Lepton-nucleus scattering within Quantum-Monte Carlo based approaches

Noemi Rocco

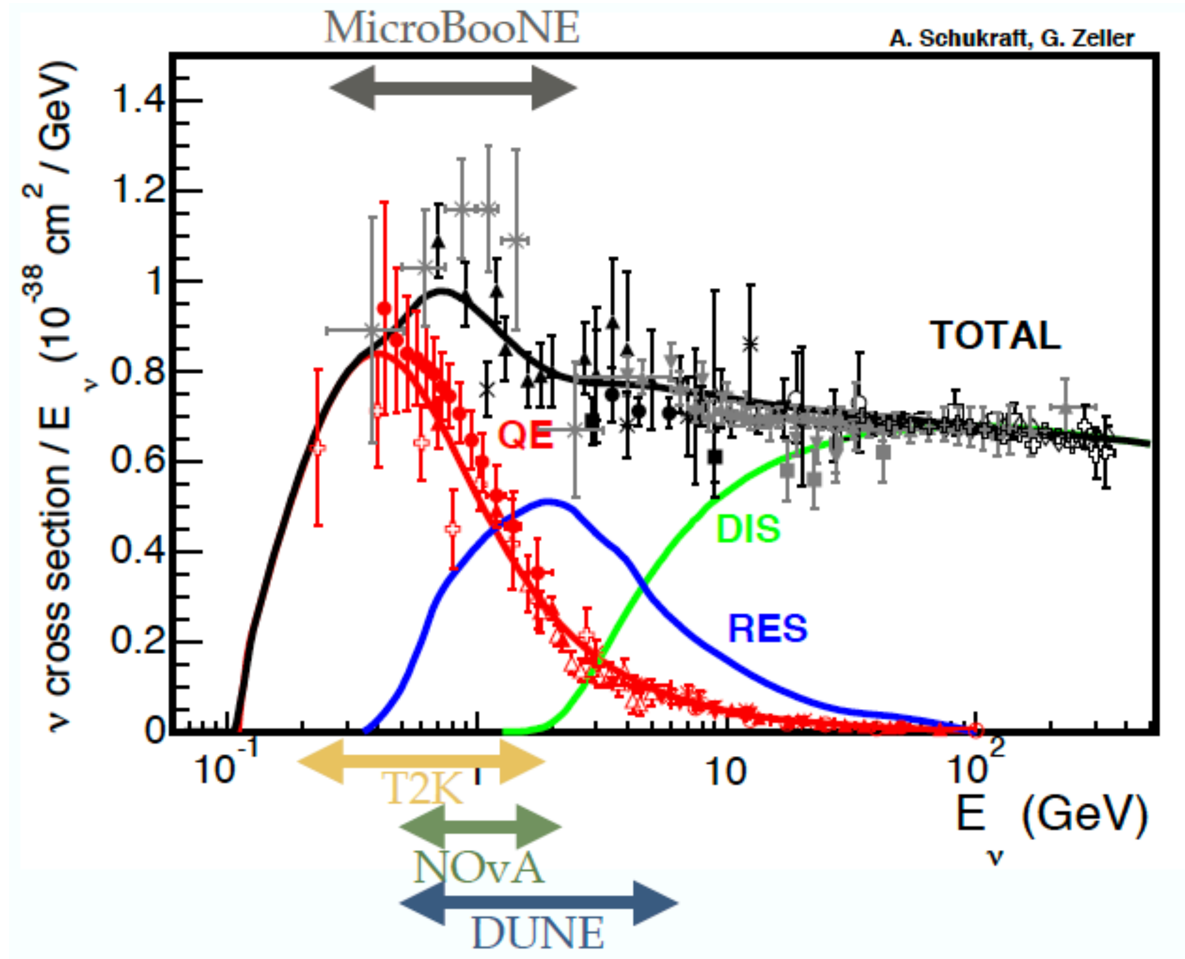
Marciana 2023 - Lepton Interactions with Nucleons and Nuclei

Elba— September 3 - 8, 2023

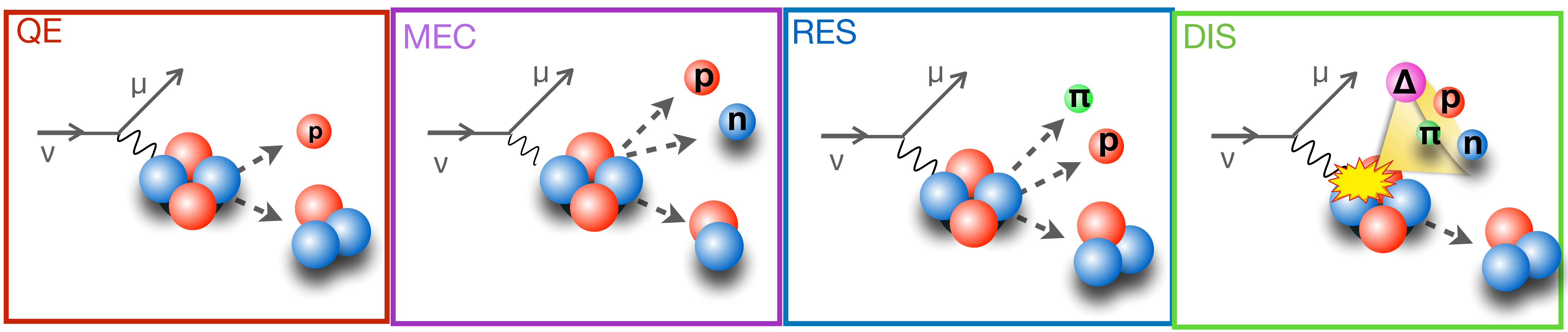
Inputs for the nuclear model

Unprecedented accuracy in the determination of **neutrino-argon cross section** is required to achieve design sensitivity to CP violation at DUNE

More than 60% of the interactions at DUNE are non-quasielastic



Theoretical tools for neutrino scattering,
Contribution to: 2022 Snowmass Summer Study



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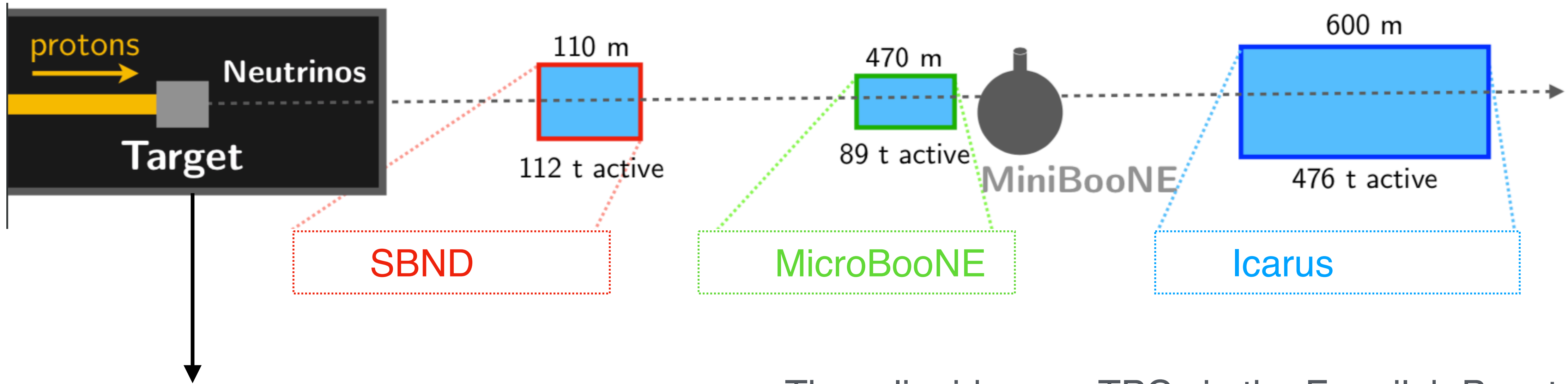
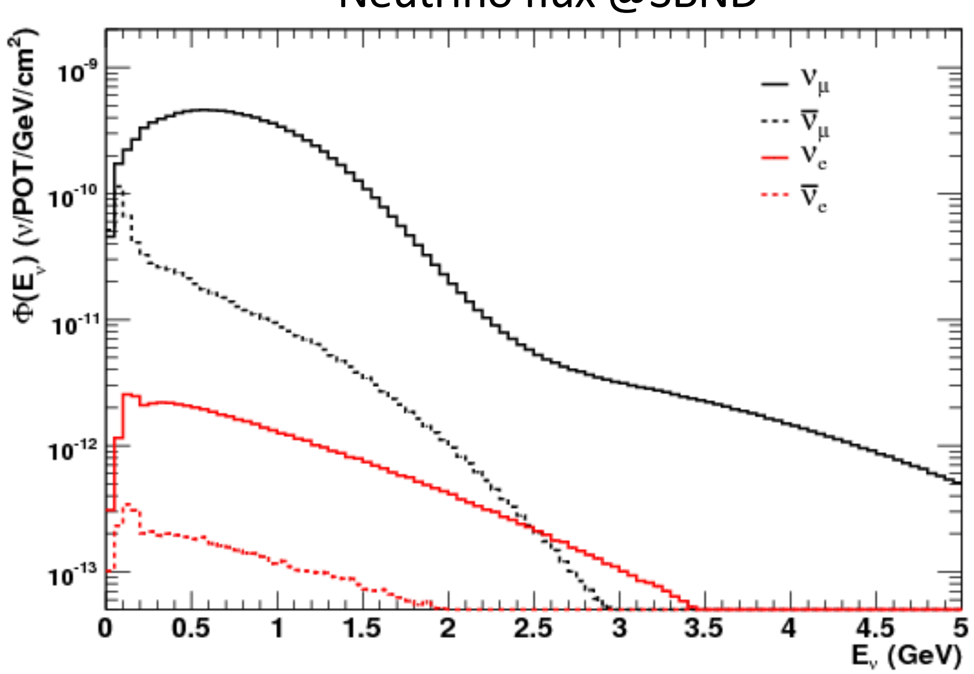
Short Baseline Neutrino program



BNB beam

Neutrino flux @SBND



Three liquid argon TPCs in the Fermilab Booster Neutrino Beam : Definitive test of LSND oscillations using three baselines

For BNB and T2K the dominant reaction mechanisms are quasi-elastic scattering

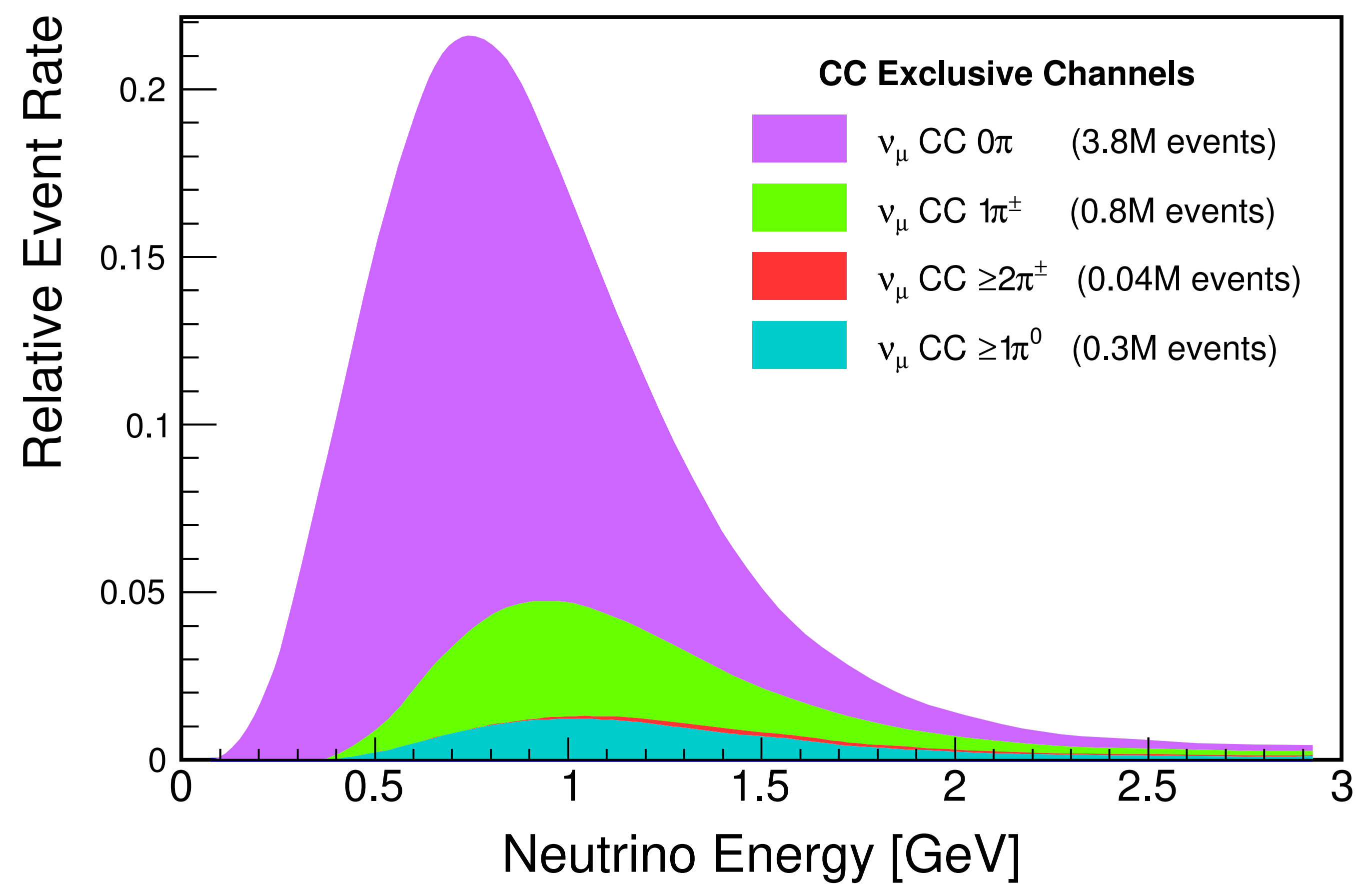
The contribution of π-production channels is ~ 25 %

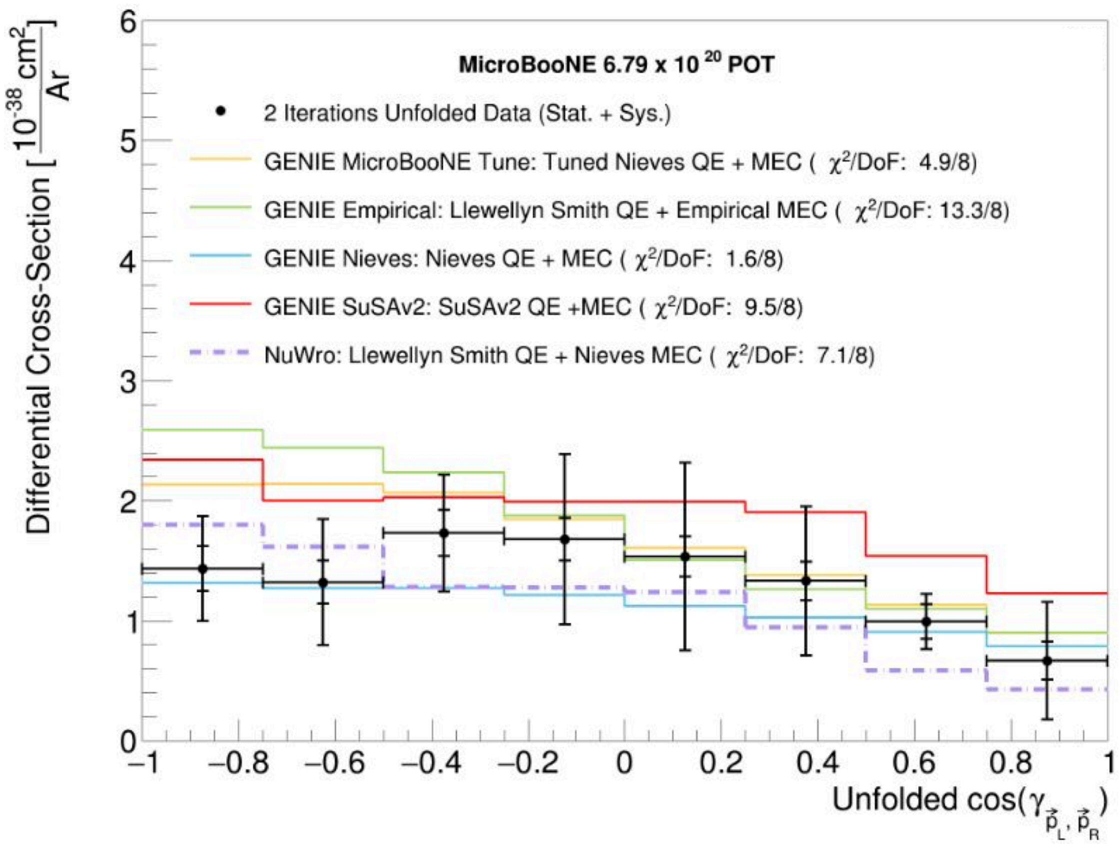
For the sub-GeV experiments the Delta is the only relevant resonance

Short Baseline Neutrino program

P. Machado et al, 1903.04608 (2019)

SBND will provide the world's highest statistics cross section measurements in LAr: 2 million events for ν_μ per year for the next 3 years

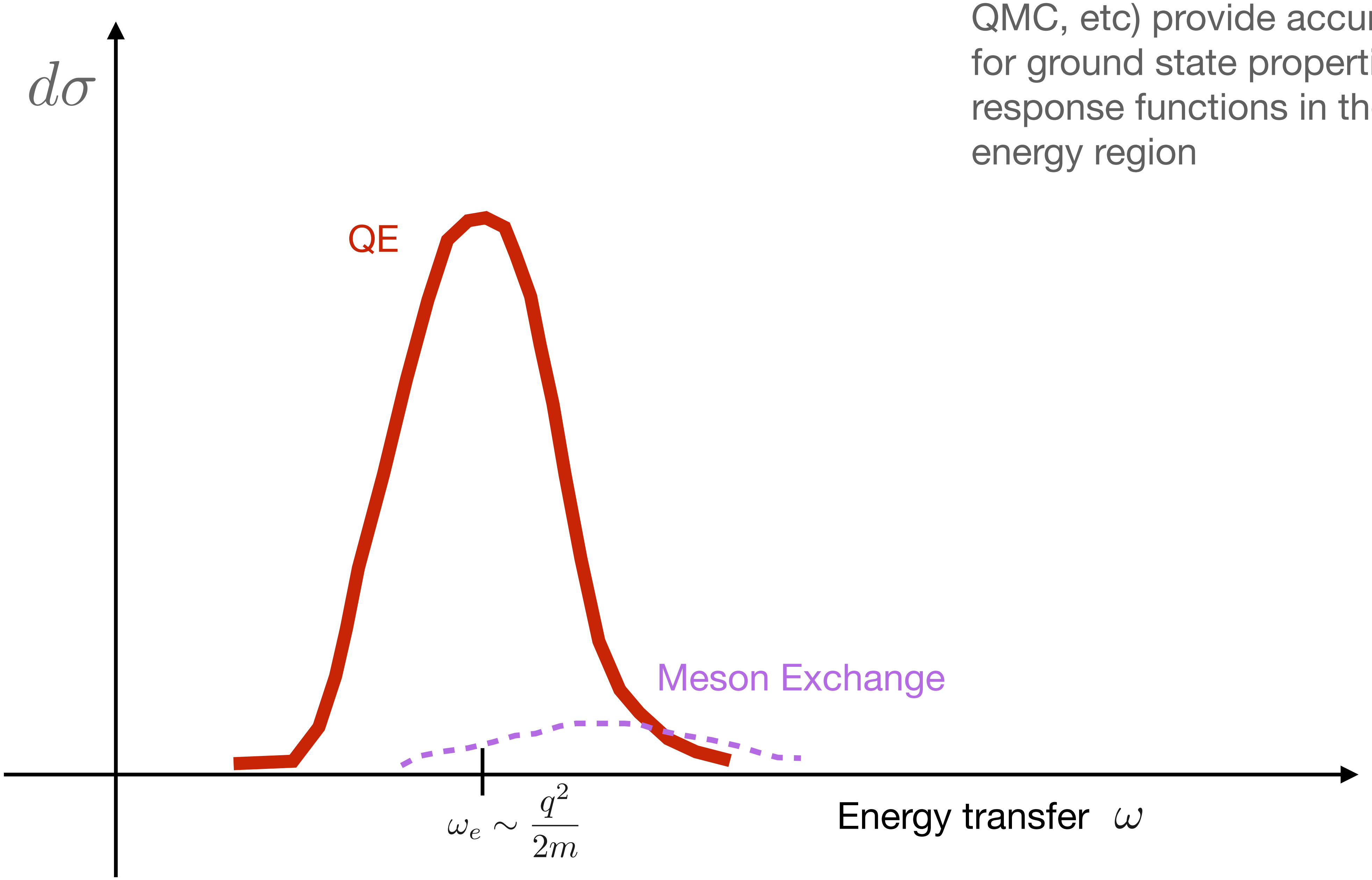




A. Papadopoulou W&C seminar June 2023 & Adi's talk yesterday

MicroBooNE provided first **two-proton knockout** single-differential **cross section on argon 2211.03734**

Ab initio Methods



Ab-initio methods (CC, IMSRG, SCGF, QMC, etc) provide accurate predictions for ground state properties of nuclei + response functions in the low/moderate energy region

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Many-Body method: GFMC

QMC techniques **projects out the exact lowest-energy state:** $e^{-(H-E_0)\tau}|\Psi_T\rangle \to |\Psi_0\rangle$

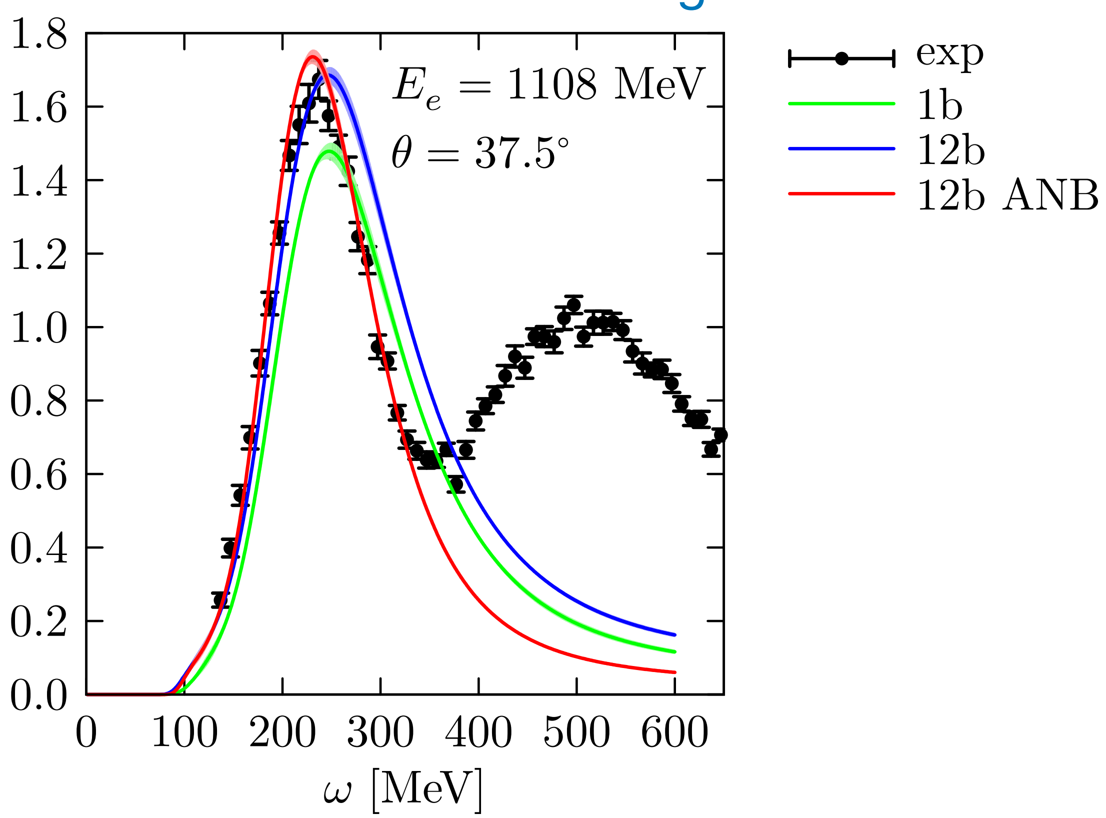
Nuclear response function involves evaluating a number of transition amplitudes.
Valuable information can be obtained from the **integral transform of the response function**

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega) R_{\alpha\beta}(\omega,\mathbf{q}) = \langle\psi_0|J^\dagger_\alpha(\mathbf{q})K(\sigma,H-E_0)J_\beta(\mathbf{q})|\psi_0\rangle$$

Inverting the Laplace transform is a complicated problem

A. Lovato et al, PRL117 (2016), 082501, PRC97 (2018), 022502

—electron-^{4}He scattering



Inclusive results which are virtually correct in the QE

Different Hamiltonians can be used in the time-evolution operator

Relies on non-relativistic treatment of the kinematics

Can not handle explicit pion degrees of freedom

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Axial form factor determination

- The axial form-factor has been fit to the dipole form

$$F_A(q^2) = \frac{g_A}{(1 - q^2/m_A^2)^2}$$

- The intercept g_A=-1.2723 is known from neutron β decay

- Different values of m_A from experiments
  - m_A =1.02 GeV q.e. scattering from deuterium
  - m_A =1.35 GeV @ MiniBooNE

- Alternative derivation based on **z-expansion** —model independent parametrization

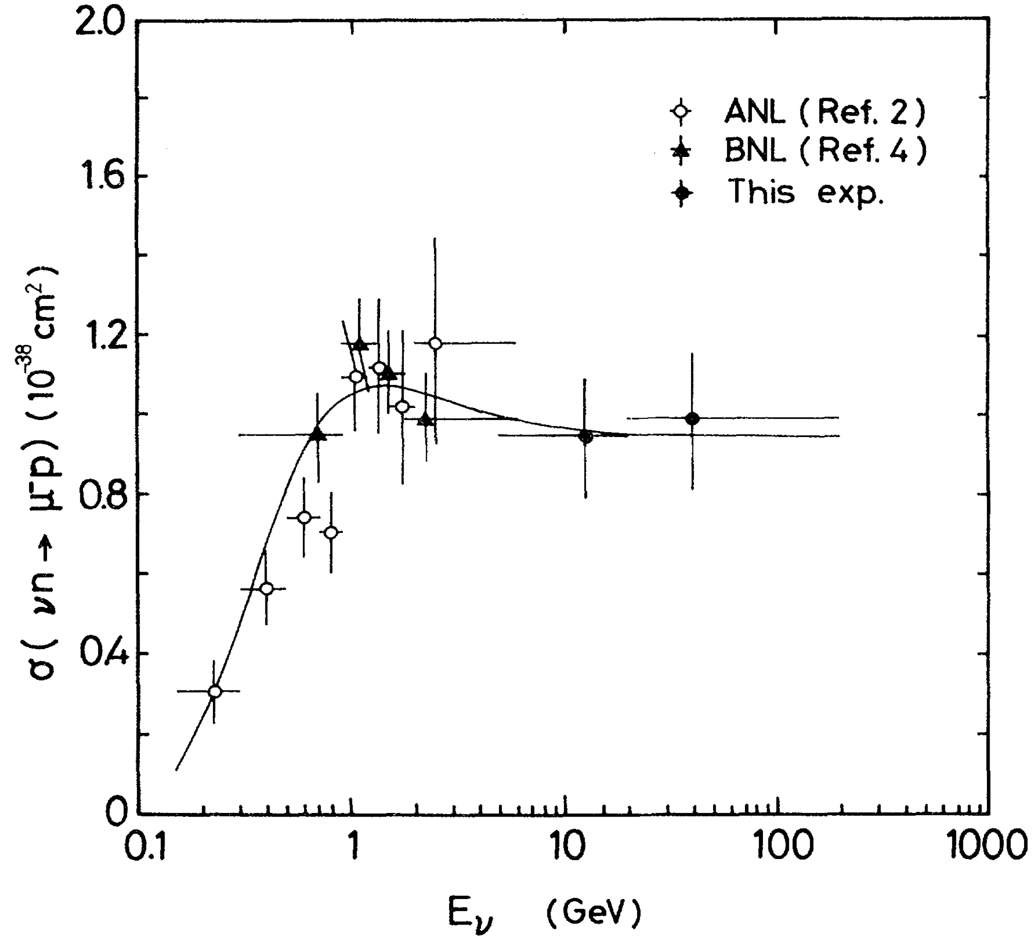
$$F_A(q^2) = \sum_{k=0}^{k_{\max}} a_k z(q^2)^k,$$

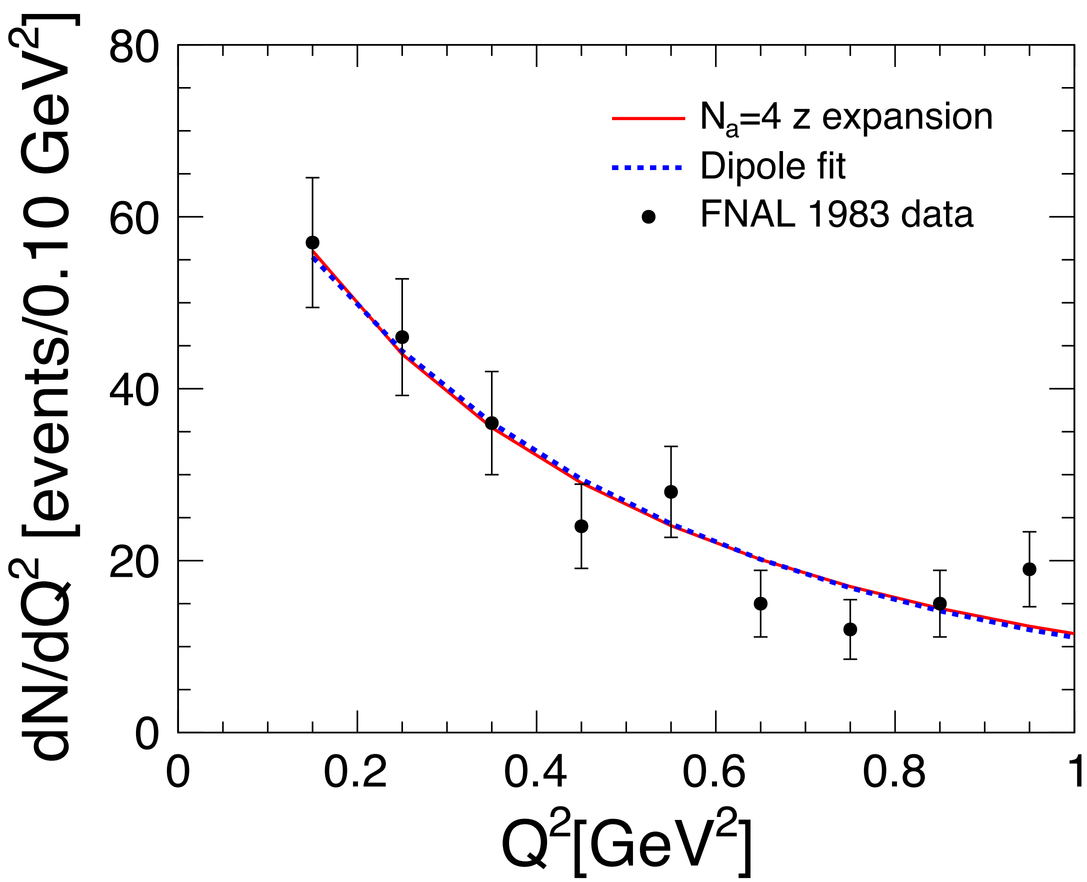
known functions

free parameters

Bhattacharya, Hill, and Paz PRD 84 (2011) 073006

A.S.Meyer et al, Phys.Rev.D 93 (2016) 11, 113015





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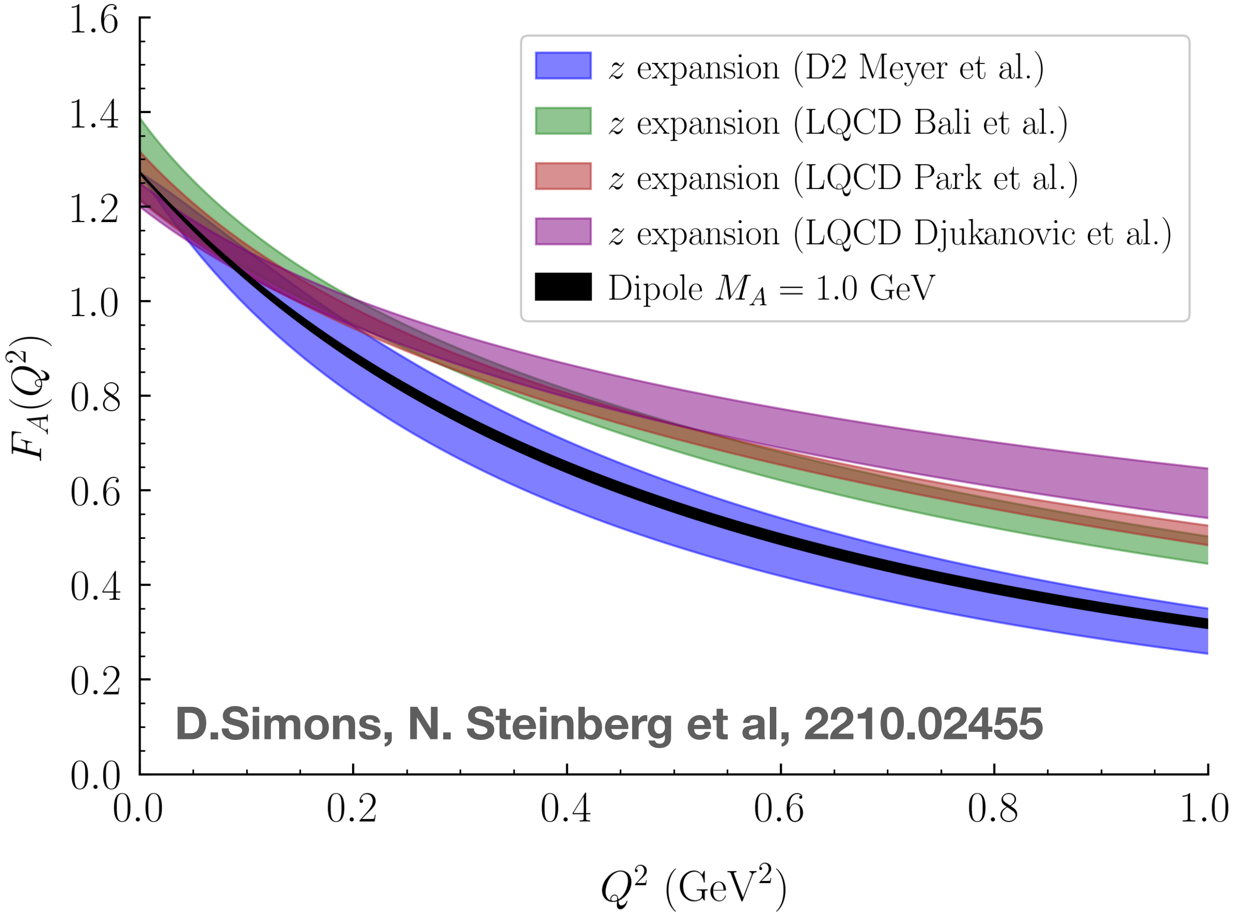
Axial form factor determination



D.Simons, N. Steinberg et al, 2210.02455

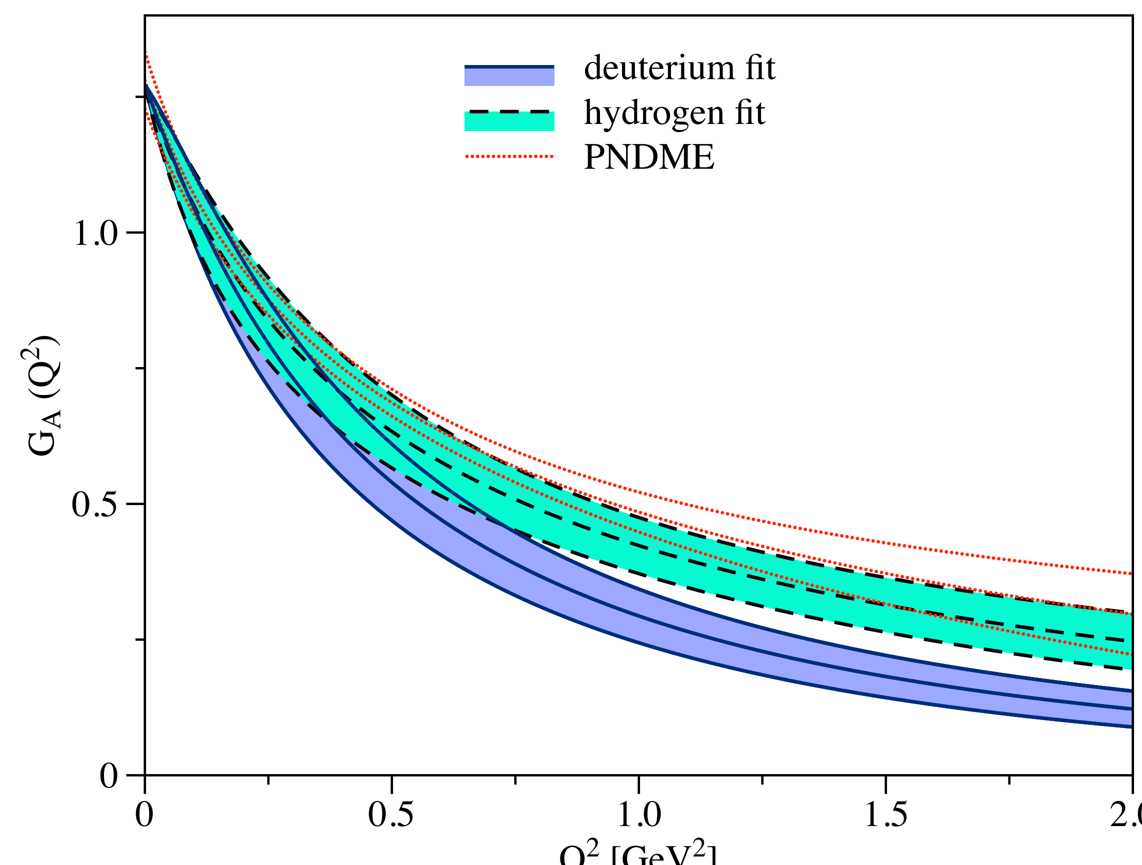
A. Meyer, A. Walker-Loud, C. Wilkinson, 2201.01839

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations

LQCD results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

O. Tomalak, R. Gupta, T. Battacharaya, 2307.14920



Comparison with recent MINERvA antineutrino-hydrogen charged-current measurements

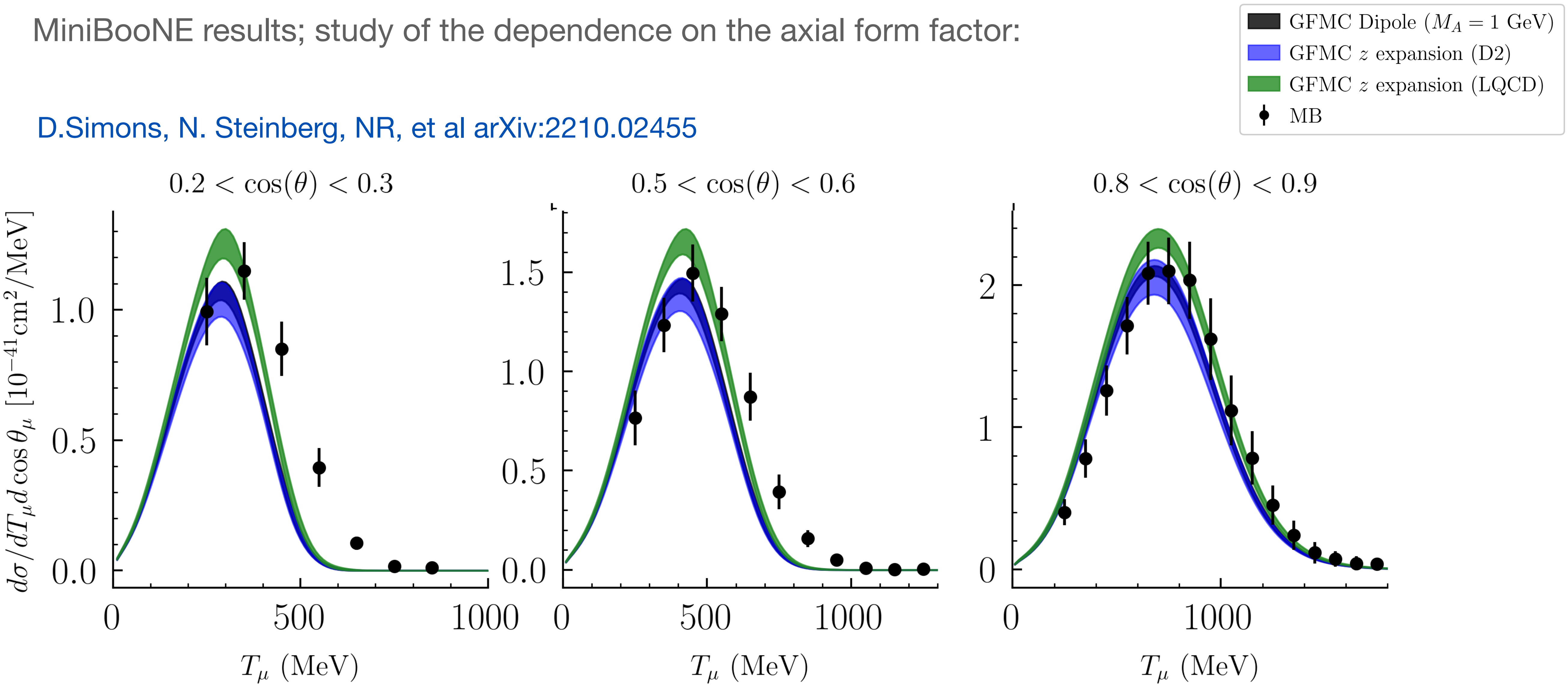
1-2σ agreement with MINERvA data and LQCD prediction by PNDME Collaboration

Novel methods are needed to remove excited-state contributions and discretization errors

Study of model dependence in neutrino predictions

MiniBooNE results; study of the dependence on the axial form factor:

D.Simons, N. Steinberg, NR, et al arXiv:2210.02455



D.Simons, N. Steinberg et al, 2210.02455

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$	$0.5 < \cos\theta_\mu < 0.6$	$0.8 < \cos\theta_\mu < 0.9$
GFMC Difference in $d\sigma_{\text{peak}}$ (%)	18.6	17.1	12.2

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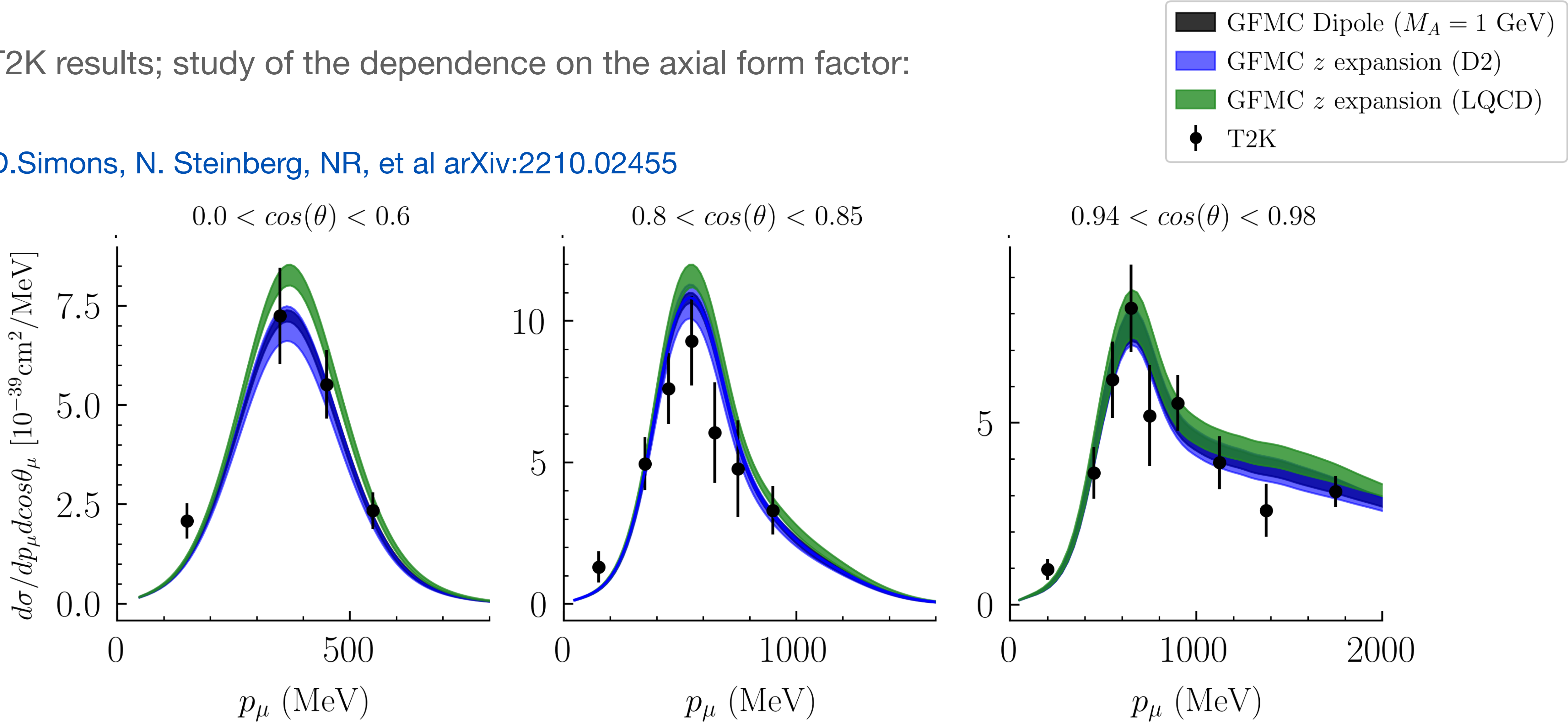
Study of model dependence in neutrino predictions

T2K results; study of the dependence on the axial form factor:

D.Simons, N. Steinberg, NR, et al arXiv:2210.02455



D.Simons, N. Steinberg et al, 2210.02455

T2K	$0.0 < \cos\theta_\mu < 0.6$	$0.80 < \cos\theta_\mu < 0.85$	$0.94 < \cos\theta_\mu < 0.98$
GFMC difference in $d\sigma_{\text{peak}}$ (%)	15.8	8.0	4.6

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Why relativity is important

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle \delta(\omega - E_f + E_0)$$

Kinematics

Currents

Covariant expression of the e.m. current:

$$j^\mu_{\gamma,S} = \bar{u}(\mathbf{p}')\Big[\frac{G_E^S + \tau G_M^S}{2(1+\tau)}\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{4m_N}\frac{G_M^S - G_E^S}{1+\tau}\Big]u(\mathbf{p})$$

Nonrelativistic expansion in powers of $\boldsymbol{p}/m_N$

$$j^0_{\gamma,S} = \frac{G_E^S}{2\sqrt{1+Q^2/4m_N^2}} - i\frac{2G_M^S - G_E^S}{8m_N^2}\mathbf{q}\cdot(\boldsymbol{\sigma}\times\mathbf{p})$$

Energy transfer at the quasi-elastic peak:

$$w_{QE} = \sqrt{\mathbf{q}^2 + m_N^2} - m_N$$

$$w_{QE}^{nr} = \mathbf{q}^2/(2m_N)$$

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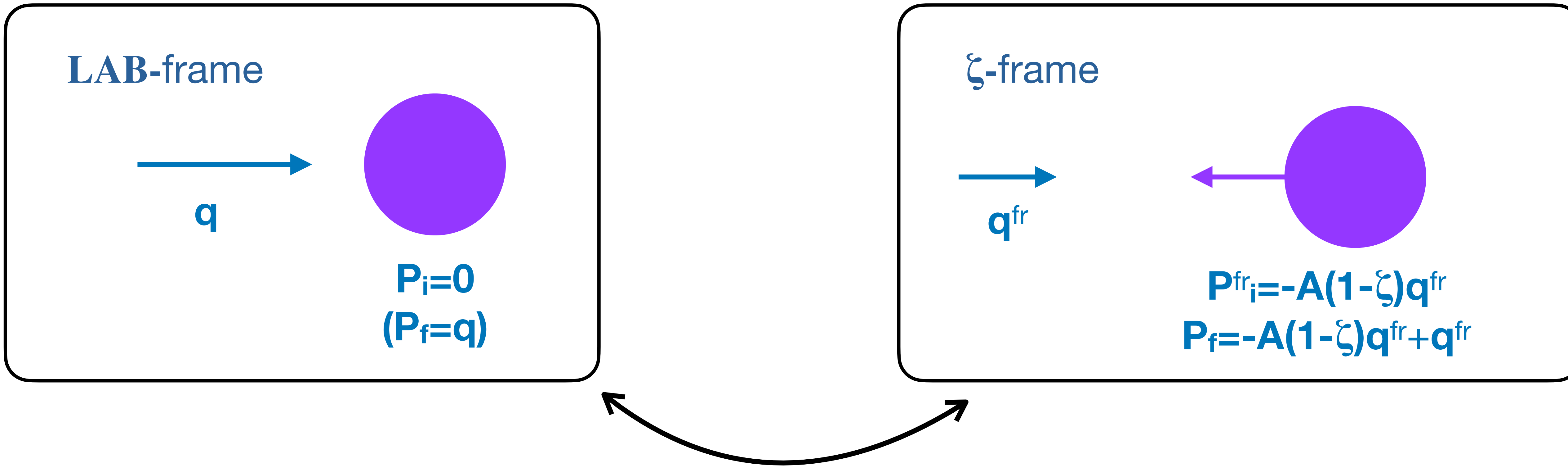
Frame dependence



Lorentz Boost connects the two frames

$$R^{\mu\nu}_{LAB}(\omega, q) = B^{\mu}{}_{\alpha}\,[\beta]\, B^{\nu}{}_{\beta}\,[\beta]\, R^{\alpha\beta}_{fr}(\omega^{fr}, \mathbf{q}^{fr})$$

The momentum and energy transfer in the different reference frames are connected:

$$\mathbf{q}^{fr} = \gamma(\mathbf{q} - \boldsymbol{\beta}\omega), \qquad \omega^{fr} = \gamma(\omega - \beta q),$$

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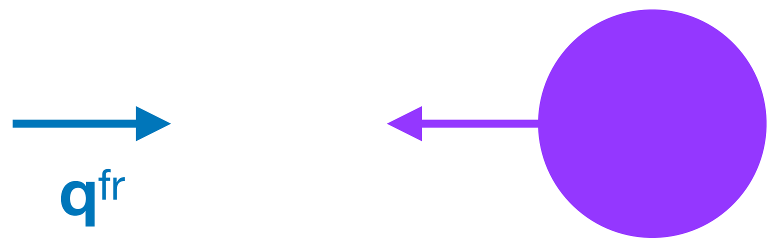
Frame dependence

Frame that minimizes relativistic corrections



$P^{fr}_i = -A/2\ q^{fr}$
$P_f = -A/2\ q^{fr} + q^{fr}$

$\zeta = 1/2$ Active nucleon Breit frame

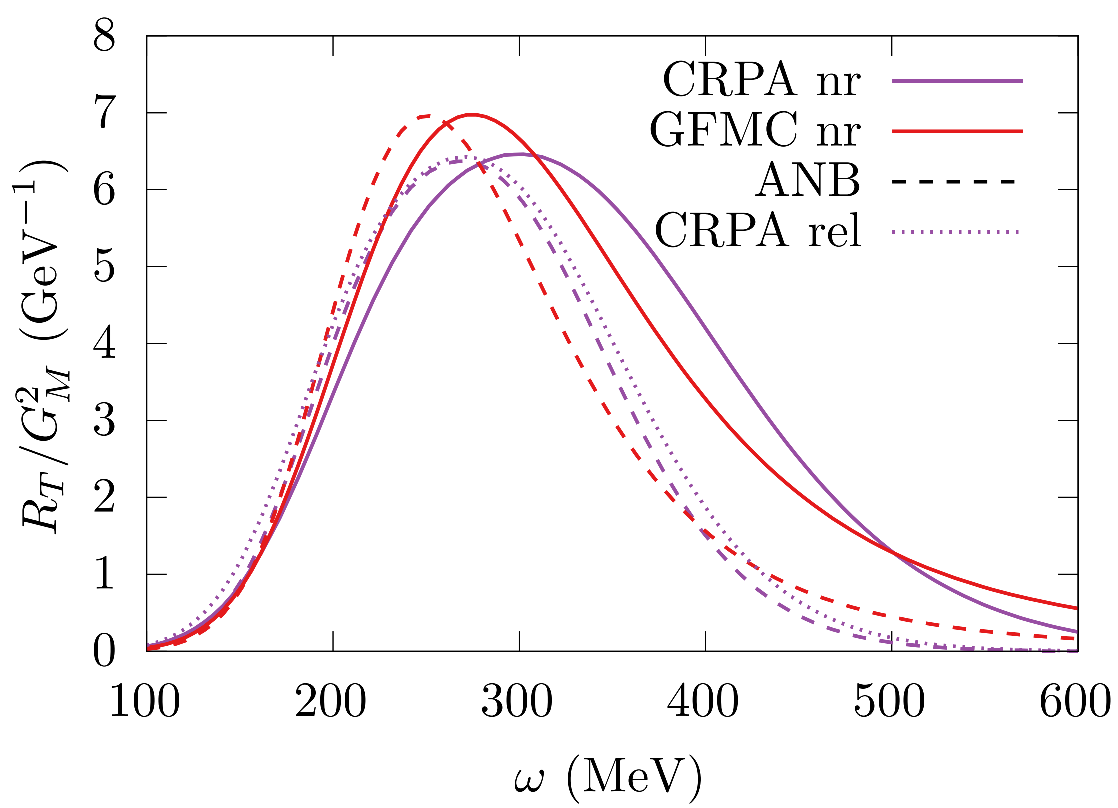
- At the single nucleon level:

$p^{fr}_i \simeq -q^{fr}/2$
$p^{fr}_f \simeq q^{fr}/2$

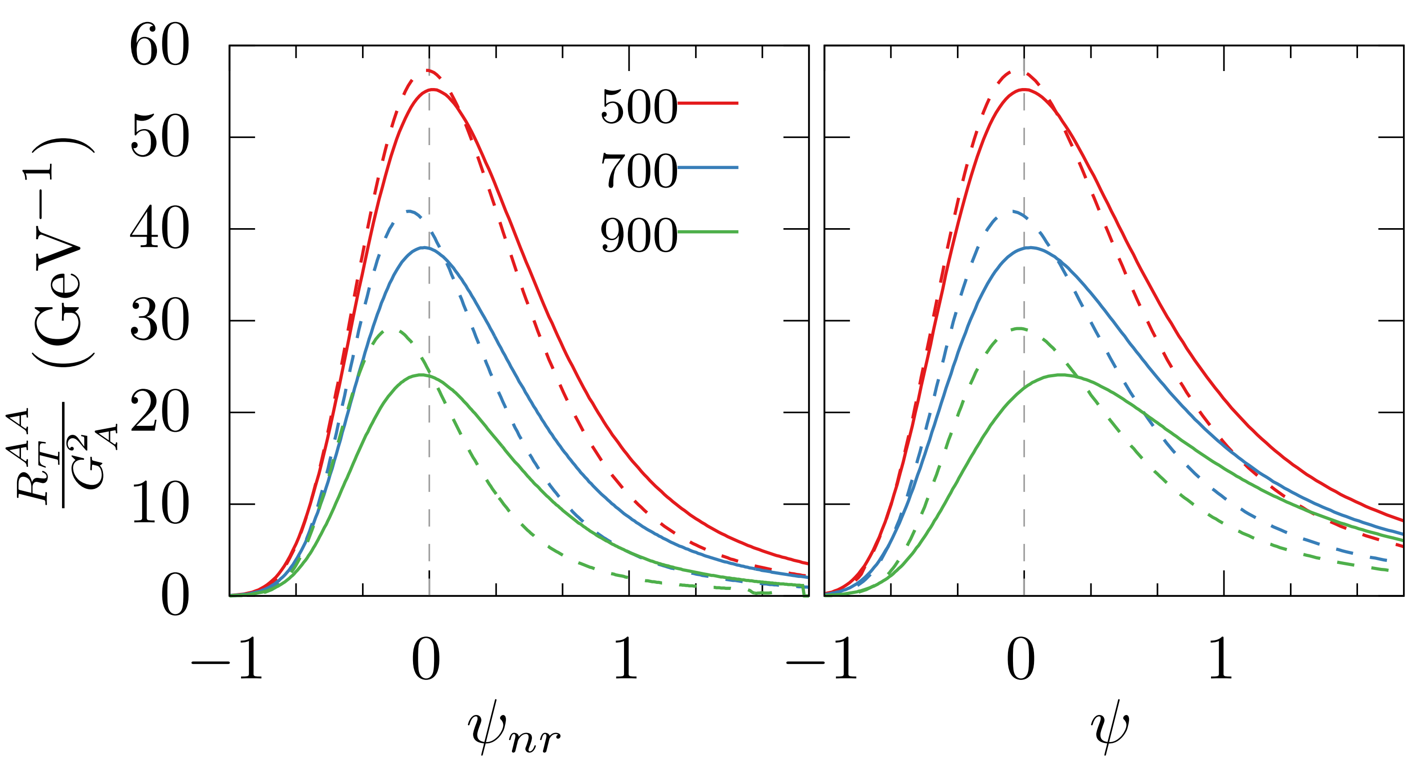
- Same position of the quasielastic peak

$$\omega_{QE} = \omega^{nr}_{QE} = 0$$

- Charge Current electroweak responses of ^{12}C :



- LAB (solid) and ANB (dashed) predictions



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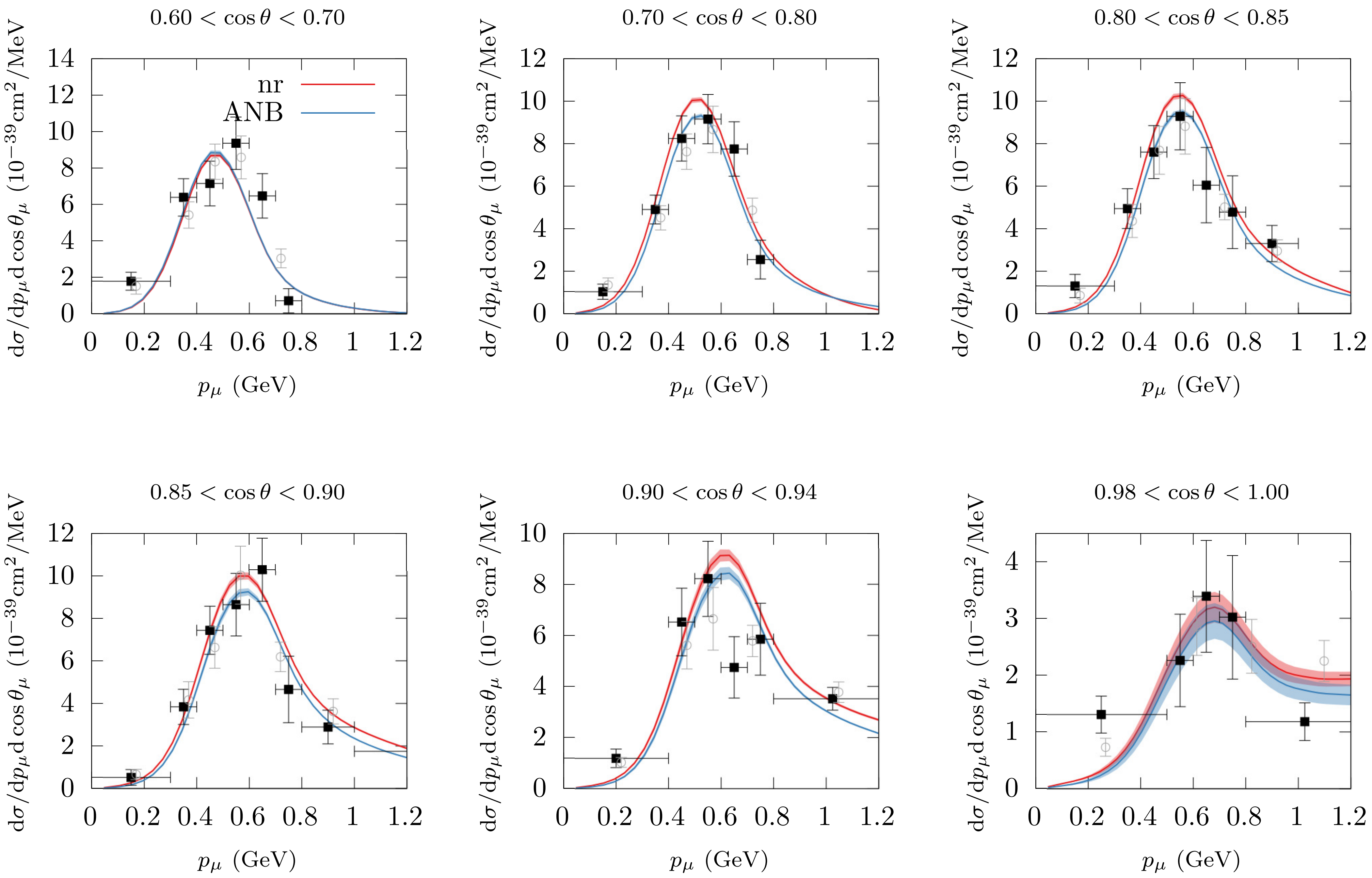
Cross sections: Green's Function Monte Carlo

T2K results including relativistic corrections

A.Nikolakopoulos, A.Lovato, NR, arXiv:2304.11772



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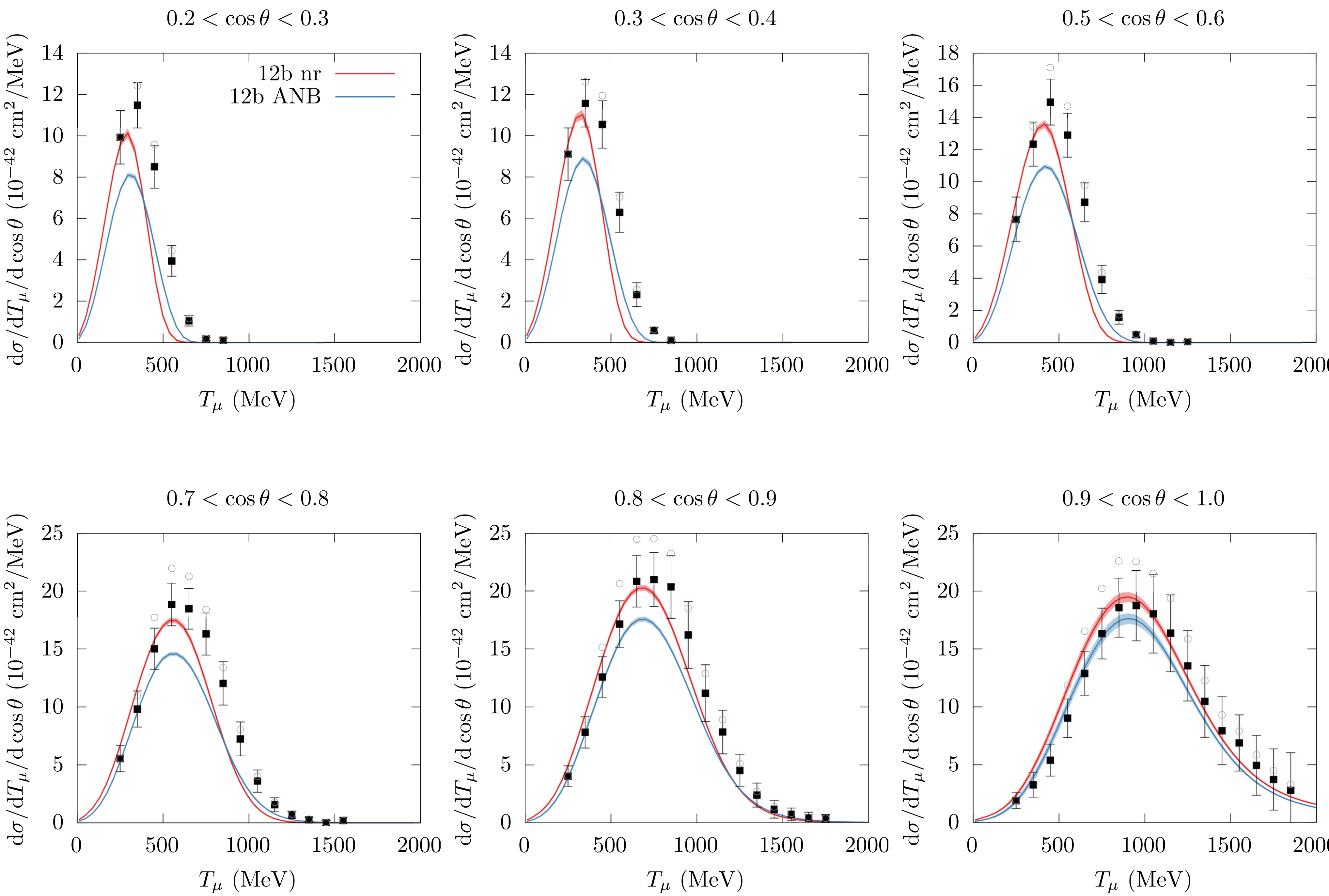
Cross sections: Green's Function Monte Carlo

MiniBooNE results including relativistic corrections

A.Nikolakopoulos, A.Lovato, NR, arXiv:2304.11772



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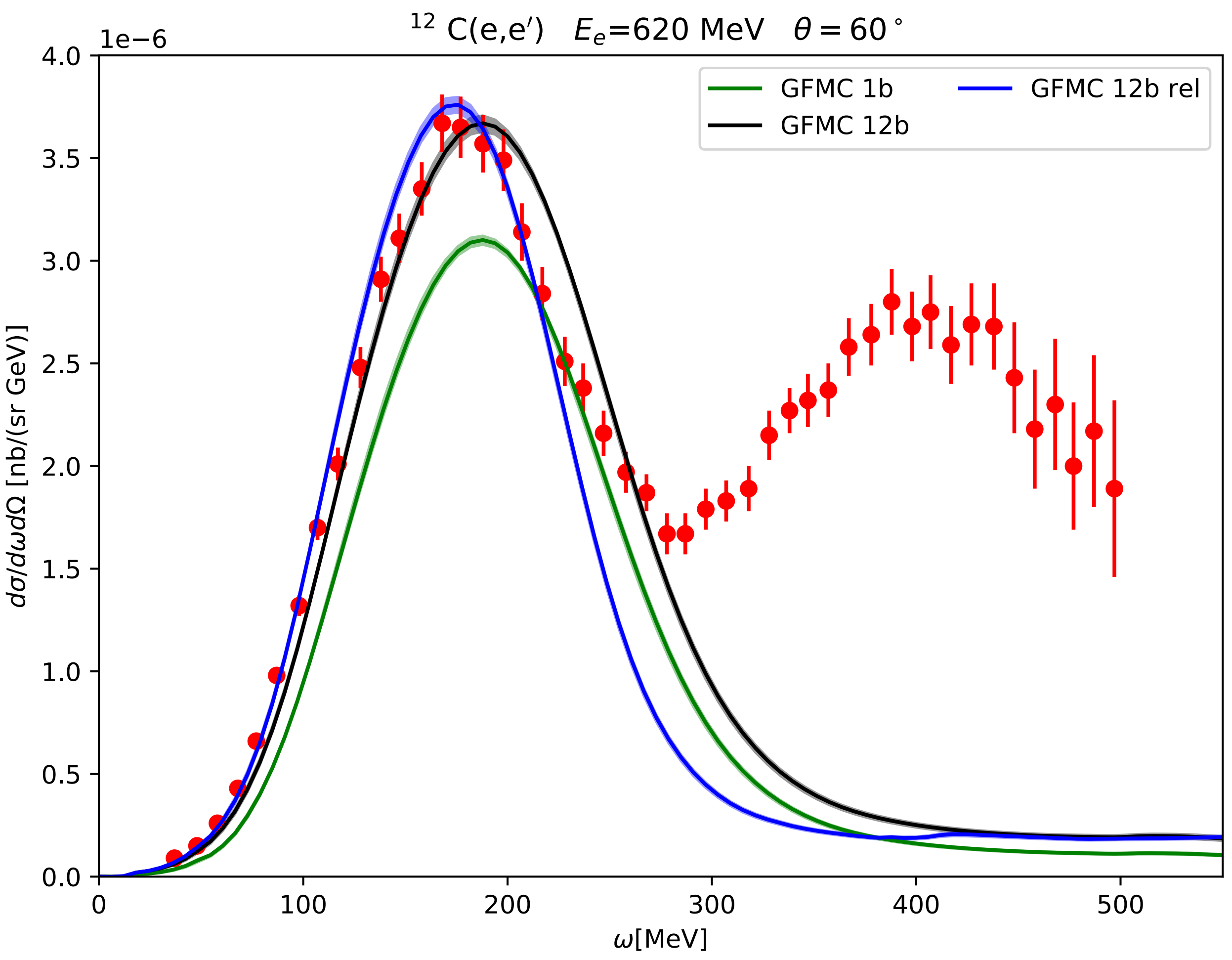
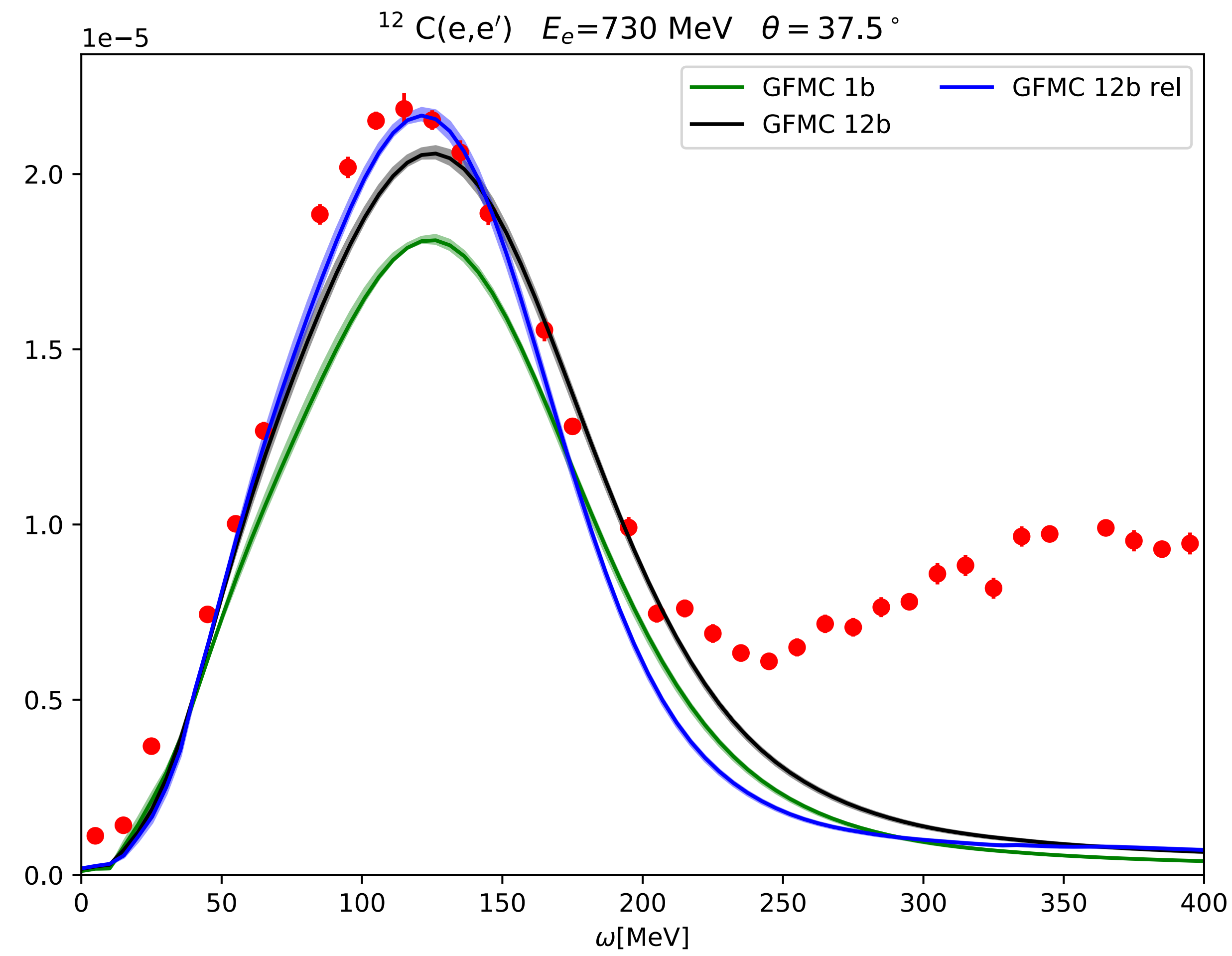
Cross sections: Green's Function Monte Carlo

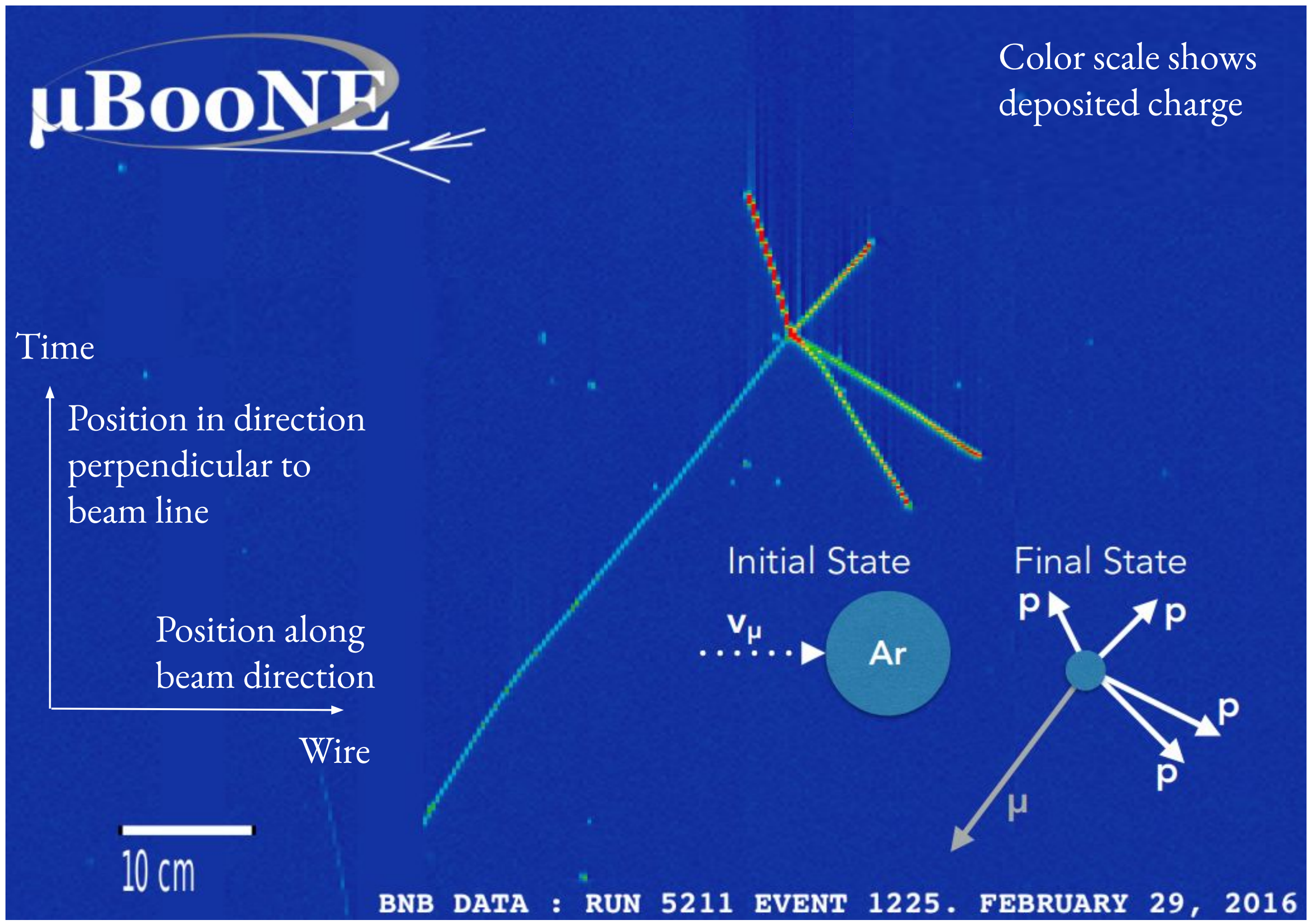
Electron scattering results including relativistic corrections for some kinematics covered by the calculated responses



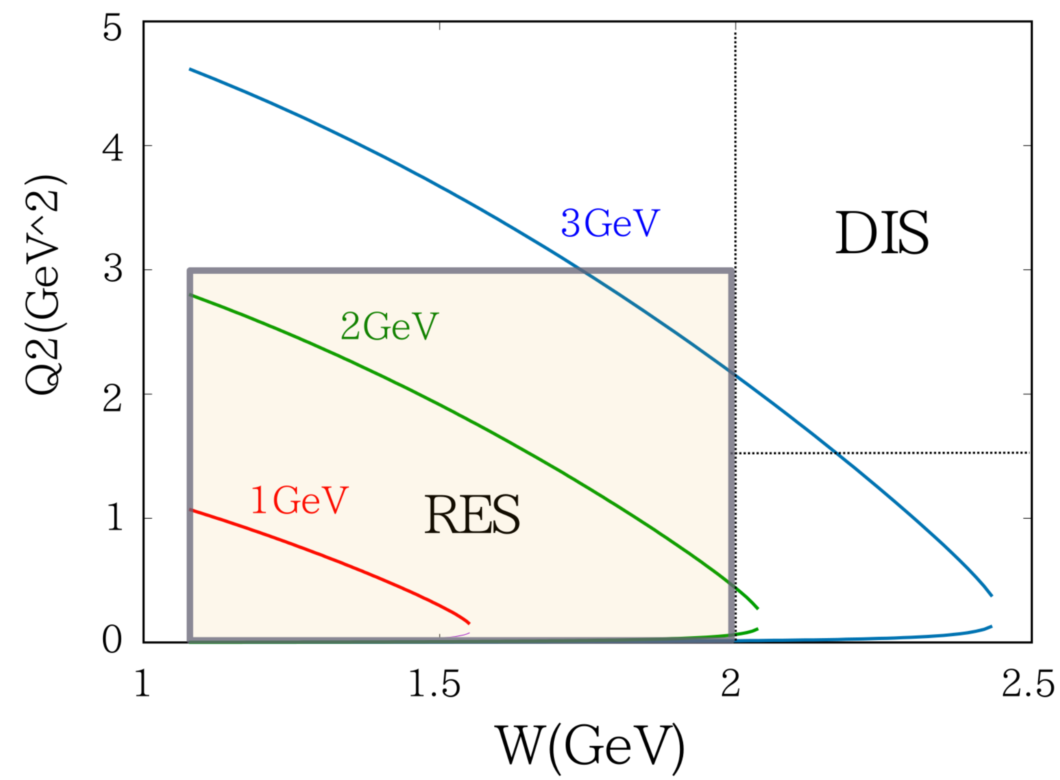


A.Lovato, A.Nikolakopoulos, NR, N. Steinberg, arXiv:2308.00736

Address new experimental capabilities



T.Sato talks @ NuSTEC Workshop on Neutrino-Nucleus Pion Production in the Resonance Region



- Excellent spatial resolution
- Precise calorimetric information
- Powerful particle identification

$$W = \sqrt{(p+q)^2}, Q^2 = -q^2 = -(p_\nu - p_l)^2$$

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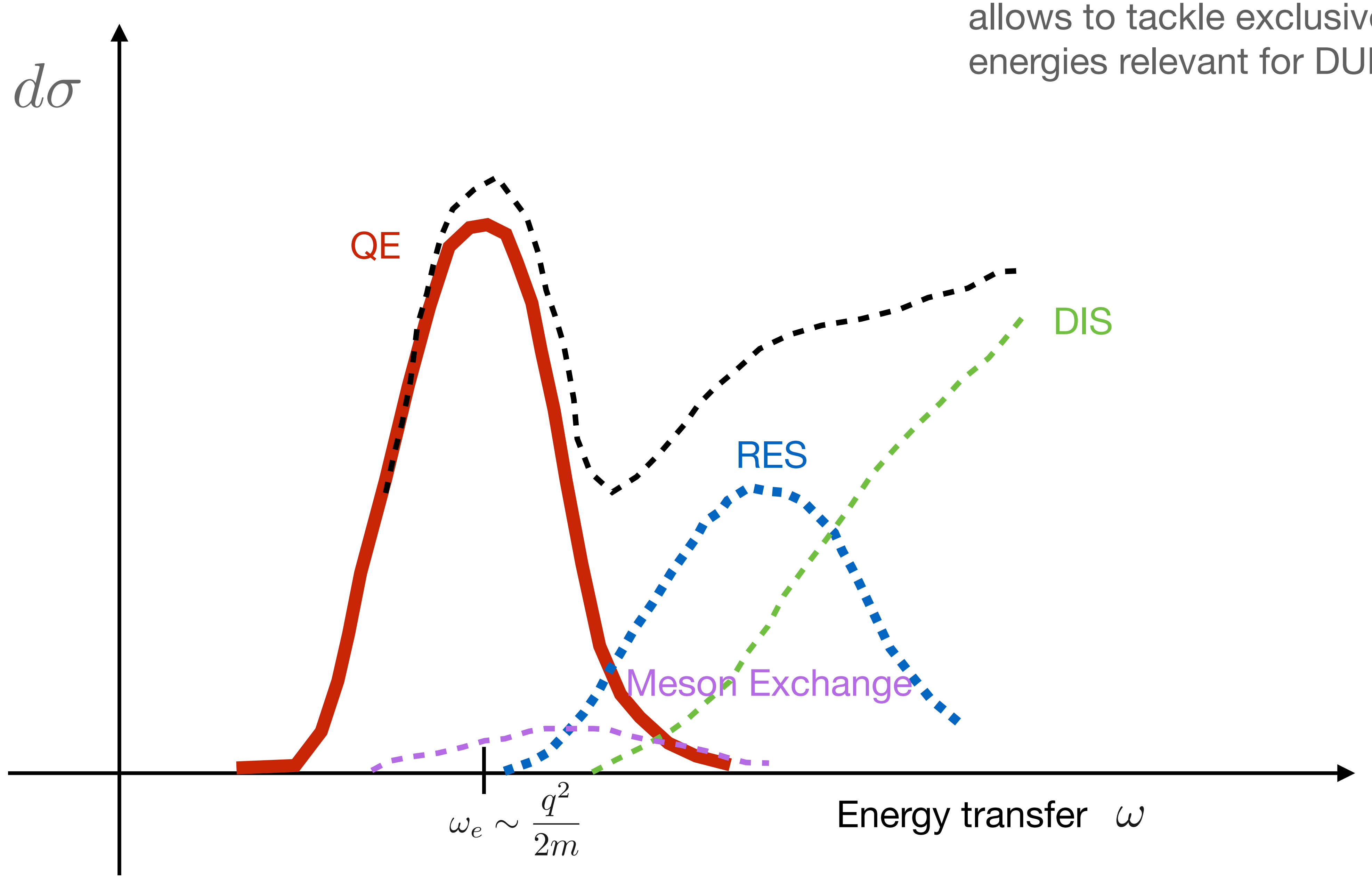
Factorization Based Approaches



Factorization of the hadronic final states: allows to tackle exclusive channels + higher energies relevant for DUNE

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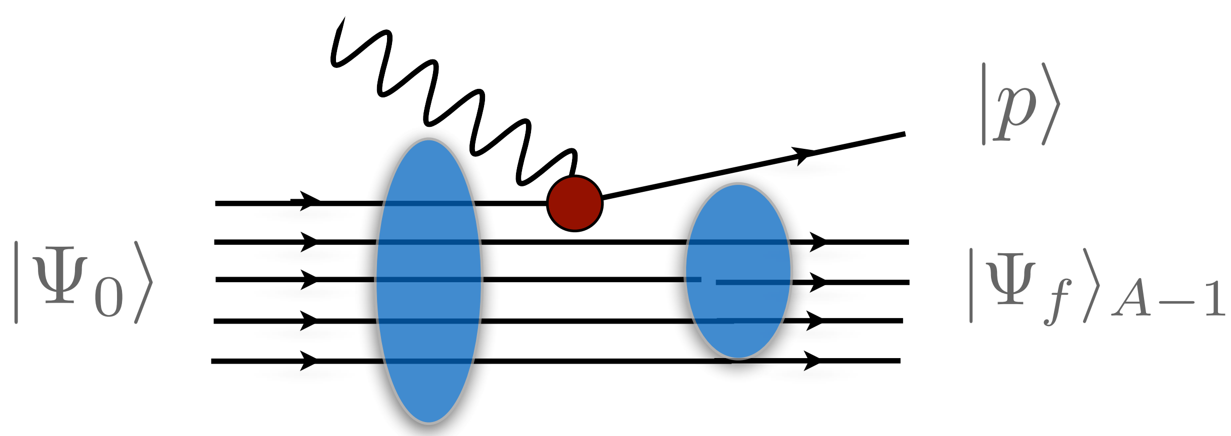
Spectral function approach

At large momentum transfer, the scattering reduces to the sum of individual terms

$$J_\alpha = \sum_i j_\alpha^i \qquad |\Psi_f\rangle \to |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

The incoherent contribution of the one-body response reads

$$R_{\alpha\beta} \simeq \int \frac{d^3k}{(2\pi)^3} dE P_h(\mathbf{k}, E) \sum_i \langle k | {j_\alpha^i}^\dagger | k+q \rangle \langle k+q | j_\beta^i | k \rangle \delta(\omega + E - e(\mathbf{k}+\mathbf{q}))$$



The Spectral Function is the imaginary part of the two point Green's Function

Different many-body methods can be adopted to determine it

I. Korover, et al Phys.Rev.C 107 (2023) 6, L061301

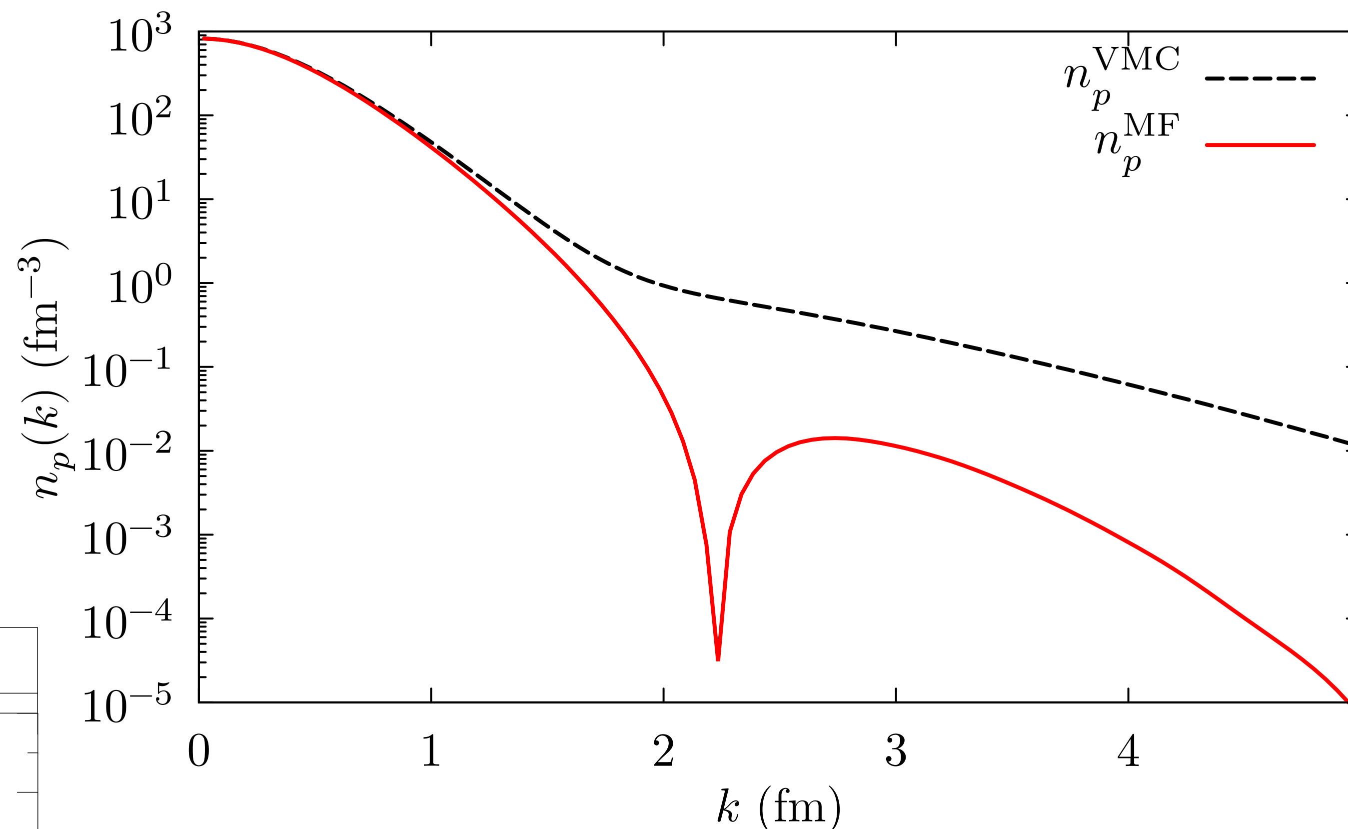
O. Benhar et al, Rev.Mod.Phys. 80 (2008)

***NR,* Frontiers in Phys. 8 (2020) 116**

QMC Spectral function of nuclei with A=3,4

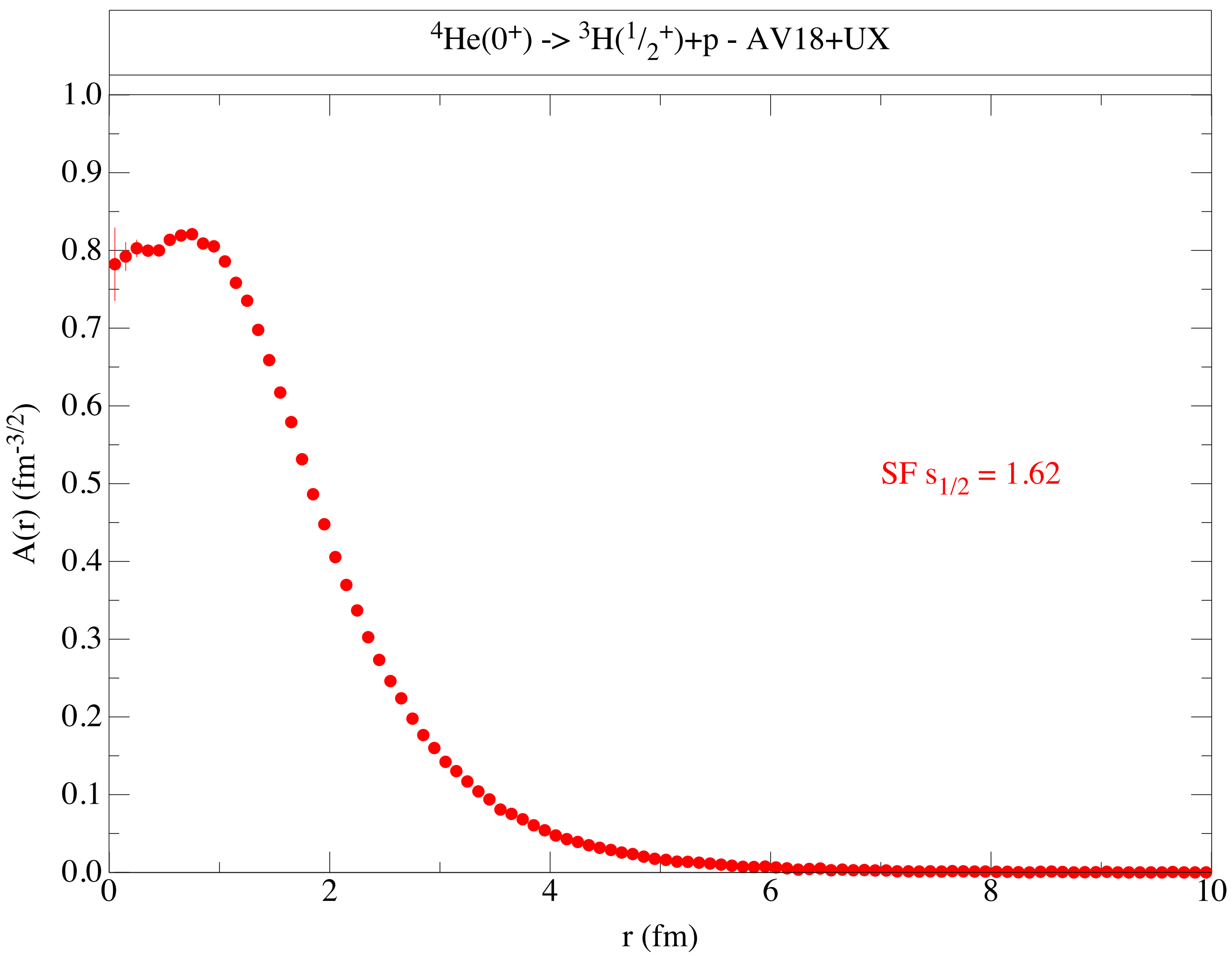
- Single-nucleon spectral function:

$$P_{p,n}(\mathbf{k},E) = \sum_n \left| \langle \Psi_0^A | [|k\rangle | \Psi_n^{A-1}\rangle] \right|^2$$
$$\times\, \delta(E + E_0^A - E_n^{A-1})$$
$$= P^{MF}(\mathbf{k},E) + P^{corr}(\mathbf{k},E)$$



$$P_p^{\rm MF}(\mathbf{k},E) = n_p^{\rm MF}(\mathbf{k})\,\delta\left(E - B_{^4\rm He} + B_{^3\rm H} - \frac{k^2}{2m_{^3\rm H}}\right)$$

$$|\langle \Psi_0^{^4\rm He} | [|k\rangle \otimes |\Psi_0^{^3\rm H}\rangle]|^2$$



- The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for)

Spectral function approach

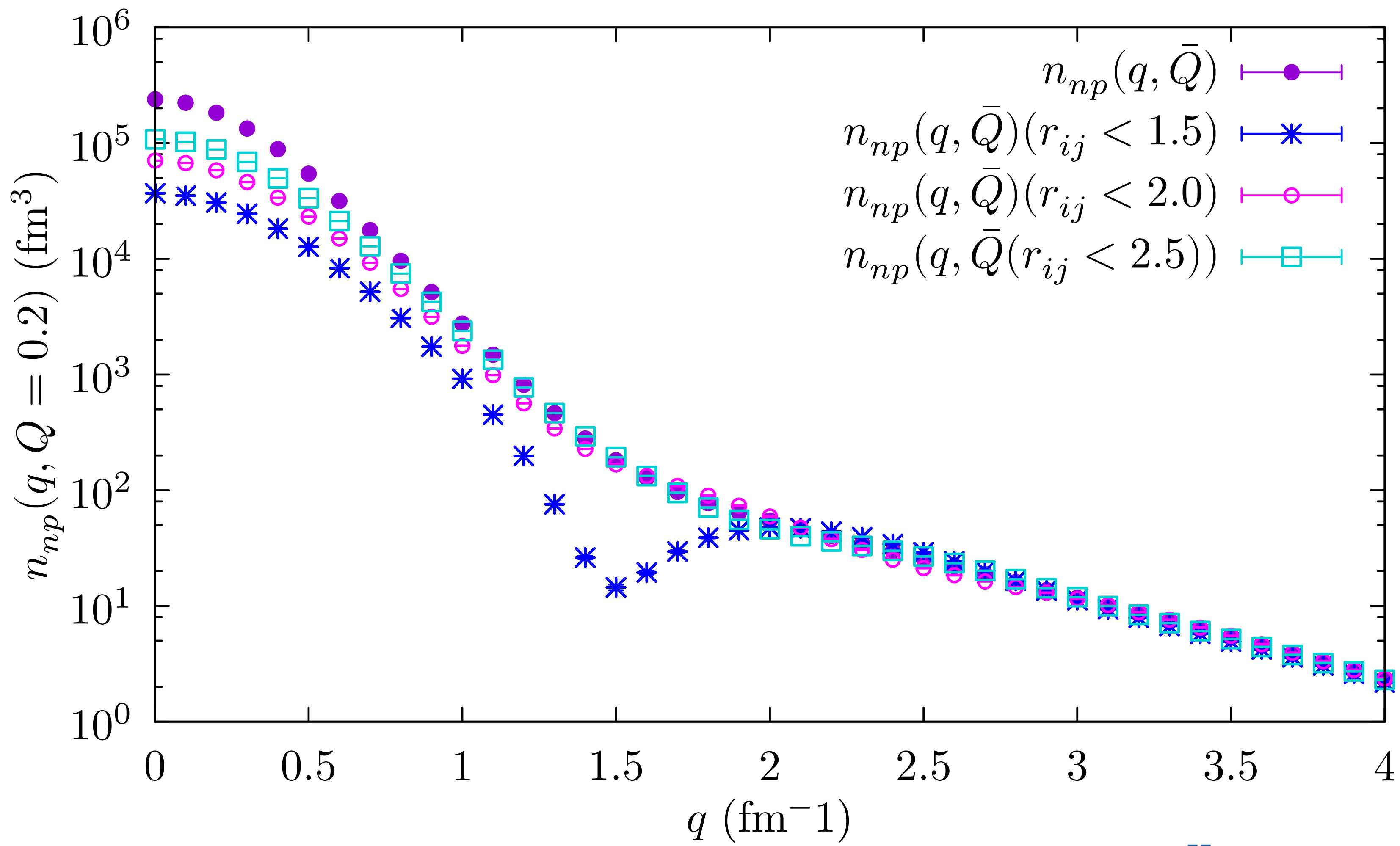
$$P_p^{\rm corr}(\mathbf{k},E)=\sum_n\int\frac{d^3k'}{(2\pi)^3}|\langle\Psi_0^A|[|k\rangle\,|k'\rangle\,|\Psi_n^{A-2}\rangle]|^2\delta(E+E_0^A-e(\mathbf{k}')-E_n^{A-2})$$

Using QMC techniques

$$\sum_{\tau_{k'}=p,n} n_{p,\tau_{k'}}(\mathbf{k},\mathbf{k}')\delta\Big(E-B_A\;-e(\mathbf{k}')+B_{A-2}-\frac{(\mathbf{k}+\mathbf{k}')^2}{2m_{A-2}}\Big)$$

Only SRC pairs should be considered: $|\Psi_0^{A-1}\rangle$ and $|k'\rangle|\psi_n^{A-2}\rangle$ be orthogonalized

One can introduce **cuts** on the **relative distance** between the particles in the two-body momentum distribution



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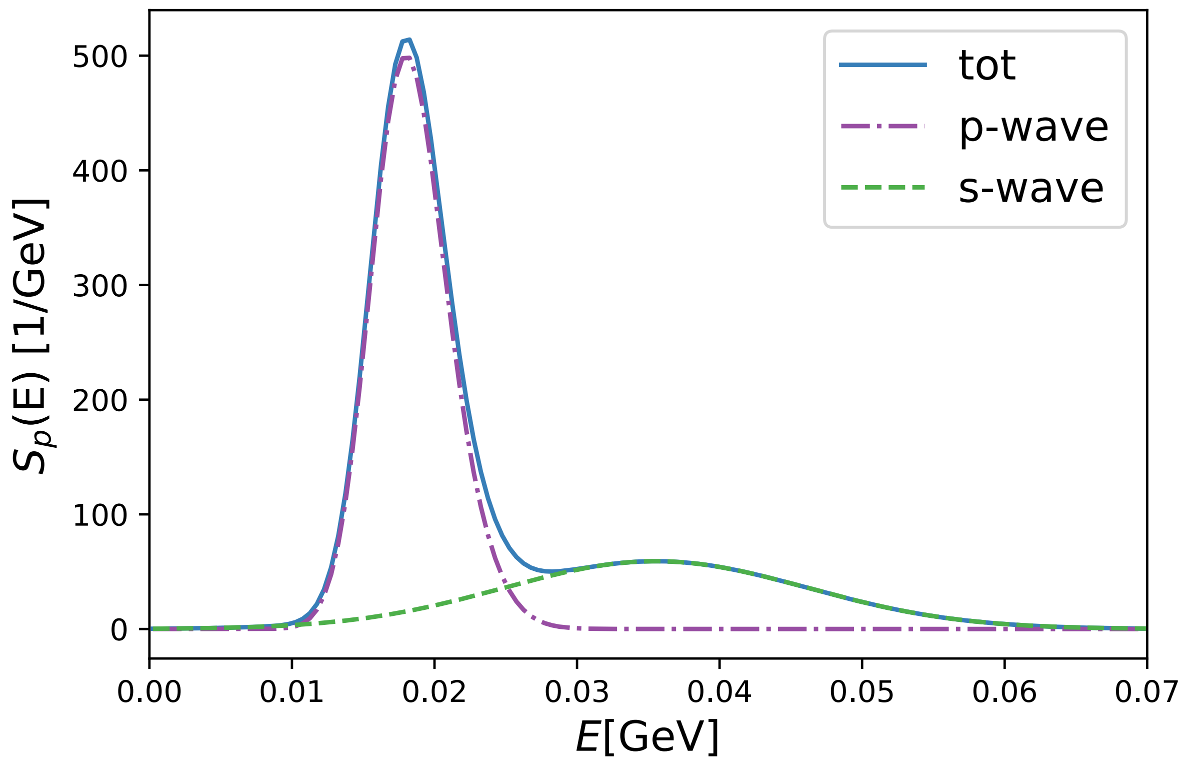
QMC Spectral Function of ^{12}C

- The p-shell contribution has been obtained by FT the radial overlaps:

$$^{12}\mathrm{C}(0^+) \rightarrow {}^{11}\mathrm{B}(3/2^-) + p$$
$$^{12}\mathrm{C}(0^+) \rightarrow {}^{11}\mathrm{B}(1/2^-) + p$$
$$^{12}\mathrm{C}(0^+) \rightarrow {}^{11}\mathrm{B}(3/2^-)^* + p\,.$$

R. Crespo, et al, Phys.Lett.B 803 (2020) 135355

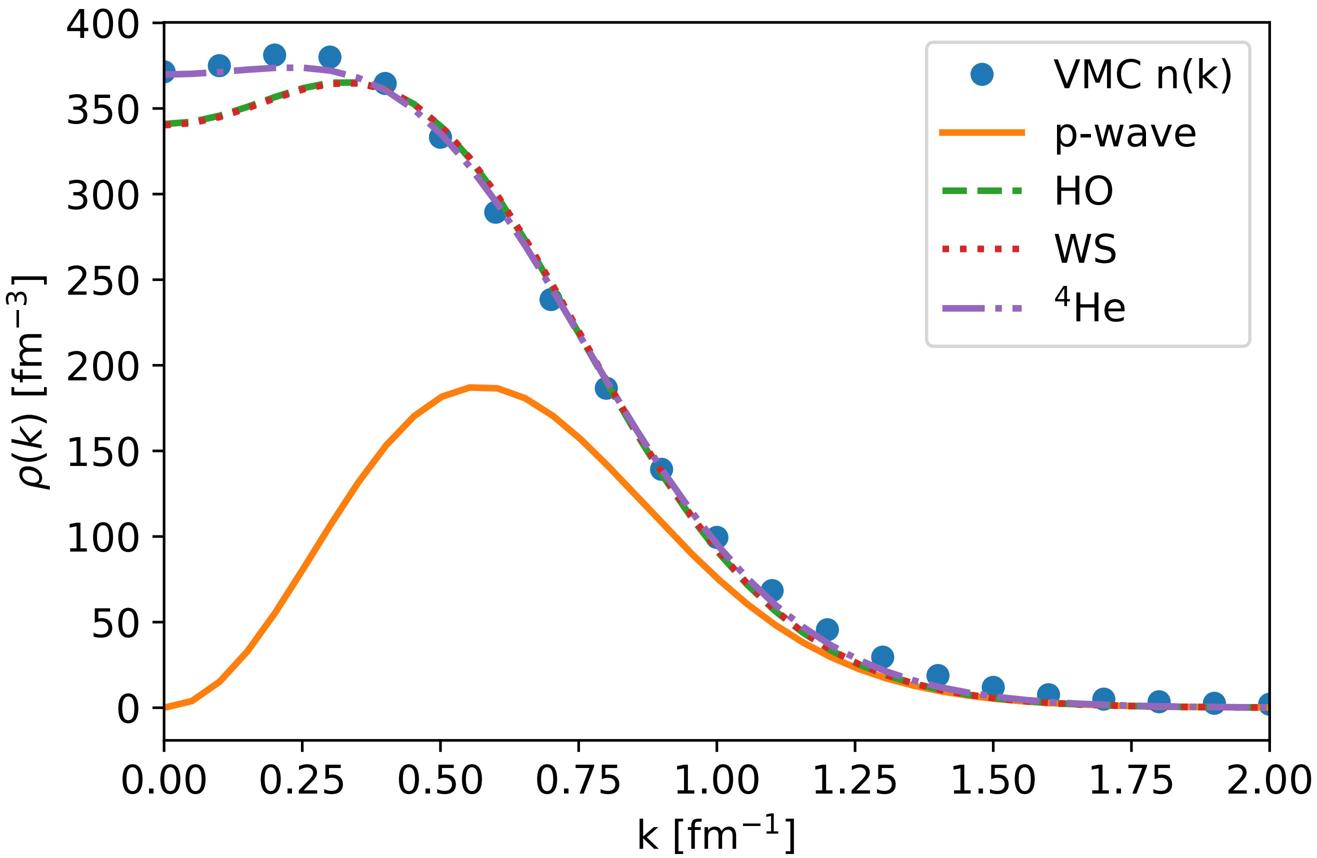


- The quenching of the spectroscopic factors automatically emerges from the VMC calculations

Computing the s-shell contribution is non trivial within VMC. We explored different alternatives:

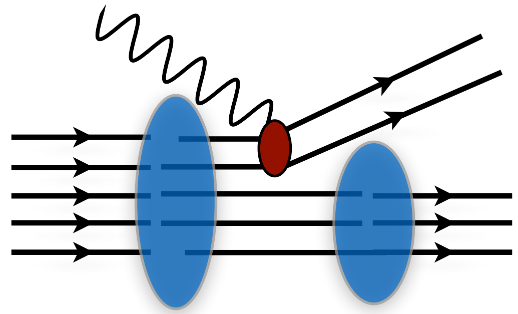
- Quenched Harmonic Oscillator
- Quenched Wood Saxon
- VMC overlap associated for the $^4\mathrm{He}(0^+) \rightarrow {}^3\mathrm{H}(1/2^+) + p$ transition

Korover, et al, CLAS collaboration PRC 107 (2023) 6, L061301



Spectral function approach

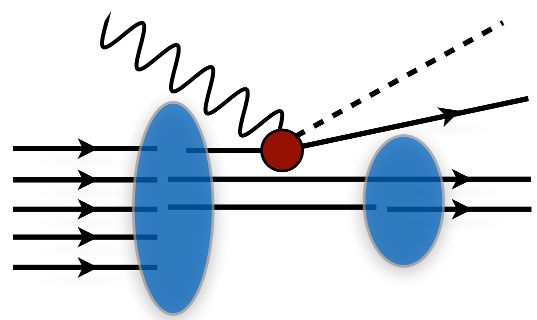
$$|f\rangle \to |pp'\rangle_a \otimes |f_{A-2}\rangle$$



The hadronic tensor for two-body current factorizes as

$$R^{\mu\nu}_{2\text{b}}(\mathbf{q},\omega) \propto \int dEd^3kd^3k' P_{2\text{b}}(\mathbf{k},\mathbf{k}',E) \times d^3pd^3p' |\langle kk'|j^{\mu}_{2\text{b}}|pp'\rangle|^2$$

$$|f\rangle \to |p_\pi p\rangle \otimes |f_{A-1}\rangle$$

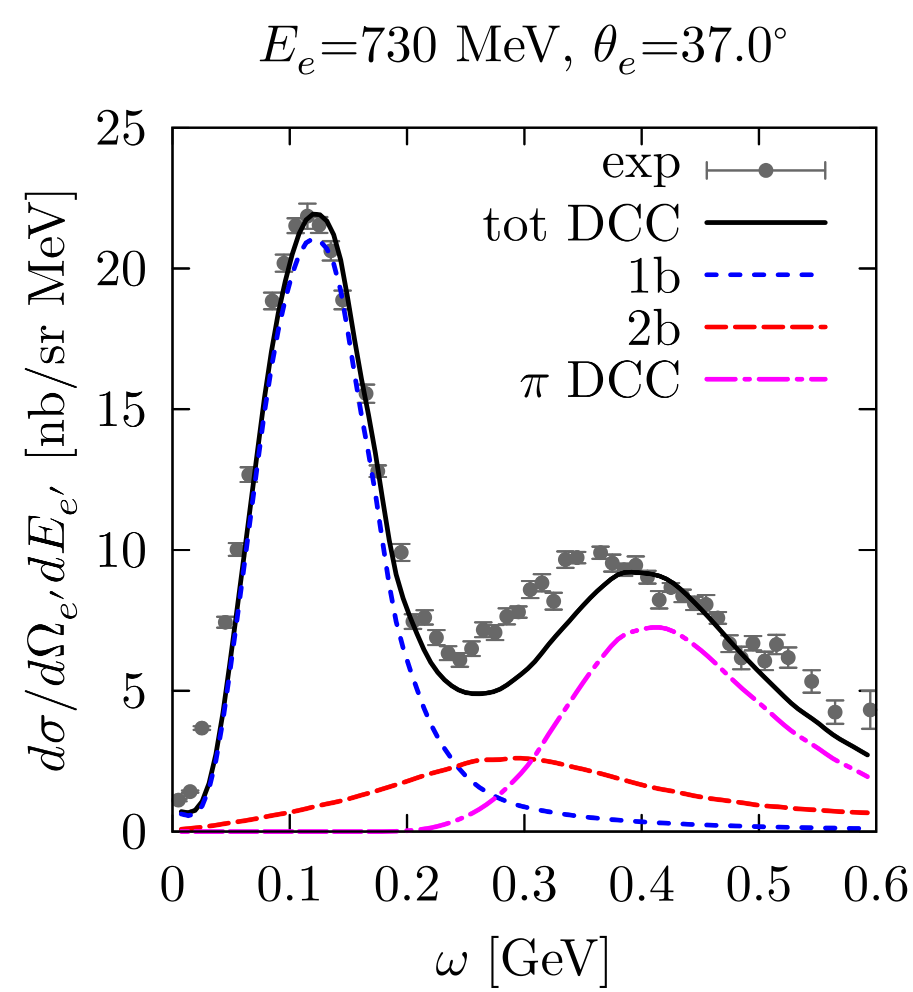


Production of real π in the final state

$$R^{\mu\nu}_{1\text{b}\pi}(\mathbf{q},\omega) \propto \int dEd^3kP_{1\text{b}}(\mathbf{k},E) \times d^3pd^3k_\pi |\langle k|j^{\mu}|pk_\pi\rangle|^2$$

* Pion production elementary amplitudes currently derived within the extremely sophisticated **Dynamic Couple Chanel approach**;

S.X.Nakamura, et al PRD92(2015)
T. Sato, et al PRC67(2003)



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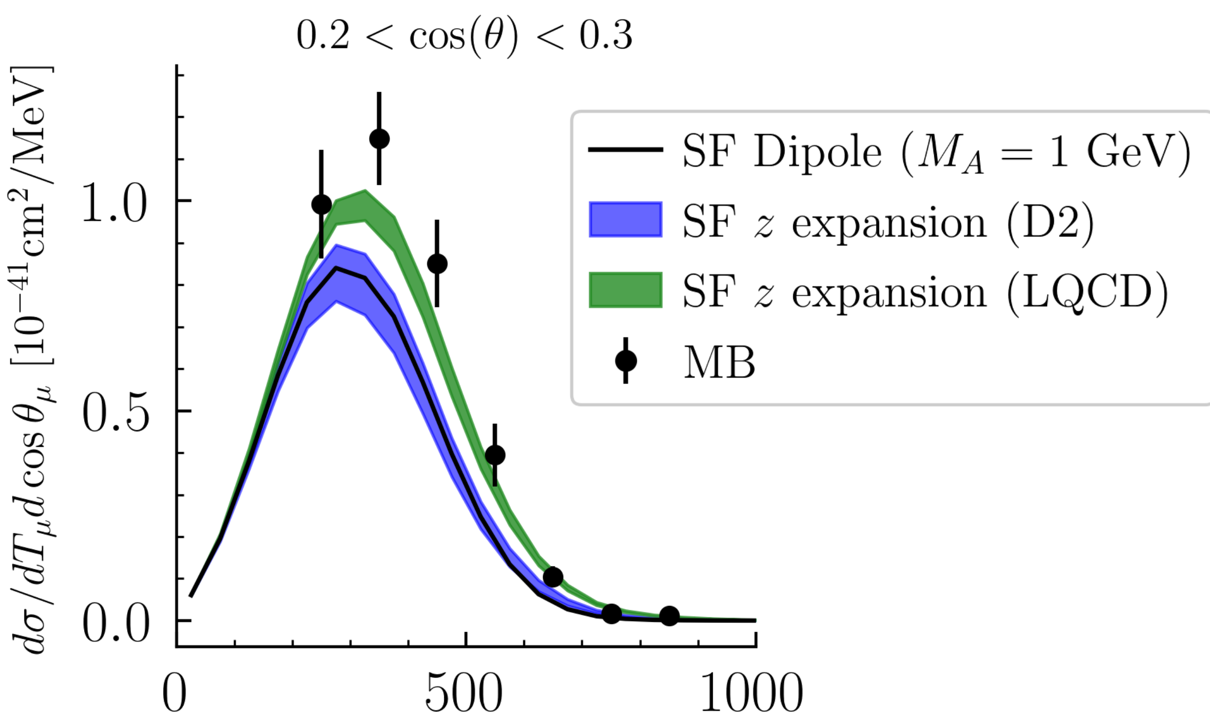
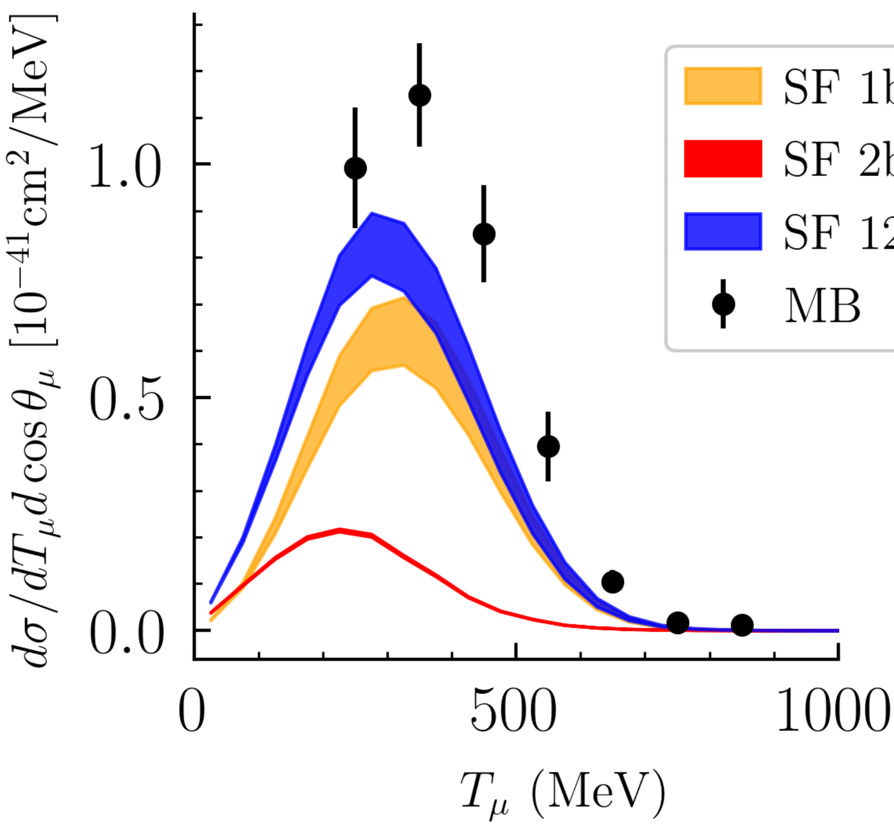
Axial Form Factors Uncertainty needs





D.Simons, N. Steinberg et al, 2210.02455

✳ Axial form factor dependence:

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
SF Difference in $d\sigma_{\text{peak}}$ (%)	16.3

✳ Many-body method dependence:

MiniBooNE	$0.2 < \cos\theta_\mu < 0.3$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	22.8

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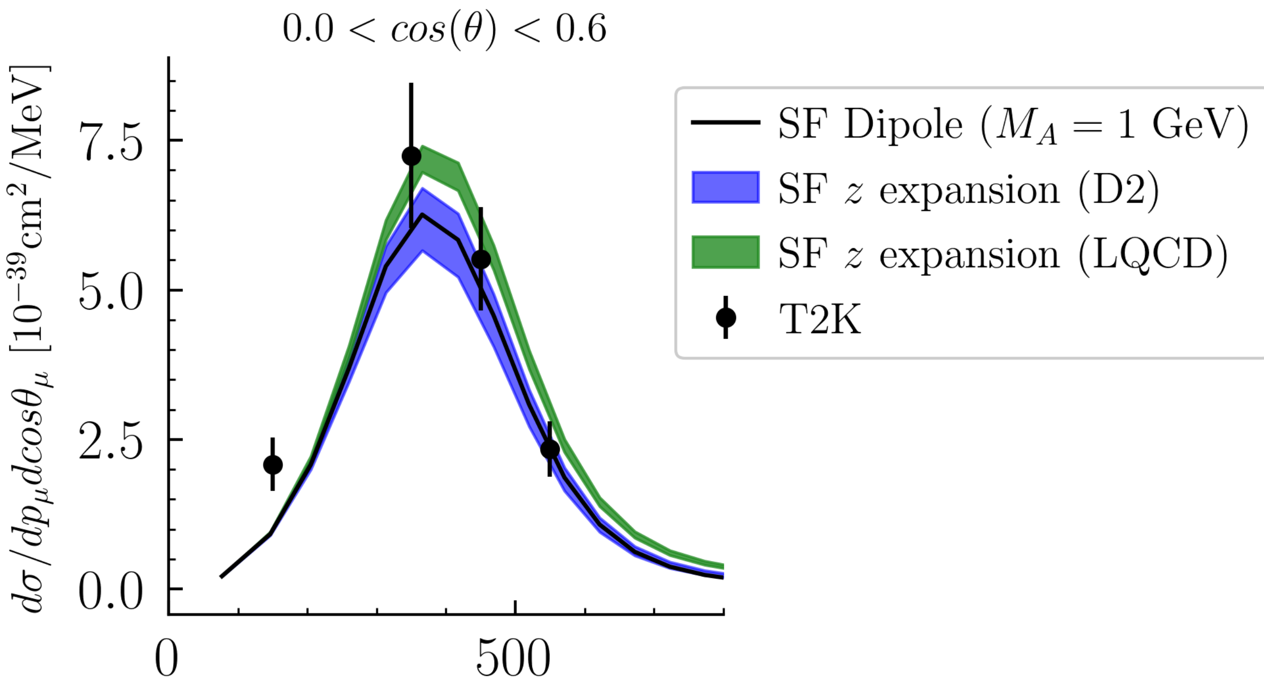
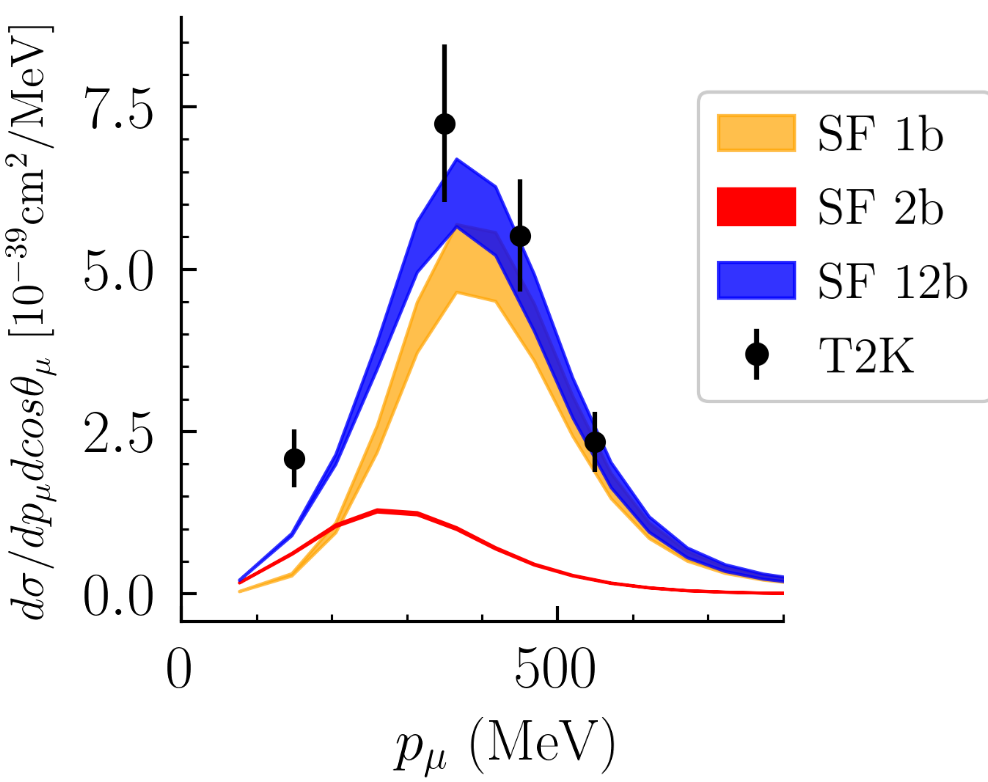
Axial Form Factors Uncertainty needs





D.Simons, N. Steinberg et al, 2210.02455

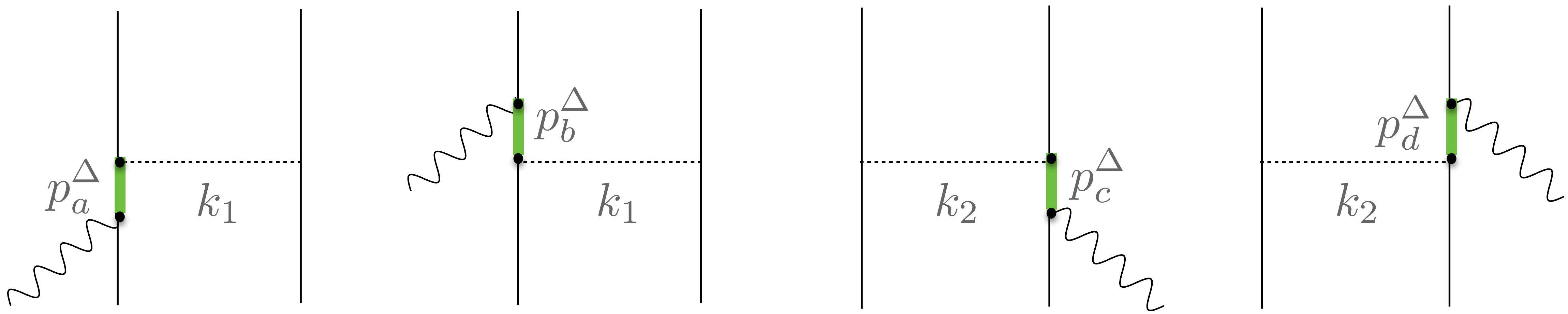
✴ Axial form factor dependence:

T2K	$0.0 < \cos\theta_\mu < 0.6$
SF difference in $d\sigma_{\text{peak}}$ (%)	15.3

✴ Many-body method dependence:

T2K	$0.0 < \cos\theta_\mu < 0.6$
GFMC/SF difference in $d\sigma_{\text{peak}}$ (%)	13.4

Two-body currents - Delta contribution



$$j^\mu_\Delta = \frac{3}{2}\frac{f_{\pi NN} f^*}{m_\pi^2}\Big\{\Pi(k_2)_{(2)}\Big[\Big(-\frac{2}{3}\tau^{(2)} + \frac{I_V}{3}\Big)_z F_{\pi NN}(k_2)F_{\pi N\Delta}(k_2)(J_a^\mu)_{(1)} - \Big(\frac{2}{3}\tau^{(2)} + \frac{I_V}{3}\Big)_z F_{\pi NN}(k_2)F_{\pi N\Delta}(k_2)(J_b^\mu)_{(1)}\Big] + (1 \leftrightarrow 2)\Big\}$$

where

Rarita Schwinger propagator

$$(J_a^\mu)_V = (k_1)^\alpha G_{\alpha\beta}(p_\Delta)\Big[\frac{C_3^V}{m_N}\Big(g^{\beta\mu}\not{q} - q^\beta\gamma^\mu\Big) + \frac{C_4^V}{m_N^2}\Big(g^{\beta\mu} q\cdot p_\Delta - q^\beta p_\Delta^\mu\Big) + \frac{C_5^V}{m_N^2}\Big(g^{\beta\mu} q\cdot k - q^\beta k^\mu + C_6^V g^{\beta\mu}\Big)\Big]\gamma_5$$

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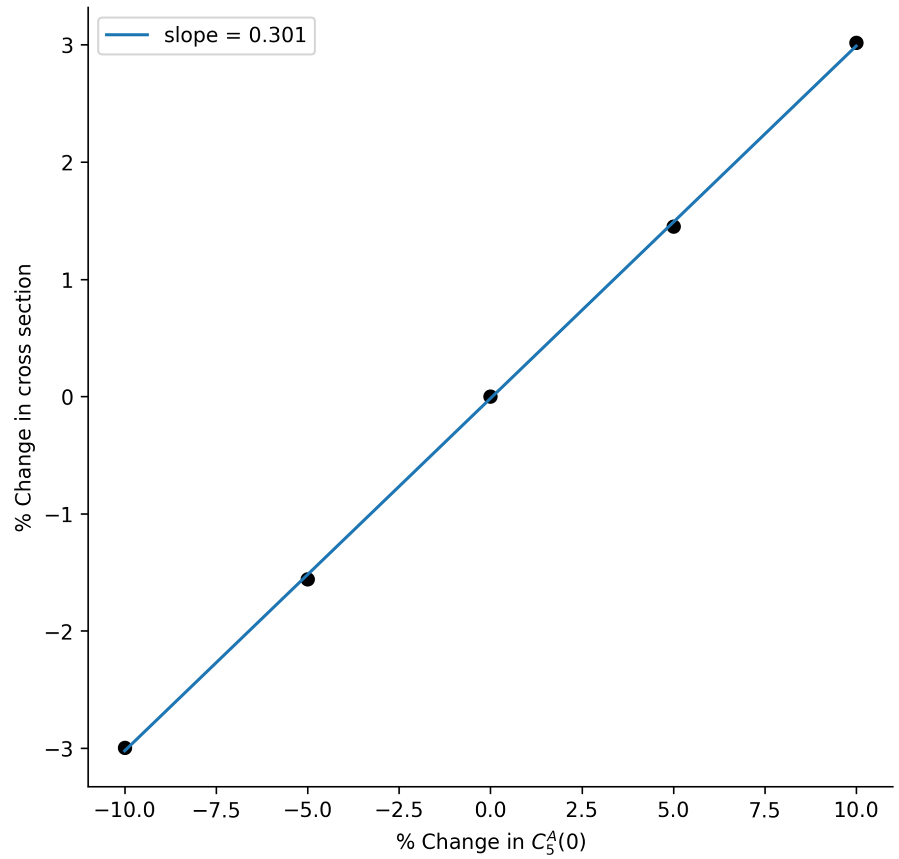
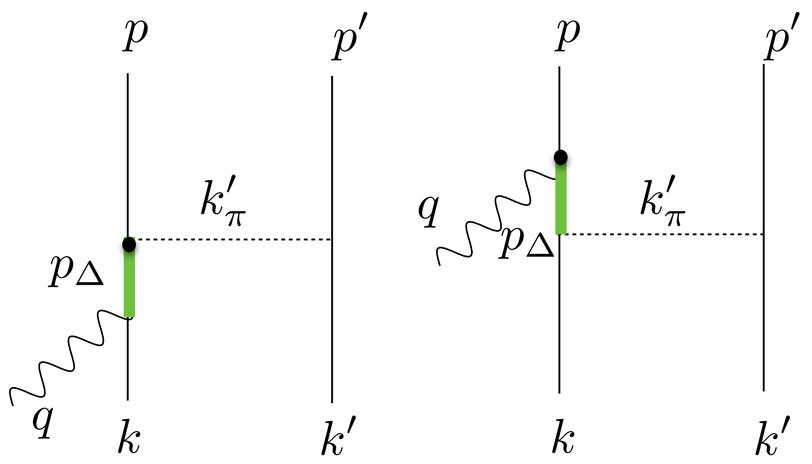
Resonance Uncertainty needs

The largest contributions to two-body currents arise from resonant $N \to \Delta$ transitions yielding pion production



D.Simons, N. Steinberg et al, 2210.02455



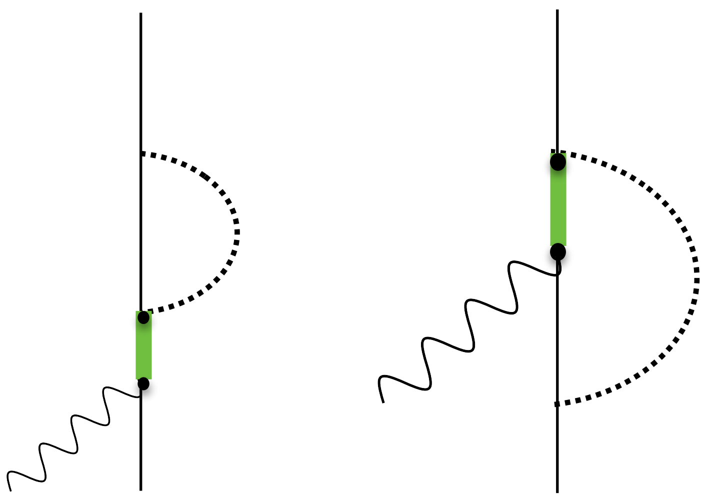
The normalization of the dominant $N \to \Delta$ transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

Hernandez et al, PRD 81 (2010)

Further constraints on $N \to \Delta$ transition relevant for two-body currents and π production will be necessary to achieve few-percent cross-section precision

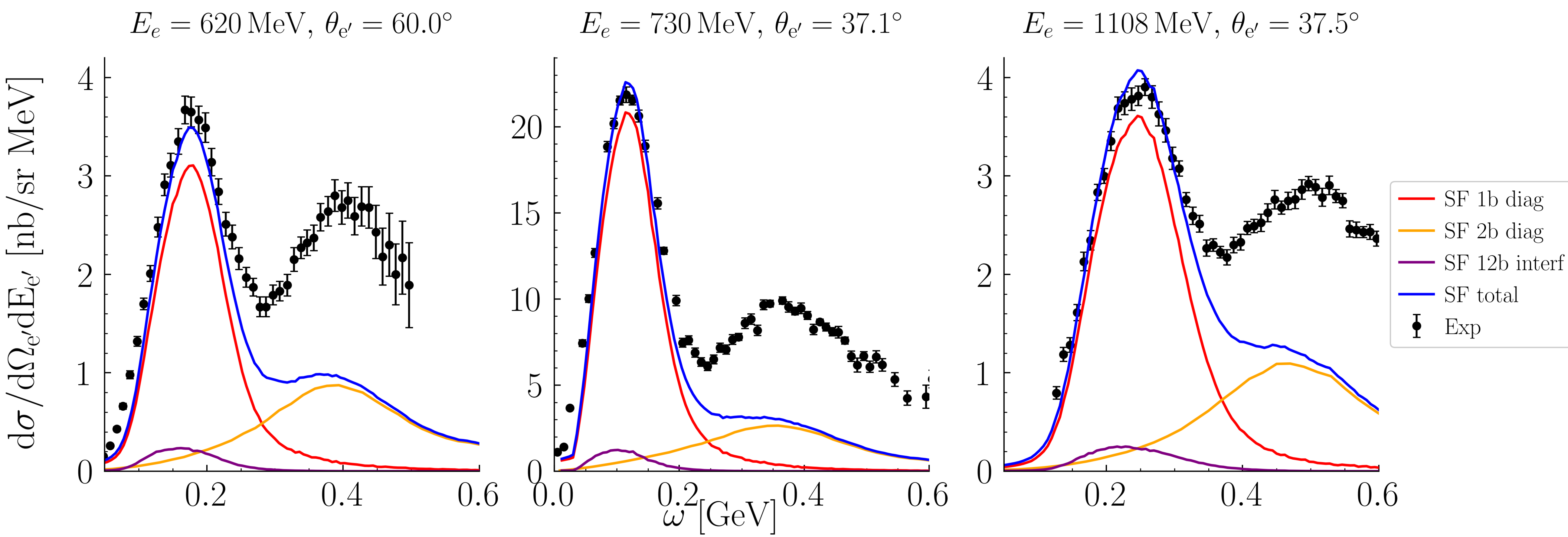
Including the one- and two-body interference



We recently included interference effects between one- and two-body currents yielding single nucleon knock-out

Observe a small quenching in the longitudinal channel and an enhancement in the q.e. peak in the transverse → agreement with the GFMC

N. Steinberg, NR, A. Lovato, in preparation



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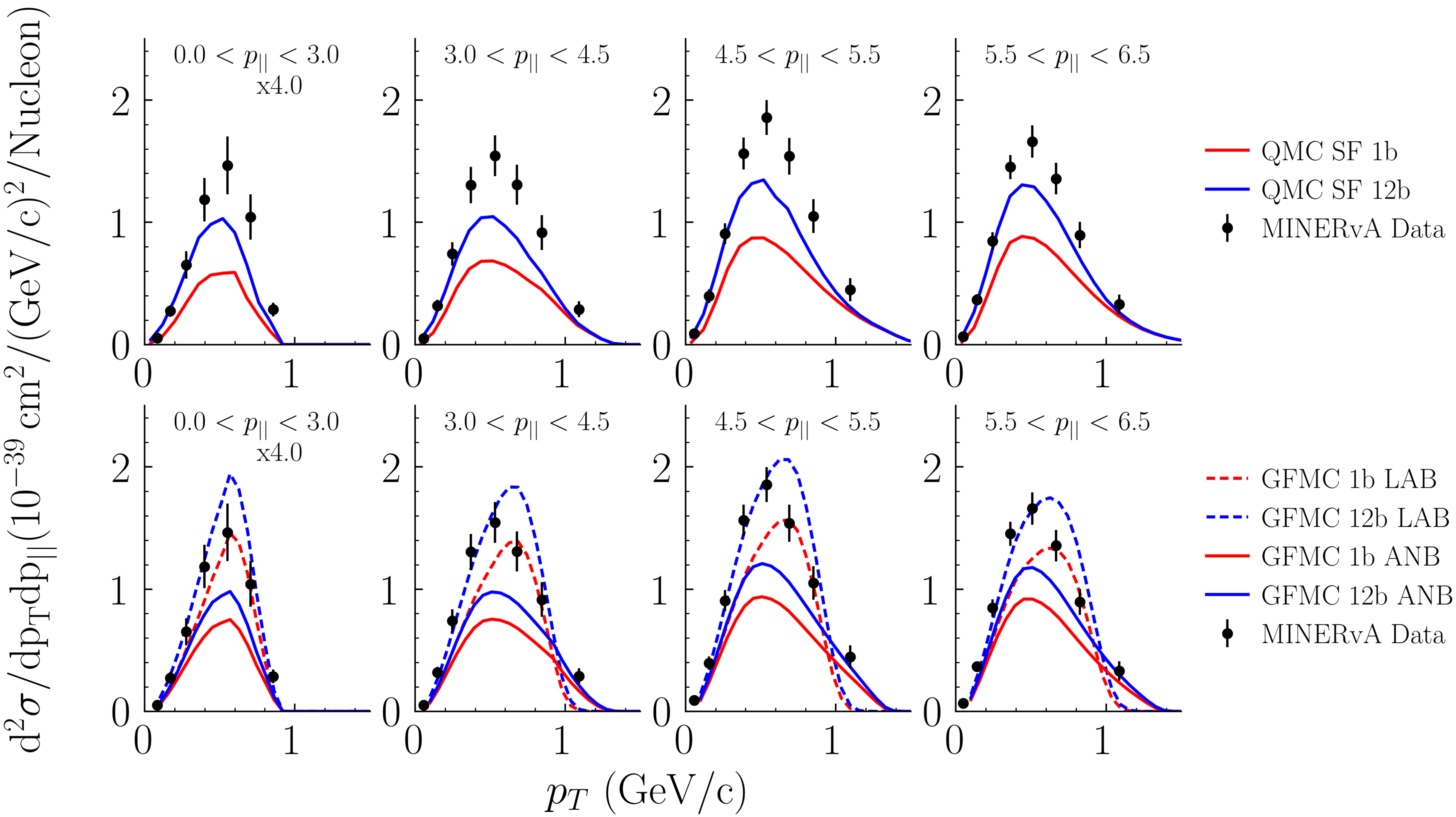
Comparing different many-body methods

— MINERvA M.E. Double Differential Cross Section in p_T, $p_{||}$. CCQE-like data on CH

N. Steinberg, A. Nikolakopoulos, A. Lovato, NR, *submitted to Universe*



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Fermilab

Conclusions

- Assessing the overall uncertainty of theory calculations requires evaluating uncertainties:

  Nuclear Hamiltonians: different efforts in place to provide UQ in chiral EFT

  Form factors: one- and two-body currents, resonance/π production

  Error of factorizing the hard interaction vertex / using a non relativistic approach

- Address neutrino precision goals requires studying relations between cross section uncertainties and input parameter uncertainties

- Additional constraints on few-nucleon inputs from experiment and lattice QCD will be crucial

- Factorized approaches ideally suited to incorporate elementary amplitudes - nucleon hadron tensor

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Thank you for your attention!