Study of dark matter scattering off $^{2}\mathrm{H}$ and $^{4}\mathrm{He}$ nuclei within Chiral effective field theory

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Motivations

- Since the '30s of last century, on galactic scales and over, a great number of gravitational anomalies has been detected.
  - The most popular explanation is the existence of a new kind of particles: the dark matter (DM) [Bertone *et al.*, Phys. Rept 405 (2005)]
- In order to analyze the results of the various direct detection experiments, which are currently attempting to detect DM, an accurate description of the nuclear response is required
- Light nuclei are great testing laboratories as they can be described from first principles to high precision
  - Helium isotopes are potential experimental target as they are sensitive to DM particles with mass ≤ 10 GeV [W. Guo *et al.*, Phys. Rev. D 87 (2013)]

Our purpose is the study of the ${}^2\mathrm{H}$-${}^4\mathrm{He}$ nuclear response to DM scattering, assumed to be composed by Weak Interacting Massive Particles (WIMPs)

- The WIMPs can be assumed to be nonrelativistic (they are gravitationally bound by the galaxy) $|\frac{v_\chi}{c}| \sim 10^{-3}$
- The typical momentum and transferred energy are small and the nucleus does not break apart
- To describe this type of scattering, the Chiral effective field theory (χEFT) approach to nuclear dynamics can be used

Effective quark-WIMP interactions

We start from the general dimension 6 effective Lagrangian for the interaction between quark and WIMP, the latter assumed to be a Dirac fermion. The Lagrangian can be cast in the form [M. Hoferichter et al., Phys. Rev. Lett. (2016)]

$$\begin{aligned}\mathcal{L}_q &= \mathcal{L}_{\mathrm{QCD}}^{\mathcal{M}=0} + \bar{q}(x)\gamma^\mu\left(v_\mu(x) + \frac{1}{3}v_\mu^{(s)}(x) + \gamma^5 a_\mu(x)\right)q(x) \\ &\quad - \bar{q}(x)\left(s(x) - i\gamma^5 p(x)\right)q(x) + \bar{q}(x)\sigma^{\mu\nu}t_{\mu\nu}(x)q(x)\end{aligned}$$

$$s(x) = -\frac{1}{\Lambda_S^2}\left(C_{S+} + C_{S-}\tau_z\right)\bar{\chi}\chi \qquad p(x) = \frac{1}{\Lambda_S^2}\left(C_{P+} + C_{P-}\tau_z\right)\bar{\chi}i\gamma_5\chi$$

$$\frac{1}{3}v^{\mu(s)}(x) = \frac{1}{\Lambda_S^2}C_{V+}\bar{\chi}\gamma^\mu\chi \qquad v^\mu(x) = \frac{1}{\Lambda_S^2}C_{V-}\tau_z\bar{\chi}\gamma^\mu\chi$$

$$t^{\mu\nu}(x) = \frac{1}{\Lambda_S^2}\left(C_{T+} + C_{T-}\tau_z\right)\bar{\chi}\sigma^{\mu\nu}\chi \qquad a^\mu(x) = \frac{1}{\Lambda_S^2}C_{A-}\tau_z\bar{\chi}\gamma^\mu\gamma_5\chi$$

with $C_{X\pm}$ adimensional coupling constant to be determined by experimental data and $\Lambda_S = 1$ GeV inserted for dimensional reasons

Differences with respect to other approaches

- Presence of an isoscalar part $a_\mu^{(s)}(x) \to SU(3)$
- Presence of tensor current $t^{\mu\nu} \to$ new terms in the χEFT Lagrangian

Nucleon-WIMP interactions

[Bishara et al. J. High Energ. Phys. (2016)]

- For each interaction case, the interaction vertices between nucleon and WIMP are derived substituting the expression of $s, p, v, \ldots$ in term of the WIMP field in the hadron χEFT Lagrangians
  - Example: Scalar interaction

$$\mathcal{L}_{int} = c_1 \bar{N}\langle \xi_+ \rangle N + c_5 \bar{N}\hat{\xi}_+ N + \frac{f_\pi^2}{4}\left\langle \xi(x)U^\dagger(x) + U(x)\xi^\dagger(x)\right\rangle + \cdots$$

$$\begin{cases} U = e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi} \quad u = \sqrt{U} \\ \xi_+ = u^\dagger \xi\, u^\dagger + u\,\xi^\dagger u \\ \xi(x) = 2B_c\big(s(x) + ip(x)\big) \\ c_1, c_5, B_c \text{ are LECs} \end{cases}$$

$$\mathcal{L}_{int} \approx -\frac{8B_c c_1}{\Lambda_S^2} C_{S+}\bar{N}N\bar{\chi}\chi - \frac{4B_c c_5}{\Lambda_S^2} C_{S-}\bar{N}\tau_z N\bar{\chi}\chi + \frac{C_{S+}}{\Lambda_S^2}\bar{\chi}\chi\pi^2 + O(\pi^3)$$

- Using the Legendre transformation, write the H_{int} in the Schrödinger picture

$$H_{int} = H^{NN\chi\chi,00} + H^{\pi\pi\chi\chi,02} + H^{\pi\pi\chi\chi,11} + H^{\pi\pi\chi\chi,20} + H^{\pi NN,01} + H^{\pi NN,10} + \cdots$$

$$H^{NN\chi\chi,00} = \frac{1}{\Omega}\sum_{\mathbf{p}'s't',\mathbf{p}st,\mathbf{k}'r',\mathbf{k},r} b^\dagger_{\mathbf{p}',s',t'} b_{\mathbf{p},s,t} B^\dagger_{\mathbf{k}',r'} B_{\mathbf{k},r}\, M^{NN\chi\chi,00}_{\mathbf{p}'s't'\mathbf{p}st\mathbf{k}'r'\mathbf{k}r}\, \delta_{\mathbf{p}'+\mathbf{k}',\mathbf{p}+\mathbf{k}}$$

$b^\dagger$, b, $B^\dagger$, $B=$ creation and annihilation operators, $\mathbf{p}, s, t, \mathbf{p}', s', t'=$ nucleon states, $\mathbf{k}, s, \mathbf{k}', s'=$ WIMP states

- Example: NR expansion of vertex function M for the scalar interaction up to $O(1/M^2)$

$$M^{NN\chi\chi,00}_{\alpha'\alpha\mathbf{k}'r'\mathbf{k}r} \approx \left(\frac{8B_c c_1 C_{S+}}{\Lambda_S^2} + \frac{4B_c c_5 C_{S-}}{\Lambda_S^2}\tau_z\right)_{t't}\left(1 - \frac{(\mathbf{p}+\mathbf{p}')^2}{8M^2} - \frac{i(\mathbf{p}'\times\mathbf{p})\cdot\boldsymbol{\sigma}}{4M^2}\right)_{s's} \times \left(1 - \frac{(\mathbf{k}+\mathbf{k}')^2}{8M_\chi^2} - \frac{i(\mathbf{k}'\times\mathbf{k})\cdot\boldsymbol{\sigma}}{4M_\chi^2}\right)_{r'r}$$

$(\tau_i)_{t't}, (\boldsymbol{\sigma})_{s's}=$ matrix elements of spin isospin Pauli matrices

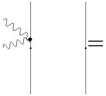
- The amplitude for the elastic scattering of a WIMP by a two-nucleon system is obtained using the time-ordered perturbation theory (TOPT) method

$$T = H_{int} + H_{int}\frac{1}{E_i - H_0}H_{int} + H_{int}\frac{1}{E_i - H_0}H_{int}\frac{1}{E_i - H_0}H_{int} + \cdots$$

- Each term contributing to the T-matrix can be visualised as a TOPT diagram
- A chiral order ν can be assigned to each diagram: its contribution is of order $(Q/\Lambda_\chi)^\nu$ ($Q \sim m_\pi$ = nucleon momentum, $\Lambda_\chi = 4\pi f_\pi \sim 1$ GeV)

$\Rightarrow$ In this study we will consider contributions up to the next-to-next-lo-leading-order (N2LO)

Example: Scalar interaction



$$= M^{NN\chi\chi}_{\mathbf{p}_1'\mathbf{p}_1\mathbf{k}'\mathbf{k}}\,\delta_{\mathbf{p}_1'+\mathbf{k}',\mathbf{p}_1+\mathbf{k}}\,\delta_{\mathbf{p}_2',\mathbf{p}_2} \sim Q^{-3}$$

Chiral order of each diagram

- NR expansion of vertex functions $M^{NN\chi\chi}$, ... in powers of Q/Λ_χ
- Energy denominators $\sim Q^{-1}$
- δ in addition to the total momentum conservation $\sim Q^{-3}$

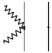
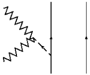
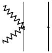
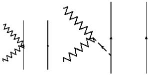
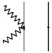
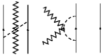
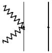
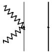
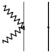
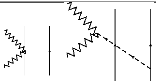
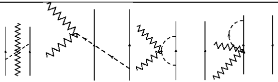
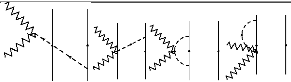
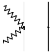
The amplitude has the following general form

$$T_{fi} = \left\{ \frac{1}{\Omega}\left(J^{(1)}_{\alpha_1,\alpha_1'}\delta_{\mathbf{p}_1'+\mathbf{k}',\mathbf{p}_1+\mathbf{k}}\delta_{\alpha_2',\alpha_2} + J^{(1)}_{\alpha_2,\alpha_2'}\delta_{\mathbf{p}_2'+\mathbf{k}',\mathbf{p}_2+\mathbf{k}}\delta_{\alpha_1',\alpha_1}\right) + \frac{1}{\Omega^2}J^{(2)}_{\alpha_1,\alpha_1',\alpha_2,\alpha_2'}\delta_{\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}-\mathbf{k}'}\right\} \cdot L_{\mathbf{k}r,\mathbf{k}'r'}$$

where α_i (α_i')$\equiv \{\mathbf{p}_i, s_i, t_i\}$ = state of nucleon i, $\mathbf{k}$ ($\mathbf{k}'$)=initial (final) WIMP momentum and r (r') its spin projection, $\mathbf{k}_i = \mathbf{p}_i' - \mathbf{p}_i$, $\mathbf{K}_i = (\mathbf{p}_i + \mathbf{p}_i')/2$, $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ and $\mathbf{Q} = (\mathbf{k} + \mathbf{k}')/2$

- $J^{(1)}$ ($J^{(2)}$) is the so-called one-body (two-body) current, while L is the so-called WIMP current
- We neglect eventual three-body transition currents

Considered diagrams

Chiral order	S	P	V	A	T
Q^{-3}					
Q^{-2}		—		—	
Q^{-1}					

Nucleus-WIMP transition amplitude

$T_{f,i}$ between the initial nucleus+WIMP state $|\mathbf{P}_i, M_i, \mathbf{k}, r\rangle$ and final nucleus+WIMP state $|\mathbf{P}_f, M_f, \mathbf{k}', r'\rangle$ is

$$\langle \mathbf{P}_f \mathbf{k}' | T | \mathbf{P}_i \mathbf{k}\rangle = \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} \int e^{i\mathbf{q}\cdot\mathbf{x}} \langle \Psi_A^{J_A,s'_A} | J^{Xa}{}_c(\mathbf{x}) | \Psi_A^{J_A,s_A}\rangle (L^{Xa})^c d\mathbf{x}\,, \qquad J_c^{Xa}(\mathbf{x}) = \sum_{i=1,A} J^{Xa,(1)}{}_c(i)\delta(\mathbf{x}-\mathbf{r}_i) + \sum_{i<j} J^{Xa,(2)}{}_c(i,j)\delta(\mathbf{x}-\mathbf{R}_{ij})\,.$$

The quantities $J_i^{Xa,(1)}$ and $J_{ij}^{Xa,(2)}$ are related to the Fourier transforms of the one- and two-body currents.
Performing the multipole expansion of the current

$$= \sum_{a=\pm} \frac{C_{Xa}}{\Lambda_S^2} \frac{1}{\Omega} \delta_{\mathbf{q}+\mathbf{P}_i,\mathbf{P}_f} (-1)^{J-J_z} \Bigg(\sum_{l\geq 0}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,0}(\varphi,\theta,-\varphi)\sqrt{4\pi}(J' J'_z J - J_z | lm) \{ L_0 X_l^C(q,r) + L_z X_l^L(q,r)\}$$

$$+ \sum_{l=1}^{\infty} \sum_{m=-l}^{l} i^l \mathcal{D}^l_{m,\lambda}(\varphi,\theta,-\varphi)\sqrt{2\pi}(J' J'_z J - J_z | lm) \times L_{-\lambda} \left\{ -\lambda X_l^M(q,r) - X_l^E(q,r) \right\} \Bigg)$$

$$\mathbf{q} = \mathbf{k} - \mathbf{k}'$$

$\mathcal{D}^l_{m,m'}(\alpha,\beta,\gamma)$ standard rotation matrices, θ, φ angles of $\mathbf{q}$, X_l^Y reduced matrix elements (RMEs)

- $\mathbf{L} = \mathbf{L} \cdot \hat{\epsilon}_{\mathbf{q},\lambda}$, $\hat{\epsilon}_{\mathbf{q},\lambda}$ = set of 3 right-handed vectors
- Nuclear wave function evaluated using the Hyperspheric Harmonics method with both phenomenological (Av18) and chiral N4LO potentials [M. Viviani et al., Phys. Rev. Lett (2023)]
- We first evaluate the matrix element and then invert the equation to determine the RMEs

Cross-section

The non-polarized cross section for this process is calculated from Fermi golden rule, by mediating over the initial polarizations and summing over the final ones,

$$\sigma_{fi} = \frac{2\pi}{2(2J_A+1)} \sum_{r'r,s'_A s_A,\mathbf{k}',\mathbf{P}'_A} \frac{1}{v} \delta_{\mathbf{k},\mathbf{P}'_A+\mathbf{k}'} |T_{fi}|^2 \delta\left(\frac{k^2}{2M_\chi} - \frac{P'^2_A}{2M_A} - \frac{k'^2}{2M_\chi}\right)$$

where J_A is the spin of the target nuclei and $\mathbf{v} = \mathbf{k}/M_\chi$ is the velocity of the incoming WIMP.
Using again the multipole expansion $d^2\sigma_{fi}$ can be written as

$$\frac{d^2\sigma}{dE'_A d\hat{\mathbf{P}}'_A} = \frac{\pi}{(2J_A+1)(2\pi)^3} \frac{C^2_{Xa}}{\Lambda^4_S} \sum_{r'r} M_A\, \delta\left(\mathbf{v}\cdot\hat{\mathbf{P}}'_A - \frac{\mathbf{P}'_A}{2\mu}\right) \frac{1}{v} \Bigg\{ (4\pi) \sum_{l\geq 0} \left[L_0 L_0{}^* |X_l^C|^2 + L_z L_z{}^* |X_l^L|^2 - 2 L_0 L_z{}^* Re\left(X_l^C X_l^{L*}\right) \right] + (4\pi) \sum_{l\geq 1} L_1 L_1^* \left(|X_l^M|^2 + |X_l^E|^2\right) \Bigg\}$$

where $E'_A = \sqrt{M_A^2 + \mathbf{P}'_A{}^2} - M_A$ is the recoiling nucleus kinetic energy.

Interaction rate

The double-differential interation rate is given by,

$$\frac{d^2R}{dE'_A\hat{\mathbf{P}}'_A} = N_\chi N_A \int d^3\mathbf{v}\, v\, f(\mathbf{v}) \frac{d^2\sigma}{dE'_A d\hat{\mathbf{P}}'_A}$$

$$N_A = \frac{100\,\text{ton}}{\text{Nucleus Mass}}\ \text{(arbitarily chosen)}$$

$$N_\chi = \frac{0.6\,\text{GeV/cm}^3}{\text{WIMP Mass}}$$ [*Cadeddu et al.* (2017)]

We assume the Standard Halo Model (SHM), i.e. a Maxwell-Boltzmann WIMP velocity distribution of width σ_v [Cadeddu *et al.*, Astro-ph (2017)]

$$f(\mathbf{v}) = \frac{1}{\sqrt{(2\pi\sigma_v^2)^3}} e^{-\frac{1}{2}\left(\frac{\mathbf{v}+\mathbf{V}}{\sigma_v}\right)^2} \qquad \sigma_V = \frac{V}{\sqrt{2}} \qquad V \approx 220\,\text{km/s (Sun velocity)}$$

Sum over the WIMP spin

$$\sum_{r'r} L_i L_j^* = a + \mathbf{b}\cdot\mathbf{u} + cu^2 + (\mathbf{d}\cdot\mathbf{u})^2 + O(\mathbf{u})^3 \qquad a, \mathbf{b}, c, \mathbf{d} = \text{constant terms} \qquad \mathbf{u} = \mathbf{v} + \mathbf{V}$$

After integrating over $\mathbf{u}$

$$\frac{d^2R}{dE'_A d\hat{\mathbf{P}}'_A} = \frac{N_\chi N_A M_A}{(2J_A+1)4\pi^2} \frac{C_{Xa}^2}{\Lambda_S^4} \frac{e^{-\frac{A^2}{2\sigma_v^2}}}{\sqrt{2\pi\sigma_v^2}} \left(a + \mathbf{b}\cdot\hat{\mathbf{q}}A + 2c\sigma_v^2 + cA^2 + d^2\sigma_v^2 - (\mathbf{d}\cdot\hat{\mathbf{q}})^2(\sigma_v^2 - A^2)\right) \sum_{\alpha=1,4} F_\alpha^X(q),$$

where $A = \mathbf{V}\cdot\hat{\mathbf{q}} + \frac{q}{2\mu}$, μ =reduced mass and

$$F_1^X(q) = 4\pi \sum_l |X_l^C|^2, \; F_2^X(q) = -4\pi \sum_l 2Re\left(X_l^C X_l^{L*}\right), \; F_3^X(q) = 4\pi \sum_l |X_l^L|^2, \; F_4^X(q) = 4\pi \sum_l \left(|X_l^M|^2 + |X_l^E|^2\right).$$

Deuteron-DM scattering

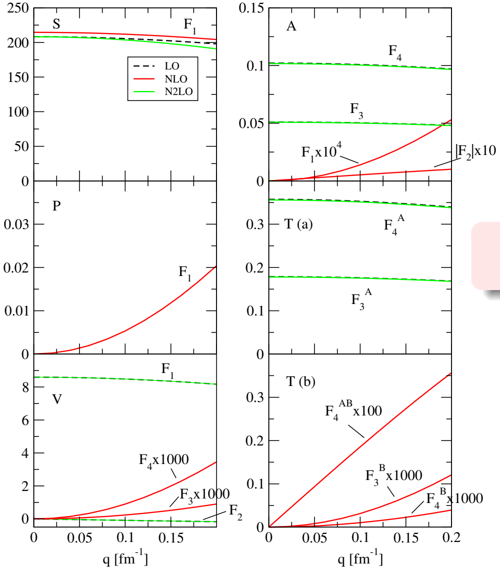
Contribution of the RMEs

$q = 0.05\ \mathrm{fm}^{-1}$

Int.	RME	order	AV18			N4LO500		
S			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	LO	$-0.144+02$		$-0.229-02$	$-0.144+02$		$-0.231-02$
		NLO	$-0.218+00$		$+0.328-03$	$-0.806-01$		$+0.336-03$
		N2LO	$+0.153+00$		$-0.467-05$	$+0.103+00$		$-0.829-05$
P			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	NLO		$-0.367-01$			$-0.377-01$	
V			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	LO	$-0.293+01$		$-0.465-03$	$-0.293+01$		$-0.470-03$
		N2LO	$+0.289-04$		$+0.320-05$	$+0.294-04$		$+0.320-05$
	X_ℓ^L	NLO	$-0.769-02$		$-0.120-05$	$-0.769-02$		$-0.120-05$
	X_ℓ^M	NLO		$-0.150-01$			$-0.152-01$	
A			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	X_ℓ^C	NLO		$+0.593-03$			$+0.609-03$	
	X_ℓ^L	LO		$+0.226+00$			$+0.232+00$	
	X_ℓ^L	N2LO		$-0.469-03$			$-0.781-03$	
	X_ℓ^E	LO		$-0.319+00$			$-0.328+00$	
	X_ℓ^E	N2LO		$+0.660-03$			$+0.110-02$	
T			$\ell=0$	$\ell=1$	$\ell=2$	$\ell=0$	$\ell=1$	$\ell=2$
	$X_\ell^L(A)$	LO		$-0.422+00$			$-0.433+00$	
	$X_\ell^L(B)$	NLO	$-0.280E-02$		$-0.290E-03$	$-0.285E-02$		$-0.314E-03$
	$X_\ell^L(A)$	N2LO		$+0.875-03$			$+0.687-03$	
	$X_\ell^E(A)$	LO		$+0.597+00$			$+0.613+00$	
	$X_\ell^M(B)$	NLO		$+0.157-02$			$+0.161-02$	
	$X_\ell^E(A)$	N2LO		$-0.124-03$			$-0.978-03$	

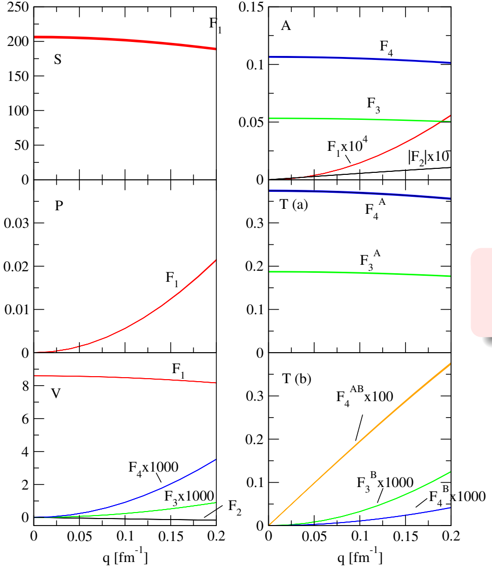
- LO transition operators give largest RMEs
- RMEs dependence from nuclear interaction is rather weak

Structure Functions calculated with AV18 potential



- NLO and N2LO components in the transition operators are tiny

Structure Functions calculated with chiral potential

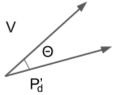


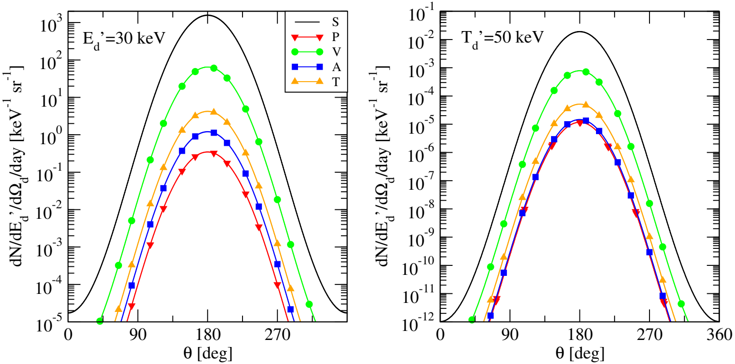
- Width band reflect the spread of theoretical results using $\Lambda = 450$, 500, or 550 MeV cutoff values

Events per day

$$\frac{d^2N}{d\hat{\mathbf{P}}'_d dE'_d} \equiv \frac{d^2R}{d\hat{\mathbf{P}}'_d dE'_d} \times 1 \;\; \text{day} = \text{events per day per unit of deuteron recoil energy and per steradiant}$$

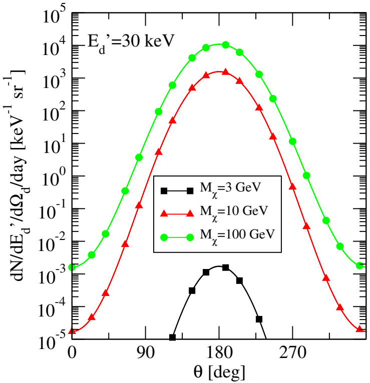
(indipendent of φ) $C_{X+} = 10^{-4}$ and $M_\chi = 10$ GeV

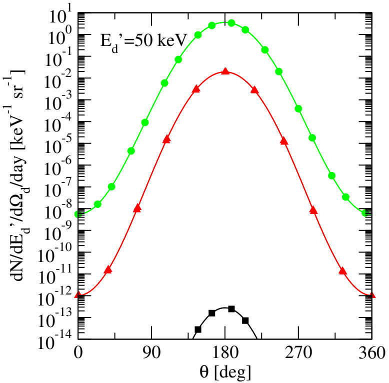




- The rates are peaked at $\theta = 180$ deg$\Rightarrow$ most of scattered deuterons observed in the direction $-\mathbf{V}$
- The number of events also depends critically on the kinetic energy of the observed deuteron$\Rightarrow$ the smaller is the better
- It also depends on the assumed interaction type $\Rightarrow$ scalar interaction largest contribution
- The results shown in the figure are bands: Rates calculated with the N4LO450, N4LO500, and N4LO550 interactions and obtained with transition currents from LO to N2LO $\Rightarrow$ weak dependence on the NN interaction

Number of events for three different values of the WIMP mass M_χ for scalar interaction





- For lighter WIMP the number of recoiling deuterons at a given energy decreases noticeably
- Dependence on the WIMP mass is particularly critical for light WIMP, with mass around 1 to 10 GeV

^{4}He-DM scattering

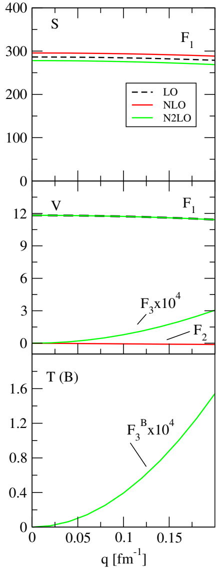
RMEs

$q = 0.05$ fm^{-1}

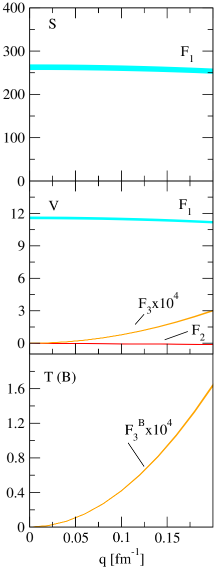
Int.	RME	order	AV18/UIX	N4LO500/3N
S			$\ell = 0$	$\ell = 0$
	X_ℓ^C	LO	$-0.169 + 02$	$-0.168 + 02$
		NLO	$-0.273 + 00$	$+0.144 + 00$
		N2LO	$+0.521 + 00$	$+0.313 + 00$
		N2LO	$+0.451 - 02$	$+0.448 - 02$
V			$\ell = 0$	$\ell = 0$
	X_ℓ^C	LO	$-0.343 + 01$	$-0.341 + 01$
		N2LO	$+0.325 - 04$	$+0.336 - 04$
	X_ℓ^L	NLO	$-0.444 - 02$	$-0.438 - 02$
T			$\ell = 0$	$\ell = 0$
	$X_\ell^L(B)$	NLO	$-0.315 - 02$	$-0.325 - 02$

- Contributions of isovector operators neglected since they are very small

Structure Functions



Av18

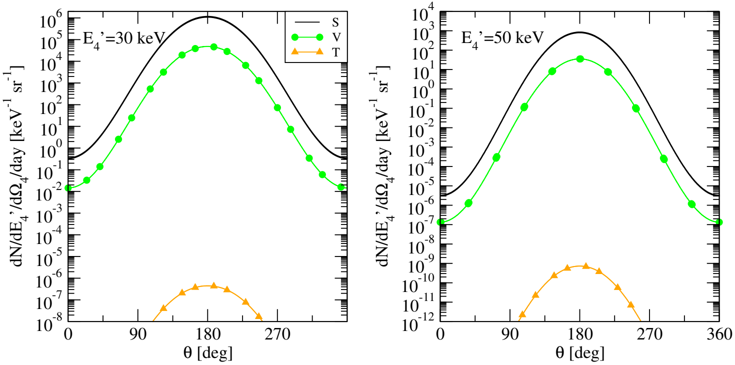


Chiral N4LO/N2LO interactions

NLO and N2LO contributions are sizeable for s case very tiny for the other cases

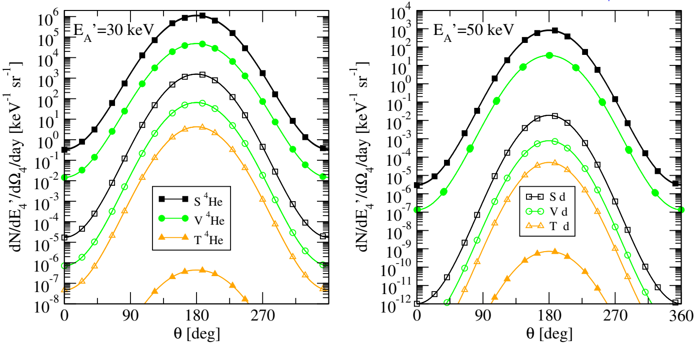
Events per day with Chiral potential

$C_{X+} = 10^{-4}$ and $M_\chi = 10$ GeV



Comparison between the rates for ^{2}H and ^{4}He

$C_{X+} = 10^{-4}$ and $M_\chi = 10$ GeV



^{4}He rate higher than the ^{2}H case $\Rightarrow$ J_A and $e^{-\frac{(\mathbf{V}\cdot\hat{\mathbf{q}}+\frac{q}{2\mu})^2}{2\sigma_v^2}}$

Conclusion & Future perspectives

With respect to other calculations in the literature

- Inclusion of two body currents, and treated $a_\mu^{(s)}$ and $t^{\mu\nu}$ currents
- Systematically study of the interation rate for each interactione type

In conclusion

- Scalar interaction dominant over the others and therefore disadvantaged by the current experimental limits
- NLO and N2LO contributions of the currents are suppressed
- Little dependence of the results on the nuclear interaction used for the wave function calculation

Future goals

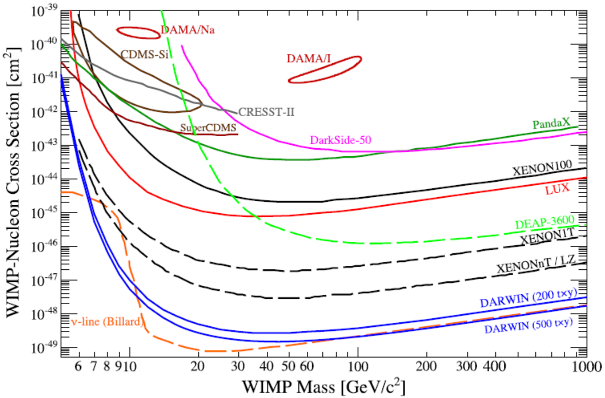
- Treatement of other cases, as the inclusion of parity and/or charge conjugation violating terms, or the cases of either the WIMP being a scalar or a Maiorana fermion
- Extend the study to heavier nuclei which are currently used in direct detention experiments, such as Argon

Thank you for your kind attention

Backup slides

Experimental situation

WIMP-nucleon cross section limits



[Baudis, 2018]

The chiral symmetry in QCD

Considering only the lightest flavour quarks, u and d [Peskin & Schroeder, 2005]

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}(x)\left(i\gamma^\mu D_\mu - \mathcal{M}\right)q(x) - \frac{1}{4}\mathcal{G}_{\mu\nu,a}(x)\mathcal{G}_a^{\mu\nu}(x) \qquad q = \begin{pmatrix} u \\ d \end{pmatrix}, \mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

If we neglet the mass term, the so-called chiral limit, $\mathcal{L}_{\mathrm{QCD}}$ is invariant under the trasformation of the group $G_\chi \equiv U(1)_R \otimes U(1)_L \otimes SU(2)_R \otimes SU(2)_L$ isomorphic to $U(1)_V \otimes U(1)_A \otimes SU(2)_V \otimes SU(2)_A$

QCD Lagrangian for massless quarks invariant under the local G_χ transformation with external currents [Gasser & Leutwyler, 1984]

$$\mathcal{L}_q = \mathcal{L}_{\mathrm{QCD}}^{\mathcal{M}=0} + \bar{q}(x)\gamma^\mu\left(v_\mu(x) + \frac{1}{3}v_\mu^{(s)}(x) + \gamma^5 a_\mu(x)\right)q(x) - \bar{q}(x)\left(s(x) - i\gamma^5 p(x)\right)q(x)$$

$$\begin{cases} v_\mu(x) = \sum_{a=x,y,z}\frac{\tau_a}{2}v_\mu^a(x) \quad a_\mu(x) = \sum_{a=x,y,z}\frac{\tau_a}{2}a_\mu^a(x) \\ s(x) = \sum_{a=0}^{3}\tau_a s^a(x) \quad p(x) = \sum_{a=0}^{3}\tau_a p^a(x) \\ \tau_0 = \mathbf{1}, \tau_{i=1,2,3} = \text{Pauli matrices} \end{cases}$$

- no $a_\mu^{(s)}$ current due to $U_A(1)$ anomaly
- coupling with EM field: $v_\mu(x) = -e\frac{\tau_z}{2}\mathcal{A}_\mu(x)$, $v_\mu^{(s)}(x) = -\frac{e}{2}\mathcal{A}_\mu(x)$
- G_χ is spontaneusly broken, pions are Goldstone boson: $s = \mathcal{M}$ the breaking is taking into account

χEFT for hadrons

- Description of hadrons using QCD is very complicated→ χEFT [Weinberg, 1969]
- Expansion in a power $(Q/\Lambda_\chi)^\nu$ [Weinberg, 1990]
  - ν =chiral order, $Q \sim m_\pi$ = typical value of nucleon momentum inside a nucleus, $\Lambda_\chi = 4\pi f_\pi \sim 1$ GeV= typical energy scale of the strong interaction
- The symmetries used to build the effective Lagrangian are
  - chiral symmetry
  - Lorentz invariance
  - C, P
- $\mathcal{L} = \sum_\nu \mathcal{L}^{(\nu)} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(2)}_{\pi\pi} + \dots$

- Nucleon and pion fields $N = \begin{pmatrix} p \\ n \end{pmatrix}$ $\quad \vec{\pi} = (\pi_x, \pi_y, \pi_z)$
- Basic quantities $U = e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi}$ $\quad U' = RUL^\dagger \quad u = \sqrt{U} \quad u' = Ruh^\dagger \quad h \equiv h(L, R, \pi)$
- N under $G_\chi \to N' = hN$
- The unknown quark dynamics is parametrized via the so called low-energy constants (LECs) which can be fixed in general from experiments

Used to describe $\pi\pi$, $N\pi$, NN and $3N$ interactions

- NN interactions derived up to order $(Q/\Lambda_\chi)^\nu$ $\nu = 5$ [Machleidt *et al*, 2017]
- 3N interactions up to $(Q/\Lambda_\chi)^2$ [Epelbaum *et al*, 2002]

Electromagnetic & Weak interactions → $v_\mu \& a_\mu$ currents [Pastore *et al*, 2009] [Baroni *et al*,2013]

Invariant terms under local G_χ [Fettes, 2000]:

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\left\langle D_\mu U(x)(D^\mu U(x))^\dagger\right\rangle + \frac{f_\pi^2}{4}\left\langle \xi(x)U^\dagger(x) + U(x)\xi^\dagger(x)\right\rangle$$

$$r_\mu = v_\mu + a_\mu\,, \quad l_\mu = v_\mu - a_\mu$$

$$D_\mu U(x) = \partial_\mu U(x) - ir_\mu(x)U(x) + il_\mu(x)U(x)$$

$$\xi(x) = 2B_c(s(x) + ip(x))$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\not{D} - M + \frac{g_A}{2}\gamma^\mu\gamma^5 u_\mu\right)N$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\pi N} = \; & c_1\bar{N}\langle\xi_+\rangle N \\ & -\frac{c_2}{8M^2}\left[\bar{N}\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\}N + \text{h.c.}\right] \\ & +\frac{c_3}{2}\bar{N}\langle u_\mu u^\mu\rangle N \\ & +\frac{i c_4}{4}\bar{N}[u_\mu, u_\nu]\sigma^{\mu\nu}N \\ & +c_5\bar{N}\hat{\xi}_+N \\ & +\frac{c_6}{8M}\bar{N}\sigma^{\mu\nu}F^+_{\mu\nu}N \\ & +\frac{c_7}{4M}\bar{N}\sigma^{\mu\nu}F^{(s)}_{\mu\nu}N\end{aligned}$$

$$u_\mu = i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu u^\dagger)\}$$

$$\xi_\pm = u^\dagger\xi\, u^\dagger \pm u\,\xi^\dagger u$$

$$F^{(s)}_{\mu\nu} = \partial_\mu v^{(s)}_\nu - \partial_\nu v^{(s)}_\mu$$

$$F^\pm_{\mu\nu} = u^\dagger F^R_{\mu\nu}u \pm u\,F^L_{\mu\nu}u^\dagger$$

where $\langle\ldots\rangle$ indicates the trace of the matrices and $\hat{\xi}_+ = \xi_+ - \frac{1}{2}\langle\xi_+\rangle$

- The unknown quark dynamics is parametrized via the so called low-energy constants (LECs) B_c, c_1, c_2,..which can be fixed in general from experiments

LECs

- B_c related to the quarks condensate and to the pion mass. From $\mathcal{L}_\pi^{(2)}$ $m_\pi^2 = 2m_q B_c \to B_c \approx 2.78\,\mathrm{GeV}$ higher order corrections from $\mathcal{L}_\pi^{(4)}$ etc. We adopted the more precise estimate from the lattice calculations [S. Aoki *et al.*, 2017]
- $c_1 - c_5$ extracted from an accurate analysis of πN scattering data [M. Hoferichter *et al.*, 2015]
- c_6 and c_7 related to the anomalous magnetic moment of the nucleons [J. Carlson and R. Schiavilla, 1988]
  $c_6 = \kappa_p - \kappa_n \,, \qquad c_7 = \kappa_p + \kappa_n \qquad \kappa_p = 1.793\,, \kappa_n = -1.913$
- $F + D \sim 1.26 \sim g_A$, $3F - D = 0.31$ [A. Hosaka *et al.*, 2009]
- $\tilde{c}_1$ and $\tilde{c}_2$ from the results of a recent lattice calculation on the tensor charges of the nucleons [C. Alexandrou *et al.*,2017]
- Other LECs fixed in similar way

LEC	Value
B_c	$2.40\,\mathrm{GeV}$
c_1	$-1.10\,\mathrm{GeV}^{-1}$
c_6	3.71
c_7	-0.12
D	0.86
F	0.39
$4\tilde{c}_1$	0.58

Incorporation of the isoscalar axial current

Due to the $U_A(1)$ anomaly, it is not possible to introduce isoscalar axial current in $SU(2)$ space$\rightarrow SU(3)$ [Scherer & Schindler, 2004]

$$\langle N|\bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d|N\rangle \rightarrow \langle N|A_\mu^{(8)}|N\rangle$$

where

$$A_\mu^{(8)} = \bar{u}\gamma_\mu\gamma^5 u + \bar{d}\gamma_\mu\gamma^5 d - 2\bar{s}\gamma_\mu\gamma^5 s = \sqrt{3}\bar{q}\gamma_\mu\gamma^5\lambda_8 q \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix} \; q(x) = \begin{pmatrix}u(x)\\ d(x)\\ s(x)\end{pmatrix}$$

Valid in the hypothesis that the content of the strange quark in the nucleon vanishes [Ellis, Nagata & Olive, 2018]

The axial current part of the quark Lagrangian in the $SU(3)$ space is

$$\mathcal{L}_q^{axial} = \sum_i \alpha_i \bar{q}\gamma_\mu\gamma^5\lambda_i q\bar{\chi}\gamma^\mu\gamma^5\chi \qquad a_\mu = a_{\mu i}\lambda_i = \alpha_i\bar{\chi}\gamma_\mu\gamma^5\chi\lambda_i$$

where the costants α_i are zero except

$$\alpha_3 = \frac{C_{A-}}{\Lambda_S^2} \qquad \alpha_8 = \frac{C_{A+}}{\Lambda_S^2} \qquad \lambda_3 = \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

- Since now $\langle\lambda_i\rangle = 0 \rightarrow$ no anomaly

Tensor current

Not treated in literature

$$t^{\mu\nu} = t_0^{\mu\nu}\mathbf{1} + t_a^{\mu\nu}\tau_a$$

Assuming

$$t^{\mu\nu} \to t^{\mu\nu\prime} = L t^{\mu\nu} R^\dagger$$

$$\mathcal{L}_q^{tens} = \bar{q}\sigma_{\mu\nu}t^{\mu\nu}q = \bar{q}_R\sigma_{\mu\nu}t^{\mu\nu}q_L + \bar{q}_L\sigma_{\mu\nu}(t^{\mu\nu})^\dagger q_R$$

In the hadron Lagrangian we construct the terms invariant under G_χ:

- Lowest order $O(Q^2)$

$$\bar{N}\sigma_{\mu\nu}T_\pm^{\mu\nu}N \qquad T_\pm^{\mu\nu} = u t^{\mu\nu\dagger} u \pm u^\dagger t^{\mu\nu} u^\dagger$$

Only term C, P invariant $\to T_+^{\mu\nu}$

$$\mathcal{L}_{\pi N}^{(2)} = \tilde{c}_1 \bar{N}\sigma_{\mu\nu}\langle T_+^{\mu\nu}\rangle N + \tilde{c}_2 \bar{N}\sigma_{\mu\nu}\hat{T}_+^{\mu\nu}N$$

where $\tilde{c}_1$ and $\tilde{c}_2$ are new LECs

- Other terms $O(Q^3)$

$$\bar{N}\gamma^\mu\gamma_5[u^\nu, T_{+\mu\nu}]N + \bar{N}\gamma^\mu\{u^\nu, T_{-\mu\nu}\}N$$

not considered for simplicity