Neutrinos, Nuclei and Quantum Computing & Information

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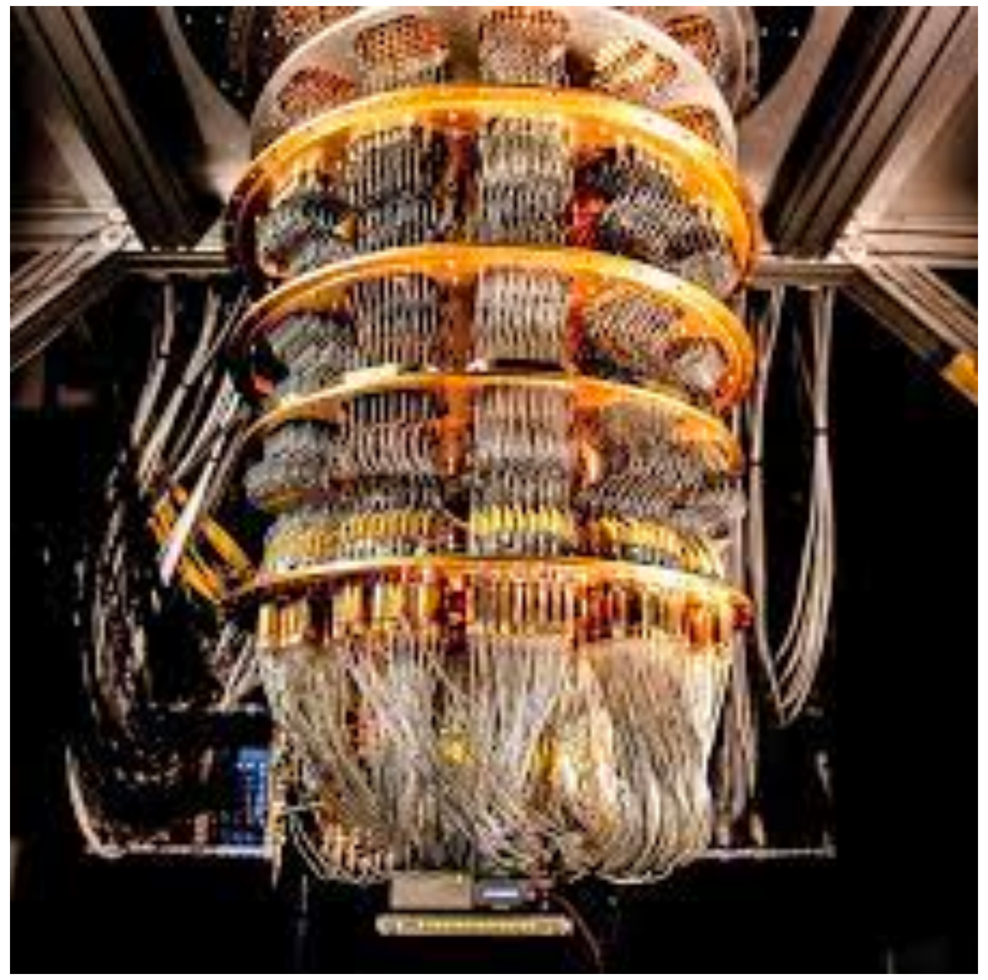
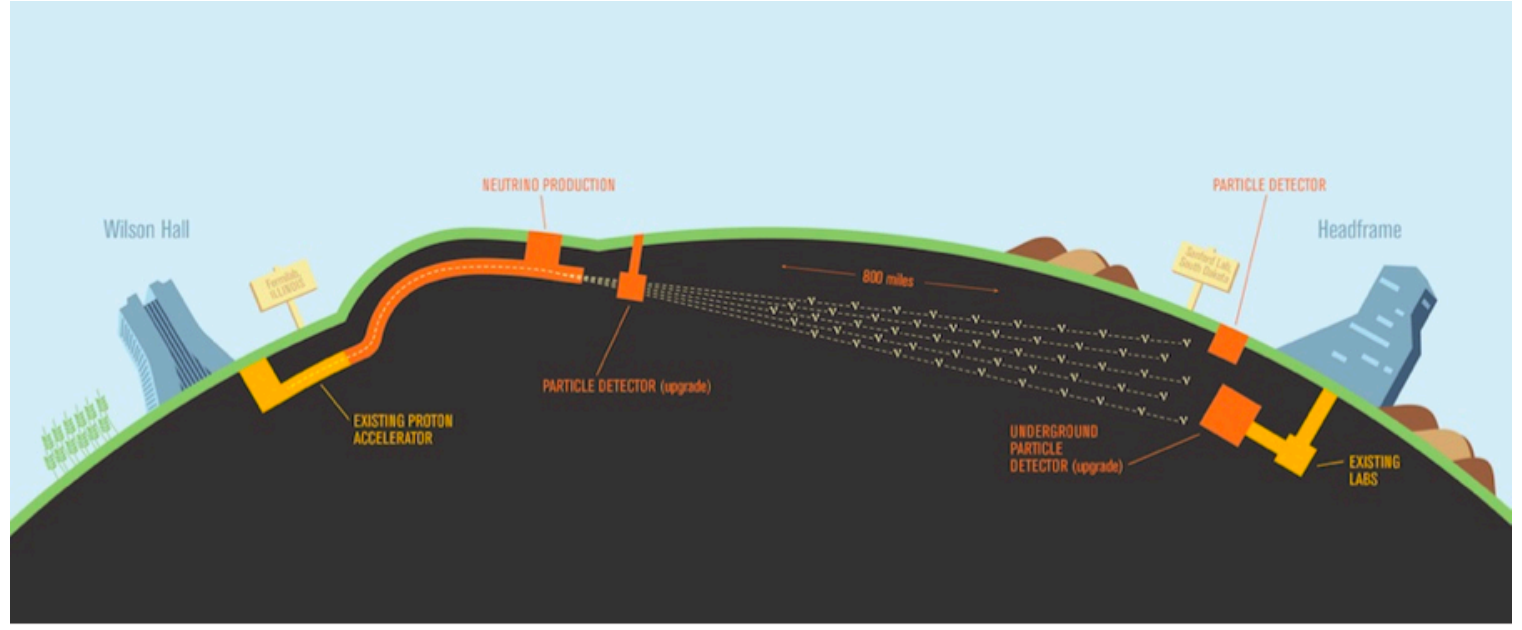
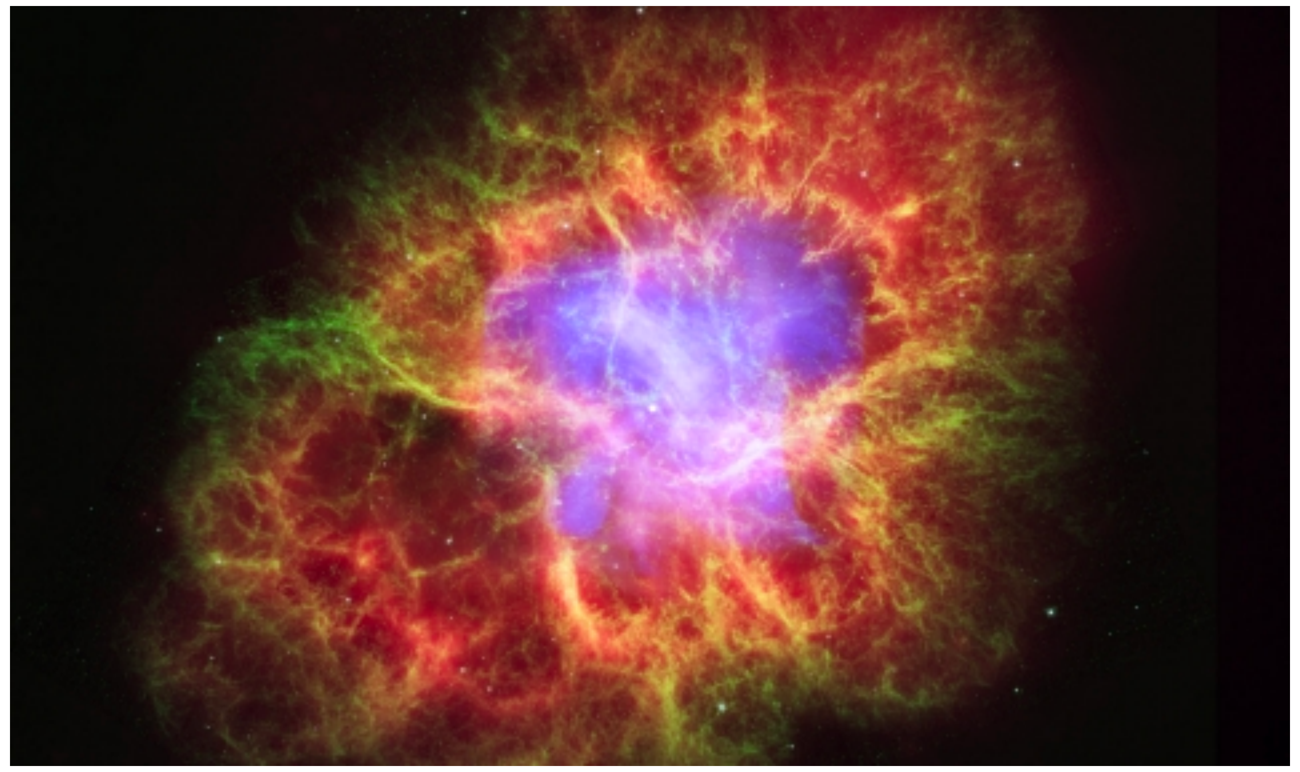
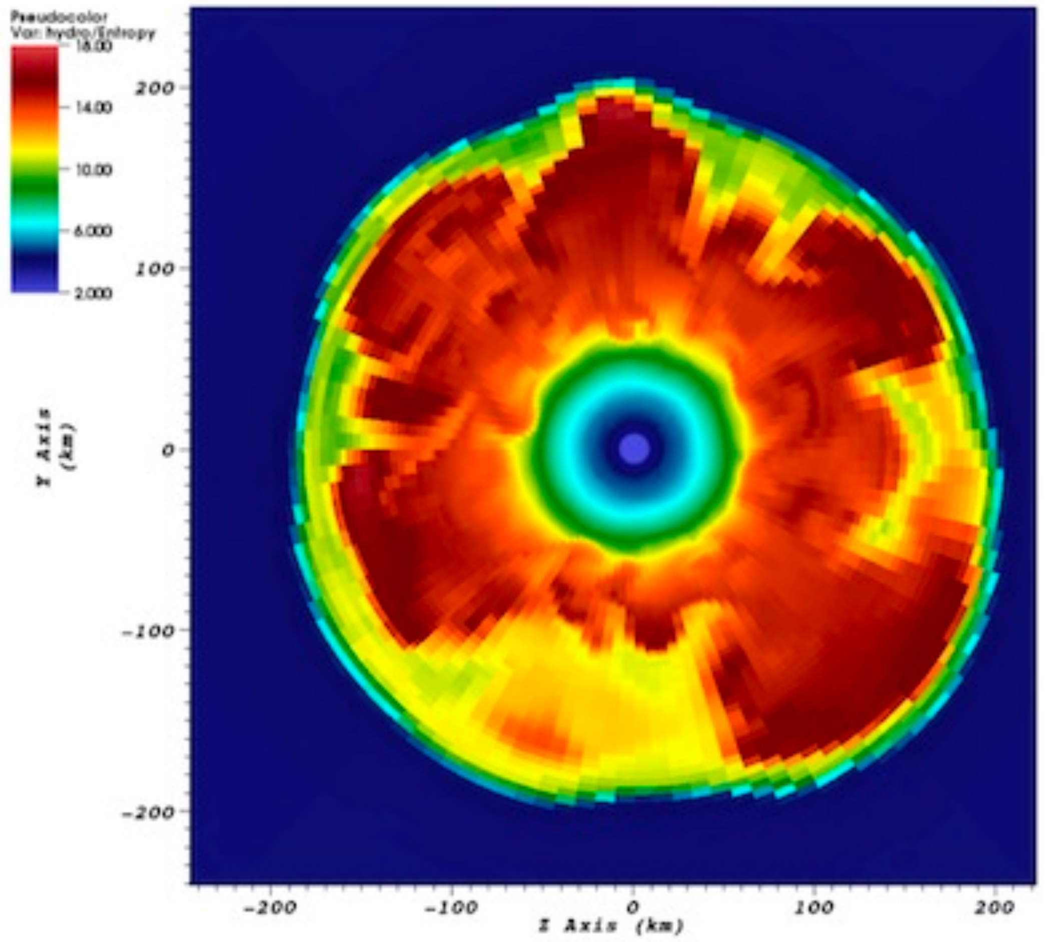
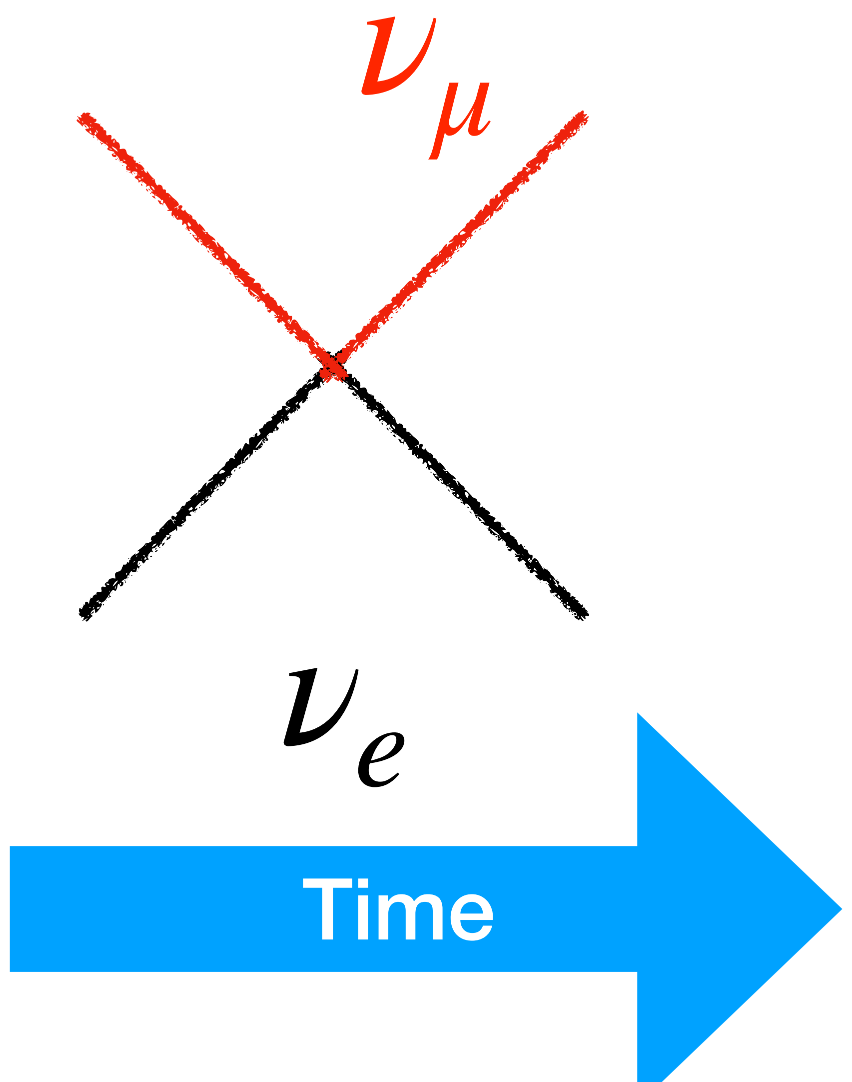


- **Brief Introduction to Quantum Computing**
- **Linear response: electron and neutrino scattering (algorithm)**
- **Ground state (or others): state preparation (algorithm)**
- **Neutrino-Neutrino Scattering: Supernovae and Neutron Star Mergers (physics!)**
- **Outlook**





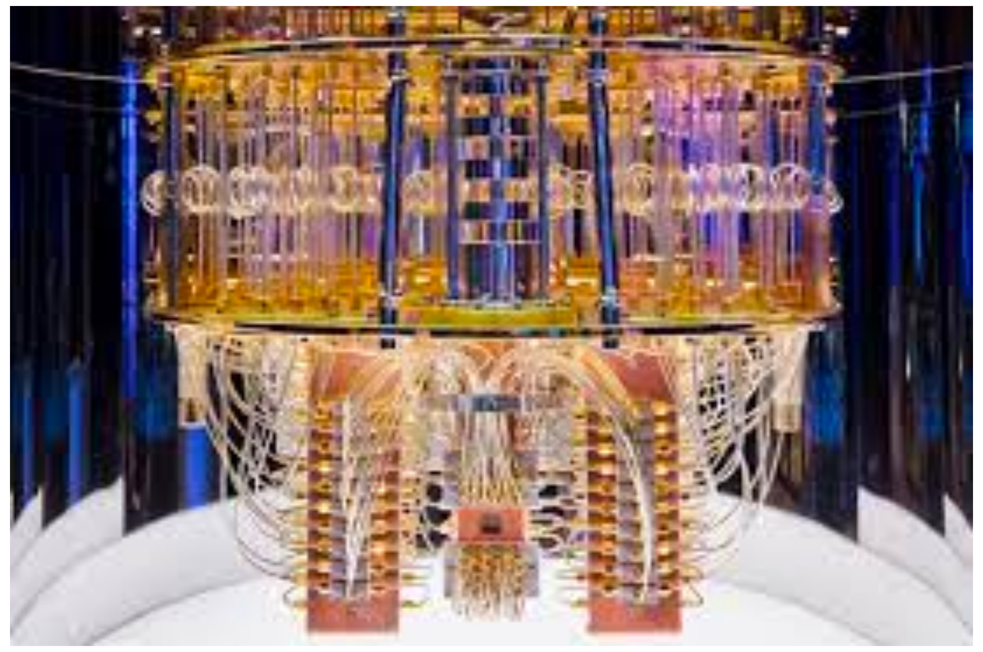




Why Quantum Computing?

- Very dense information storage:
  - N qbits store 2^N complex amplitudes
  - 40-50 qbits store more than largest conventional supercomputers

- Parallel operations:
  - 1 and 2 qbit gates are SU(2) rotations, Pauli operations
  - Well mapped to physical amplitudes (locality, 2-3 body operators,…)



- Various mappings (eg. Jordan-Wigner)
  - Hubbard or Heisenberg (lattice spin) models
  - Lattice Field Theories, Nuclear Many-Body, …

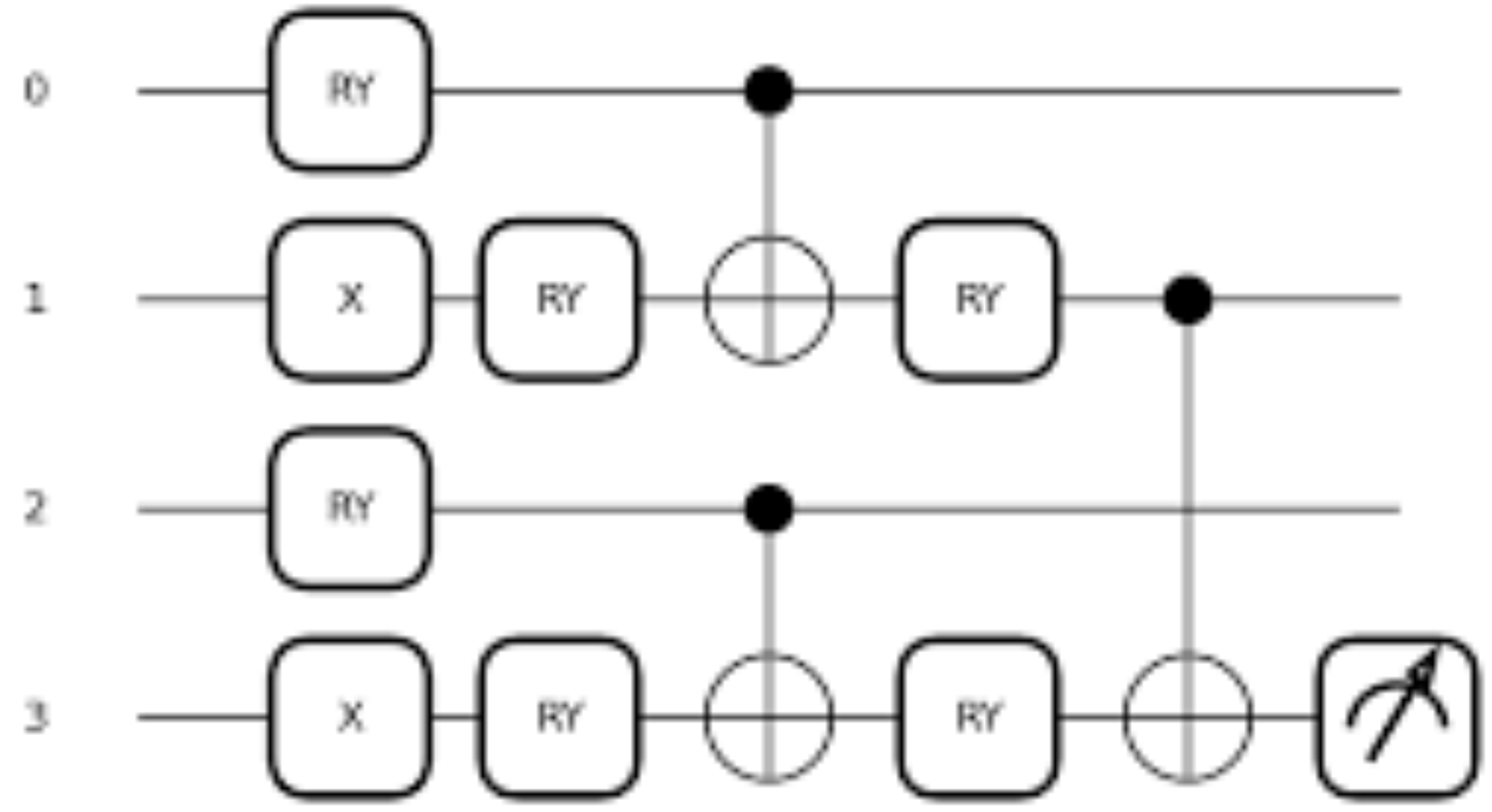
- Have to think a bit differently:
  - Bigger Hilbert space is ok
  - QC Designed for Unitary evolution: $\exp[-iHt]$
  - Measurements need to be carefully thought out

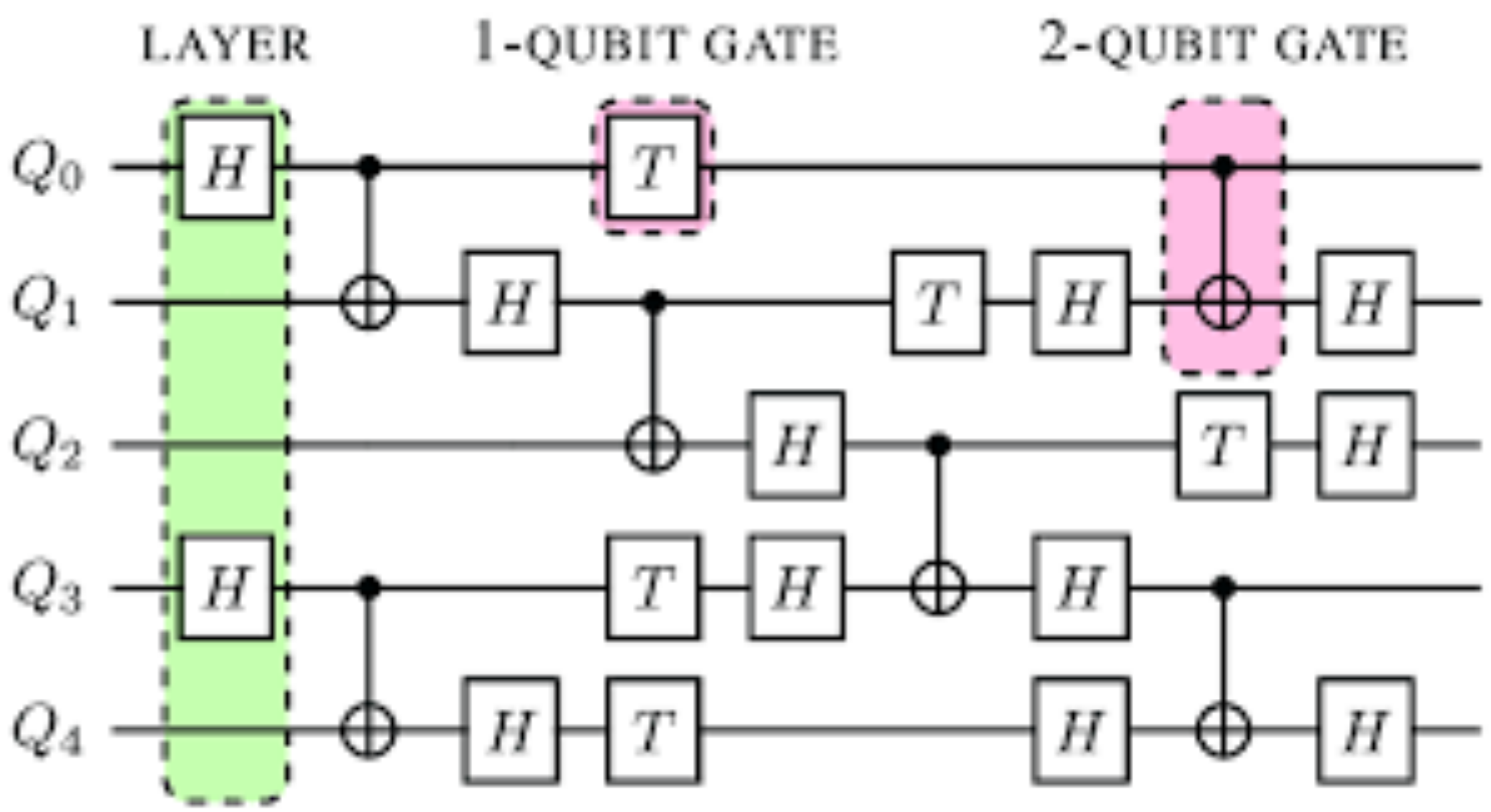
- Hardware in early stages but rapidly improving
  - Order 100 qbits (enough)
  - Error or noise ~ 0.1% (needs to improve by 3-4 orders of magnitude)

What is a Quantum Computer?
(Theorist's oversimplified view)

Set of N qbits: SU(2) spinors in e.g. up-down basis

- **Set up a simple initial simple (unentangled) state:**
  product of SU(2) spinors
- **Ordered set of gates operate on 1-2 qbits at at a time**
  (can be parallel)
- **Unitary Evolution**
- **Measurement of some or all qubits**





Linear Response on a Quantum Computer

A. Roggero & J. Carlson

Phys. Rev. C **100**, 03461 (2019)

$$S_O(\omega) = \sum_\nu |\langle \psi_\nu | \hat{O} | \psi_0 \rangle|^2 \delta(E_\nu - E_0 - \omega)$$

$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \overline{\sum_{M_i}} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q})\, e^{-iHt}\, O_\alpha(\mathbf{q}) | \Psi_i \rangle.$$

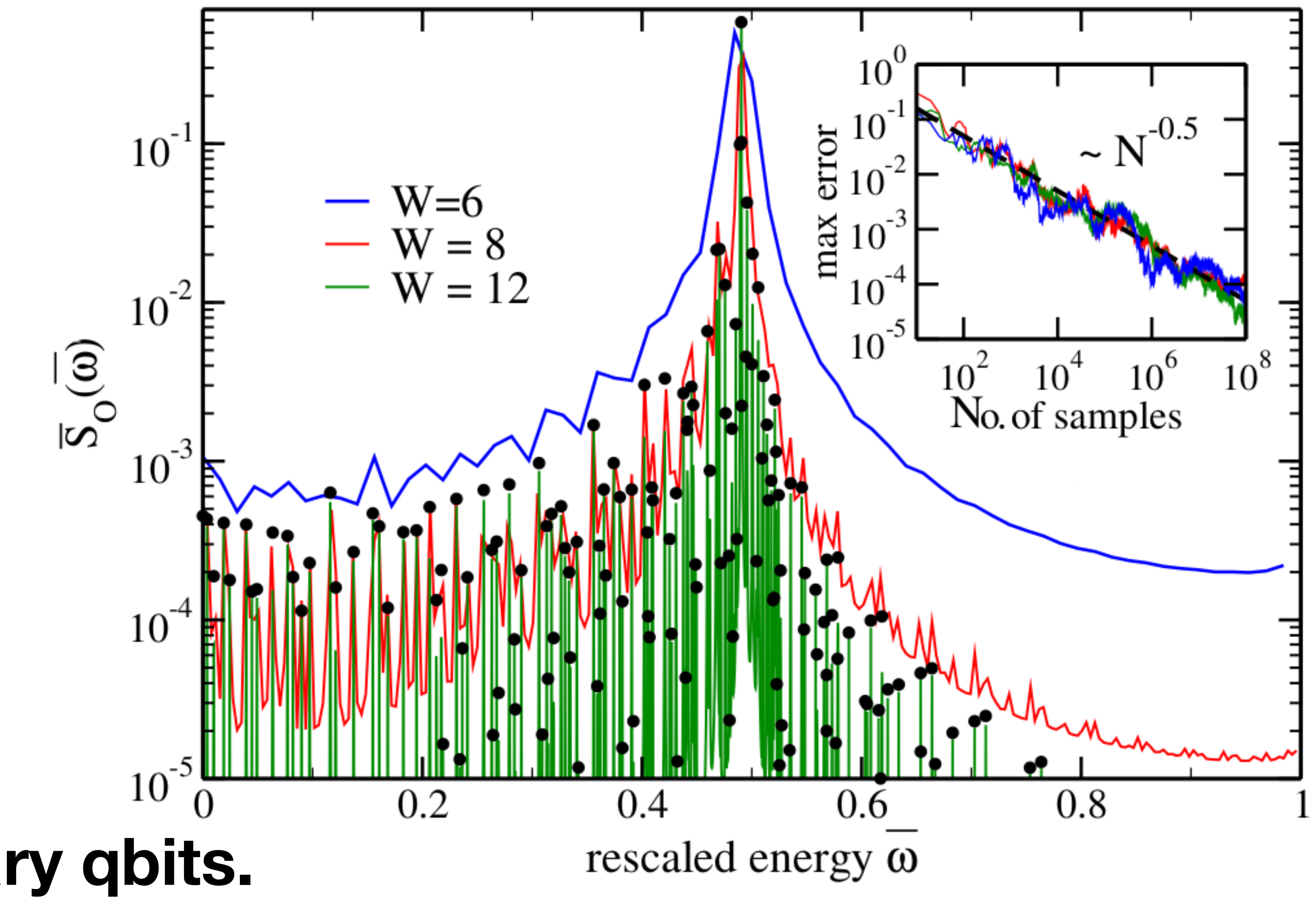
Prepare Initial State: Variational state, …

Need Initial state: variational or projection (see below)

High Energy & smooth response -> limited time evolution is sufficient

Unitary operator representing the current on the initial state:

$$\hat{U}_S^\gamma = e^{-i\gamma \hat{O}\otimes\sigma_y} = \begin{pmatrix} \cos(\gamma\hat{O}) & -\sin(\gamma\hat{O}) \\ \sin(\gamma\hat{O}) & \cos(\gamma\hat{O}) \end{pmatrix}$$



Need to evolve for time t with controlled evolution and W auxiliary qbits.

Probability of measurement of auxiliary qbits

$$P(y) = \frac{1}{2^{2W}} \sum_\nu |\langle \psi_\nu | \Phi_O \rangle|^2 \frac{sin^2\left(2^W \pi \left(\lambda_\nu - \frac{y}{2^W}\right)\right)}{sin^2\left(\pi\left(\lambda_\nu - \frac{y}{2^W}\right)\right)}$$

$$\equiv \frac{1}{2^W} \sum_\nu |\langle \psi_\nu | \Phi_O \rangle|^2 F_{2^W}\left(2\pi\left(\lambda_\nu - \frac{y}{2^W}\right)\right)$$

Only a few (~10) extra qubits gives and excellent reproduction of the response

Resolution improve exponentially with the number of ancillary qubits

State Preparation on a Quantum Computer

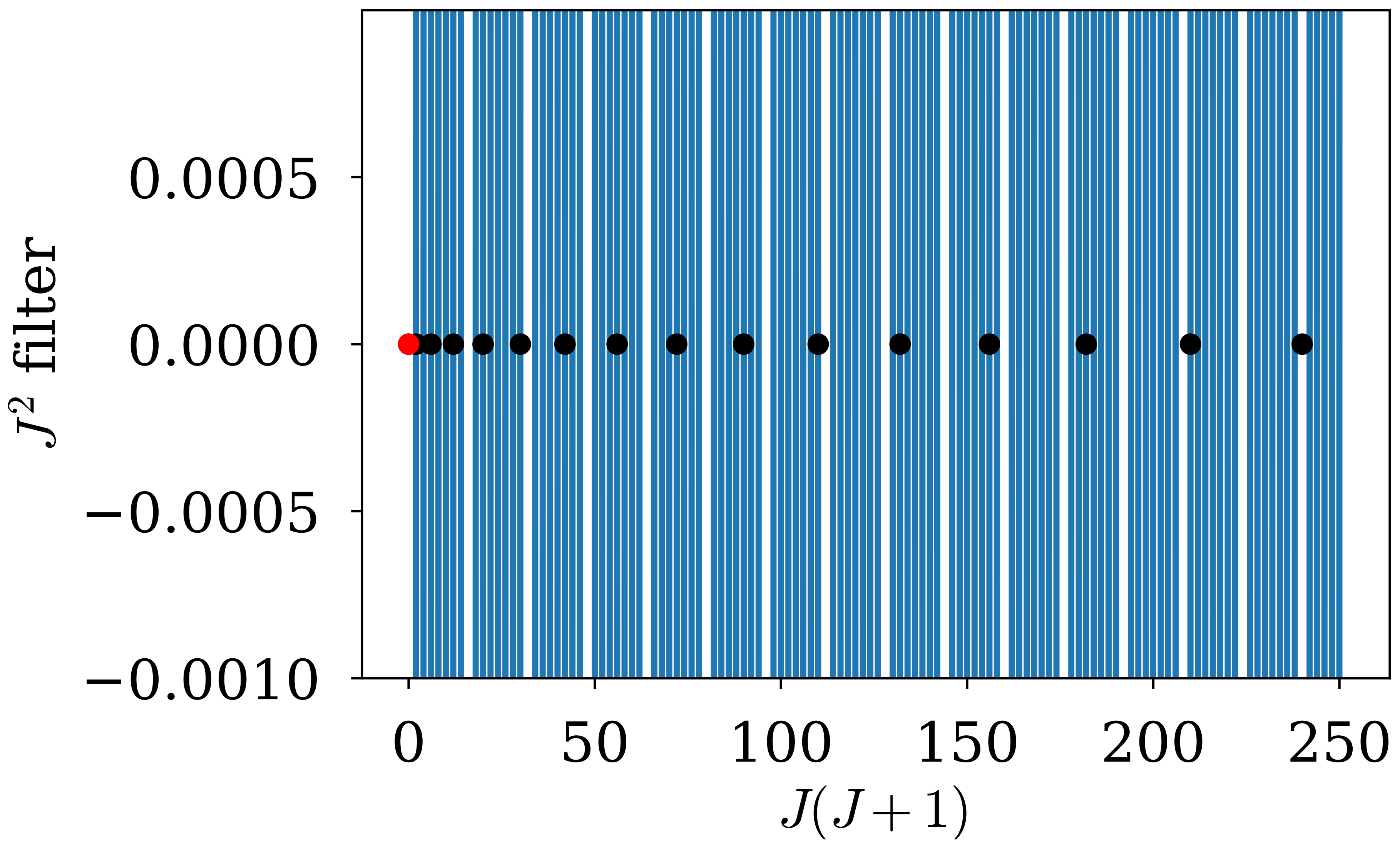
Goal: Project an Eigenstate w/ Good <H>,<J>,…
From an initial variational state

Algorithm:

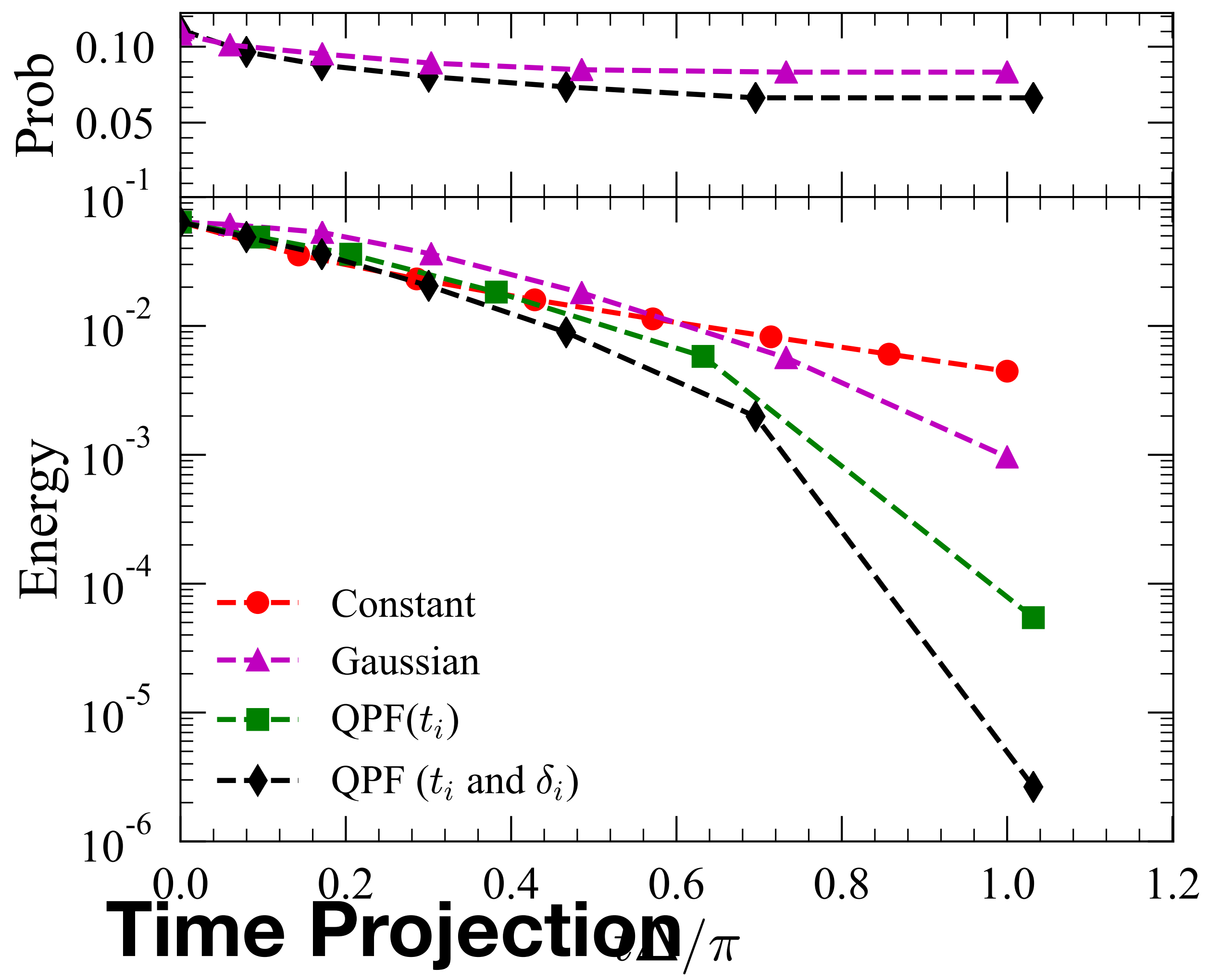
1. **Project on Quantum Numbers**

   Known Eigenspectrum, Zeros exponentially
   growing number of states with each measurement
   Increases Gap to Lowest Excitation

2. **Measure 'Response' of trial state**
3. **Calculate Optimum Times and Phases**
4. **Project to Ground State**



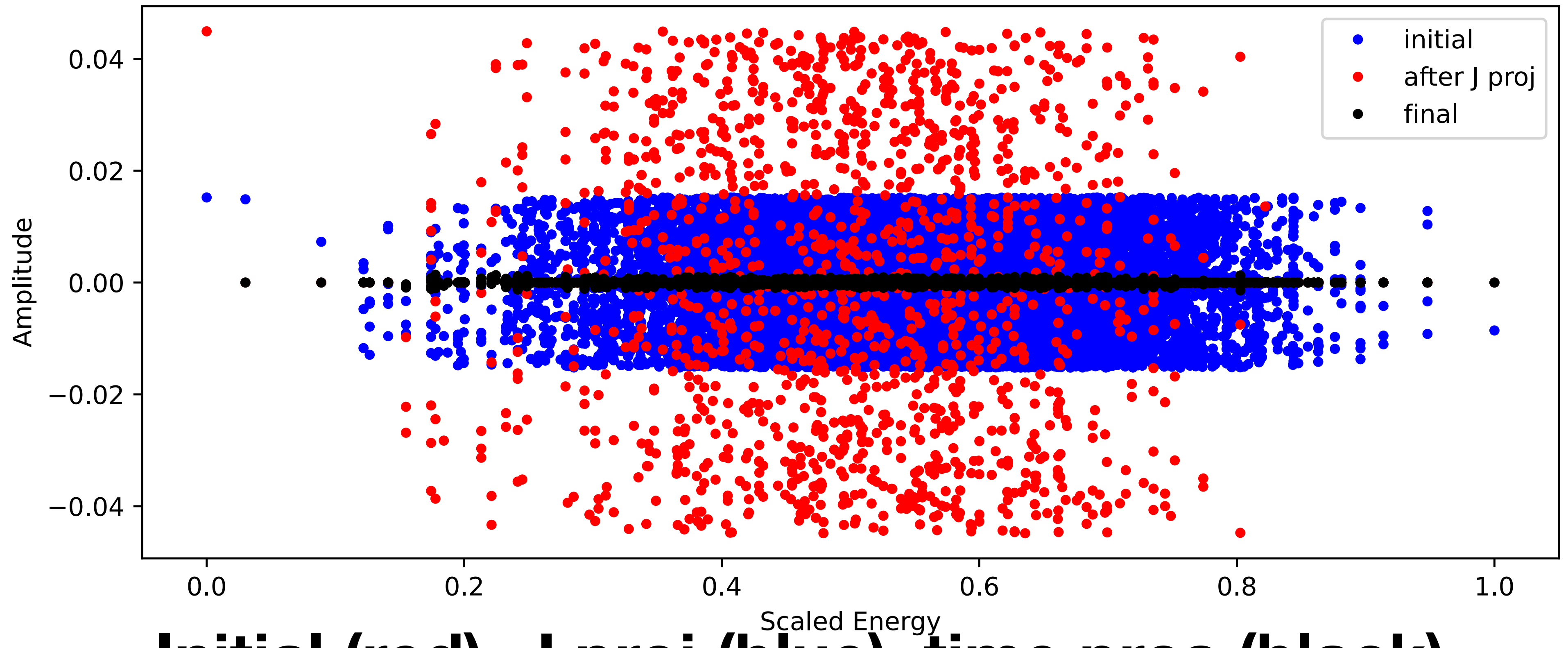
Projecting J^2 quantum number



Time Projection

$$\text{Time} \propto \frac{\pi}{\Delta^\star} \text{ (same as adiabatic)}$$

Works for arbitrary Quantum numbers, E

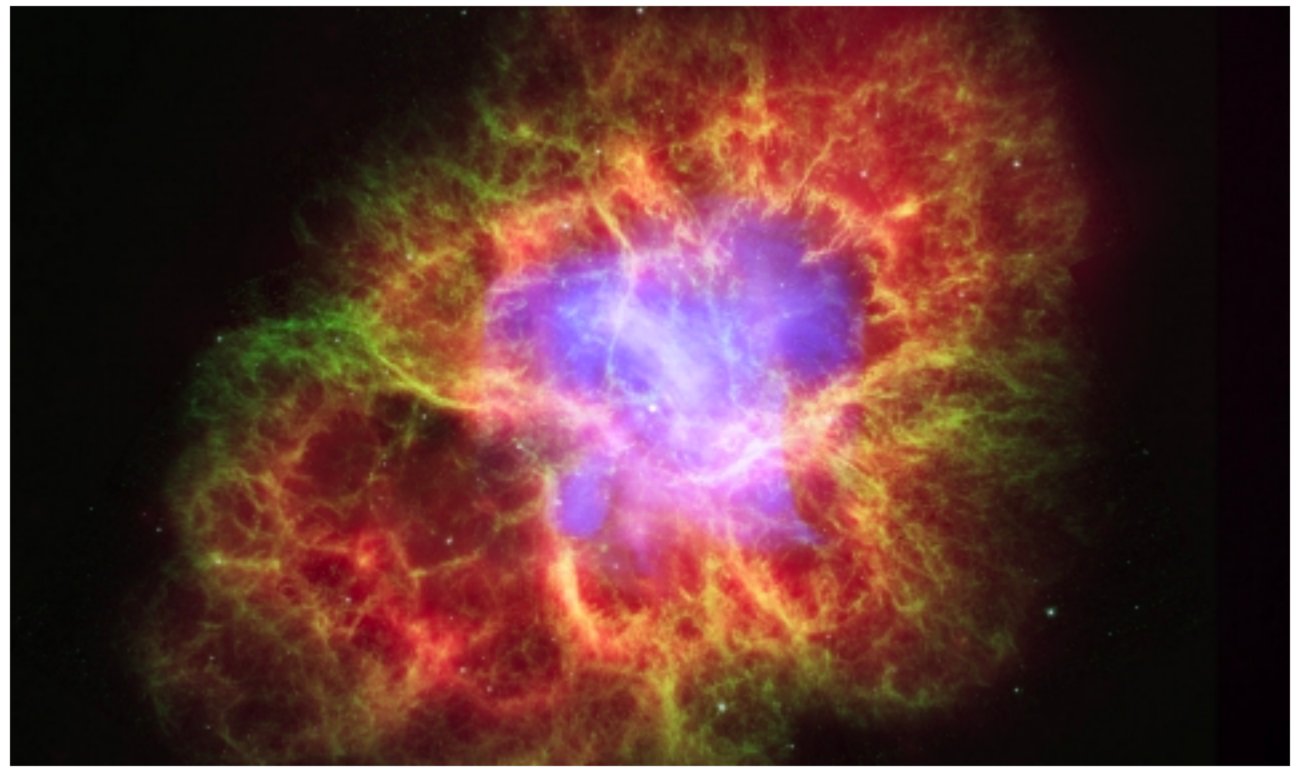


Initial (red), J proj (blue), time pros (black)

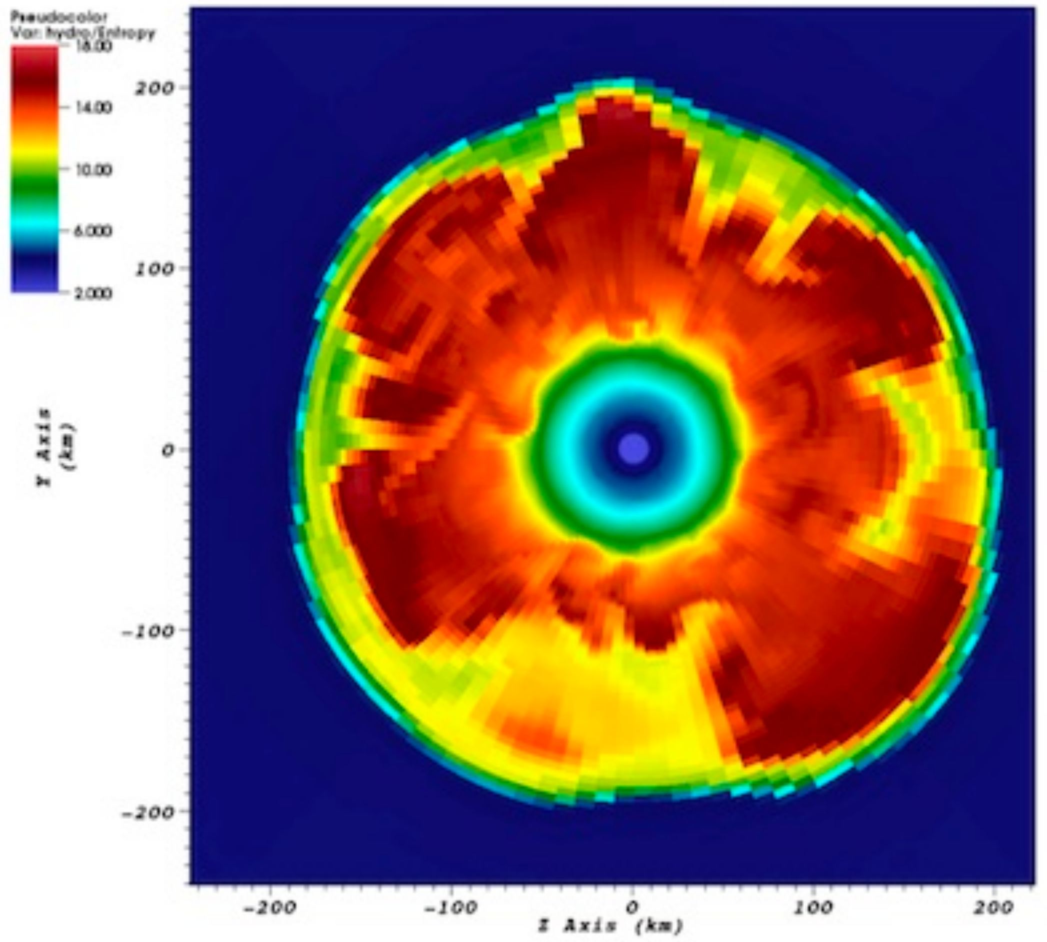
Symmetries and scramblers in dense neutrino enviroments

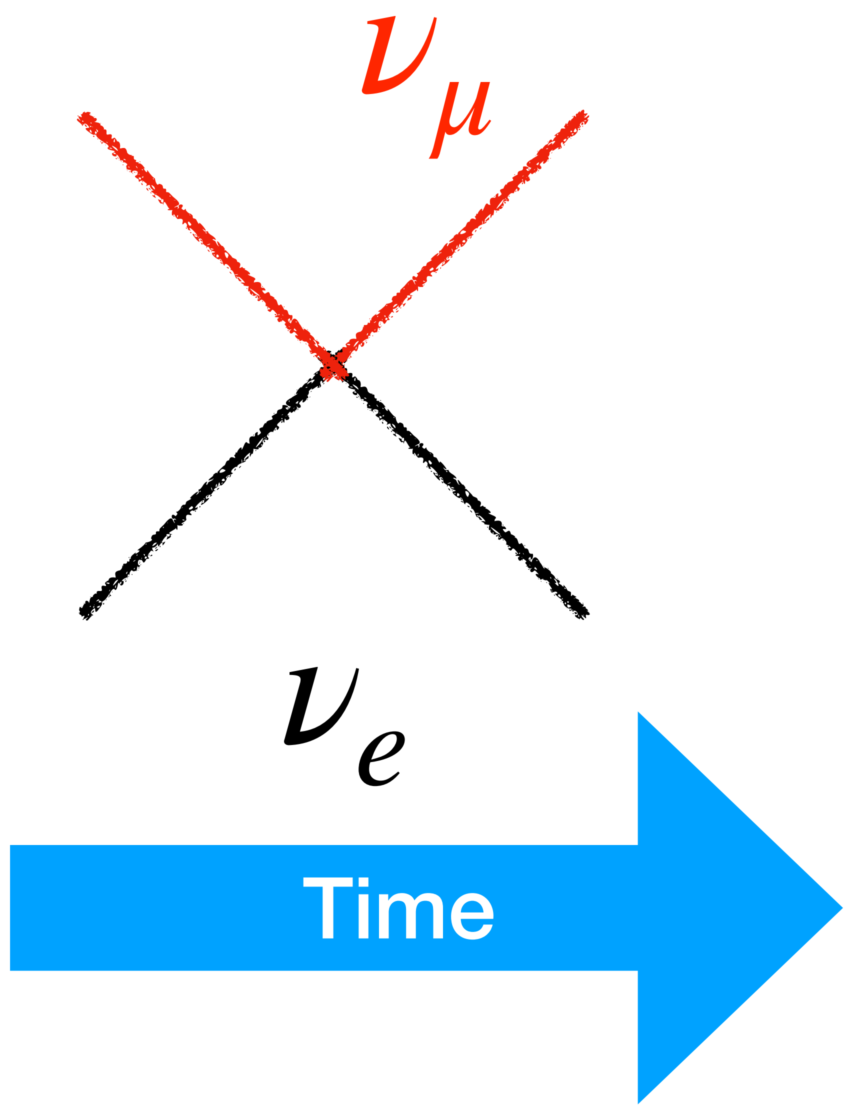
Many-body quantum mechanics of neutrinos
in regimes where $\nu - \nu$ interactions can be important.

With huge flux, $\nu - \nu$ interactions can potentially be important!
We can derive Hamiltonian describing the system
We have at least a rough idea of some relevant initial conditions
Can we do time evolution of enough neutrinos to be useful?
How to approach modeling quantum dynamics?



Fujibayashi et al (2020)



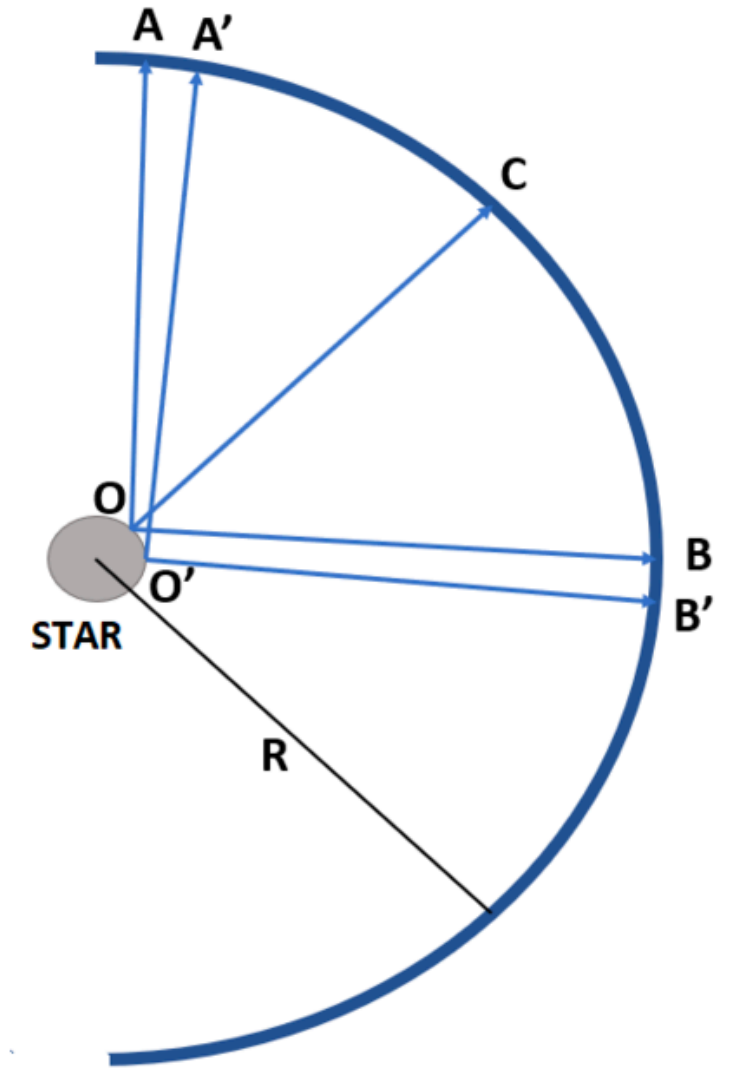


Many-body quantum mechanics of neutrinos
(JC, A. Roggero, Duff Neill, Josh Martin)

- **Simplified Hamiltonian relevant to SN, NS-NS mergers: Integrable vs. non-Integrable Hamiltonians**
- **Initial States**
- **Equilibration (thermodynamics) from real-time evolution**
  - **Eigenstate Thermalization Hypothesis (ETH)**
  - **Path Integral Representation of Dynamics**
- **Analysis of equilibration/time scales for larger N**
- **'Classical' Picture: Neutrinos and Anti-Neutrinos**

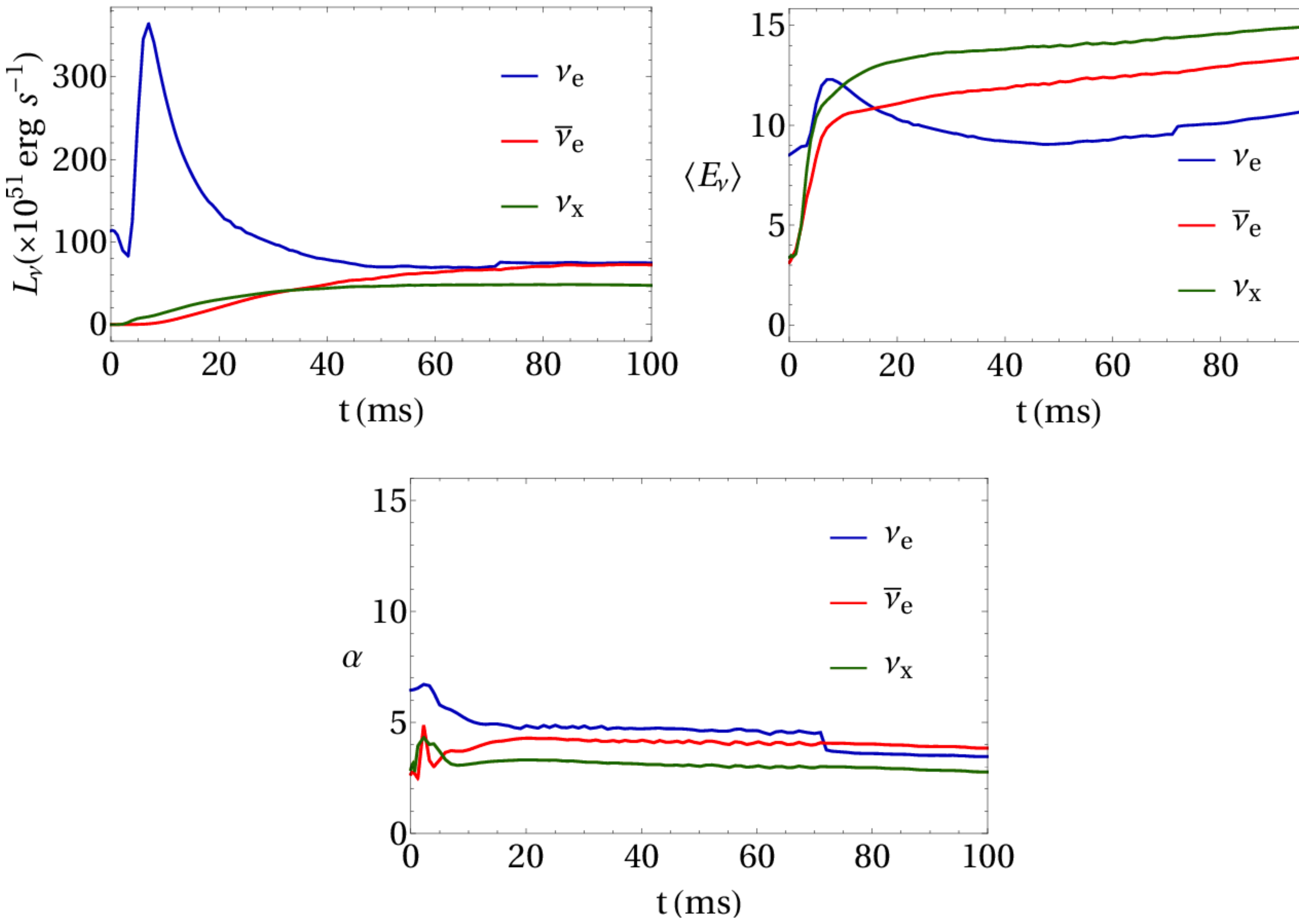
$$H_{\nu-\nu} = \frac{\sqrt{2} G_F \rho_\nu}{N} \sum_{i<j} (1 - \hat{k}_i \cdot \hat{k}_j)\ \sigma_i \cdot \sigma_j$$

arXiv: 2307.16793



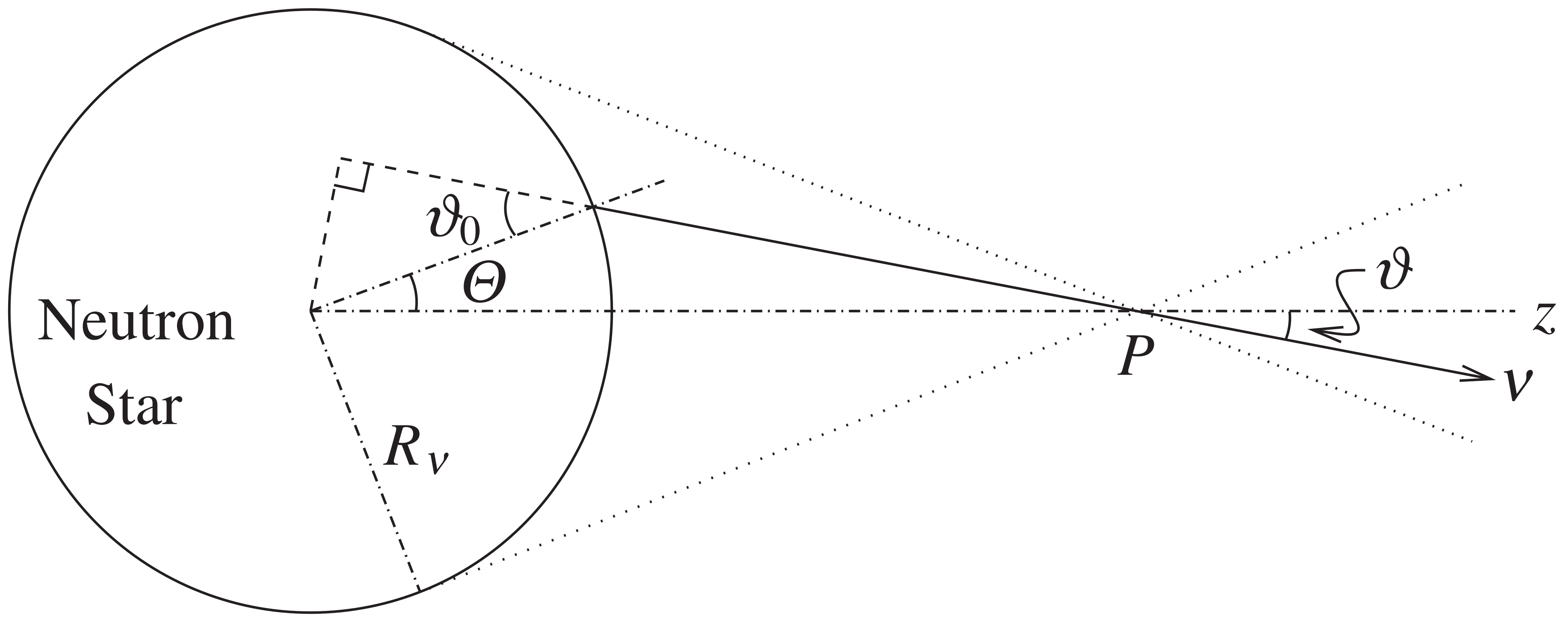
Neutrino Bulb model (Wikipedia)

Neutrino fluxes during CCS neutronization burst



Hamiltonian setup for dense neutrino environment

See Panteleone, Sigl, Raffelt, Bell, Fuller, Balantekin, Sawyer, Friedland, Lunardini, McKellar, Cirigliano,…



$$H = H_{vacuum} + H_{MSW} + H_{\nu-\nu}$$

Vacuum Oscillations: neutrinos oscillate between flavors in the vacuum : SU(2) example

$$H_{vacuum} = -\frac{\delta m^2}{2E_i}\begin{pmatrix} -\cos(2\theta) & sin(2\theta) \\ sin(2\theta) & \cos(2\theta) \end{pmatrix}$$

In matter, forward scattering on ‘ordinary’ matter introduces a potential for electron neutrinos (MSW)

$$H_{MSW} = \begin{pmatrix} \sqrt(2)G_F\rho_e & 0 \\ 0 & 0 \end{pmatrix}$$

Neutrino-Neutrino interaction ↔ Quantum Spin (here SU(2)) Hamiltonian

$$H_{\nu-\nu} = \frac{\sqrt{2} G_F \rho_\nu}{N} \sum_{i<j} (1 - \hat{k}_i \cdot \hat{k}_j)\ \sigma_i \cdot \sigma_j$$

- k_i, k_j represent neutrino directions
- ρ_ν is neutrino density
- N is total number of neutrinos
- σ_i, σ_j are Pauli matrices representing neutrino flavor states

Note unusual factor of $\frac{\rho_\nu}{N}$ because of ‘box’ normalization for neutrino single-particle states

Spectra proportional to N like standard Hamiltonian

More neutrinos at fixed density $\rightarrow$ larger volume

Big mismatch between number of neutrinos we can calculate in a quantum many-body approach and real physical system

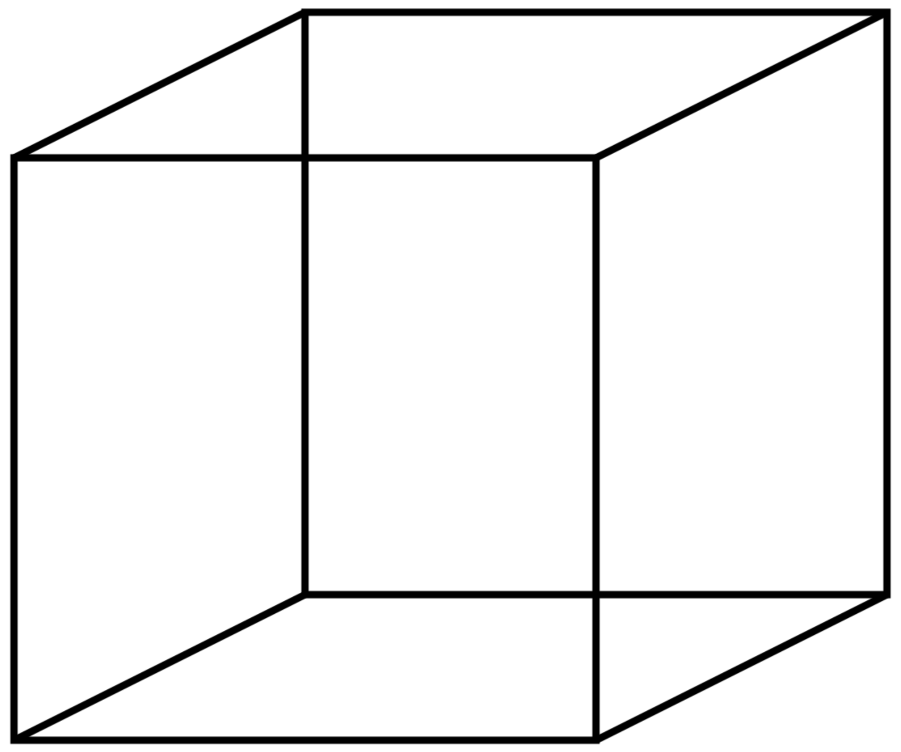
Note this is always true (neutron stars, cold atoms, liquid He,…)

Take luminosity *L* ~ 10^{53} erg/sec and an average energy E of 10 MeV at a radius of ~ 50 km gives $\rho_\nu \approx 6.6 \times 10^{-7} \mathrm{fm}^{-3}$

Number of quantum many-body states = 2^N

For N= 20 at this density the box length (cubic) is only L ~ 300 fm

L	N
300 fm	20
3000 fm	2000
3 Angstrom	2×10^{10}



More particles is only a partial answer

Density low compared to degeneracy, can treat as distinguishable spins

Convert part of dynamics to statistical mechanics (finite T)

Neutrino-Neutrino interaction ↔ Quantum Spin (here SU(2)) Hamiltonian

$$H_{\nu-\nu} = \frac{\mu}{N} \sum_{i<j} (1 - \hat{k}_i \cdot \hat{k}_j)\, \sigma_i \cdot \sigma_j$$

For conditions described previously $\mu \approx 1 cm^{-1}$

Initial Condition (product state): ~A complex amplitudes

$$\psi(0) = \prod_{i=1}^{N} \chi(i)$$

In general are determined by sampling from an energy, angle and flavor dependent distributions

Dynamics rapidly introduces many complex amplitudes

$$|\Psi(t)\rangle = \exp[-iHt]\,|\Psi(0)\rangle$$

$$\sigma_i \cdot \sigma_j = 2P_{ij} - 1$$ P_{ij} exchanges spins

mappings to quantum computer SU(2) spin ↔ qubit

Major difficulty on a QC is all to all coupling :Savage, Roggero, Hall, Illa, …

Integrability is a key component of the Hamiltonian determined by choice of directions/couplings k

In all cases considered J^2, J_x, J_y and J_z are symmetries as they commute with the Hamiltonian

- ***Integrable :*** *Many symmetries: eg. all equal couplings*

$$H = \sum_{i<j} \sigma_i \cdot \sigma_j$$

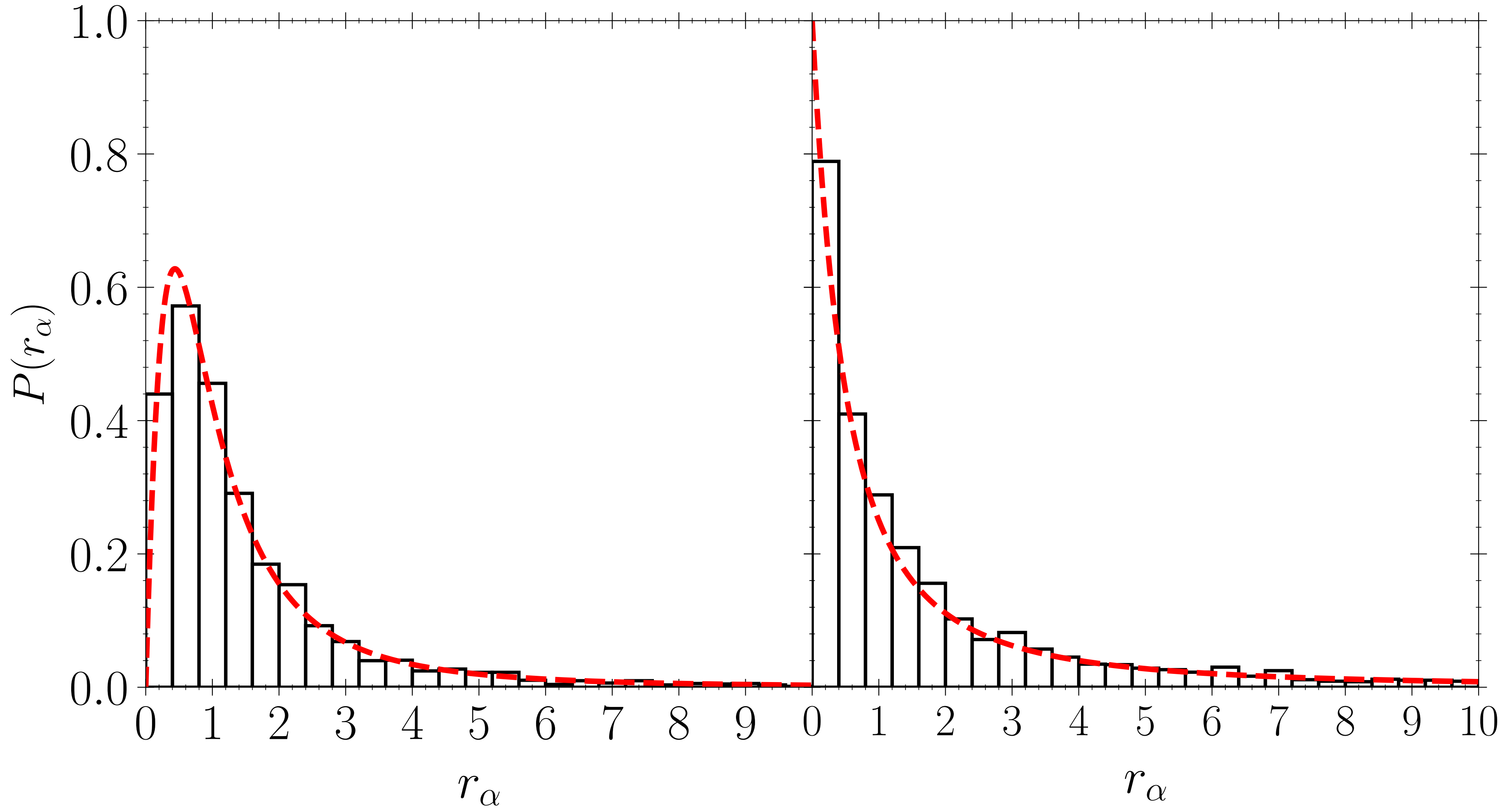
cases solved by Bethe Ansatz (see eg. Somma, NPB, 2005)

- ***Non-Integrable :*** *Almost all cases*
  Very few symmetries, level repulsion, …

Here we consider random unit vectors in a hemisphere

Only total angular momentum and projections are preserved, along with moments of H (time independent Hamiltonian)

Histograms of ratios of level spacings



Non-Integrable GOE

Integrable Poisson

Initial states assumed to be product states
Indepence of quantum phases from different source directions

$$|\Psi(t=0)\rangle = \prod_{i=1,N} |\chi(i)\rangle$$

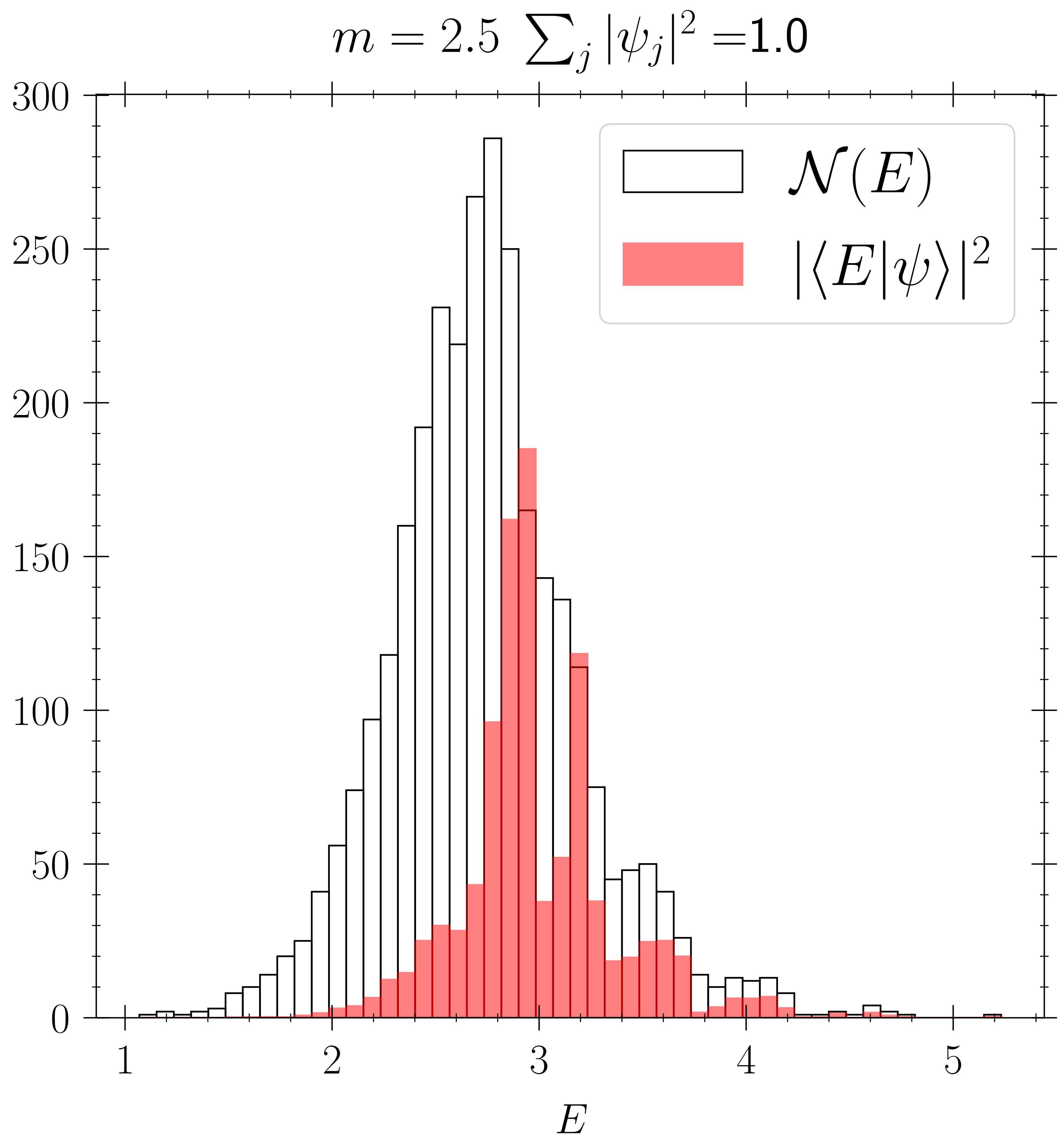
$$|\chi(i)\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} : |\alpha|^2 + |\beta|^2 = 1$$

No quantum phase orientation (coherence) between different neutrinos
Individual spins may be drawn from same classical distributions in flavor versus energy and angle

Can easily calculate expectation value s <H>, <H^2>, <J^2>, <Jx>, <Jy>, <Jz>
These are all symmetries and hence time independent quantities for a Hamiltonian constant in time

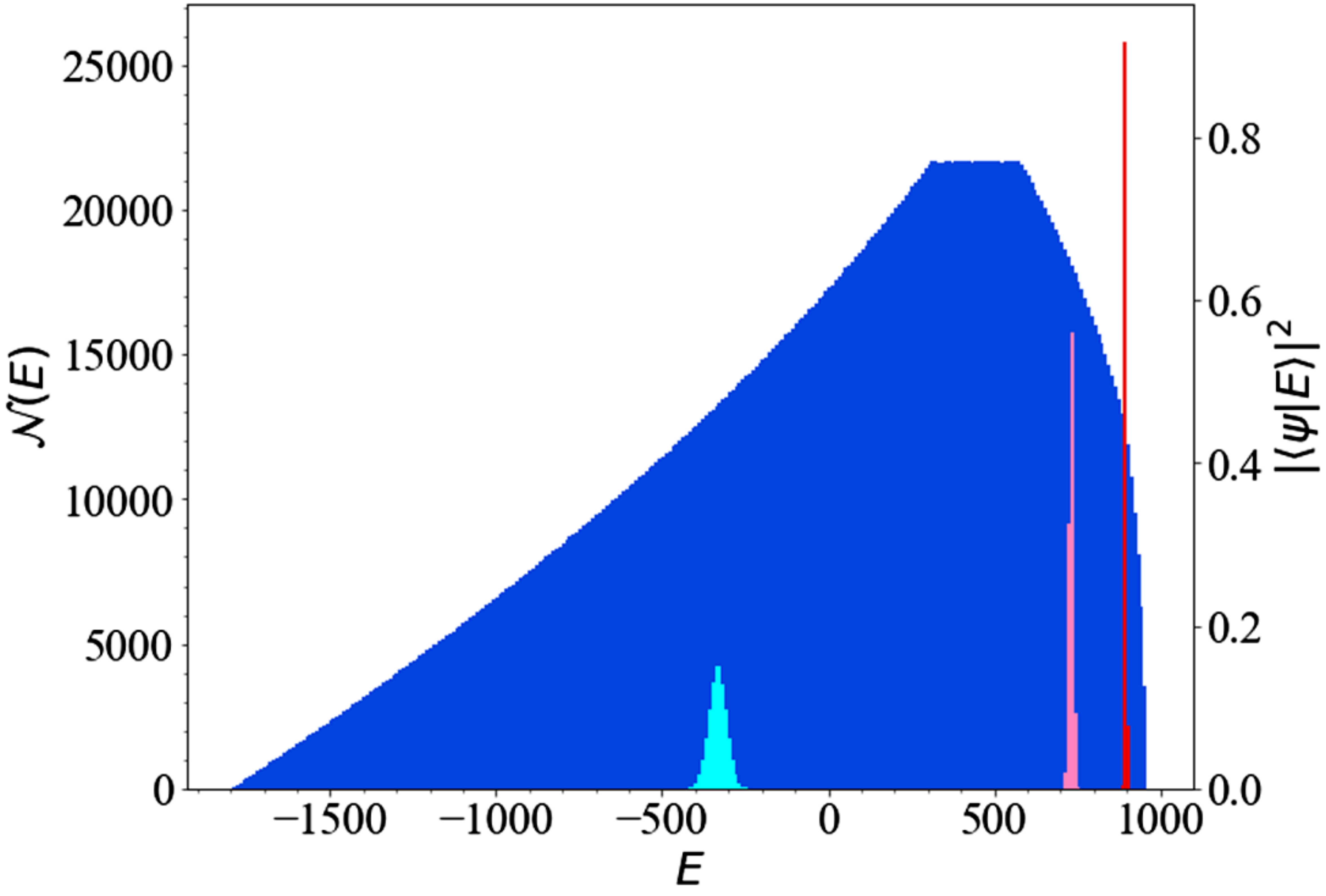
Density of States and Initial State Overlaps with product states

N=15
Non-integrable



Open rectangles: Full density of states
Filled rectangles: including overlap with product state

N=3600 (2 beams)
Integrable



PRD 105, 083020 (2022)
J. Martin et. al

Time dependence of observables and ~statistical observables

$$|\Psi(t)\rangle = \exp[-iHt]\,|\Psi(0)\rangle$$

$$\langle O(t)\rangle = \langle \Psi(0)\,|\exp[iHt]\,|\,O\,|\exp[-iHt]\,|\,\Psi(0)\rangle$$

Expanding in eigenstates $|\Phi_i\rangle$

$$\langle O(t)\rangle = \langle \Psi(0)\,|\,\Phi_i\rangle\ \langle \Phi_i\,|\exp[iE_it]O\exp[-iE_jt]\,|\,\Phi_j\ \rangle\langle \Phi_j\,|\,\Psi(0)\rangle$$

Integrating over long times from t_1 to $t_1 + \Delta t$

$$\bar{O} = \frac{1}{\Delta t}\int_{t1}^{t1+\Delta t} \langle O(t)\rangle = \sum_i |\langle \Psi(0)\,|\,\Phi_i\rangle|^2 \langle \Phi_i\,|\,O\,|\,\Phi_i\rangle$$

Thermal-like property, incoherent sum over eigenstates with positive definite coefficients

ETH (eigenstate thermalization hypothesis) replace explicit sum with thermal average

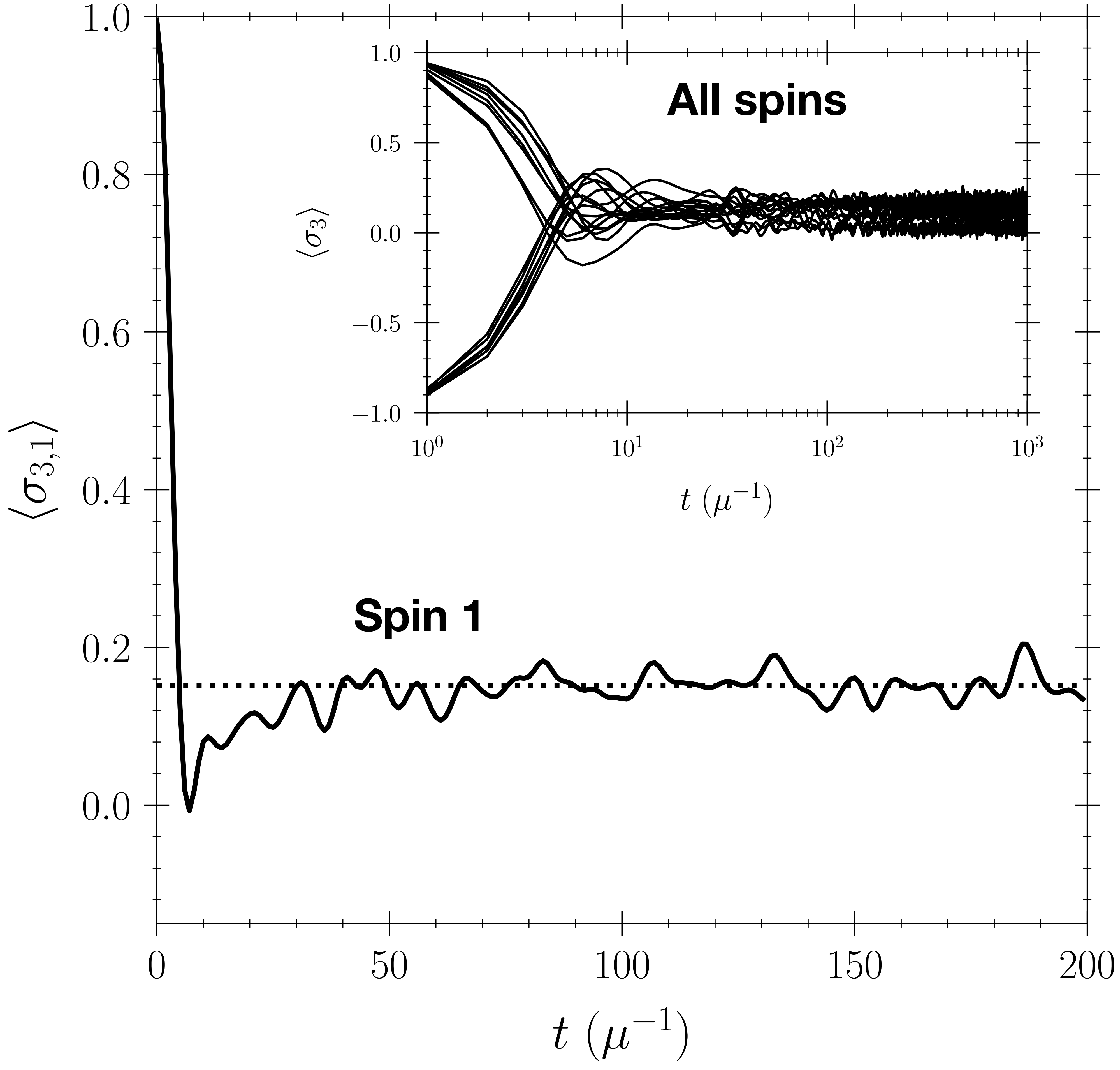
Typically applied with state near micro-canonical ensemble (fixed energy)

$$\bar{O} = \frac{\sum \exp[-\beta(E_i-\mu)]\langle \Phi_i\,|\,O\,|\,\Phi_i\rangle}{\sum \exp[-\beta(E_i-\mu)]}$$

ETH: Deutsch PRA, 1991; Srednicki PRE, 1994, …..

Calculated Time dependence of $\langle \sigma_z(i) \rangle$

16 spins; 9 ↑+ 7 ↓ initially; random directions in hemisphere

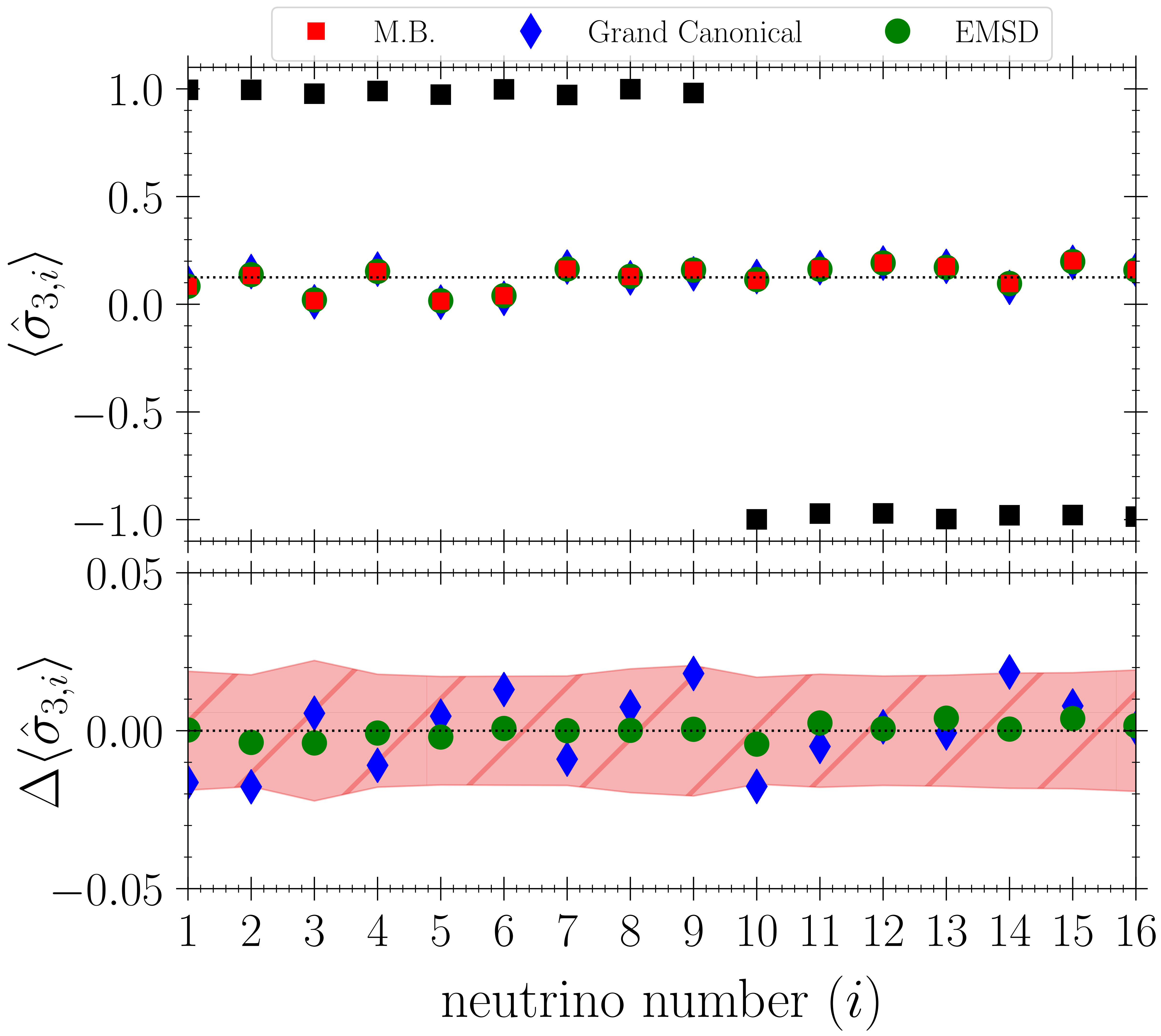


'Rapid' Equilibration of all spins

Time scale discussed later, here of order 5-10 μ

Some small memory of initial state energy

Example: 16 spins; 9 ↑+ 7 ↓ initially;
Couplings from random directions within hemisphere



- **Initial Conditions**
- **Red Integration of exact many-body solutions**
- **Blue Symbols: ETH**
- **Green Symbols: EMSD**
- **Dashed line equilibrium value for fixed total J_Z**

Time averaged final values well predicted by EMSD and ETH

Equilibration of spins to a high degree

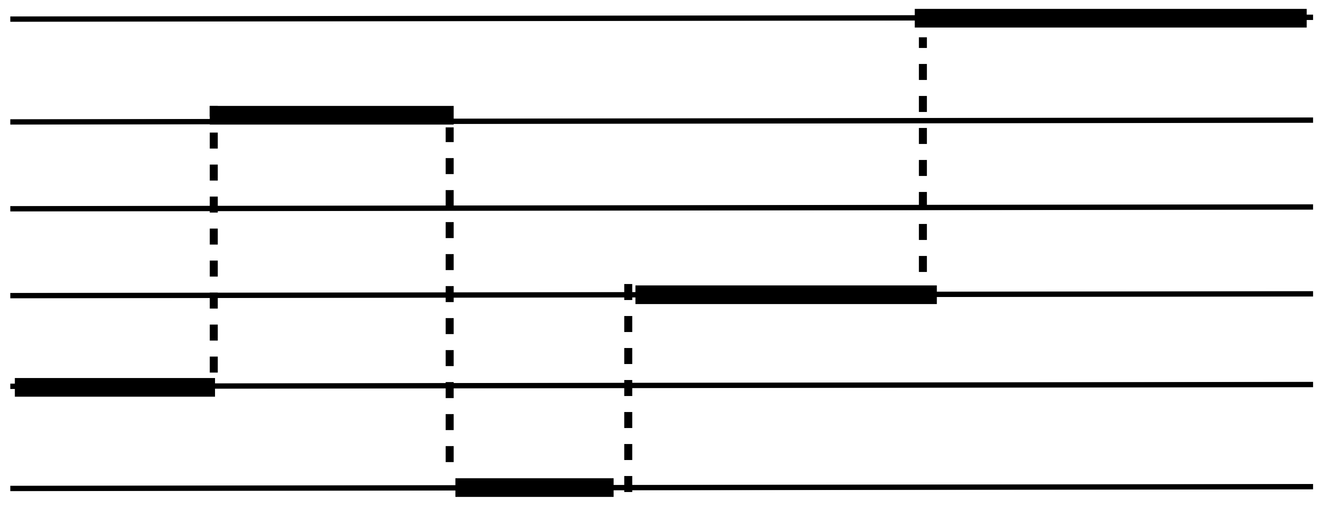
Both EGH and EMSD require many-body simulations (QMC will work for ETH in some cases)

Path Integral description of time evolution:

Write path integral in up/down spin basis describing time evolution: Expand in terms of matrix elements of off-diagonal (exchange) operators

Basis states
Permutation of N with $N_\uparrow$

↓



←Time

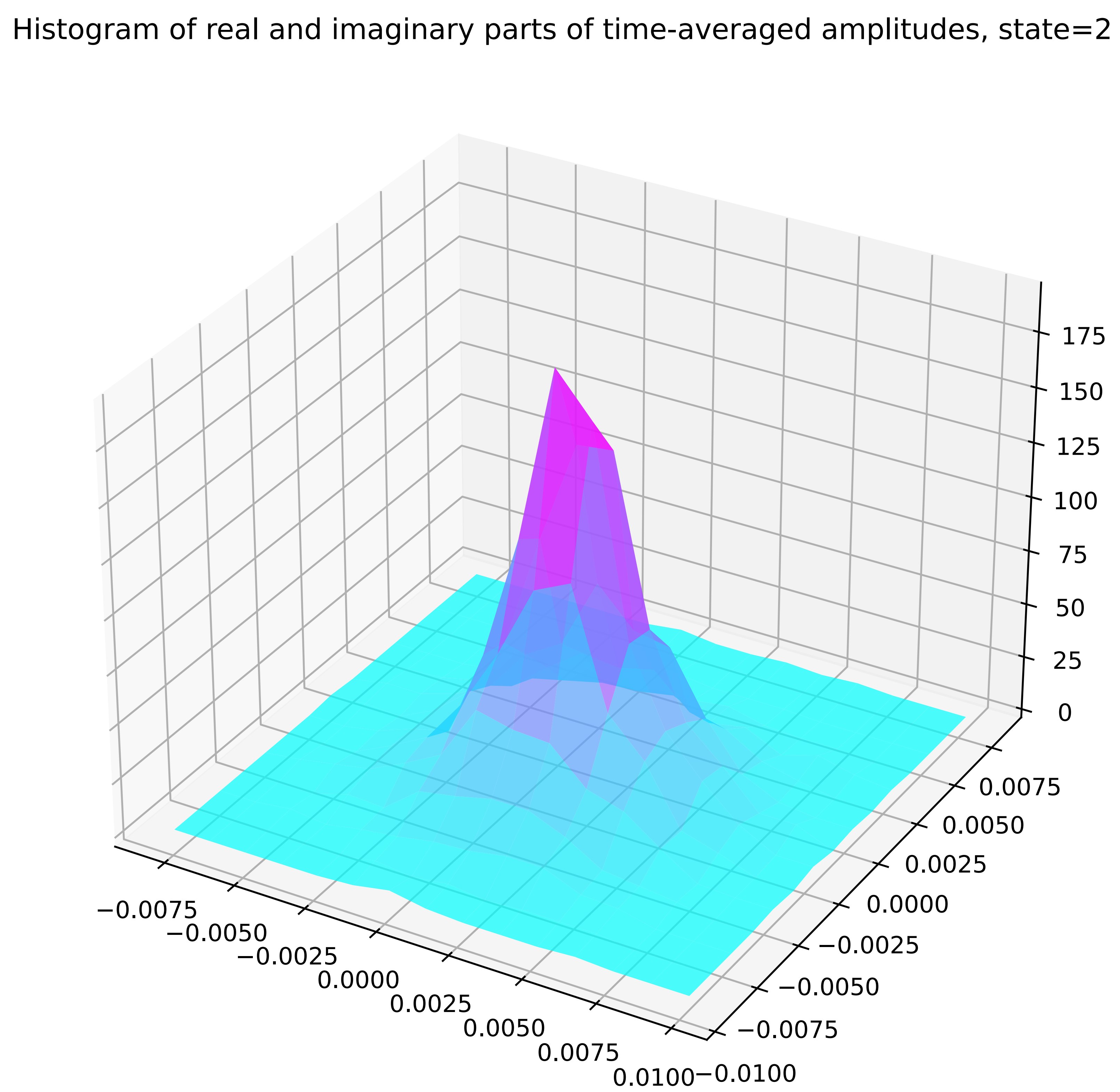
Off-diagonal H connects states with exchange of two Anti-parallel spins

$$\langle n|\exp[-\mathrm{i}\hat{H}_{\nu\nu}t]|\psi_0\rangle = \sum_m \frac{(-\mathrm{i})^m}{m!}\sum_{n_i\cdots n_m}\int \mathrm{d}t_0\cdots\mathrm{d}t_m \langle n|\exp[-\mathrm{i}\hat{H}^{\mathrm{d}}_{\nu\nu}t_m]\hat{H}^{\mathrm{od}}_{\nu\nu}|n_m\rangle$$

$$\langle n_m|\exp[-\mathrm{i}\hat{H}^{\mathrm{d}}_{\nu\nu}t_{m-1}]\hat{H}^{\mathrm{od}}_{\nu\nu}|n_{m-1}\rangle\cdots\langle n_1|\exp[-\mathrm{i}\hat{H}^{\mathrm{d}}_{\nu\nu}t_1]\hat{H}^{\mathrm{od}}_{\nu\nu}|n_0\rangle\langle n_0|\exp[-\mathrm{i}\hat{H}^{\mathrm{d}}_{\nu\nu}t_0|\psi_0\rangle.$$

Propagation is product of phases (solid lines) with insertion of swaps (dashed lines)
Can approximate resulting state as an incoherent sum over all up-down product states
Random phase approximation for each line gives incoherent sum
Magnitudes can be defined to reproduce symmetries

Exact time integrated complex amplitudes



Amplitude in up/down state I at time t + δt

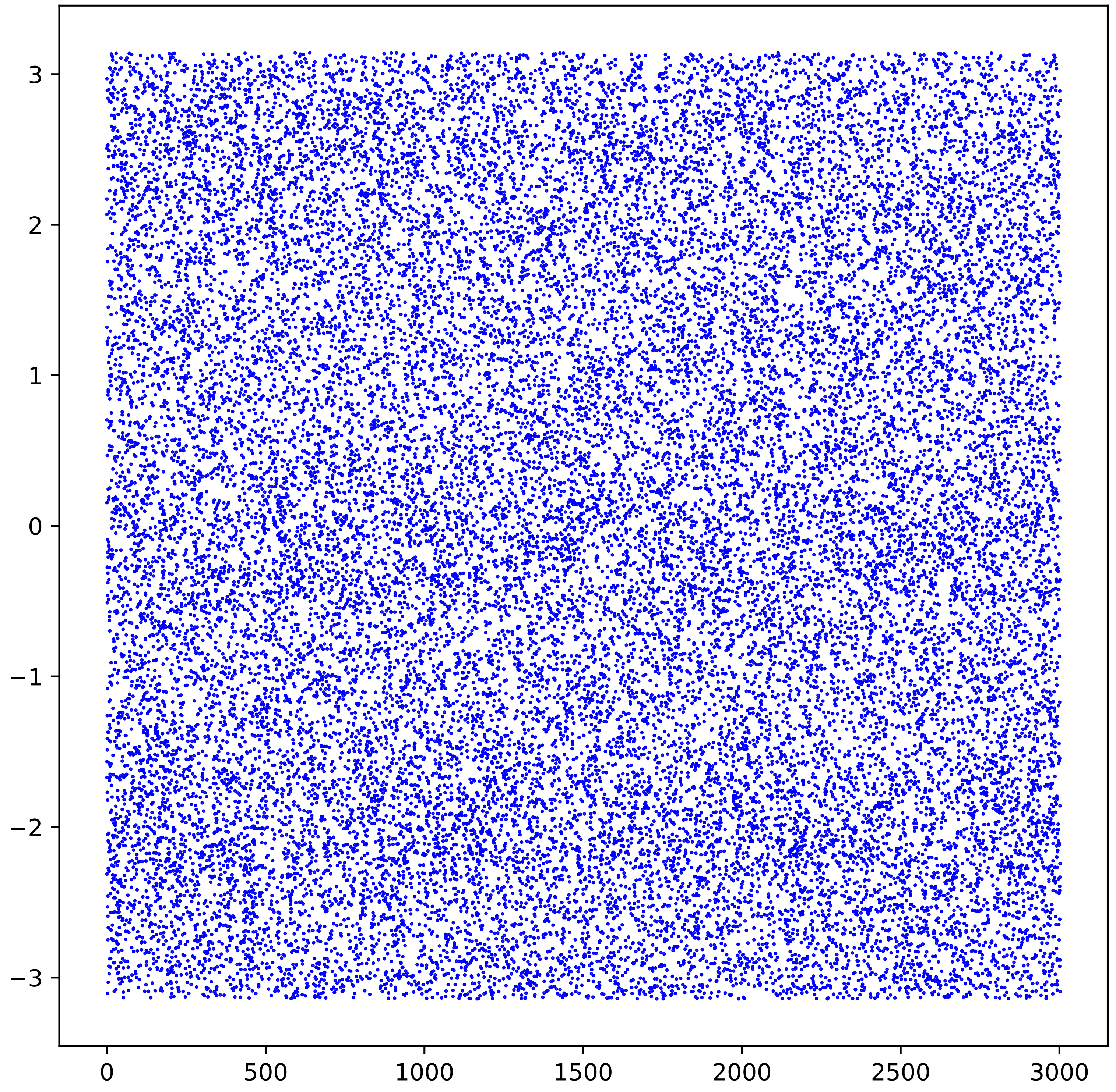
$$a_i(t+\delta t) = \exp[-iH_d(i)t]a_i(t) + (-i)\sum_{j\neq i} a_j(t)H_{od}(i,j)\delta t$$

Rotation from diagonal Hamiltonian plus
Displacement from off-diagonal transitions
N(N-1)/2 terms in sum, each of same magnitude as 1st term
Rapidly leads to random phases

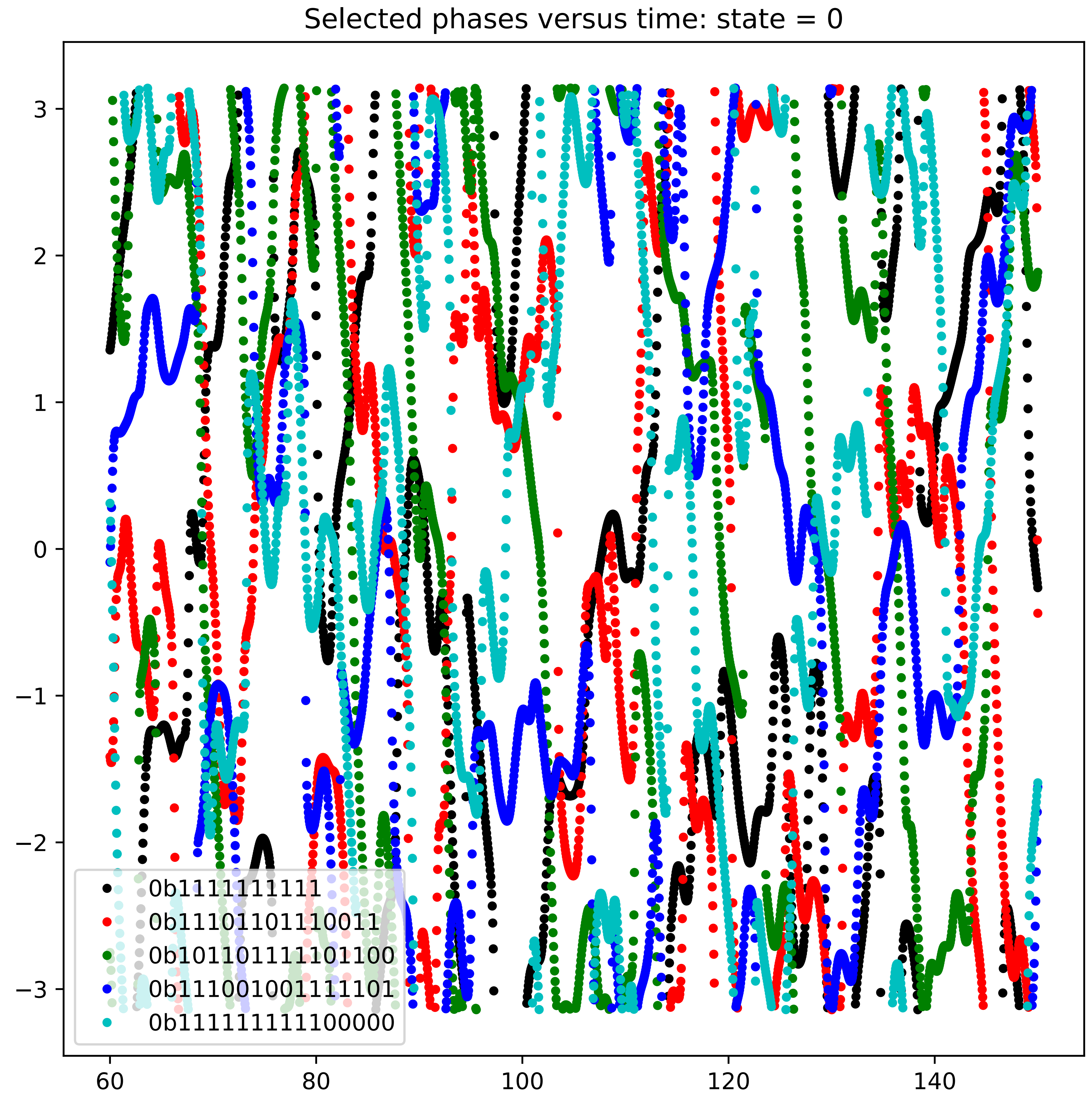
Assuming $a_j(t)$ from random gaussian distribution
Yields gaussian distribution

Snapshot of phases, instantaneous and time-dependence

Phases from snapshot at 10 separated times

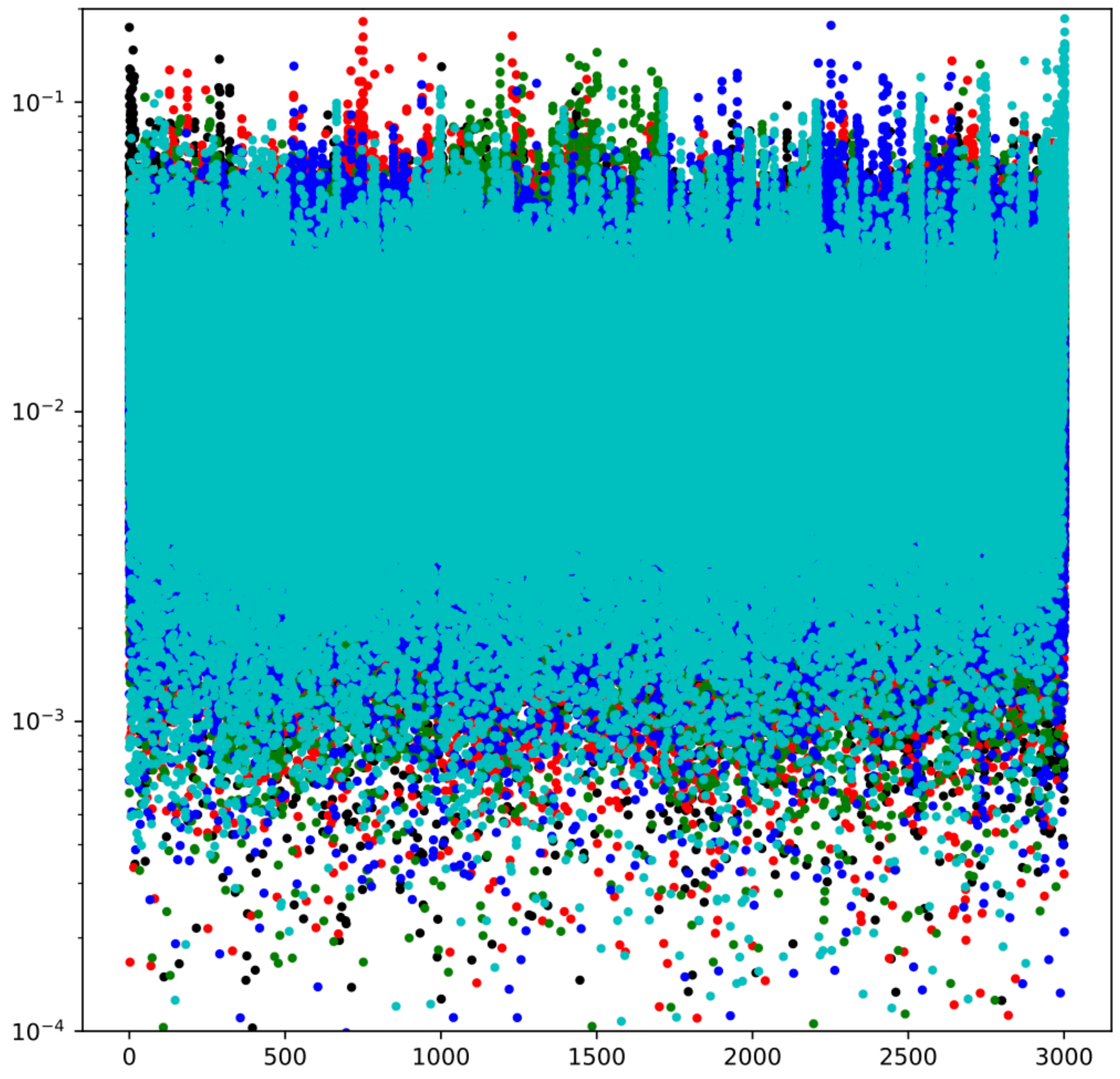


Time-dependent phases



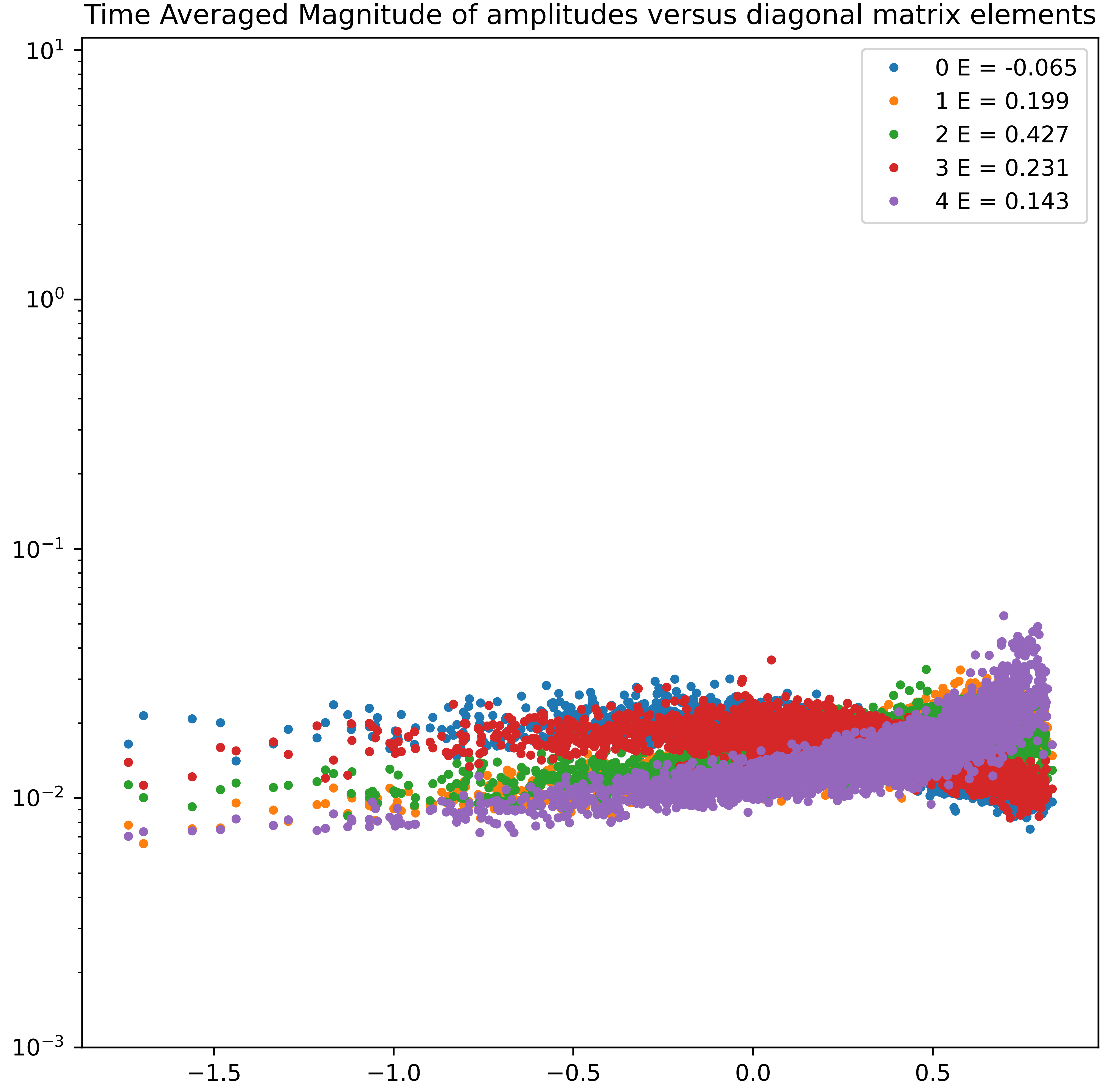
Time-Averaged magnitudes

10 μ^{-1}



Magnitudes vs. state index

100 μ^{-1}



Magnitudes vs. H_d

Time Scales (For spin problem)

Note this is for a system initially spatially homogeneous,
We are looking at time scales for spin exchange
Not yet any spacial information
Starting with an up-down product spin state

Loschmidt Amplitudes: $\langle \Psi(t) \,|\, \Psi(0) \rangle$

Any antiparallel spin exchange gives zero zero

$$1 \sim c\ t\langle H_{od} \rangle = c\ t\ N(N-1)\frac{\mu}{N}$$

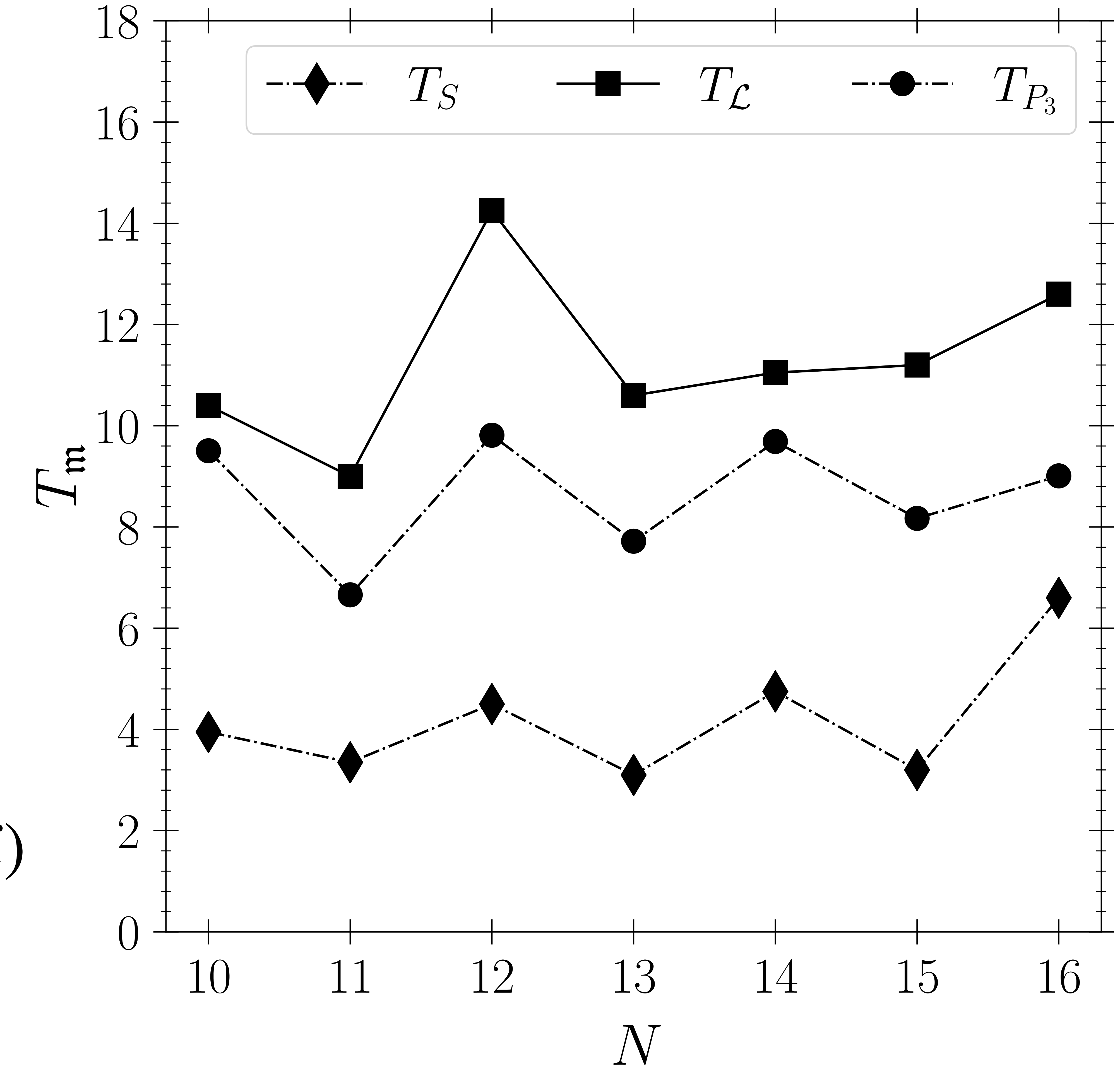
$$t = \mu^{-1}/N$$

Two-point Correlation function

$$< \psi(0)\,|\exp[iHt]\sigma_z(i)\exp[-iHt]\sigma_z(i)\psi(0)\rangle = 1 - \beta t^2 \approx \sigma_{z,eq}(i)$$

$$t \propto \mu^{-1}\sqrt{N}$$

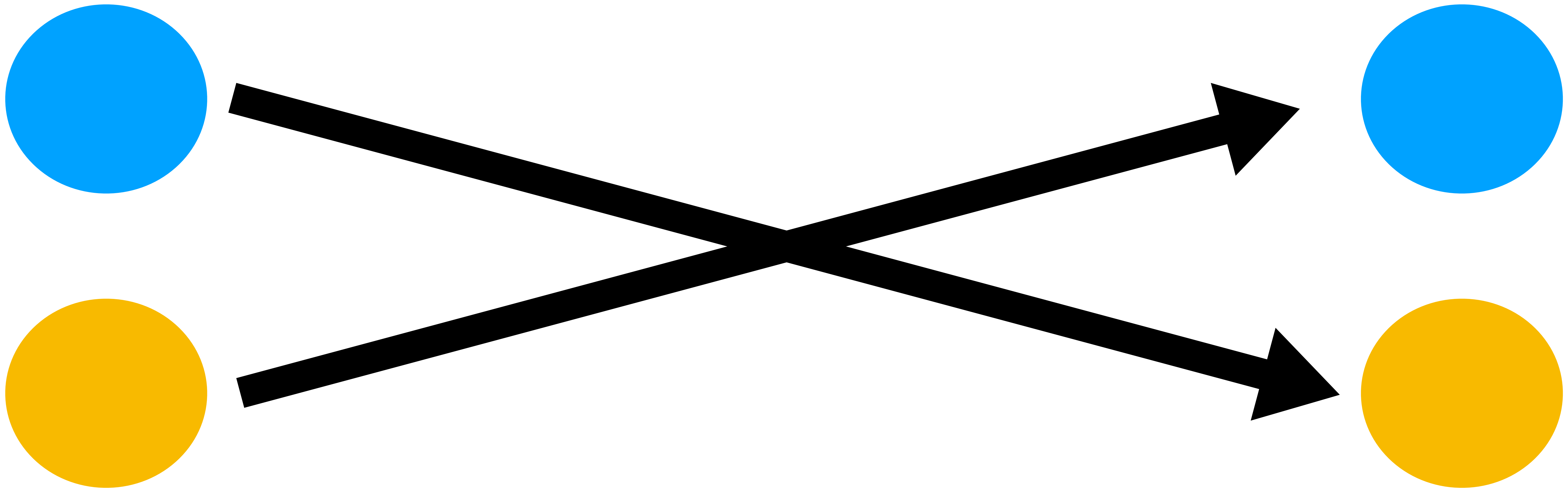
Larger time scales for bigger systems



Symmetries and Conserved Quantities

- Path integral representation only allows permutations of original spins with arbitrary complex amplitudes
- Any arbitrary sum of permutations of the original state gives correct <J2>, <Jx>, <Jy>, <Jz>
- Numerical evidence suggests that for non-integrable H phases are random giving an incoherent sum
- Time averages indicate that the time averaged |amplitudes| are smooth function F(E) of $\langle H_d \rangle$
  Initial expectation values of $\langle H^n \rangle$ can be calculated from initial state, to get F(E)

Classical swap network swapping spins can easily be implemented to reach large number of spins



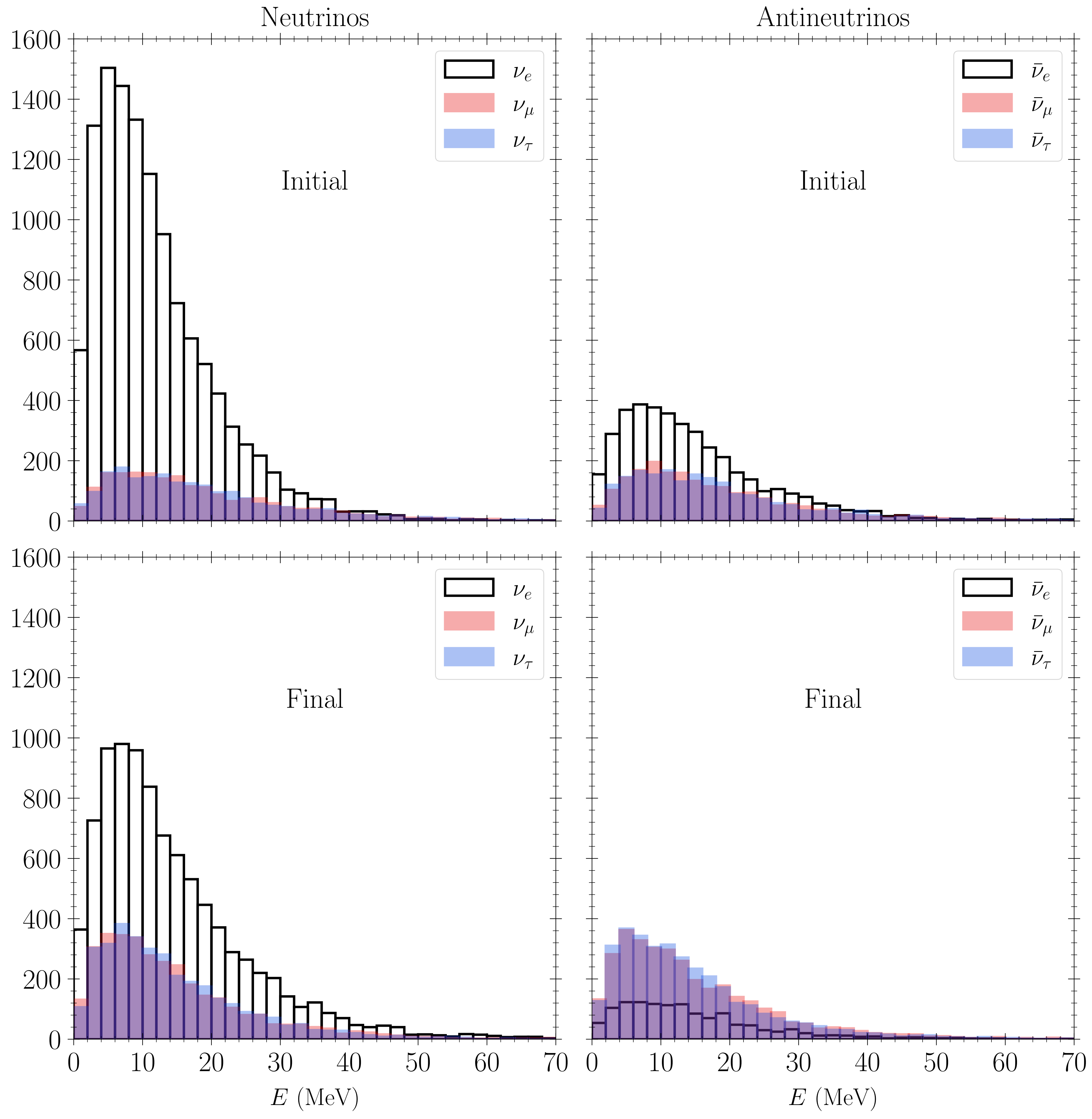
Equilibration of Neutrinos and anti-neutrinos with multiple flavors

Total # of neutrinos conserved (1 constraint)
Lepton # conservation for 3 flavors (3 constraints)
Need 6 total

$\nu_e\bar{\nu}_e \leftrightarrow \nu_\mu\bar{\nu}_\mu$ leads to conservation of product of neutrino and anti-neutrino densities

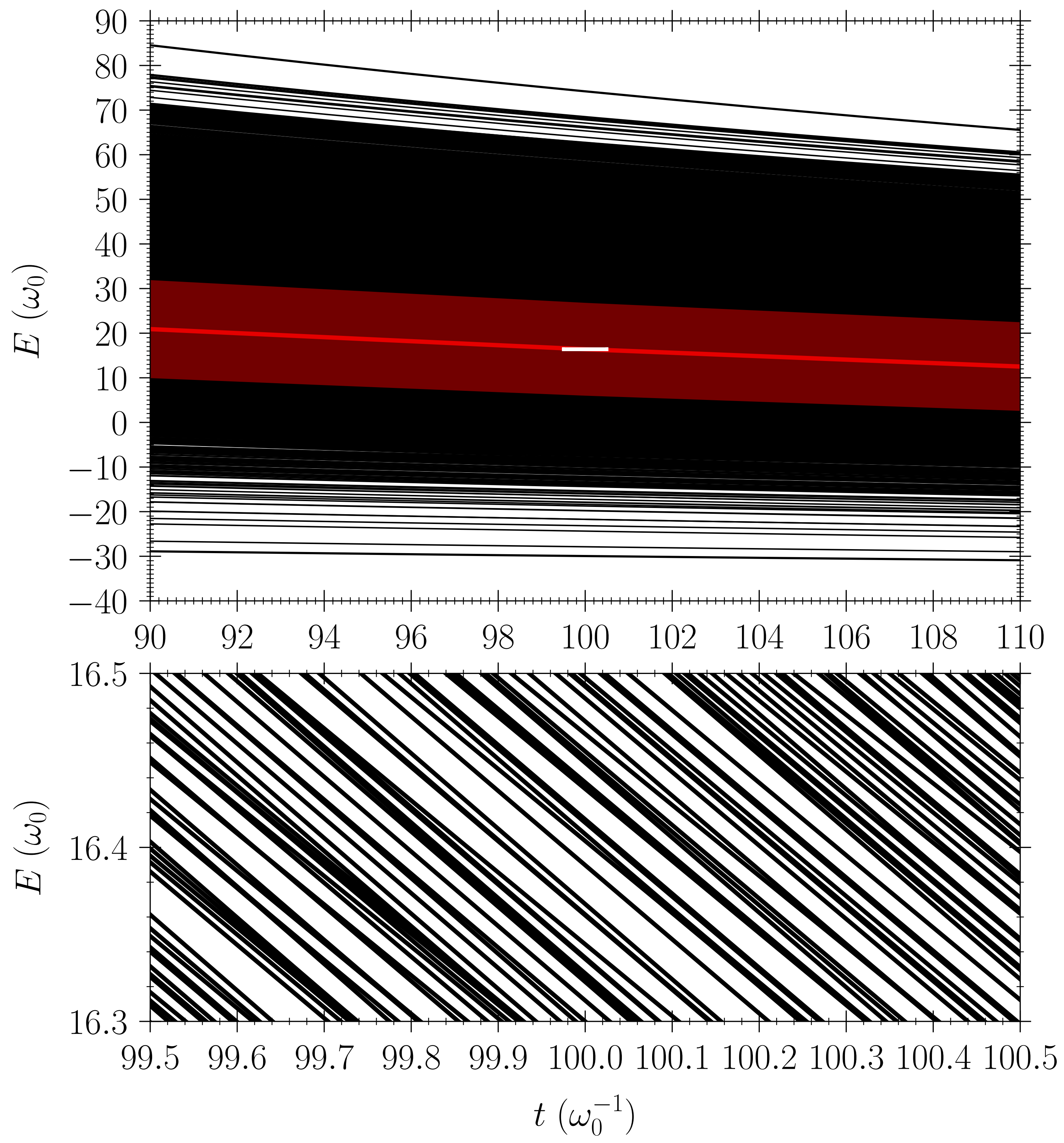
$\rho_e\rho_{\bar{e}} = \rho_\mu\rho_{\bar{\mu}} = \rho_\tau\rho_{\bar{\tau}}$ (2 constraints)

Sample Initial/Final Distributions

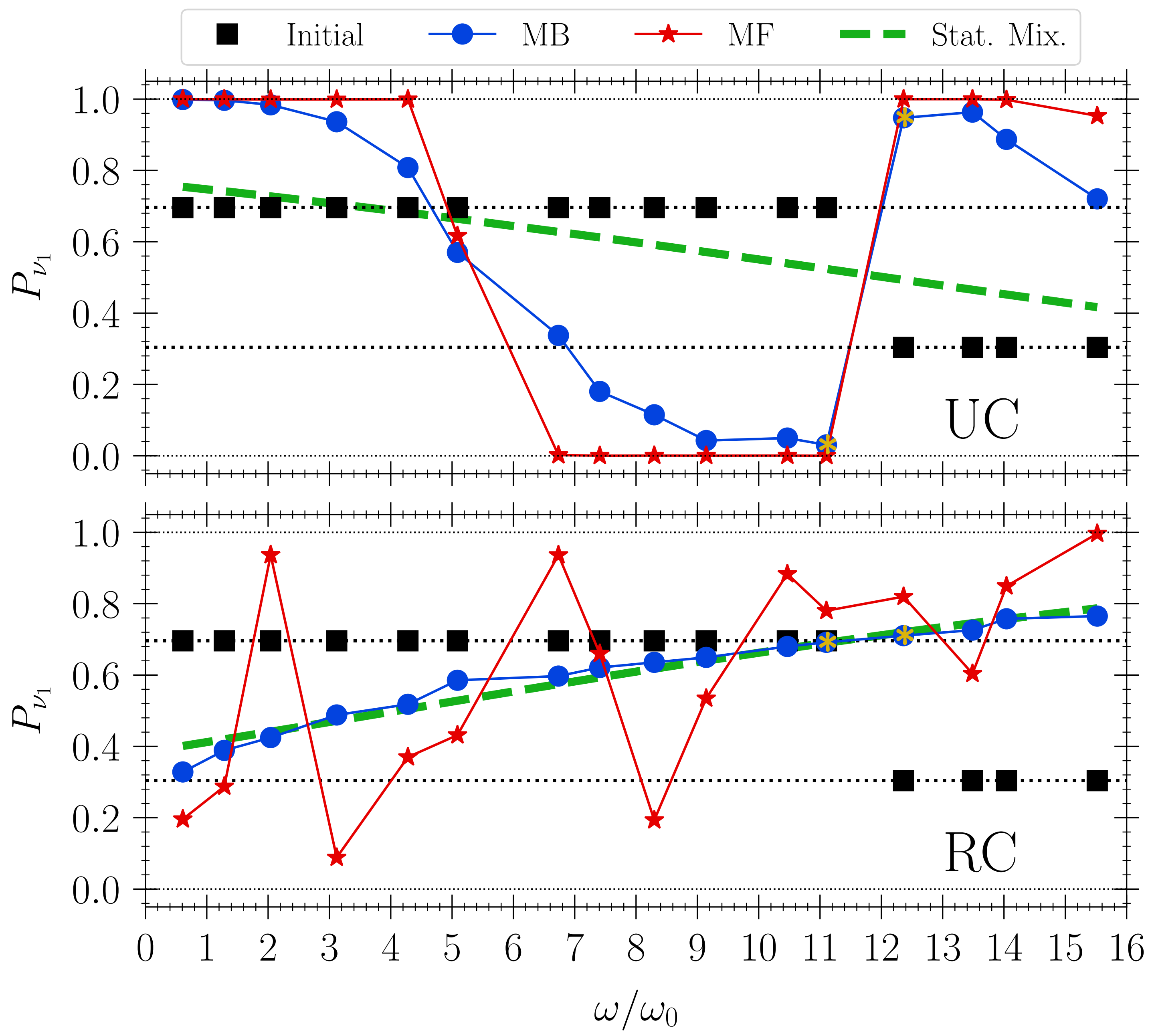


Future Directions:

- **Real-Time dynamics on simulators**
  - **Inclusive**
  - **`Detectors' on QC**
- **Time-dependent H for $\nu - \nu$: MSW + vacuum**
- **Beyond Forward Scattering**



Time-dependent H: MF vs Many-Body



Uniform Coupling

Random coupling