Electron and neutrino scattering off the deuteron: elastic and breakup channels.

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IN KRAKÓW

Outline

Based on:

- J. Golak, R. Skibiński, K. Topolnicki, H. Witała, A. Grassi, H. Kamada, L. E. Marcucci,
  Phys. Rev. C 98, 015501 (2018),
  Momentum space treatment of inclusive neutrino scattering off the deuteron and trinucleons.

- J. Golak, R. Skibiński, K. Topolnicki, H. Witała, A. Grassi, H. Kamada, L. E. Marcucci,
  Phys. Rev. C 100, 064003 (2019),
  From response functions to cross sections in neutrino scattering off the deuteron and trinucleons.

- A. Grassi, J. Golak, W. N. Polyzou , R. Skibiński , H. Witała, H. Kamada,
  Phys. Rev. C107, 024617 (2023),
  Electron and neutrino scattering off the deuteron in a relativistic framework

1. Nonrelativistic approach: neutrino induced processes
2. Relativistic approach: electron/neutrino induced processes

Introduction

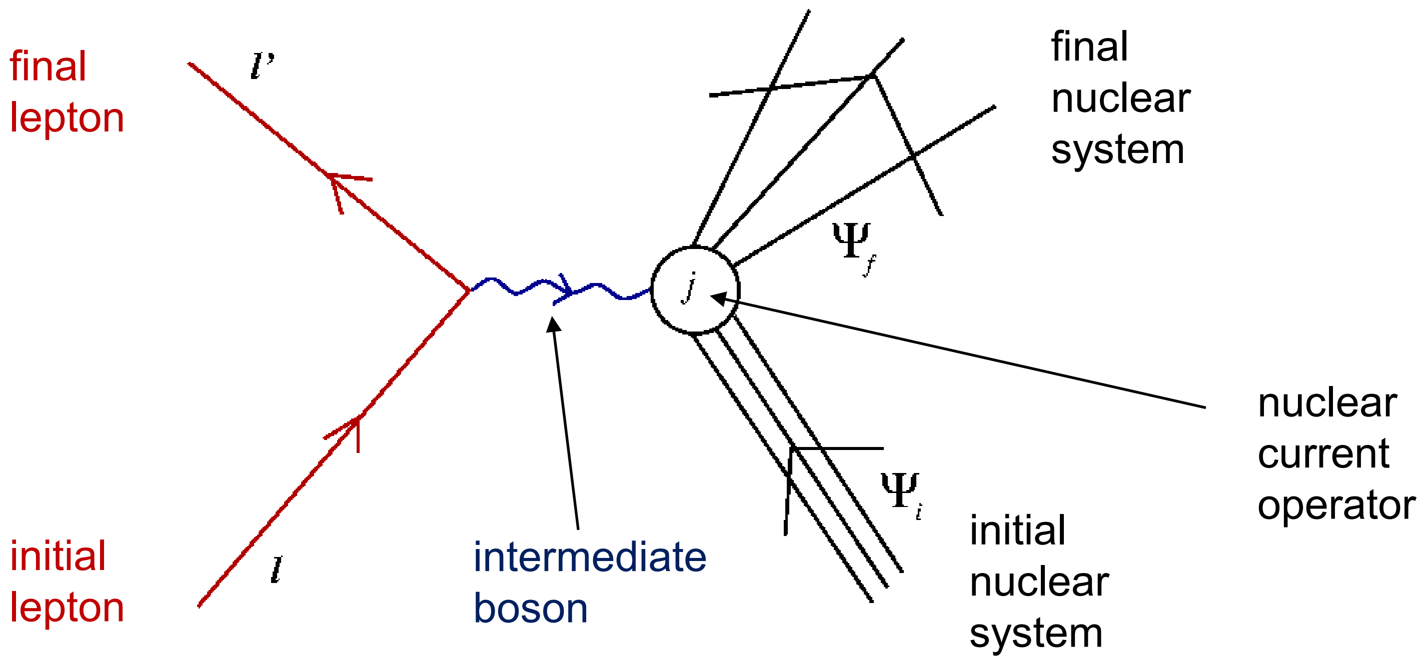
- We started from the study of 2N and 3N scattering, including N+d breakup channels (W.Glöckle, H.Witała, H.Kamada, …)
  → knowledge how to compute the nuclear states
  → opportunity to study relativistic approach

- But the same formalism can be used to study electroweak processes (+ J.Golak, …)
  → firstly we have attacked processes with electrons and photons, later we included muons, pions, neutrinos

- There is a hope to combine both directions by using consistent forces and currents from χEF (+ E.Epelbaum, H.Krebs, …)

- Our formalism is quite flexible, which makes computations easier

Introduction - Formalism



$$\left|T_{fi}\right|^2 \propto L_{\alpha\beta}\, N^{\alpha}\left(N^{\beta}\right)^{*} \qquad N^{\alpha} \equiv \left\langle \Psi_f \left| j^{\alpha} \right| \Psi_i \right\rangle$$

$L_{\alpha\beta}$ known analytically ! (spinors and gamma matrices)

Introduction - Formalism

$N^{\alpha} = \langle \Psi_{f m_f} | j^{\alpha} | \Psi_{i m_i} \rangle$ from *ab initio* calculations in momentum space

Dynamical ingredients:

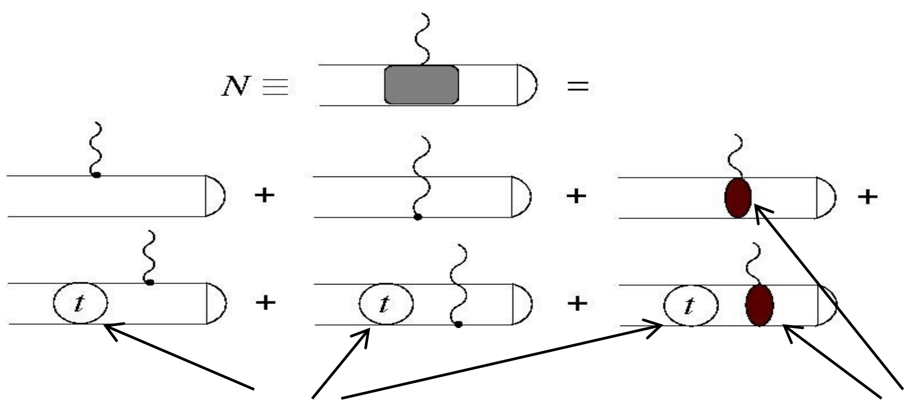
(1): 2N and 3N Hamiltonians $H_{2N} = H_0^{2N} + V_{12}$

$$H_{3N} = H_0^{3N} + V_{23} + V_{13} + V_{12} + V_{123}$$

used to generate nuclear bound and scattering states contain 2N and 3N potentials.

(2): nuclear EM and weak single-nucleon, 2N current operators, and many-body currents for A>2

$j_{2N} = j_1 + j_2 + j_{12}$ describe interactions of the electroweak probe with nuclear system.



full amplitude

plane wave part

rescattering (FSI) part

2N t-matrix half-shell

2N mechanism
of photon absorption (MEC)

Introduction - Formalism

$$H_{2N}\left|\psi_d\right\rangle = E_d\left|\psi_d\right\rangle$$ deuteron state with $E_d < 0$

$$N^\alpha \equiv \left\langle \psi'_d \right| j^\alpha_{2N} \left|\psi_d\right\rangle$$ elastic channel

$$N^\alpha \equiv \left\langle \psi^{(-)} \right| j^\alpha_{2N} \left|\psi_d\right\rangle = {}_a\left\langle \vec{p}_o \right| \left(1 + t_{12} G_0^{2N}\right) j^\alpha_{2N} \left|\psi_d\right\rangle$$ break-up channel

$$H_{2N}\left|\psi^{(-)}\right\rangle = E\left|\psi^{(-)}\right\rangle, \quad E = \frac{p_0^2}{m} > 0$$

$$t_{12} = V_{12} + t_{12}\, G_0^{2N}\left(E + i\varepsilon\right) V_{12}$$ Lippmann-Schwinger equation

$$G_0^{2N}(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E + i\varepsilon - H_0^{2N}}$$ free 2N propagator

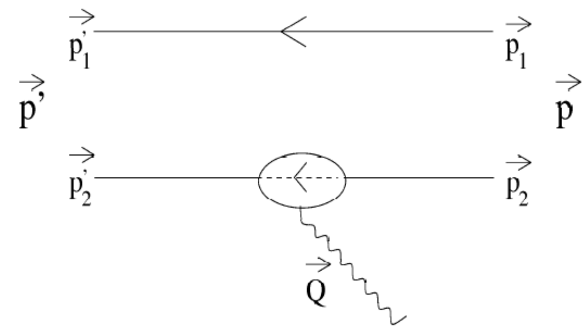
In results shown here the Argonne V18 interaction is applied.

For 3N scattering states (nucleon-deuteron or three free nucleons) we solve much more complex Faddeev equations.

Introduction - matrix elements of the EM current operator

We deal with the (relatively simple) single nucleon current, where

$$j^{\mu}(\vec{Q}) = j_1^{\ \mu}(\vec{Q}) + j_2^{\ \mu}(\vec{Q}) \longrightarrow$$



single nucleon momenta

Examples:

charge density

$$\left\langle \vec{p}\,' \left| \frac{1}{e} J_1^0(0) \right| \vec{p} \right\rangle = \left(G_E^p \Pi^p + G_E^n \Pi^n \right),$$

$$\left\langle \vec{p}\,' \left| \frac{1}{e} \vec{J}_1(0) \right| \vec{p} \right\rangle = \frac{\vec{p} + \vec{p}\,'}{2M_N} \left(G_E^p \Pi^p + G_E^n \Pi^n \right) + \frac{i}{2M_N} \left(G_M^p \Pi^p + G_M^n \Pi^n \right) \vec{\sigma} \times (\vec{p}\,' - \vec{p}).$$

convection current

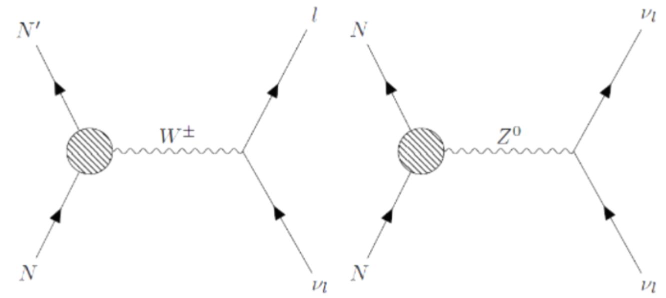
spin current

$$\Pi^p \equiv \frac{1}{2}\left(1 + (\tau)_3\right) \qquad \Pi^n \equiv \frac{1}{2}\left(1 - (\tau)_3\right)$$

single nucleon momenta

Reactions with (anti)neutrinos

- S. Nakamura *et al.*, Phys. Rev. C 63, 034617 (2001); Erratum Phys. Rev. C 73, 049904 (2006)
- G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012)
- A. Baroni and R. Schiavilla, C 96, 014002 (2017)



Simple recipe: If one starts with $e + d \rightarrow e + p + n$ proces: replace for the charged current (CC) driven reactions

$$\bar{u}(k',s')\gamma_\alpha u(k,s)\frac{e^2}{q^2} N^\alpha_{EM}$$

$$\rightarrow \bar{u}(k',s')\gamma_\alpha(1-\gamma_5)\, u(k,s)\frac{G_F \cos\Theta_C}{\sqrt{2}} N^\alpha_{CC,+1}$$

$$\nu_l + d \rightarrow l^- + p + p$$

$$\rightarrow \bar{v}(k,s)\gamma_\alpha(1-\gamma_5)\, v(k',s')\frac{G_F \cos\Theta_C}{\sqrt{2}} N^\alpha_{CC,-1}$$

$$\bar{\nu}_l + d \rightarrow l^+ + n + n$$

Reactions with (anti)neutrinos

$$\langle \mathbf{p}' \mid j^0_{\mathrm{NR+RC}}(1) \mid \mathbf{p} \rangle = \Big(g_1^V - (g_1^V - 4M g_2^V) \frac{(\mathbf{p}'-\mathbf{p})^2}{8M^2} + (g_1^V - 4M g_2^V)\; i \frac{(\mathbf{p}' \times \mathbf{p}) \cdot \boldsymbol{\sigma}}{4M^2}$$

$$+ g_1^A \frac{\boldsymbol{\sigma} \cdot (\mathbf{p}+\mathbf{p}')}{2M} + g_2^A \frac{(\mathbf{p}'^2 - \mathbf{p}^2)}{4M^2} \boldsymbol{\sigma} \cdot (\mathbf{p}' - \mathbf{p}) \Big) \tau_-$$

$$\langle \mathbf{p}' \mid \mathbf{j}_{\mathrm{NR+RC}}(1) \mid \mathbf{p} \rangle = \Big(g_1^V \frac{\mathbf{p}+\mathbf{p}'}{2M} - \frac{1}{2M} (g_1^V - 2M g_2^V)\; i\, \boldsymbol{\sigma} \times (\mathbf{p} - \mathbf{p}') + g_1^A \Big(1 - \frac{(\mathbf{p}+\mathbf{p}')^2}{8M^2} \Big) \boldsymbol{\sigma} +$$

$$+ \frac{g_1^A}{4M^2} [(\mathbf{p} \cdot \boldsymbol{\sigma}) \mathbf{p}' + (\mathbf{p}' \cdot \boldsymbol{\sigma}) \mathbf{p} + i\, (\mathbf{p} \times \mathbf{p}')]) + g_2^A (\mathbf{p} - \mathbf{p}') \frac{\boldsymbol{\sigma} \cdot (\mathbf{p} - \mathbf{p}')}{2M} \Big) \tau_-$$

Above currents base on L.E.Marcucci, …, L.Girlanda, …, R.Schiavilla, Phys. ReV C83 (2012) 014002

For reactions with the neutral current (NC), construct the corresponding nuclear current operator and replace

$$\bar{u}(k',s')\gamma_\alpha\, u(k,s) \frac{e^2}{q^2} N^\alpha_{EM}$$

→

$$\bar{u}(k',s')\gamma_\alpha (1-\gamma_5)\, u(k,s) \frac{G_F}{\sqrt{2}} N^\alpha_{NC}$$

$$\nu_l + d \rightarrow \nu_l + p + n$$

→

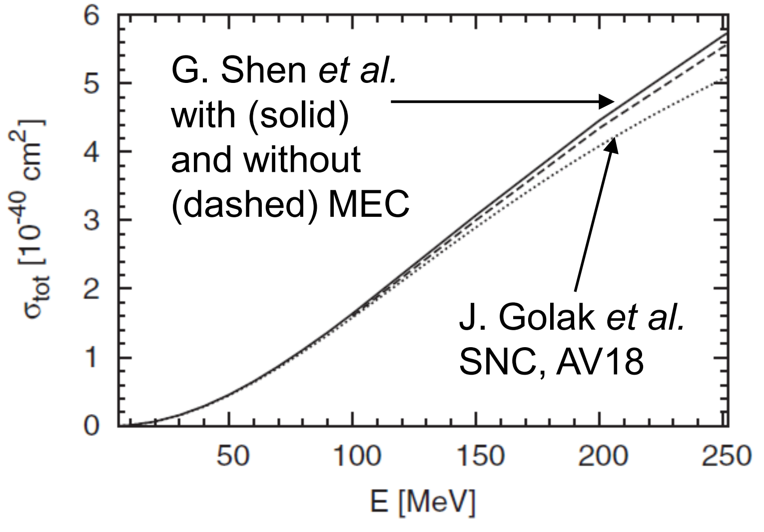
$$\bar{v}(k,s)\gamma_\alpha (1-\gamma_5)\, v(k',s') \frac{G_F}{\sqrt{2}} N^\alpha_{NC}$$

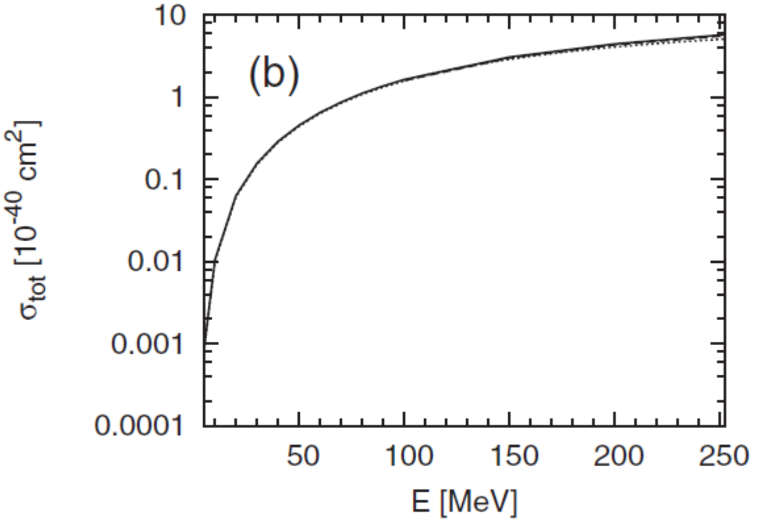
$$\bar{\nu}_l + d \rightarrow \bar{\nu}_l + p + n$$

Reactions with (anti)neutrinos

$$\bar{\nu}_e + {}^2H \rightarrow e^+ + n + n$$

Total CC cross section as a function of the antineutrino energy



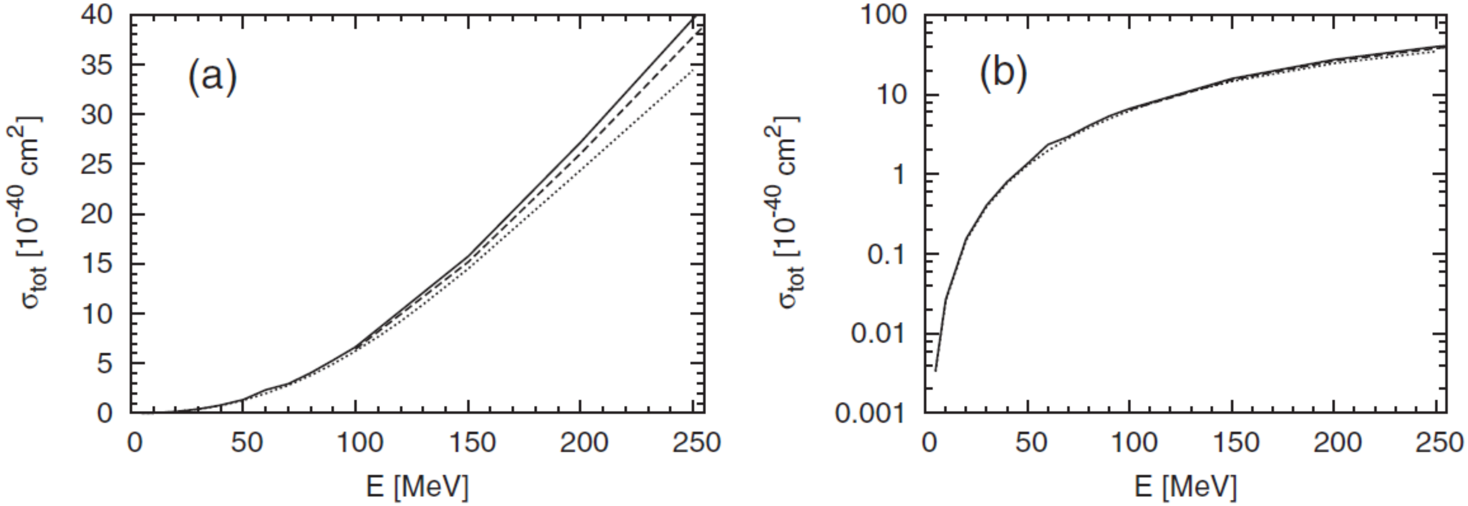


Comparison with G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012): difference between dotted and dashed curves is due relativistic kinematics for nucleons.

Reactions with (anti)neutrinos

$$\nu_e + {}^2H \rightarrow e^- + p + p$$

Total CC cross section as a function of the neutrino energy



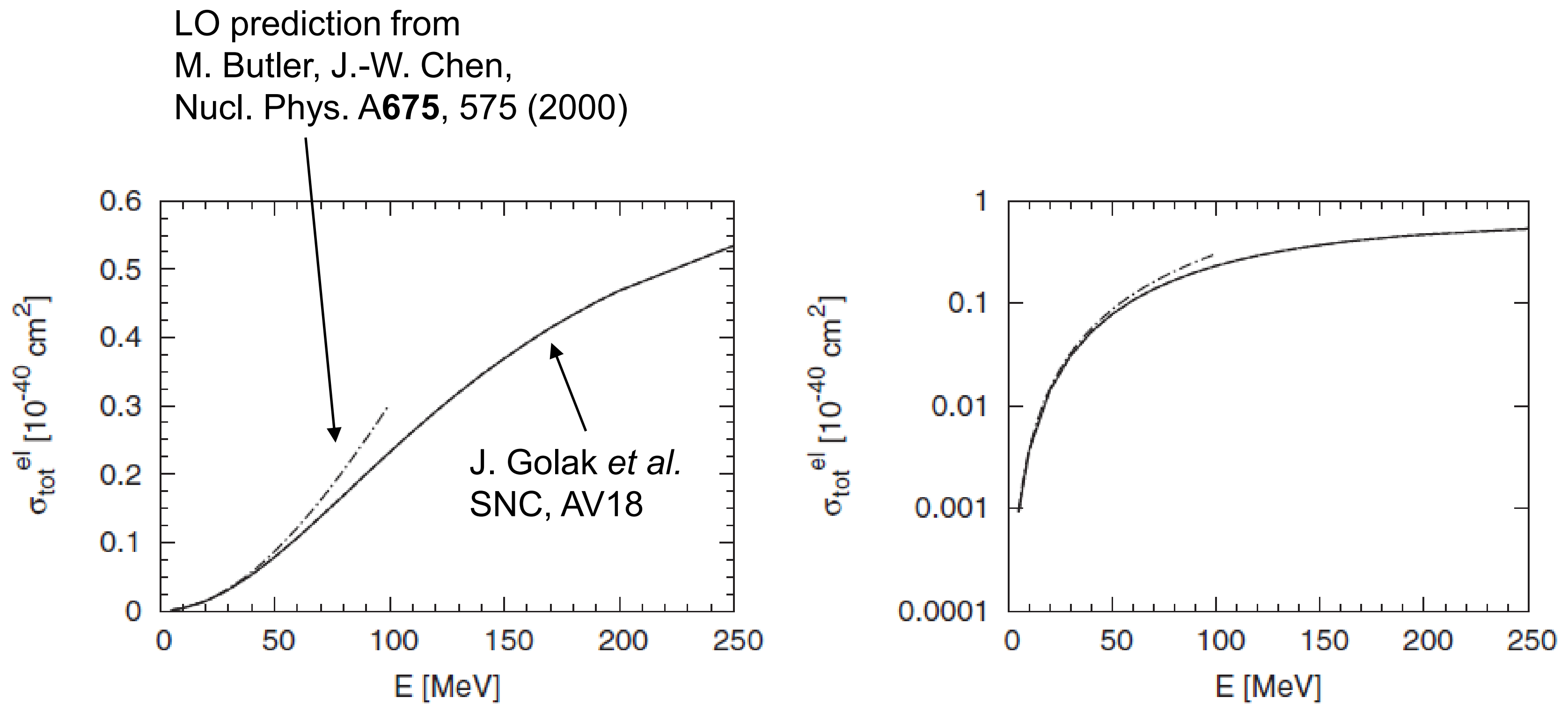
Comparison with G. Shen *et al.*, Phys. Rev. C 86, 035503 (2012)

For the p+p final state we include the Coulomb interaction
by the sharply cut off Coulomb force (R=40 fm).

Reactions with (anti)neutrinos

$$\nu_l + {}^2H \to \nu_l + {}^2H \quad \text{and} \quad \bar{\nu}_l + {}^2H \to \bar{\nu}_l + {}^2H$$

Total NC (flavor independent !) cross section as a function of the neutrino energy

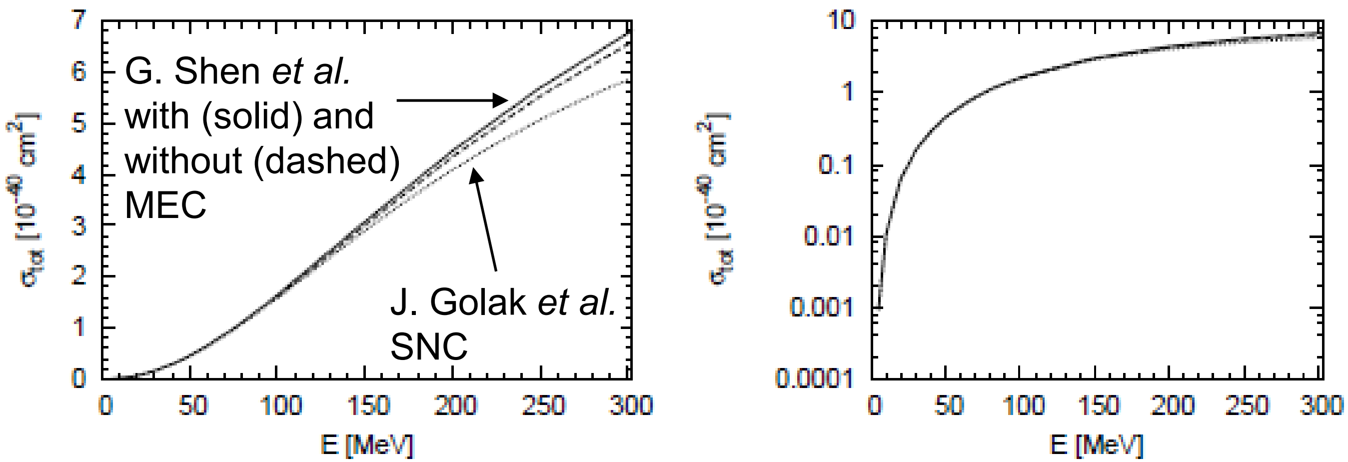


Agreement at low energies, at higher energies LO results can be misleading.

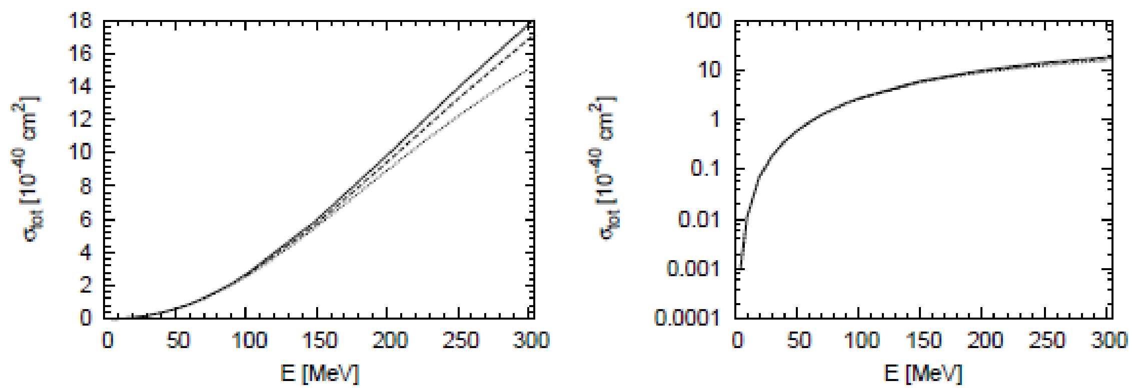
Reactions with (anti)neutrinos

Total NC cross section as a function of the antineutrino energy

$$\bar{\nu}_e + {}^2H \rightarrow \bar{\nu}_e + p + n$$



$$\nu_e + {}^2H \rightarrow \nu_e + p + n$$



Picture similar to that for the CC cross section – most of components is the same: similar kinematics, except ν and e masses.

Reactions with (anti)neutrinos

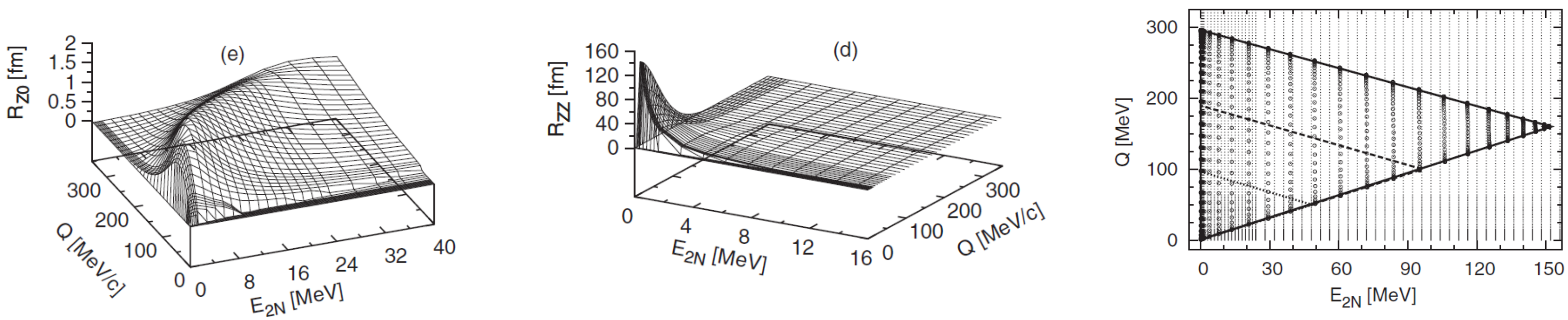
Beyond the total cross section:

$$\frac{d^3\sigma}{dE'\,d\Omega'} = \frac{G_F^2 \cos^2(\theta_C)}{(2\pi)^2} F(Z,E') \frac{|\vec{k}'|}{8E} \left(V_{00}R_{00} + V_{MM}R_{MM} + V_{PP}R_{PP} + V_{ZZ}R_{ZZ} + V_{Z0}R_{Z0}\right)$$

$$R_{AB} = R_{AB}(E_{c.m.}, Q) = \sum_{m_i, m_f} \int df\ \delta(E_{c.m.} - E_f) \left\langle \Psi_f \vec{P}_f m_f \left| j^A \right| \Psi_i \vec{P}_i m_i \right\rangle \left(\left\langle \Psi_f \vec{P}_f m_f \left| j^B \right| \Psi_i \vec{P}_i m_i \right\rangle \right)^*$$

$$R_{00} \propto \left|N^0\right|^2 \qquad R_{ZZ} \propto \left|N_Z\right|^2 \qquad R_{MM} \propto \left|N_{-1}\right|^2 \qquad R_{PP} \propto \left|N_{+1}\right|^2 \qquad R_{Z0} \propto \mathrm{Re}\left(N^0 (N_Z^*)\right)$$

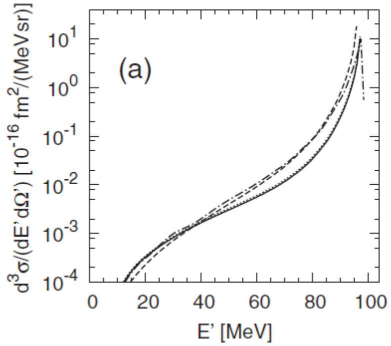
Examples for $\nu_e + {}^2H \rightarrow e + p + p$: (AV18, SNC+RC)

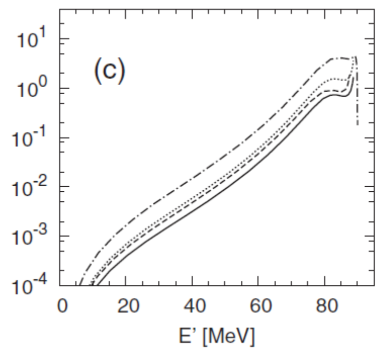


Reactions with (anti)neutrinos

- Differential cross section $\frac{d^3\sigma}{dE'\,d\Omega'}$
  Examples at:

E=100 MeV, θ'=27.5°   θ'=152.5°





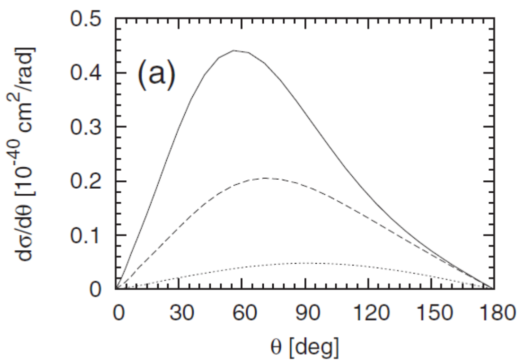
$$\bar{\nu}_e + {}^2H \rightarrow e^+ + n + n$$

$$\nu_e + {}^2H \rightarrow e^- + p + p$$

$$\bar{\nu}_e + {}^2H \rightarrow \bar{\nu}_e + p + n$$

$$\nu_e + {}^2H \rightarrow \nu_e + p + n$$

- Differential cross section $\frac{d\sigma}{d\theta'}$
  Example for
  $\bar{\nu}_e + {}^2H \rightarrow e^+ + n + n$



$E = 50\,\mathrm{MeV}$

$E = 100\,\mathrm{MeV}$

$E = 150\,\mathrm{MeV}$

- Integration over θ' reproduces σ_{tot} computed directly.

Extension to relativistic regime

A.Grassi et al., Phys. Rev. C107 (2023) 024617

New ingredients in 2N system:

- Relativistic kinematics

Nuclear states:

- Relativistic NN potential in the 2N total momentum zero frame, obtained from the AV18 by H.Kamada & W.Glöckle method (Phys. Lett. B655 (2007) 119)
- „Boosted” NN potential for the nonzero momentum of the 2N system
- Construction of spin states with inclusion of Wigner rotations

→ Multiparticle eigenstates transform like the single-particle states with the mass m replaced by the eigenvalue of $M=M_0+V$ (Bakamjian-Thomas construction; Dirac's instant-form dynamics)

Currents:

- treated using the one-boson exchange approximation
- We use SNC only

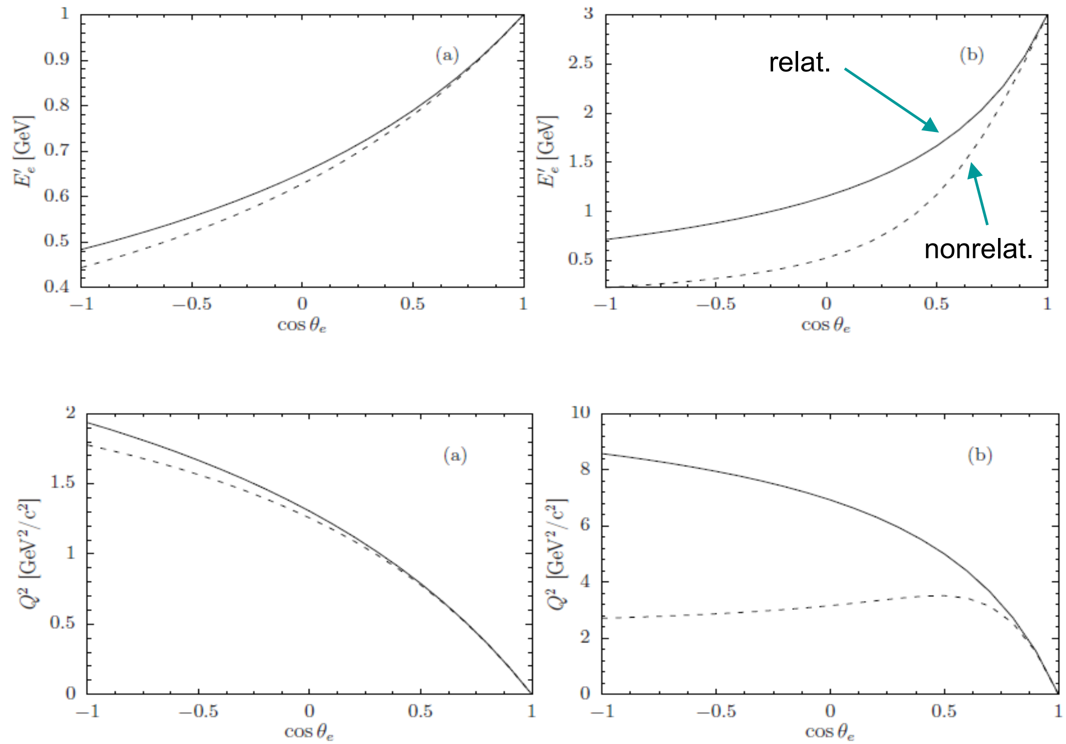
$$\langle \mathbf{p}', \mu', \tau' | J^{\mu}_{k,\mathrm{EM}}(0) | \mathbf{p}, \mu, \tau \rangle = \delta_{\tau'\tau} \frac{1}{(2\pi)^3} \sqrt{\frac{m^2}{E(\mathbf{p}')E(\mathbf{p})}} \bar{u}(\mathbf{p}', \mu') \left(\gamma^{\mu} F_{1,\tau}(Q^2) + i \frac{(p'_\alpha - p_\alpha)\sigma^{\mu\alpha}}{2m} F_{2,\tau}(Q^2) \right) u(\mathbf{p}, \mu)$$

- Transition matrix is contraction of leptonic and nuclear matrix elements $g_{\mu\nu} J^{\mu}_{nucl} J^{\nu}_{e}$.

Extension to relativistic regime

Elastic electron scattering off the deuteron: kinematics

E_e = 1 GeV $\quad\quad$ E_e = 3 GeV



final electron energy

four-momentum transfer squared

For neutrino scattering off the deuteron:
relativistic effects in kinematics are negligible below E_ν=300 MeV,
e.g. maximal lepton energy is the same (up to <1%)

Extension to relativistic regime

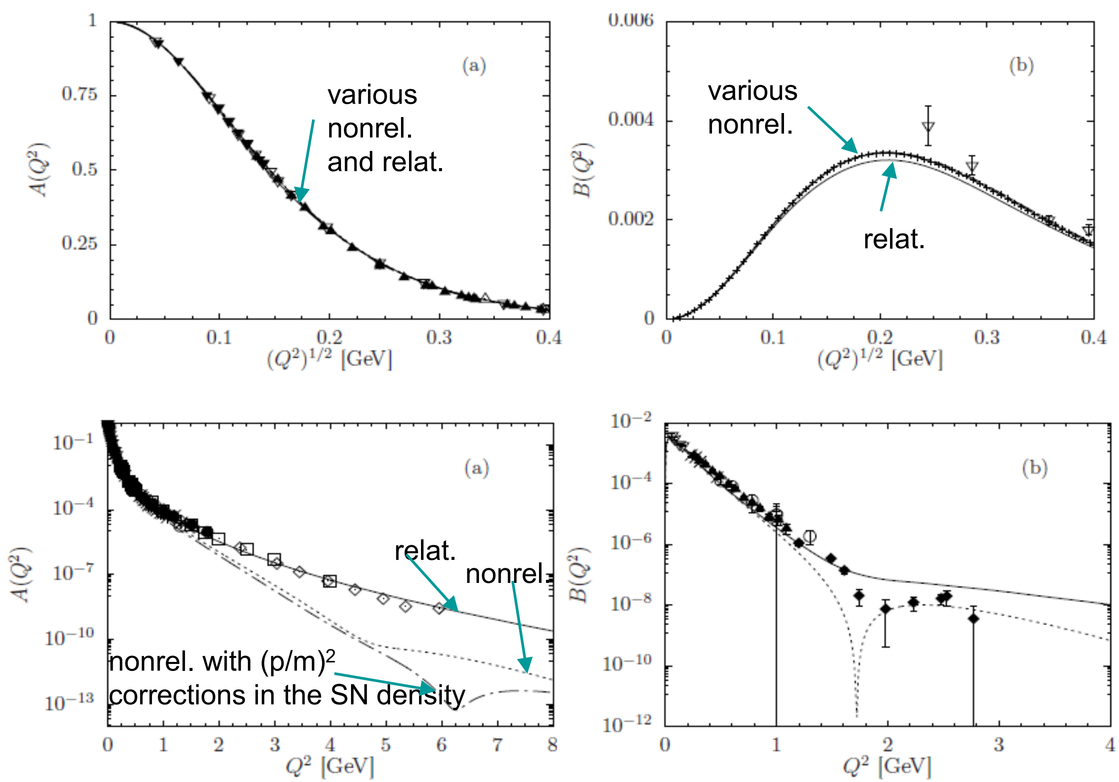
e+d→e'+d : unpolarized cross section: structure functions

E_e=3 GeV, various θ_e → various Q

$$\frac{d\sigma}{d\hat{\mathbf{p}}'_e}(Q^2,\theta_e) = \sigma_{\rm Mott}\left[A(Q^2) + B(Q^2)\tan^2\left(\frac{\theta_e}{2}\right)\right]\frac{|\mathbf{p}'_e|}{|\mathbf{p}_e|}$$

$$A(Q^2) = \frac{E_{\mathbf{p}'}}{M_d}\frac{1}{3}\sum_{\mu'_d\mu_d}\left(v_L R_L + \frac{1}{2}\frac{Q^2}{q^2}R_T\right)$$

$$B(Q^2) = \frac{E_{\mathbf{p}'}}{M_d}\frac{1}{3}\sum_{\mu'_d\mu_d} R_T\,.$$



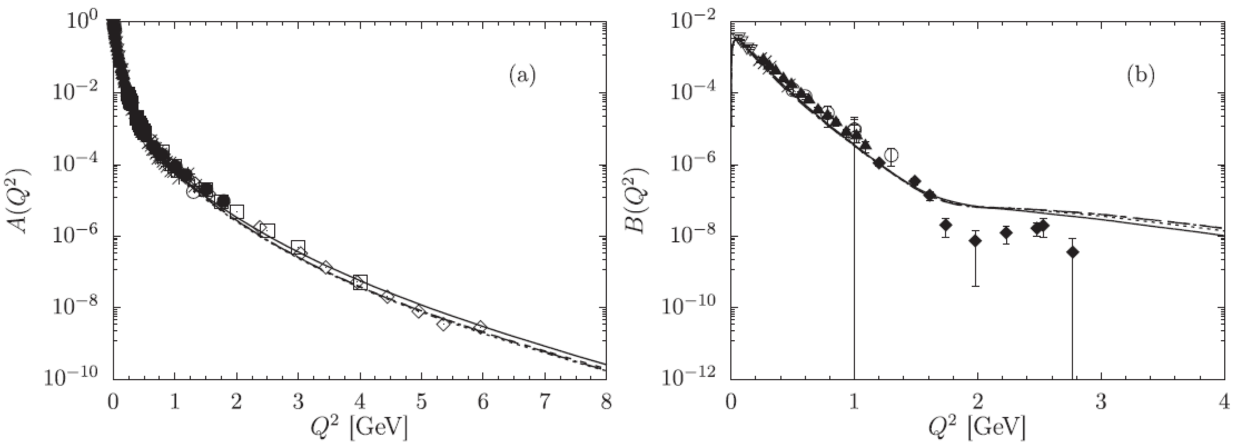
Q<0.4 GeV

Q^2<4-8 GeV^2

→ 2N currents required for Q^2>1 GeV^2

Extension to relativistic regime

- Dependence on the model of the nucleon EM form factors



dash-dotted: H. Budd, A. Bodek, and J. Arrington, arXiv:hep-ex/0308005v2 (2003)
dashed: J. J. Kelly, Phys. Rev. C **70**, 068202 (2004)
dotted: E. L. Lomon, Phys. Rev. C **66**, 045501 (2002)

solid: G. Shen, L. E. Marcucci, J. Carlson, S. Gandolfi, and R. Schiavilla, Phys. Rev. C **86**, 035503 (2012)

dipole parametrization of FF sticks out

- Effects off the Wigner spin rotations are negligible.

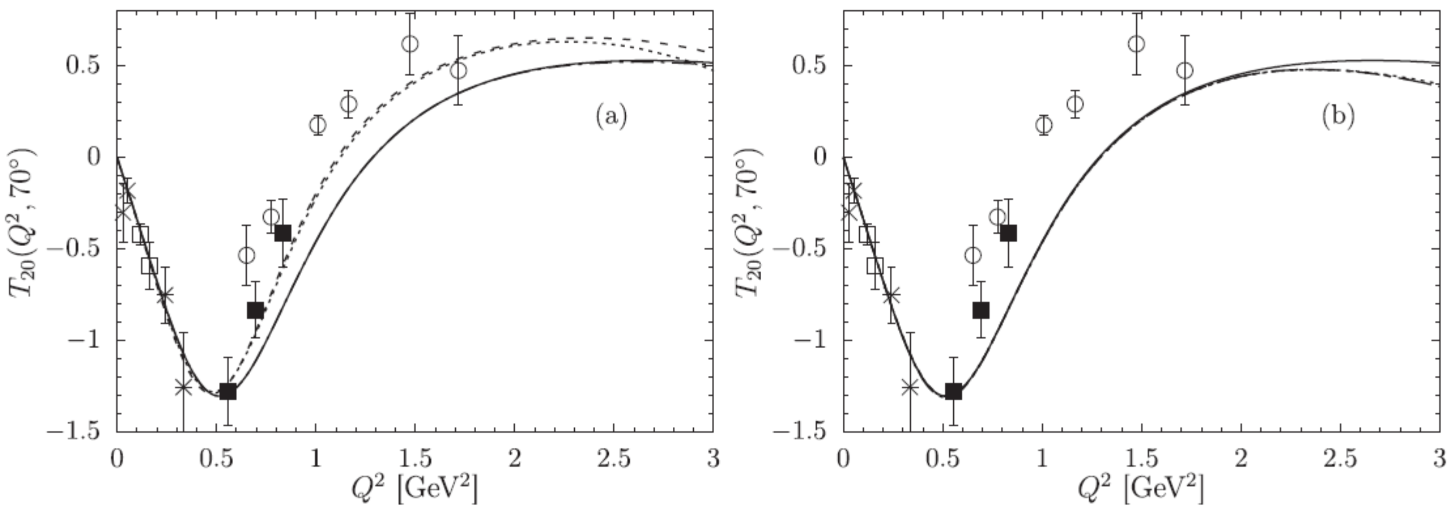
Extension to relativistic regime

Deuteron tensor analyzing power
θ'_e=70^o, various E_e

$$T_{20}(Q^2,\theta') = \sqrt{2}\,\frac{\sum_{d'} \frac{d^2\sigma_{dd'}}{d\hat{\mathbf{k}}'}\Big|_{\mu_d=1} - \sum_{d'} \frac{d^2\sigma_{dd'}}{d\hat{\mathbf{k}}'}\Big|_{\mu_d=0}}{\frac{d^2\sigma}{d\hat{\mathbf{k}}'}}$$

dashed: nonrelat. SNC
dotted: nonrelat. SNC+RC
dashed-dotted relat. without Wigner rotations
solid: relat. calculations

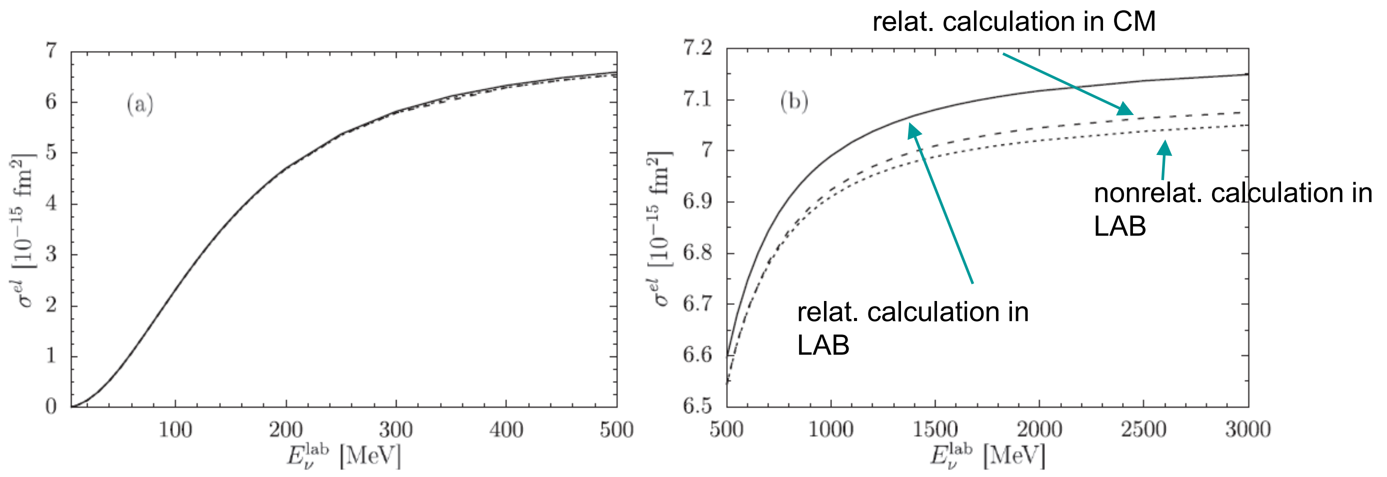
various models of the nucleon
EM form factors



Extension to relativistic regime

Elastic neutrino (NC) scattering off the deuteron: $\nu+d\rightarrow\nu+d$

Total elastic cross section



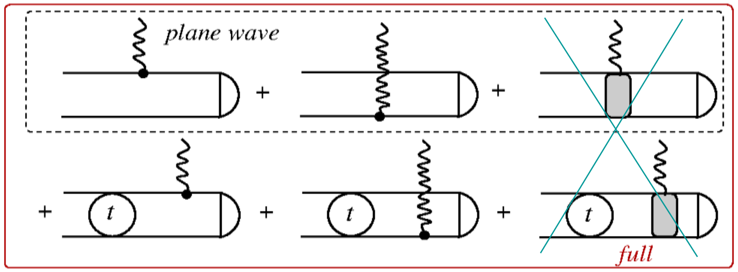
Lack of covariance in the nuclear current matrix element !
The effect amounts to approx. 1.5 % at 3 GeV

For elastic neutrino (NC) scattering off **the proton**: $\nu+p\rightarrow\nu+p$ we reproduce the same value of σ^{tot} both in LAB and CM.

Extension to relativistic regime

Inelastic electron scattering off the deuteron: *e + d → e' + p + n*

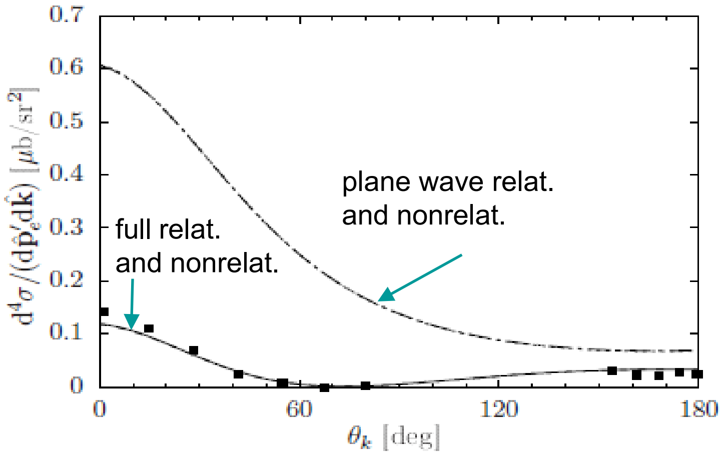
Exclusive cross section: $\frac{d^5\sigma}{d\hat{\mathbf{p}}'_e d|\mathbf{p}'_e| d\hat{\mathbf{p}}'_1}$



For nucleon

$$\langle \mathbf{p}'_1 \mu'_1 \tau'_1 \mathbf{p}'_2 \mu'_2 \tau'_2 | j(1) | \phi_d \mu_d \mathbf{P}_i \rangle$$

$$\langle \mathbf{p}'_1 \mu'_1 \tau'_1 \mathbf{p}'_2 \mu'_2 \tau'_2 | t\, G_0\, j(1) | \phi_d \mu_d \mathbf{P}_i \rangle$$



Exp: P. Von Neumann-Cosel et al., PRL 88 (2002) 202304

$$\frac{d^4\sigma}{d\hat{p}'_e d\hat{k}} = \int_{E_e'^{\min}}^{E_e'^{\max}} dE'_e \frac{d^5\sigma}{dE'_e d\hat{p}'_e d\hat{k}}$$

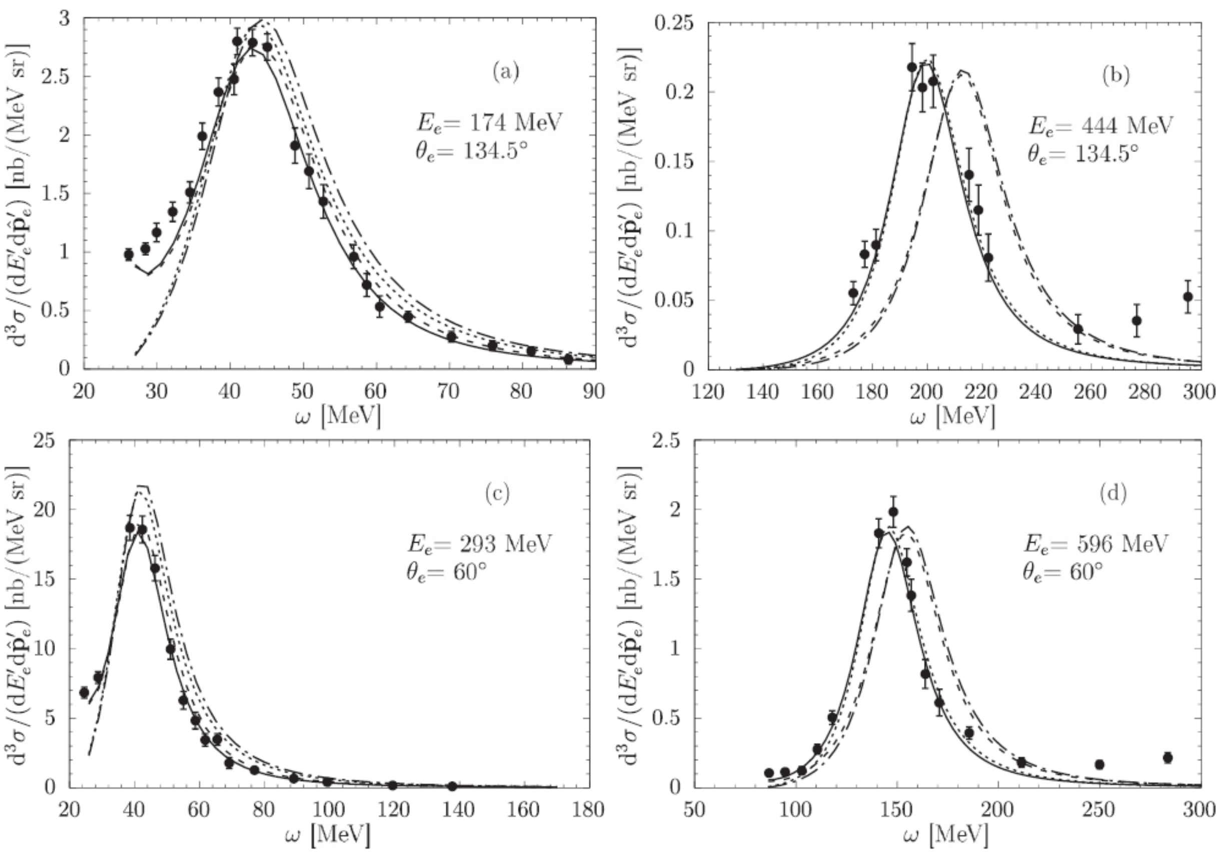
E=85 MeV, $\theta_e = 40^o$

Rescattering part is important (at least in that kinematics).

Extension to relativistic regime

Semi-inclusive cross section for the ^{2}H(e,e') reaction $\frac{d^3\sigma}{dE'_e\, d\hat{p}'_e}$ as a function of energy transfer ω (only electron in the final state is detected)

Exp: B.P.Quinn et al., Phys. Rev. C37 (1988) 1609



Nonrel PW

Nonrel FULL

Relat PW

Relat FULL

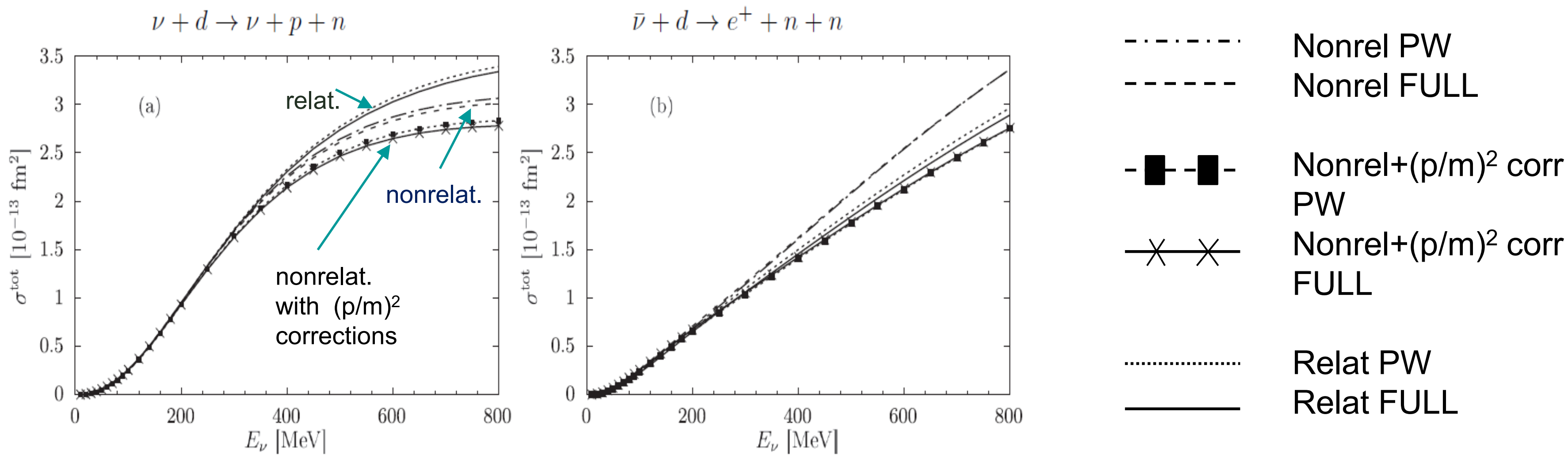
→rescattering part is important at low energy transfer

→at high energy transfer some dynamic is missing

Extension to relativistic regime

Inelastic neutrino NC and CC scattering off the deuteron

Total breakup cross section



→ small rescattering effects

Summary and outlook

1. Based on our experience from nucleon-deuteron scattering we built nonrelativistic and relativistic framework to describe electron and neutrino scattering off the deuteron under the one-boson-exchange approximation.
2. Lack of 2N currents suggests caution in its use for specific kinematics.
3. Our relativistic framework is formally applicable to calculations at energies below the pion production threshold and at arbitrary magnitudes of the three-momentum transfer.
4. We can deal with various kinematics (exclusive, semi-exclusive, inclusive) and make predictions for unpolarized cross sections, response functions and various polarization observables.
5. Various, extensive test of numerics have been successfully performed

1. Next step: include vector and axial 2N-currents, hopefully from χEFT, consistent with NN and 3N interaction.
2. Together with using chiral forces and currents reliable uncertainty quantification is possible
3. Works on scattering off ^3H and ^3He are ongoing (nonrelativistic scheme already completed)

Summary and outlook

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4. We can deal with various kinematics (exclusive, semi-exclusive, inclusive) and make predictions for unpolarized cross sections, response functions and various polarization observables.
5. Various, extensive test of numerics have been successfully performed

1. Next step: include vector and axial 2N-currents, hopefully from χEFT, consistent with NN and 3N interaction.
2. Together with using chiral forces and currents reliable uncertainty quantification is possible
3. Works on scattering off ^{3}H and ^{3}He are ongoing (nonrelativistic scheme already completed)

Thank you for your attention!