The ¹²C(α , γ)¹⁶O reaction: an evaluator's perspective

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What's so important about ${}^{12}C(\alpha,\gamma){}^{16}O?$

- "Holy Grail of nuclear astrophysics" -Willie Fowler
- Nobel prize in Physics 1983, which he shared with Chandrasekhar





Hydrostatic equilibrium

Stars produce energy by fusing **charged particles** below the Iron Peak

- The tunneling probability is dominated by the Coulomb barrier
- Nuclear properties determine the (relatively) small, but important, modifications



- Its initial mass determines the star's evolution
- That is, the heaviest elements it can burn



Hydrogen burning reactions



"Low Mass"



"High Mass"

Helium Burning



Close encounters of the alpha kind, Sofia Quaglioni, Nature 528, 42-43 (2015)

bottleneck





The ¹²C(α,γ)¹⁶O reaction determines the composition of remnants



Motivation Highlight: Black Hole Mass Gap Link to LIGO

Farmer et al. (2020), Mehta et al. (2022), Chidester et al. (2022), Shen et al. (2020)



Maybe something interesting here?



Mehta *et al*. (2022)

What energy range must be measured?



TABLE I. Astrophysical environments and burning stages where the ${}^{12}C(\alpha, \gamma){}^{16}O$ reaction plays an important role. The temperatures of these environments dictate the energy ranges where the ${}^{12}C(\alpha, \gamma){}^{16}O$ cross section must be well known for an accurate calculation of the reaction rate.

Burning stages	Astrophysical sites	Temperature range (GK)	Gamow energy range (MeV)
Core helium burning	AGB stars and massive stars	0.1-0.4	0.15-0.65
Core carbon and oxygen burning	Massive stars	0.6–2.7	0.44-2.5
Core silicon burning	Massive stars	2.8-4.1	1.1–3.4
Explosive helium burning	Supernovae and x-ray bursts	≈ 1	0.6-1.25
Explosive oxygen and silicon burning	Supernovae	> 5	> 1.45

If the ¹²C(α,γ)¹⁶O cross section is so important, why haven't we measured it?



The S-factor



¹⁶O level structure

- 2⁺ and 1⁻ bound state act as subthreshold resonances for the ground state transition that greatly enhance the low energy cross section
- Note that only natural parity state (J=l) can be populated and only electric multipoles can be excited



Ground state transition dominates at low energies



Reaction components

- The cross section is often modeled using phenomenological *R*-matrix
- Level wave functions are added based on the constraints of experimental data
- Interference between resonances is reproduced and fundamental principles like time-reversibility, unitarity, and angular momentum are conserved
- Levels are parameterized in terms of J^π, energy, partial decay widths



What has been done? Direct Measurements

- Several measurements between 1970 and present
- Ground state transition dominates
- Commonly divided into "E1" and "E2" components



What has been done?: ERNA recoil separator

- Schürmann et al. (2005, 2011)
- High precision total cross section measurement over a wide energy range
- Cascade transition measurements over a difficult range to access using standard methods
- Is extremely helpful in constraining background contributions in the *R*-matrix fit







What has been done? ${}^{16}N(\beta\alpha){}^{12}C$





What has been done? ${}^{12}C(\alpha, \alpha)$ scattering

- Very comprehensive measurement by Tischhauser *et al*. (2002) and (2010)
- Only a very small amount of the data is shown here
- High cross section, "easy" to measure





What has been done? Transfer reaction studies

 SubCoulomb measurements of Asymptotic Normalization Coefficients (C_{λc})



$$C_{\lambda c} = \frac{(2\mu_{\alpha}a_c)^{1/2}}{\hbar W_c(a_c)} \frac{\tilde{\gamma}_{\lambda c}}{\left[1 + \sum_{c'} \tilde{\gamma}_{\lambda c'}^2 (dS_{c'}/dE)(\tilde{E}_{\lambda})\right]^{1/2}},$$



	$ANC_{\alpha} (fm^{-1/2})$	
Reference	6.92 MeV, 2 ⁺	7.12 MeV, 1 ⁻
Transfer Brune et al. (1999) Belhout et al. (2007) Oulebsir et al. (2012) Avila et al. (2015)	$\begin{array}{c} 1.14(10)\times 10^{5}\\ 1.40(50)\times 10^{5\mathrm{c}}\\ 1.44(28)\times 10^{5}\\ 1.22(7)\times 10^{5} \end{array}$	$\begin{array}{c} 2.08(20)\times 10^{14}\\ 1.87(54)\times 10^{14}\\ 2.00(35)\times 10^{14}\\ 2.09(14)\times 10^{14} \end{array}$

Combine as much experimental information as possible



Where are we at?

- The uncertainty is dominated by systematics
 - Model
 - Experiment
- Both nuclear models and experimental techniques are both currently improving, but in a slow but steady fashion
 - Computational speed
 - Detection methods



What's on the horizon?



On the horizon: ${}^{16}O(\gamma,\alpha){}^{12}C$

- Lots of effort at the High Intensity γ-ray source (HIγS)
 - Smith *et al*. (2021)
 - O-TPC
 - Data is tough to analyze, but looks really promising
- Experimental measurements planned for ELI-NP



On the horizon: ${}^{16}O(\gamma, \alpha){}^{12}C$

- Jefferson Laboratory
- Bubble Chamber + Bremsstrahlung beam
- Previous tests at HIγS
- Not sure on current status?



Ugalde *et al*. (2013)

On the horizon: ${}^{16}O(e,e'\alpha){}^{12}C$

- MIT group
 - See Friščić et al. (2019)
- FAIR at GSI
 - Experiment underway!
 - Kathrin Göbel
 - Aims to get 10% uncertainty at 1 MeV
 - Will cover a wide energy range



measure

- charge
- track points
- time
- photon from ¹²C*

to determine

excitation energy

Figure courtesy of Rene Reifarth

Underground Facilities

JUNA

Connec tronged &

Auxiliary tunnel

Drainage tunnel

Traffic tunnel

Low background environments

LUNA-MV

- 0.2 to 3.5 MV
- Can probably make improved low energy measurements but can also over a wide energy range!
- Long term, as most of the background is beam induced
- JUNA
 - Can reach 700+ keV with He⁺⁺
 - Milliamps of beam available
 - Target stability?
- CASPAR
 - Coming back online soon!



Weiping Liu, Nuclei in the Cosmos XV

A new evaluation?



AZURE is an R-Matrix minimization code developed under the support of the Joint Institute for Nuclear Astrophysics (JINA). The code has been designed specifically, but not exclusively, for the needs of the Nuclear Astrophy been optimized for low energy charged particle reactions relevant to stellar nucleosynthesis. AZURE is ideal for the analysis of complex multi-level multi-channel reactions with an interface constructed for ease of use and the compound nucleus mechanism, AZURE also supports the analysis of reactions containing direct capture components.

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The original version of the AZURE source code, written in Fortran 77, has been replaced by a new C++ version, AZURE2, with improved functionality, performance and stability. The original version can still be obtained from Versions of Internet Explorer before 8.0 are known to have some compatibility problems when rendering this website.

- deBoer *et al*. RMP (2017)
- Not much new data since 2017
- A Bayesian uncertainty analysis framework has been developed for *R*-matrix fits by Daniel Odell at Ohio University, but it is very computationally intensive
- Time and computation time are the current hold ups, but its probably coming in the next few years

brick-james 0.2.2 pip install brick-james Released: Jul 6, 2023 A Python interface to AZURE2 that makes it easy to sample R-matrix parameters. Navigation Project description

	BRICK
3 Release history	
	Bayesian R-Matrix Inference Code Kit
🛓 Download files	BRICK is a Python package that serves as an interface to AZURE2 and readily permits the sampling of R-matrix parameters.
Project links	It <i>accompanies</i> AZURE2. The primary goal is to allow the user to deploy Markov Chain Monte Carlo (MCMC) to sar parameters that are typically optimized in a χ ² -minimization analysis.
📌 Homepage	
👬 Bug Tracker	Requirements
Statistics	AZURE2 must be installed and available at the command line via AZURE2. Currently, command-line execution is available on Windows or macOS.
GitHub statistics:	NumPy and Matplotlib must be available in order to run the test script in test directory.
🚖 Stars: 0	emcee is the MCMC sampler that is used in the test scripts. BRICK is intentionally designed such that other sample
P Forks: 0	can be used with little effort.

Questions?

My current estimate of S(300 keV): 140 ± 21 (MC) +18/-11 (model) keV b

see deBoer *et al*. (2017) and Shen *et al*. (2020)





¹²C/¹⁶O in the stellar core

Because the ¹²C(α,γ)¹⁶O reaction determines the ratio of ¹²C to ¹⁶O in the stellar core, it impacts nearly all later stages of evolution

Core nuclear reactions in massive stars

lifetime remaining	core temperature	core reaction
10,000,000 years		$^{1}H \Rightarrow {}^{4}He$
1,000,000 years	170,000,000 K	⁴ He \Rightarrow ¹² C, ¹⁶ O
1,000 years		$^{12}C \Rightarrow ^{20}Ne, ^{24}Mg$
10 years	1,500,000,000 K	²⁰ Ne ⇒ ¹⁶ O, ²⁴ Mg
1 year	2,000,000,000 K	$^{16}O \Rightarrow ^{28}Si, ^{32}S$
1 day	3,000,000,000 K	²⁸ Si, ³² S \Rightarrow ⁵⁶ Fe, ⁵⁶ Ni
1 s	explosi neutror	ve fusion \Rightarrow light elements h capture \Rightarrow heavy elements

Strategy in 2017 (and earlier)

Used a broader distribution

function for data point

than a Gaussian)

uncertainties (larger tails

 Use χ² minimization to find a best description of the data within the R-matrix framework (used MINUIT₂)

 $\chi^2 = \sum_{i} \left(\frac{\sum_{j} R_{ij}^2}{N_i} \right) + \frac{(n_i - 1)^2}{n_i^2 \sigma_{\text{syst},i}^2}$

$$\chi^2 = \sum_i \left(\frac{\sum_j R_{ij}^2}{N_i - \nu}\right) + \frac{(n_i - 1)^2}{n_i^2 \sigma_{\text{syst},i}^2}$$

$$R_{ij} = \frac{f(x_{i,j}) - n_i y_{i,j}}{n_i \sigma_{i,j}}$$

Additional data normalization fit parameter, with common systematic uncertainty. Also fitting to reduced χ^2 .

0 /

$$\frac{(n_i - 1)^2}{n_i^2 \sigma_{\text{syst},i}^2}$$

$$+ \sum \frac{(P_{\text{fit},k} - P_{\text{exp},k})^2}{\sigma^2}$$
Add additional terms for subthreshold ANC to include uncertainty constraints from transfer reactions.



$$L = \sum_{j} \log \left[\frac{1 - e^{-R_{ij}^2/2}}{R_{ij}^2} \right]$$

Sivia and Skilling (2006)

Kind of a Frequentist / Bayesian hybrid method

Gamow Energy



R-matrix framework

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_{\lambda} - E},$$

$$\mathbf{U} = \rho^{\frac{1}{2}} \mathbf{O}^{-1} (1 - \mathbf{R} \mathbf{L}_0)^{-1} (1 - \mathbf{R} \mathbf{L}_0^*) \mathbf{I} \rho^{-\frac{1}{2}}$$

$$T_{cc'} = e^{2i\omega_c}\delta_{cc'} - U_{cc'},$$

$$\sigma_{\alpha\alpha'} = \frac{\pi}{k_{\alpha}^2} \sum_{Jll'ss'} g_J \left| T_{cc'}^J \right|^2,$$



Bardayan et al. (2015)

Simple Monte Carlo

- Created thousands of "synthetic" data sets by assuming that the error bars on the data represented an underlying Gaussian (probably should have used lognormal) Probability Density Function.
- Refit
- Histogrammed S-factor calculated at many energies to get uncertainty.
- Calculated many different variations on assumptions about the *R*-matrix fit and included those as well.
- Even more computationally expensive



Monte Carlo of experimental data Model uncertainties

Model assumptions

- External capture contribution for GS?
 - Often neglected completely
 - May play a significant role in the E2 cross section (given recent improvements in the uncertainty of this cross section)
- Inconsistent measurements of the ground state α -particle ANC in ¹⁶O
- Additional measurements needed!

TABLE I. Present ANC of the ¹⁶O GS and other available results in the literature.

Reference	ANC $(fm^{-1/2})$	Method
Adhikari (2009) [14]	13.9 ± 2.4	¹⁶ O + Pb breakup
Morais (2011) [16]	3390 (WS1)	$^{12}C(^{16}O, ^{12}C)^{16}O$
	1230 (WS2)	
	750 (FP)	
Sayre (2012) [11]	709	R matrix
Adhikari (2017) [15]	637 ± 86	${}^{12}C(^{7}Li, t){}^{16}O$
Present	337 ± 45	$^{12}C(^{11}B, ^{7}Li)^{16}O$



Background contributions



Best solution: Fit experimental data up in energy until the cross section gets smaller or there is a natural gap in the level structure.

Fit to higher energy

- Challenges
 - Multiple particle decay channels
 - Multiple entrance channel data
- Additional advantages
 - Combine ¹²C(α,γ), ¹⁵N(p,γ), and
 ¹⁵N(p,α) R-matrix analyses into single consistent analysis
 - Check on systematic uncertainties like effective energy determination and partial width consistency



R-matrix LEGO blocks (reaction components)

Resonance
Subthreshold state
HS External Capture



ANCs: Strength of subthreshold states and external capture

ANC of bound state 1⁻



Data inconsistencies



- "World data sets"
- Normalized to one another
- Large discrepancies in ground state E2 data

"E1 and E2" data

- Can in principle measure the "E1 cross section" by measuring the differential cross section at 90 degrees.
 - Problems when E2 >> E1
- To get E₂ one needs to measure the angular distribution.
- Main issue: often only the E1 and E2 derived cross sections have been reported in the literature, and the original angular distributions are omitted.
- Problems with extracting E2 data
 - Number of parameters to use for fit.
 - Systematic problems with data
 - Can often be corrected/understood if original angular distribution data are given
 - See Brune and Sayre (2013)



Measurement Highlights: TAMU ANCs

- Avila et al. (2015)
- Built on the ground breaking sub-Coulomb transfer measurements by Brune et al. (1999)
- ¹²C(⁶Li,d)¹⁶O
- Improved measurements of the 2⁺ and 1⁻ ANCs, first measurements of the o⁺ and 3⁻ ANCs
 - Around 10% uncertainties achieved



Measurement Highlights: High Precision ${}^{12}C(\alpha,\gamma){}^{16}O$ low energy measurements

- Makii et al. (2009)
- Used time-of-flight to separate out background from (n,γ) reactions produced by neutrons from the ¹³C(α,n)¹⁶O reaction
- Only two energies, but highly accurate and precise
- Just wish they could have done more measurements



Measurement Highlight: High Precision ${}^{12}C(\alpha,\gamma){}^{16}O$ low energy measurements

- Plag et al. (2012)
- Used time-of-flight to separate out background from (n,γ) reactions produced by neutrons from the ¹³C(α,n)¹⁶O reaction
- BaF₂ array for detailed angular distribution measurements
- All angular distribution data is given, not just E1 and E2 data



Alternative transfer reactions

- Shen et al. (2019, 2020)
- ¹²C(¹¹B,⁷Li)¹⁶O
- Probing the model and reaction sensitivity of ANC determination
- 2⁺ ANC is remeasured in (2019)
- GS ANC is measured in (2020)



TABLE I.Present ANC of the ¹⁶O GS and other availableresults in the literature.Shen et al. (2020)

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This ends up being a big multidimensional fit problem

- Fit parameters
 - Level energies
 - Level widths (or reduced width amplitudes)
 - Asymptotic Normalization Coefficients (or reduced width amplitudes)
 - Data normalization factors
- For the case described in the deBoer *et al.* (2017), there were 64 level parameters and over 100 data normalization fit parameters
- >10,000 experimental data points
- Computationally challenging



Improving uncertainty estimation for R-matrix fits

- A more general problem
- Bayesian methods provide a way to improve and gain more detailed information
- See de Souza et al. (2020) for an application to ³H(d,n)⁴He
- Computationally intensive, but probably doable
- Daniel Odell at Ohio University has developed the Bayesian R-matrix Inference Code Kit (BRICK) for use with the AZURE2 R-matrix code



Figures courtesy of Daniel Odell

Improvement in reaction rate tabulation!

- Reaction rates are tabulated over "standard" temperature grids.
- Astrophysics codes then interpolate the values given in tables
- Have to watch out for too few temperature steps!
- Frank Timmes and Ebraheem Farag



MESA interpolation is off by more than 1σ !!!

Some interesting estimates

- Holt and Filippone (2019)
 - ¹⁶O(e,e'α)¹²C
 - Measures only the E2 component
- Holt et al. (2019)
 - ¹⁶O(γ,α)¹²C
 - Gives the total GS





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Summary

- Improvements in the uncertainty of the low energy S-factor are hampered by inconsistent data
 - Newer data are much more consistent! (a general trend) We're on the right track.
 - Make new measurements using new techniques (reduce systematic uncertainties)
 - ${}^{16}O(\gamma, \alpha){}^{12}C$ (HI γ S, Jefferson Lab, ELI-NP)
 - ¹⁶O(e,e'α)¹²C
 - Additional types of transfer measurements
 - Improved traditional measurements, but in low background environments (reduce statistical / outlier uncertainties)
 - LUNA, JUNA
- Improve uncertainty estimation
 - Bayesian uncertainty estimation
 - Improved computational resources