Nuclear physics for the precise extraction of Vud

Chien-Yeah Seng

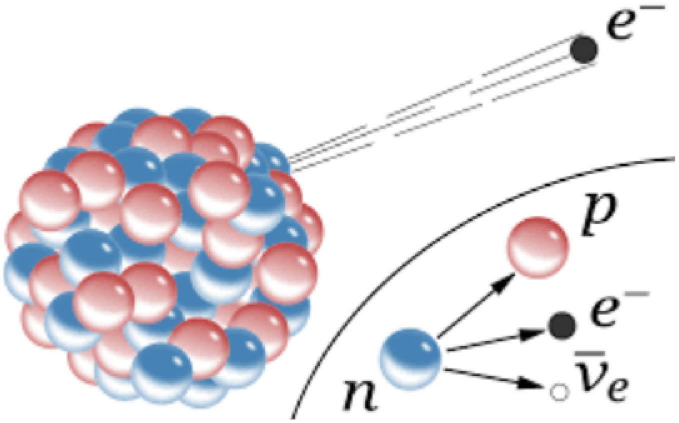
University of Washington
and
FRIB, Michigan State University

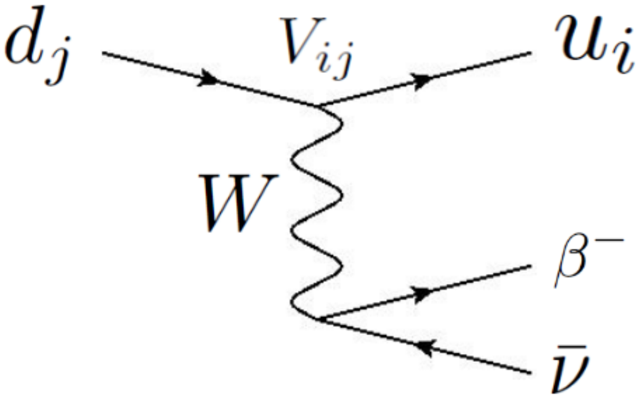
seng@frib.msu.edu

Marciana 2023 – Lepton Interactions with Nucleons and Nuclei

5 September, 2023

Nuclear beta decay





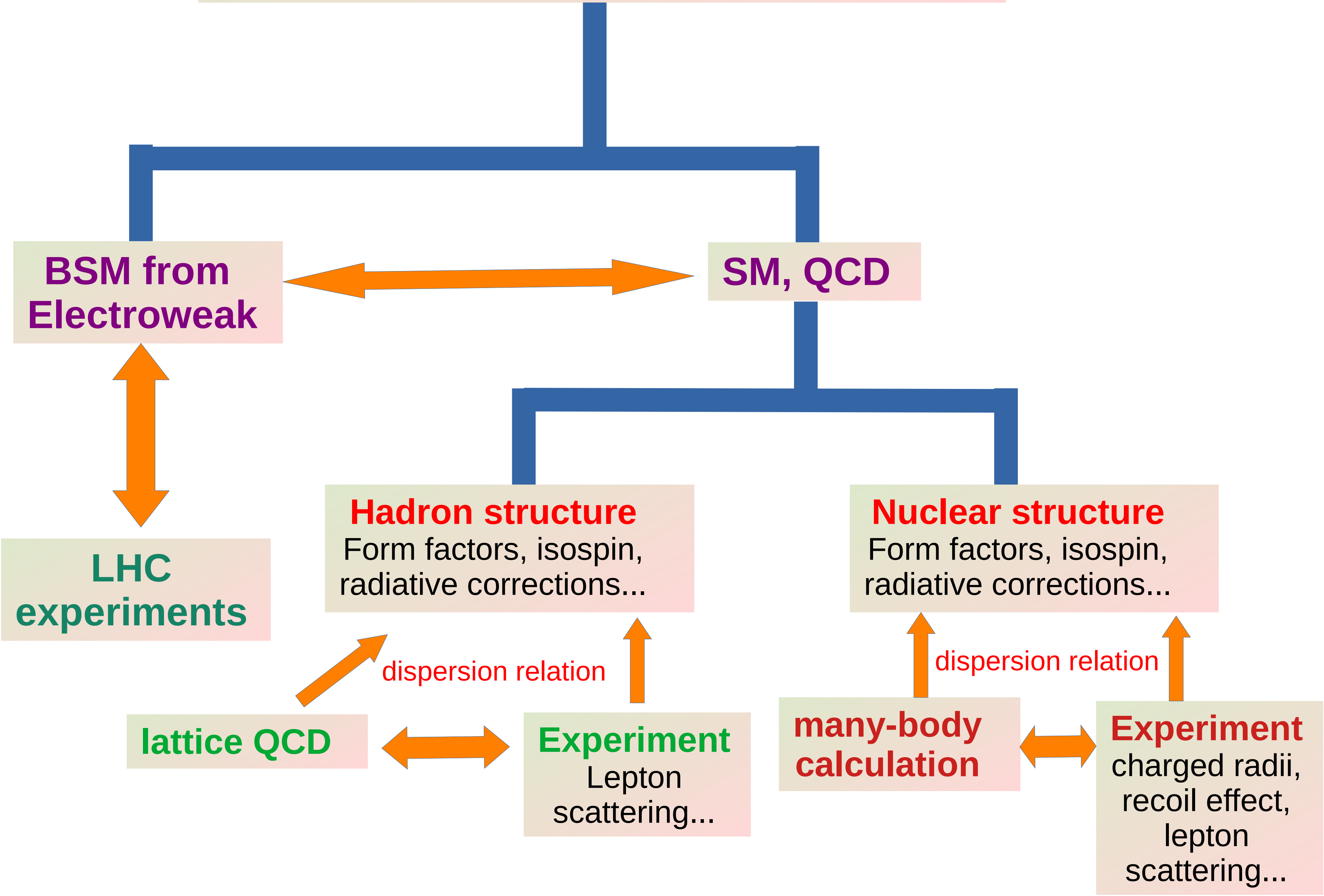
Observables measured at **0.01%** level probes new physics at the scale:

$$\left(\frac{v_{\mathrm{H}}}{\Lambda_{\mathrm{BSM}}}\right)^2 \sim 0.01\% \quad \Longrightarrow \quad \Lambda_{\mathrm{BSM}} \sim 20\ \mathrm{TeV}$$

Competitive to high-energy experiments at LHC!

See Vincenzo's talk!

Nuclear Beta Decays as Precision Laboratories



BSM from Electroweak

SM, QCD

LHC experiments

Hadron structure
Form factors, isospin,
radiative corrections...

Nuclear structure
Form factors, isospin,
radiative corrections...

dispersion relation

lattice QCD

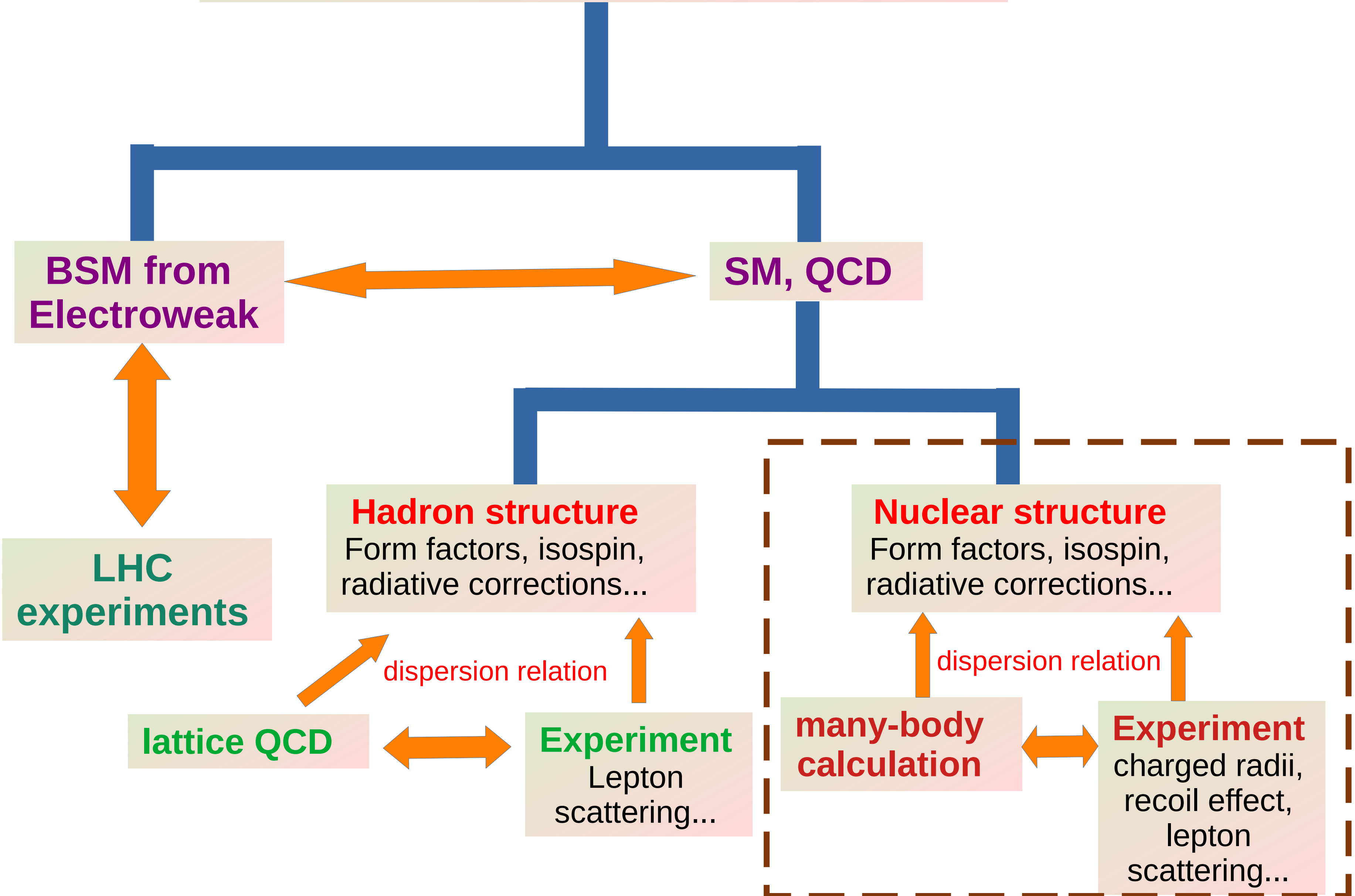
Experiment
Lepton
scattering...

dispersion relation

many-body calculation

Experiment
charged radii,
recoil effect,
lepton
scattering...

Nuclear Beta Decays as Precision Laboratories



BSM from Electroweak

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Hadron structure
Form factors, isospin, radiative corrections...

Nuclear structure
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dispersion relation

dispersion relation

lattice QCD

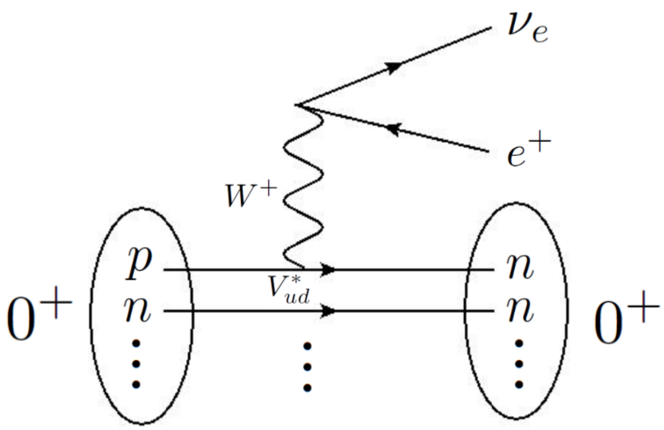
Experiment
Lepton scattering...

many-body calculation

Experiment
charged radii, recoil effect, lepton scattering...

“Superallowed” beta decays of **T=1, $J^p=0^+$ nuclei**

$$i(0^+) \rightarrow f(0^+) + e^+ + \nu_e$$



$T_z = -1$
${}^{10}_{6}\mathrm{C} \rightarrow {}^{10}_{5}\mathrm{B}$
${}^{14}_{8}\mathrm{O} \rightarrow {}^{14}_{7}\mathrm{N}$
${}^{18}_{10}\mathrm{Ne} \rightarrow {}^{18}_{9}\mathrm{F}$
${}^{22}_{12}\mathrm{Mg} \rightarrow {}^{22}_{11}\mathrm{Na}$
${}^{26}_{14}\mathrm{Si} \rightarrow {}^{26}_{13}\mathrm{Al}$
${}^{30}_{16}\mathrm{S} \rightarrow {}^{30}_{15}\mathrm{P}$
${}^{34}_{18}\mathrm{Ar} \rightarrow {}^{34}_{17}\mathrm{Cl}$
${}^{38}_{20}\mathrm{Ca} \rightarrow {}^{38}_{19}\mathrm{K}$
${}^{42}_{22}\mathrm{Ti} \rightarrow {}^{42}_{21}\mathrm{Sc}$
${}^{46}_{24}\mathrm{Cr} \rightarrow {}^{46}_{23}\mathrm{V}$
${}^{50}_{26}\mathrm{Fe} \rightarrow {}^{50}_{25}\mathrm{Mn}$
${}^{54}_{28}\mathrm{Ni} \rightarrow {}^{54}_{27}\mathrm{Co}$

$T_z = 0$
${}^{26m}_{13}\mathrm{Al} \rightarrow {}^{26}_{12}\mathrm{Mg}$
${}^{34}_{17}\mathrm{Cl} \rightarrow {}^{34}_{16}\mathrm{S}$
${}^{38m}_{19}\mathrm{K} \rightarrow {}^{38}_{18}\mathrm{Ar}$
${}^{42}_{21}\mathrm{Sc} \rightarrow {}^{42}_{20}\mathrm{Ca}$
${}^{46}_{23}\mathrm{V} \rightarrow {}^{46}_{22}\mathrm{Ti}$
${}^{50}_{25}\mathrm{Mn} \rightarrow {}^{50}_{24}\mathrm{Cr}$
${}^{54}_{27}\mathrm{Co} \rightarrow {}^{54}_{26}\mathrm{Fe}$
${}^{62}_{31}\mathrm{Ga} \rightarrow {}^{62}_{30}\mathrm{Zn}$
${}^{66}_{33}\mathrm{As} \rightarrow {}^{66}_{32}\mathrm{Ge}$
${}^{70}_{35}\mathrm{Br} \rightarrow {}^{70}_{34}\mathrm{Se}$
${}^{74}_{37}\mathrm{Rb} \rightarrow {}^{74}_{36}\mathrm{Kr}$

Provides the **best measurement of V_{ud} :**

- **23** measured transitions
- **15** with lifetime precision better than **0.23%**

Hardy and Towner, 2020 PRC

“Trajectory of V_{ud} ” in the past few years

CKM unitarity good

Early 2018: 0.97417(21)

Single-nucleon RC by Marciano and Sirlin, 2006

~4σ unitarity violation

Late 2018: 0.97370(14)

Single-nucleon RC revisited with dispersion relation

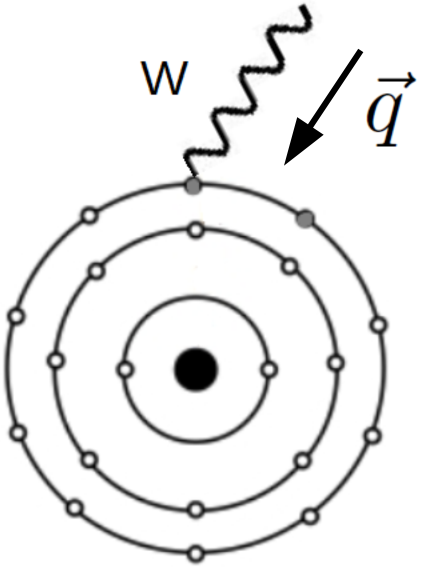
~3σ unitarity violation

Late 2020: 0.97373(31)

New nuclear effects discovered, final review by Hardy/Towner

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1}$$

Superallowed decays at tree level



Charged weak decay form factor:

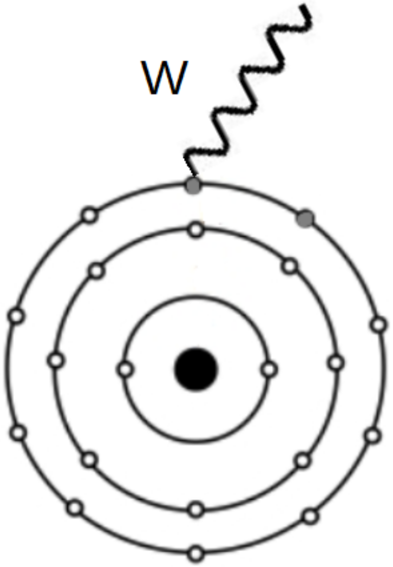
$$\boxed{f(\vec{q}^{\,2}) = M_F\left(1 + \overline{f}(\vec{q}^{\,2})\right)}$$

Fermi matrix element

q^2-dependence

Nuclear (charged) weak distribution

$\rho_{\mathrm{cw}}(r)$: Distribution of “active” nucleons eligible for weak transitions in a nucleus



Old : Impulse approximation in **non-relativistic (NR) nuclear models**.

$$\langle f|O|i\rangle = \sum_{\alpha\beta} \langle\alpha|O|\beta\rangle\langle f|a_\alpha^\dagger a_\beta|i\rangle$$

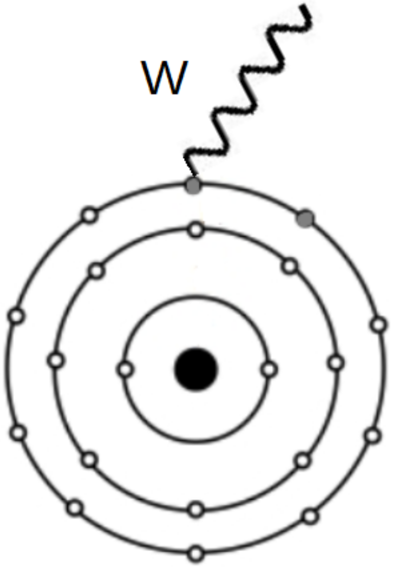
α,β : single-nucleon states

Hardy and Towner, 2005 PRC

How reliable is such approximation at **0.01% level**?

Nuclear (charged) weak distribution

$\rho_{\rm cw}(r)$: Distribution of “active” nucleons eligible for weak transitions in a nucleus



New: Utilize **CVC** to relate (charged) weak distribution to nuclear **charge distributions**

Hostein, 1974 RMP; CYS, 2023 PRL

$$\begin{aligned}\rho_{\rm cw}(r) &= \rho_{\rm ch,1}(r) + Z_0\left(\rho_{\rm ch,0}(r) - \rho_{\rm ch,1}(r)\right) \\ &= \rho_{\rm ch,1}(r) + \frac{Z_{-1}}{2}\left(\rho_{\rm ch,\text{-}1}(r) - \rho_{\rm ch,1}(r)\right)\end{aligned}$$

Form factor expansion:

$$F(q^2) = 1 + \frac{q^2}{6}\langle r^2 \rangle + \ldots$$

M.S. radius

Data + isospin predicts **unexpectedly large (charged) weak radii**!

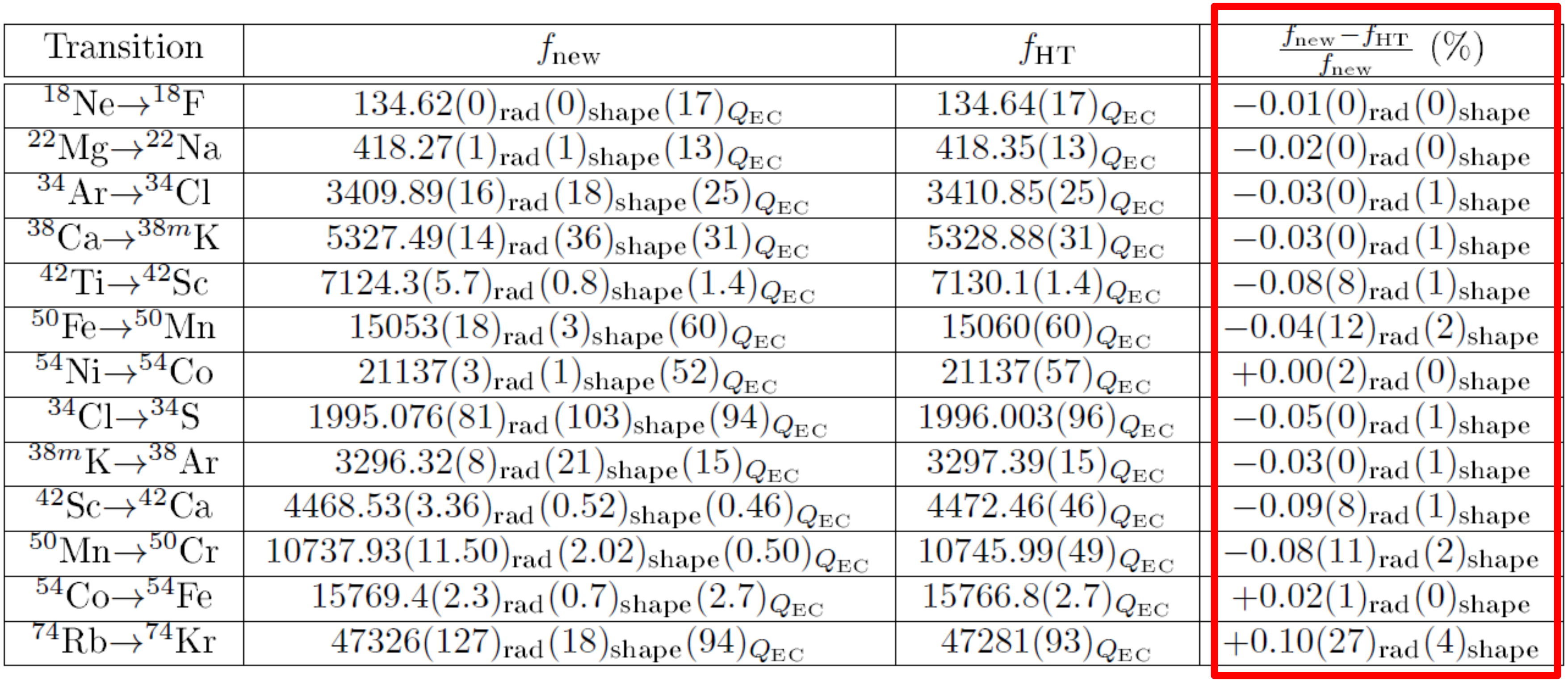
E.g.
$$\begin{aligned} \langle r_{\mathrm{Ch}}^2 \rangle_{^{38}\mathrm{Ar}}^{1/2} &= 3.4028(19)\ \mathrm{fm} \\ \langle r_{\mathrm{Ch}}^2 \rangle_{^{38}\mathrm{Ca}}^{1/2} &= 3.467(1)\ \mathrm{fm} \end{aligned} \quad \Rightarrow \quad \langle r_{\mathrm{cw}}^2 \rangle^{1/2} = 4.00(4)\ \mathrm{fm}$$

Simplified shell model prediction: $\langle r_{\mathrm{cw}}^2 \rangle^{1/2} \approx 3.62\ \mathrm{fm}$

Wilkinson, 1993 Nucl.Instrum.Meth.A

$$|V_{ud}|^2 \propto \frac{1}{ft}$$

(Preliminary) data-driven re-analysis of the **"statistical rate function" f**:



Transition	$f_{\rm new}$	$f_{\rm HT}$	$\frac{f_{\rm new}-f_{\rm HT}}{f_{\rm new}}$ (%)
^{18}Ne$\rightarrow^{18}$F	$134.62(0)_{\rm rad}(0)_{\rm shape}(17)_{Q_{\rm EC}}$	$134.64(17)_{Q_{\rm EC}}$	$-0.01(0)_{\rm rad}(0)_{\rm shape}$
^{22}Mg$\rightarrow^{22}$Na	$418.27(1)_{\rm rad}(1)_{\rm shape}(13)_{Q_{\rm EC}}$	$418.35(13)_{Q_{\rm EC}}$	$-0.02(0)_{\rm rad}(0)_{\rm shape}$
^{34}Ar$\rightarrow^{34}$Cl	$3409.89(16)_{\rm rad}(18)_{\rm shape}(25)_{Q_{\rm EC}}$	$3410.85(25)_{Q_{\rm EC}}$	$-0.03(0)_{\rm rad}(1)_{\rm shape}$
^{38}Ca$\rightarrow^{38m}$K	$5327.49(14)_{\rm rad}(36)_{\rm shape}(31)_{Q_{\rm EC}}$	$5328.88(31)_{Q_{\rm EC}}$	$-0.03(0)_{\rm rad}(1)_{\rm shape}$
^{42}Ti$\rightarrow^{42}$Sc	$7124.3(5.7)_{\rm rad}(0.8)_{\rm shape}(1.4)_{Q_{\rm EC}}$	$7130.1(1.4)_{Q_{\rm EC}}$	$-0.08(8)_{\rm rad}(1)_{\rm shape}$
^{50}Fe$\rightarrow^{50}$Mn	$15053(18)_{\rm rad}(3)_{\rm shape}(60)_{Q_{\rm EC}}$	$15060(60)_{Q_{\rm EC}}$	$-0.04(12)_{\rm rad}(2)_{\rm shape}$
^{54}Ni$\rightarrow^{54}$Co	$21137(3)_{\rm rad}(1)_{\rm shape}(52)_{Q_{\rm EC}}$	$21137(57)_{Q_{\rm EC}}$	$+0.00(2)_{\rm rad}(0)_{\rm shape}$
^{34}Cl$\rightarrow^{34}$S	$1995.076(81)_{\rm rad}(103)_{\rm shape}(94)_{Q_{\rm EC}}$	$1996.003(96)_{Q_{\rm EC}}$	$-0.05(0)_{\rm rad}(1)_{\rm shape}$
^{38m}K$\rightarrow^{38}$Ar	$3296.32(8)_{\rm rad}(21)_{\rm shape}(15)_{Q_{\rm EC}}$	$3297.39(15)_{Q_{\rm EC}}$	$-0.03(0)_{\rm rad}(1)_{\rm shape}$
^{42}Sc$\rightarrow^{42}$Ca	$4468.53(3.36)_{\rm rad}(0.52)_{\rm shape}(0.46)_{Q_{\rm EC}}$	$4472.46(46)_{Q_{\rm EC}}$	$-0.09(8)_{\rm rad}(1)_{\rm shape}$
^{50}Mn$\rightarrow^{50}$Cr	$10737.93(11.50)_{\rm rad}(2.02)_{\rm shape}(0.50)_{Q_{\rm EC}}$	$10745.99(49)_{Q_{\rm EC}}$	$-0.08(11)_{\rm rad}(2)_{\rm shape}$
^{54}Co$\rightarrow^{54}$Fe	$15769.4(2.3)_{\rm rad}(0.7)_{\rm shape}(2.7)_{Q_{\rm EC}}$	$15766.8(2.7)_{Q_{\rm EC}}$	$+0.02(1)_{\rm rad}(0)_{\rm shape}$
^{74}Rb$\rightarrow^{74}$Kr	$47326(127)_{\rm rad}(18)_{\rm shape}(94)_{Q_{\rm EC}}$	$47281(93)_{Q_{\rm EC}}$	$+0.10(27)_{\rm rad}(4)_{\rm shape}$

CYS, **PRELIMINARY RESULTS**

Shifts at 0.01% level are observed!

Upward shift of V_{ud} ?

Fermi matrix element and isospin-symmetry breaking (ISB)

$$|M_F|^2 = |\langle f|\hat{\tau}_+|i\rangle|^2 = 2(1-\delta_\mathrm{C})$$

$$|V_{ud}|^2 \propto \frac{1}{1-\delta_\mathrm{C}}$$

Starts at **second order** in ISB interaction:

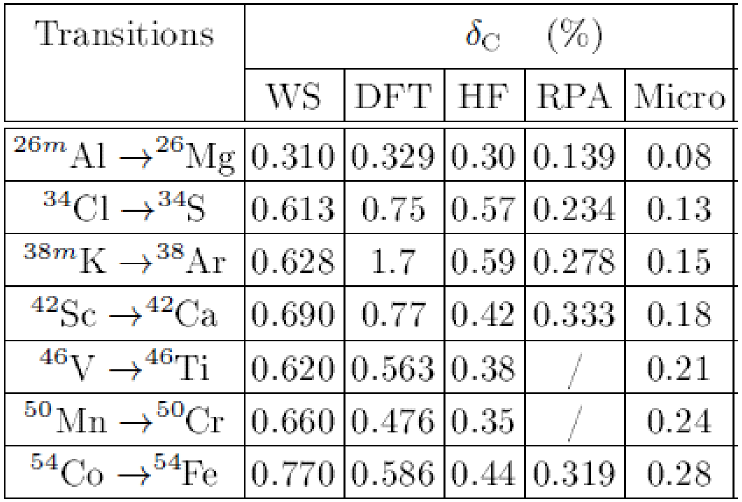
Behrends and Sirlin, 1960 PRL;
Ademollo and Gatto, 1964 PRL

$$H = H_0 + \enclose{circle}{V}$$ **ISB**

$$\delta_\mathrm{C} = \mathcal{O}(V^2)$$ **0.1%~1%**

- Computing δ_C: Classic problem over ***6 decades***!

MacDonald, 1958 Phys.Rev

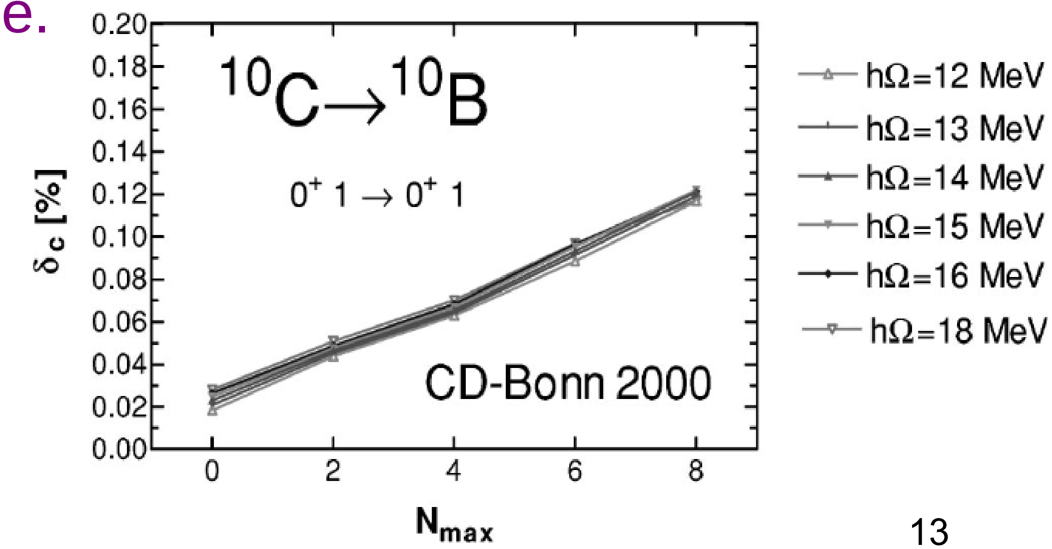


Transitions	δ_C (%)				
	WS	DFT	HF	RPA	Micro
$^{26m}\mathrm{Al} \rightarrow {}^{26}\mathrm{Mg}$	0.310	0.329	0.30	0.139	0.08
$^{34}\mathrm{Cl} \rightarrow {}^{34}\mathrm{S}$	0.613	0.75	0.57	0.234	0.13
$^{38m}\mathrm{K} \rightarrow {}^{38}\mathrm{Ar}$	0.628	1.7	0.59	0.278	0.15
$^{42}\mathrm{Sc} \rightarrow {}^{42}\mathrm{Ca}$	0.690	0.77	0.42	0.333	0.18
$^{46}\mathrm{V} \rightarrow {}^{46}\mathrm{Ti}$	0.620	0.563	0.38	/	0.21
$^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}$	0.660	0.476	0.35	/	0.24
$^{54}\mathrm{Co} \rightarrow {}^{54}\mathrm{Fe}$	0.770	0.586	0.44	0.319	0.28

(Selected results)

Caveats:

- Significant model dependence.
- **No direct experimental constraint**
- Ab-initio calculations still in preliminary stages



Caurier et al., 2002 PRC

Probing isospin mixing in experiment?

Nuclear mass splitting: $\delta E_n = {}_0\langle n|V|n\rangle_0$ **X**

$$|n\rangle = |n\rangle_0 + \underbrace{\sum_{k\neq n} |k\rangle_0 \frac{{}_0\langle k|V|n\rangle_0}{E_n^{(0)} - E_k^{(0)}}}_{\textbf{We need this}} + \dots$$

Second-class current: doubly-suppressed (kinematics*ISB), null results so far

Minamisono et al., 2011 PRC

$$V_\mu = \bar{u}(p_2)\left(g_V\gamma_\mu - i\frac{g_M - g_V}{2M}\sigma_{\mu\nu}q^\nu + \frac{\boxed{g_S}}{2M}q_\mu\right)u(p_1),$$

$$A_\mu = -\bar{u}(p_2)\gamma_5\left(g_A\gamma_\mu - i\frac{\boxed{g_T}}{2M}\sigma_{\mu\nu}q^\nu + \frac{g_P}{2M}q_\mu\right)u(p_1).$$

Isospin mixing can be probed through measurements of **electroweak nuclear radii**

The non-zeroness of

cw radius
(directly
measured)

related to neutron skin

$$\Delta M_A^{(1)} \equiv -\underline{\langle r_{\mathrm{cw}}^2 \rangle} + \underline{\left(\langle r_{n,1}^2 \rangle - \langle r_{p,1}^2 \rangle\right)}$$

$$\Delta M_B^{(1)} \equiv \underline{\frac{1}{2}\left(Z_1 \langle r_{\mathrm{ch},1}^2 \rangle + Z_{-1} \langle r_{\mathrm{ch},\text{-}1}^2 \rangle\right) - Z_0 \langle r_{\mathrm{ch},0}^2 \rangle}$$

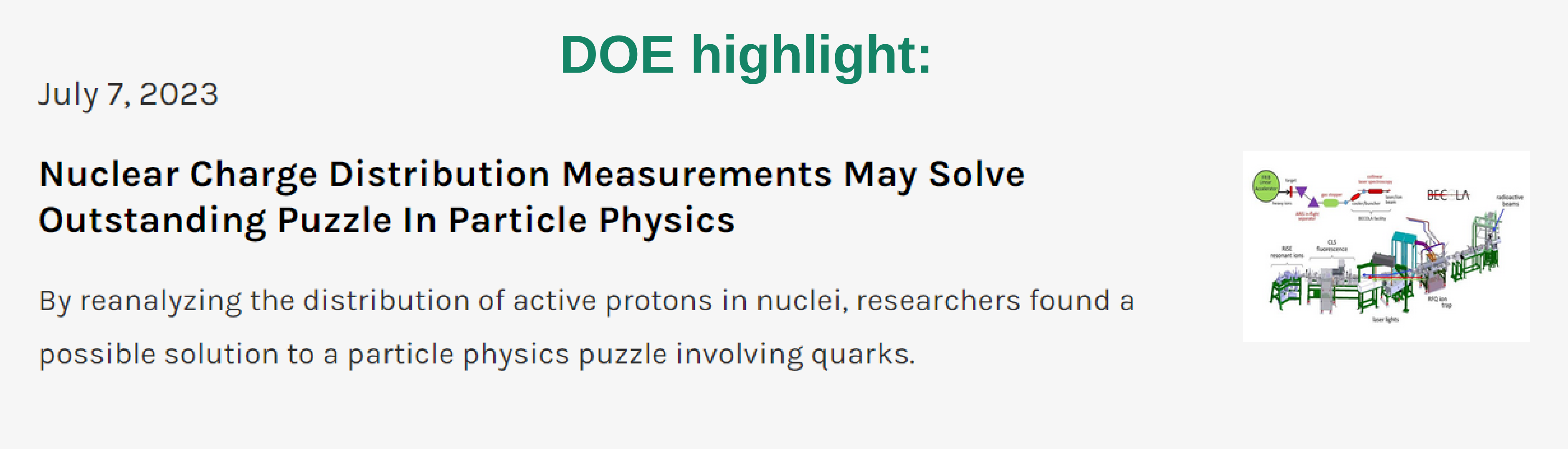
THREE charge radii

signify ISB. No double suppression!

Seng and Gorchtein, 2023 PLB

A	$R_{\mathrm{Ch},-1}$ (fm)	$R_{\mathrm{Ch},0}$ (fm)	$R_{\mathrm{Ch},1}$ (fm)
10	${}^{10}_{6}\mathrm{C}$	${}^{10}_{5}\mathrm{B}(\mathrm{ex})$	${}^{10}_{4}\mathrm{Be}$: 2.3550(170)[a]
14	${}^{14}_{8}\mathrm{O}$	${}^{14}_{7}\mathrm{N}(\mathrm{ex})$	${}^{14}_{6}\mathrm{C}$: 2.5025(87)[a]
18	${}^{18}_{10}\mathrm{Ne}$: 2.9714(76)[a]	${}^{18}_{9}\mathrm{F}(\mathrm{ex})$	${}^{18}_{8}\mathrm{O}$: 2.7726(56)[a]
22	${}^{22}_{12}\mathrm{Mg}$: 3.0691(89)[b]	${}^{22}_{11}\mathrm{Na}(\mathrm{ex})$	${}^{22}_{10}\mathrm{Ne}$: 2.9525(40)[a]
26	${}^{26}_{14}\mathrm{Si}$	${}^{26m}_{13}\mathrm{Al}$	${}^{26}_{12}\mathrm{Mg}$: 3.0337(18)[a]
30	${}^{30}_{16}\mathrm{S}$	${}^{30}_{15}\mathrm{P}(\mathrm{ex})$	${}^{30}_{14}\mathrm{Si}$: 3.1336(40)[a]
34	${}^{34}_{18}\mathrm{Ar}$: 3.3654(40)[a]	${}^{34}_{17}\mathrm{Cl}$	${}^{34}_{16}\mathrm{S}$: 3.2847(21)[a]
38	${}^{38}_{20}\mathrm{Ca}$: 3.467(1)[c]	${}^{38m}_{19}\mathrm{K}$: 3.437(4)[d]	${}^{38}_{18}\mathrm{Ar}$: 3.4028(19)[a]
42	${}^{42}_{22}\mathrm{Ti}$	${}^{42}_{21}\mathrm{Sc}$: 3.5702(238)[a]	${}^{42}_{20}\mathrm{Ca}$: 3.5081(21)[a]
46	${}^{46}_{24}\mathrm{Cr}$	${}^{46}_{23}\mathrm{V}$	${}^{46}_{22}\mathrm{Ti}$: 3.6070(22)[a]
50	${}^{50}_{26}\mathrm{Fe}$	${}^{50}_{25}\mathrm{Mn}$: 3.7120(196)[a]	${}^{50}_{24}\mathrm{Cr}$: 3.6588(65)[a]
54	${}^{54}_{28}\mathrm{Ni}$: 3.738(4)[e]	${}^{54}_{27}\mathrm{Co}$	${}^{54}_{26}\mathrm{Fe}$: 3.6933(19)[a]
62	${}^{62}_{32}\mathrm{Ge}$	${}^{62}_{31}\mathrm{Ga}$	${}^{62}_{30}\mathrm{Zn}$: 3.9031(69)[b]
66	${}^{66}_{34}\mathrm{Se}$	${}^{66}_{33}\mathrm{As}$	${}^{66}_{32}\mathrm{Ge}$
70	${}^{70}_{36}\mathrm{Kr}$	${}^{70}_{35}\mathrm{Br}$	${}^{70}_{34}\mathrm{Se}$
74	${}^{74}_{38}\mathrm{Sr}$	${}^{74}_{37}\mathrm{Rb}$: 4.1935(172)[b]	${}^{74}_{36}\mathrm{Kr}$: 4.1870(41)[a]

DOE highlight:



July 7, 2023

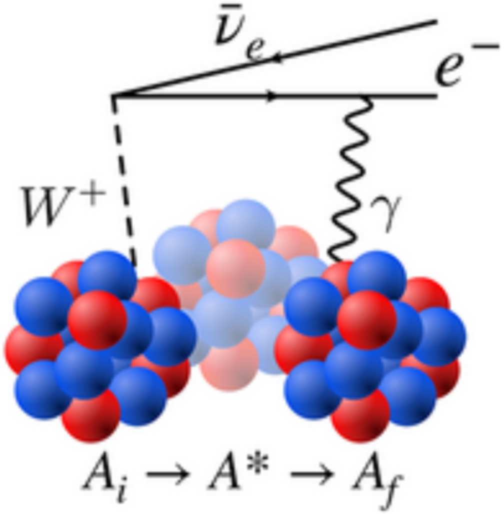
Nuclear Charge Distribution Measurements May Solve Outstanding Puzzle In Particle Physics

By reanalyzing the distribution of active protons in nuclei, researchers found a possible solution to a particle physics puzzle involving quarks.

Experimental opportunities:

- Nuclear charge radii measurements: **FRIB, TRIUMF, PSI** (muX) ...
- Recoil effects in nuclear beta decay: **FRIB** (SALER)…
- Neutron skin: **JLab, Mainz** (P2)...

Superallowed decays at loop level

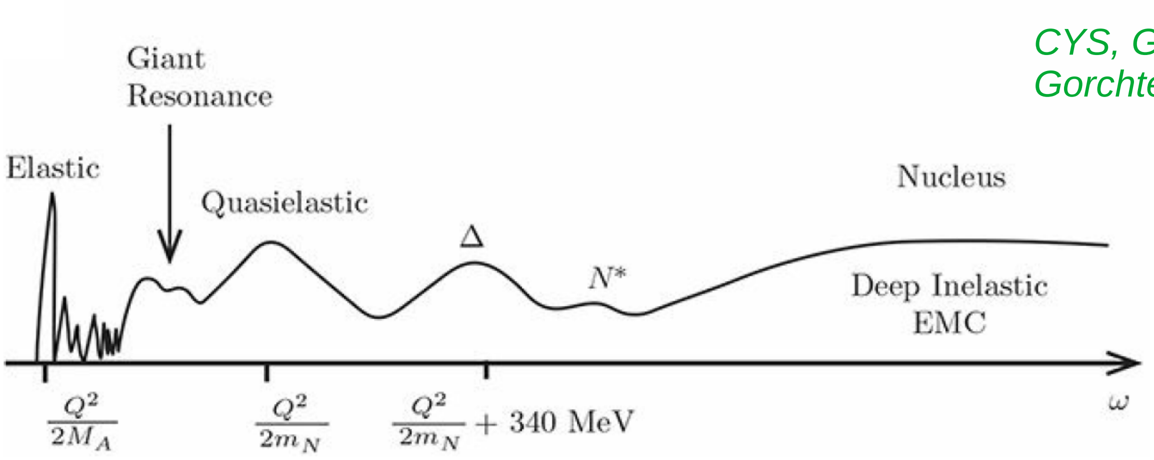


One-loop **radiative corrections** probe QCD at all scales:

- Parton scale (>> 1GeV) : Theoretically calculable
- Hadron scale (~ 1GeV) : Data from lepton-nucleon scattering + lattice QCD
- Nuclear scale (~100 MeV) : Most uncertain, requires many-body calculations

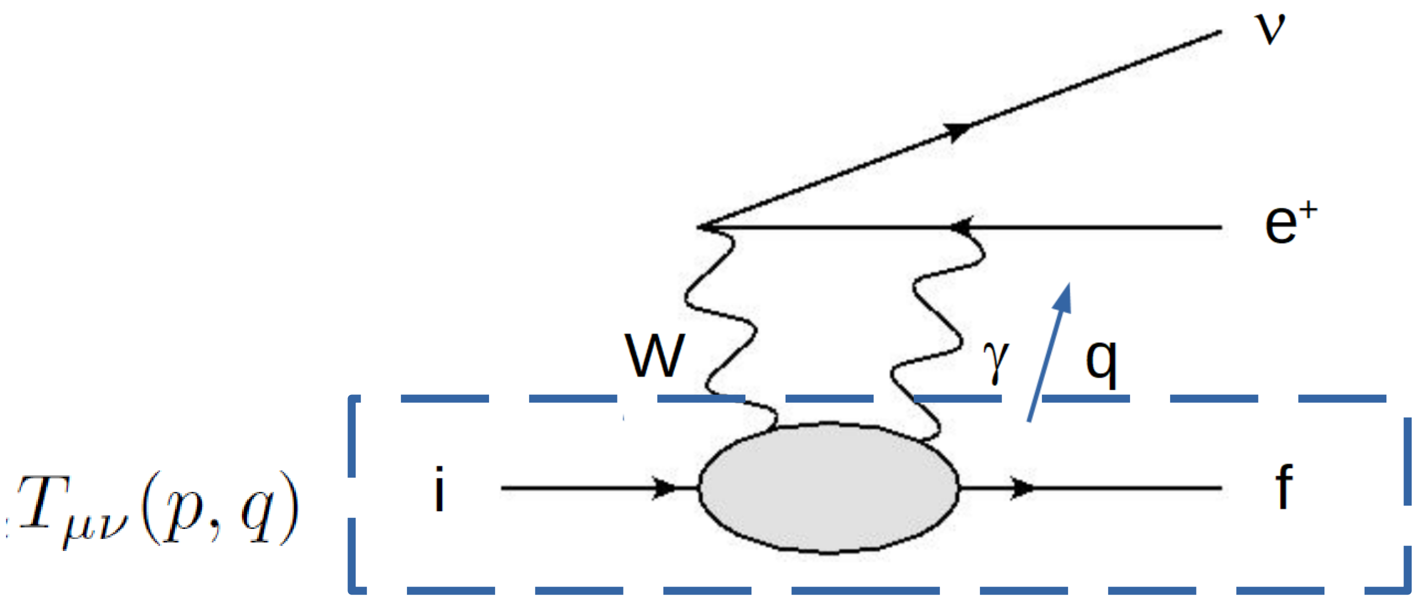
Previous shell-model-based calculations had missed important nuclear effects

CYS, Gorchtein and Ramsey-Musolf, 2019 PRD; Gorchtein, 2019 PRL



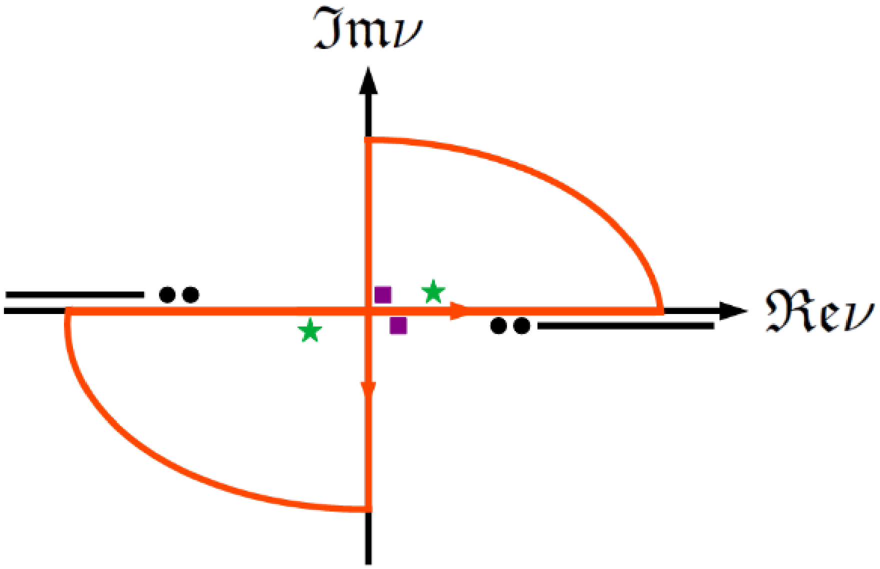
“Generalized forward Compton tensor”:

$$\begin{aligned} T^{\mu\nu}(p,q) &= \frac{1}{2}\int d^4x e^{iq\cdot x}\langle\phi_f(p)|T[J^{\mu}_{\text{em}}(x)\left(J^{\dagger\nu}_{W}(0)\right)_A]|\phi_i(p)\rangle \\ &= \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2M\nu}T_3(\nu,Q^2) \end{aligned}$$



The loop diagram depends on the **invariant function T_3**

Performing the q_0-integral using **Wick rotation**



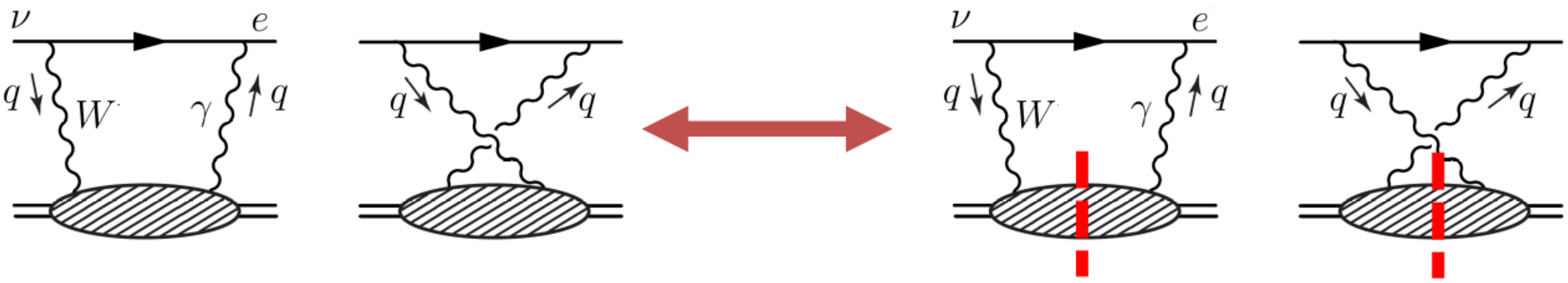
$(\nu \equiv q_0)$

$$\boxed{\Box^b_{\gamma W} = \underbrace{(\Box^b_{\gamma W})_{\mathrm{Wick}} + (\Box^b_{\gamma W})_{\mathrm{res},e}}_{\text{Regular to } E_e} + \underbrace{(\Box^b_{\gamma W})_{\mathrm{res},T_3}}_{\text{“Residue contribution”, singular to } E_e}}$$

Expansion of the **“regular” terms** in powers of electron energy:

$$\boxed{(\Box^b_{\gamma W})_{\text{Wick}} + (\Box^b_{\gamma W})_{\text{res},e} = \boxminus_0 + \boxminus_1 E_e + \mathcal{O}(E_e^2)}$$

The coefficients can be related to **nuclear response functions** through **dispersion relation** *Seng and Gorchtein, 2023 PRC*



$$R^{xy}(\nu, Q^2) \sim \sum_X \delta(M + q_0 - E_X)\langle f|J^x_{\text{em}}|X\rangle\langle X|J^y_W|i\rangle$$

which serves as starting point for ab-initio nuclear theory calculation

- **Light nuclei:** NCSM, QMC...
- **Medium nuclei:** IMSRG, Coupled-Cluster, NLEFT...

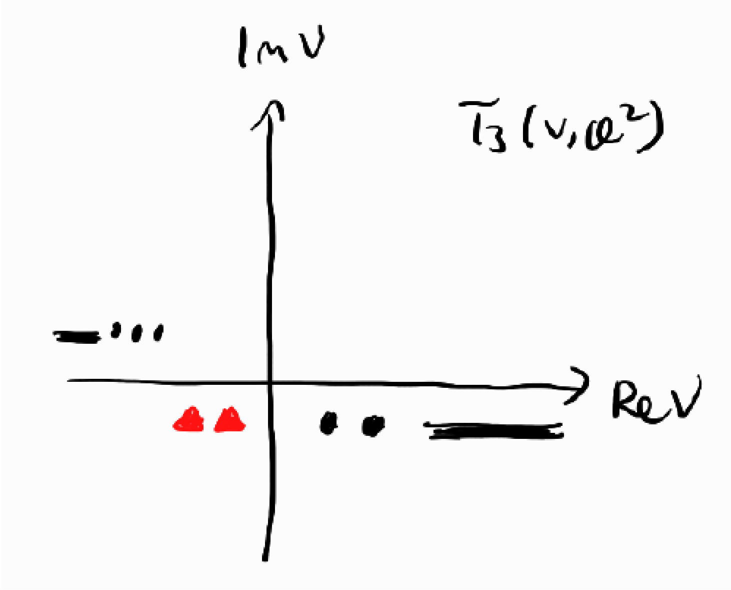
Residue contribution

(First pointed out by Michael Gennari, TRIUMF)

For A=10,14,18,22,26,30 and 38, the $T_z=0$, $J^p=0^+$ state is an excited state.

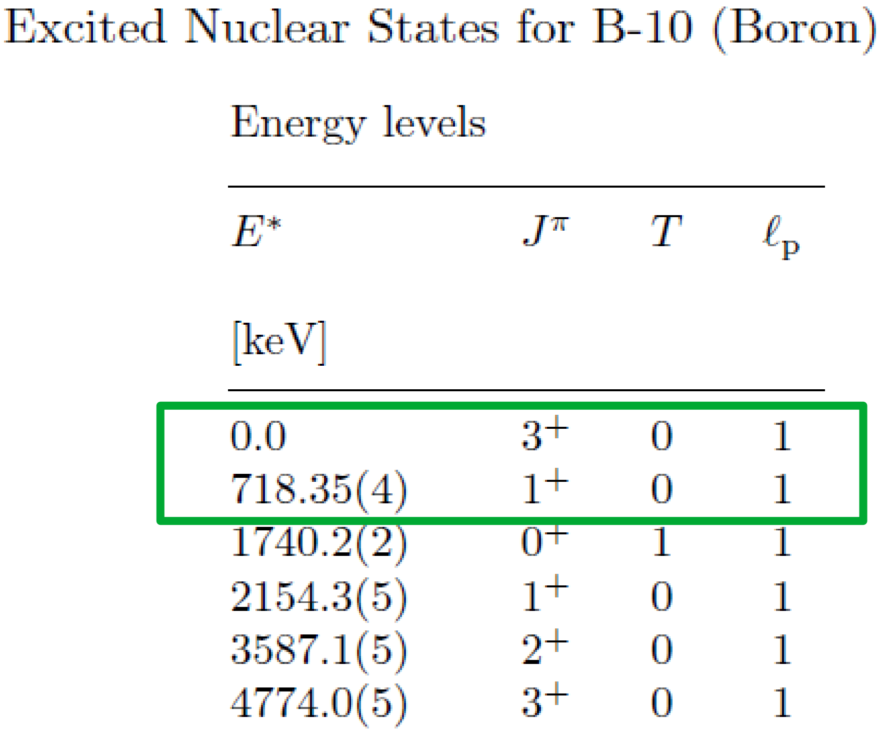
$T_z=0$ initial nucleus → extra poles of T_3 in the first quadrant;
$T_z=0$ final nucleus → extra poles of T_3 in the third quadrant

For $^{10}C \rightarrow {}^{10}B$:



$$\boxed{\nu_k = \sqrt{M_k^2 + \mathbf{q}^2} - M - i\varepsilon}$$

In third quadrant if $\mathbf{q}^2 < M^2 - M_k^2$


Excited Nuclear States for B-10 (Boron)

Energy levels

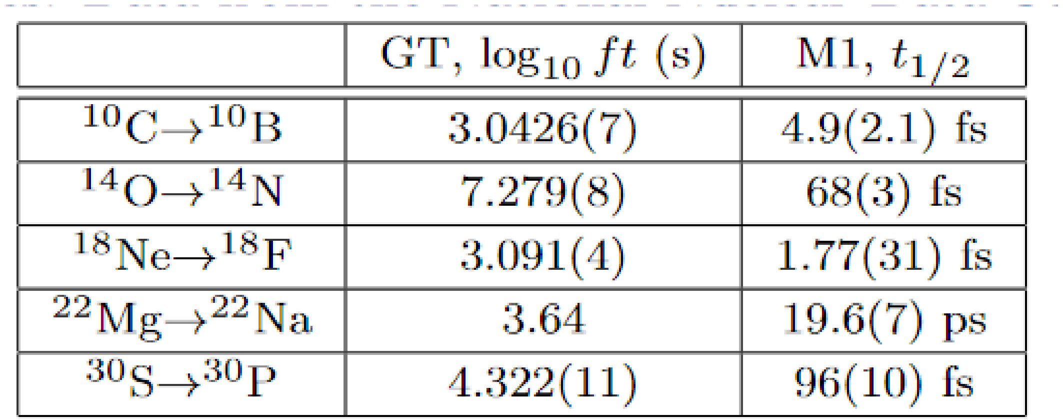
E^* [keV]	J^π	T	ℓ_p
0.0	3^+	0	1
718.35(4)	1^+	0	1
1740.2(2)	0^+	1	1
2154.3(5)	1^+	0	1
3587.1(5)	2^+	0	1
4774.0(5)	3^+	0	1

Residue of T_3 due to a low-lying $J^P=1^+$ state:

$$\mathfrak{Res}\,T_3 \propto \langle f(0^+)|J^x_{\mathrm{em}}(\vec{q})|f(1^+)\rangle\langle f(1^+)|J^y_W(-\vec{q})|i(0^+)\rangle$$

M1-transition **GT-transition**

Matrix elements can be inferred from **M1** and **GT**-transition rates!



	GT, $\log_{10} ft$ (s)	M1, $t_{1/2}$
^{10}C$\to^{10}$B	3.0426(7)	4.9(2.1) fs
^{14}O$\to^{14}$N	7.279(8)	68(3) fs
^{18}Ne$\to^{18}$F	3.091(4)	1.77(31) fs
^{22}Mg$\to^{22}$Na	3.64	19.6(7) ps
^{30}S$\to^{30}$P	4.322(11)	96(10) fs

From IAEA / NNDC website

Future experimental opportunities??

Ab-initio study of ^{10}C → ^{10}B radiative corrections with **no-core shell model**

Collaboration: Michael Gennari, Petr Navratil (TRIUMF)

$$|V_{ud}|^2 \propto \frac{1}{1+\delta_{\mathrm{NS}}}$$

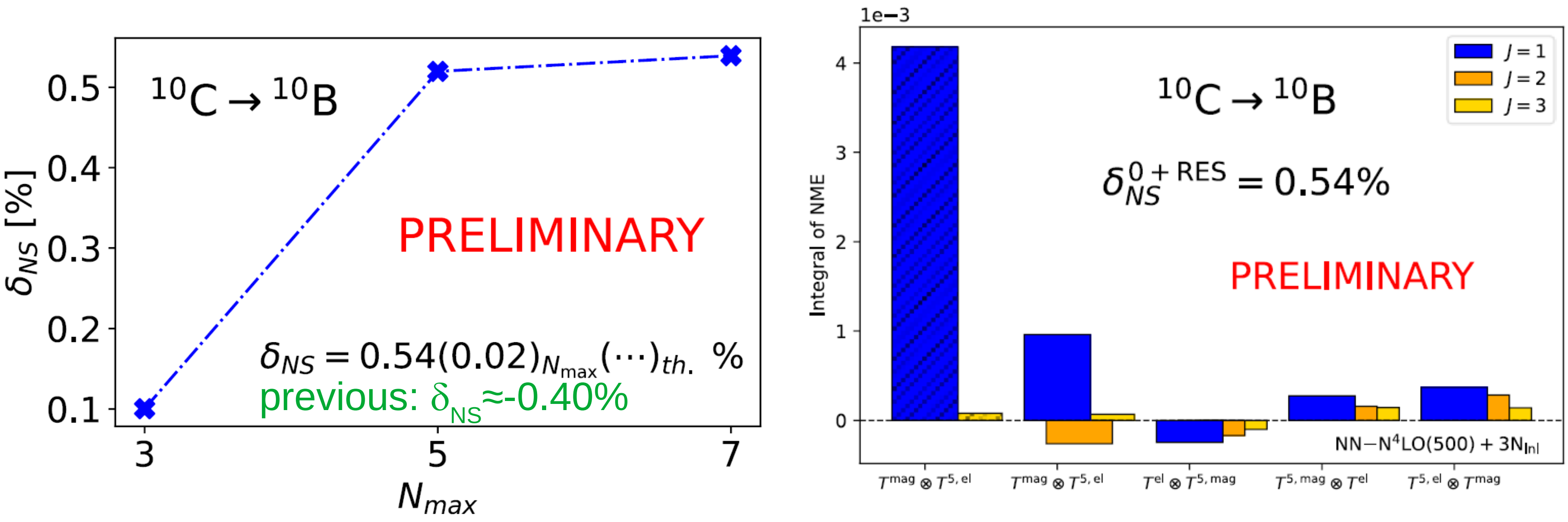


Image credit: Michael Gennari

Dominated by the **"residue contribution"**, which agrees with experiment, but was **completely absent in earlier studies!**

Summary

- **Nuclear beta decays** offer precision tests of SM
  - **Various anomalies** hint at the existence of **BSM physics**
  - **High-precision calculations** test our understanding of **QCD**
- Theory foundations are developed to relate the nuclear decay form factor (Fermi matrix element and q^2-dependence) to measurable **electroweak nuclear radii**
- Rigorous dispersive formalism is constructed to enable **ab-initio studies** of **nucleus-dependent radiative corrections**; **M1** and **GT** rates for light nuclei are useful experimental inputs
- Research may foster new theory-experiment interplay, and could significantly alter the landscape of precision physics in the charged weak sector

Thanks for your attention!