Helium burning nuclear reactions and stellar evolution: low-mass stars

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He-burning reactions

- Triple α: ignition T $\simeq$ 1.2 x 10^8 K, energy $\simeq$ 7,27 MeV

$$^4He + {}^4He \rightarrow {}^8Be$$
$$^8Be + {}^4He \rightarrow {}^{12}C + \gamma$$

- $^{12}C(\alpha,\gamma)^{16}O$: energy $\simeq$ 7,16 MeV

- ^4He converted into a $^{12}C/^{16}O$ mixture

Triple α

The uncertainty in the reaction rate at the He-burning temperatures of low mass stars:

- NACRE (Angulo et al. 1999): 25%
- Fynbo et al. 2005: 12%

We adopted:

- Fynbo et al. 2005 for the reference models
- Perturbed models: ± 12%, ± 24%
- Additional models: + 34% (Kibedi et al. 2020)

$^{12}C(\alpha, \gamma)^{16}O$

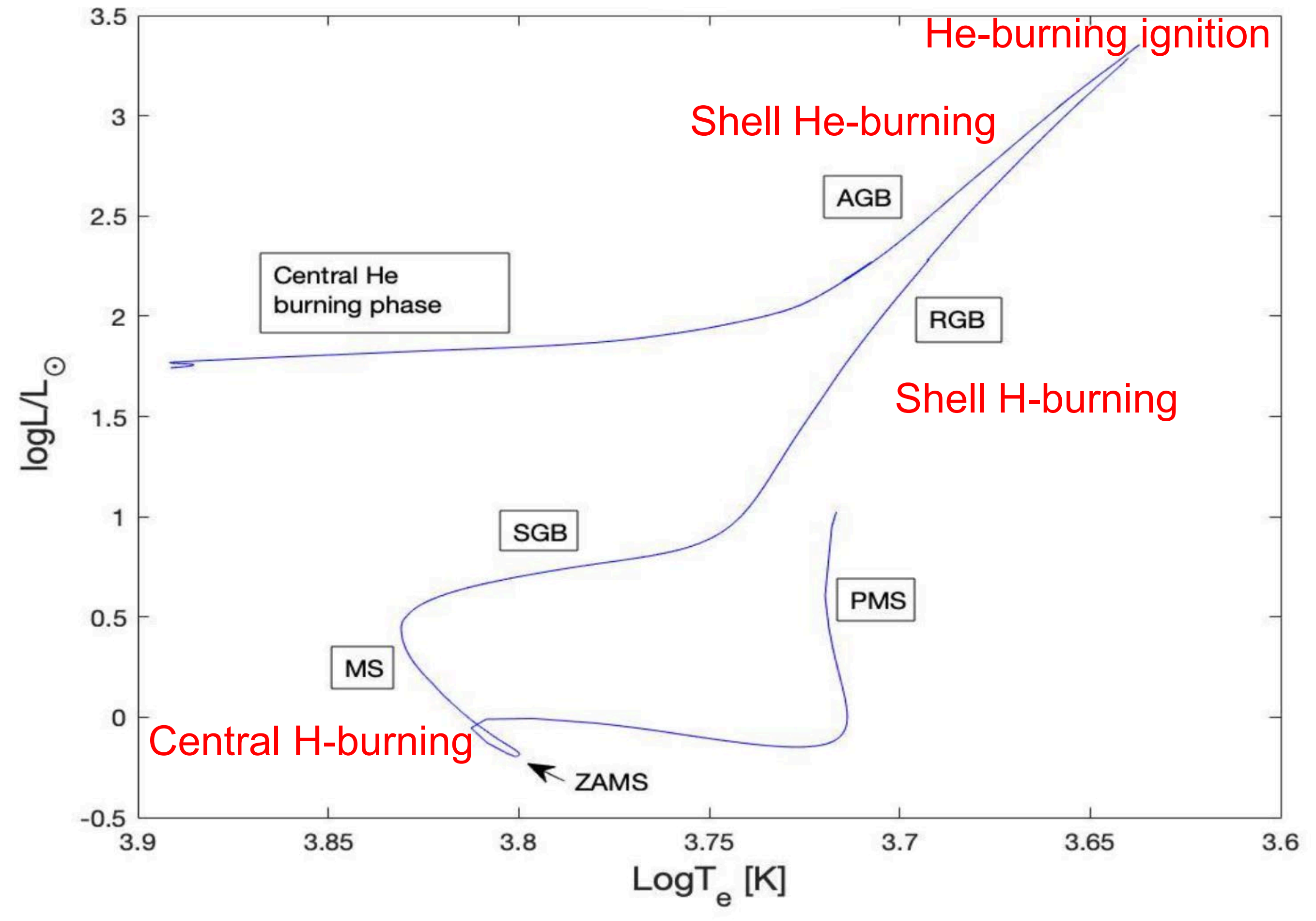
The uncertainty of the reaction rate at the He-burning temperatures of low mass stars:

- NACRE (Angulo et al. 1999): 40%
- Kunz et al. 2002: 35%
- Hammer et al. 2005: 30%
- deBoer et al. 2017: 20%

We adopted:

- deBoer et al. 2017 for the reference models
- Perturbed models: ± 20% , ± 28%, ±35%

Stellar evolution in a nutshell: low-mass stars



M= 0.8 M_o Z=0.0001 Y=0.246

Stellar evolution in a nutshell

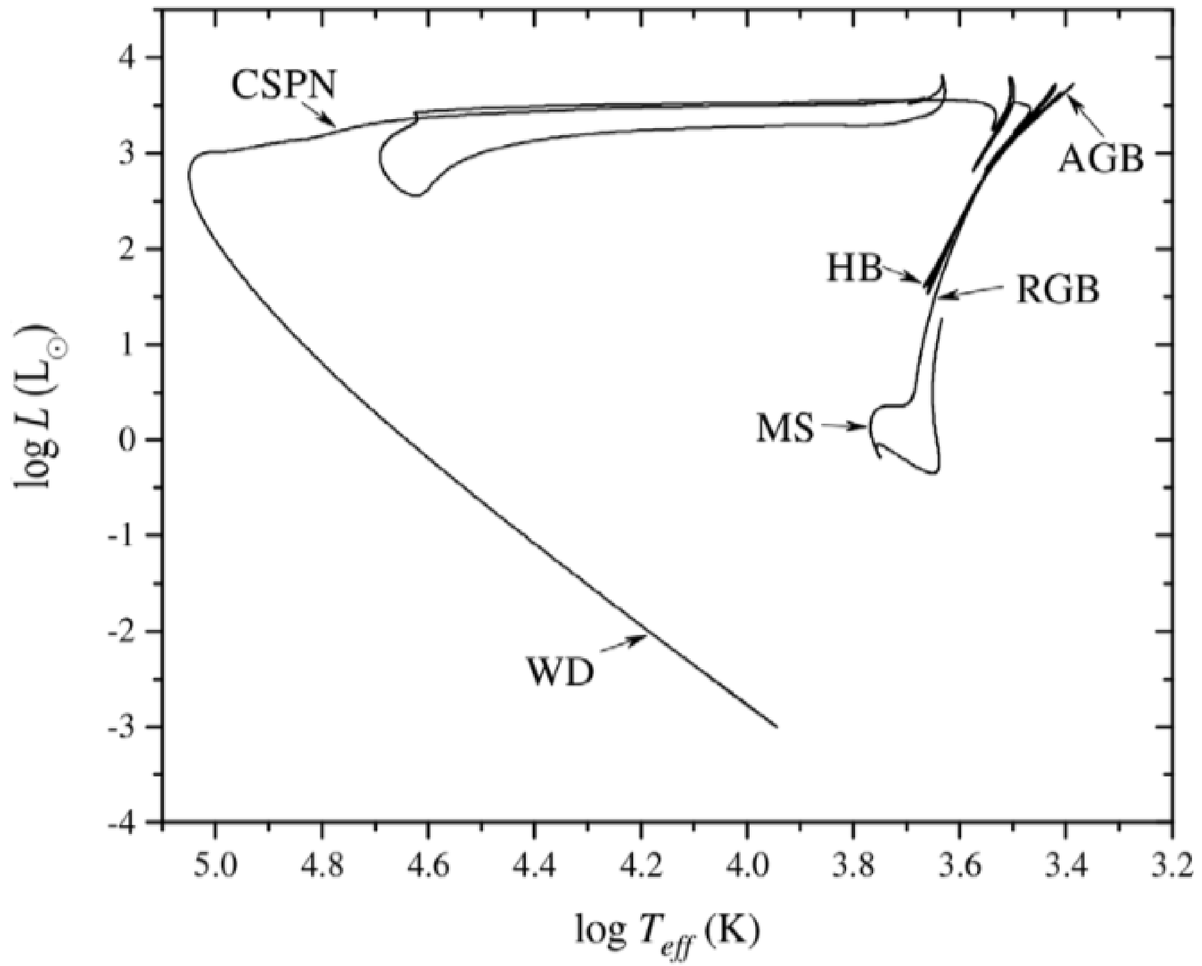
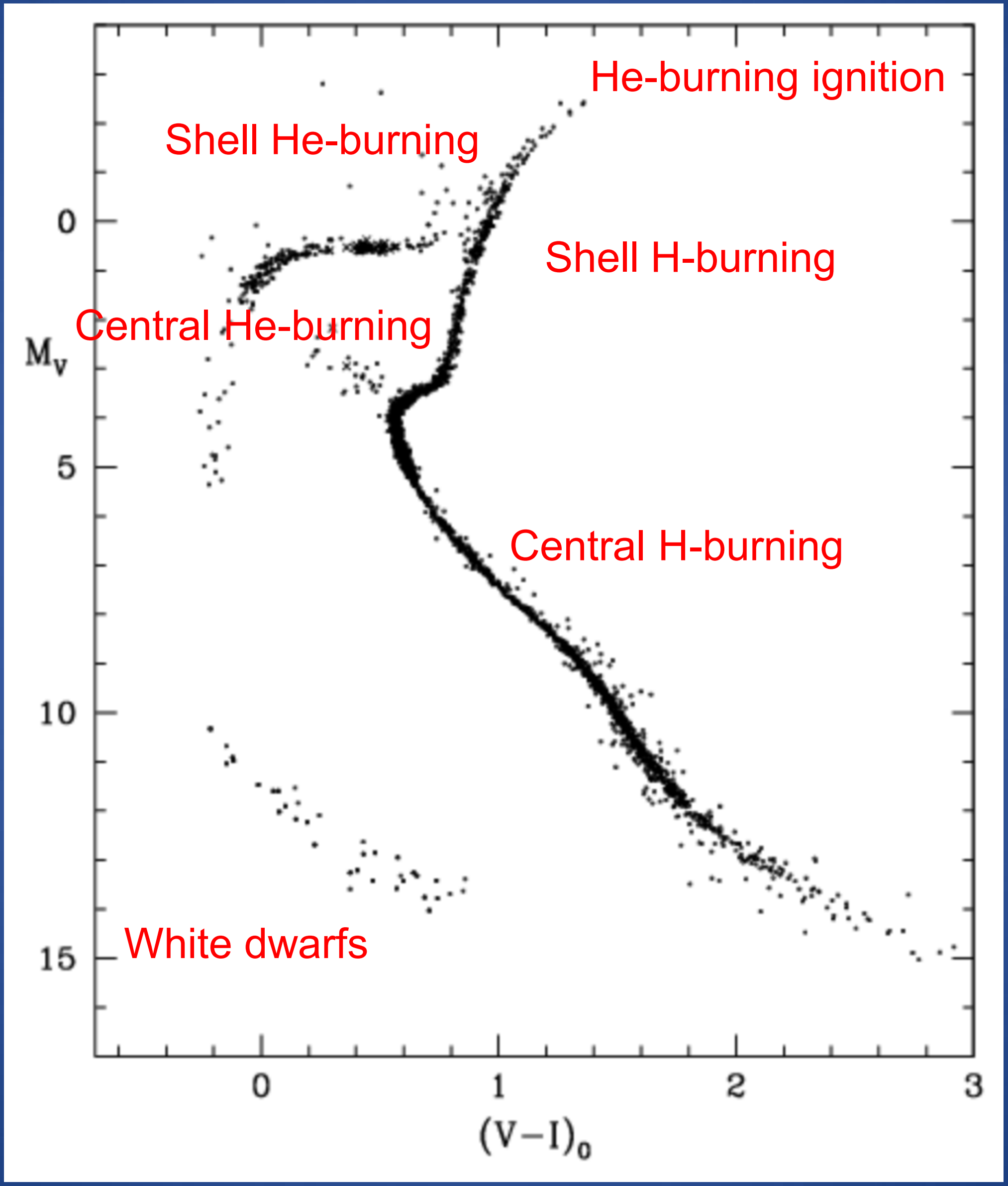


Figure 20.1. Complete evolutionary path in the HRD of a 1 $M_\odot$ model from the PMS to a cool WD.

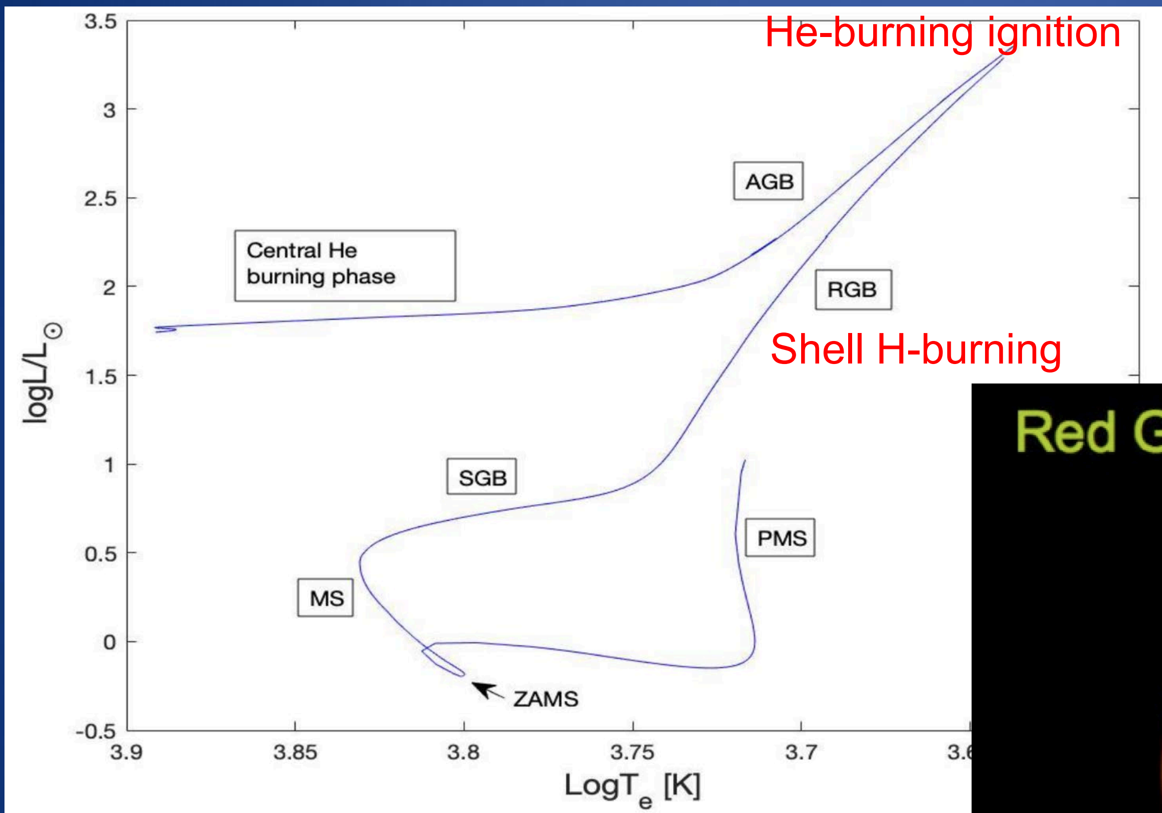
McDonald

Globular clusters

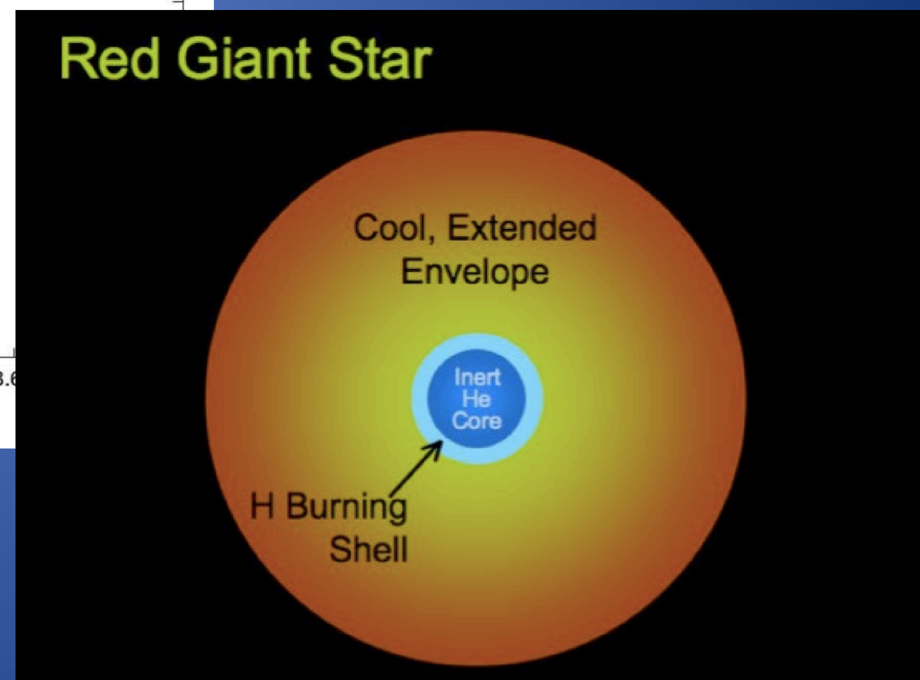


Template CMD M3, M55, M68, NGC6397, NGC2419 (Harris 2003)

He-burning ignition in low-mass stars: He flash



- Important distance indicator
- Affected by 3α but not by $^{12}C(\alpha,\gamma)$



An increase of the 3α rate leads to a decrease of the RGB tip luminosity

Not to scale

He-burning ignition in low-mass stars
Impact of 3α uncertainty on RGB tip

	Standard rate	Rate +12%	Rate +24%	Rate +34%	Rate −12%	Rate −24%
$\log L_{tip}/L_{\odot}$	3.2878	3.2835	3.2796	3.2767	3.2927	3.2982
P. V.		−0.13%	−0.25%	−0.34%	+0.14%	+0.33%

M= 0.67 M_o, progenitor M= 0.8 M_o Z=0.0001 Y=0.246
(Tognini et al 2023, submitted)

The effect is rather modest

How does it compare to other uncertainty sources?

Stellar models depend on…

- input physics (EOS, radiative and conductive opacity, nuclear reaction cross sections, neutrino emission rates, etc.)

- Macroscopic processes (super-adiabatic convection, overshooting, diffusion, etc.)

- initial chemical composition (He abundance, metallicity, elements mixture)

Cumulative theoretical uncertainty

In Valle et al. 2013, we estimated the cumulative uncertainty affecting stellar models computing a very large number of models:

- Sistematically and simultaneously varying the main input physics within their current range of uncertainty
- Covering all the possible combinations of simultaneously perturbed input physics

(see also Chaboyer et al. 1998, Fields et al. 2016, Tognelli et al. 2011)

Cumulative theoretical uncertainty

Table 1. Physical inputs perturbed in the calculations and their assumed uncertainty.

parameter	description	uncertainty
p_1	$^1H(p,\nu e^+)^2H$ reaction rate	3%
p_2	$^{14}N(p,\gamma)^{15}O$ reaction rate	10%
p_3	radiative opacity k_r	5%
p_4	microscopic diffusion velocities	15%
p_5	triple-α reaction rate	20%
p_6	neutrino emission rate	4%
p_7	conductive opacity k_c	5%

Valle, Dell'Omodarme, Prada Moroni, Degl'Innocenti 2013, A&A 549, A50

Cumulative theoretical uncertainty

A recent update of Valle et al. 2013

Table 1. Physical inputs perturbed in the calculations and their assumed uncertainty.

parameter	description	uncertainty	
p_1	$^{1}H(p,\nu e^{+})^{2}H$ reaction rate	3%	1%
p_2	$^{14}N(p,\gamma)^{15}O$ reaction rate	10%	8%
p_3	radiative opacity k_{r}	5%	
p_4	microscopic diffusion velocities	15%	
p_5	triple-α reaction rate	20%	12%
p_6	neutrino emission rate	4%	
p_7	conductive opacity k_{c}	5%	

He-burning ignition in low-mass stars
Cumulative uncertainty

Quantity	Range half width	%
Log L_{tip}/L_o	0,026 dex	±1

Luminosity of the RGB tip:

1. Radiative opacity impact: ∓ 0,015 dex
2. Triple α impact: ∓ 0,0046 dex ⇒ small contribution

He-burning ignition in low-mass stars
Cumulative uncertainty

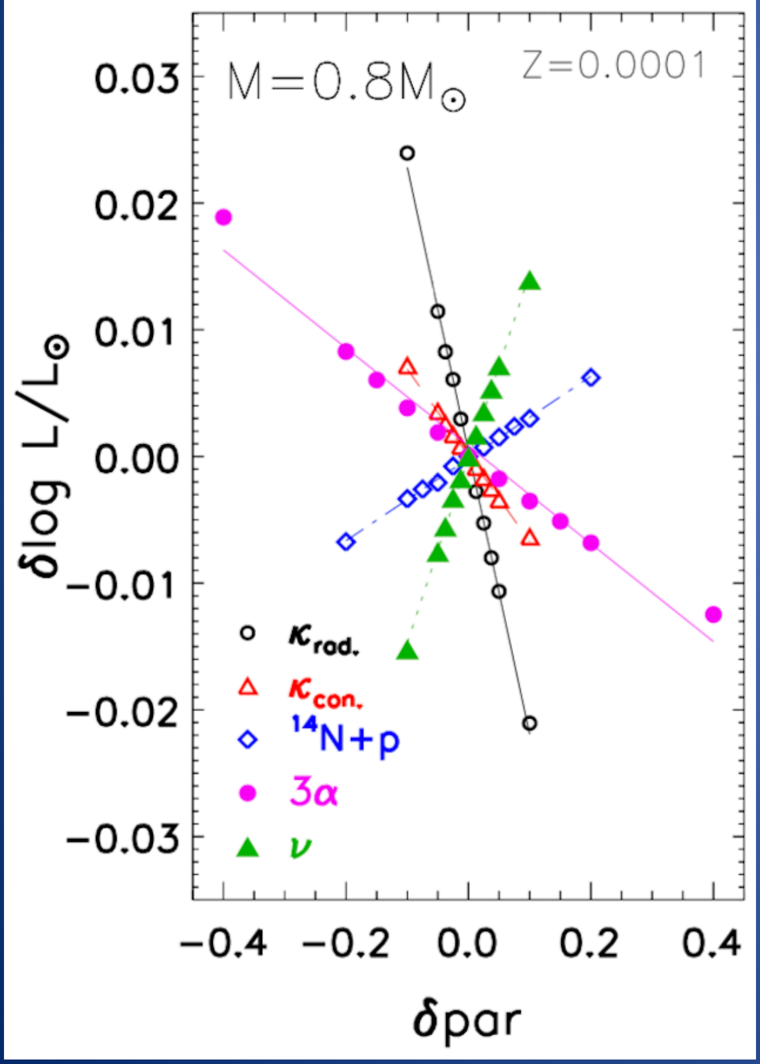
Saltas & Tognelli 2022

Table 1. List of the quantities analysed in this work with the related uncertainty.

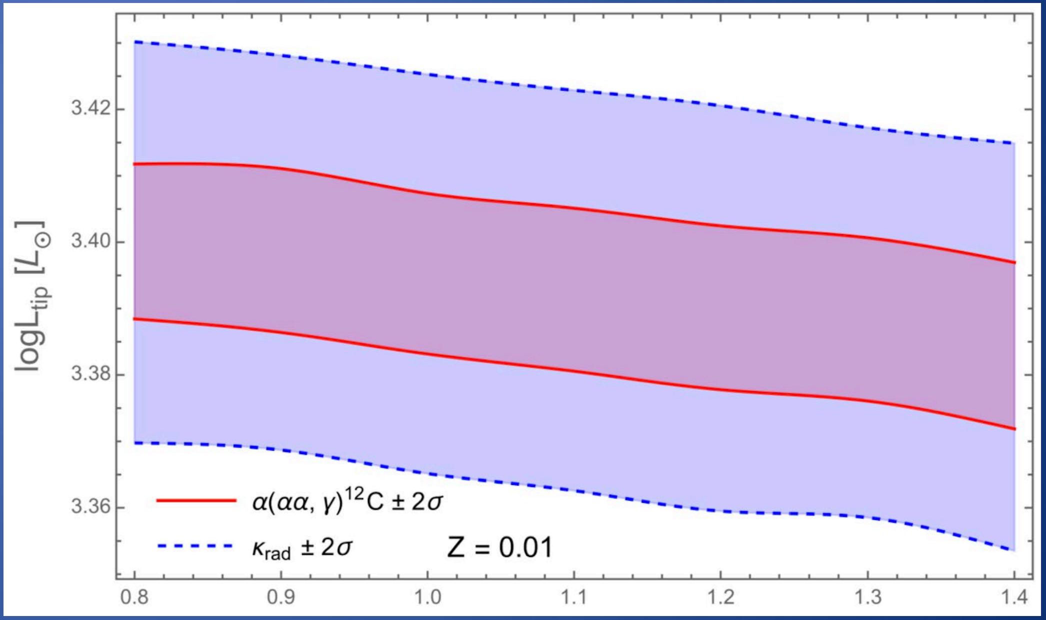
Quantity (G_j)	Perturbation (δ_{G_j})	References
Radiative opacity ($\kappa_{\rm rad}$)	$\pm$5 per cent	VD13, T18, S20
Conductive opacity ($\kappa_{\rm con}$)	$\pm$5 per cent	VD13, S20
^{14}N(p,γ)^{15}O	$\pm$10 per cent	LUNA05, VC13, VD13, S20
$\alpha(\alpha\alpha,\gamma)^{12}$C	$\pm$20 per cent	NACRE99, VC13, VD13, S20
p(p, e$^+$, ν)d	$\pm$1 per cent	AD11, MSV13
^{3}He(^{3}He, 2p)α	$\pm$4 per cent	AD11
^{3}He(α, γ)^{7}Be	$\pm$7 per cent	AD11
^{7}Be(e$^-$, ν)^{7}Li	$\pm$ 2 per cent	AD11
^{7}Be(p, α)α	$\pm$10 per cent	AD11
Electron screening [$f_{\rm sc}(3\alpha)$]	Weak	(See text)
Electron screening [$f_{\rm sc}(^{14}{\rm N})$]	Weak	(See text)
Neutrino energy loss (ν)	$\pm$5 per cent	VC13, VD13, S20
Outer BCs	$\pm$5 per cent	VC13, S20
Diffusion coefficients ($v_{\rm dif}$)	$\pm$15 per cent	TBL94, VD13
Core overshooting ($\beta_{\rm ov}$)	0, 0.15	(See text)
Mixing length ($\alpha_{\rm ML}$)	$\pm$0.2, $\pm$0.4	(See text)
Mass-loss ($\eta_{\rm Reim}$)	0, 0.2, 0.3, 0.4	(See text)
Primordial helium abundance ($\delta Y_{\rm p}$)	$\pm$0.0015	(See text)
Helium enrichment, $\delta(\Delta Y/\Delta Z)$	$\pm$1	(See text)

Notes. References: AD11 (Adelberger et al. 2011); LUNA05 (Imbriani et al. 2005); MSV13 (Marcucci, Schiavilla & Viviani 2013); NACRE99 (Angulo et al. 1999); VC13 (Viaux et al. 2013); VD13 (Valle et al. 2013a); T18 (Tognelli, Prada Moroni & Degl'Innocenti 2018); S20 (Straniero et al. 2020); and TBL94 (Thoul et al. 1994).

He-burning ignition in low-mass stars
Cumulative uncertainty



Saltas & Tognelli 2022



$1\sigma_{3\alpha}$= 20% $1\sigma_{k}$= 5%

(See also Serenelli et al. 2017, Straniero et al. 2020)

Zero Age Horizontal Branch (ZAHB)

initial stage of central He-burning of low-mass stars

- Important distance indicator
- Affected by 3α but not by $^{12}C(\alpha,\gamma)$

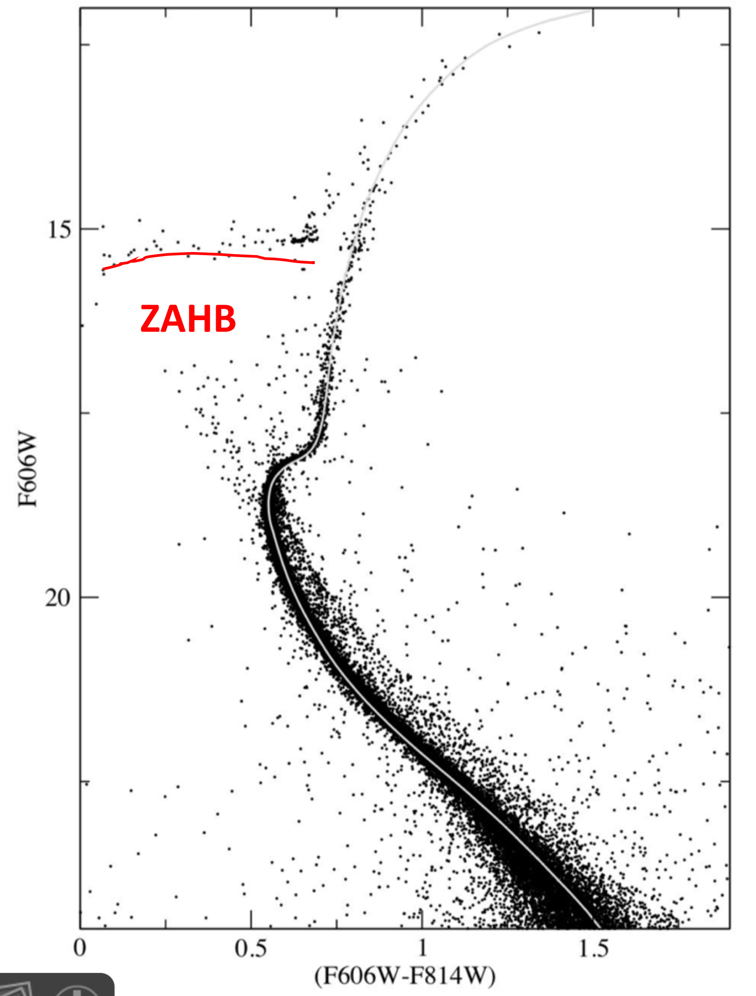


Figure 7. CMD for NGC 6362. The observations are fit with a [Fe/H] = −1.08 14.0 Gyr isochrone with an apparent distance modulus of 14.70 and a color excess of 0.09.

Paust et al. 2010

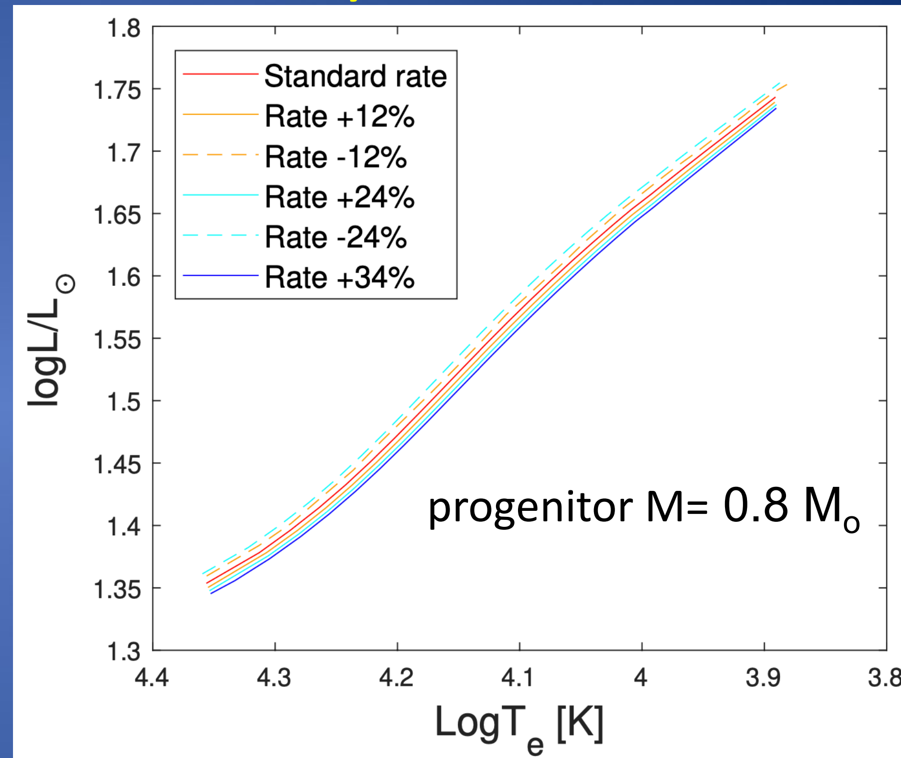
Zero Age Horizontal Branch (ZAHB)

initial stage of central He-burning of low-mass stars

Impact of 3α uncertainty

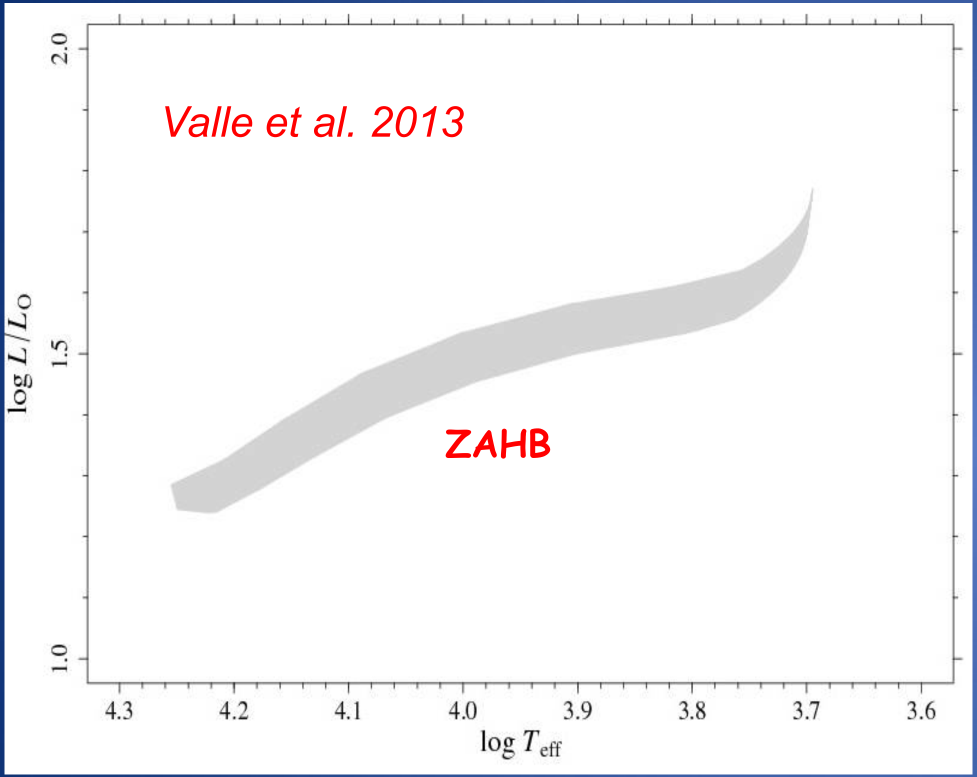
Tognini et al 2023, submitted

The effect is rather modest



$M = 0.67\ M_\odot$, $Y = 0.246$, $Z = 0.0001$	Rate +34%	Rate +24%	Rate +12%	Standard rate	Rate −12%	Rate −24%
$\log L_{\rm ZAHB}/L_\odot$	1.6880 −0.6%	1.6905 −0.5%	1.6943 −0.2%	1.6986	1.7032 +0.2%	1.7085 +0.6%

ZAHB
Cumulative uncertainty



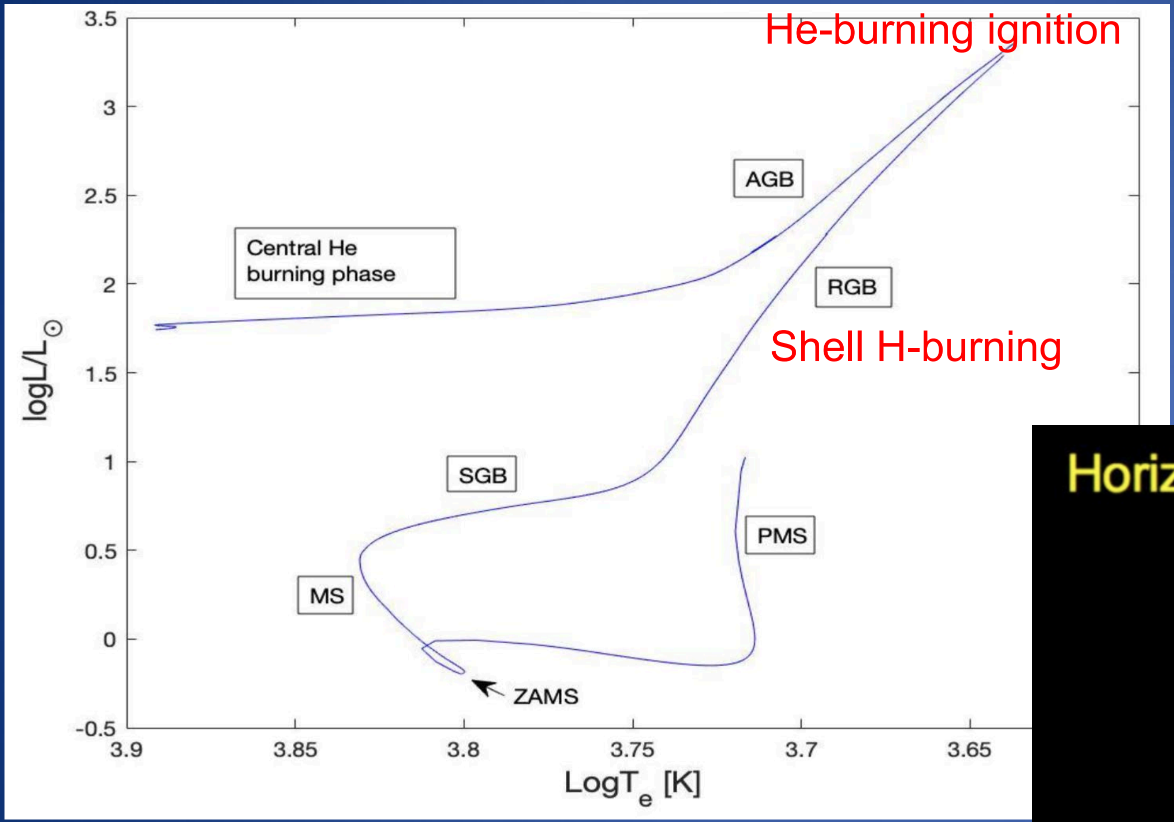
Update of Valle et al 2013:

Δ log $L_{ZAHB}/L_o = \pm 0.04$ dex

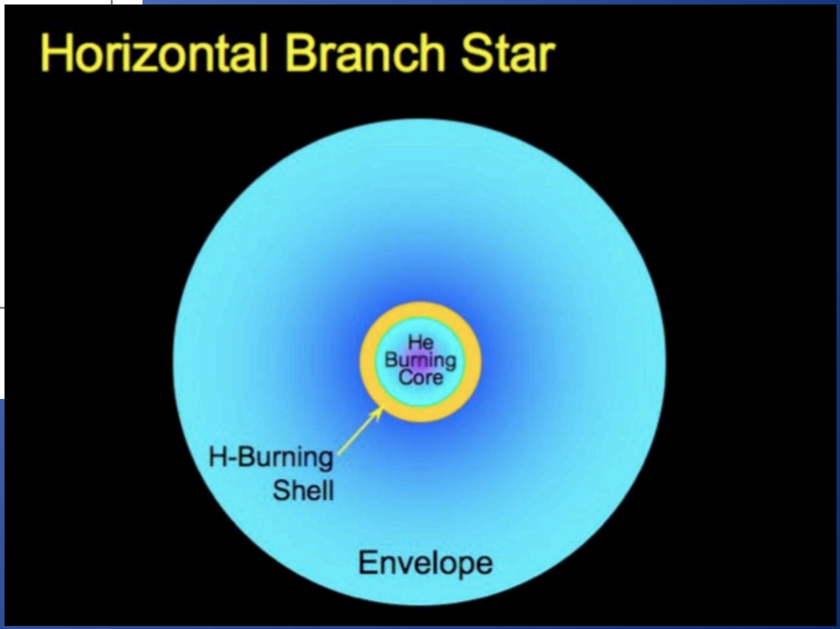
1. Radiative opacity impact: $\mp$ 0,022 dex
2. Triple α impact: $\mp$ 0,0044 dex ➡ small contribution

The current uncertainty in the triple α reaction rate is less relevant than that of the opacity for the ZAHB luminosity

Central He-burning phase

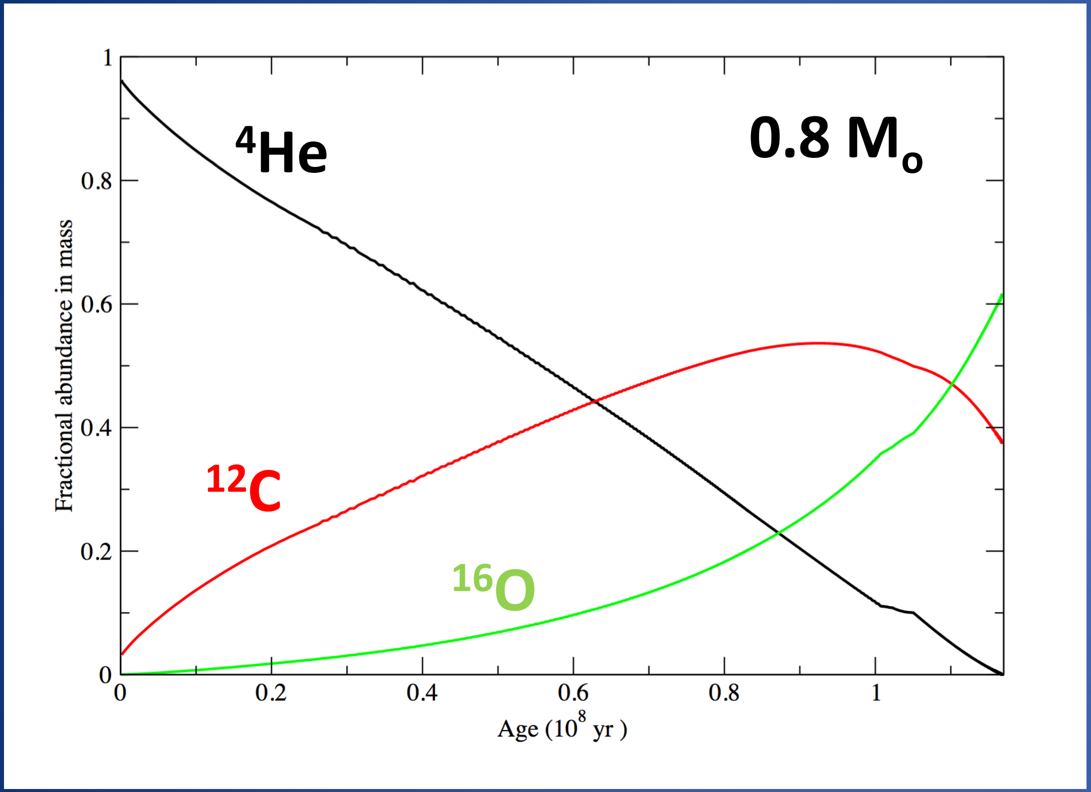


- Affected by both 3α and $^{12}C(\alpha,\gamma)$



Not to scale

Central He-burning phase



At the beginning:

- triple α dominates over the $^{12}C(\alpha,\gamma)$
- ^4He is primarily depleted by triple α and converted in ^{12}C

In the late part:

- $^{12}C(\alpha,\gamma)$ overcomes the triple α
- ^{12}C abundance decreases

Central He-burning phase

Triple α and $^{12}C(\alpha,\gamma)$ release a similar amount of energy per reaction but the latter consumes one α particle

An increase of the triple α rate leads to a decrease of:

- central He-burning lifetime
- CO core mass at the central He exhaustion
- O abundance at the central He exhaustion

An increase of the $^{12}C(\alpha,\gamma)$ rate leads to an increase of:

- central He-burning lifetime
- CO core mass at the central He exhaustion
- O abundance at the central He exhaustion

(see also Cassisi et al. 1998, 2001, 2003; Imbriani et al. 2001; Straniero et al. 2003; Weiss et al. 2005; Dotter & Paxton 2009; Valle et al. 2009, 2013; Fields et al. 2016; Pepper et al. 2022)

Central He-burning lifetime
Impact of changing the rate of:

triple α

$M = 0.67\ M_\odot$, $Y = 0.246$, $Z = 0.0001$	Rate +34%	Rate +24%	Rate +12%	Standard rate	Rate −12%	Rate −24%
t_{HB} [Myr]	97.9 −0.5%	98.0 −0.4%	98.2 −0.2%	98.4	99.8 +1.4%	98.8 +0.4%

$^{12}C(\alpha,\gamma)$

$M = 0.67\ M_\odot$, $Y = 0.246$, $Z = 0.0001$	Rate +35%	Rate +28%	Rate +20%	Standard rate	Rate −20%	Rate −28%	Rate −35%
t_{HB} [Myr]	101.2 +2.8%	100.8 +2.4%	100.1 +1.8%	98.4	96.4 −2.0%	95.4 −3.0%	94.8 −3.6%

progenitor M= 0.8 M_o Z=0.0001 Y=0.246
(Tognini et al 2023, submitted)

Central He-burning lifetime

The present uncertainties in the triple α and $^{12}C(\alpha,\gamma)$ reaction rates have a negligible effect on both the central He-burning lifetime and the final CO core mass

Central ^{12}C and ^{16}O abundances at the central He exhaustion Impact of changing the rate of:

triple α

$M = 0.67\ M_\odot,\ Y = 0.246,\ Z = 0.0001$						
	Rate +34%	Rate +24%	Rate +12%	Standard rate	Rate −12%	Rate −24%
$X_{^{12}C}$	0.440 +15%	0.426 +11%	0.405 +5%	0.384	0.360 −6%	0.331 −14%
$X_{^{16}O}$	0.560 −9%	0.574 −7%	0.595 −3%	0.616	0.640 +4%	0.669 +8%

^{12}C(α,γ)

$M = 0.67\ M_\odot,\ Y = 0.246,\ Z = 0.0001$							
	Rate +35%	Rate +28%	Rate +20%	Standard rate	Rate −20%	Rate −28%	Rate −35%
$X_{^{12}C}$	0.292 −22%	0.309 −18%	0.329 −13%	0.384	0.453 +18%	0.482 +26%	0.514 +34%
$X_{^{16}O}$	0.708 +15%	0.691 +12%	0.671 +9%	0.616	0.547 −11%	0.517 −16%	0.485 −21%

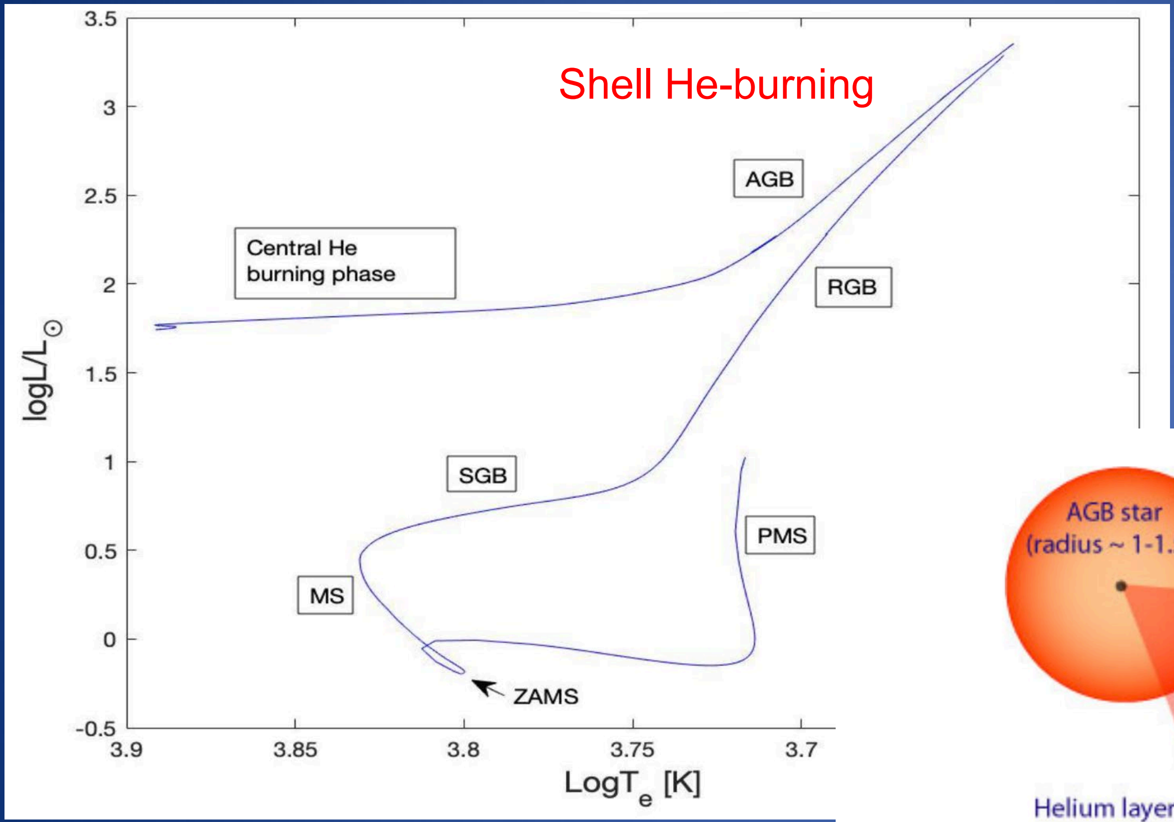
Tognini et al 2023, submitted

Central ^{12}C and ^{16}O abundances at the central He exhaustion

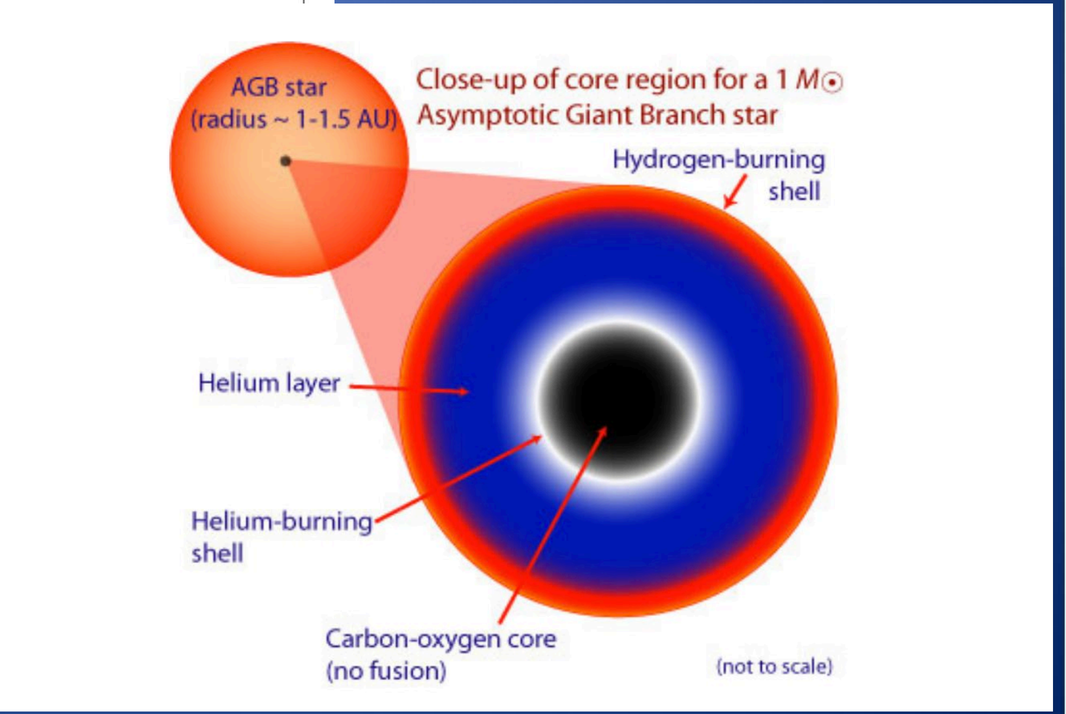
- The uncertainty in the $^{12}C(\alpha,\gamma)$ reaction rate has a larger impact than the uncertainty in the triple α rate

- The effect is less relevant than in the past

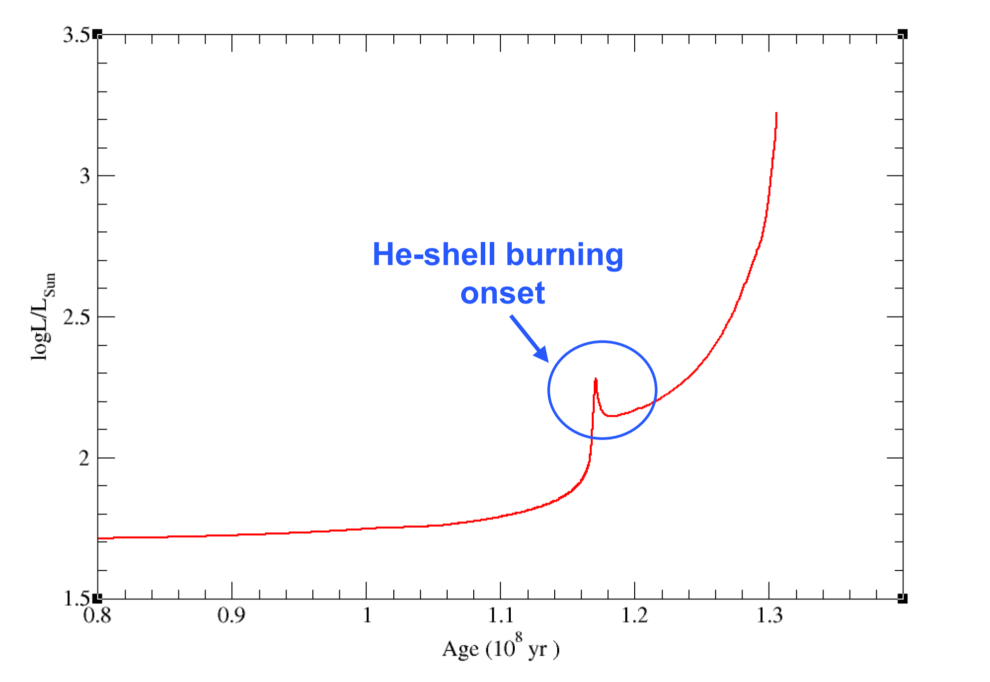
Early Asymptotic Giant Branch

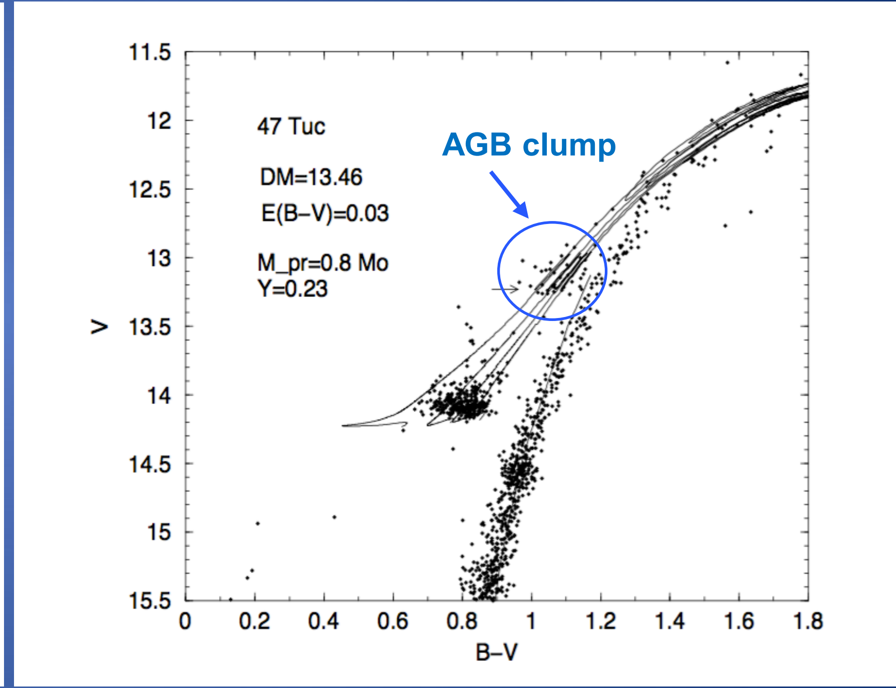


- Affected by both 3α and $^{12}C(\alpha,\gamma)$



Early Asymptotic Giant Branch





Cassisi et al. 2001

(see also Caputo et al. 1978; Catelan 2007; Cassisi & Salaris 2013)

Early AGB phase

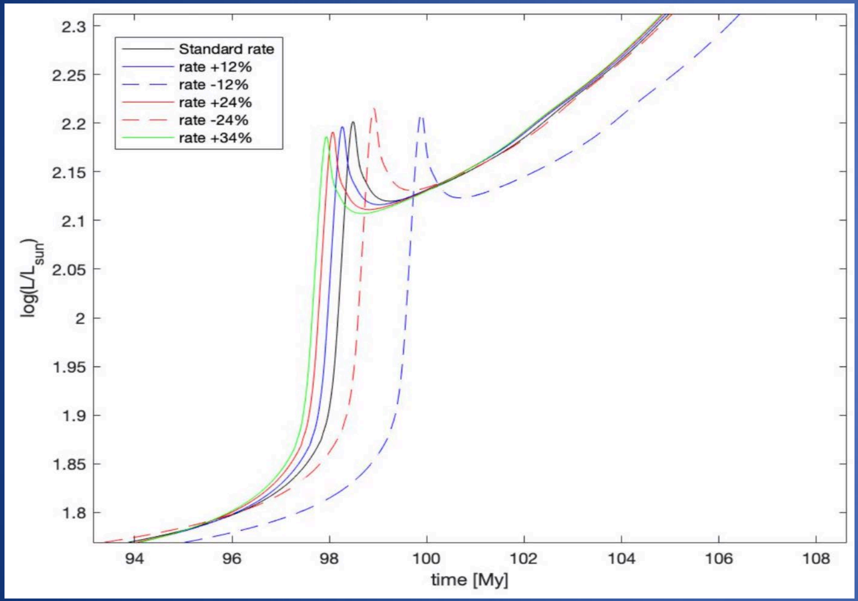
The competition between the triple α and the $^{12}C(\alpha,\gamma)$ affects:

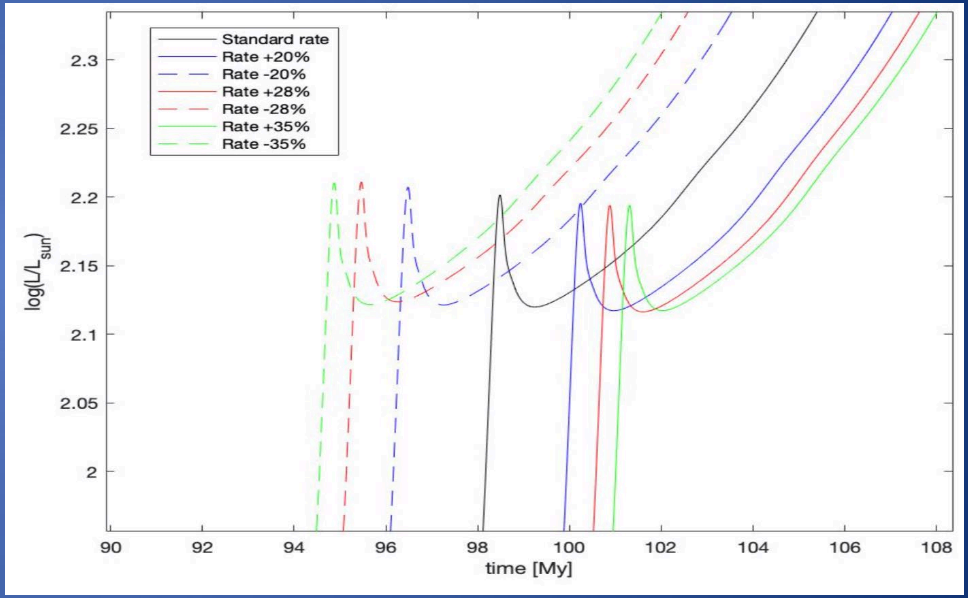
- AGB lifetime
- CO core mass at the 1^{st} thermal pulse
- C/O abundance profile at the 1^{st} thermal pulse

What's the impact of the present uncertainties in the reaction rates of the triple α and the $^{12}C(\alpha,\gamma)$?

AGB clump luminosity
Impact of changing the rate of:

triple α | $^{12}C(\alpha,\gamma)$

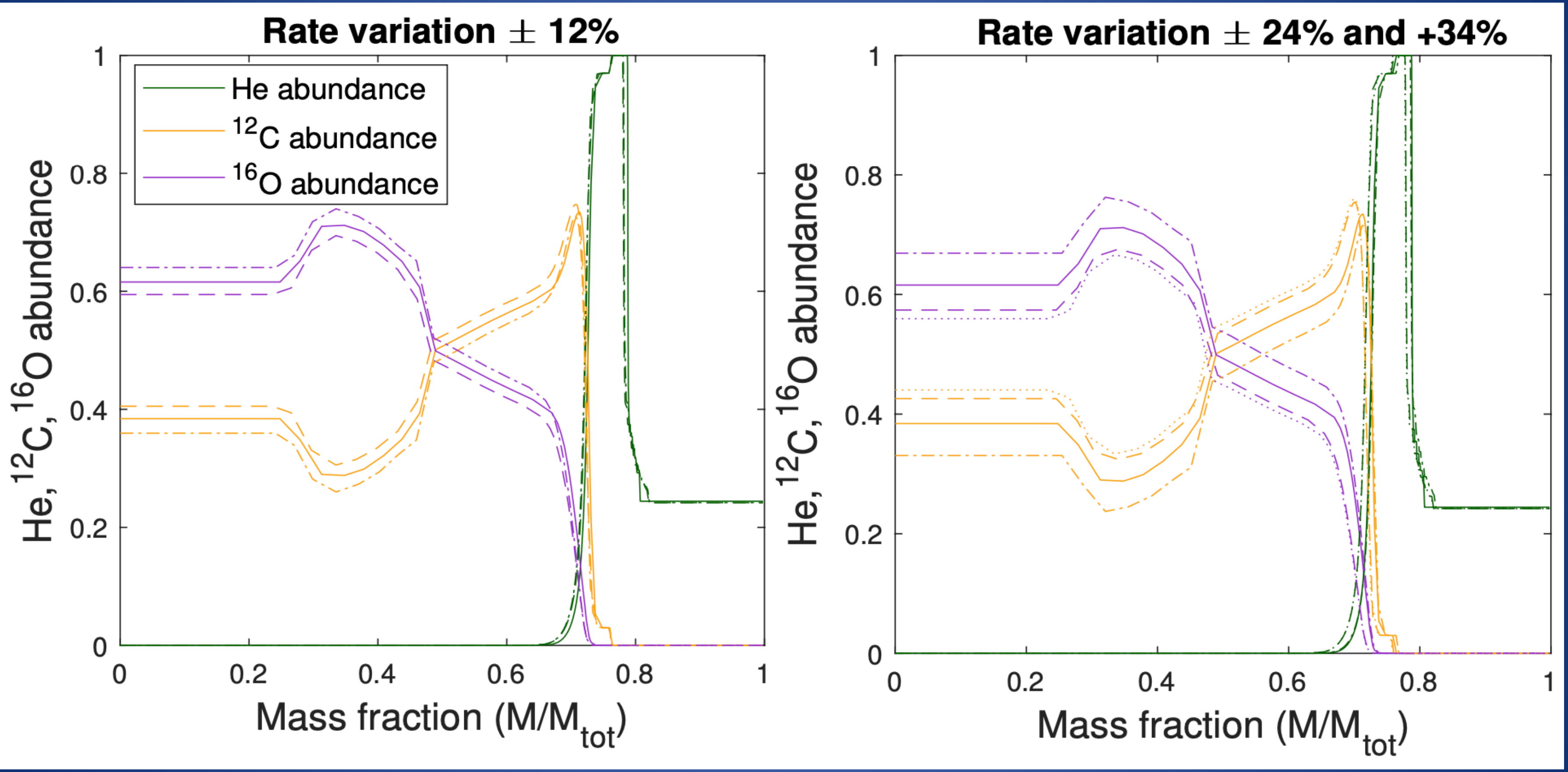




The impact on the AGB clump luminosity of the present uncertainty in the 3α and $^{12}C(\alpha,\gamma)$ reaction rates is negligible

Abundance profiles at the 1st thermal pulse

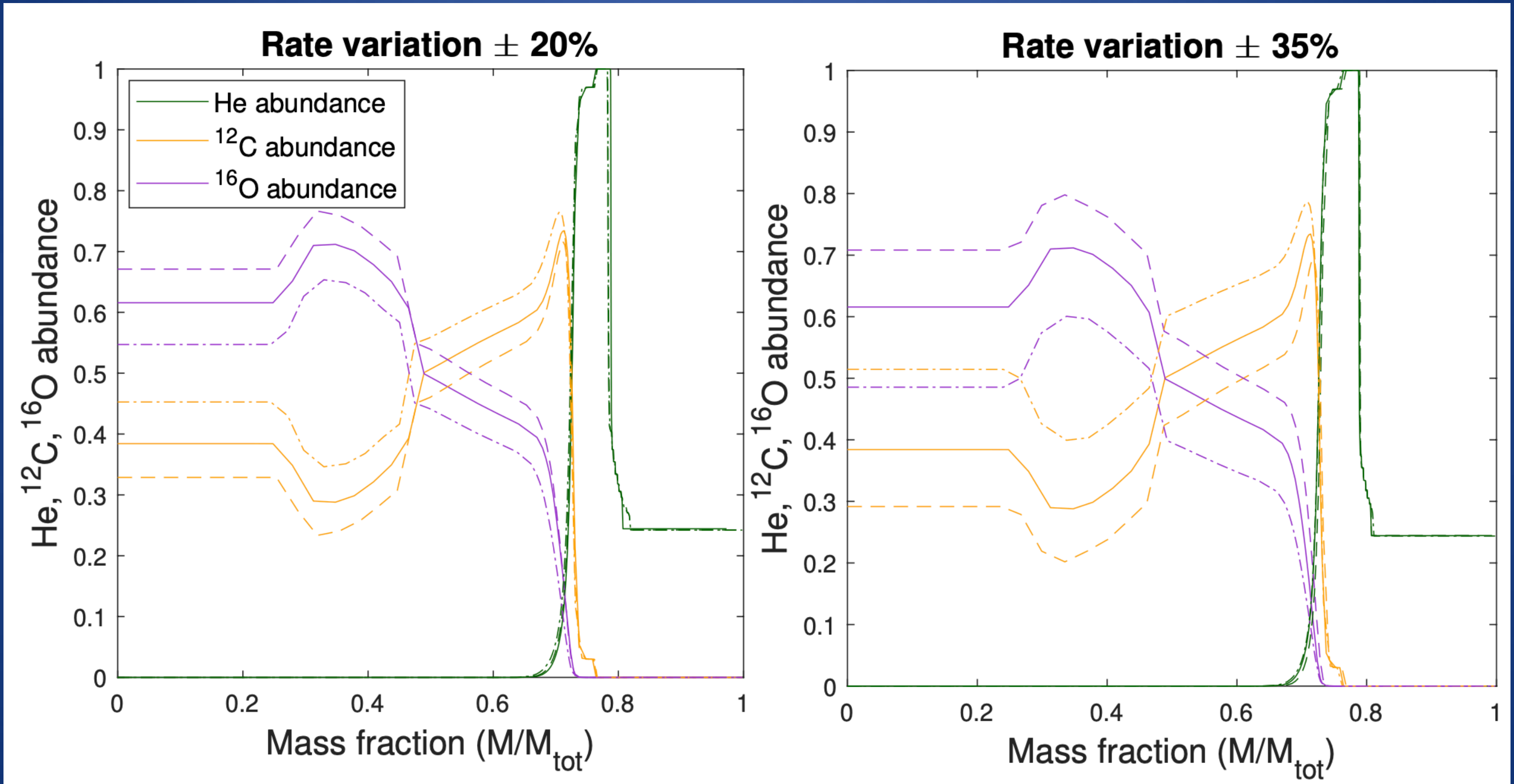
Impact of 3α uncertainty



M=0.67 M_o, progenitor M= 0.8 M_o Z=0.0001 Y=0.246

(Tognini et al 2023, submitted)

Abundance profiles at the 1st thermal pulse
Impact of $^{12}C(\alpha, \gamma)$ uncertainty



M=0.67 M_o, progenitor M= 0.8 M_o Z=0.0001 Y=0.246

(Tognini et al 2023, submitted)

Early AGB lifetime
Impact of changing the rate of:

triple α

$M = 0.67\ M_{\odot}$, $Y = 0.246$, $Z = 0.0001$	Rate +34%	Rate +24%	Rate +12%	Standard rate	Rate −12%	Rate −24%
t_{AGB} [Myr]	12.66	12.63	12.6	12.52	12.50	12.38
	+1,1%	+0.9%	+0.6%		-0.2%	-1.1%

$^{12}C(\alpha,\gamma)$

$M = 0.67\ M_{\odot}$, $Y = 0.246$, $Z = 0.0001$	Rate +35%	Rate +28%	Rate +20%	Standard rate	Rate −20%	Rate −28%	Rate −35%
t_{AGB} [Myr]	12.35 −1.4%	12.37 −1.2%	12.38 −1.1%	12.52	12.57 +0.4%	12.62 +0.8%	12.62 +0.8%

progenitor M= 0.8 M_o Z=0.0001 Y=0.246 *(Tognini et al 2023, submitted)*

Early AGB lifetime

The current uncertainties on both the triple α and $^{12}C(\alpha,\gamma)$ reaction rates have a negligible effect on the early-AGB lifetime

Are there other significant sources of uncertainty?

Central He-burning occurs in a convective core…

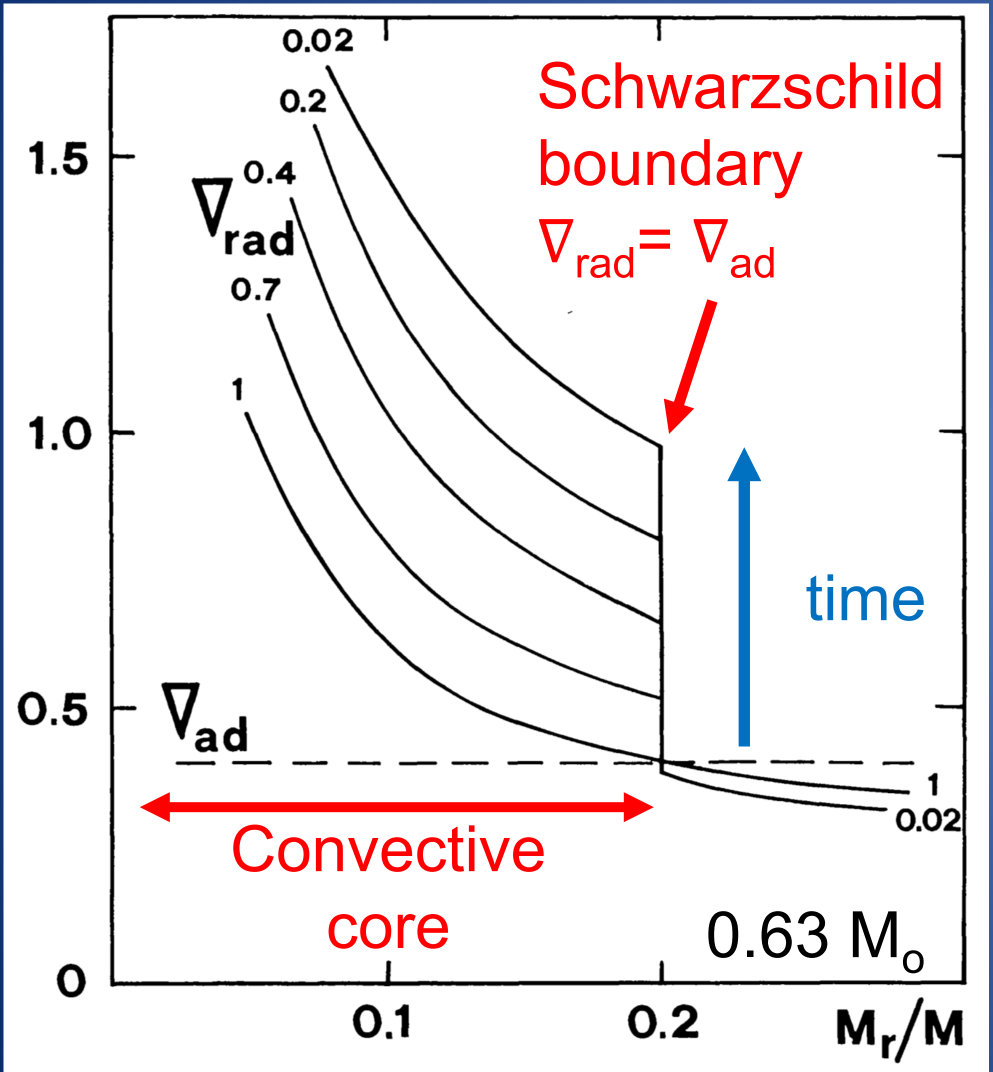
Convection

One of the major and long-standing weaknesses in stellar models

Stellar models are not yet able to accurately predict:

- the extension of convective regions
- the temperature gradient

Mixing in core He-burning



Castellani, Giannone & Renzini 1971a

He-burning in the convective core produces a:

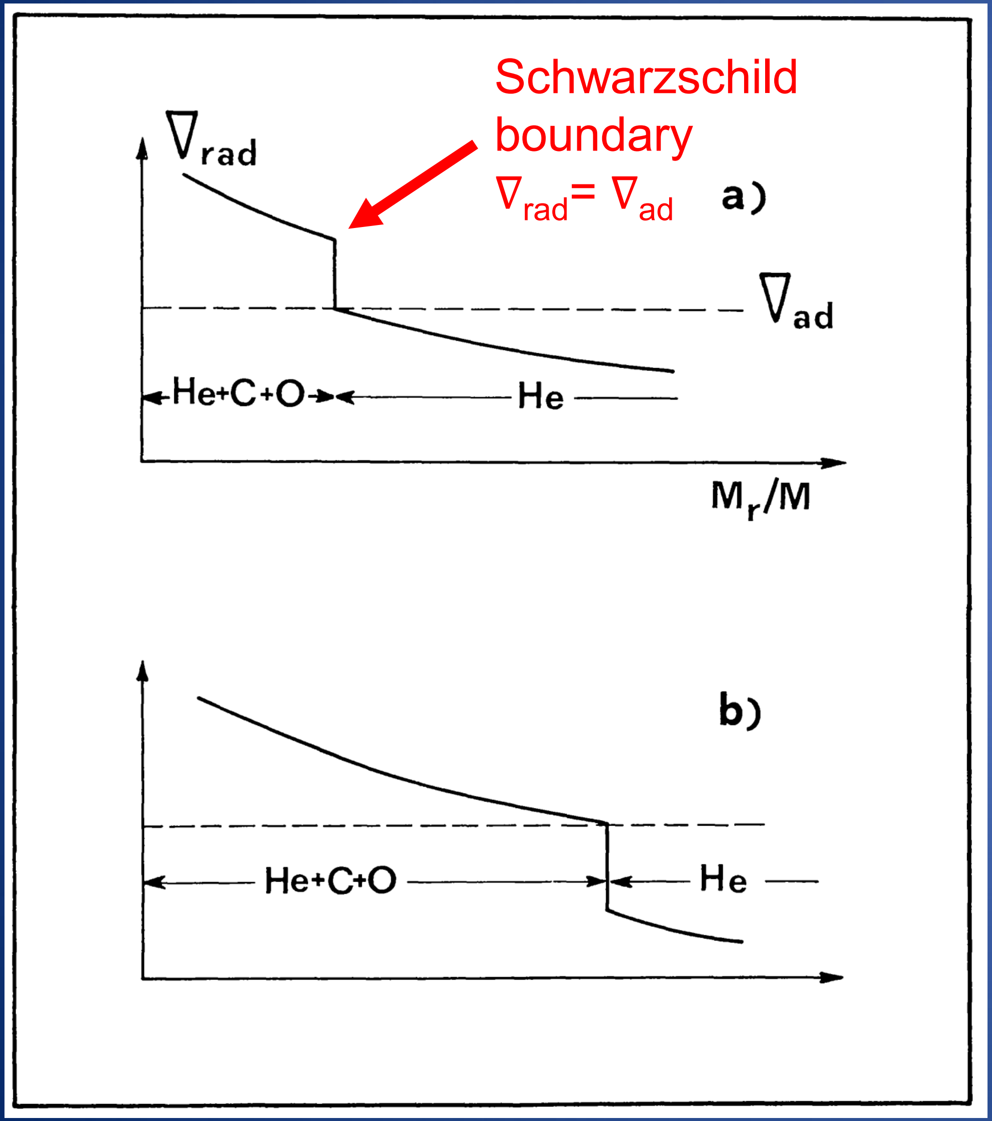
- Decrease of the ^{4}He abundance
- Increase of the ^{12}C and ^{16}O abundances

Growing discontinuity at the convective core boundary in the:

- C-O abundance
- Radiative opacity
- Radiative temperature gradient ∇_{rad}

- Unstable equilibrium (Schwarzschild 1958)

Mixing in core He-burning: Growth of the convective core



Castellani, Giannone & Renzini 1971a

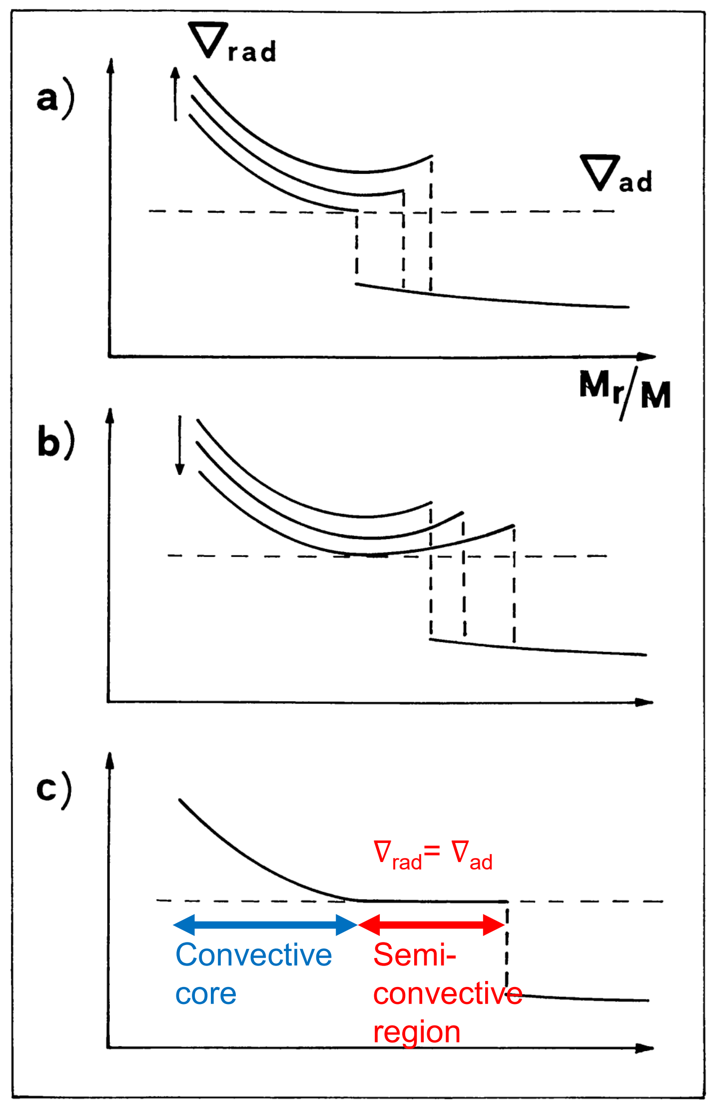
Mixing of the radiative shell from convective overshoot leads to an:

- increase of the opacity
- Increase of the radiative temperature gradient ∇_{rad}

- Self-driving mechanism

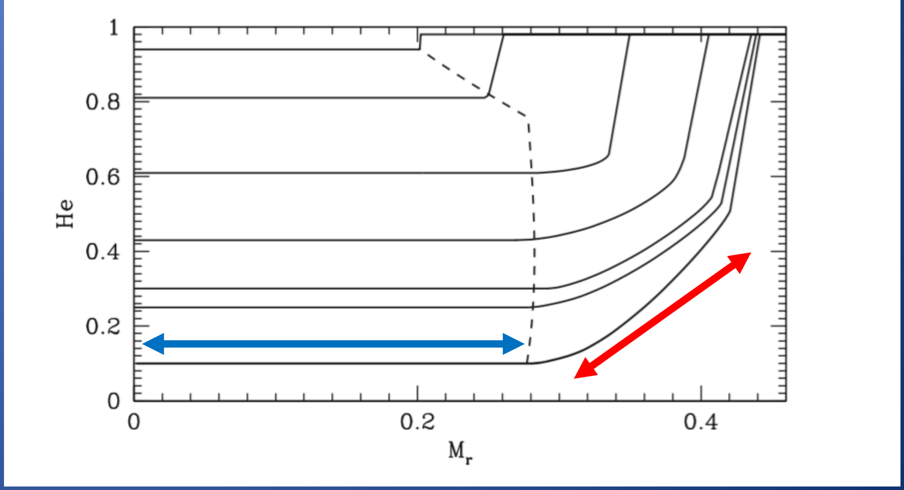
- growth of the convective core

Mixing in core He-burning: Induced semi-convection



Minimum in the radiative temperature gradient ∇_{rad} , leads to the decoupling between:

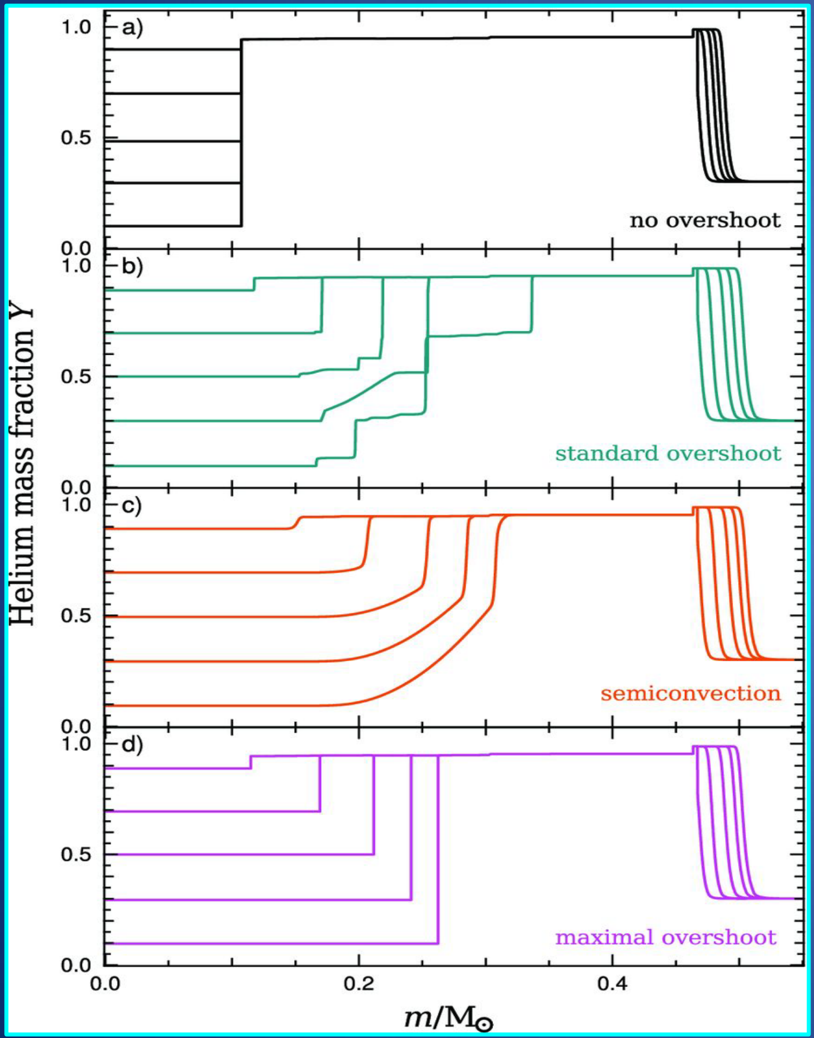
- Fully mixed convective core
- Partially mixed semi-convective region

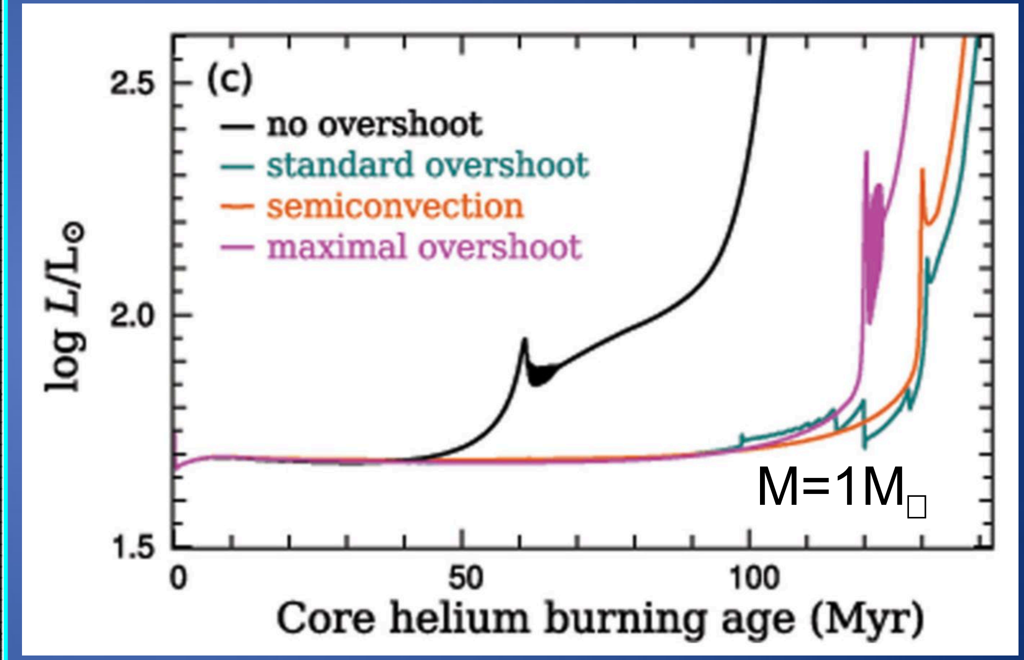


Salaris & Cassisi 2017

Castellani, Giannone & Renzini 1971b (see also Castellani et al. 1985)

Effect of changing the mixing scheme in core He-burning

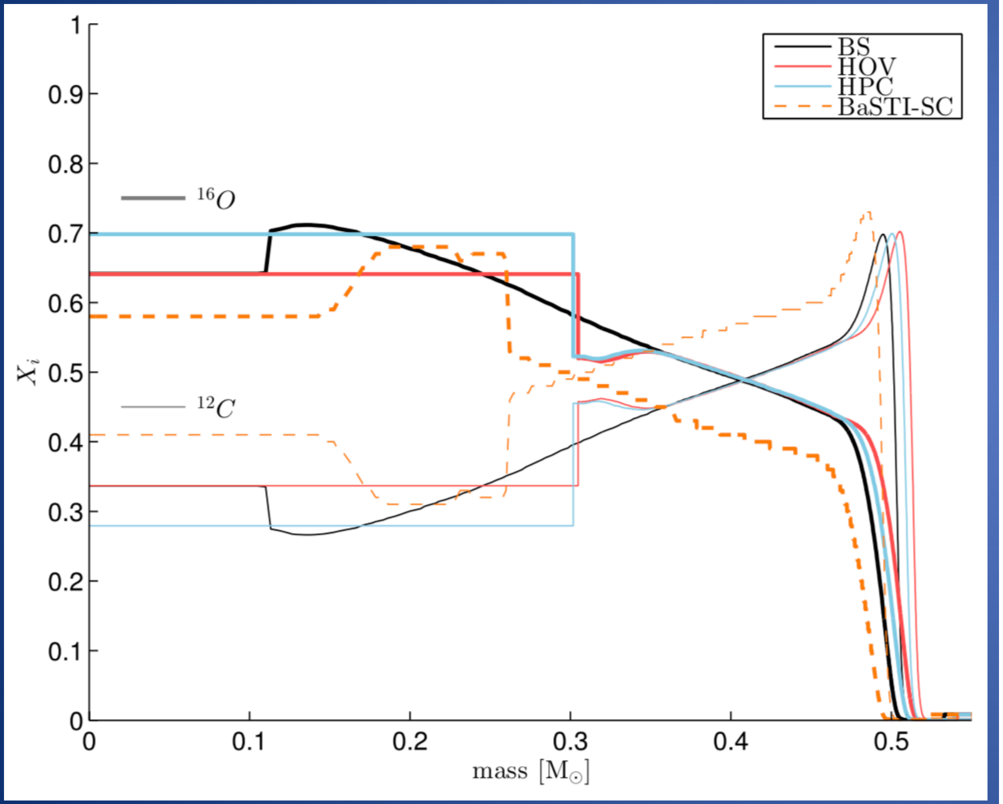




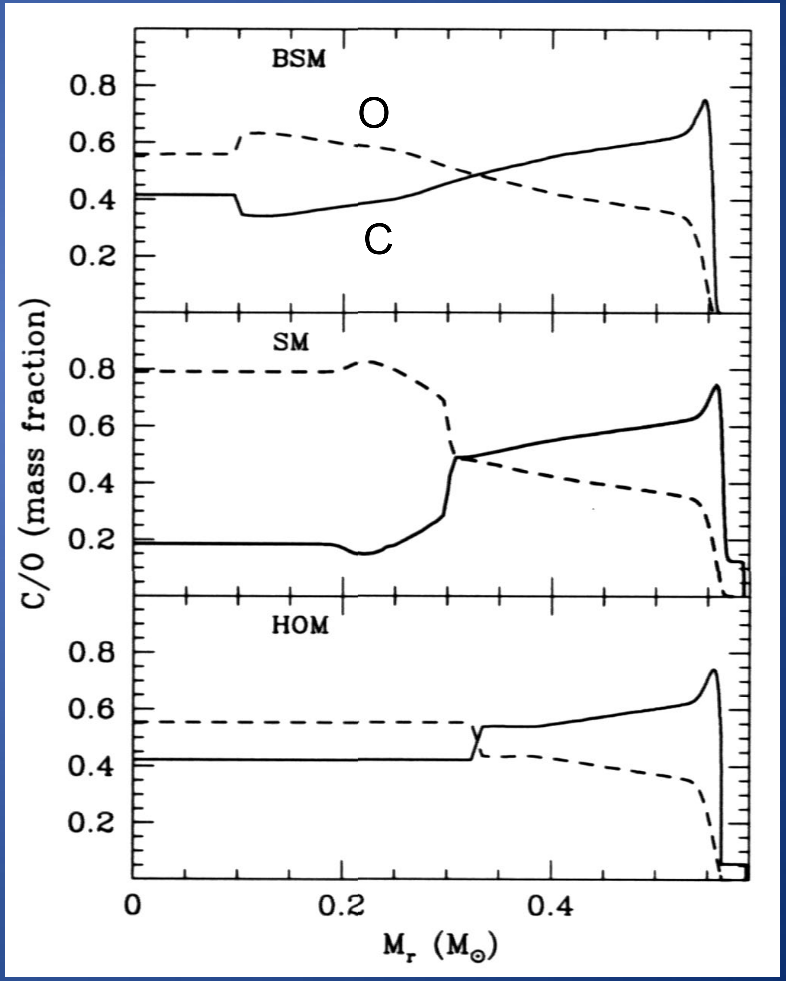
Costantino et al. 2015

Effect of changing the mixing scheme in core He-burning

C/O profile at the 1° thermal pulse

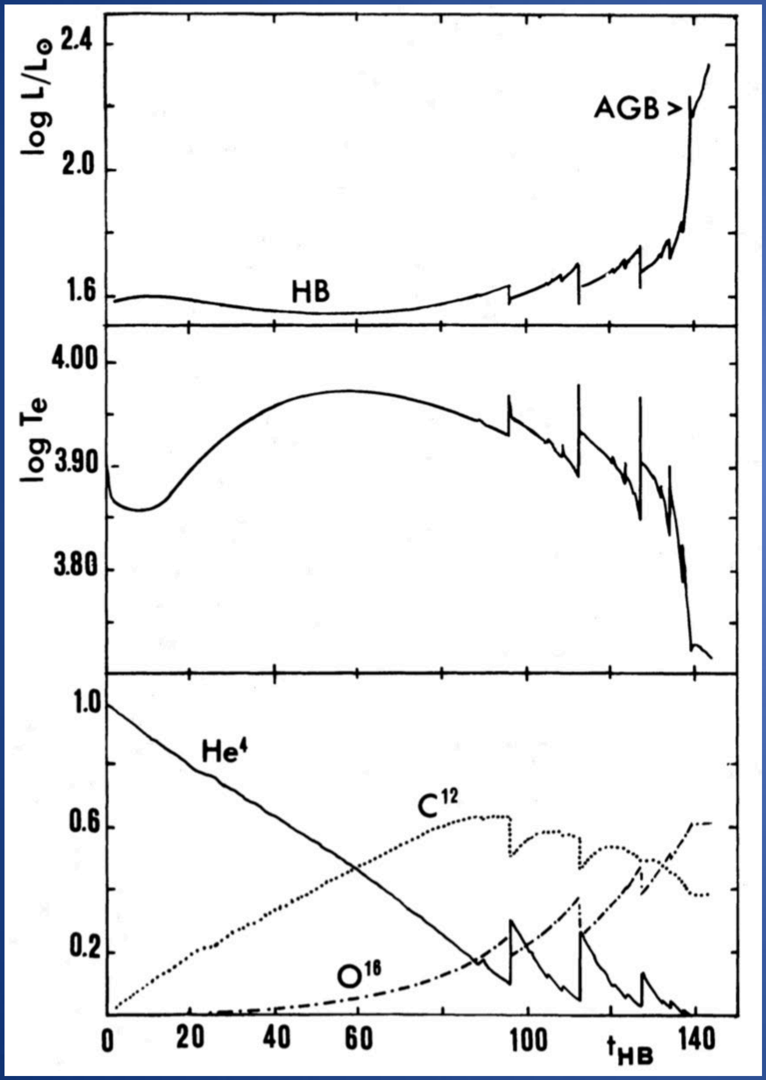


Bossini et al. 2015



Straniero et al. 2003

Mixing in core He-burning: Breathing pulses



Castellani et al. 1985

When the central He-abundance < 0.1, the ingestion of fresh He induced by overshoot

Nuclear energy increase

More efficient mixing

Larger amount of He ingested

Convective runaway: breathing pulses

(see Castellani et al. 1971; Sweigart & Demarque 1972, 1973; Castellani et al. 1985, Caputo et al. 1989, Dorman & Rood 1993, Costantino et al. 2015, 2017)

Impact of breathing pulses

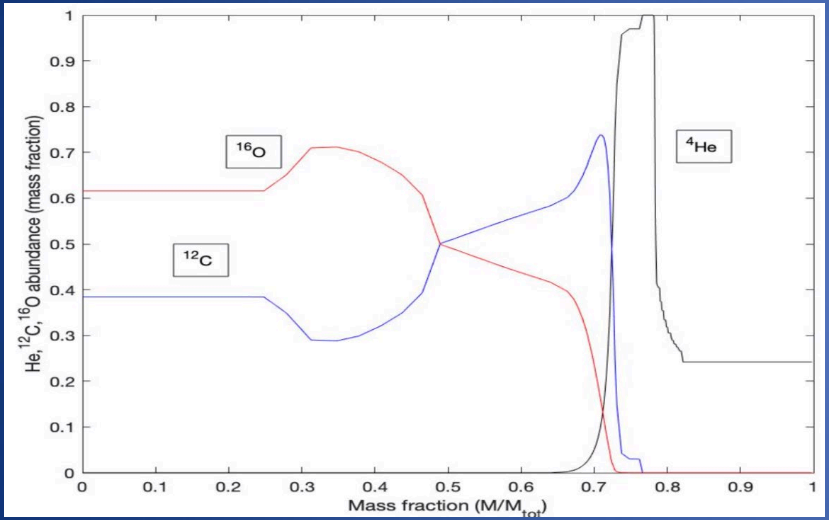
	standard	Breathing pulses	$^{12}C(\alpha,\gamma)$ +20%	3α +12%
t_{HB}[Myr]	98.4	124.04	100.1	98.2
		+26%	+1.8%	-0.2%
R parameter	0.9487	1.1959	0.965	0.9425
		+26%	+1.8%	-0.7%
X_{12C}	0.3842	0.2505	0.3290	0.4050
		-35%	-13%	+5%
X_{16O}	0.6157	0.7494	0.6710	0.5950
		+22%	+9%	-3%
t_{AGB}[Myr]	12.53	6.54	12.38	12.6
		-48%	-1.1%	0.6%
R2 parameter	0.1274	0.0528	0.1237	0.1284
		-59%	-2.8%	+0.7%

M=0.67 M_o, progenitor M= 0.8 M_o Z=0.0001 Y=0.246

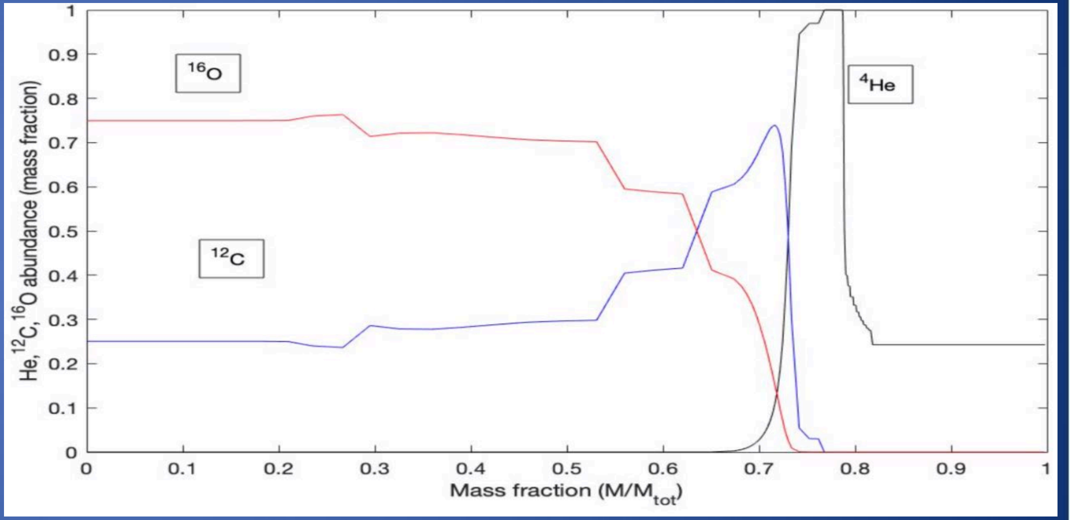
(Tognini et al 2023, submitted)

Abundance profiles at the 1st thermal pulse Impact of breathing pulses

standard



with breathing pulses



M=0.67 M_o, progenitor M= 0.8 M_o Z=0.0001 Y=0.246
(Tognini et al 2023, submitted)

(see also Cassisi et al. 2001; Imbriani et al. 2001; Prada Moroni & Straniero 2002; Straniero et al. 2003; Costantino et al. 2017)

Mixing in core He-burning

With the present uncertainties, a change in the efficiency of the convective mixing has a larger effect on the C and O abundances than a change of the He-burning reaction rates.

The attempts to constrain the He-burning reaction rates on astrophysical grounds are hampered by the fact that central He-burning occurs in a convective core.

Conclusions

- The efforts of nuclear physicists in reducing the uncertainties in the reaction rates of 3α and $^{12}C(\alpha,\gamma)$, allowed to significantly reduce their impact on He-burning stellar models with respect to the past

- The current uncertainties mainly affect the ^{12}C and ^{16}O abundance profile at the end of He-burning phase

- To better constrain with astronomical observations the efficiency of convective mixing during the central He-burning phase, it would be very important to further improve the He-burning reaction rates, primarily the $^{12}C(\alpha,\gamma)$

Thanks

Cumulative theoretical uncertainty

Table 5. Fit of RGB tip log-luminosity (dex).

	Estimate	Std. error	t value	Impact (dex)	
β_0	3.68	6.41×10^{-4}	5743.68		
β_1 (pp)	-2.56×10^{-3}	4.11×10^{-4}	-6.23^*	0.0000	
β_2 (^{14}N)	2.74×10^{-2}	1.23×10^{-4}	222.28	0.0027	
β_3 (k_r)	-2.98×10^{-1}	2.47×10^{-4}	–1209.61	–0.0149	1
β_4 (v_d)	1.72×10^{-3}	8.22×10^{-5}	20.91	0.0002	
β_5 (3α)	-3.05×10^{-2}	6.16×10^{-5}	–494.01	–0.0061	2
β_6 (ν)	8.22×10^{-2}	3.08×10^{-4}	266.77	0.0033	3
β_7 (k_c)	-5.81×10^{-2}	2.47×10^{-4}	–235.71	–0.0029	4
$\sigma = 4.7 \times 10^{-4}$ dex; $R^2 = 0.9988$					

Valle, Dell'Omodarme, Prada Moroni, Degl'Innocenti 2013, A&A 549, A50

ZAHB

Cumulative theoretical uncertainty

Table 7. Fit of the ZAHB log-luminosity $\log L_{\mathrm{HB}}$ (dex) at $\log T_{\mathrm{eff}} = 3.83$.

	Estimate	Std. error	t value	Impact (dex)	
β_0	2.03	1.20×10^{-3}	1696.88		
β_1 (pp)	6.93×10^{-3}	7.68×10^{-4}	9.03	0.0002	
β_2 (^{14}N)	1.20×10^{-2}	2.30×10^{-4}	52.04	0.0012	
β_3 (k_{r})	-4.33×10^{-1}	4.61×10^{-4}	–939.64	–0.0216	1
β_4 (v_{d})	-1.12×10^{-2}	1.54×10^{-4}	–73.17	–0.0017	
β_5 (3α)	-5.78×10^{-2}	1.15×10^{-4}	–501.83	–0.0115	2
β_6 (ν)	5.77×10^{-2}	5.76×10^{-4}	100.22	0.0023	3
β_7 (k_{c})	-4.36×10^{-2}	4.61×10^{-4}	–94.70	–0.0022	4
$\sigma = 8.8 \times 10^{-4}$ dex; $R^2 = 0.9981$					

Notes. All the p-values of the tests are $<2 \times 10^{-16}$. The column legend is the same as in Table 3.

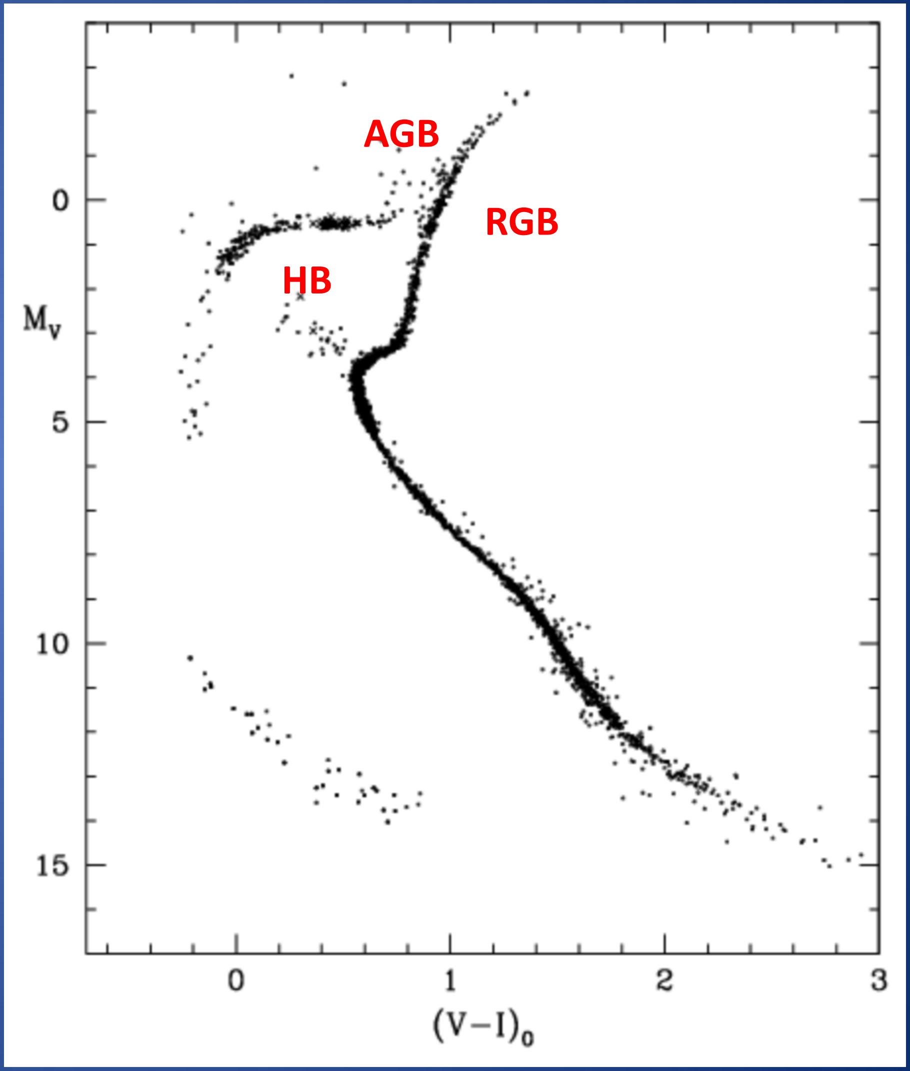
Valle, Dell'Omodarme, Prada Moroni, Degl'Innocenti 2013, A&A 549, A50

R and R_2 parameters

$$R = \frac{N_{HB}}{N_{RGB}} = \frac{t_{HB}}{t_{RGB}}$$

Iben 1968

$$R2 = \frac{N_{AGB}}{N_{HB}} = \frac{t_{AGB}}{t_{HB}}$$



Harris 2003

R and R_2 parameters
Impact of changing the rate of:

triple α

	Rate +34%	Rate +24%	Rate +12%	Standard rate	Rate −12%	Rate −24%
R parameter	0.9285 −2.0%	0.9372 −1.2%	0.9425 −0.7%	0.9487	0.9681 +2.0%	0.9709 +2.3%
*R*2 parameter	0.1293 +1.4%	0.1289 +1.1%	0.1284 +0.7%	0.1275	0.1258 −1.3%	0.1253 −1.7%

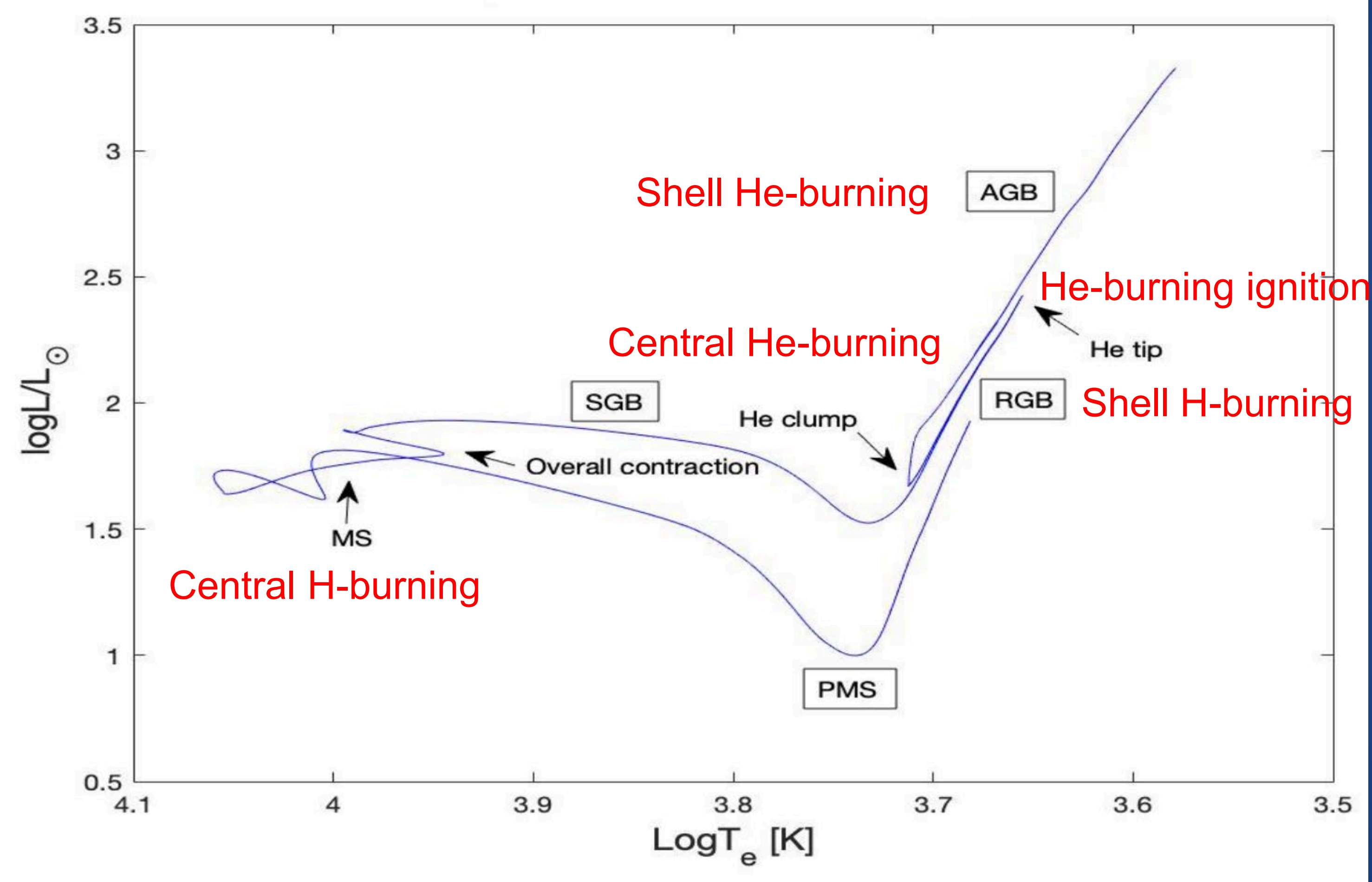
$^{12}C(\alpha,\gamma)$

0.67 $M_\odot$	Rate +35%	Rate +28%	Rate +20%	Standard rate	Rate −20%	Rate −28%	Rate −35%
R parameter	0.976 +2.8%	0.972 +2.4%	0.965 +1.8%	0.949	0.929 −2.0%	0.920 −3.0%	0.914 −3.6%
*R*2 parameter	0.1229 −3.5%	0.1231 −3.3%	0.1237 −2.8%	0.1273	0.1305 +2.5%	0.1323 +4.0%	0.1326 +4.5%

M=0.67 M_o, progenitor M= 0.8 M_o Z=0.0001 Y=0.246

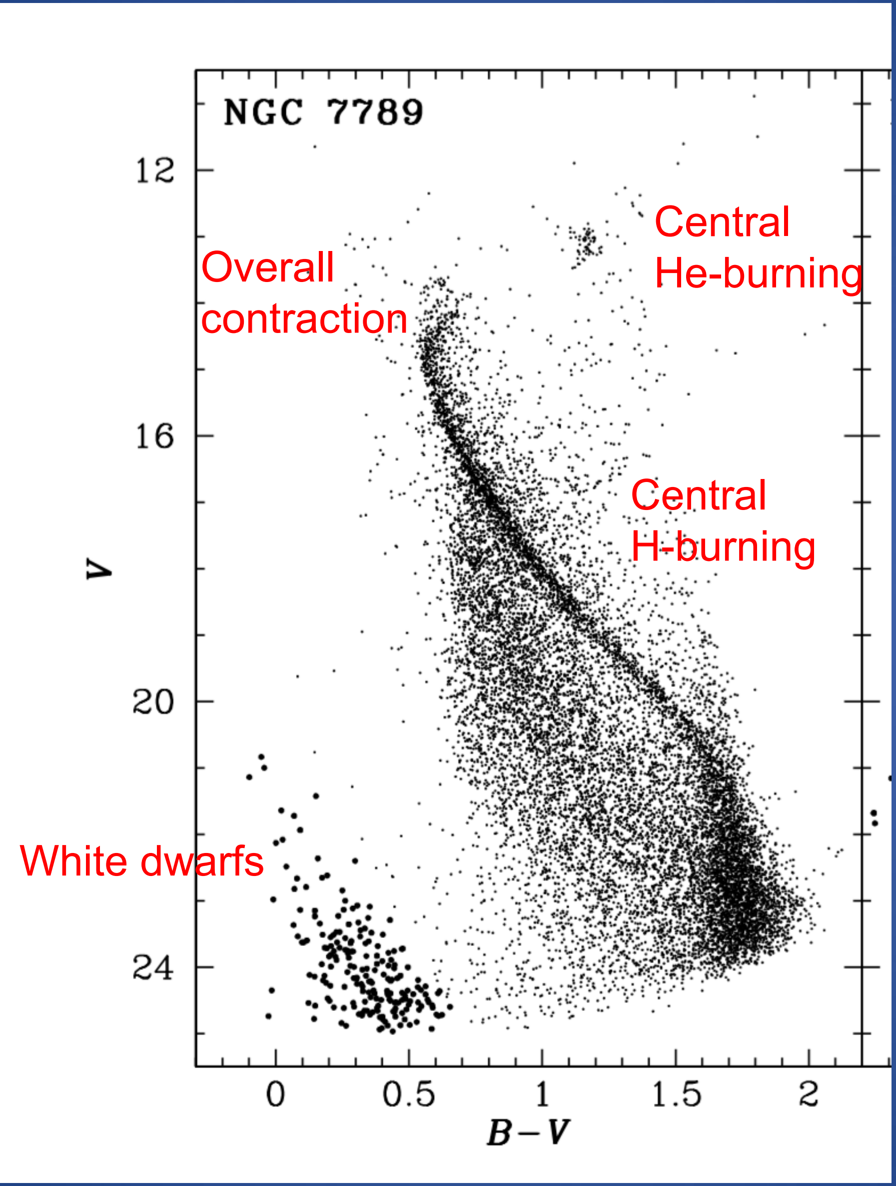
(Tognini et al 2023, submitted)

Stellar evolution in a nutshell: intermediate-mass stars



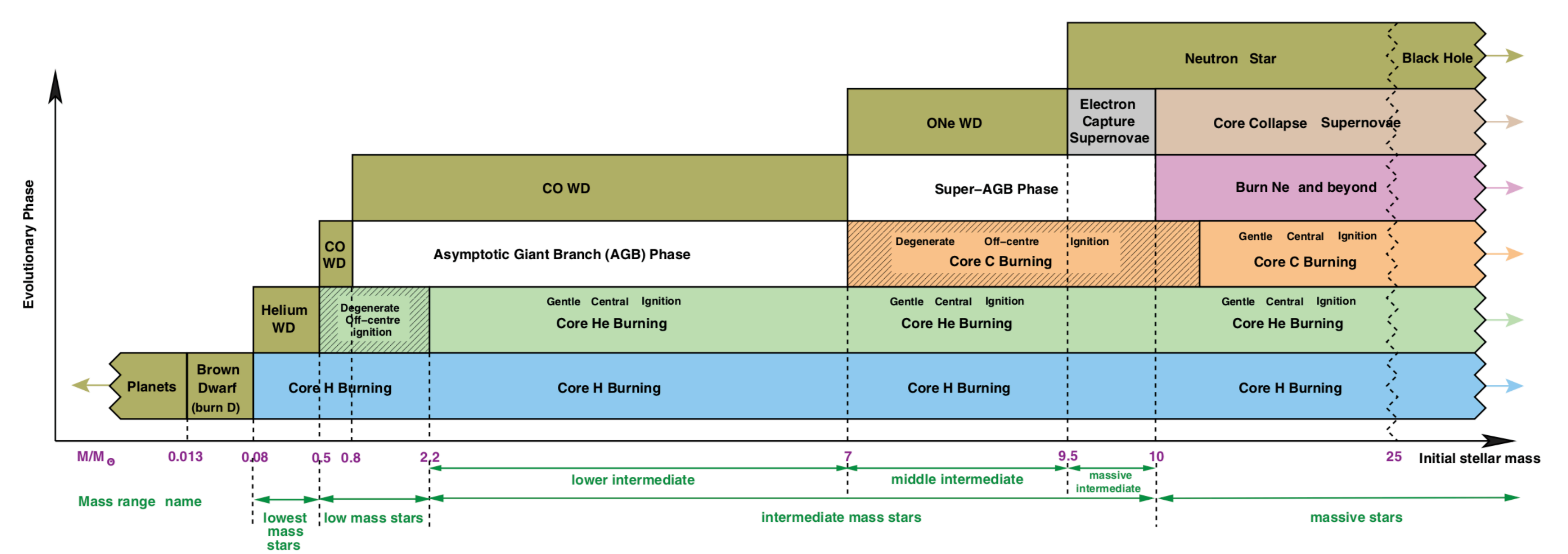
M= 2.5 M_o Z=0.015 Y=0.28

Open clusters



Kalirai et al. 2008

Stellar evolution in a nutshell



Karakas et al. 2014

Triple α

			$M = 1.5\ M_{\odot}, Y = 0.28, Z = 0.015$			
	Rate +34%	Rate +24%	Rate +12%	Original rate	Rate −12%	Rate −24%
t_{HB} [Myr]	117.1 −0.3%	117.2 −0.2%	117.3 −0.1%	117.4	117.55 +0.1%	117.60 +0.2%
$X_{^{12}C}$	0.432 +14%	0.418 +11%	0.399 +6%	0.377	0.354 −6%	0.327 −13%
$X_{^{16}O}$	0.548 −9%	0.562 −7%	0.581 −4%	0.603	0.626 +4%	0.653 +8%
t_{AGB} [Myr]	13.63 +3.8%	13.52 +3%	13.29 +1.3%	13.17	12.97 −1.1%	12.8 −2.5%
			$M = 2.5\ M_{\odot}, Y = 0.28, Z = 0.015$			
	Rate +34%	Rate +24%	Rate +12%	Standard rate	Rate −12%	Rate −24%
t_{HB} [Myr]	237.5 −1.1%	237.8 −1.0%	238.6 −0.6%	240.2	240.8 +0.3%	241.4 +0.5
$X_{^{12}C}$	0.403 +16%	0.389 +12%	0.369 +6%	0.348	0.325 −7%	0.299 −14%
$X_{^{16}O}$	0.577 −8.6%	0.591 −6.4%	0.611 −3.3%	0.631	0.655 +4.0%	0.682 +8.0%
t_{AGB} [Myr]	19.99 +5.7%	19.78 +4.6%	19.47 +3.0%	18.95	18.69 −1.1%	18.42 −2.5%

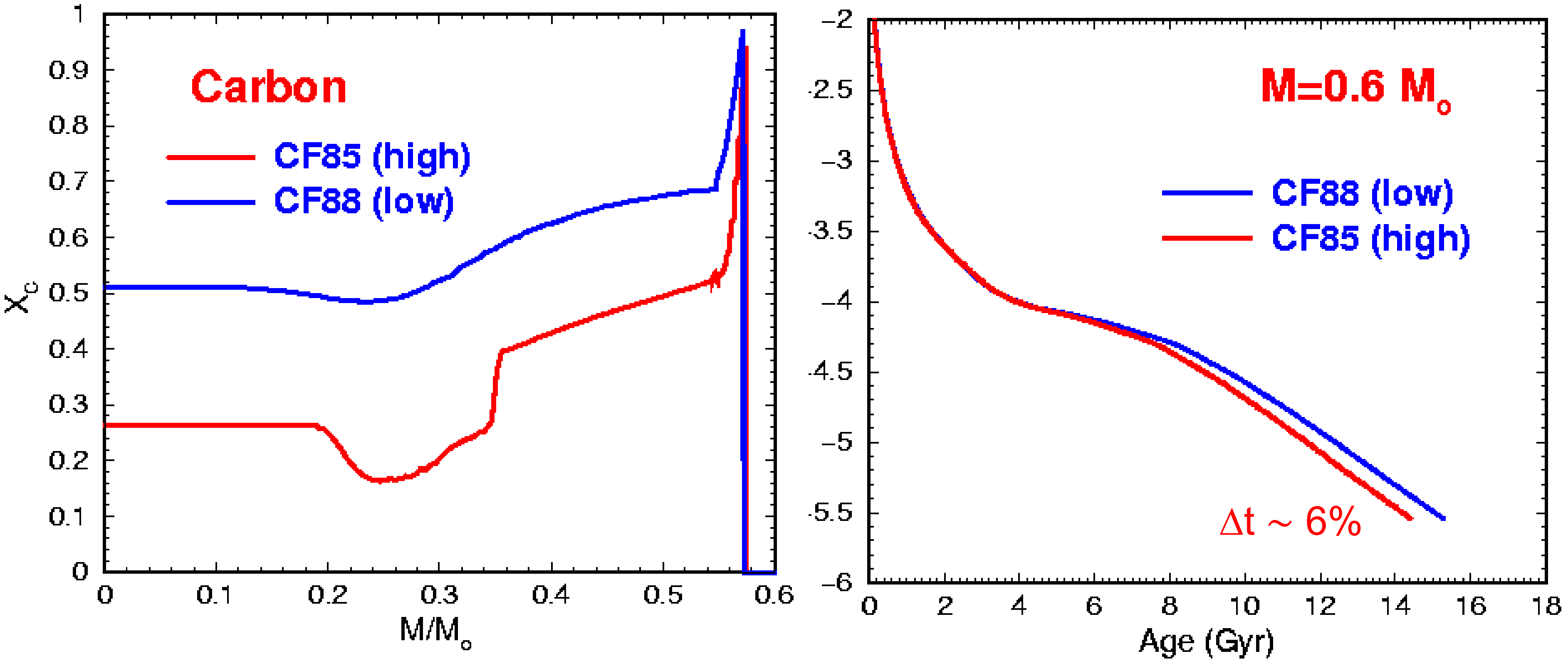
Z=0.015 Y=0.28 *(Tognini et al 2023, submitted)*

$^{12}C(\alpha, \gamma)^{16}O$

			$M = 1.5\ M_\odot$, $Y = 0.28$, $Z = 0.015$				
	Rate +35%	Rate +28%	Rate +20%	Standard rate	Rate −20%	Rate −28%	Rate −35%
t_{HB} [Myr]	120.3 +2.9%	119.9 +2.1%	119.3 +1.6%	117.4	115.2 −1.8%	114.2 −3.2%	113.2 −3.6%
$X_{^{12}C}$	0.289 −23%	0.304 −19%	0.323 −14%	0.377	0.444 +18%	0.475 +26%	0.504 +34%
$X_{^{16}O}$	0.691 +15%	0.676 +12%	0.657 +10%	0.603	0.536 −11%	0.505 −16%	0.475 −24%
t_{AGB} [Myr]	12.99 −1.4%	13.03 −1.1%	13.06 −0.8%	13.17	13.38 +1.5%	13.46 +2.2%	13.55 +2.8%
			$M = 2.5\ M_\odot$, $Y = 0.28$, $Z = 0.015$				
	Rate +35%	Rate +28%	Rate +20%	Standard rate	Rate −20%	Rate −28%	Rate −35%
t_{HB} [Myr]	244.5 +1.8%	243.8 +1.5%	242.9 +1.2%	240.2	237.1 −1.3%	235.9 −1.8%	234.3 −2.4%
$X_{^{12}C}$	0.259 −25.6%	0.274 −21%	0.293 −16%	0.348	0.415 +20%	0.444 +27%	0.475 +36%
$X_{^{16}O}$	0.721 +14%	0.706 +12%	0.687 +9%	0.631	0.565 −10%	0.536 −15%	0.505 −20%
t_{AGB} [Myr]	18.25 −3.6%	18.27 −3.5%	18.33 −3.0%	18.95	19.34 +2.0%	19.37 +2.2%	19.52 +3.0%

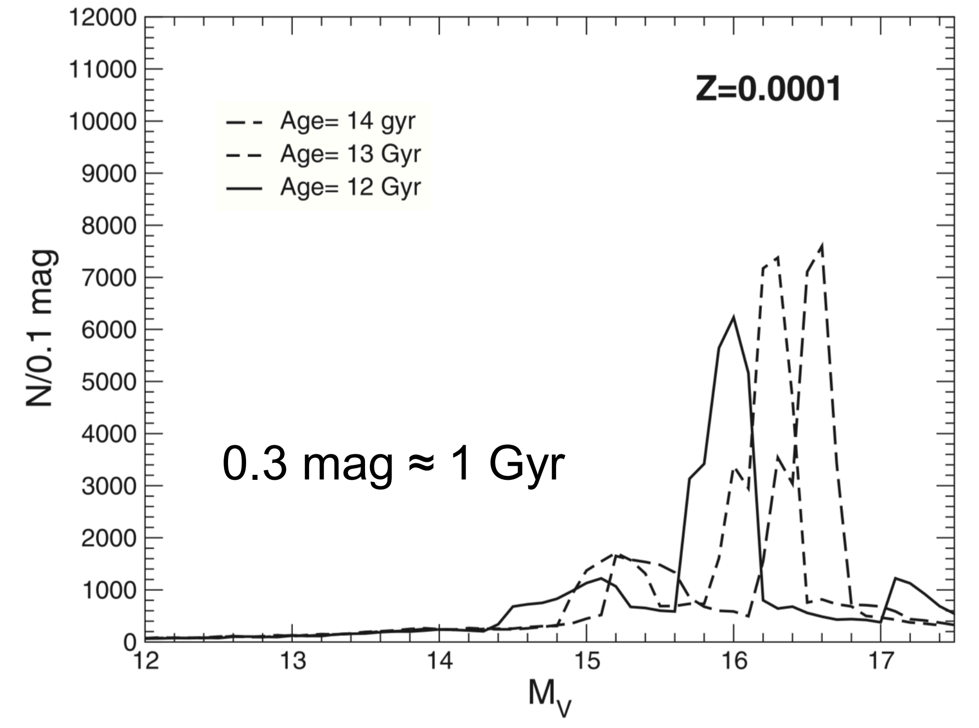
Z=0.015 Y=0.28 *(Tognini et al 2023, submitted)*

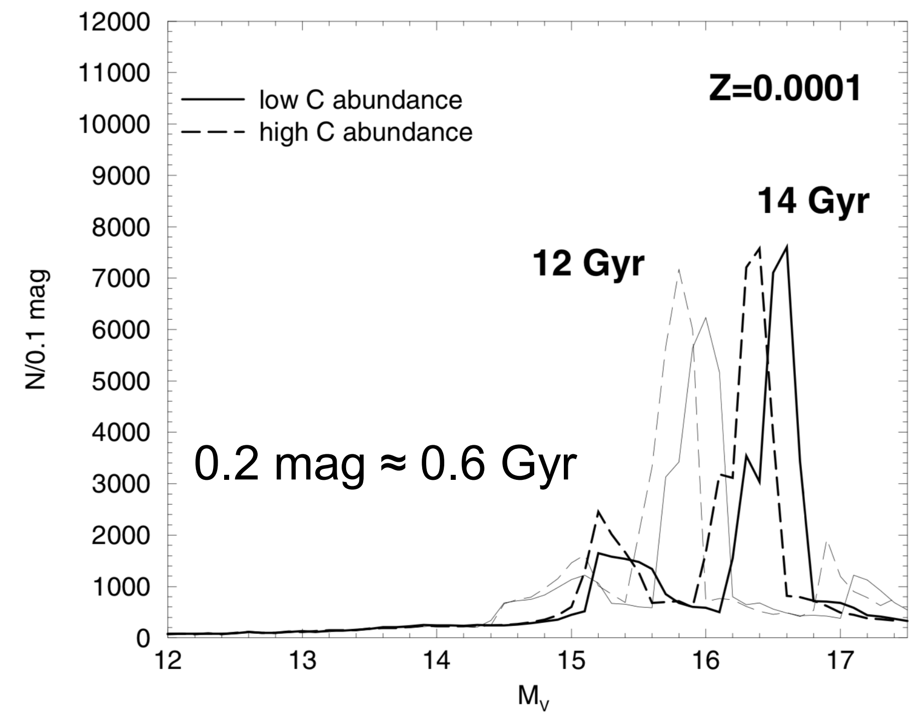
$^{12}C(\alpha, \gamma)^{16}O$:
White dwarf evolution



Prada Moroni & Straniero 2002

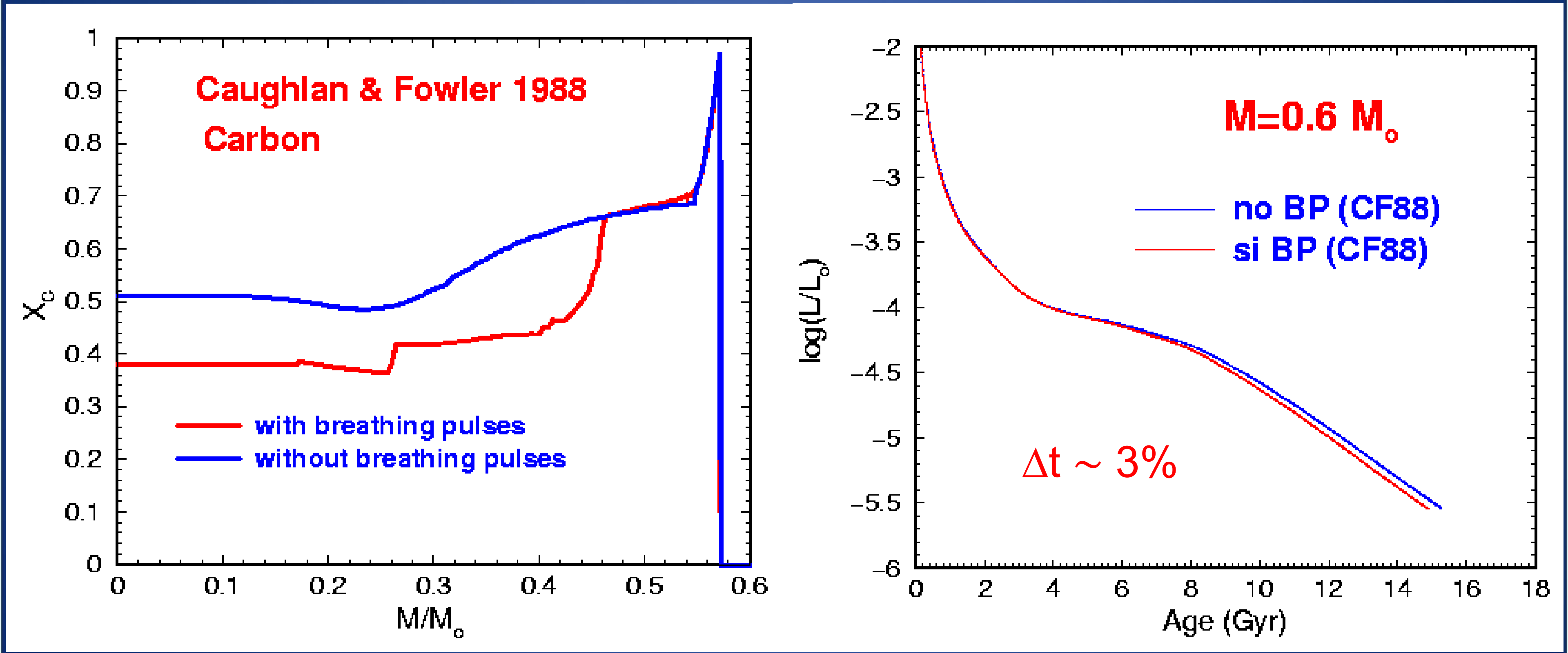
$^{12}C(\alpha, \gamma)^{16}O$:
White dwarf evolution





Prada Moroni & Straniero 2007

Breathing pulses: White dwarf evolution



Prada Moroni & Straniero 2002