LENSES in GRAIN: 3D Reconstruction for light Points and Tracks

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and

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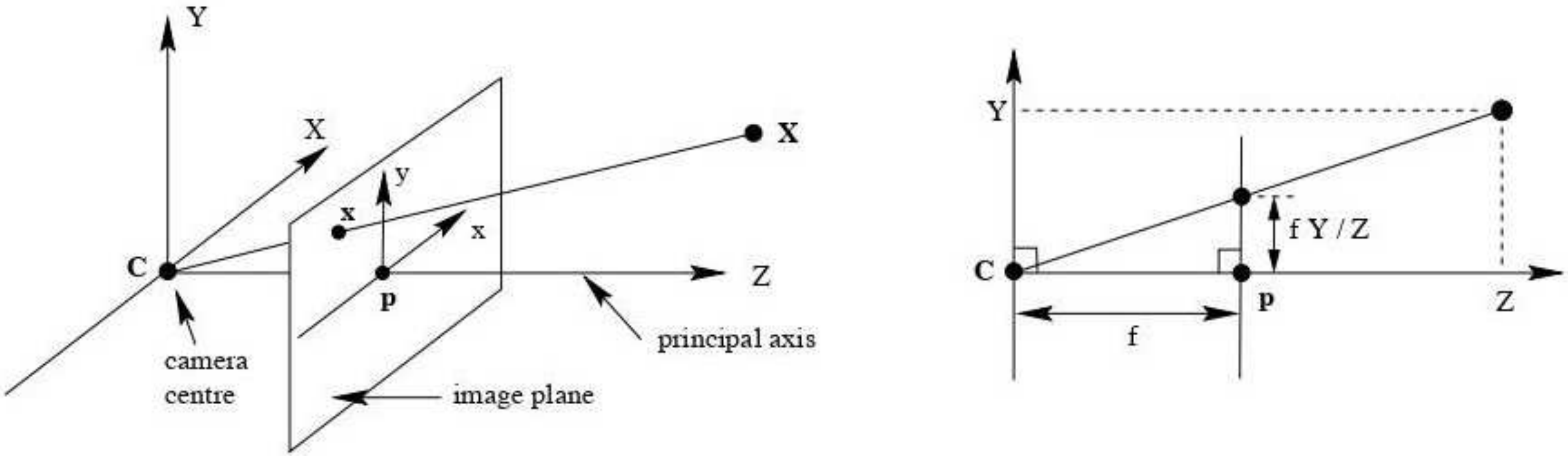
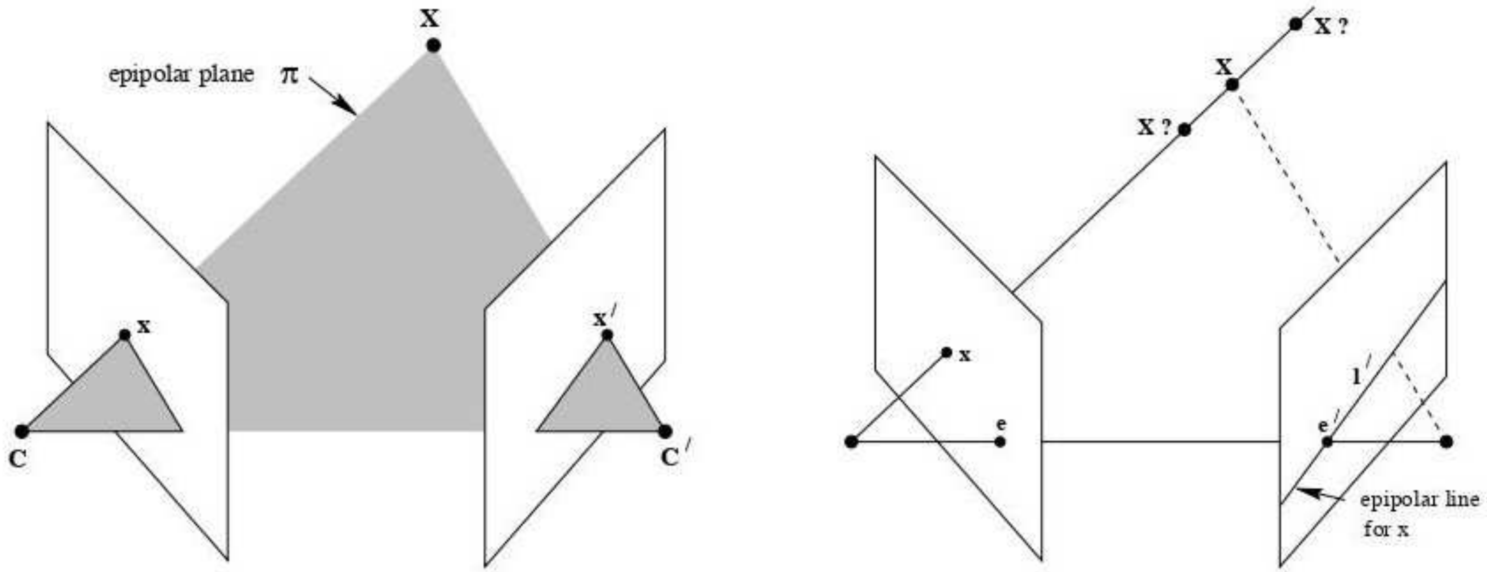
MATHEMATICAL FRAMEWORK:
MULTIPLE VIEW PROJECTIVE GEOMETRY IN 3D

SINGLE VIEW GEOMETRY



X: source point, **x**: reconstructed image on the image plane

DOUBLE VIEW GEOMETRY

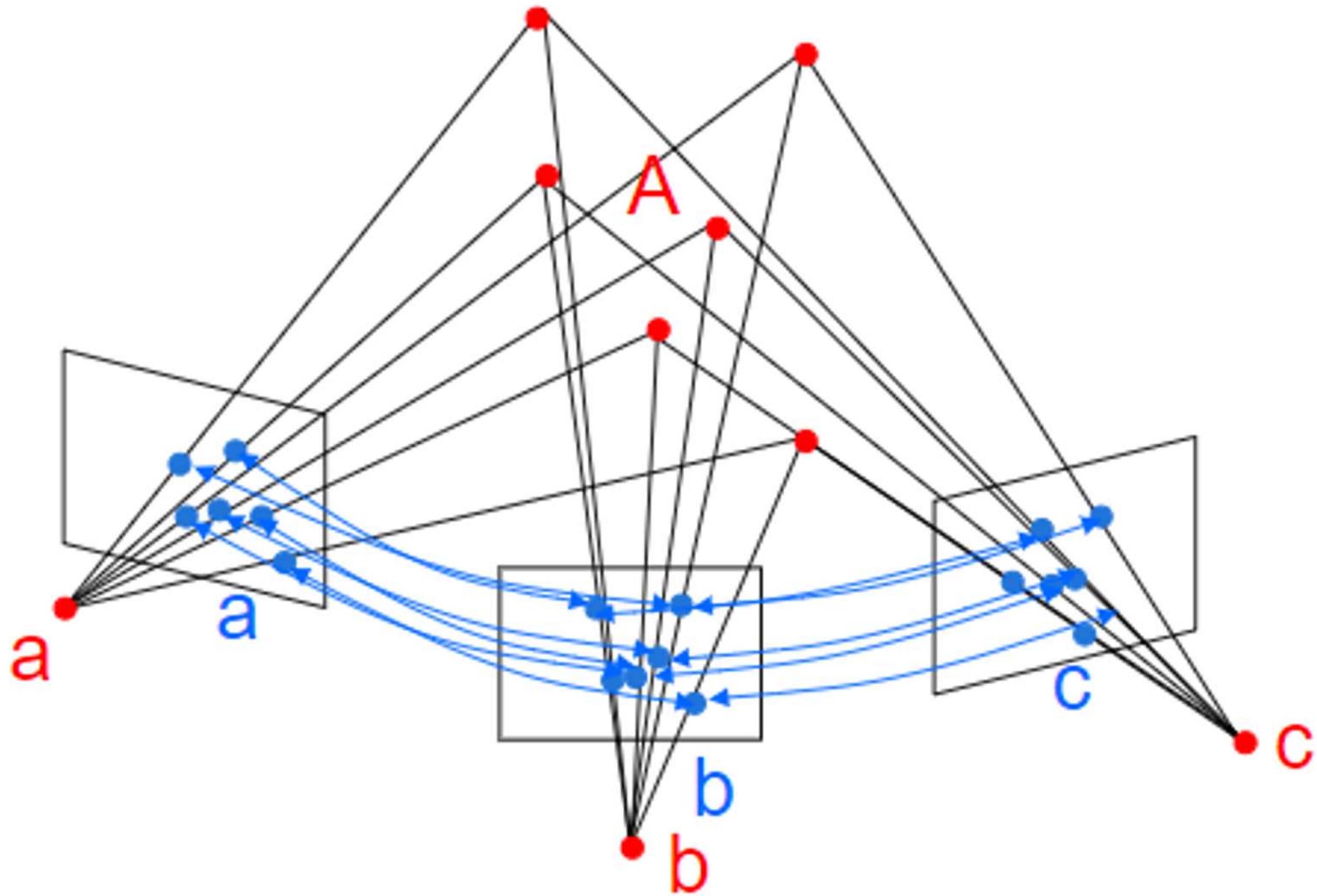


X: source point

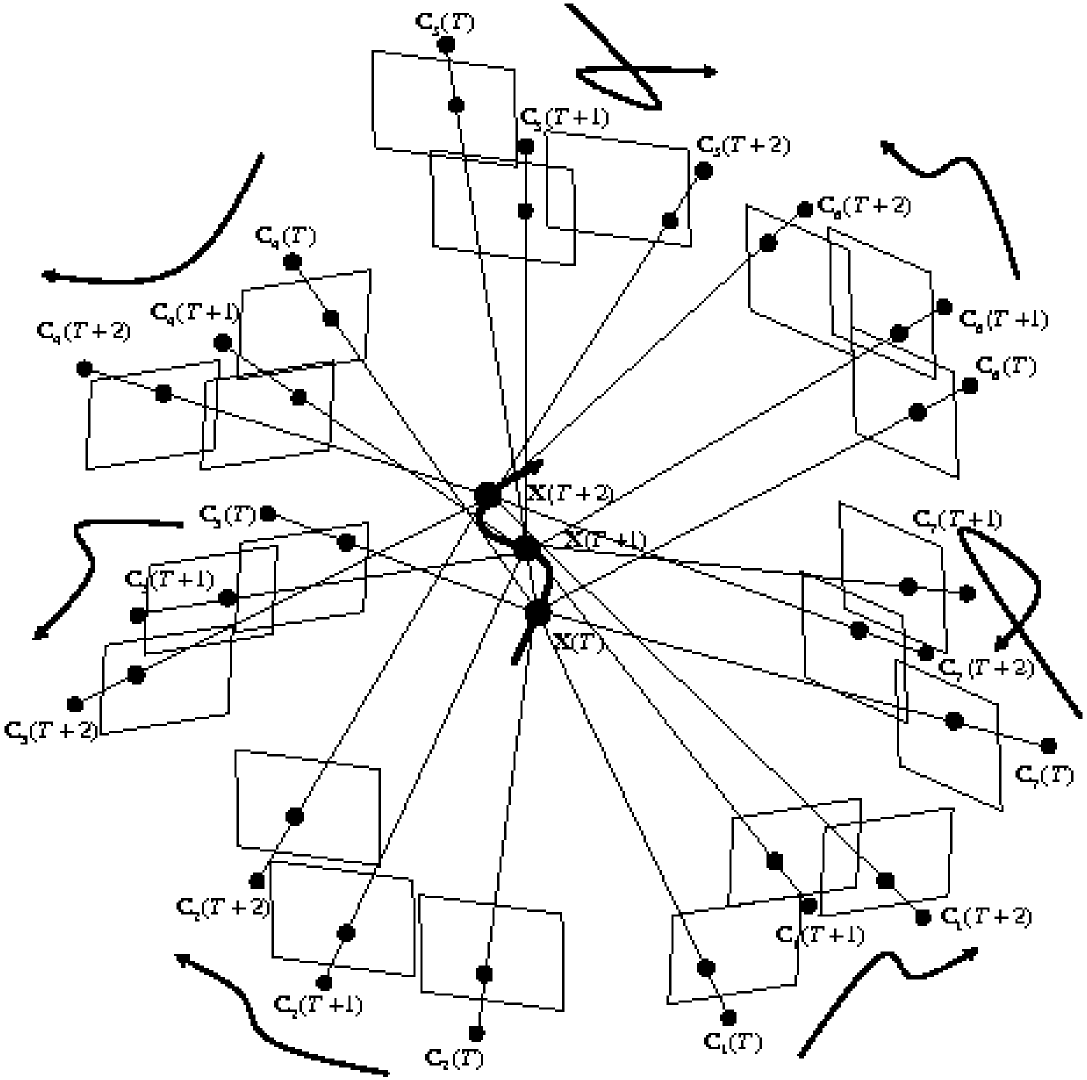
x: reconstructed image on the first image plane

x′: reconstructed image on the second image plane

TRIPLE VIEW GEOMETRY



MULTIPLE VIEW GEOMETRY



3D Reconstruction: system of coupled double-view geometry

$$\mathbf{x} = \mathbf{P}\mathbf{X} \tag{1}$$

$$\mathbf{x}' = \mathbf{P}'\mathbf{X} \tag{2}$$

$\mathbf{X}$: 3D source point

$\mathbf{x}$: 2D reconstructed image on the first image plane

$\mathbf{x}'$: 2D reconstructed image on the second image plane

$\mathbf{P}$ and $\mathbf{P}'$ projection/camera matrices

Use of projective coordinates

3D reconstruction

$$\mathbf{0} = \mathbf{x} \times \mathbf{x} = \mathbf{x} \times \mathbf{P}\mathbf{X} \tag{3}$$

$$\mathbf{0} = \mathbf{x}' \times \mathbf{x}' = \mathbf{x}' \times \mathbf{P}'\mathbf{X} \tag{4}$$

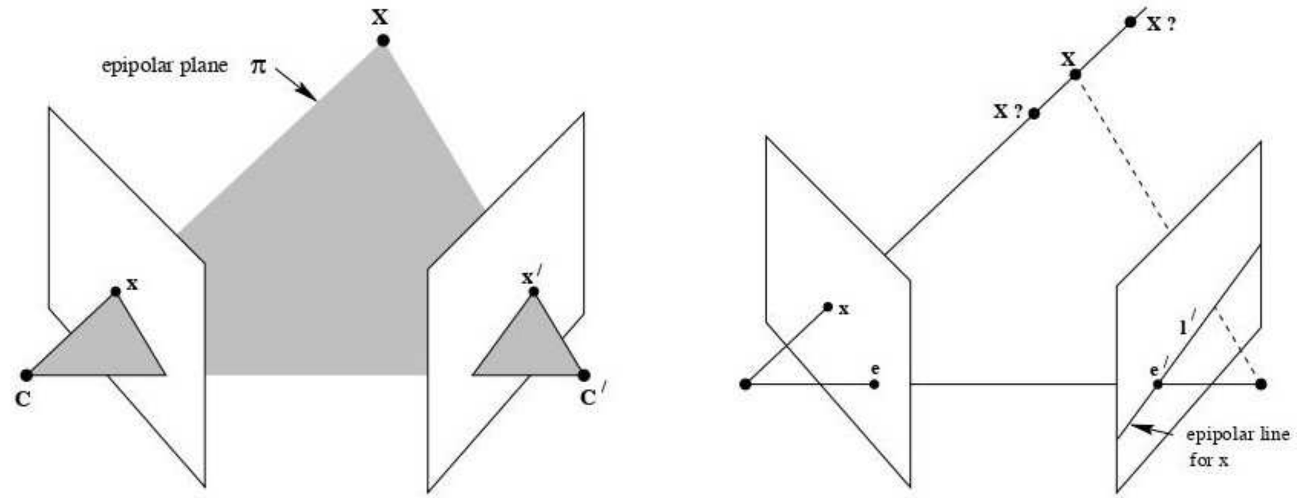
$$\mathbf{X} = \mathbf{P}^{+}\mathbf{x} + \left[\frac{(\mathbf{P}'\mathbf{P}^{+}\mathbf{x} \times \mathbf{x}') \cdot (\mathbf{x}' \times \mathbf{P}'C)}{(\mathbf{x}' \times \mathbf{P}'C) \cdot (\mathbf{x}' \times \mathbf{P}'C)} \right] C \tag{5}$$

$$\mathbf{X} = \mathbf{P}'^{+}\mathbf{x}' + \left[\frac{(\mathbf{P}\mathbf{P}'^{+}\mathbf{x}' \times \mathbf{x}) \cdot (\mathbf{x} \times \mathbf{P}C')}{(\mathbf{x} \times \mathbf{P}C') \cdot (\mathbf{x} \times \mathbf{P}C')} \right] C' \tag{6}$$

$$\mathbf{P}^{+} = \mathbf{P}^{T} \left(\mathbf{P}\mathbf{P}^{T} \right)^{-1} \tag{7}$$

$$\mathbf{P}'^{+} = \mathbf{P}'^{T} \left(\mathbf{P}'\mathbf{P}'^{T} \right)^{-1} \tag{8}$$

The Fundamental Matrix



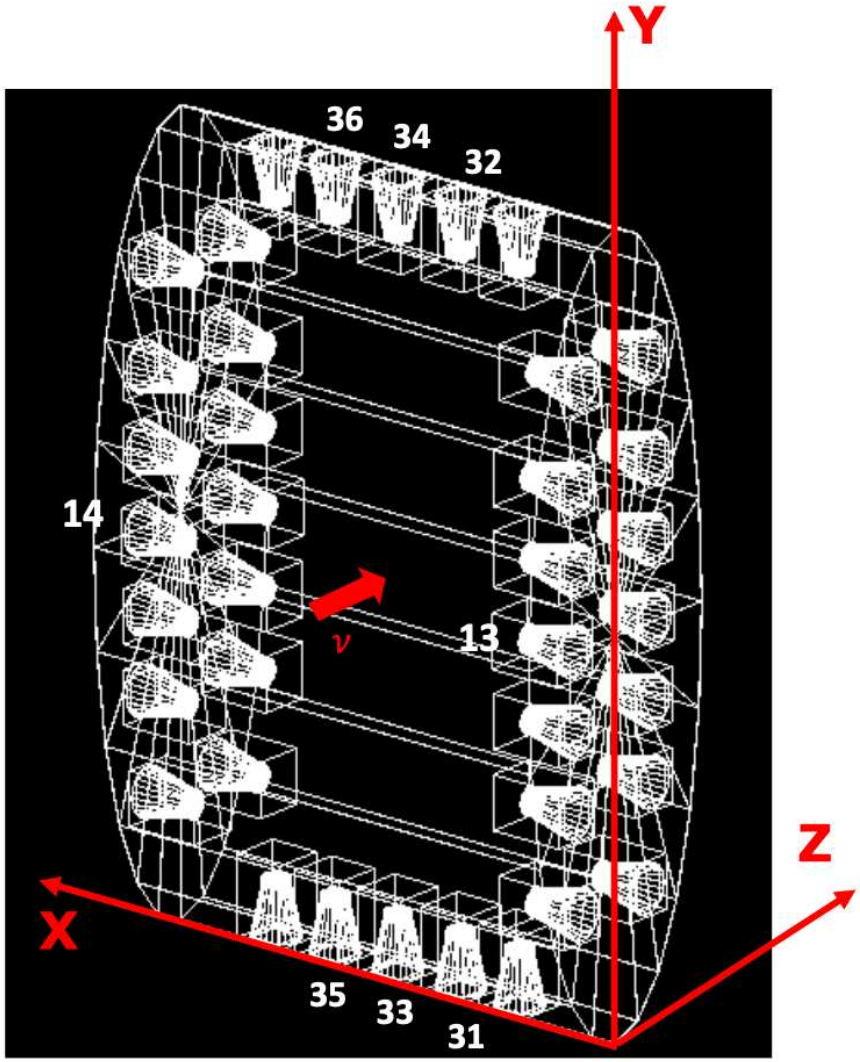
$$\mathbf{F} = [\mathbf{P}'C]_\times \, \mathbf{P}'\mathbf{P}^+ \tag{9}$$

$$\det\mathbf{F} = 0 \tag{10}$$

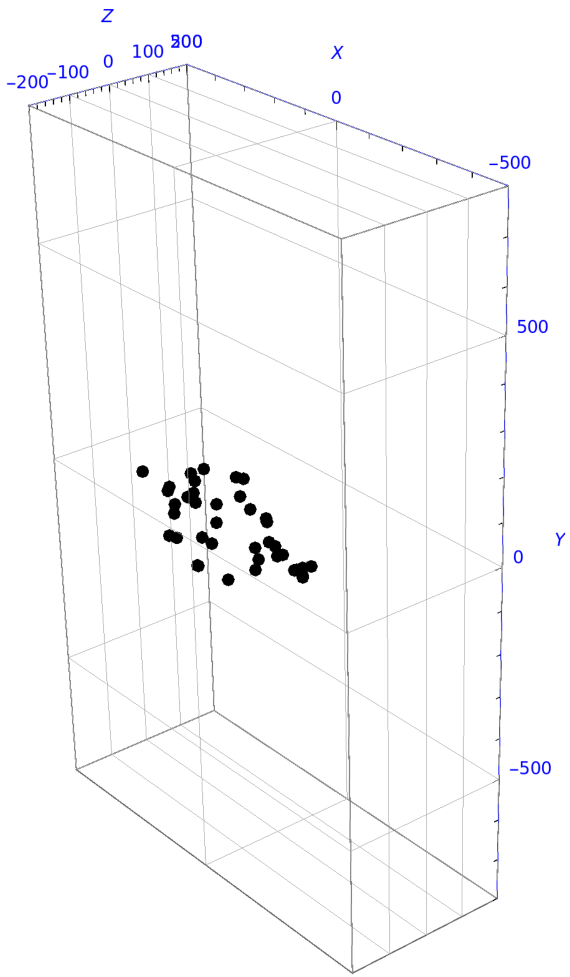
Compatibility Condition

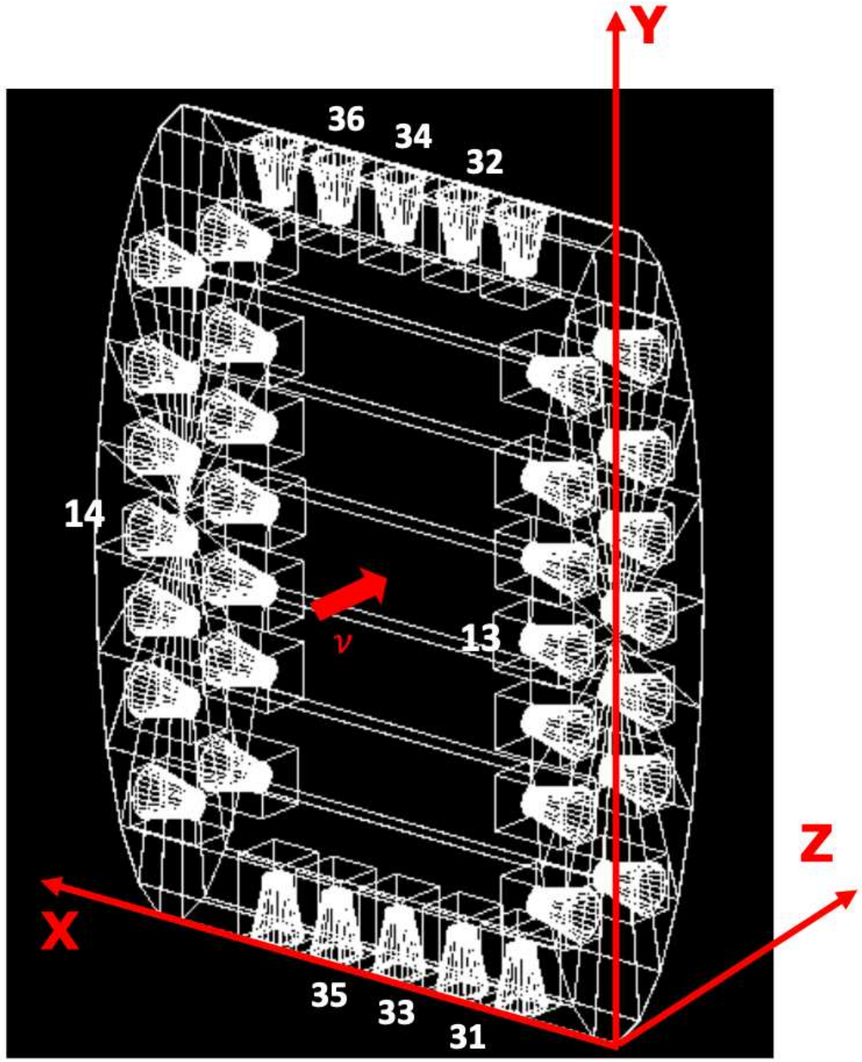
$$\mathbf{x}'^T \cdot \mathbf{F}\mathbf{x} = 0 \tag{11}$$

GRAIN: lens system

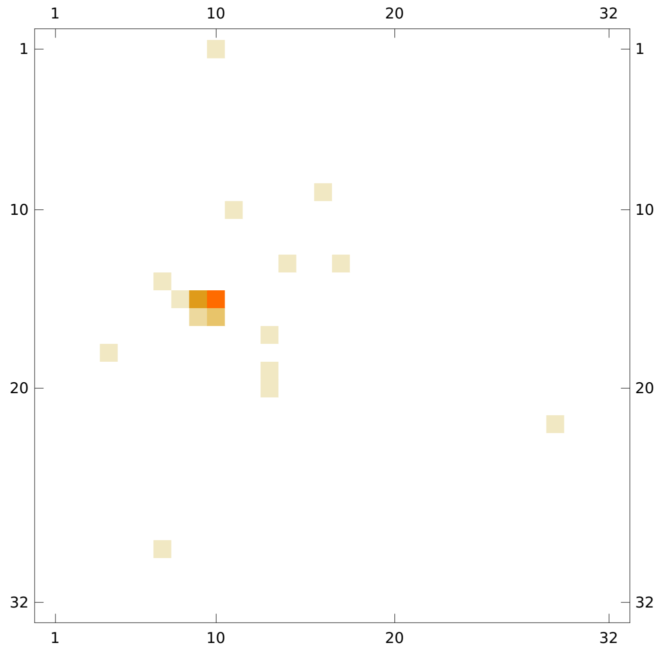


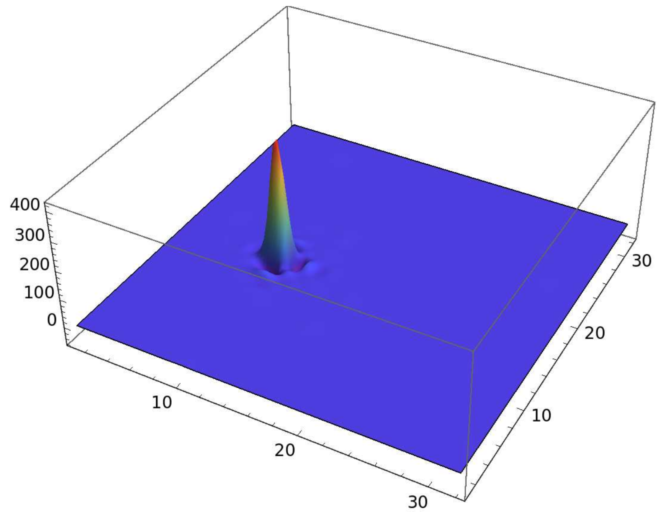
3D representation of 39 selected source points

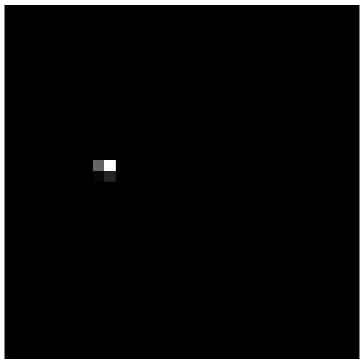




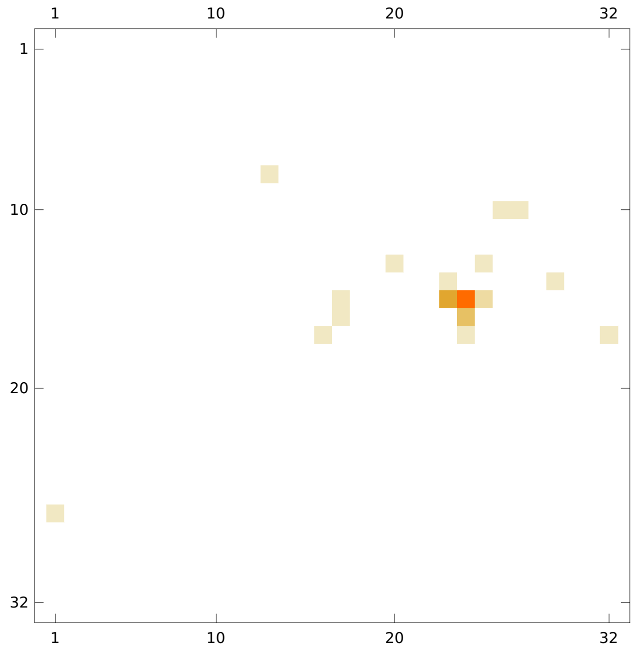
2D representation of source points (point 0)
Image on lens-sensor 13

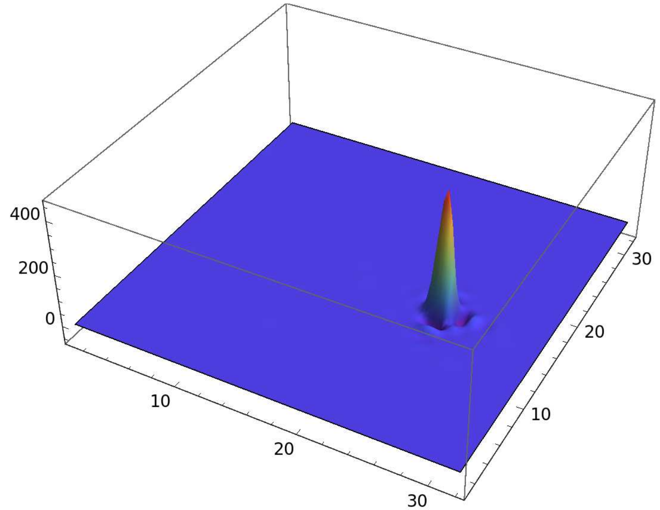


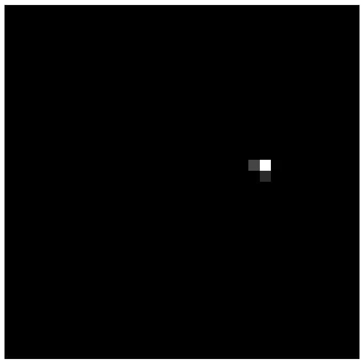




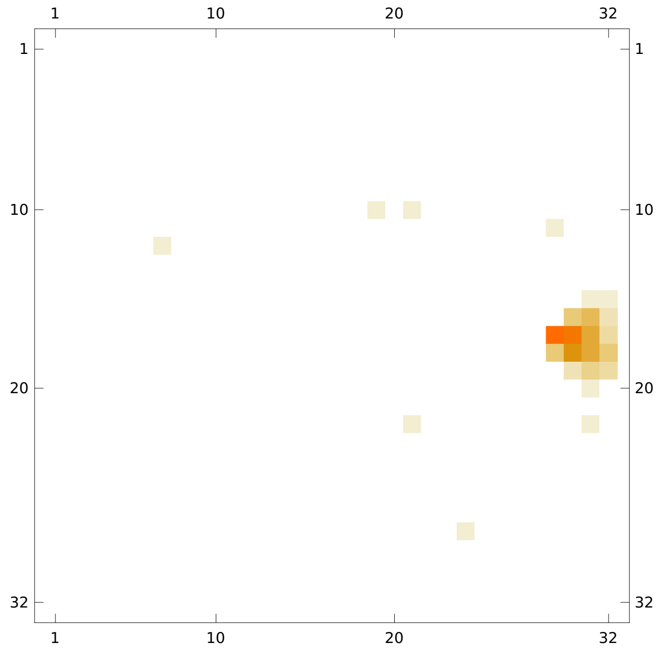
2D representation of source points (point 0)
Image on lens-sensor 14

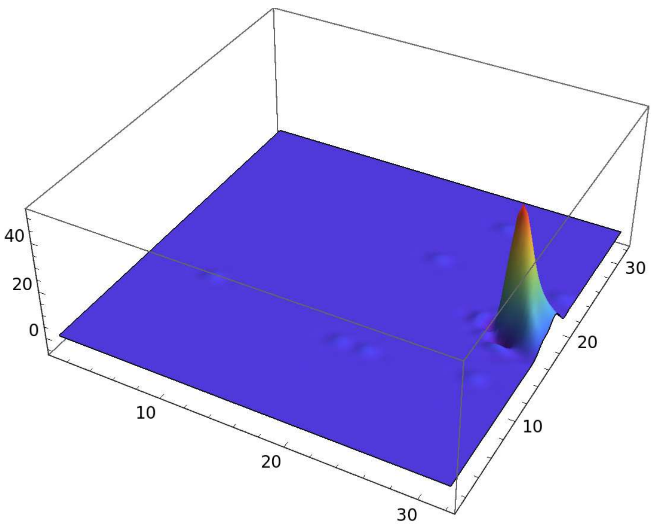


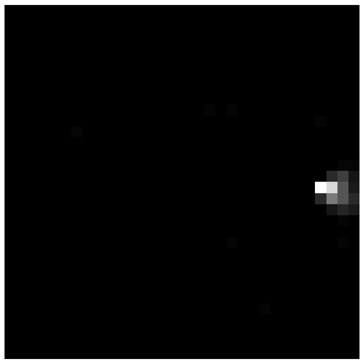




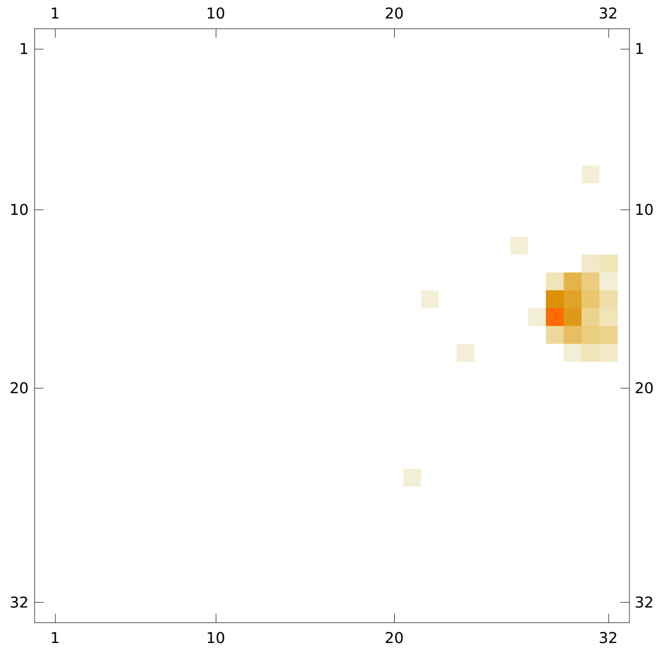
2D representation of source points (point 0)
Image on lens-sensor 33

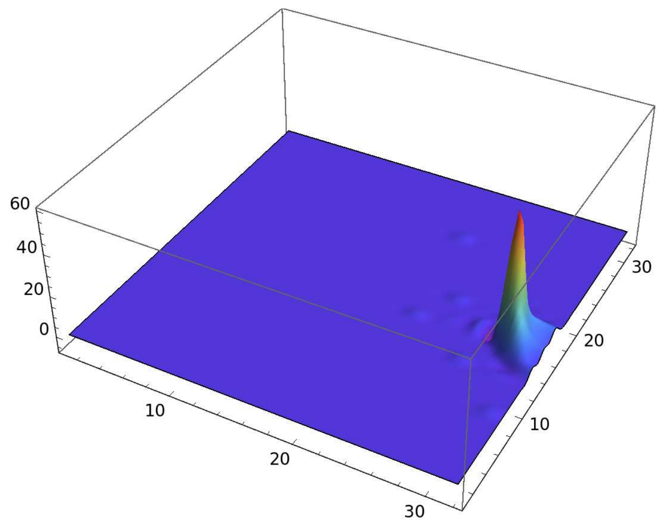


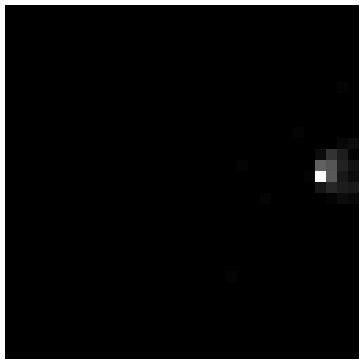




2D representation of source points (point 0)
Image on lens-sensor 34







Centroids

Image points $\mathbf{x}$ and $\mathbf{x}'$ are the centroids of the distribution of photons

$$\mathbf{x} = \frac{\sum_{ij} \gamma_{ij} \mathbf{x}_{ij}}{\sum_{ij} \gamma_{ij}} \tag{12}$$

$$\mathbf{x}' = \frac{\sum_{ij} \gamma'_{ij} \mathbf{x}'_{ij}}{\sum_{ij} \gamma'_{ij}} \tag{13}$$

$\mathbf{x}_{ij}$, $\mathbf{x}'_{ij}$ image pixels

γ_{ij}: number of photons on pixel (i, j) of the first camera

γ'_{ij}: number of photons on pixel (i, j) of the second camera

3D reconstruction - General Framework

$$\mathbf{0} = \mathbf{x} \times \mathbf{x} = \mathbf{x} \times \mathbf{P}\mathbf{X} \tag{14}$$

$$\mathbf{0} = \mathbf{x}' \times \mathbf{x}' = \mathbf{x}' \times \mathbf{P}'\mathbf{X} \tag{15}$$

$$\mathbf{X} = \mathbf{P}^{+}\mathbf{x} + \left[\frac{(\mathbf{P}'\mathbf{P}^{+}\mathbf{x} \times \mathbf{x}') \cdot (\mathbf{x}' \times \mathbf{P}'C)}{(\mathbf{x}' \times \mathbf{P}'C) \cdot (\mathbf{x}' \times \mathbf{P}'C)} \right] C \tag{16}$$

$$\mathbf{X} = \mathbf{P}'^{+}\mathbf{x}' + \left[\frac{(\mathbf{P}\mathbf{P}'^{+}\mathbf{x}' \times \mathbf{x}) \cdot (\mathbf{x} \times \mathbf{P}C')}{(\mathbf{x} \times \mathbf{P}C') \cdot (\mathbf{x} \times \mathbf{P}C')} \right] C' \tag{17}$$

$$\mathbf{P}^{+} = \mathbf{P}^{T} \left(\mathbf{P}\mathbf{P}^{T} \right)^{-1} \tag{18}$$

$$\mathbf{P}'^{+} = \mathbf{P}'^{T} \left(\mathbf{P}'\mathbf{P}'^{T} \right)^{-1} \tag{19}$$

The Fundamental Matrix

$$\mathbf{F} = [\mathbf{P}'C]_\times \mathbf{P}'\mathbf{P}^+ \tag{20}$$

$$\det\mathbf{F} = 0 \tag{21}$$

Compatibility Condition

$$\mathbf{x}'^T \cdot \mathbf{F}\mathbf{x} = 0 \tag{22}$$

3D reconstruction: front-to-front lenses and using theoretical $\mathbf{P}$, $\mathbf{P}'$ and $\mathbf{F}$

$$X_s = -\frac{2cx'(x^2+y^2)}{f(x^2+y^2+xx'+yy')}$$

$$Y_s = -\frac{2cy'(x^2+y^2)}{f(x^2+y^2+xx'+yy')}$$

$$\mathbf{x} = (x, y), \qquad \mathbf{x}' = (x', y')$$

c distance of the center of the lens from the center of GRAIN

f lens-sensor distance

WARNING: the labels X_s and Y_s are meant as the transversal 3D coordinates to the projection direction

Once the correspondence is implemented: $\mathbf{x}'^T \cdot \mathbf{F}\mathbf{x} = 0$

3D reconstruction

$$X_s = -\frac{2cxx'}{f(x+x')}$$

$$Y_s = -\frac{2cyy'}{f(y+y')}$$

$$\mathbf{x} = (x, y), \qquad \mathbf{x}' = (x', y')$$

c distance of the center of the lens from the center of GRAIN

f lens-sensor distance

WARNING: the labels X_s and Y_s are meant as the transversal 3D coordinates to the projection direction

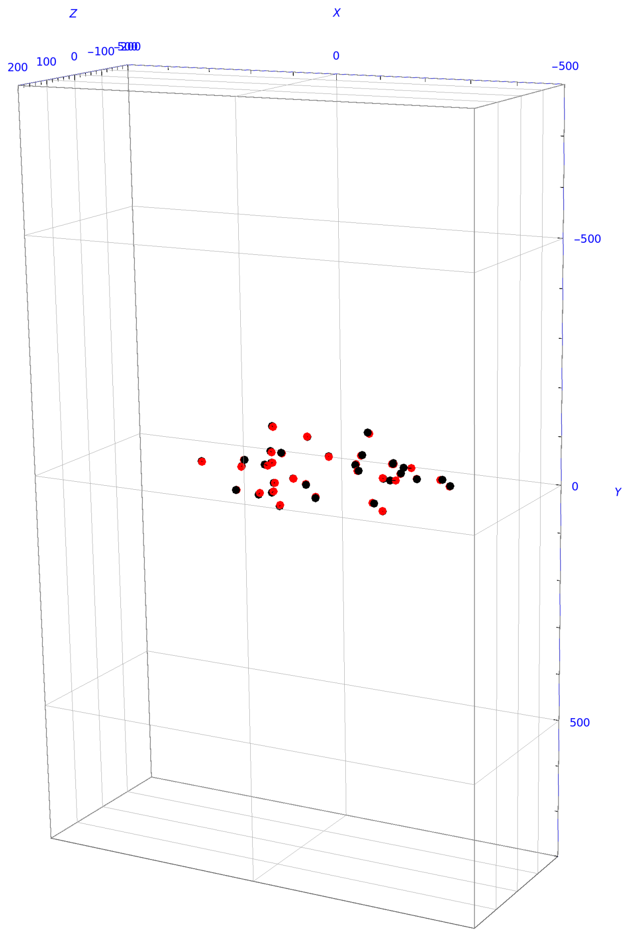
APPLICATION TO SELECTED 39 3D SOURCE POINTS

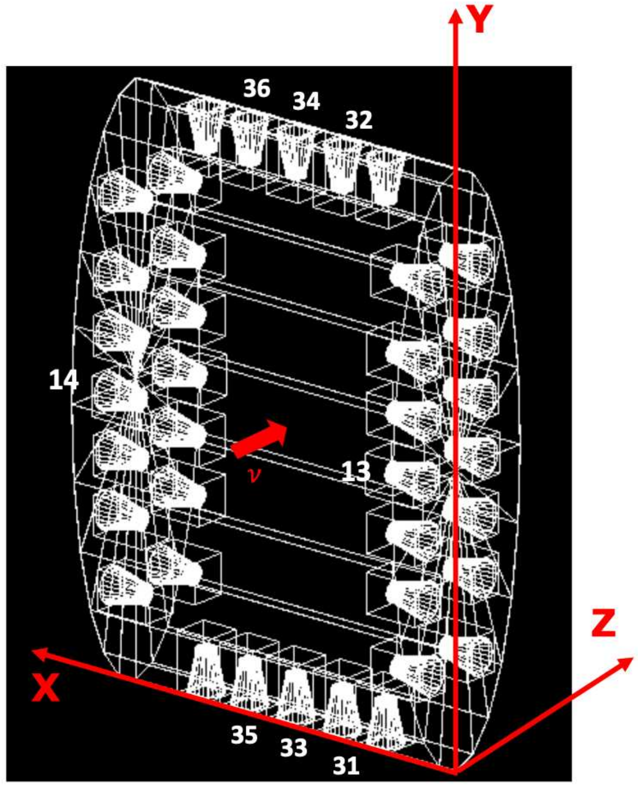
POINT 0: Explicit Example

Point0 $(10.22, -10.31, -164.16)$ in the global frame (mm) TRUTH

	X	Y	Z
TRUTH	10.22	-10.31	-164.16
13-14		-11.08	-166.09
33-34	11.39		-165.48

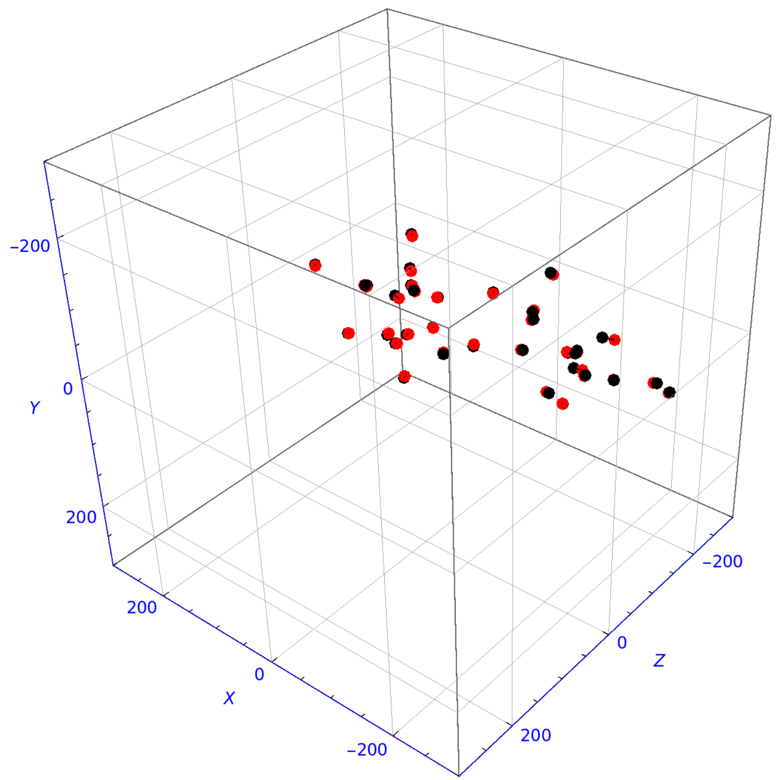
3D global representation of the reconstructed points





Black points: original sources Red points: reconstructed points

3D representation of the reconstructed points



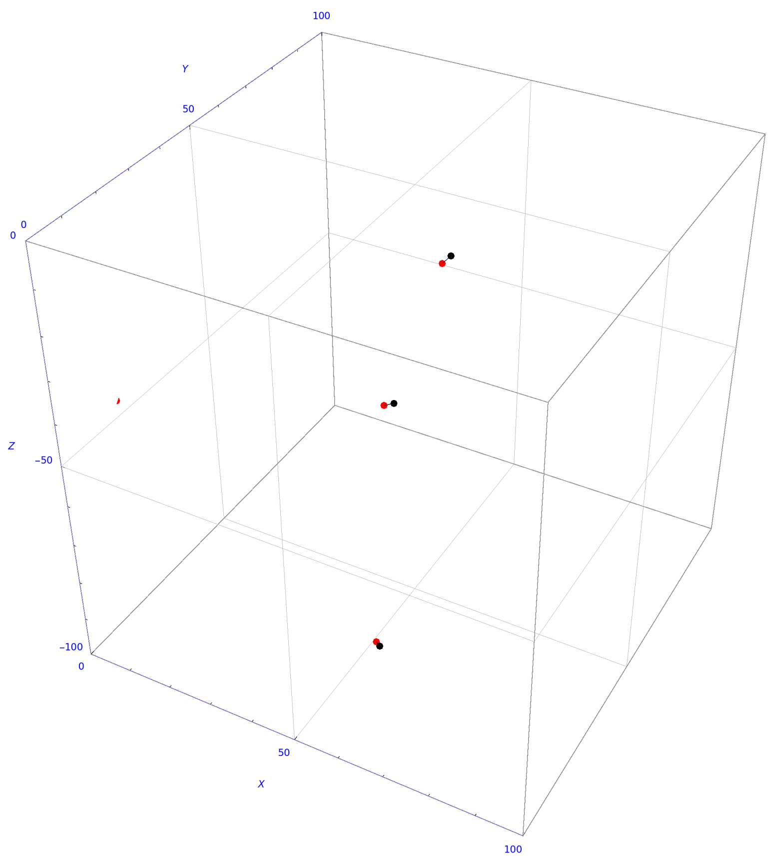
Black points: original sources Red points: reconstructed points

Segments connecting two corresponding points (original and reconstructed) are drawn but not visible at this scale

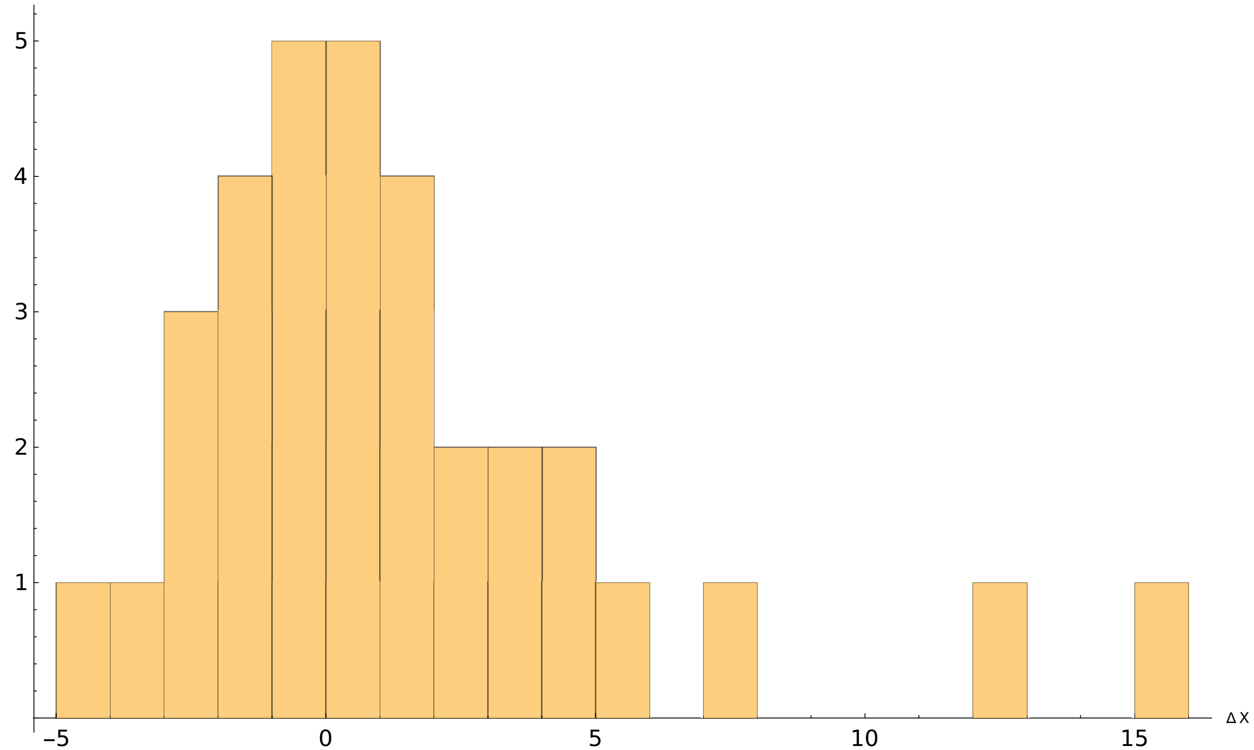
3D representation of the reconstructed points

Black points: original sources Red points: reconstructed points

Segments connecting two corresponding points (original and reconstructed) are drawn



Histograms: $\Delta X = X_c - X_t$

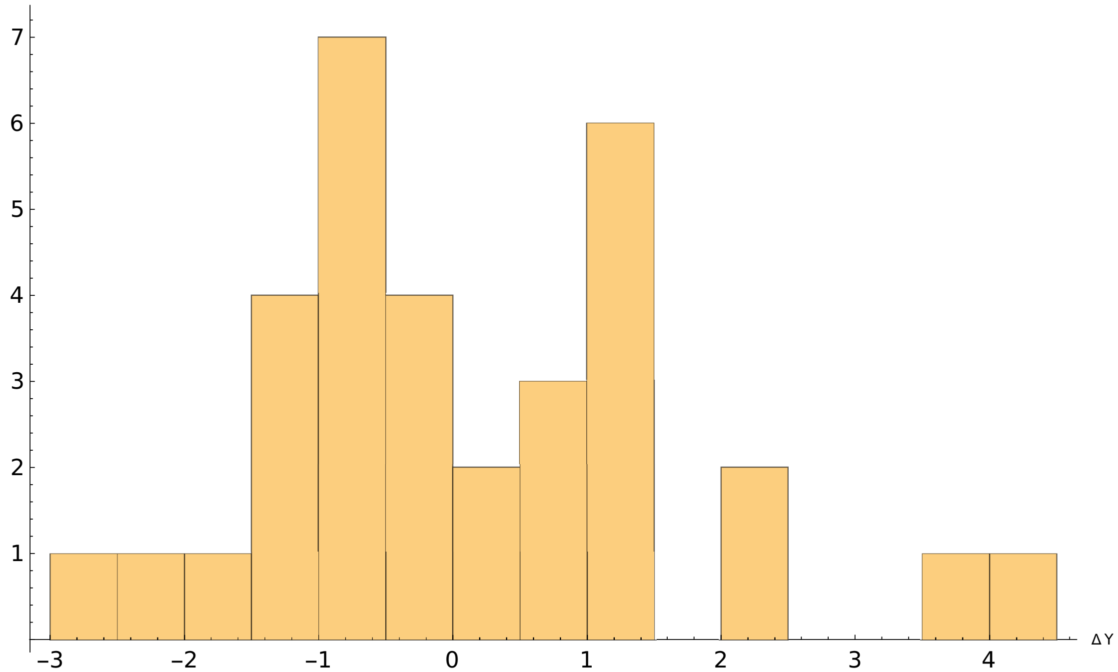


X_c: calculated X coordinate

X_t: original X coordinate

$$\text{Mean}(\Delta X) = 1.4 \quad \text{mm}$$

Histograms: $\Delta Y = Y_c - Y_t$

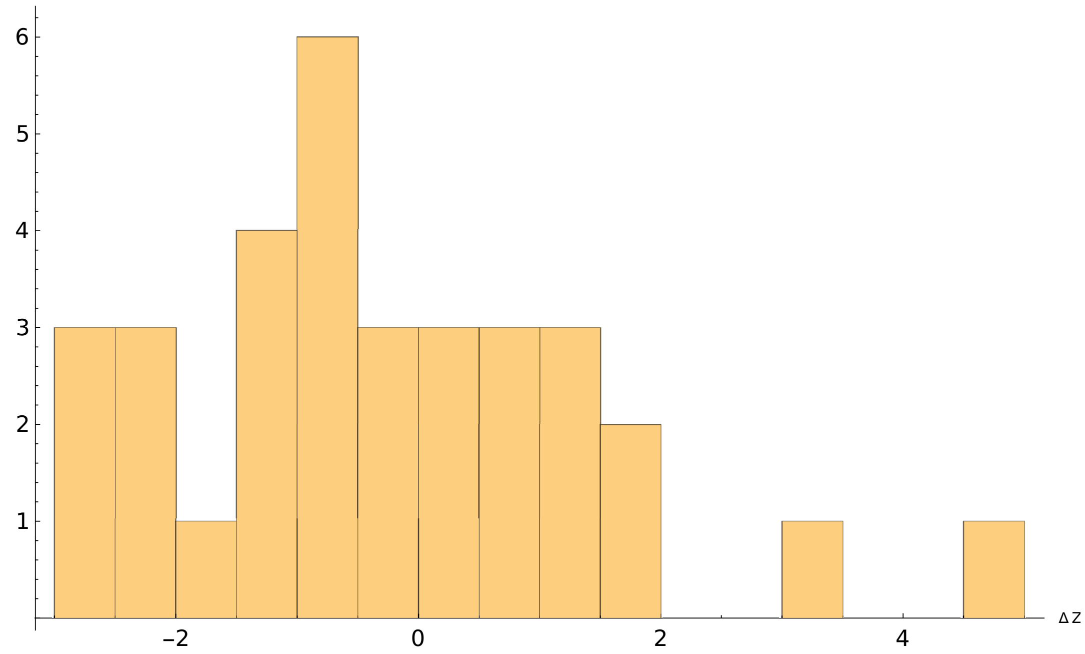


Y_c: calculated Y coordinate

Y_t: original Y coordinate

$$\text{Mean}(\Delta Y) = 0.1 \quad \text{mm}$$

Histograms: $\Delta Z = Z_c - Z_t$

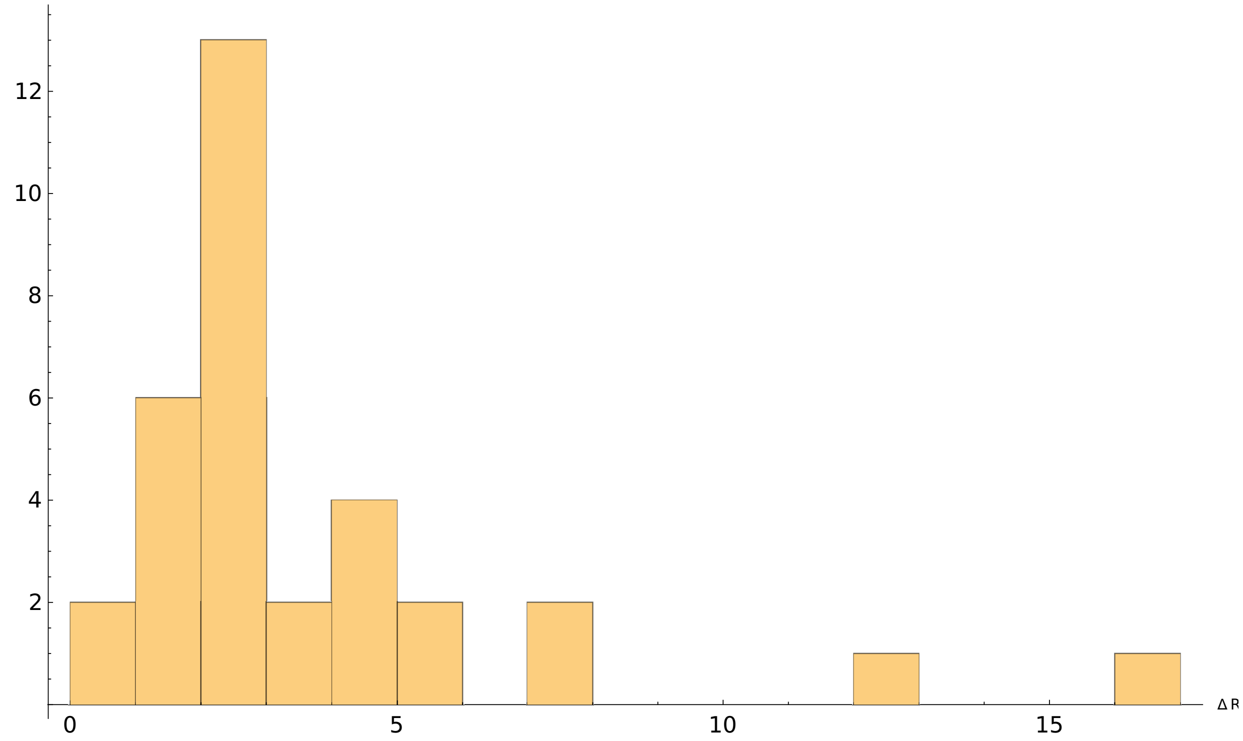


Z_c: calculated Z coordinate

Z_t: original Z coordinate

$$\text{Mean}(\Delta Z) = -0.2 \quad \text{mm}$$

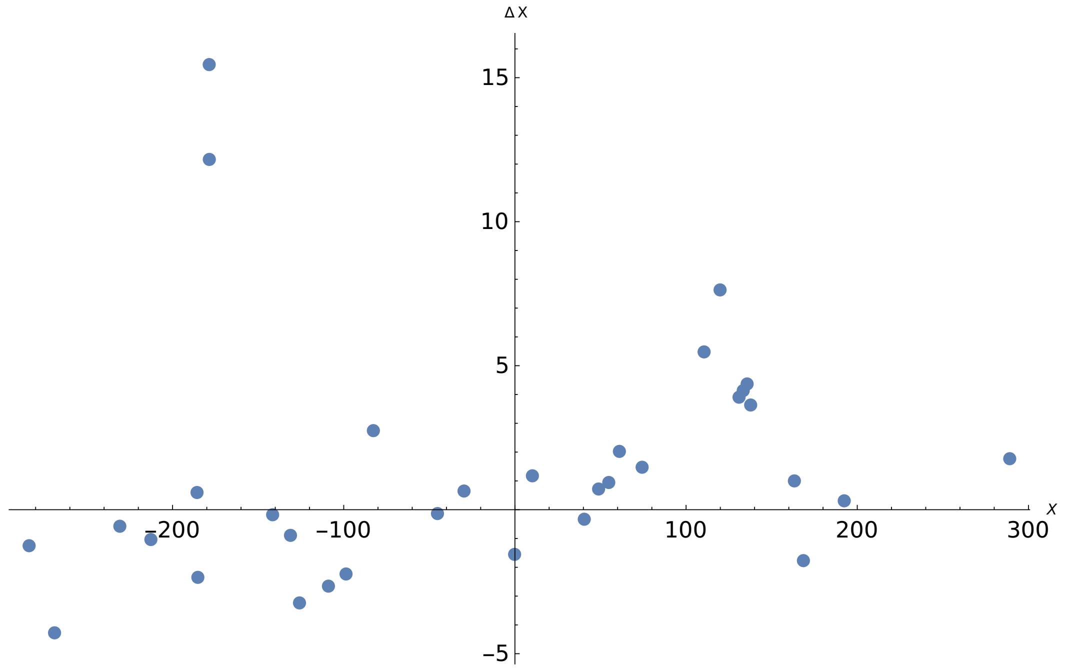
Histograms: ΔR



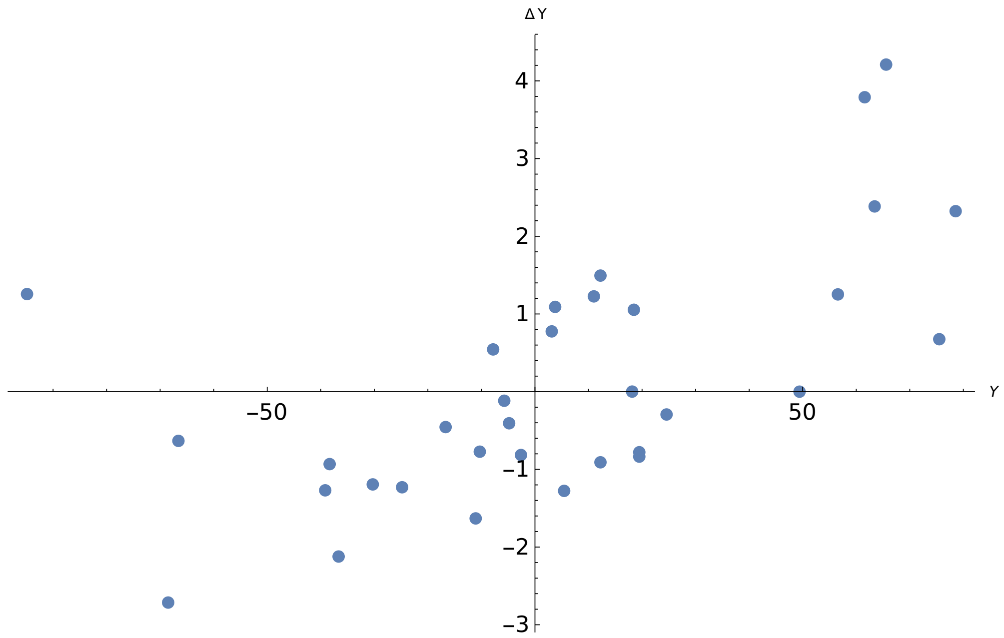
$$\Delta R = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Y)^2}$$

$$\text{Mean}(\Delta R) = 3.7 \quad \text{mm}$$

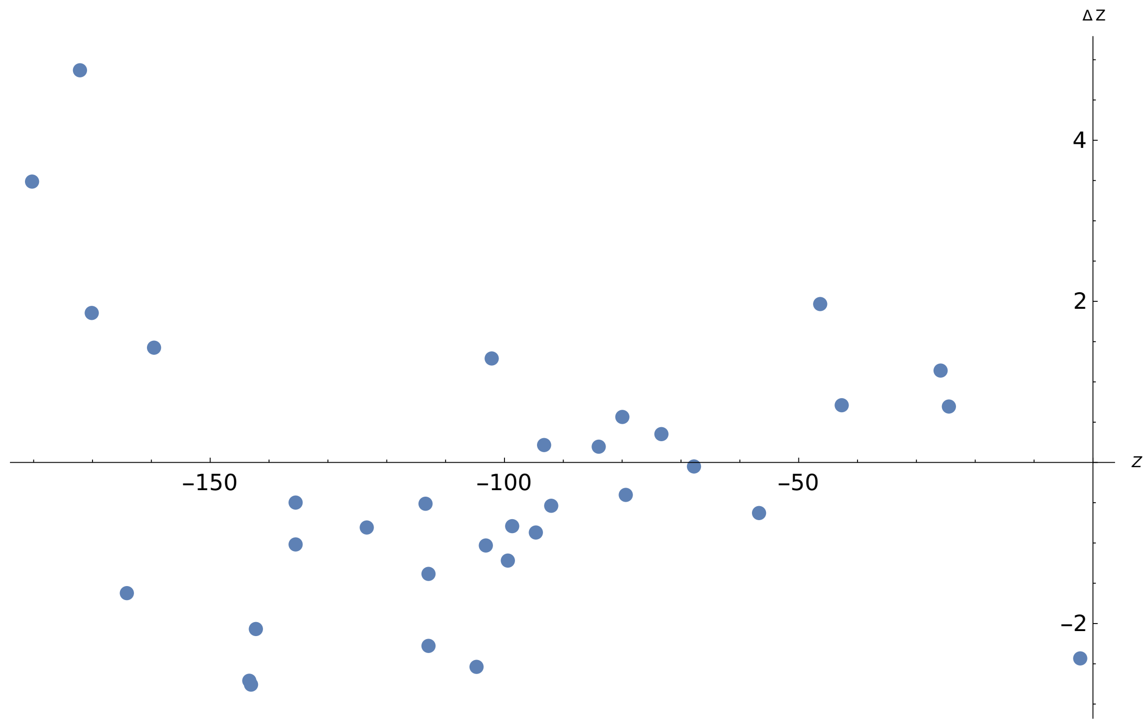
ΔX vs X_t



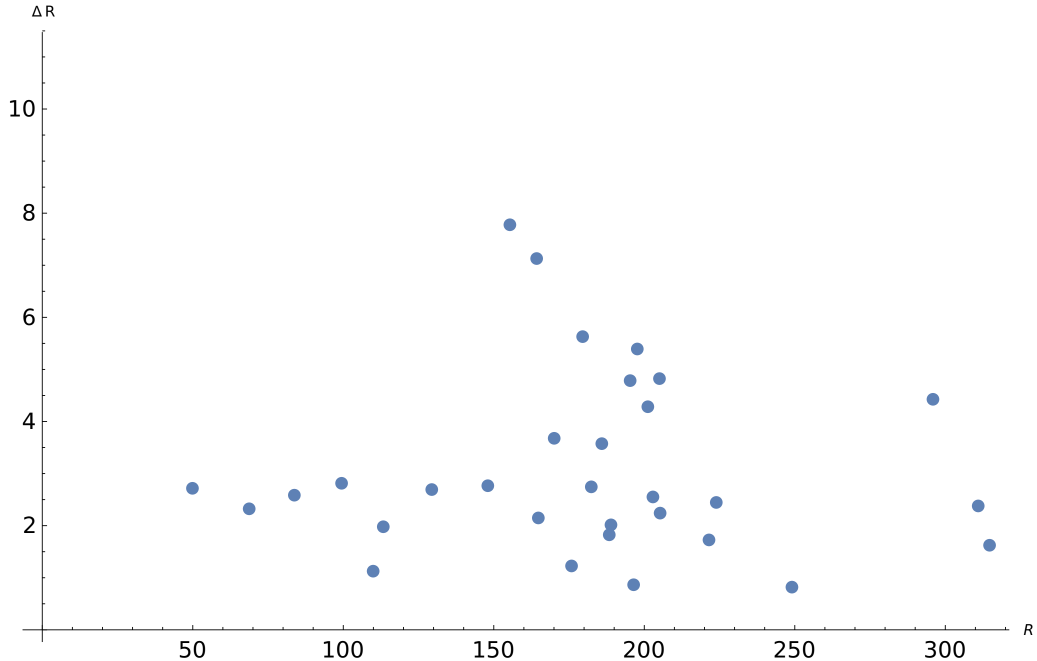
ΔY vs Y_t



ΔZ vs Z_t



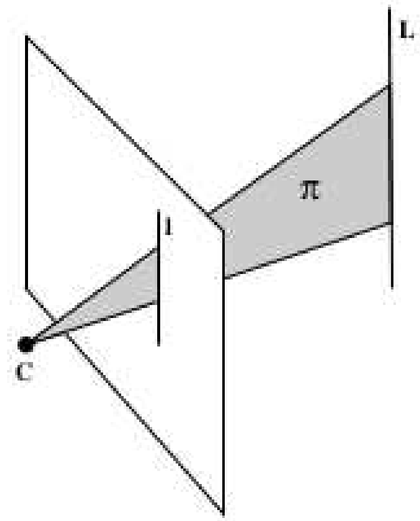
ΔR vs R_t



$$R_t = \sqrt{X_t^2 + Y_t^2 + Z_t^2}$$

Towards Tracks reconstruction: theoretical preliminaries

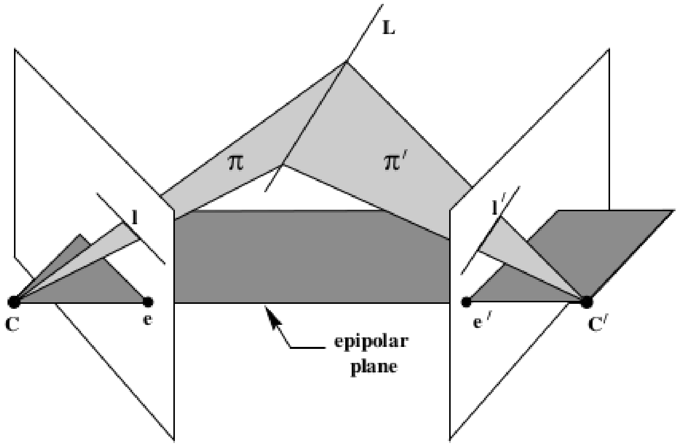
Back-projection of lines



$$\pi = \mathbf{P}^T \mathbf{l}$$

π vector of plane parameters in 3D space, $\mathbf{P}$ camera matrix, $\mathbf{l}$ vector of line parameters on the sensor, $\mathbf{L}$ infinite line in 3D space to be reconstructed

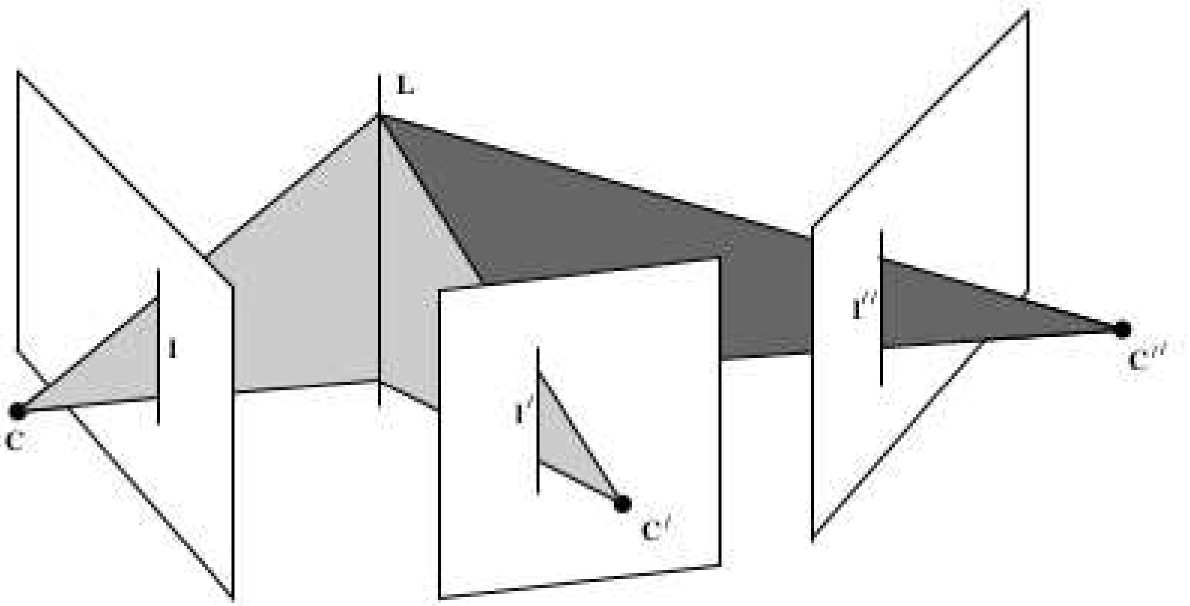
Line Reconstruction



$$\mathbf{L} = \begin{pmatrix} \mathbf{l}^T\mathbf{P} \\ \mathbf{l}'^T\mathbf{P}' \end{pmatrix}$$

π vector of plane π parameters in 3D space,π' vector of plane π' parameters in 3D space, $\mathbf{P}, \mathbf{P}'$ camera matrices, $\mathbf{l}, \mathbf{l}'$ vectors of lines l, l' parameters on the sensor, $\mathbf{L}$ infinite line in 3D space (reconstructed)

Triple-View Geometry and image correspondences: the Trifocal Tensor



CONCLUSIONS

- Double Double-view-geometry for **3D light source points** reconstruction in GRAIN
- Reconstruction for a couple of front-to-front mutually orthogonal double views
- Use of centroids for image points
- Resolution of order 1 mm
- Possible systematics (1 mm)

TO BE DONE

- **Extension to a larger set of source points uniformly distributed over GRAIN volume (with M. Vicenzi and L. Di Noto)**
- **APPLICATION TO TRACKS, Infinite Lines, Semilines and Vertices (with M. Vicenzi and L. Di Noto)**
- Optimization/Calibration of **F** and camera matrices **P**
- Correction to **F** and **P** for coded masks
- Lens distortions
- Error analysis
- Application to lenses and masks simultaneously
- Triple view geometry for IMAGE TRANSFER: Trifocal Tensor

...AND

THANK YOU

FOR YOUR ATTENTION!