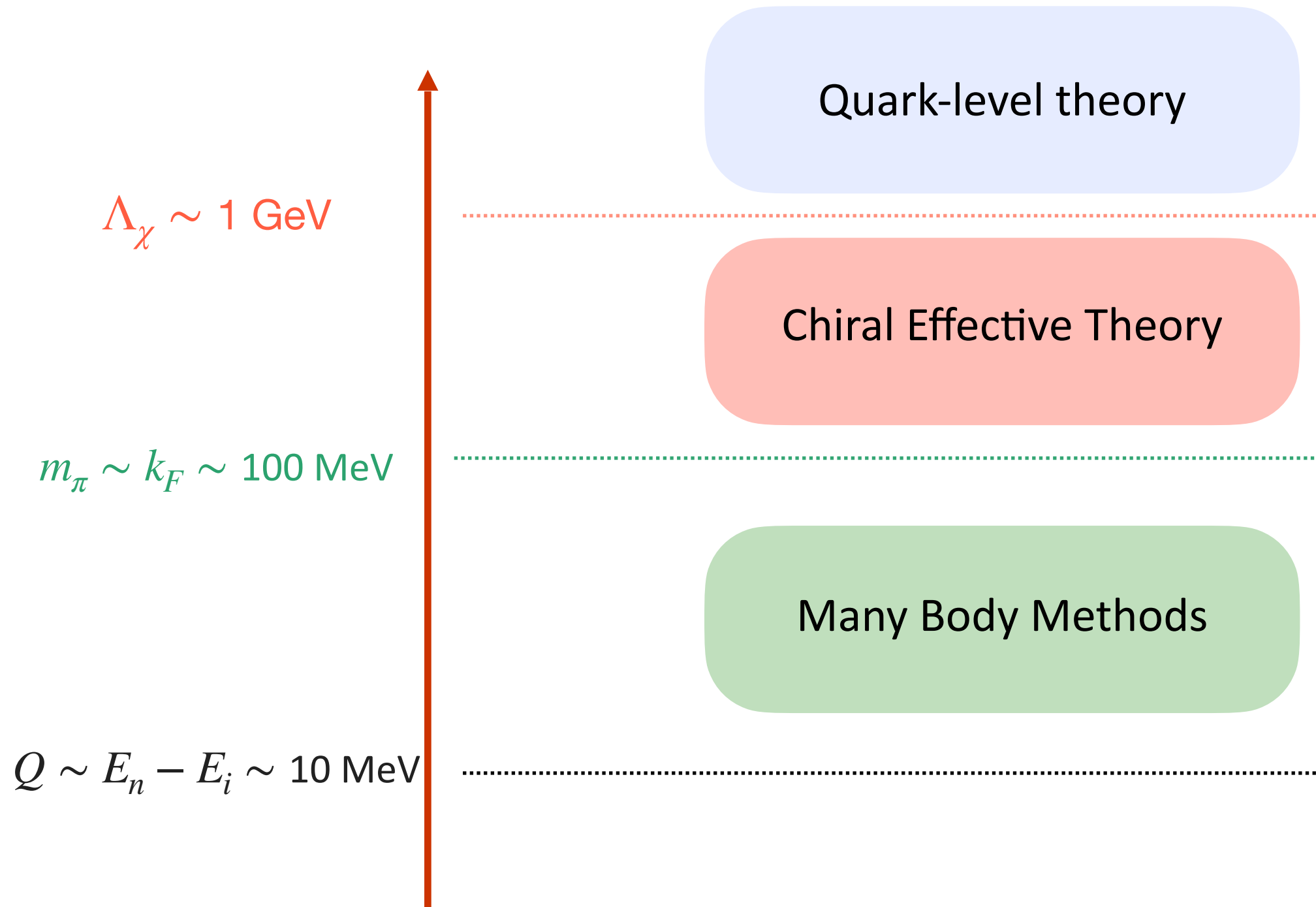
Including cancellation mechanism



EFT approach

Non-negligible masses

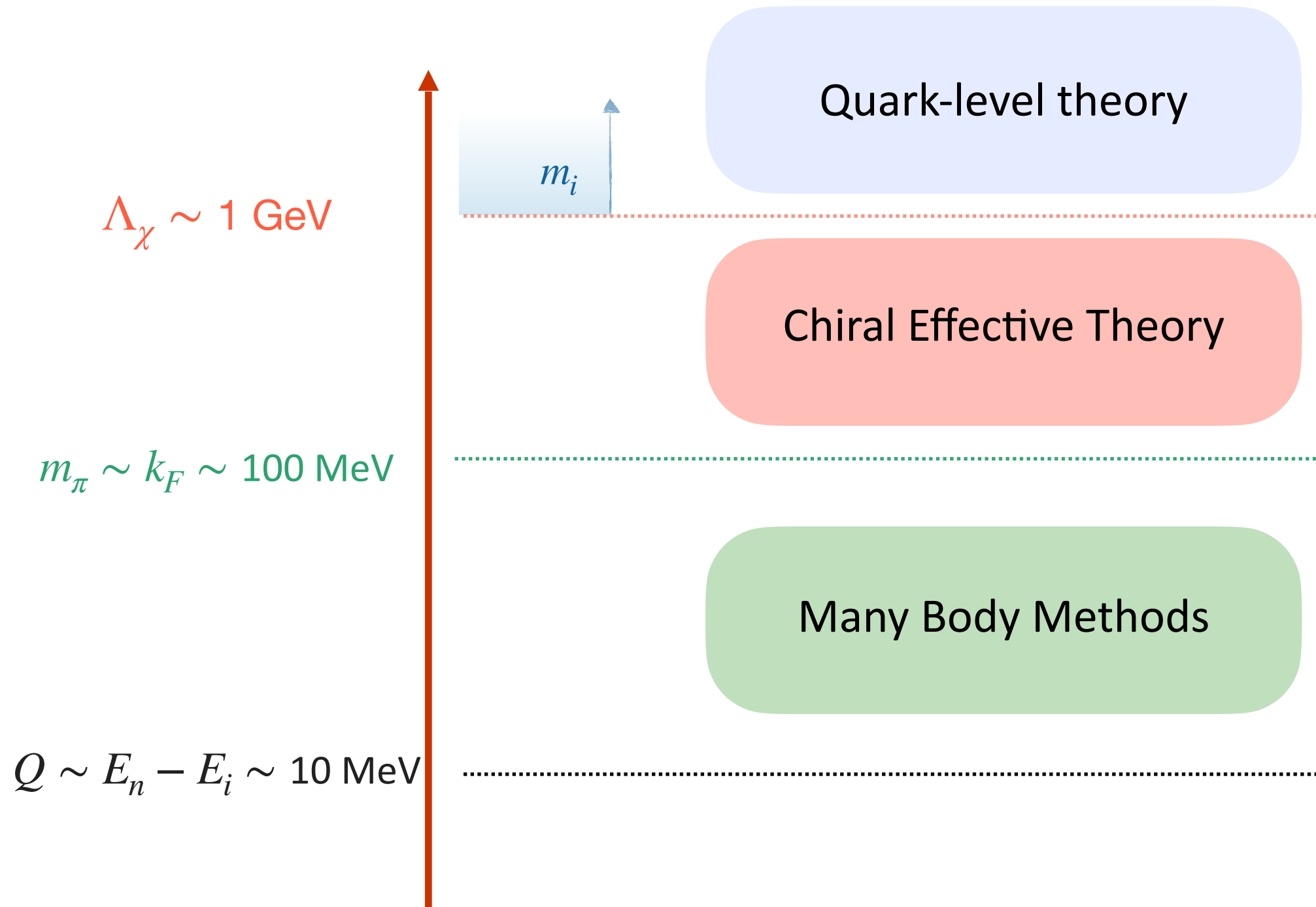
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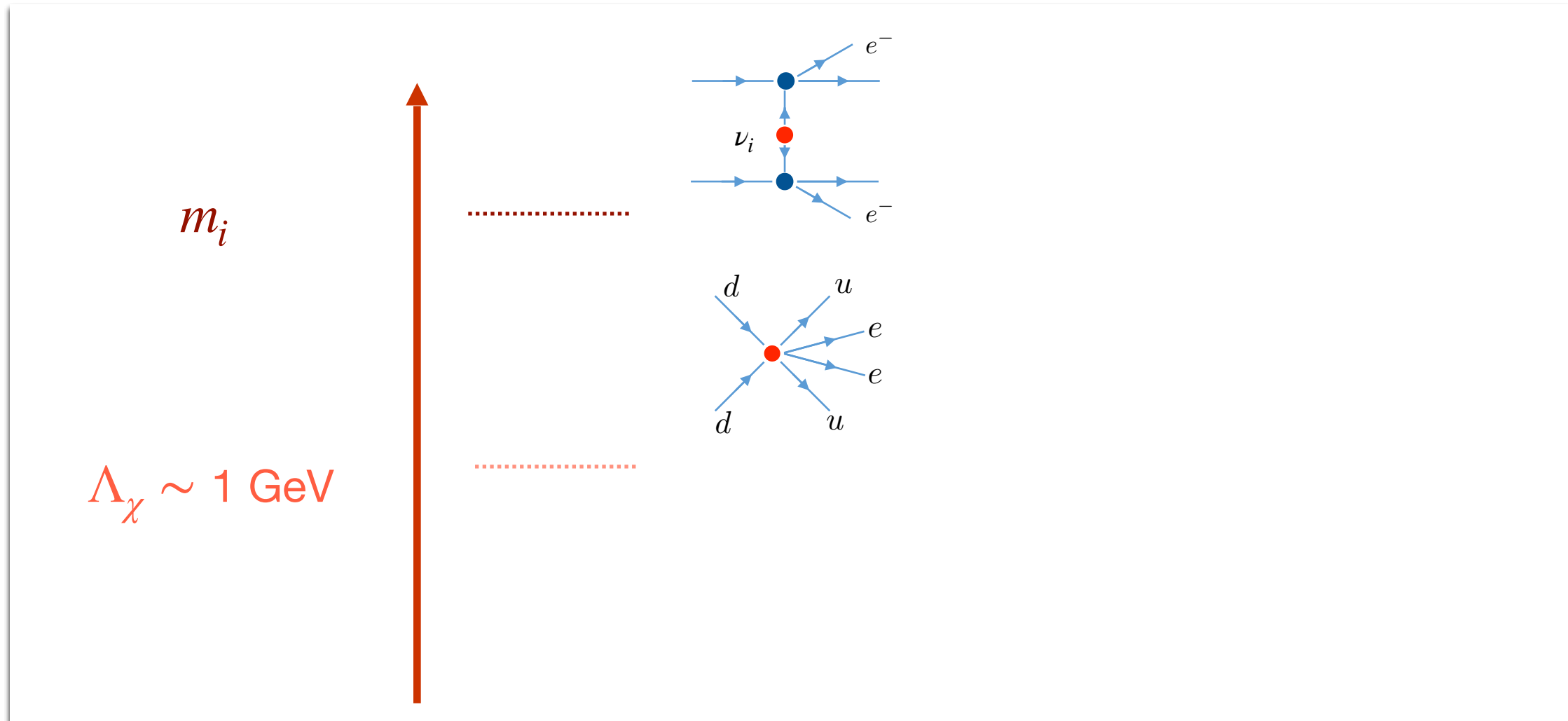
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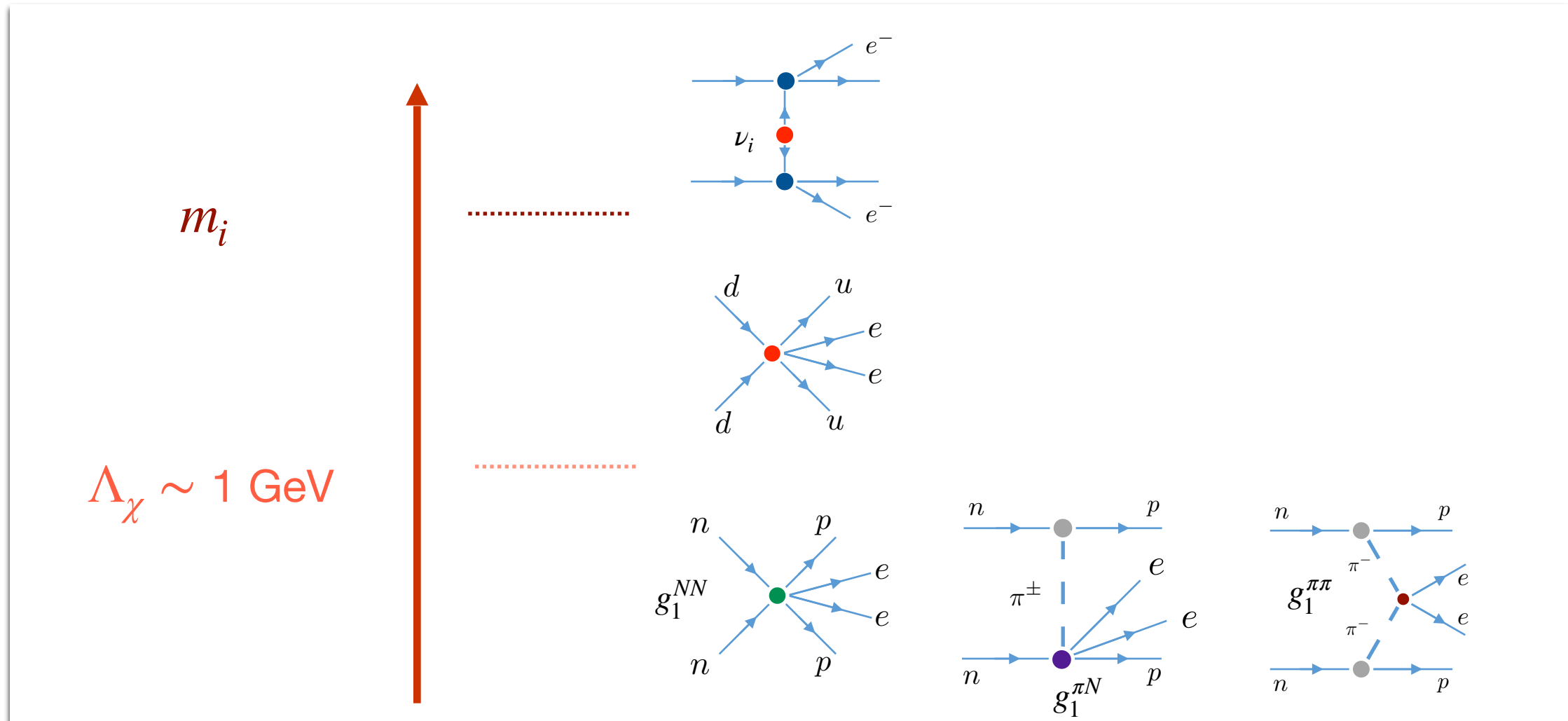


$$m_i \gg \Lambda_\chi$$



- ν_i can be integrated-out at quark level
- Determines m_i dependence: $A_\nu(m_i) \sim U_{ei}^2/m_i^2$

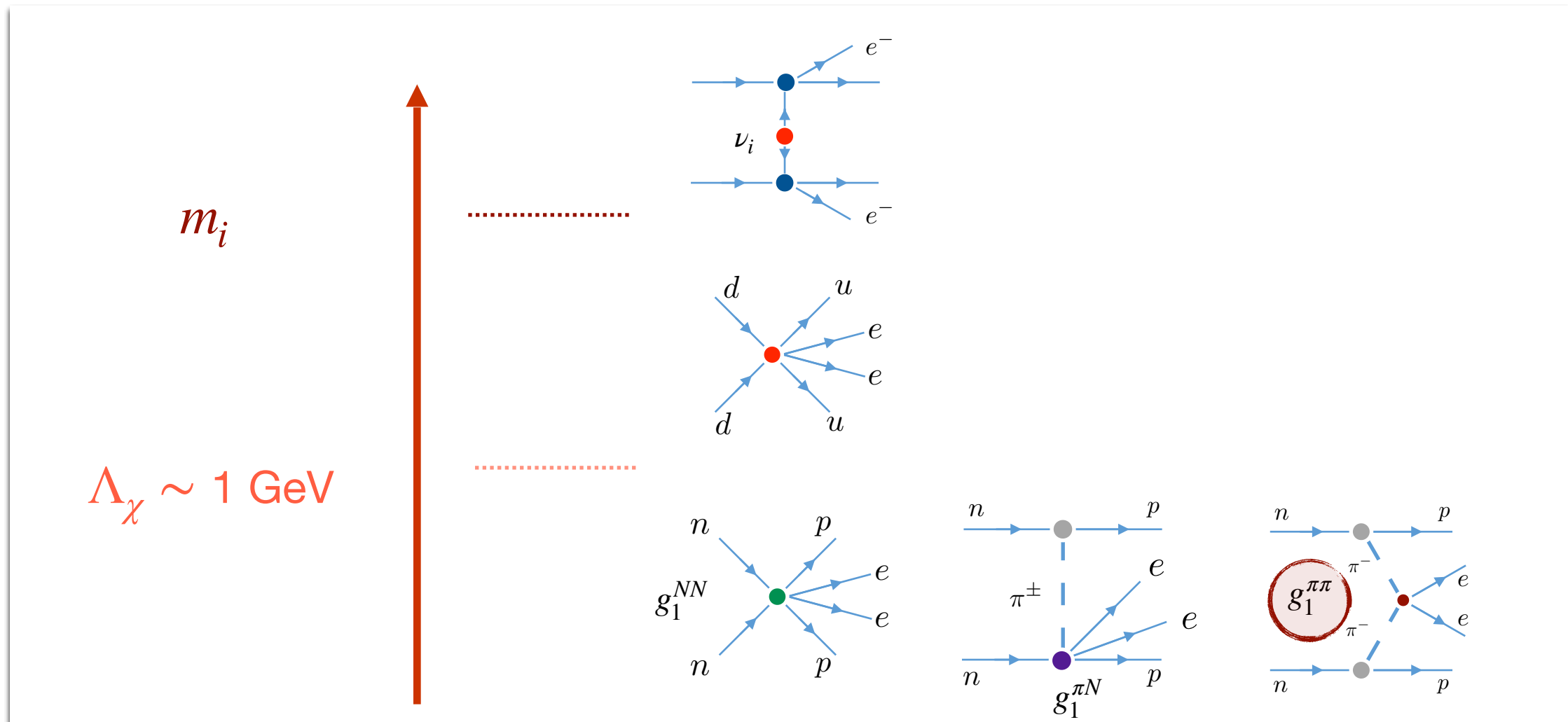
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- Match to chiral EFT without ν_i
- Involves several LECs

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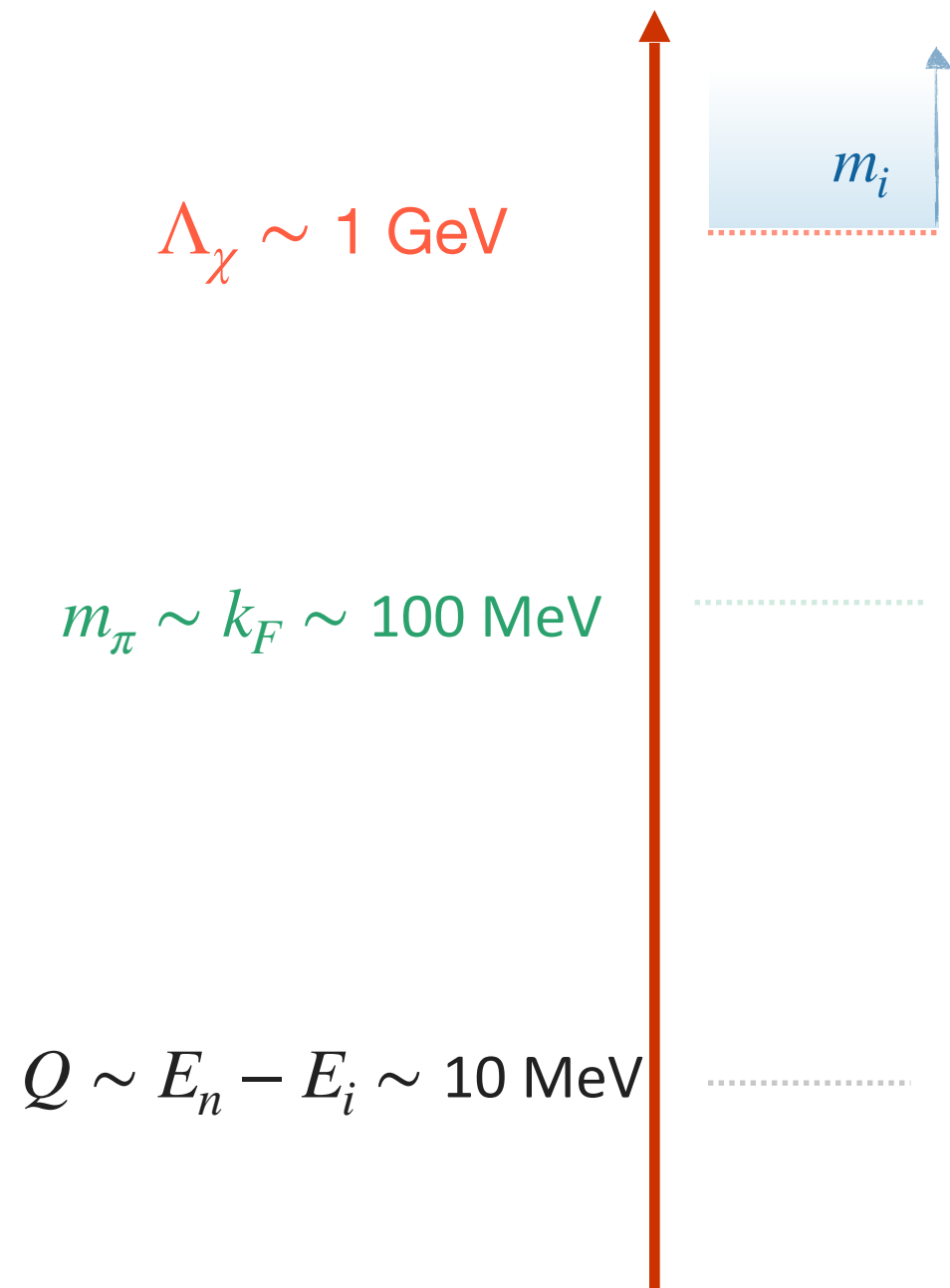


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 - Only $g_1^{\pi\pi}$ known

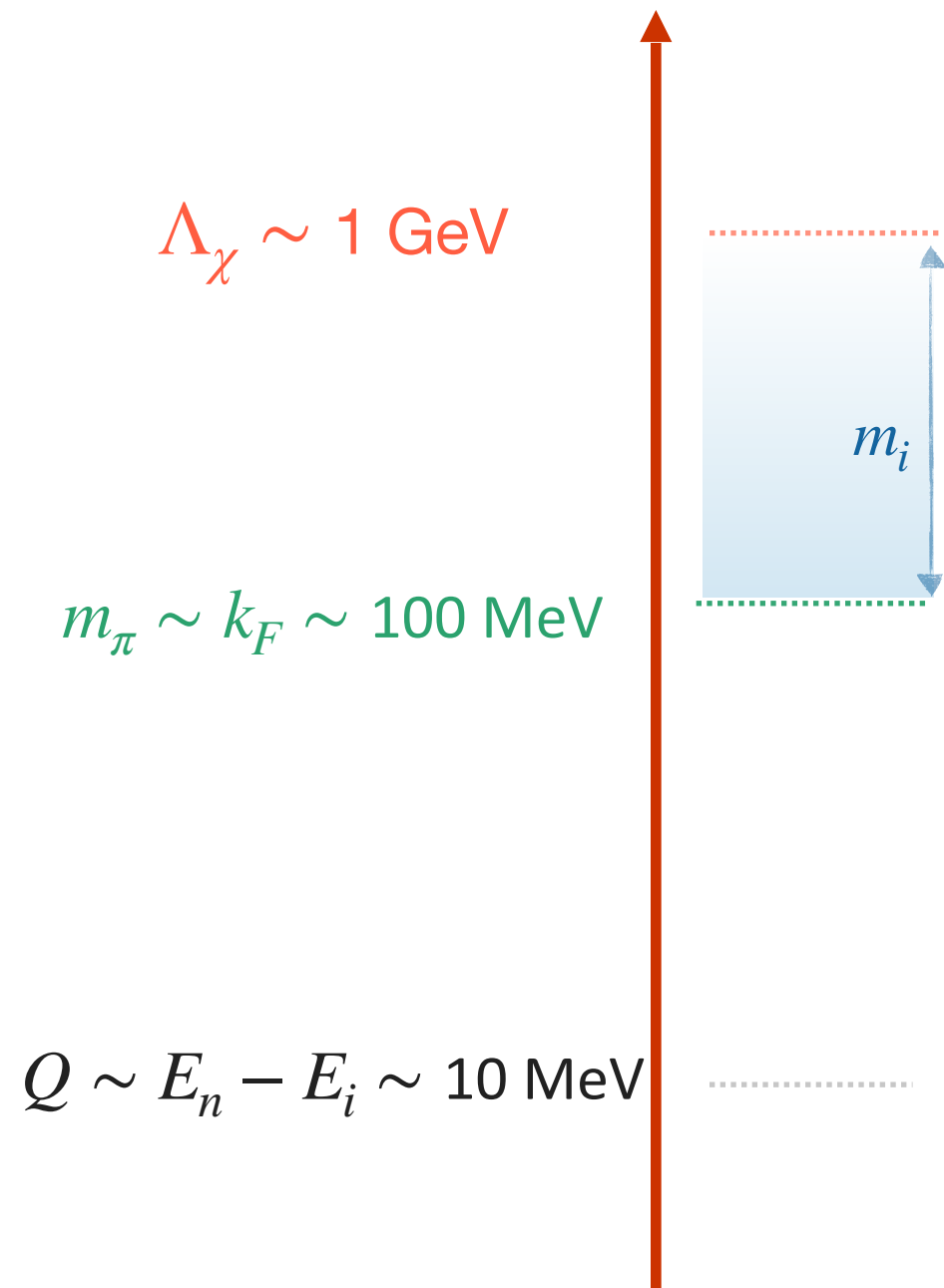
EFT approach

One momentum scale at a time

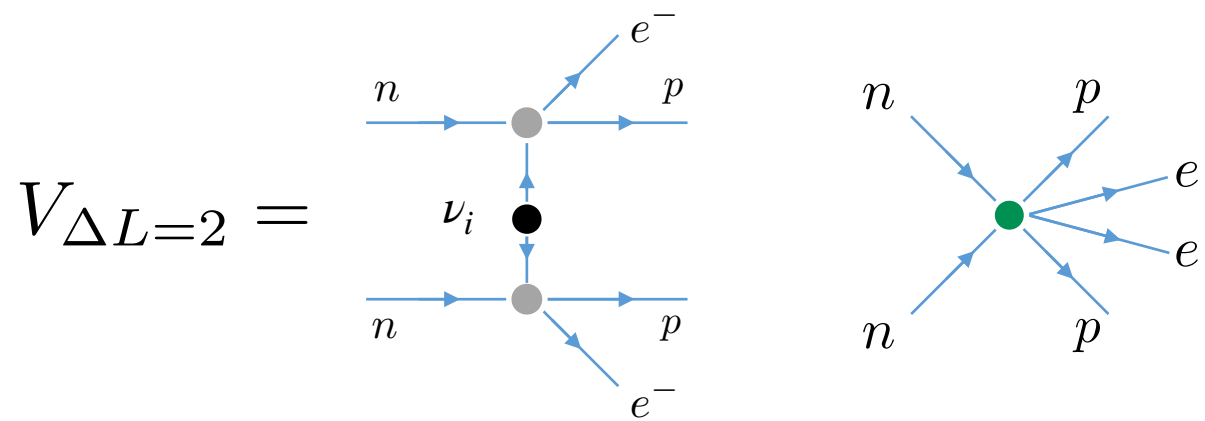


EFT approach

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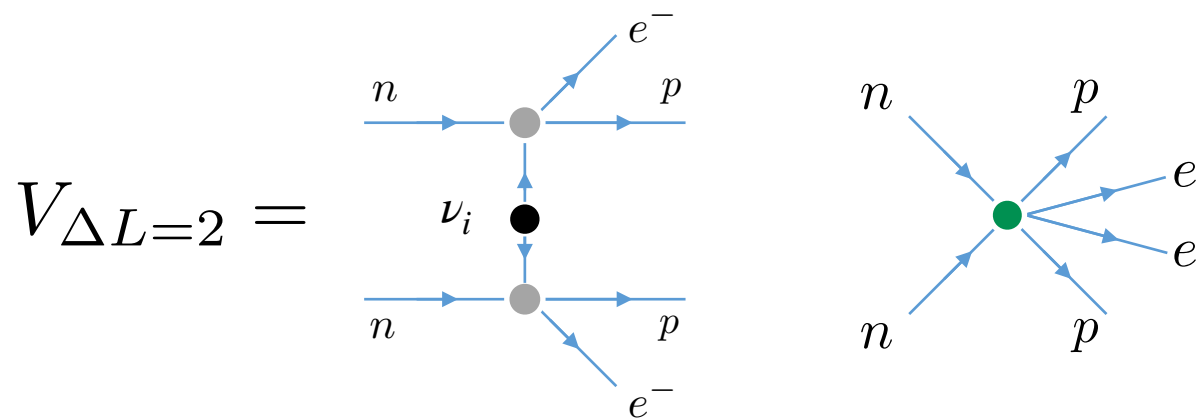


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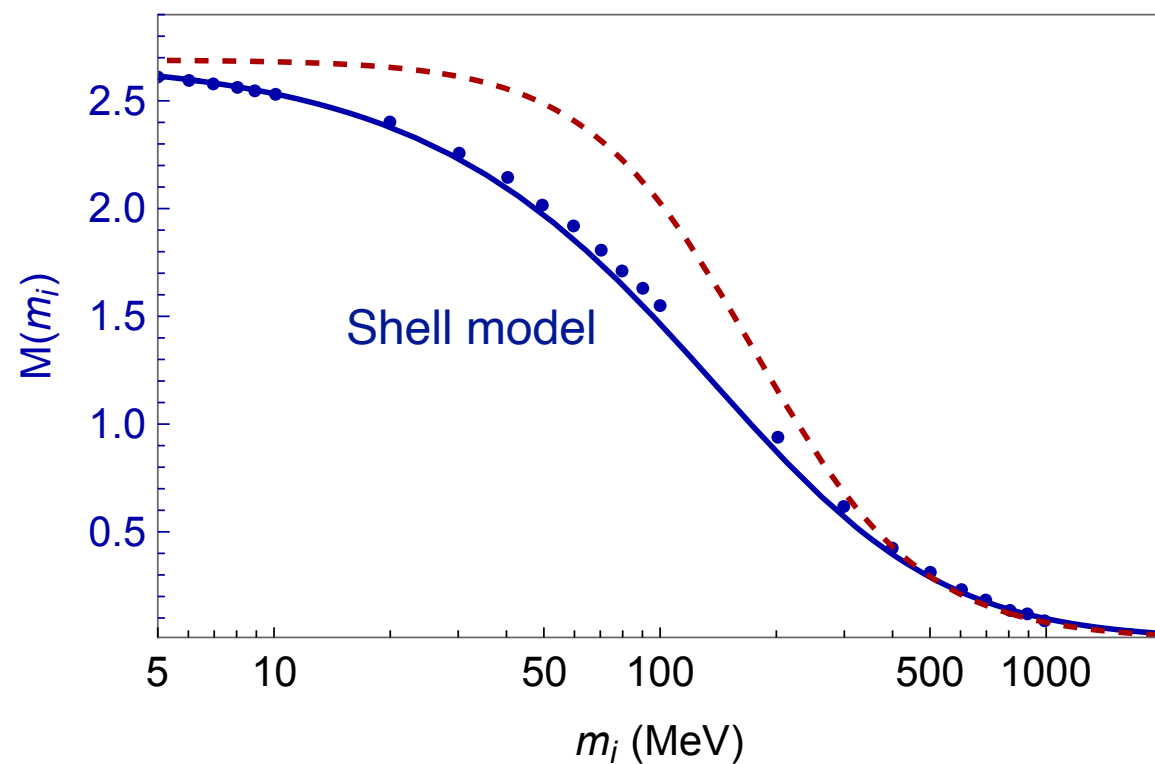


- Have to keep ν_i in the chiral theory
- Again have 'potential' + 'hard' contributions
- m_i dependence in NMEs and g_ν^{NN}

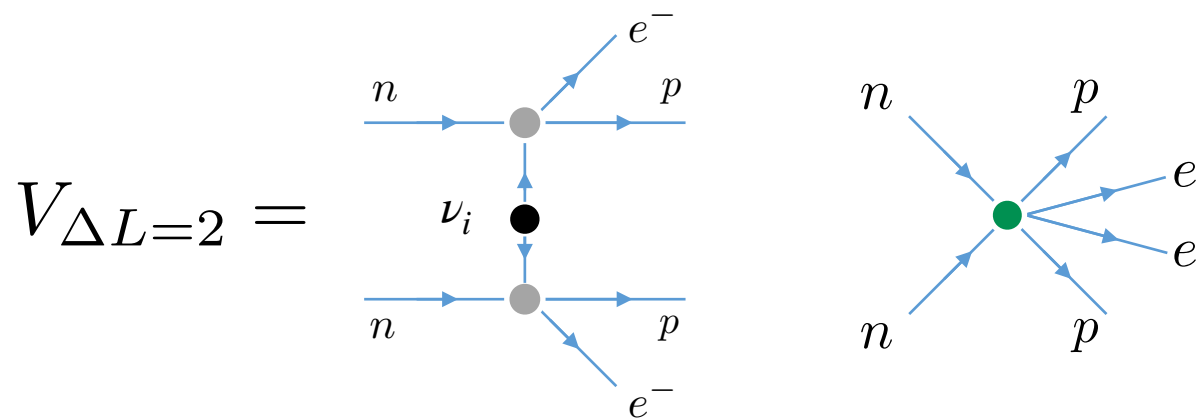
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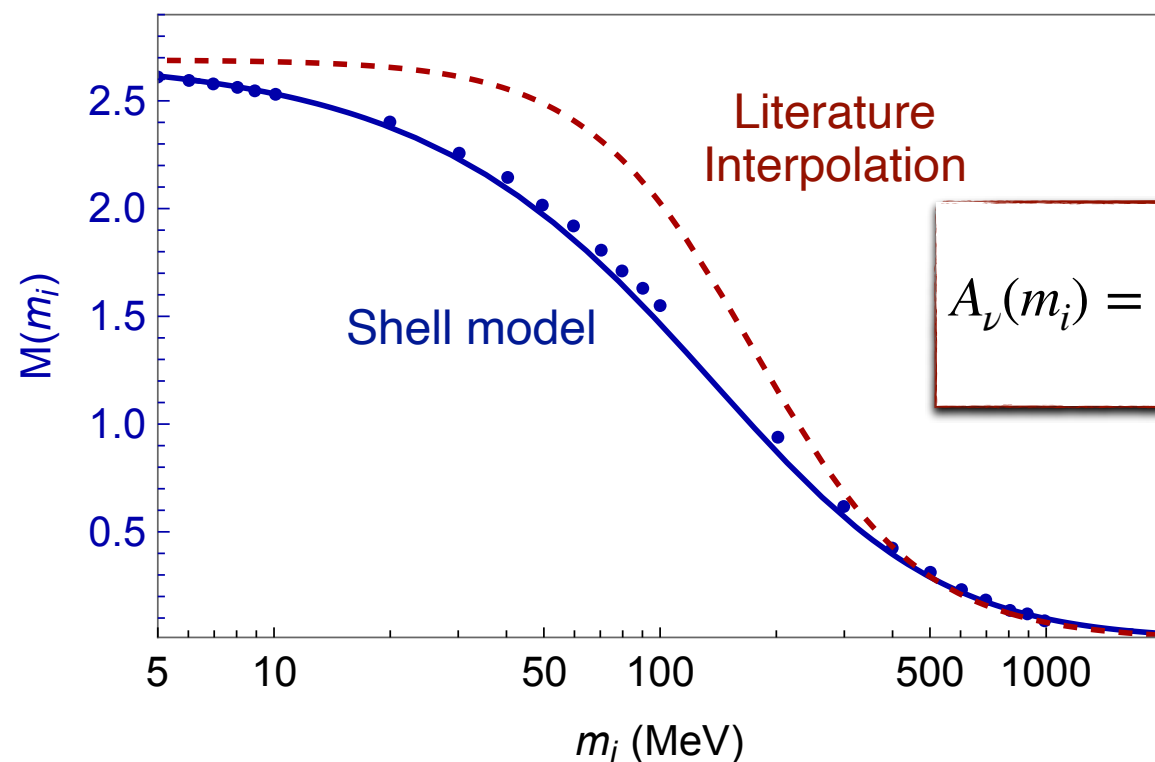
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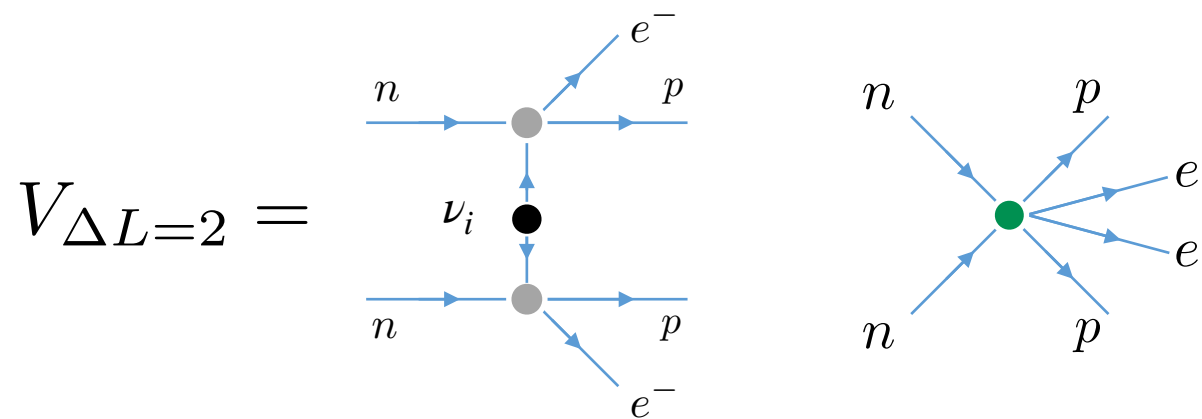


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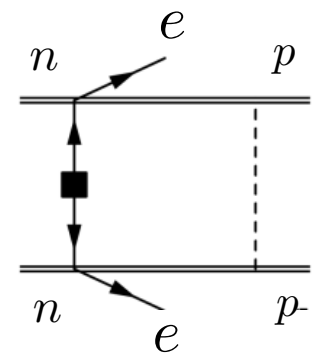


$$A_\nu(m_i) = A_\nu(0) \frac{\langle p^2 \rangle}{\langle p^2 \rangle + m_i^2}$$

$$\Lambda_\chi \gtrsim m_i \gtrsim k_F$$

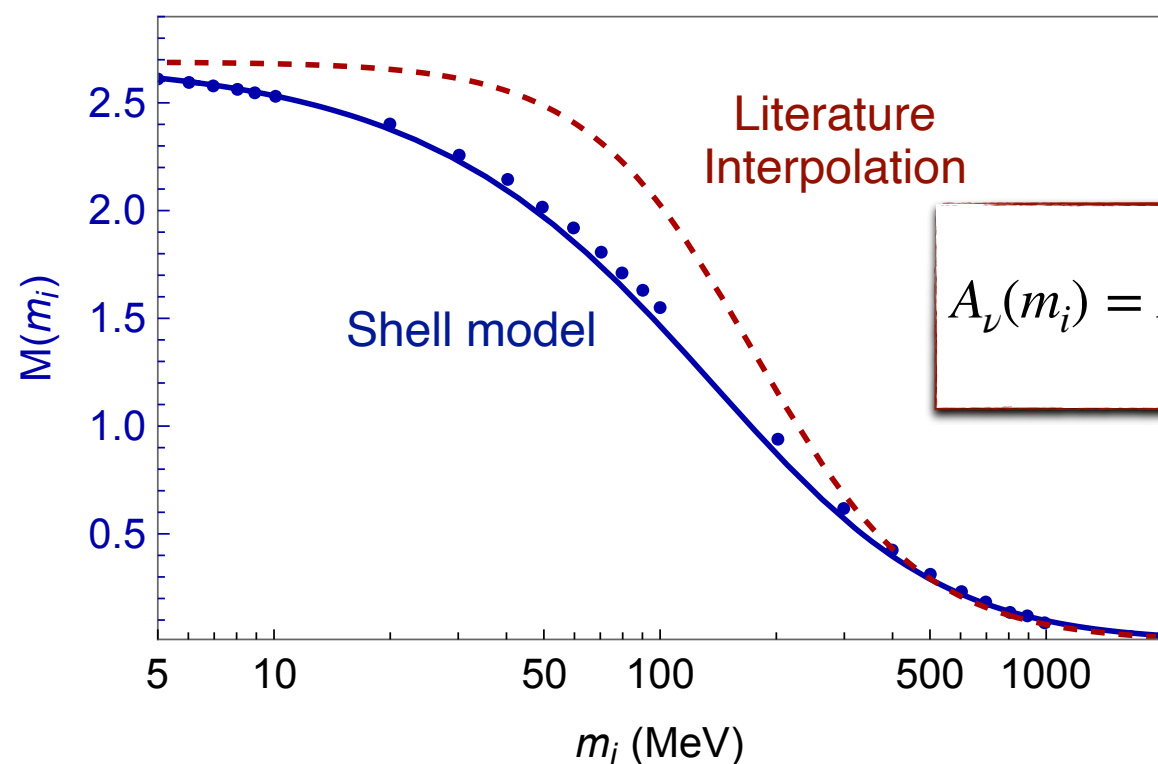


Soft contributions $\mathcal{O}\left(\frac{m_i^2}{\Lambda_\chi^2}\right)$



- Have to keep ν_i in the chiral theory
- Again have 'potential' + 'hard' contributions
- m_i dependence in NMEs and g_ν^{NN}

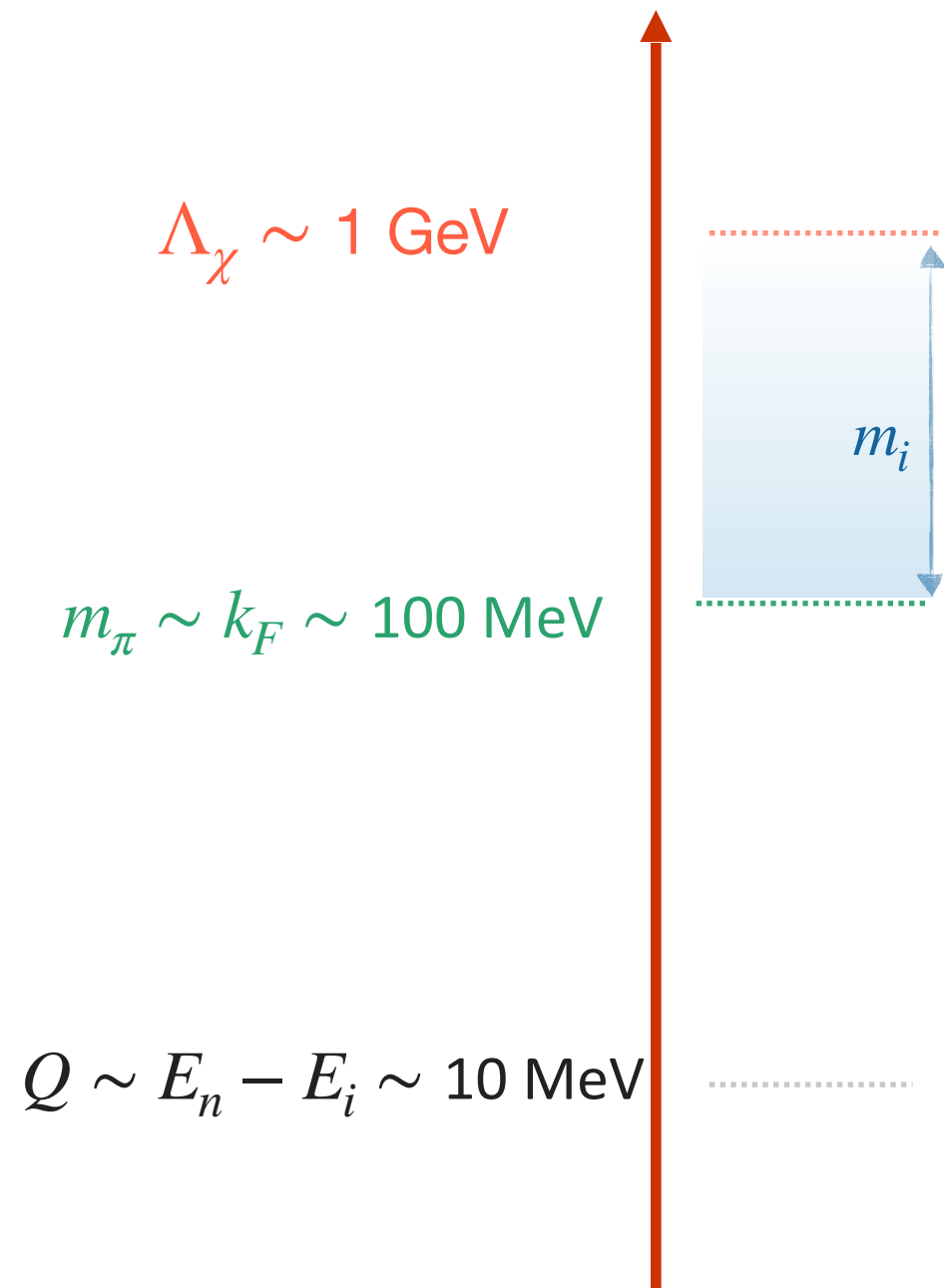
- 'soft' contributions can be significant



$$A_\nu(m_i) = A_\nu(0) \frac{\langle p^2 \rangle}{\langle p^2 \rangle + m_i^2}$$

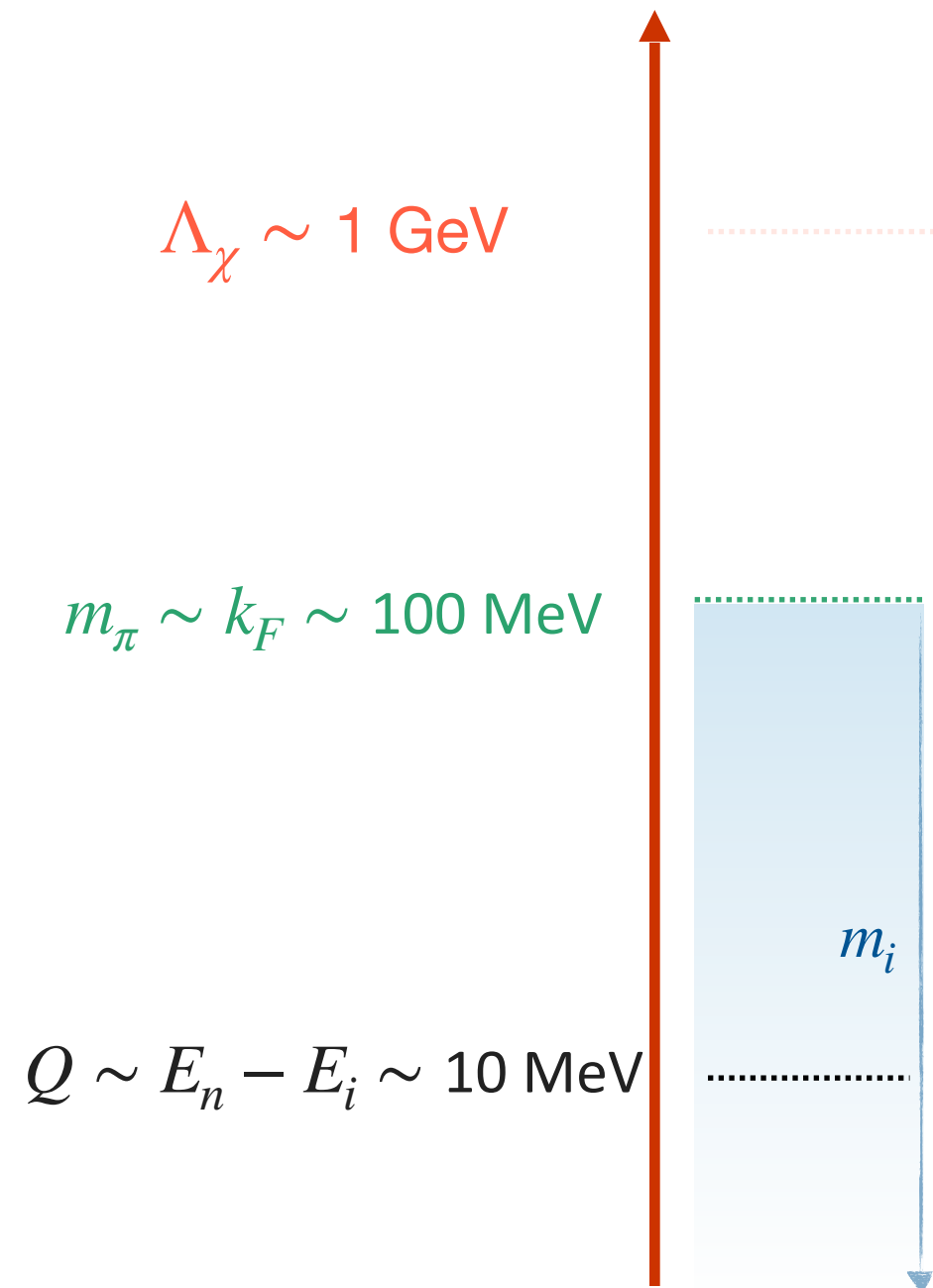
EFT approach

One momentum scale at a time



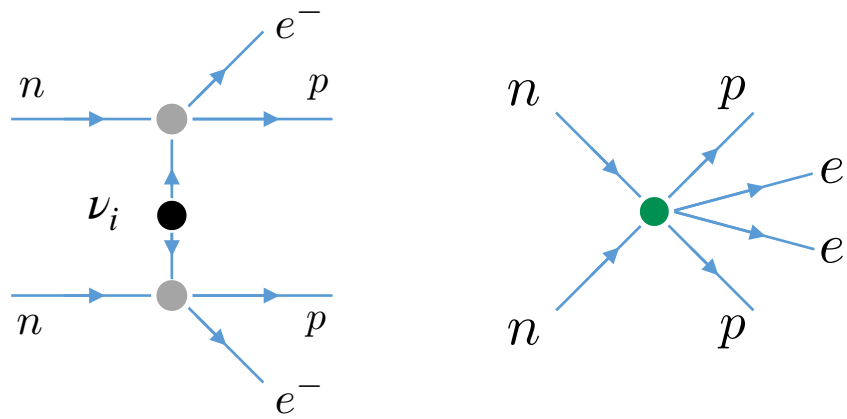
EFT approach

One momentum scale at a time



$$k_F \gtrsim m_i$$

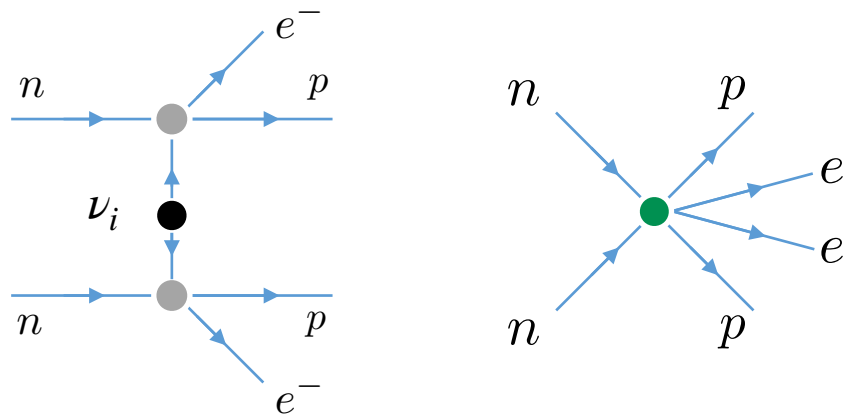
$$V_{\Delta L=2} =$$



- Similar to previous case:
- Contributions from potential + hard regions
- Soft loop contributions are now negligible

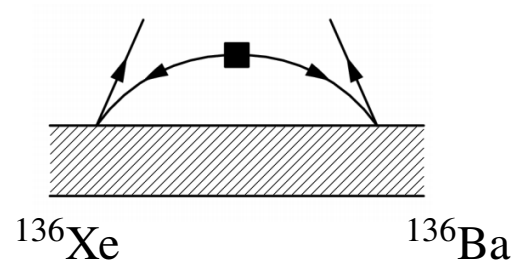
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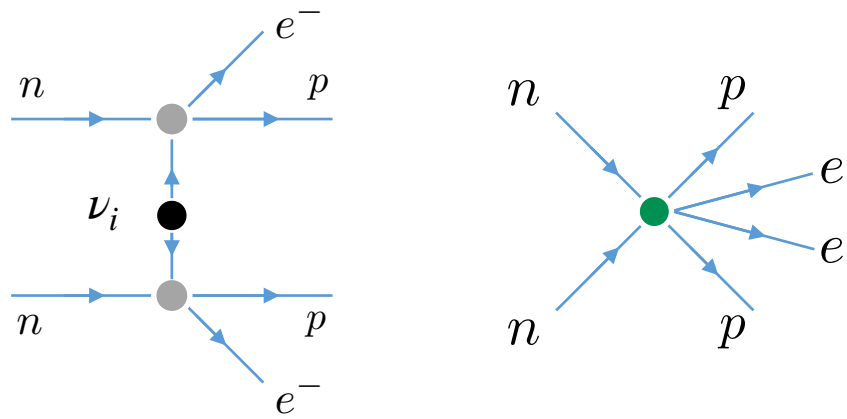
Ultrasoft contributions



- Usually N2LO effect, now leading
- Depend on:
 - **Overlap integrals**
 - **Intermediate state energies, $\Delta E \equiv E_n + E_e - E_i$**

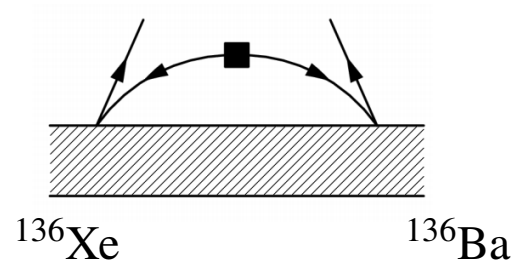
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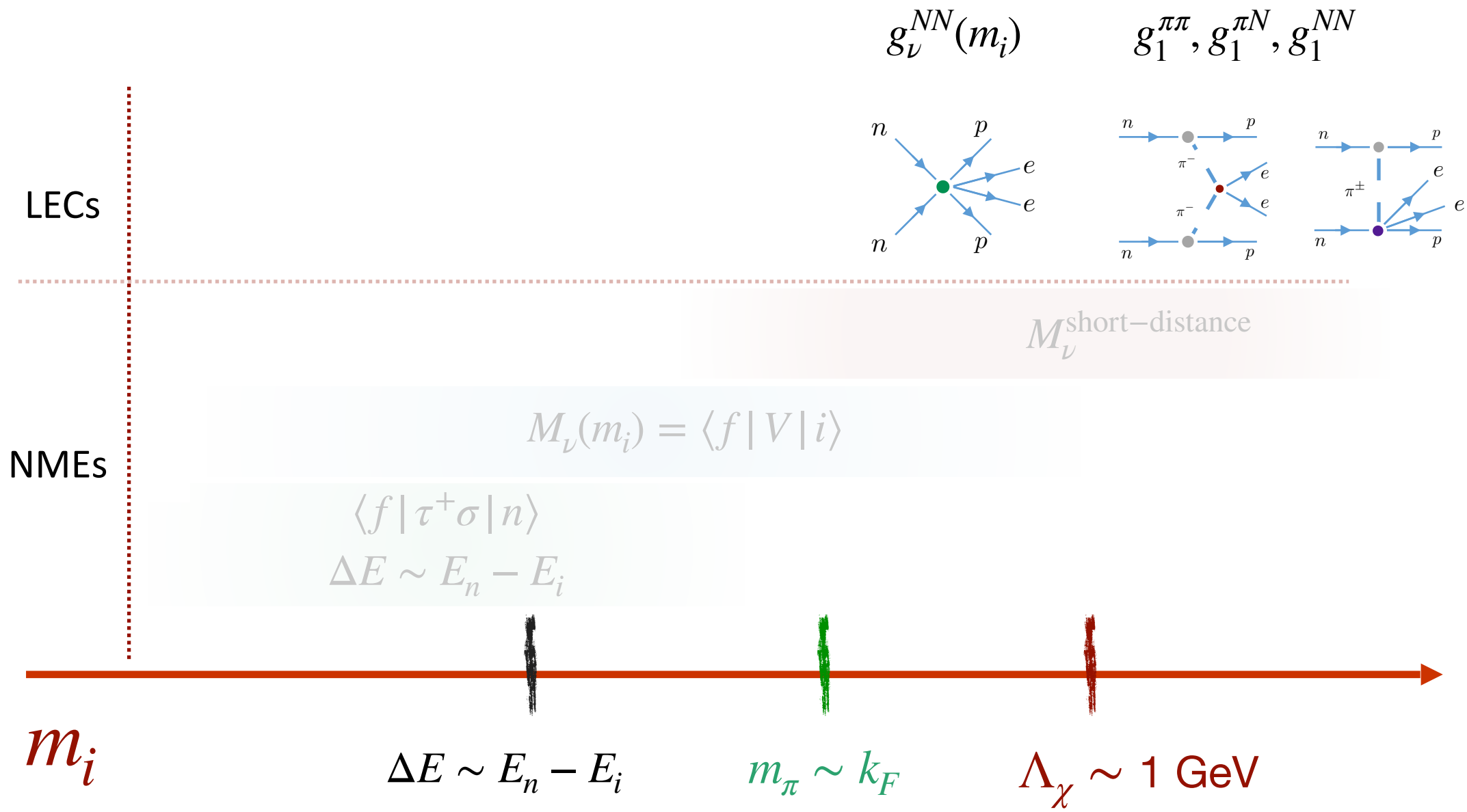
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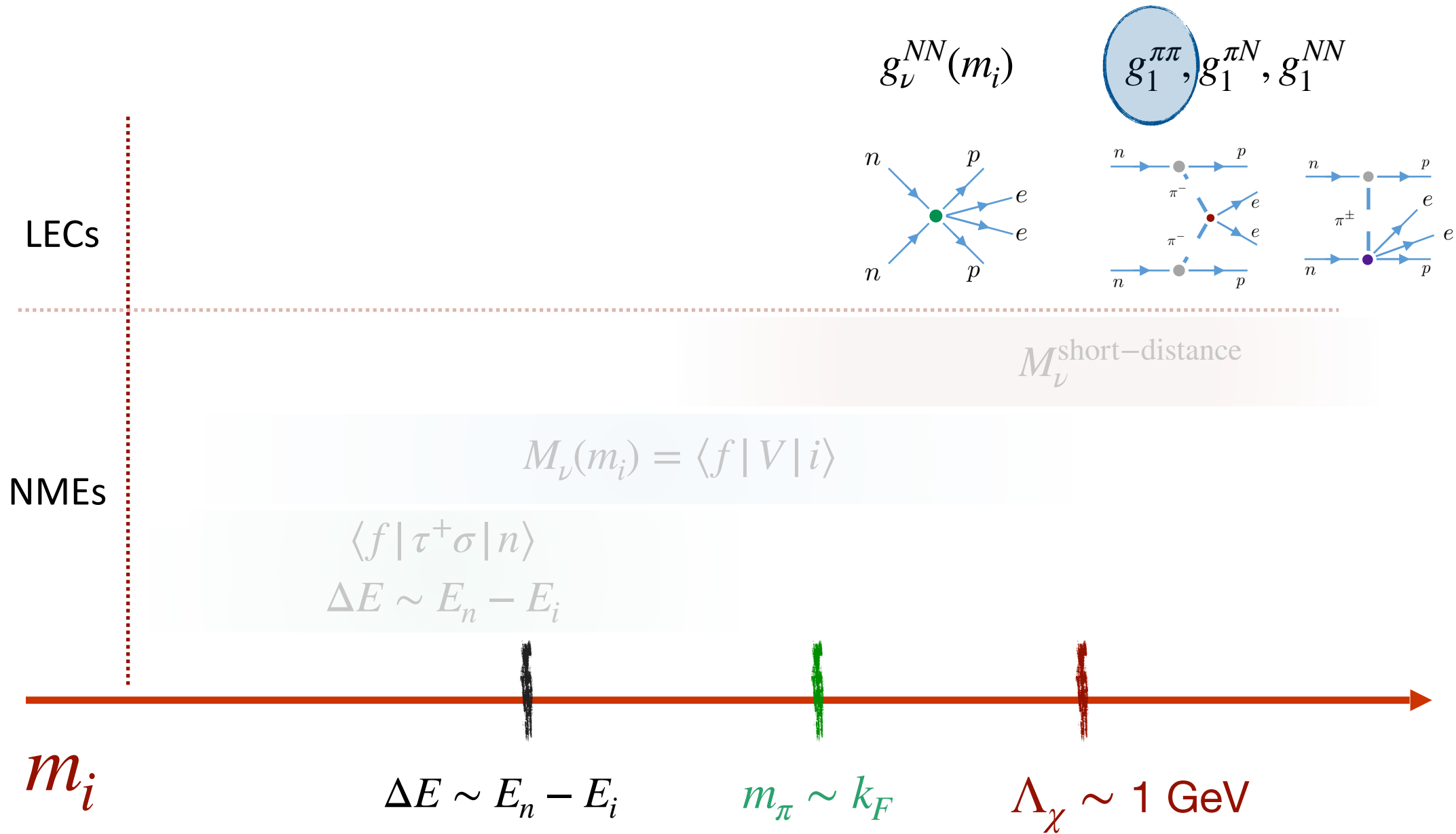
$$A_\nu^{\text{usoft}} \sim \sum_n \langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle \times \begin{cases} \frac{m_i}{k_F}, & \Delta E \lesssim m_i \lesssim k_F \\ \frac{m_i^2}{4\pi k_F \Delta E} \ln \frac{m_i}{\Delta E}, & m_i \lesssim \Delta E \end{cases}$$

Hadronic/nuclear input

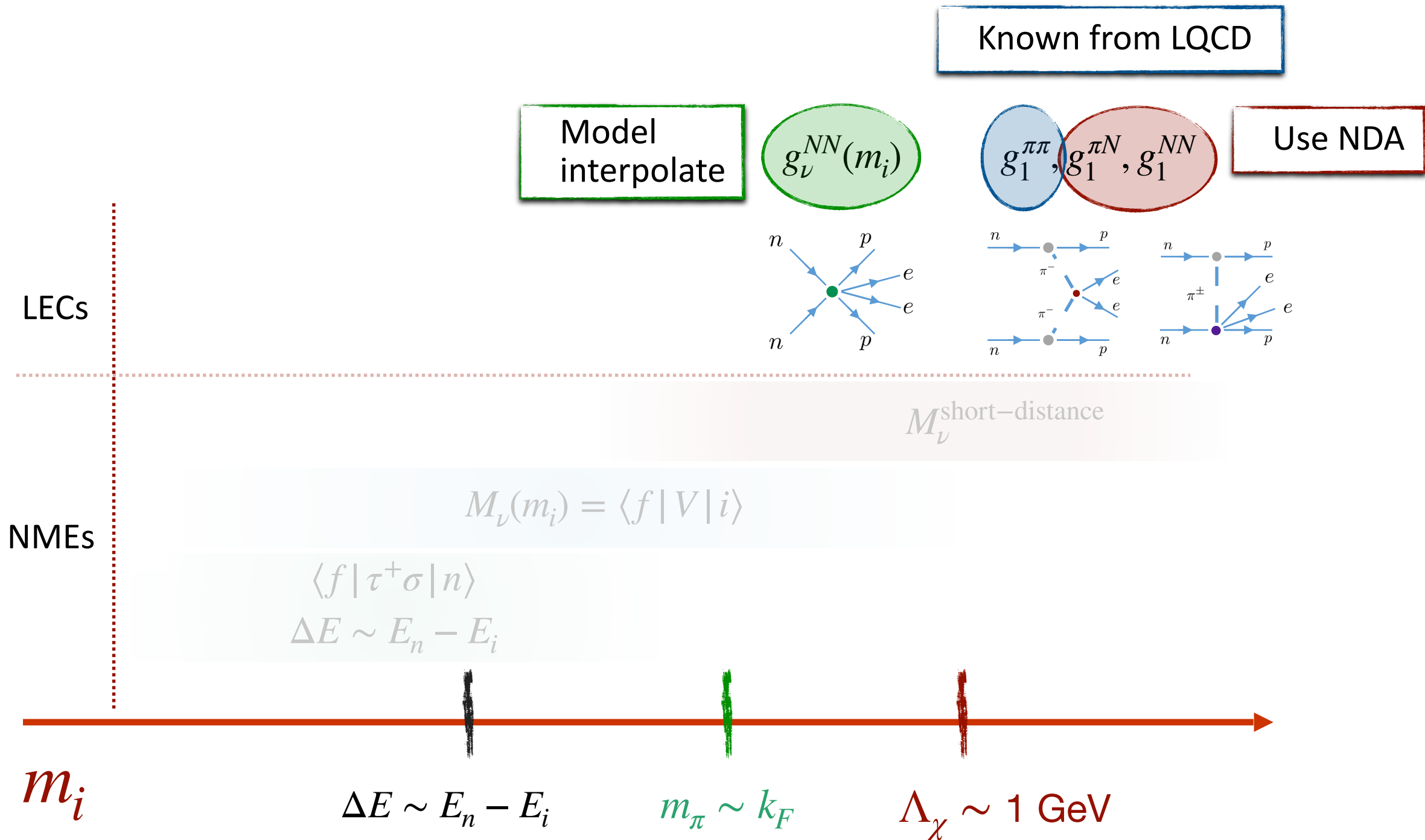


Hadronic/nuclear input

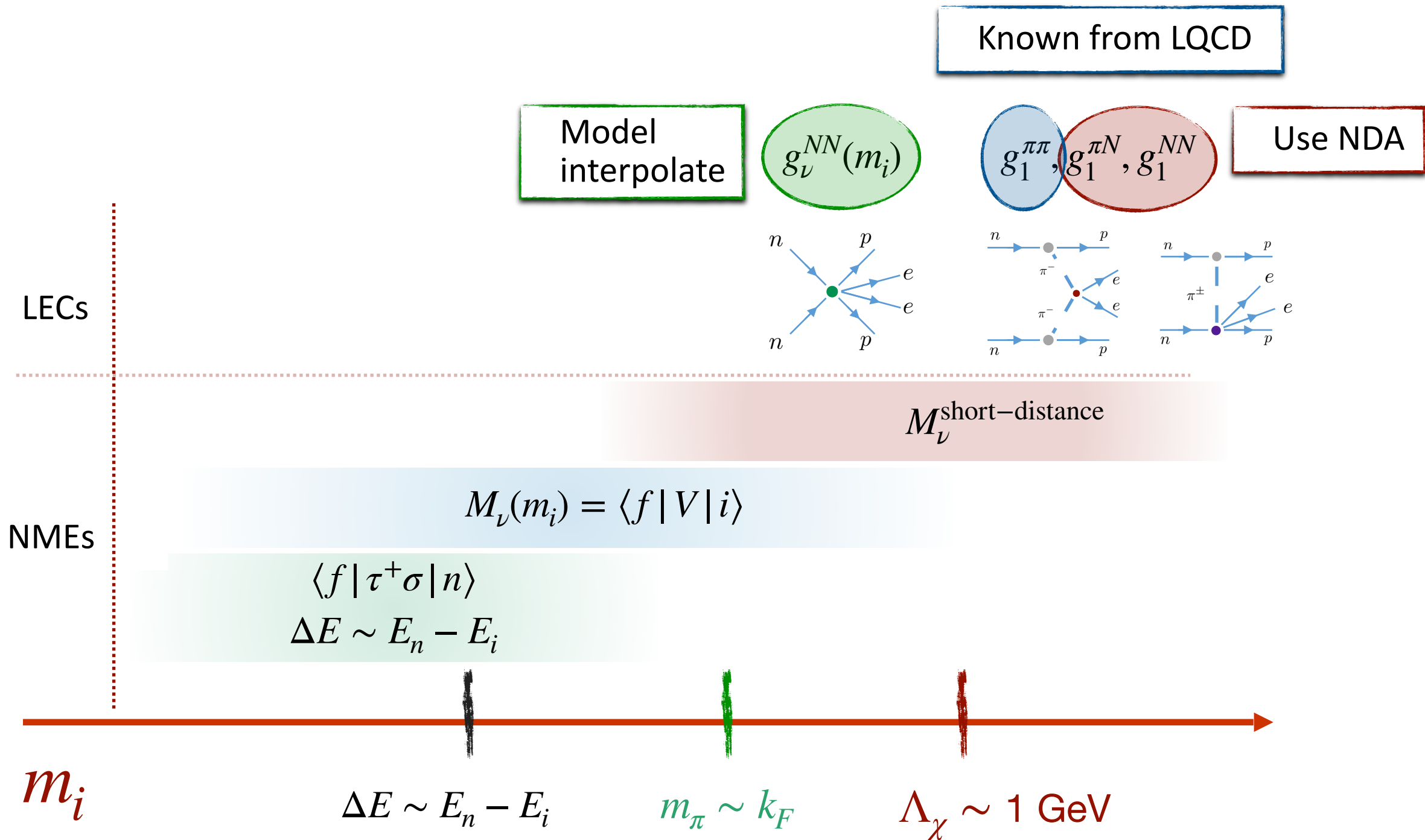
Known from LQCD



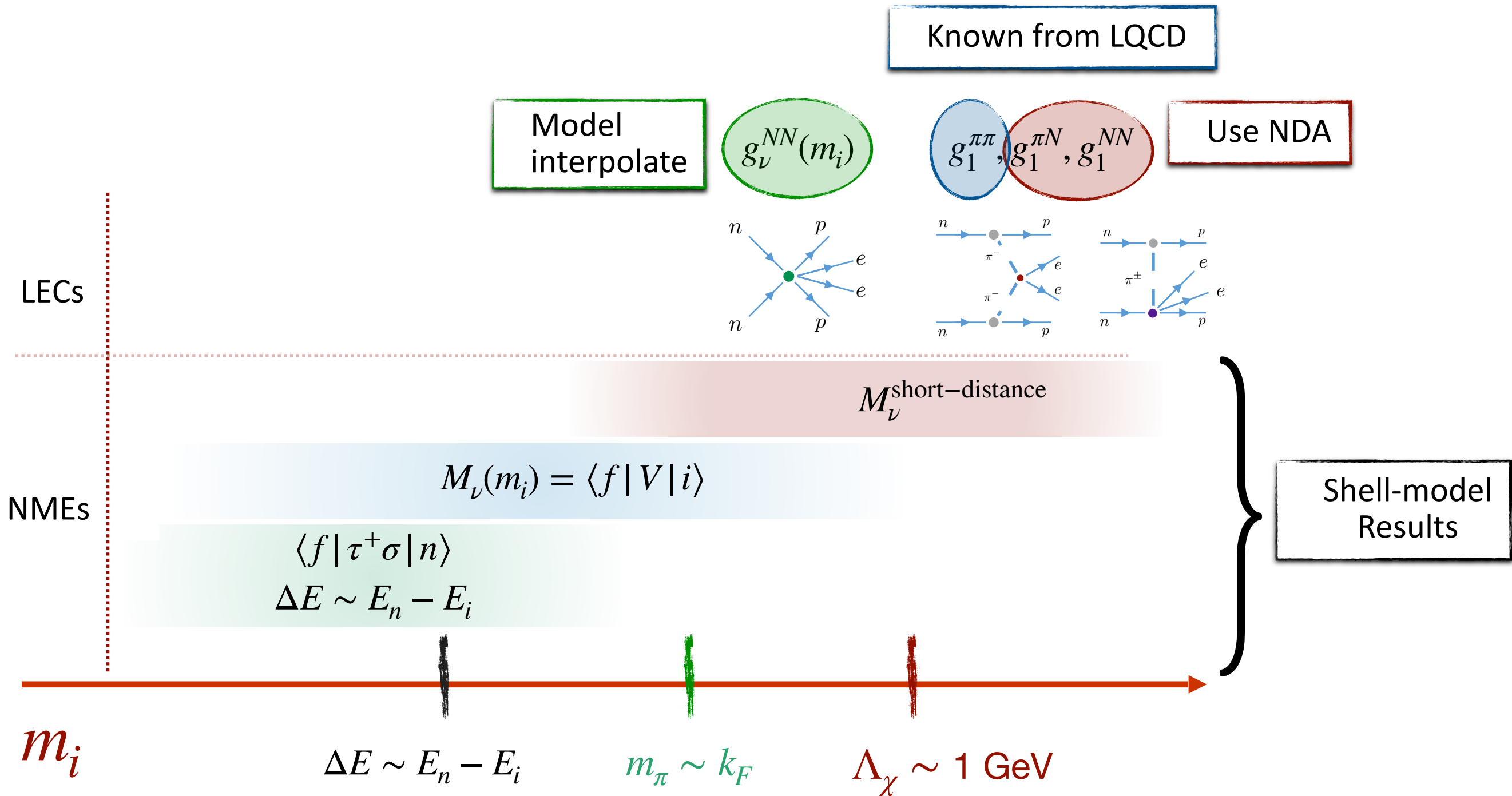
Hadronic/nuclear input



Hadronic/nuclear input



Hadronic/nuclear input



Phenomenology

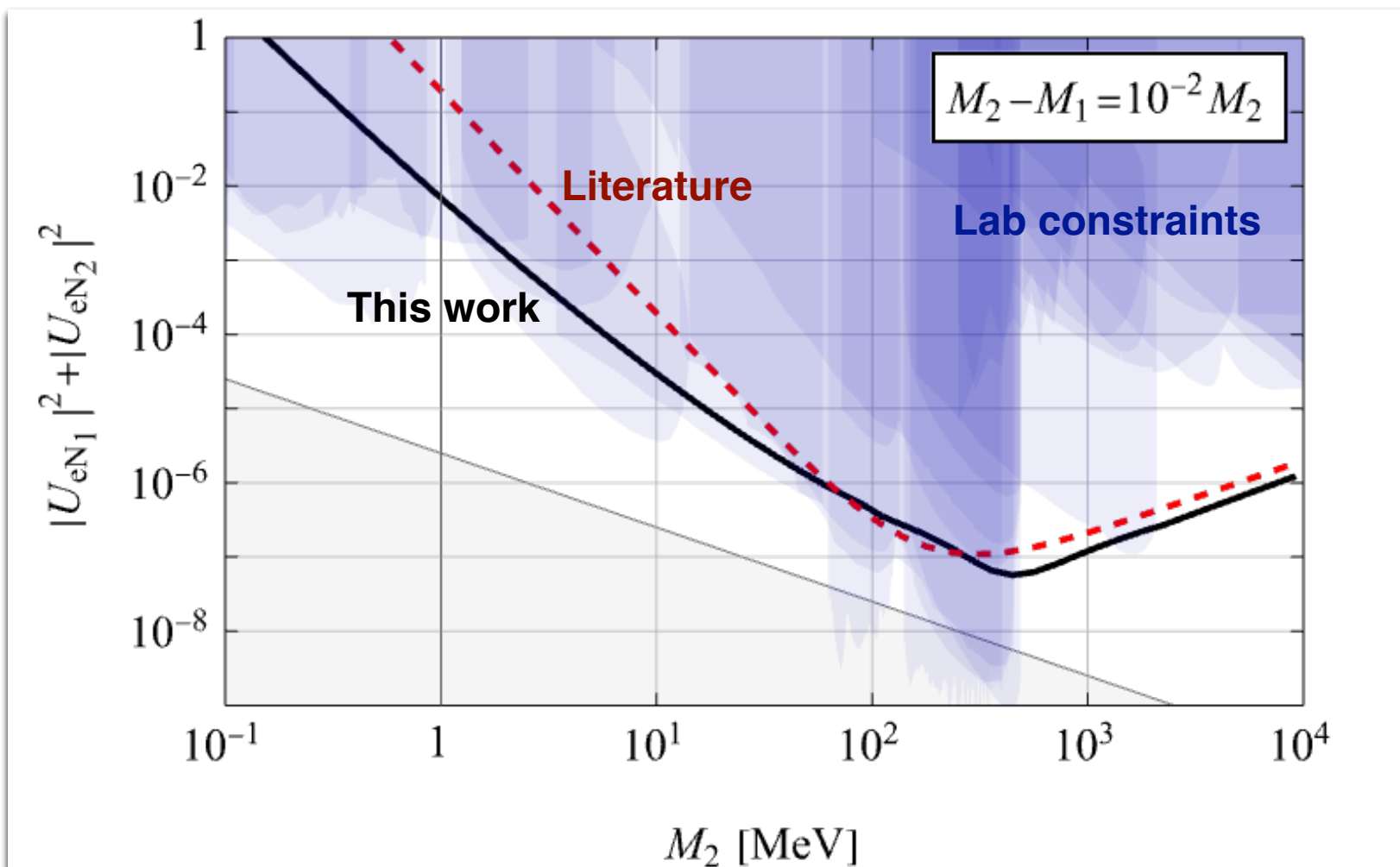


Toy model: 1+1+1 pseudo-Dirac

- Involves 1 active, two sterile neutrinos
 - Assume steriles much heavier than the active neutrinos; $M_1 \simeq M_2 \gg m_\nu$
 - Two heavier ν 's, form a pseudo-Dirac pair
 - Light neutrino mass proportional to LNV parameter (opposite to seesaw)

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Summary

- Sterile neutrinos are motivated by
 - Neutrino masses
 - Leptogenesis
 - Dark matter candidate
- Generally induce $0\nu\beta\beta$
 - Minimal extension of the SM leads to a cancellation:

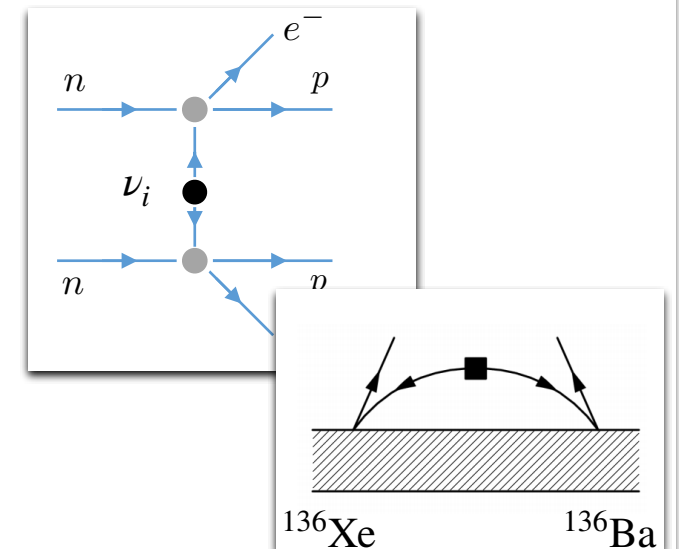
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$$\sum_{i=1}^{3+n} m_i U_{ei}^2 = 0$$

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 - Can be captured in EFT approach
- Usually subleading contributions can become important
 - Ultrasoft contributions promoted from N2LO to LO for $m_i \lesssim k_F$

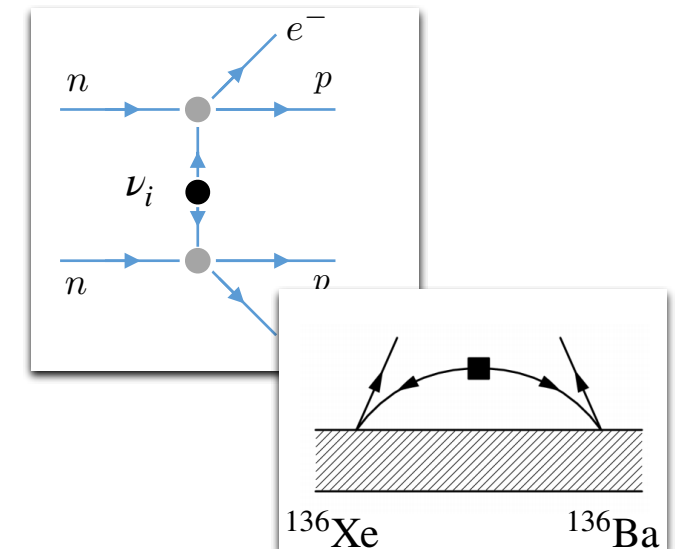


Summary

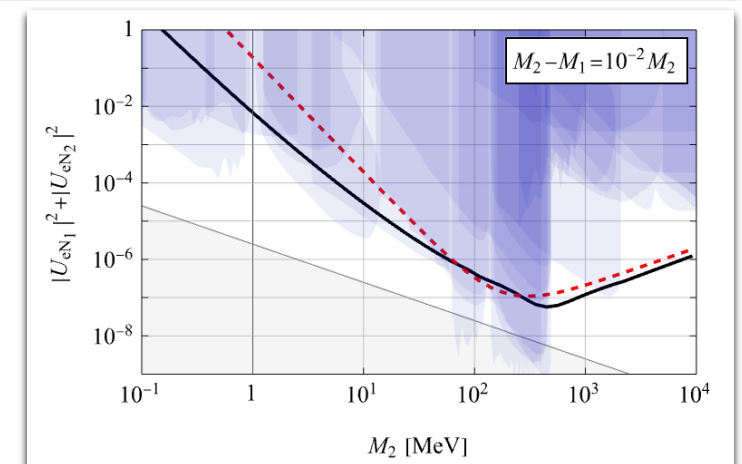
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- Significant changes compared to usual approach
 - Can already be seen in simple toy models



Back up slides



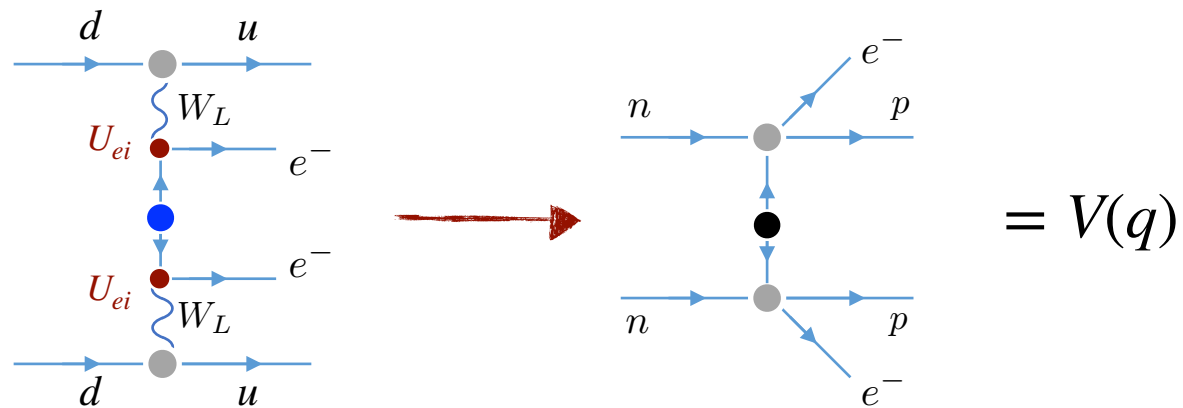
Literature approach



$0\nu\beta\beta$

Commonly used approach:

- Assume quark currents factorize:



$$A_\nu(m_i) \sim \langle {}^{136}\text{Ba} | V(q) | {}^{136}\text{Xe} \rangle$$

- Approximate amplitude by

$$A_\nu(m_i) = A_\nu(0) \frac{\langle p^2 \rangle}{\langle p^2 \rangle + m_i^2} \quad \langle p^2 \rangle \simeq k_F^2$$

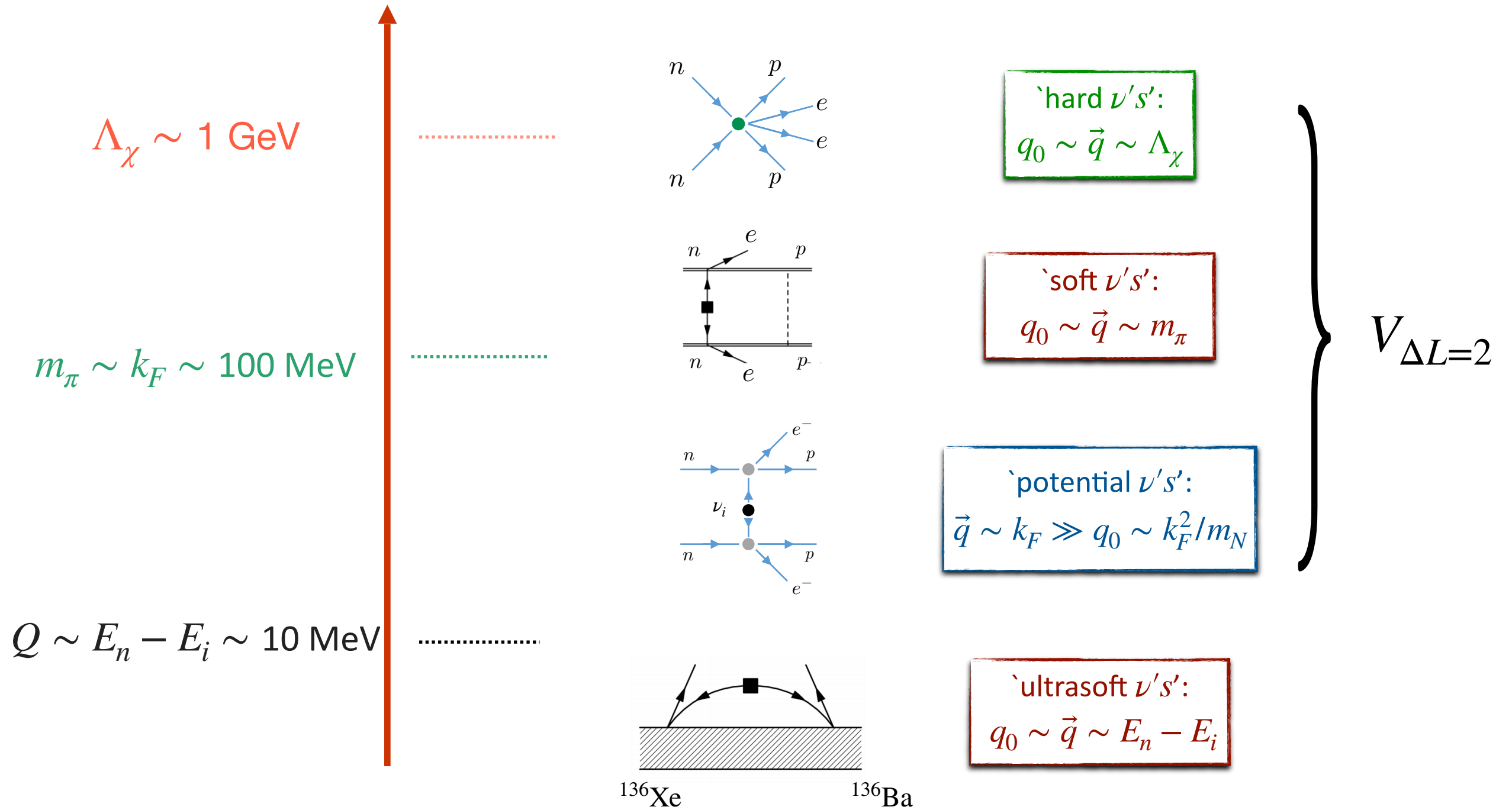
- m_i dependence seems reasonable, however,
 - Does not have the right QCD behavior for $m_i \gg \Lambda_\chi \sim \text{GeV}$
 - Misses several effects for $m_i \leq \Lambda_\chi$

Overview of Contributions



Momentum scales

$$m_i = 0$$



$$A_\nu = \langle ^{136}\text{Ba} | V_{\Delta L=2} | ^{136}\text{Xe} \rangle + A_\nu^{\text{usoft}}$$

Overview

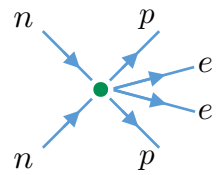
Leading m_i dependence

$$m_i \ll \Delta E$$

$$\Delta E \ll m_i \ll k_F$$

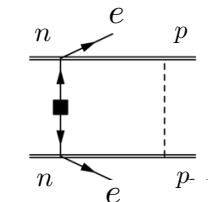
$$k_F \ll m_i \ll \Lambda_\chi$$

$$\Lambda_\chi \ll m_i$$



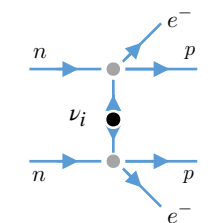
Hard

$$\frac{k_F^2}{m_i^2}$$



Soft

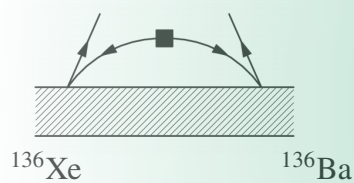
$$\frac{m_i^2}{\Lambda_\chi^2}$$



Potential

$$\frac{m_i^2}{k_F^2}$$

$$\frac{k_F^2}{m_i^2}$$

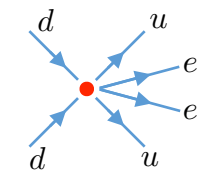


Ultrasoft

$$\frac{m_i^2}{4\pi\Delta E k_F} \ln \frac{m_i}{\Delta E}$$

$$\frac{m_i}{k_F}$$

Ultrasoft dominant for small masses



Perturbative

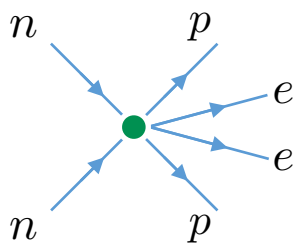
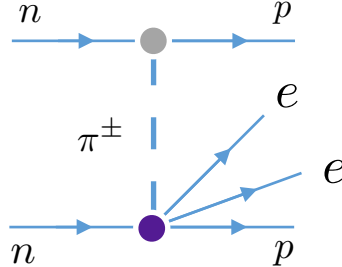
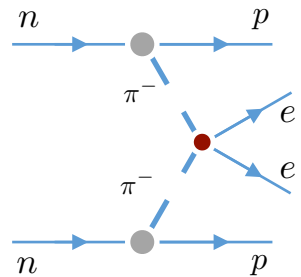
$$\frac{k_F^2}{m_i^2}$$

Hadronic matrix elements



Required LECs

$$m_i \gg \Lambda_\chi$$

	 <p style="text-align: center;">g_1^{NN}</p>	 <p style="text-align: center;">$g_1^{\pi N}$</p>	 <p style="text-align: center;">$g_1^{\pi\pi}$</p>
NDA	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Used value	$\frac{1 + 3g_A^2}{4}$	0	0.36
			LQCD: Nicholson et al '18

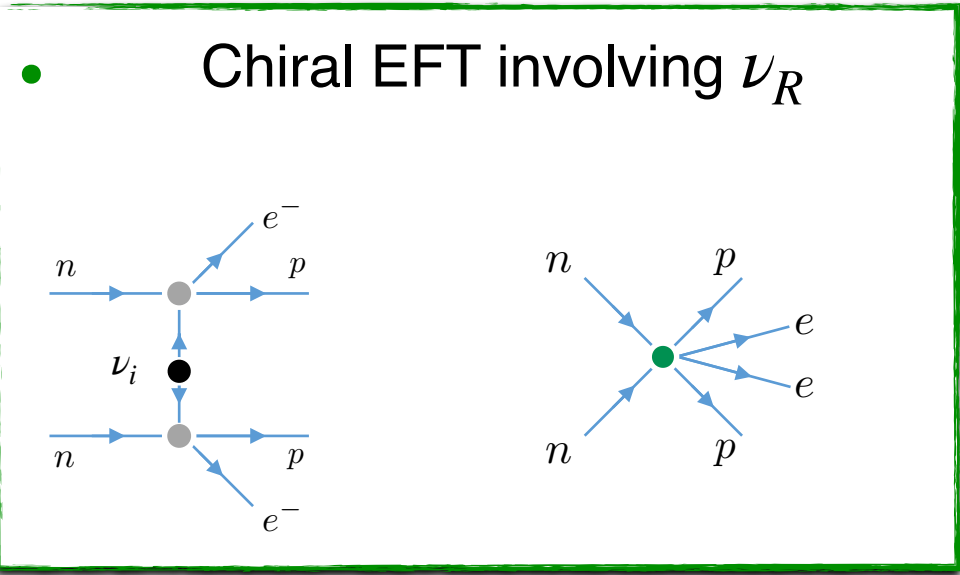
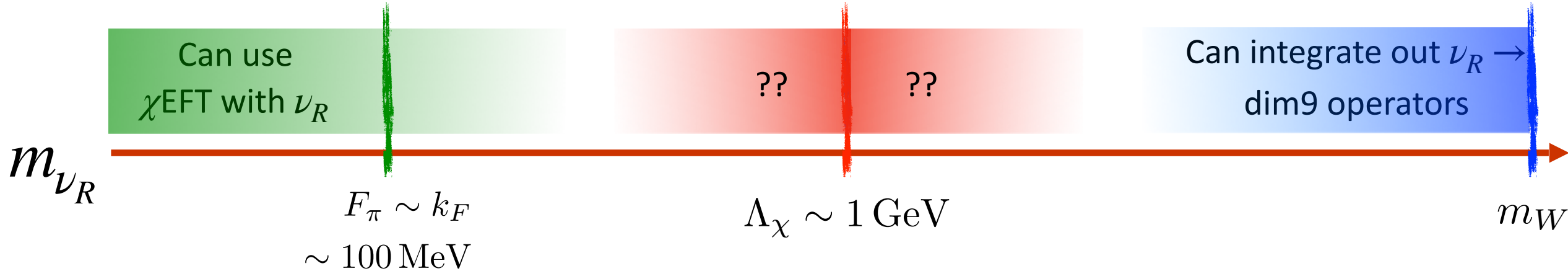
Required LECs

$$m_i \lesssim \Lambda_\chi$$

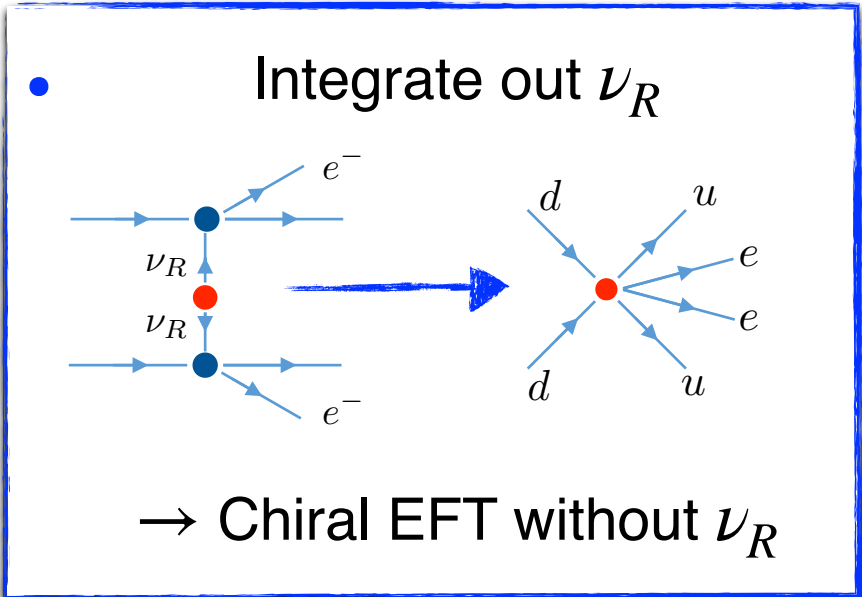
Interpolation

$$g_\nu^{NN}(m_i) = g_\nu^{NN}(0) \frac{1 + (m_i/m_c)^2}{1 + (m_i/m_c)^2(m_i/m_d)^2},$$

- NDA gives $m_c \sim 1 \text{ GeV}^2$
- Model esteems imply $g_\nu^{NN}(0) \sim - \text{fm}^2$



Match for $m_i \sim 2 \text{ GeV} \Rightarrow m_d$

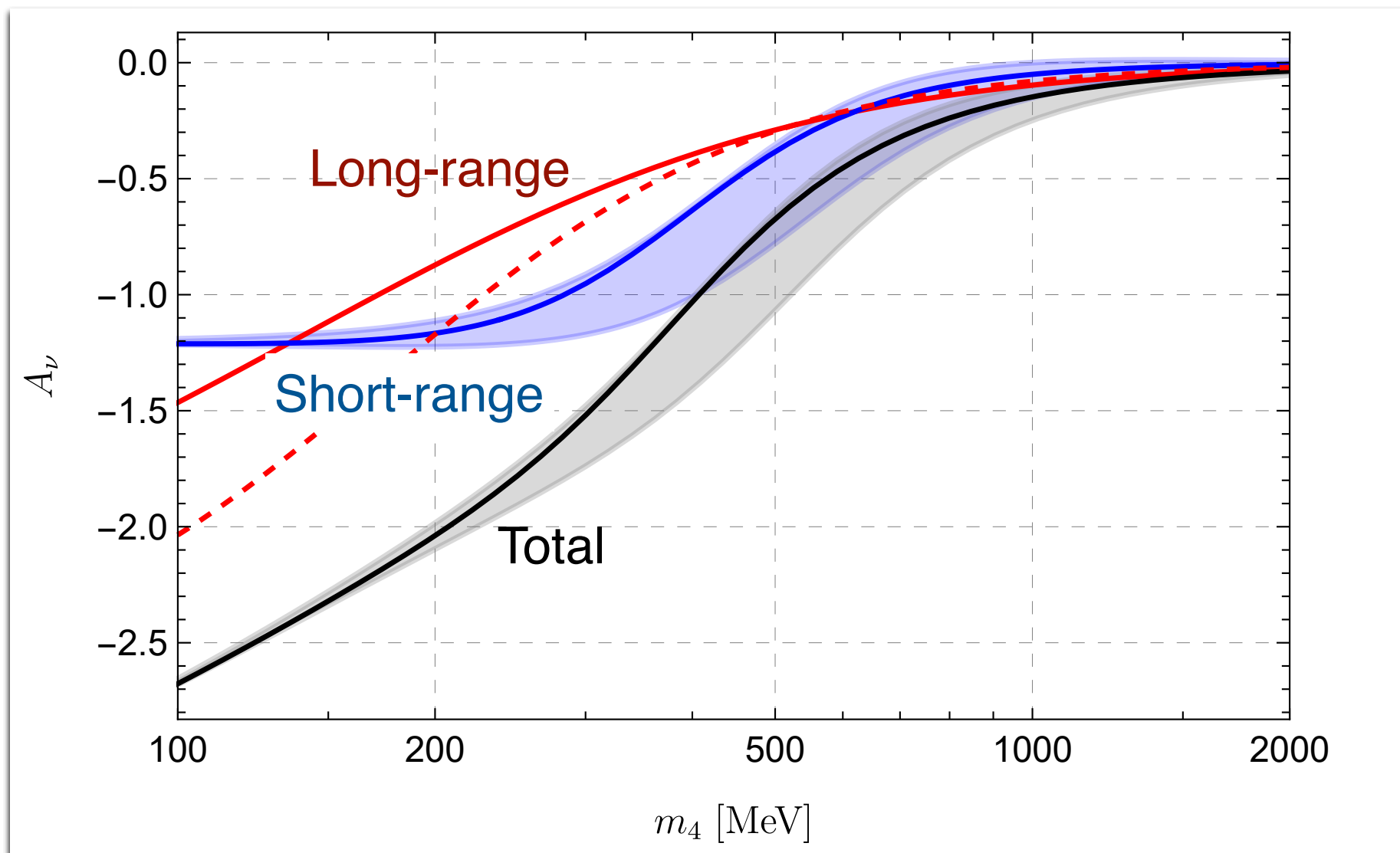
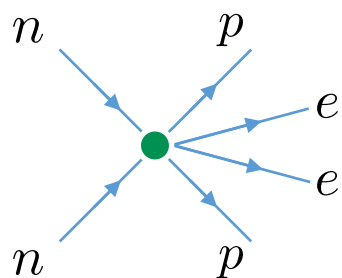


Uncertainties



LEC uncertainties

Varying g_1^{NN}



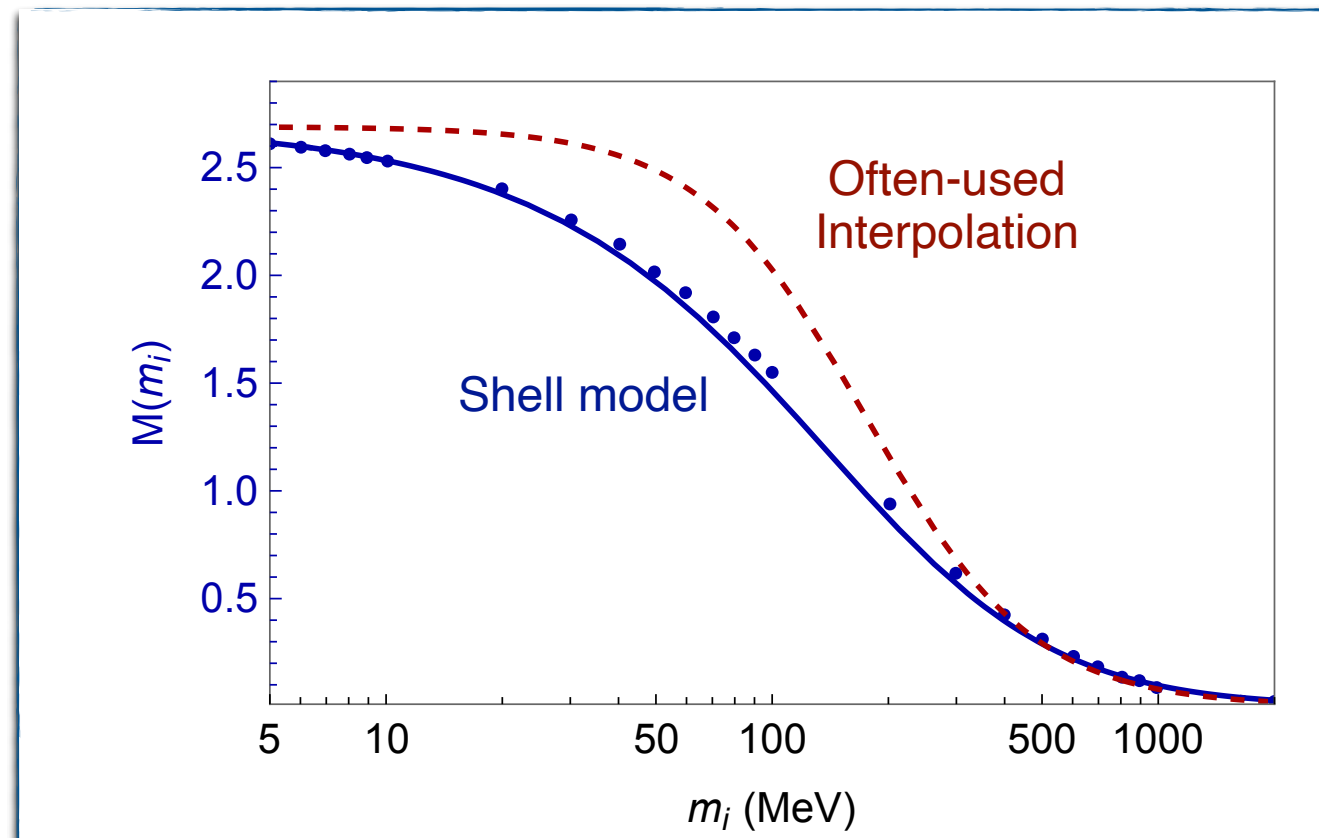
Nuclear matrix elements



Required NMEs

Potential contribution

$$A_\nu = \langle {}^{136}\text{Ba} | V(m_i) | {}^{136}\text{Xe} \rangle$$



Required NMEs

Ultrasoft contribution

$\frac{E_n - E_i}{\text{MeV}}$	$\langle n \sigma \tau^+ 0_i^+ \rangle$	$\langle 0_f^+ \sigma \tau^+ n \rangle$
0.17	1.0	0.13
0.63	-0.19	-0.0063
0.89	-0.25	-0.016
1.02	0.30	0.036
1.05	0.23	0.025
1.1	-0.13	-0.00076
1.2	0.12	-0.0052
1.3	0.16	-0.0028
1.4	-0.23	-0.0098
1.5	0.20	-0.012
1.6	-0.36	0.0084
1.7	-0.24	0.00058
1.9	0.22	0.011
2.0	0.34	0.0070
2.2	0.35	0.0060
2.3	-0.49	-0.0086
2.6	0.62	0.021
2.7	-0.91	-0.024
2.9	0.37	0.0064
3.1	0.30	0.0013

$\frac{E_n - E_i}{\text{MeV}}$	$\langle n \sigma \tau^+ 0_i^+ \rangle$	$\langle 0_f^+ \sigma \tau^+ n \rangle$
3.3	0.39	-0.0013
3.6	0.39	0.0021
3.8	0.45	-0.013
4.0	-0.44	-0.0032
4.3	-0.35	-0.0038
4.6	-0.36	-0.0067
4.8	0.44	0.0083
5.1	0.44	0.0066
5.4	-0.55	-0.0093
5.7	0.63	0.012
6.1	0.85	0.013
6.3	-1.2	-0.016
6.7	-1.3	-0.014
7.0	-1.9	-0.016
7.3	3.1	0.023
7.5	-4.0	-0.028
7.7	2.6	0.017
8.1	1.4	0.0091
8.4	-1.0	-0.0057
8.8	-0.93	-0.0064

$\frac{E_n - E_i}{\text{MeV}}$	$\langle n \sigma \tau^+ 0_i^+ \rangle$	$\langle 0_f^+ \sigma \tau^+ n \rangle$
9.1	0.80	0.0038
9.4	0.59	0.0014
9.8	-0.50	0.0027
10.1	0.35	-0.0027
10.5	0.26	-0.00053
10.9	-0.22	-0.00021
11.3	0.17	-0.00037
11.7	-0.16	-0.00054
12.0	-0.16	-0.0010
12.4	0.14	0.00092
12.8	0.12	-0.00014
13.1	0.092	-0.00040
13.5	-0.079	-0.00019
13.9	0.071	-0.00026
14.2	-0.070	0.000031
14.6	-0.035	0.00021
15.1	-0.051	-0.00015
16.2	-0.039	0.00011
17.3	-0.043	-0.000091
17.7	0.11	-0.000029

$$A_\nu^{(\text{usoft})} = -\frac{R_A}{2\pi} \sum_n \langle 0^+ | \tau^+ \sigma | n \rangle \langle n | \tau^+ \sigma | 0^+ \rangle \times [f(m_i, \Delta E_1) + f(m_i, \Delta E_2)] ,$$

$$f(m, E) = -2 \left[E \left(1 + \ln \frac{\mu_{us}}{m} \right) + \sqrt{m^2 - E^2} \times \left(\frac{\pi}{2} - \tan^{-1} \frac{E}{\sqrt{m^2 - E^2}} \right) \right] , \quad k_F \gtrsim m_i \gtrsim k_F^2 / m_N$$

$$f(m, E) = -2 \left[E \left(1 + \ln \frac{\mu_{us}}{m} \right) - \sqrt{E^2 - m^2} \ln \frac{E + \sqrt{E^2 - m^2}}{m} \right] . \quad m_i \lesssim \Delta E$$

Required NMEs

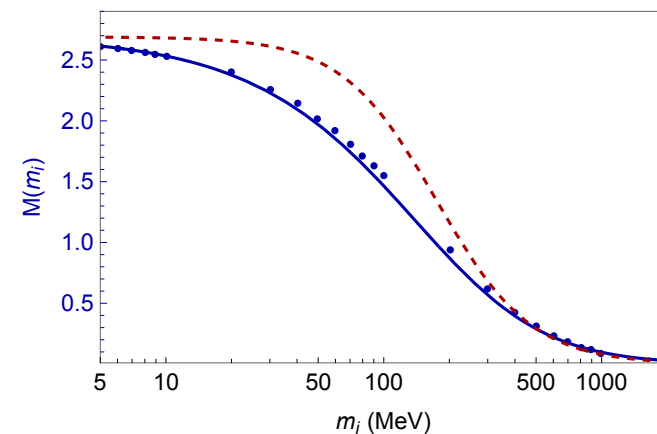
Ultrasoft/potential contributions

- Part of the ultrasoft and potential contributions are related:
 - For $m_\pi \gtrsim m_i \gtrsim \Delta E$

$$A_\nu^{\text{usoft}} \simeq \frac{R_A}{2\pi} m_i \sum_n \langle 0^+ | \tau^+ \sigma | n \rangle \langle n | \tau^+ \sigma | 0^+ \rangle$$

- This linear term is also present in

$$A_\nu^{\text{pot}} = \langle {}^{136}\text{Ba} | V(m_i) | {}^{136}\text{Xe} \rangle$$



- Have to make sure not to double count
 - In practice we remove the linear term from the potential contributions
- Allows for a cross check of the form

$$A_\nu^{\text{usoft}} \simeq m_i \frac{d}{dm_i} A_\nu^{\text{pot}}$$

- Numerically works to $\sim 20\%$

Renormalization arguments



Checking the power counting

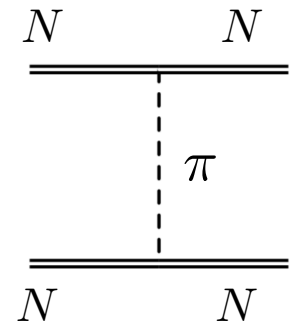
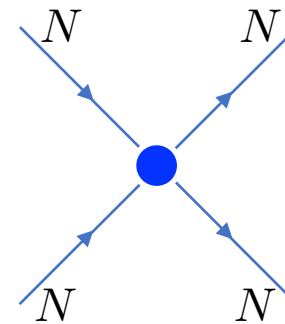
Check that $\mathcal{A}(nn \rightarrow ppee)$ is finite

Checking the power counting

Check that $\mathcal{A}(nn \rightarrow ppee)$ is finite

- Requires inclusion of the strong interaction

$$\mathcal{L}_\chi = C (N^T P_{1S_0} N)^\dagger N^T P_{1S_0} N - \frac{g_A}{2F_\pi} \nabla \pi \cdot \bar{N} \boldsymbol{\tau} \boldsymbol{\sigma} N$$

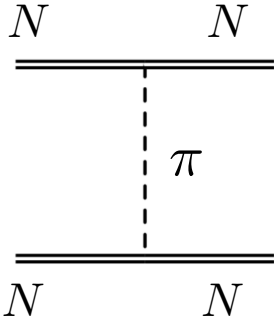
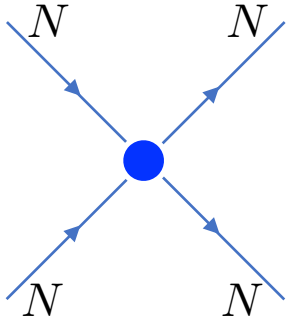


Checking the power counting

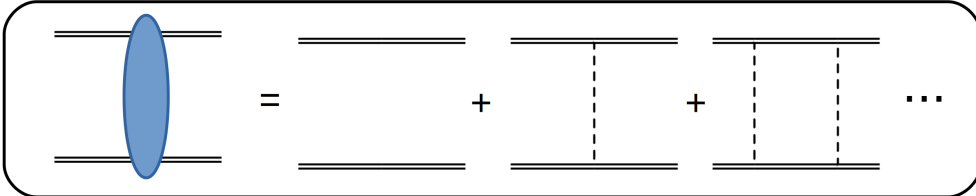
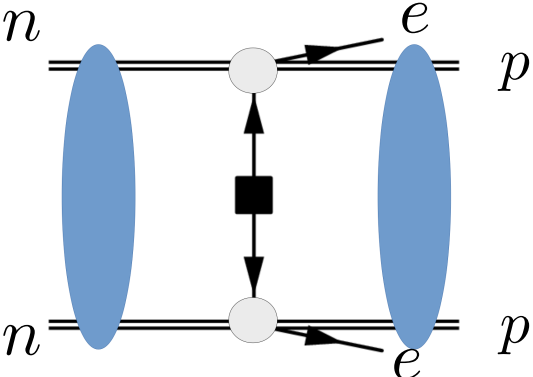
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Dress the $\Delta L=2$ potential with (renormalized) strong interactions:



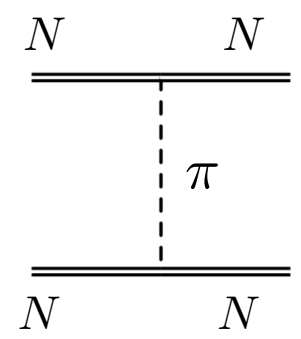
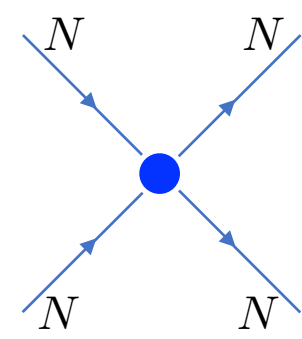
✓ finite

Checking the power counting

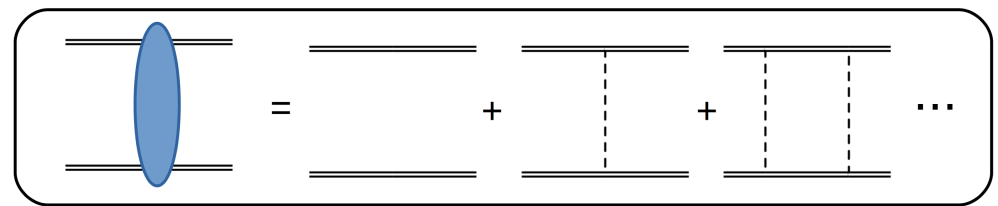
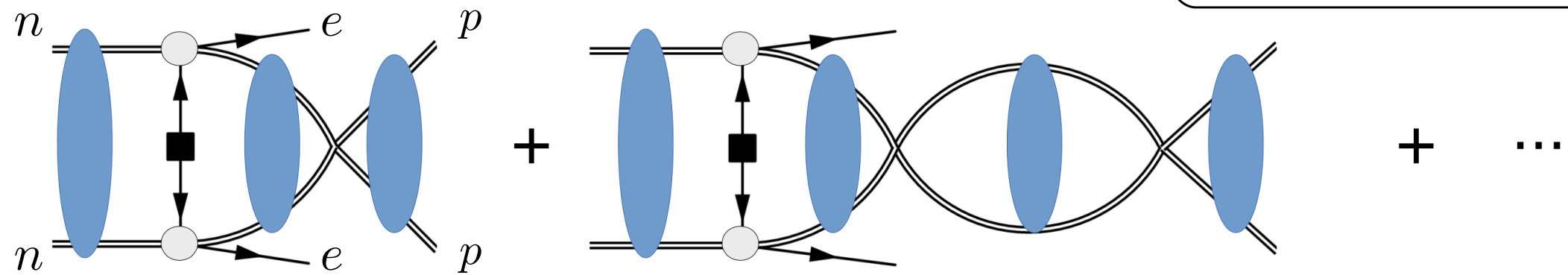
Check that $\mathcal{A}(nn \rightarrow ppee)$ is finite

- Requires inclusion of the strong interaction

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Dress the $\Delta L=2$ potential with (renormalized) strong interactions:



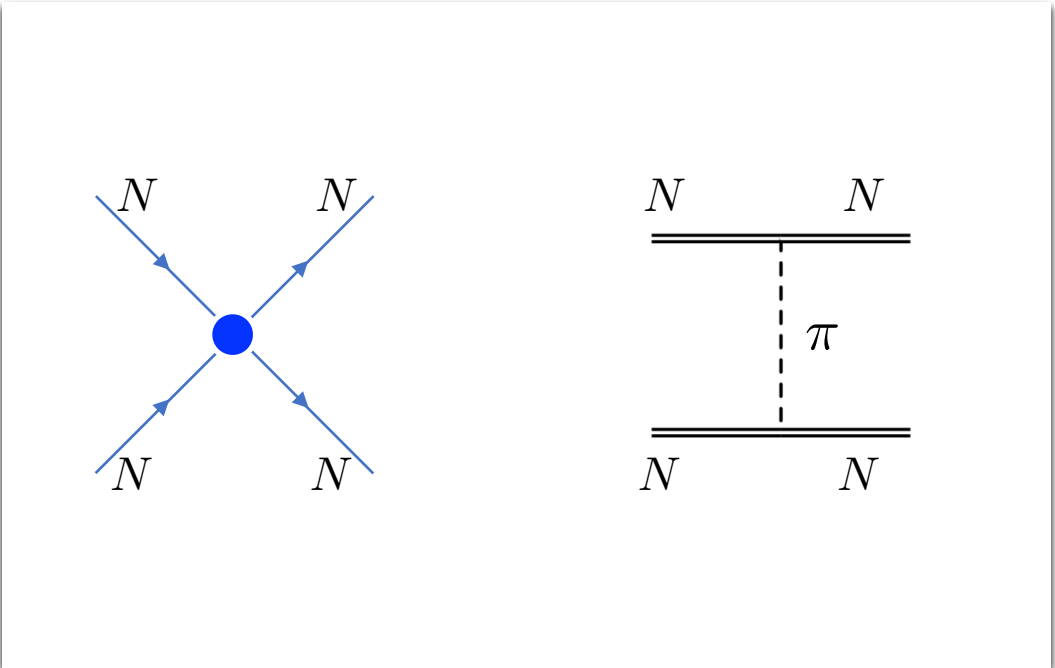
✓ finite

Checking the power counting

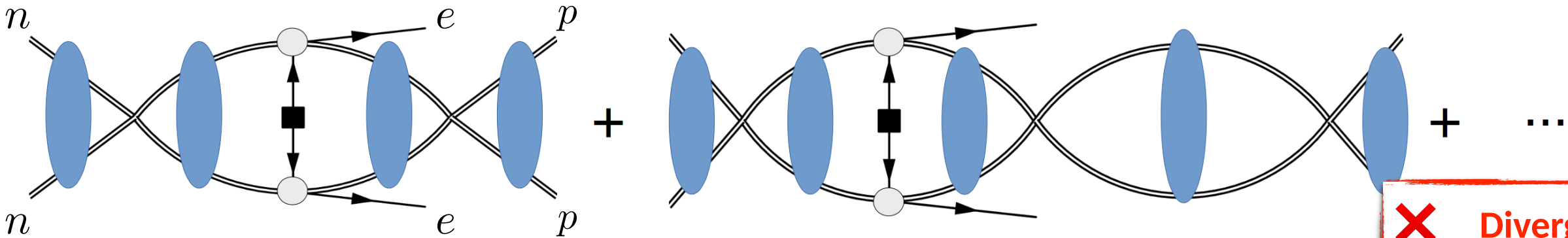
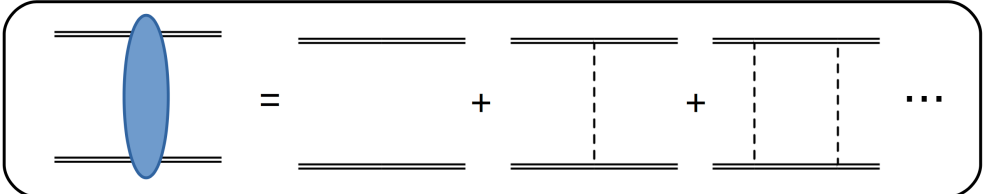
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$$\mathcal{L}_\chi = C (N^T P_1 S_0 N)^\dagger N^T P_1 S_0 N - \frac{g_A}{2F_\pi} \nabla \pi \cdot \bar{N} \tau \sigma N$$



Dress the $\Delta L=2$ potential with (renormalized) strong interactions:



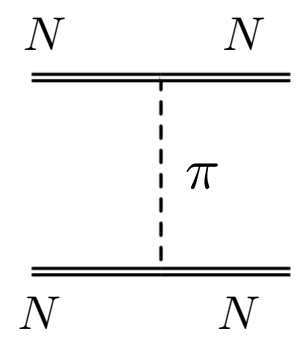
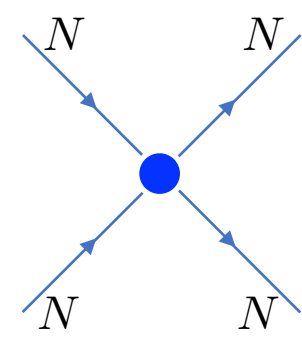
X Divergent

Checking the power counting

Check that $\mathcal{A}(nn \rightarrow ppee)$ is finite

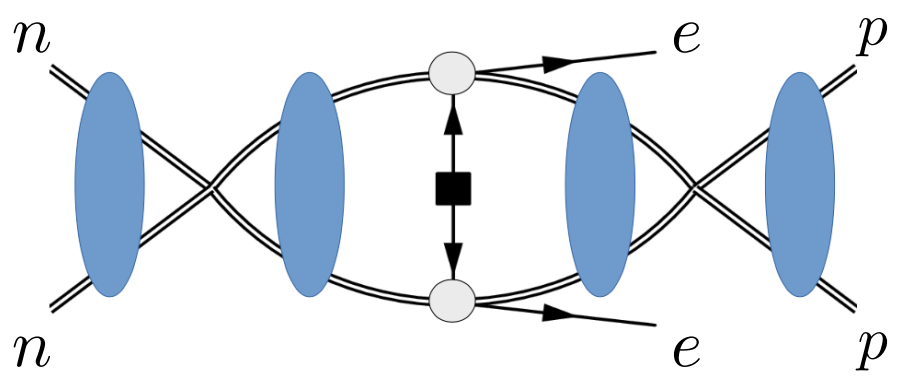
- Requires inclusion of the strong interaction

$$\mathcal{L}_\chi = C (N^T P_{1S_0} N)^\dagger N^T P_{1S_0} N - \frac{g_A}{2F_\pi} \nabla \pi \cdot \bar{N} \tau \sigma N$$



Dress the $\Delta L=2$ potential with (renormalized) strong interactions:

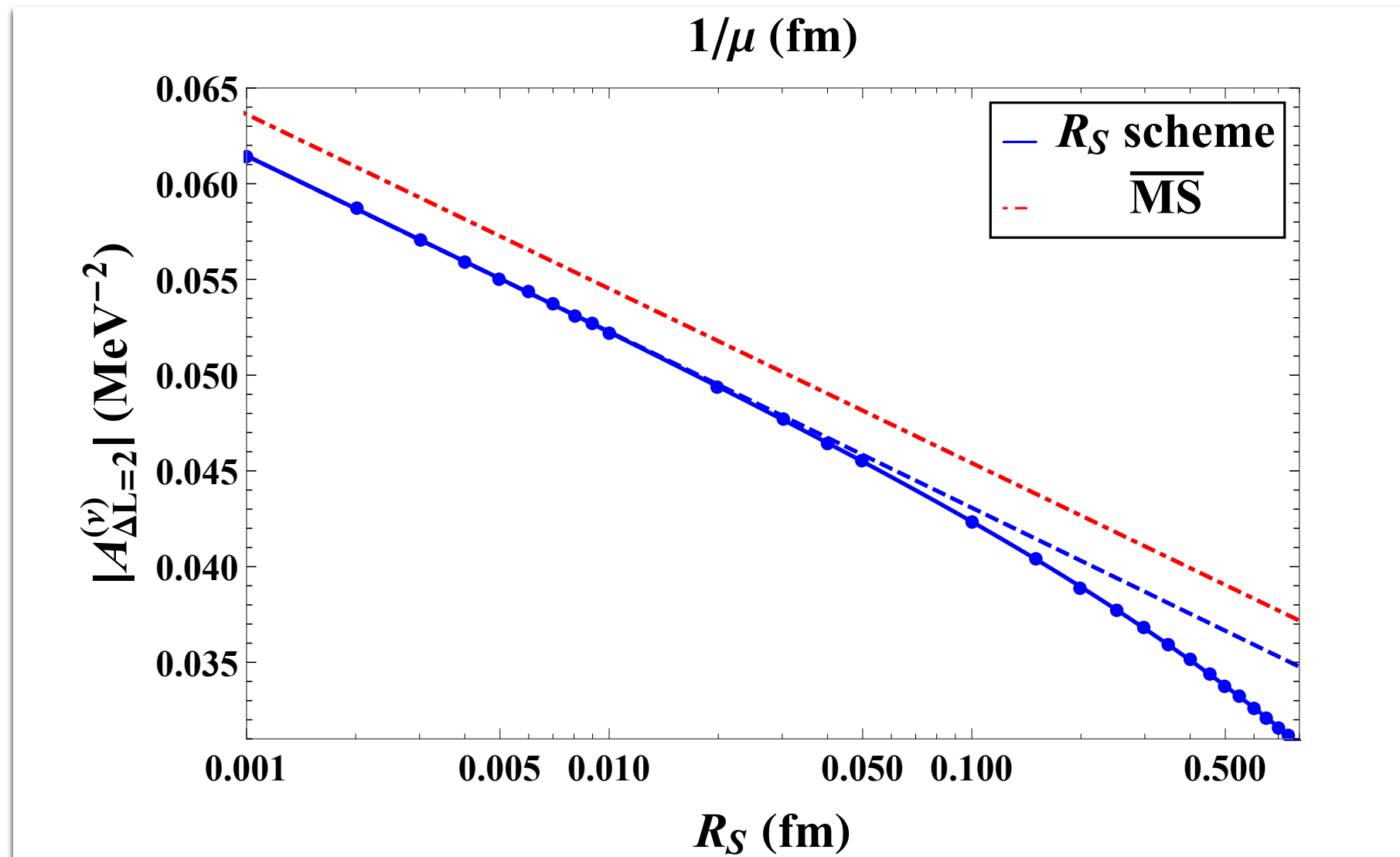
In MS-bar:



$$= - \left(\frac{m_N}{4\pi} \right)^2 (1 + 2g_A^2) \frac{1}{2} \left(\log \frac{\mu^2}{-(|\mathbf{p}| + |\mathbf{p}'|)^2 + i0^+} + 1 \right) + \text{finite}$$

Regulator dependent

Numerical results



- Amplitudes obtained using
 - MS-bar
 - Coordinate-space cut-off

• Clear μ or R_S dependence

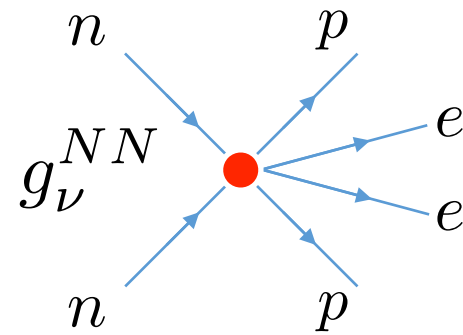
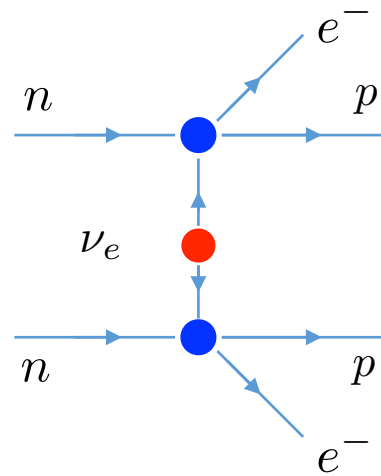
$$\tilde{C} \delta^{(3)}(\mathbf{r}) \rightarrow \frac{\tilde{C}(R_S)}{(\sqrt{\pi}R_S)^3} \exp\left(-\frac{r^2}{R_S^2}\right)$$

Need for a counter term

New interaction needed at leading order to get physical amplitudes:

$$\mathcal{L}_{CT} = 2G_F^2 V_{ud}^2 m_{\beta\beta} g_\nu^{NN} \bar{p}n \bar{p}n \bar{e}_L C \bar{e}_L^T$$

$$V_{\Delta L=2} = V_\nu + V_{\nu,CT} =$$

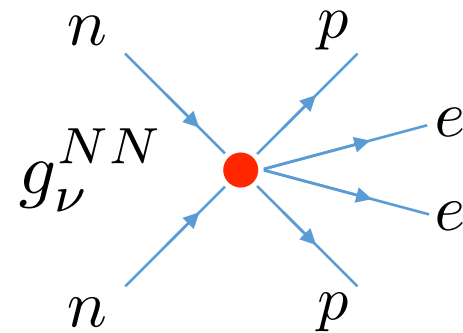
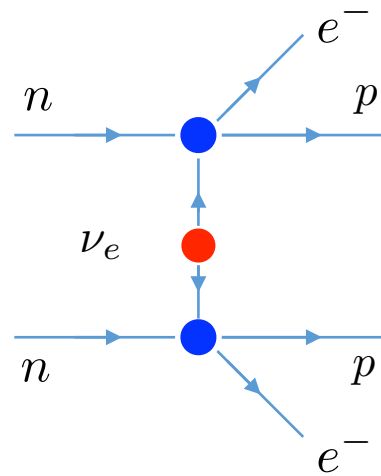


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- g_ν^{NN} to be determined from a lattice calculation of $\mathcal{A}(nn \rightarrow ppe^- e^-)$

- Area of active research

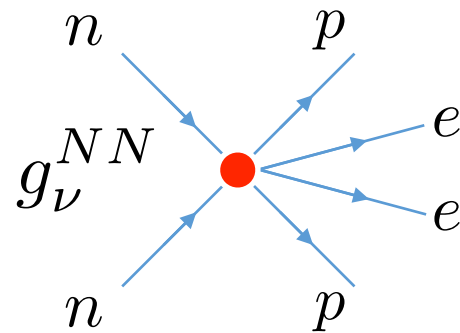
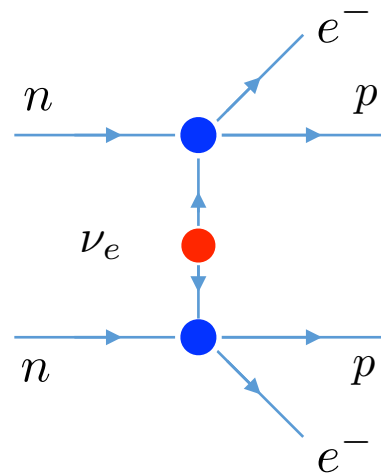
Davoudi and Kadam, '20, '21
Feng et al, '20

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- g_ν^{NN} to be determined from a lattice calculation of $\mathcal{A}(nn \rightarrow ppe^- e^-)$

- Area of active research

Davoudi and Kadam, '20, '21
Feng et al, '20

- Several estimates give $\tilde{g}_\nu^{NN} = \mathcal{O}(1)$

- Comparison with isospin-breaking observables

- Model (Cottingham) estimate

Cirigliano, et al, '19,'20, '21

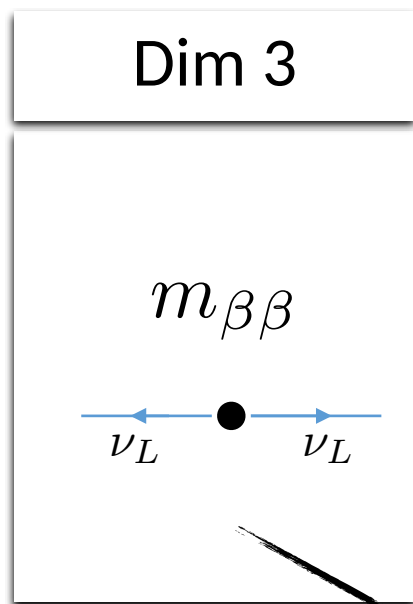
- Large-Nc estimate

Richardson et al, '21

[See backup](#)

Chiral EFT

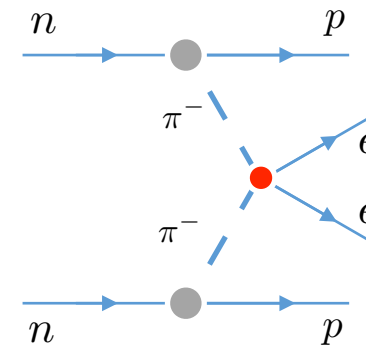
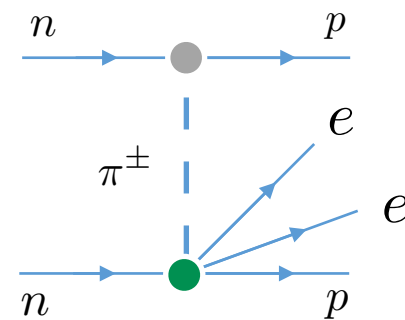
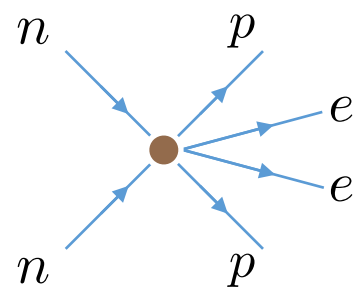
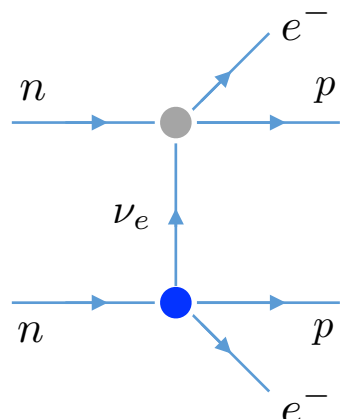
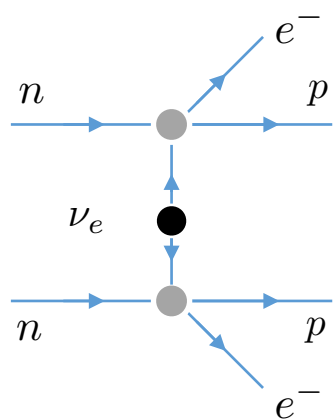
Non-Weinberg counting



M_{QCD}
1 GeV

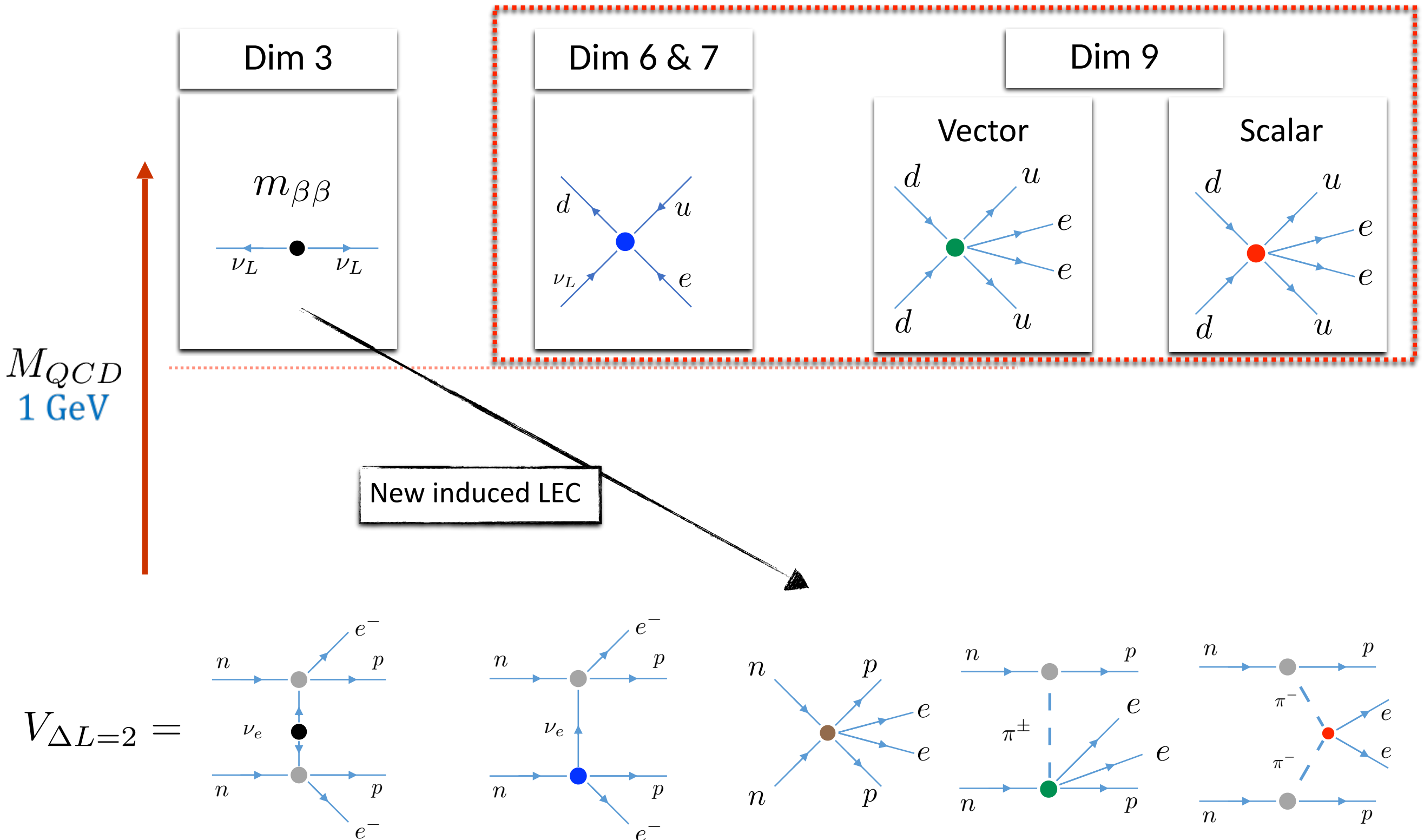
New induced LEC

$V_{\Delta L=2} =$



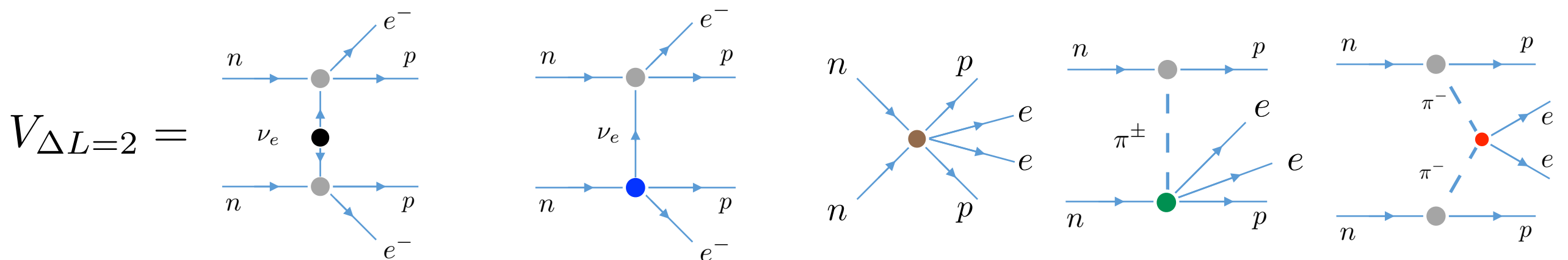
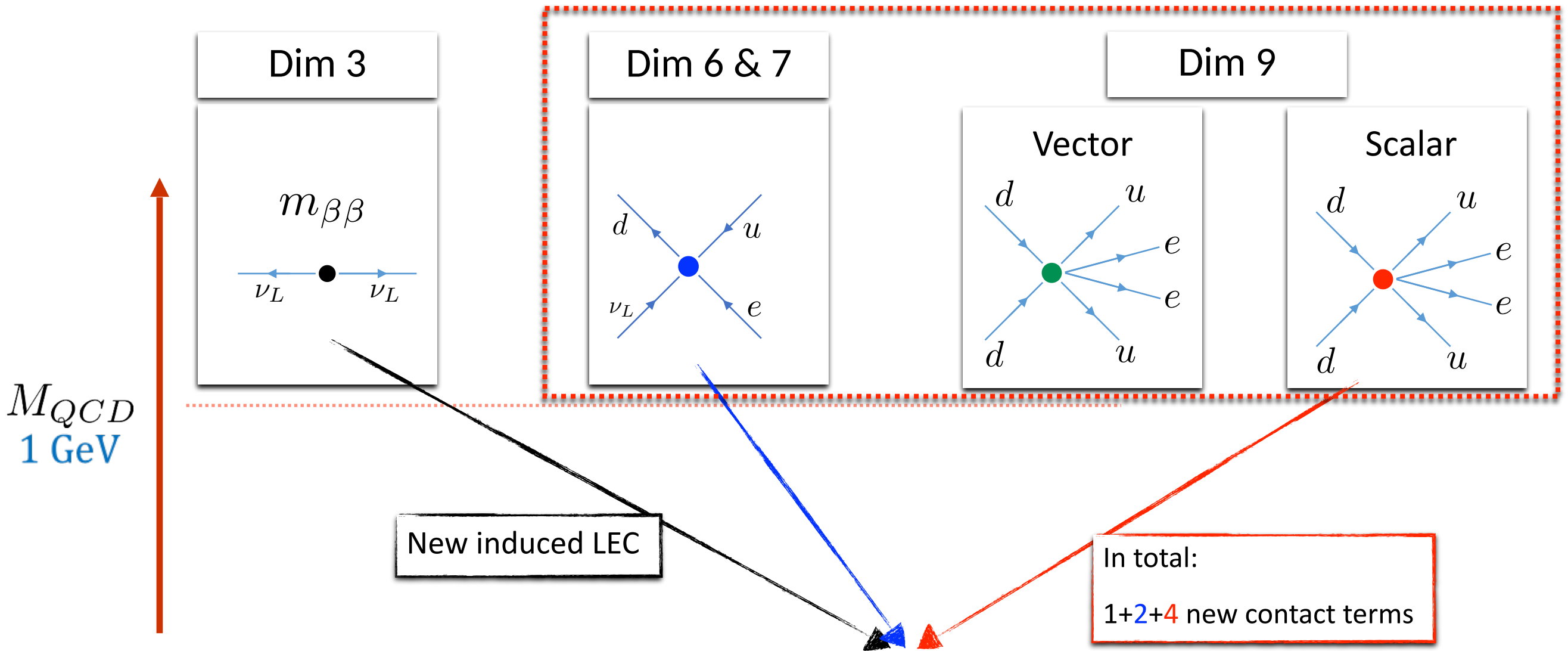
Chiral EFT

Non-Weinberg counting affects higher dimensional interactions as well

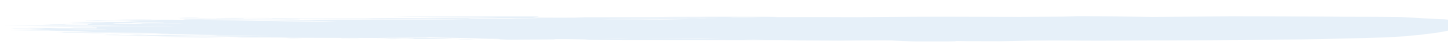


Chiral EFT

Non-Weinberg counting affects higher dimensional interactions as well



Estimate of impact in light nuclei



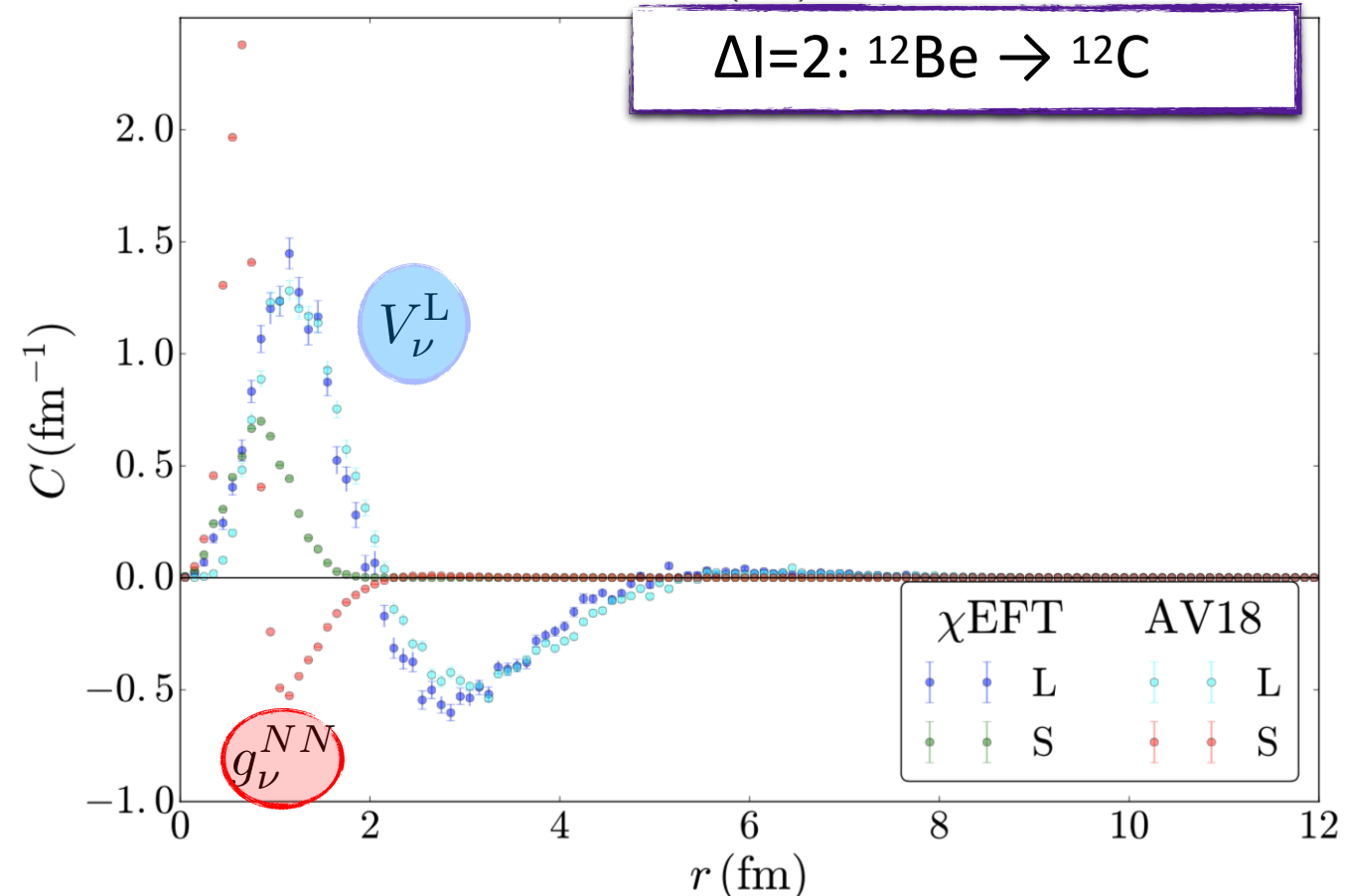
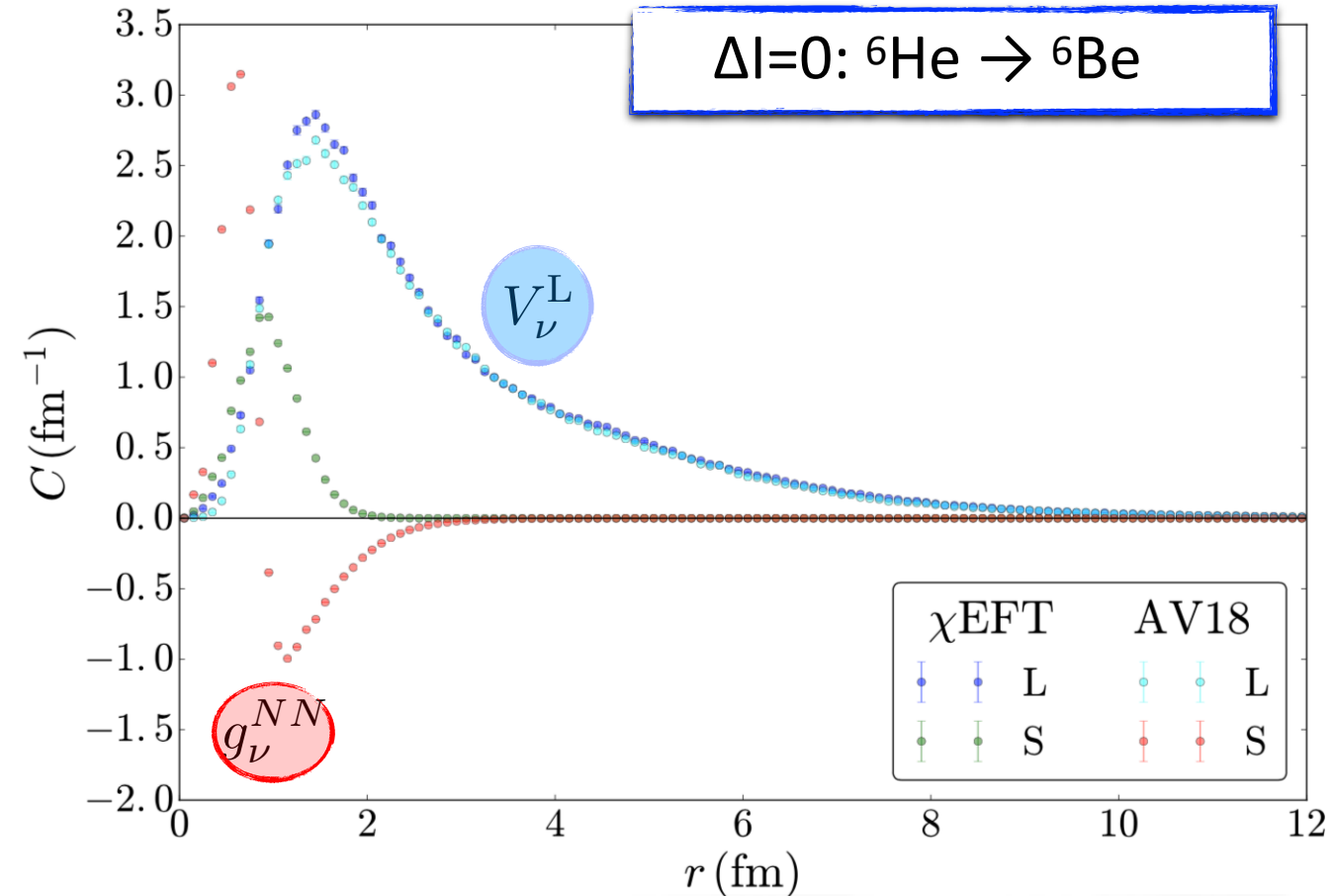
Estimate of impact

Light nuclei

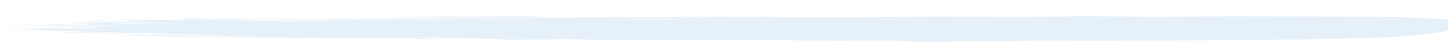
M. Piarulli, R. Wiringa, S. Pastore

- Combine estimate $g_\nu = (C_1 + C_2)/2$
- With wavefunctions:
 - From Chiral potential
M. Piarulli et. al. '16
 - Obtained from AV18 potential
R. Wiringa, Stoks, Schiavilla, '95

- ~10% effect in ${}^6\text{He} \rightarrow {}^6\text{Be}$
- ~60% effect in ${}^{12}\text{Be} \rightarrow {}^{12}\text{C}$
 - Due to presence of a node
 - Feature in realistic $0\nu\beta\beta$ candidates



More simple models

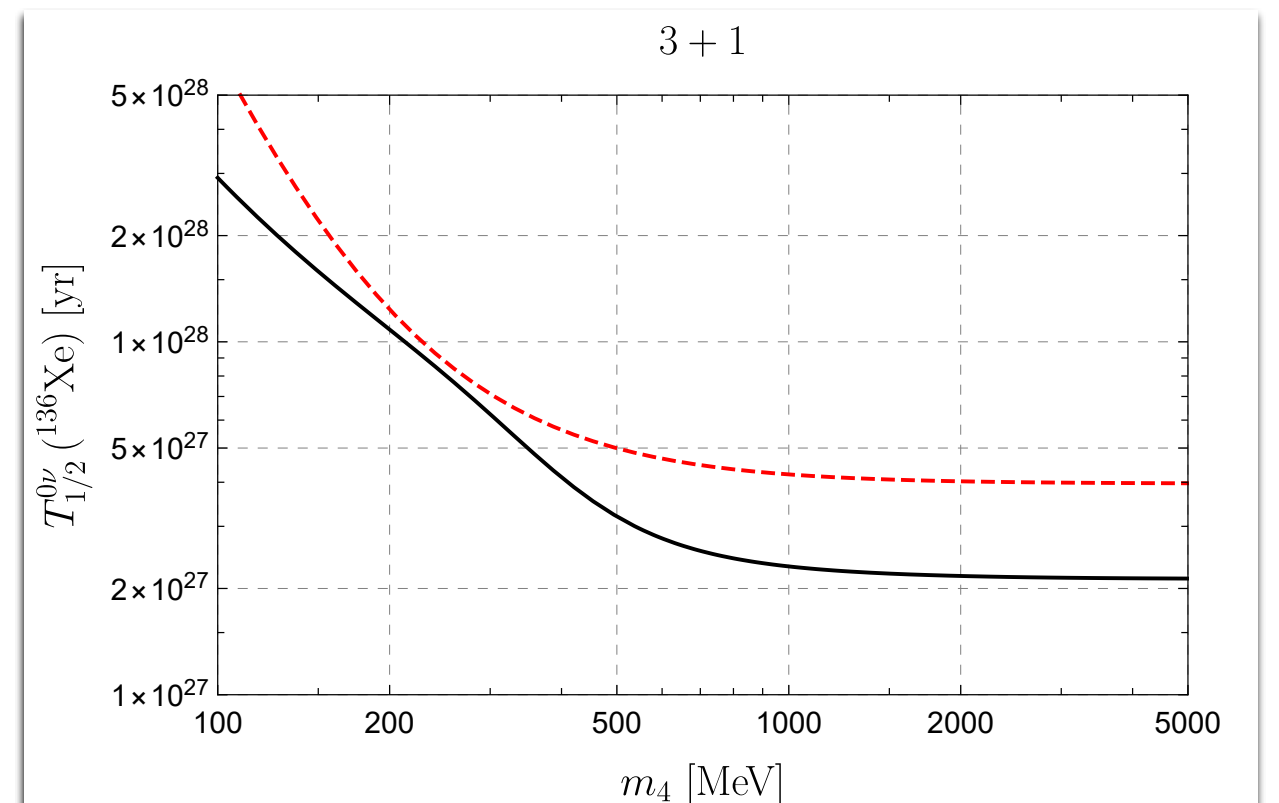
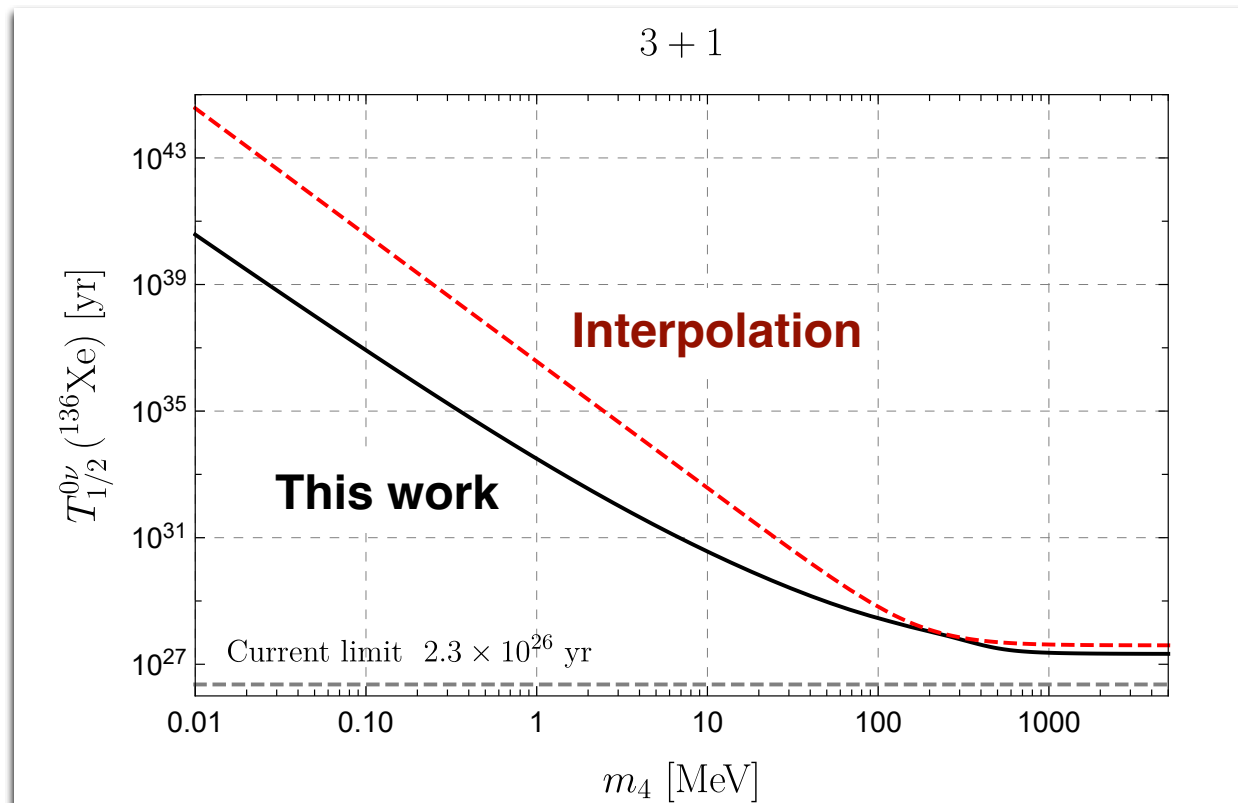


Toy model: 3+1

- Add just one sterile neutrino to the SM
 - Assume mass matrix of the form

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & M_D \\ 0 & 0 & 0 & M_D \\ 0 & 0 & 0 & M_D \\ M_D & M_D & M_D & M_R \end{pmatrix}$$

- Not realistic:
 - Only two nonzero ν masses
 - Does not reproduce mixing angles
- Simplest case to test differences with usual approach



3+2 scenario

- Add 2 sterile neutrinos to the SM
 - Can now fit the oscillation data
 - Involves more parameters, for some choice can derive bounds:

- Normal hierarchy

- Inverted hierarchy

